

MUSSA PUZZLE REDUX*

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UNDER CONSTRUCTION

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Abstract

The [Mussa \(1986\)](#) puzzle — a sharp and simultaneous increase in the volatility of both nominal and real exchange rates after the end of the [Bretton Woods System](#) of pegged exchange rates in early 1970s — is commonly viewed as a central piece of evidence in favor of monetary non-neutrality. Indeed, a change in the *monetary* regime has caused a dramatic change in the equilibrium behavior of a *real* variable — the real exchange rate. The Mussa fact is further interpreted as direct evidence in favor of models with *nominal rigidities* in price setting (sticky prices). We show that this last conclusion is not supported by the data, as there was no simultaneous change in the properties of the other macro variables — neither nominal like inflation, nor real like consumption, output or net exports. We show that the extended set of Mussa facts equally falsifies both flexible-price RBC models and sticky-price New Keynesian models. We present a resolution to this broader puzzle based on a model of segmented financial market — a particular type of financial friction by which the bulk of the *nominal* exchange rate risk is held by a small group of financial intermediaries and not shared smoothly throughout the economy. We argue that rather than discriminating between models with sticky versus flexible prices, or monetary versus productivity shocks, the Mussa puzzle provides sharp evidence in favor of models with monetary non-neutrality arising due to *financial market segmentation*. Sticky prices are neither necessary, nor sufficient for the qualitative success of the model.

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1 Introduction

The [Mussa \(1986\)](#) puzzle is the fact that the end of the [Bretton Woods System](#) and the change in the monetary policy regime in the early 1970s away from pegged towards floating exchange rates had naturally increased the volatility of the nominal exchange rates (by an order of magnitude), but had also instantaneously increased the volatility of the *real* exchange rate almost by the same proportion (see [Figure 1](#)). This fact is commonly viewed by economists as a central piece of evidence in favor of monetary non-neutrality, since a change in the monetary regime has caused a dramatic change in the equilibrium behavior of a real variable — the real exchange rate.¹ Indeed, in models with complete monetary neutrality, the property of the real exchange rate should not be affected by the change in the monetary rule, absent other contemporaneous changes.² However, the Mussa fact is further interpreted as the direct evidence in favor of models with *nominal rigidities* in price setting (sticky prices). We show that this last conclusion is not supported by the data and provide an alternative explanation to the puzzle.

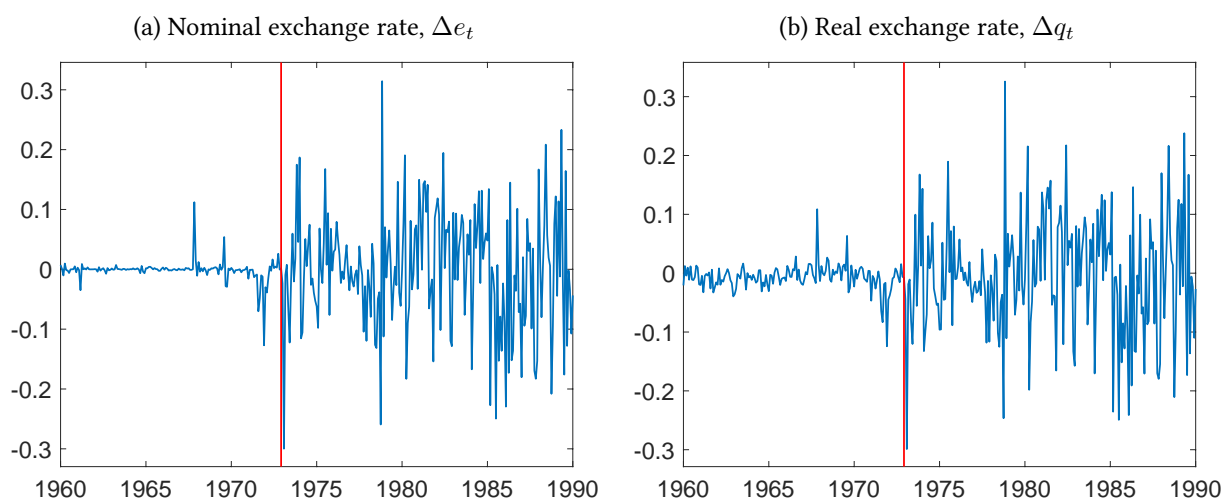


Figure 1: Nominal and real exchange rates, log changes

Note: US vs the rest of the world (defined as G7 countries except Canada plus Spain), monthly data from IFM IFS database. See [Appendix Figure A1](#) for the comparison of volatilities and the correlation of the two exchange rate series over time.

We start by documenting empirically that while there was a change in the properties of the real exchange rate, there was no change in the properties of other macro variables — neither nominal like inflation, nor real like consumption, output or net exports (see [Figure 2](#), which exhibits no evident structural break). One could interpret this as an extreme form of *neutrality*, where a major shift in the

¹When [Nakamura and Steinsson \(2018, pp.69–70\)](#) surveyed “prominent macroeconomists [on what is the most convincing evidence for monetary nonneutrality], the three most common answers have been: the evidence presented in [Friedman and Schwartz \(1963\)](#) regarding the role of monetary policy in the severity of the Great Depression; the [Volcker disinflation](#) of the early 1980s and accompanying twin recession; and the sharp break in the volatility of the US real exchange rate accompanying the breakdown of the [Bretton Woods System](#) of fixed exchange rates in 1973.” See also a textbook treatment of the Mussa puzzle in [Uribe and Schmitt-Grohé \(2017, Chapter 9.12\)](#) from the perspective of discriminating between flexible-price and sticky-price models.

²The argument here relies on the timing and the sharp discontinuity in the behavior of the exchange rates (see [Figure 1](#)), absent other immediate major changes in the environment.

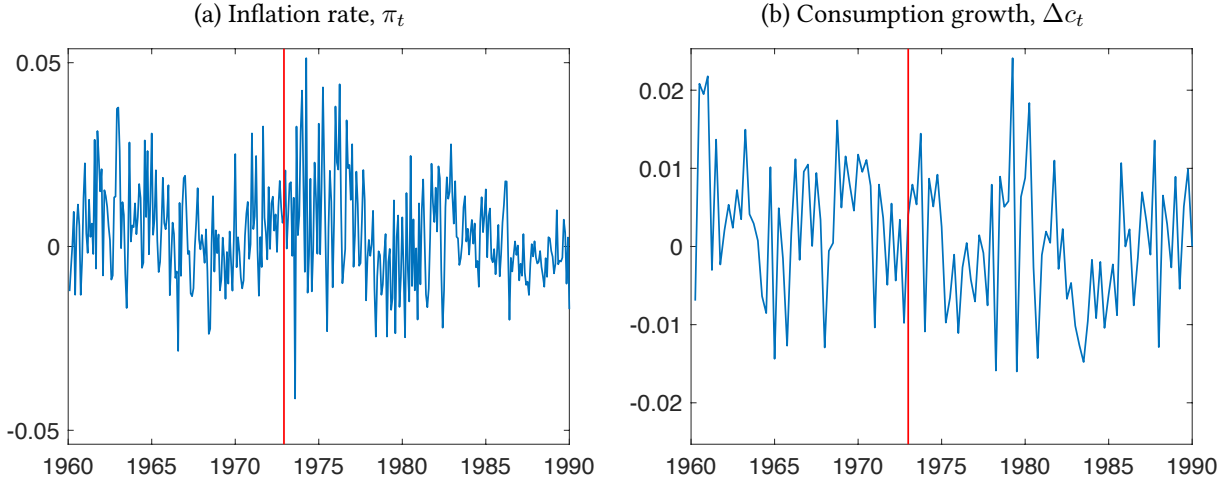


Figure 2: Inflation and consumption growth

Note: average inflation rates (monthly) and consumption growth rates (quarterly) for G7 countries except Canada plus Spain.

monetary regime, which increased the volatility of the nominal exchange rate by an order of magnitude, fails to affect the equilibrium properties of any macro variables, apart from the real exchange rate. In fact, this is a considerably more puzzling part of the larger set of “Mussa facts”: while the lack of change in the volatility of nominal variables, like inflation, is inconsistent with models of monetary neutrality, the lack of change in the volatility of real variables, like consumption and output, is inconsistent with sticky-price models. Therefore, if we take the combined evidence, it does not seem to favor one type of models over the other, but rather rejects both types.

To provide immediate intuition for this logic, consider two equilibrium conditions. The first is simply the definition of the real exchange rate (in logs):

$$q_t = e_t + p_t^* - p_t, \quad (1)$$

where p_t and p_t^* are consumer price levels at home and abroad, and e_t and q_t are the nominal and real exchange rates respectively. In models with monetary neutrality (e.g., international RBC), a change to the monetary policy rule should *not* affect the process for q_t , and therefore (1) necessary implies that the volatility of $\pi_t - \pi_t^* \equiv \Delta p_t - \Delta p_t^*$ must change along with the volatility of Δe_t . In the data, the volatility of Δq_t and Δe_t increased simultaneously, while the volatility of $\pi_t - \pi_t^*$ remained stable and low (see Figure 3 and Table 1). This pattern can, however, be consistent with the conventional sticky-price models (see e.g. Monacelli 2004). This observation is at the core of the traditional interpretation of the Mussa puzzle, suggesting that sticky price models (NKOE) beat RBC models, and monetary policy must have real effects due to nominal rigidities.

This interpretation, however, misses the second half of the picture. Equilibrium dynamics in a general class of models satisfies the following equilibrium cointegration relationship between relative consumption (with the rest of the world) and real exchange rate:

$$\sigma(c_t - c_t^*) = q_t + \zeta_t, \quad (2)$$

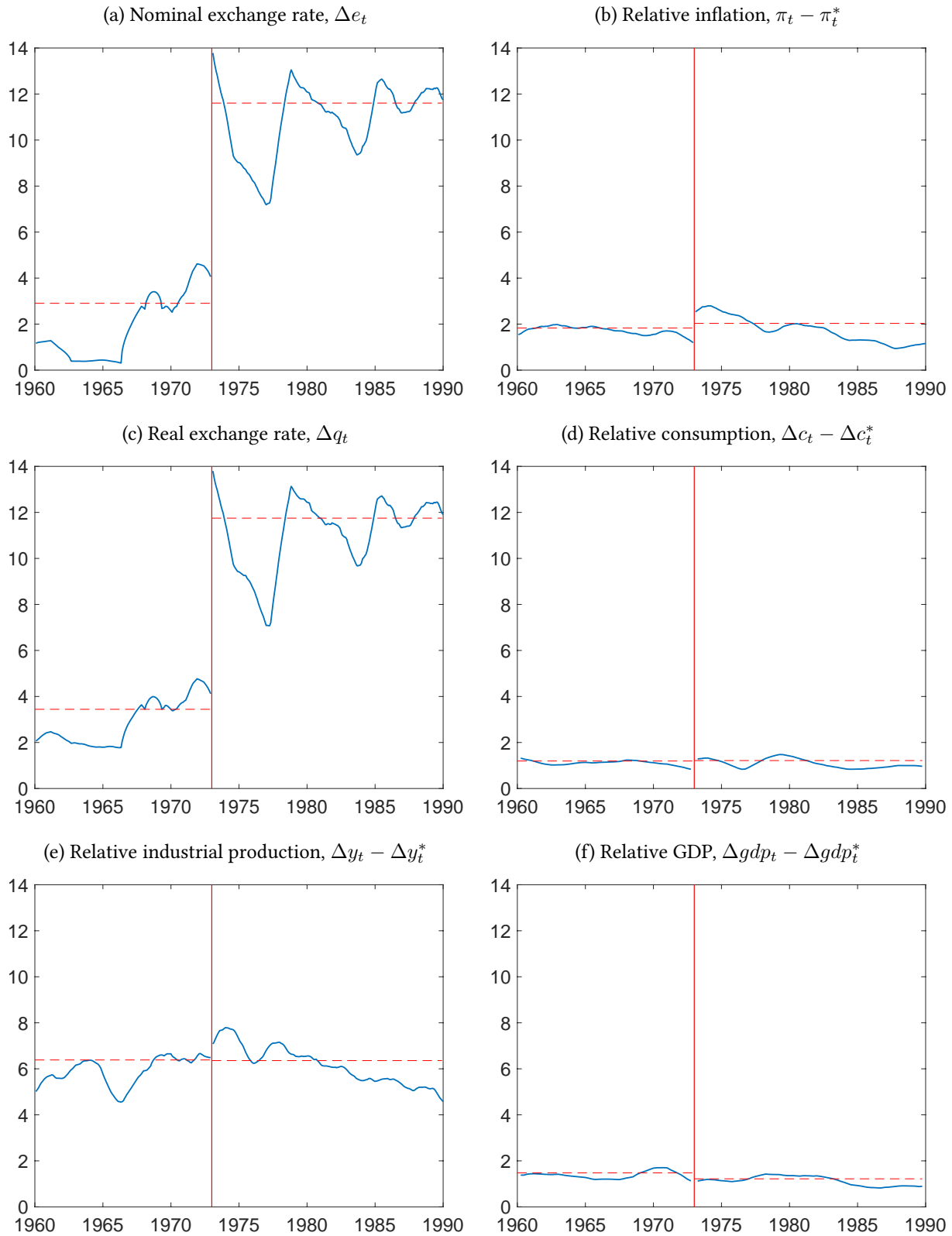


Figure 3: Volatility of macroeconomic variables over time

Note: All panels plot annualized standard deviations (of the log changes), estimated as triangular moving averages with a window over 18 months (or 10 quarters for quarterly data) before and after, treating 1973:01 as the end point for the two regimes; the dashed lines correspond to standard deviations measured over the entire subsamples (before and after 1973). US vs the rest of the world (as in Figure 1); monthly data from IFM IFS for panels a, b and e, and quarterly data from OECD in panels c, d and f. Appendix Figure A2 zooms in on the range of variation in panels b, d, e and f; Table 1 provides further details.

where $\sigma > 0$ and ζ_t can be interpreted as the equilibrium departure from the optimal international risk sharing.³ Indeed, equation (2) with $\zeta_t \equiv 0$ corresponds to the classic [Backus and Smith \(1993\)](#) condition under separable utility with constant relative risk aversion σ . We show that equation (2) is considerably more general and emerges as an equilibrium cointegration relationship independently of asset market completeness and other features of the model. Furthermore, we show that in a large class of conventional models – including both IRBC and NKOE – the structural residual ζ_t is independent of the monetary policy regime. Therefore, a shift in the monetary policy regime, which changes dramatically the volatility of Δq_t , should necessarily change the volatility of $\Delta c_t - \Delta c_t^*$. In the data, however, the volatility of relative consumption growth, just like that of inflation, remained both stable and small (see [Figure 3](#)).

To summarize, the models of monetary neutrality are consistent with the observed lack of change in the volatility of consumption, but for the wrong reason – as they fail to predict the change in the volatility of the real exchange rate. In contrast, models with nominal rigidities can explain the changing behavior of the real exchange rate, but have the counterfactual implications for the missing change in the volatility of the real variables. Therefore, the extended Mussa facts falsify the conventional RBC and New Keynesian models alike.

We then present a new resolution to the Mussa puzzle, which is simultaneously consistent with all the empirical facts. In particular, we show that in a model with segmented financial markets and limits to arbitrage developed in [Itskhoki and Mukhin \(2019\)](#), shifts in monetary policy regime affect the volatility of both nominal and real exchange rates, *even* when prices are fully flexible. Intuitively, the unpredictable movements in *nominal* exchange rate are the main source of uncertainty for financial intermediaries, who as a result are less aggressive under free floating exchange rates in taking large currency positions to ensure that uncovered interest parity (UIP) holds. By consequence, the equilibrium UIP violations are larger under the floating exchange rate regime, consistent with the data (see [Kollmann 2005](#)). A pegged exchange rate, in contrast, decreases uncertainty and stimulates arbitrageurs to take larger positions. As a result, the real exchange rate is less sensitive to shocks in the financial market and has lower volatility. At the same time, the financial shocks do not constitute the main source of volatility in the other macro variables under *either* monetary regime, and thus the model is consistent with nearly no change in the macroeconomic volatility, apart from the exchange rates.⁴

This logic is summarized in the *modified UIP* condition, which holds in equilibrium of our economy with a segmented financial market:

$$\frac{i_t - i_t^* - \mathbb{E}_t \Delta e_{t+1}}{\omega \sigma_e^2} = \psi_t - b_{t+1}. \quad (3)$$

³Note the parallel between ζ_t and q_t , which can be viewed as defined by identifies (2) and (1) respectively: just like q_t is the departure from *parity* in the goods market (namely, the purchasing power parity), ζ_t can be viewed as the departure from “parity” in the financial market (namely, the optimal risk sharing).

⁴This additionally requires that economies are sufficiently closed to international trade – an important feature of the world as argued by [Obstfeld and Rogoff \(2001\)](#) – so that real exchange rate volatility does not translate into a large volatility increase in the price level, production and consumption. This is consistent with the *exchange rate disconnect* mechanism under the floating (Taylor rule) regime developed in [Itskhoki and Mukhin \(2019\)](#).

The left-hand side is the expected carry trade return scaled by the price of risk, where $\sigma_e^2 \equiv \text{var}_t(\Delta e_{t+1})$ is the quantity of risk and ω is the risk-aversion of the financiers intermediating the currency demand in the economy. The right hand side of (3) is the demand for foreign currency (bonds), with net foreign assets b_{t+1} representing the 'fundamental' demand due to international merchandise trade and ψ_t denoting the 'liquidity' (or noise-trader) demand. The departures from the UIP are necessary for the financiers to be willing to intermediate the currency demand in the market, and crucially the equilibrium extent of these departures is simultaneously determined with the volatility of the *nominal* exchange rate.

Equation (3) is the only unconventional equilibrium condition in an otherwise standard international DSGE model. A change in the monetary regime has a direct effect on the equilibrium in the financial market – via σ_e^2 in (3) – and this is what allows the model to be consistent with the umbrella of Mussa facts, independently of the presence of nominal rigidities and the source of the other shocks, as long as ψ_t is an important contributor to the exchange rate volatility under a floating regime.⁵ Intuitively, as the government pledges to stabilize the volatility of the nominal exchange rate, this changes the incentives in the financial sector by encouraging risk-averse intermediaries to take on large currency exposure without requiring large UIP deviations.

We conclude that rather than discriminating between models with sticky versus flexible prices, and monetary versus productivity shocks, the Mussa puzzle provides a strong evidence in favor of models with monetary non-neutrality arising due to *financial market segmentation* – a particular type of financial friction by which the bulk of the *nominal exchange rate risk* is held by a small group of financial intermediaries and not shared smoothly throughout the economy. Sticky prices are neither necessary, nor sufficient ingredient for the qualitative success of this model. Nonetheless, realistic price and wage stickiness can improve the model's quantitative fit. Our analysis emphasizes that monetary non-neutrality is *not exclusive* to nominal rigidities in price setting, as changes in equilibrium properties of the nominal variables – such as the nominal exchange rate – can change the degree of financial market (in)completeness, and hence have real consequences for the real equilibrium outcomes.

Overidentifying moments Backus-Smith correlation, Fama regression coefficient, Balassa-Samuelson...

Related literature Stockman (1988), Baxter and Stockman (1989), Flood and Rose (1995), Jeanne and Rose (2002), Kollmann (2005), Alvarez, Atkeson, and Kehoe (2007), Colacito and Croce (2013), Berka, Devereux, and Engel (2012), Devereux and Hnatkowska (2014), Kollmann, Frenkel-Levich...

⁵Itskhoki and Mukhin (2017) show that financial shocks ψ_t are essential for a successful model of exchange rate disconnect under a Taylor rule regime, and resolve a variety of exchange rate puzzles, including Meese-Rogoff, PPP, Backus-Smith and UIP puzzles. As we show below, their presence *per se* is not sufficient to resolve the Mussa puzzle. We further show the relationship between (2) and (3), and in particular how (3) implies the endogeneity of ζ_t to the exchange rate regime. Importantly, (3) does not rely on the assumption that an Euler equation holds for a representative consumer, or that there is any direct asset trades between home and foreign households.

2 Empirical Facts

Data We start by briefly describing the construction of our dataset, and provide further details in Appendix A.2. All monthly data (for nominal exchange rate, consumer prices and production index) come from the [IFM IFS database](#), while all quarterly data (for GDP, consumption, imports and exports) are from the [OECD database](#). All quantity variables (GDP, consumption, imports and exports) are real and seasonally-adjusted. Production index is also seasonally-adjusted, while nominal exchange rates and consumer price indexes are not. The *net export* variable is defined as the ratio of exports minus imports to the sum of exports and imports, in order to counter a mechanical increase in the volatility of net exports to GDP due to higher openness of the economies in later periods. All data are annualized to make volatilities (standard deviations) comparable across series.

There is ambiguity associated with identifying the exact end of the Bretton Woods System. In particular, during the Bretton Woods period, there are already large devaluations in the U.K. and Spain in November 1967, a devaluation in France and an appreciation in Germany in August–October 1969. While all countries officially allowed their exchange rates to float in February 1973, most of them were already adjusting their exchange rates since the “Nixon shock” in August 1971, which limited the direct convertibility of dollar to gold. Therefore, we label the period from 1960:01–1971:07 as “peg” and the period from 1973:01–1989:12 as “float”, as used in tables and scatter plots below (which exclude the intermediate period 1971:08–1972:12).⁶ The “regression discontinuity” graphs are done for two alternative break points – 1973:01 in the main figures and 1971:08 in the robustness figures in the appendix.

The rest of the world for the U.S. is constructed as a weighted average of percent changes in series across France, Germany, Italy, Japan, Spain and the U.K. (G7 countries except Canada plus Spain). Average GDP shares during the sample period are used to construct country weights.

Macroeconomic volatility Figure 3 displays the main empirical results of the paper. In the spirit of the regression discontinuity design (RDD), we estimate standard deviations of the variables using a rolling window that starts at 1973:01 and goes either forward or backward. In line with the seminal [Mussa \(1986\)](#) paper, the end of the Bretton Woods System is associated with a dramatic change in the volatility of both nominal and real exchange rates, from around 2% to 10% (more precisely, in units of annualized standard deviations of log changes). What makes this fact much more puzzling, however, is the absence of any comparable change in the volatility of the other variables – either nominal like inflation, or real like production and consumption.⁷ Thus, while under the peg regime, the volatility of the real exchange rate is of the same order of magnitude as for the other macroeconomics variables, there is a clear “disconnect” between the real exchange rate and macroeconomic fundamentals in the floating regime. We emphasize the relative magnitudes of volatilities across different variables and regimes by keeping the same scale for standard deviations of all variables in Figure 3, while Appendix

⁶In Canada, the two exchange rate regimes occurred over different periods with free floating before 1962:06 and after 1970:05, and a peg in between. This is why we exclude Canada from the construction of the “rest of the world” in the figures.

⁷Figure 3 presents the *relative* volatilities of macroeconomic variables between the US and the rest of the world, as these are the relevant objects emphasized by the theory in Section 3. Table 1 also present the results for individual country variables across a range of countries, which exhibit similar patterns as the relative variables for the US vs the ROW.

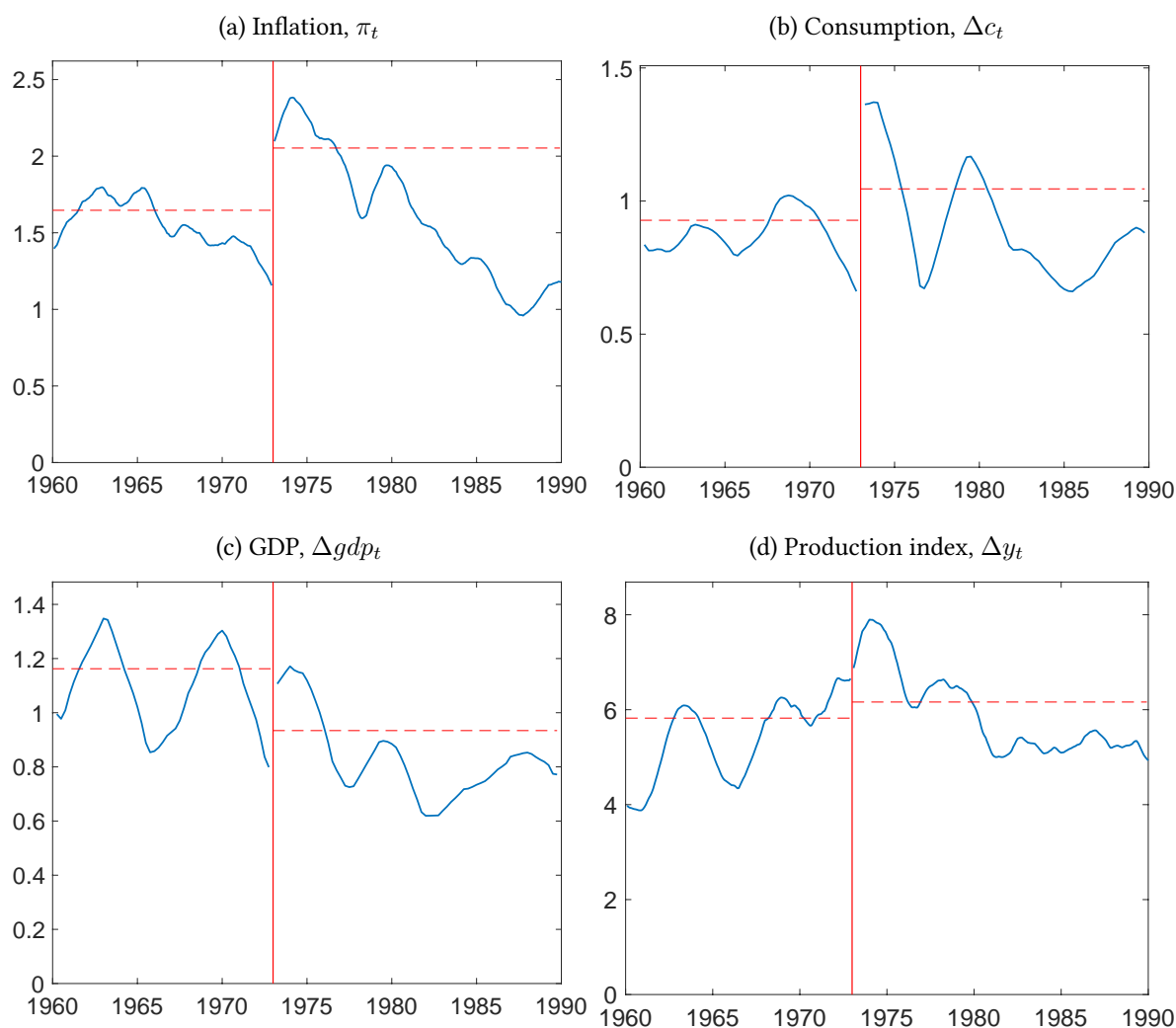


Figure 4: Macroeconomic volatility over time: country-level variables

Note: Triangular moving averages of the standard deviations of macro variables, treating 1973:01 as the break point; average across countries (G7 except Canada plus Spain).

Figure A2 zooms in on the range of variation of individual macroeconomic variables to see (the lack of) the discontinuity in the behavior of inflation, consumption and output.⁸

The rest of the pictures and tables expand on this finding and provide some additional details. Figure A3 provides a robustness check using 1971:08 as the alternative break point. There is no evidence of changing volatility for macroeconomic variables in this case either. The missing change in the volatility is true not only for the cross-country differences of variables, but also for the fundamentals at the country level. In particular, from Figure 4, there is almost no differences in the volatilities of macro variables.

We now unpack the rest of the world (RoW) into separate countries and show that the main results

⁸Note a slight increase in the volatility of consumer price inflation in the brief period after the break up of the Bretton Woods System, which quickly comes back down so that the average relative inflation volatility before and after 1973 is about the same. This increase in the volatility of inflation in the second half of 1970s is likely a response to the two large oil price shocks. There is also a slight increase in the volatility of consumption briefly after 1973, due to the 1974 recession in Japan.

Table 1: Empirical moments: standard deviations

	Δe_t			Δq_t			$\pi_t - \pi_t^*$			$\Delta c_t - \Delta c_t^*$		
	peg	float	ratio	peg	float	ratio	peg	float	ratio	peg	float	ratio
Canada	0.8	4.4	5.7*	1.5	4.7	3.0*	1.3	1.4	1.1	0.8	1.1	0.9
France	3.4	11.8	3.5*	3.7	11.8	3.2*	1.3	1.3	1.0	1.2	0.9	0.7*
Germany	2.4	12.3	5.0*	2.7	12.5	4.7*	1.4	1.3	0.9	1.3	1.2	0.9
Italy	0.5	10.4	18.8*	1.5	10.4	6.9*	1.4	1.9	1.3*	1.0	1.1	1.0
Japan	0.8	11.7	13.8*	2.7	11.9	4.4*	2.7	2.8	1.0	1.1	1.3	1.2
Spain	4.4	10.8	2.5*	4.7	10.8	2.3*	2.7	2.6	0.9	1.2	1.0	0.8
U.K.	4.1	11.5	2.8*	4.4	12.0	2.7*	1.7	2.5	1.5*	1.4	1.5	1.1
RoW	1.2	9.8	8.0*	1.8	9.9	5.6*	1.3	1.4	1.1	0.9	0.9	1.0
	$\Delta gdp_t - \Delta gdp_t^*$			$\Delta y_t - \Delta y_t^*$			Δnx_t			$\sigma(\Delta c_t - \Delta c_t^*) - \Delta q_t$		
	peg	float	ratio	peg	float	ratio	peg	float	ratio	peg	float	ratio
Canada	1.0	1.0	1.0	3.8	4.9	1.3	1.7	1.6	0.9	2.4	4.5	1.9*
France	1.2	1.0	0.8	5.3	5.6	1.1	1.5	1.4	0.9	4.4	12.2	2.7*
Germany	1.8	1.2	0.7*	6.7	6.0	0.9	1.8	1.7	0.9	3.9	13.7	3.5*
Italy	1.5	1.3	0.8	8.1	9.7	1.2	2.5	2.2	0.9	2.8	11.4	4.1*
Japan	1.5	1.3	0.8	5.5	5.0	0.9	2.4	2.2	0.9	2.8	13.1	4.7*
Spain	1.6	1.2	0.7*	10.1	10.4	1.0	5.4	2.1	0.4*	5.8	11.4	2.0*
U.K.	1.4	1.4	0.9	3.9	6.0	1.5*	2.2	1.9	0.9	5.2	11.8	2.2*
RoW	1.1	1.0	0.8	3.9	3.5	0.9	1.1	1.0	0.9	2.5	10.7	4.3*
	π_t			Δc_t			Δgdp_t			Δy_t		
	peg	float	ratio	peg	float	ratio	peg	float	ratio	peg	float	ratio
Canada	1.3	1.4	1.1	0.8	0.9	1.1	0.9	0.9	1.0	4.1	5.1	1.2
France	1.1	1.3	1.2*	0.9	0.8	0.9	0.9	0.6	0.6*	4.2	5.4	1.3*
Germany	1.2	1.1	0.9	1.0	1.0	1.0	1.5	1.0	0.7	6.2	5.7	0.9
Italy	1.0	2.1	2.0*	0.7	0.8	1.2	1.3	1.0	0.8	7.5	9.5	1.3*
Japan	2.6	2.9	1.1	1.0	1.3	1.3	1.1	1.1	0.9	4.6	4.9	1.1
Spain	2.5	2.5	1.0	1.0	0.7	0.7	1.4	0.7	0.5*	10.1	10.1	1.0
U.K.	1.6	2.6	1.6*	1.2	1.4	1.2	1.0	1.3	1.2	3.5	5.9	1.7*
U.S.	0.9	1.3	1.5*	0.7	0.8	1.1	0.9	1.0	1.1	2.9	2.9	1.0

Average

Note: "peg" corresponds to the period from 1960:01-1971:07 (except for Canada where it is from 1962:04-1970:01); "float" is from 1973:08-1989:12. The RoW corresponds to France, Germany, Italy, Japan, Spain, and the U.K., aggregated using 2010 GDP as weights. nx_t is the ratio of export minus imports to the sum of imports and exports. We use $\sigma = 2$ in panel eight. * indicates significance of the difference between peg and float at the 5% level (robvar test in Stata).

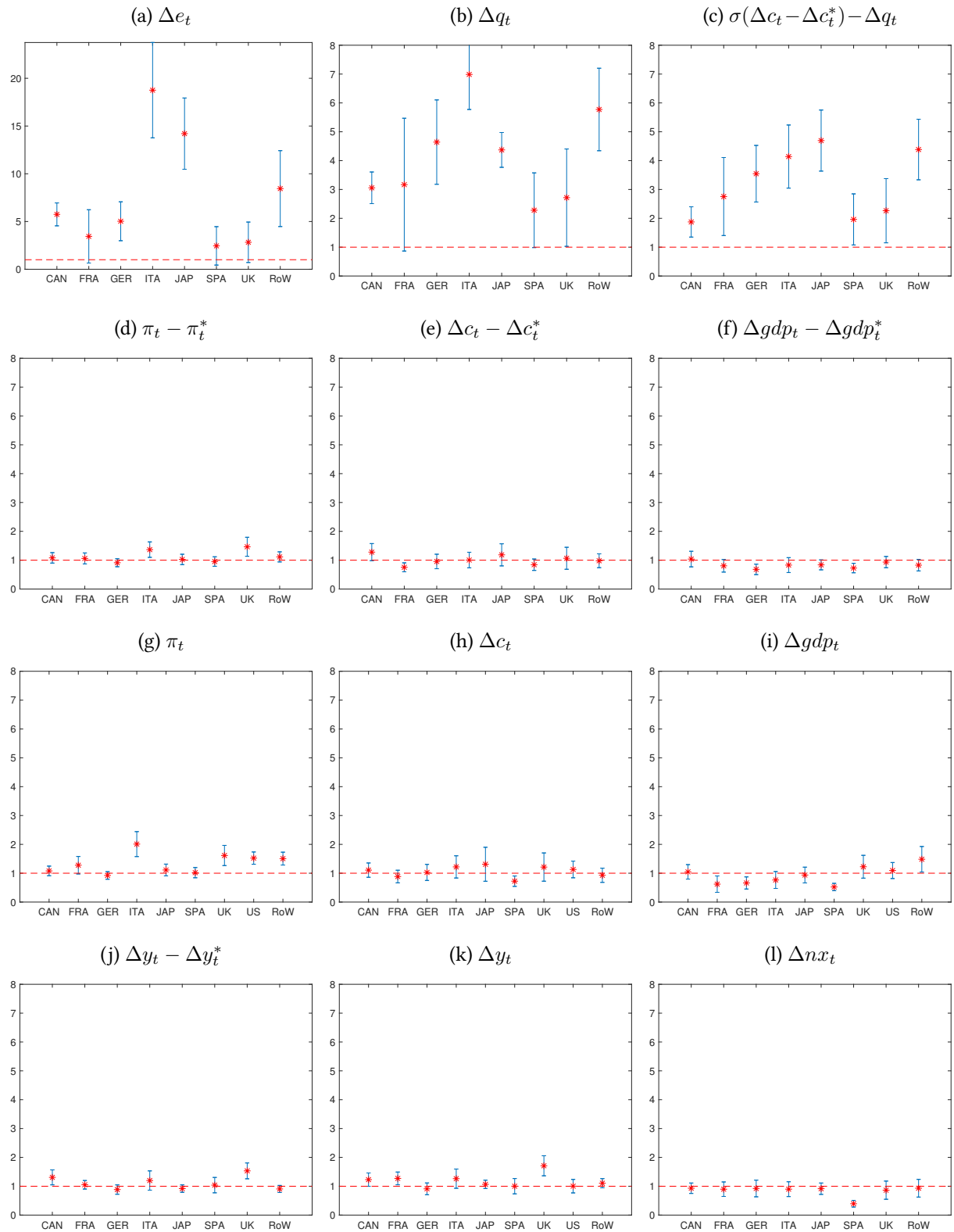


Figure 5: Volatility ratio float/peg, across variables and countries

Note: plots show the ratios of standard deviations under floating and peg regimes across individual countries with 90% confidence intervals estimated using Newey-West (HAC) standard errors. y -axis has the same scale for all plots except Δe_t .

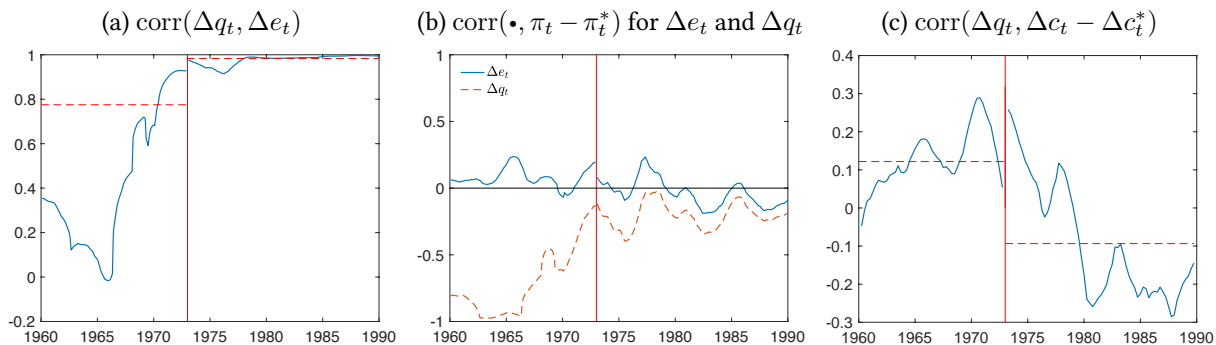


Figure 6: Correlations of exchange rates over time

Note: Triangular moving average correlations, treating 1973:01 as the end point for the two regimes.

hold in the panel as well. Table 1 summarizes the standard deviation of various variables for each country in our sample across the two monetary regimes, as well as provides a formal test of the equality of the variances of variables under the two regimes. We confirm that the change in the volatility of the exchange rates was large and highly significant in every country, while changes in the volatility of the other variables were small and generally insignificant. Note that rather than emphasizing the lack of any change in other variables, we emphasize the difference in the order of magnitude. Table 1 shows that while nominal and real exchange rate volatility increased on average by about 8 and 6 times respectively, the volatility of the other variables changed in different direction across countries and by an order of magnitude of about 10% — a stark difference. We further illustrate this point in Figure 5 which plots the ratio of the volatilities under the two regimes for each variable and country, on a common scale for comparability, with a Newey-West (HAC) robust 90% confidence interval (see also Appendix Figures A5 and A6).

Perhaps most surprisingly, there is no increase in the volatility of net exports, despite a large increase in the volatility of the real exchange rate (see Figures A2(e) and 5(l) and Table 1). Systematic data on terms of trade is not available for this period, however, in the FRED data we see that the volatility of the US terms of trade increased only twofold, while the volatility of the US real exchange rate increased six times. Therefore, we conjecture that the lack of the increased volatility in net exports is in part due to a much muted response of the behavior of the terms of trade and in part due to muted response of net exports themselves to international relative prices.

Correlations Figure 6 plots the correlations between the nominal and real exchange rates, as well as their correlations with macro variables — relative inflation and relative consumption growth — over time, calculated as triangular moving average with a break point at 1973:01. The first two panels identify two clear shifts with the break of the fixed exchange rate regime. In particular, the correlation between nominal and real exchange rates is positive but not very strong during the peg, where nominal exchange rates barely moved before 1967, and it becomes virtually perfect (0.98) after the early 1970s. The latter correlation is around 0.7 and does not change significantly with the end of the Bretton Woods System. In contrast, the real exchange rate is tightly correlated with the relative inflation during the peg, yet it quickly becomes nearly orthogonal with relative inflation as soon as nominal exchange rates begin to

Table 2: Empirical moments: correlations

	$\Delta q_t, \Delta e_t$		$\Delta q_t, \Delta c_t - \Delta c_t^*$		$\Delta q_t, \Delta n x_t$		$\Delta g d p_t, \Delta g d p_t^*$		$\Delta c_t, \Delta c_t^*$		$\Delta c_t, \Delta g d p_t$	
	peg	float	peg	float	peg	float	peg	float	peg	float	peg	float
Canada	0.77	0.92	0.03	-0.07	0.01	0.05	0.31	0.47	0.40	0.25	0.28	0.57
France	0.96	0.99	0.05	-0.08	0.23	0.12	0.09	0.30	-0.24	0.29	0.51	0.48
Germany	0.87	0.99	0.04	-0.19	-0.06	0.00	-0.01	0.28	-0.11	0.11	0.57	0.58
Italy	0.54	0.97	0.07	-0.13	0.02	-0.01	0.04	0.17	-0.18	0.13	0.64	0.45
Japan	0.76	0.98	0.21	-0.00	0.03	0.21	-0.08	0.24	0.11	0.23	0.70	0.71
Spain	0.83	0.96	-0.09	-0.18	-0.06	0.16	0.05	0.09	-0.06	0.05	0.56	0.63
U.K.	0.94	0.96	0.09	-0.10	-0.39	-0.16	-0.11	0.30	-0.02	0.22	0.59	0.71
RoW	0.80	0.98	0.05	-0.19	-0.20	0.21	-0.03	0.39	-0.11	0.31	0.68	0.72

Note: see notes to Table 1. Cross-country correlation are with the U.S. as the foreign counterpart indicated with a star.

float. At the same time, the nominal exchange rate is orthogonal with the relative inflation both before and after the 1973. While the pattern of correlation during the peg is mechanical – since $\Delta e_t \approx 0$ and thus $\Delta q_t \approx -(\pi_t - \pi_t^*)$ – the comovement of variables during the float is rather puzzling, as nominal depreciations if anything are negatively correlated with relative domestic inflation.

The last panel of Figure 6 shows that the correlation between real exchange rate and relative consumption growth, while somewhat positive under the peg, has become noticeably negative under the float, suggesting that the Backus-Smith puzzle became more pronounced with floating exchange rates.⁹ While the correlations are not very large, this pattern is observed robustly across the countries, as we document in Table 2.

Table 2 reports correlations between various variables under the two exchange rate regimes, while Appendix Figure A4 plots the evolution of these correlation over time. Table 2 identifies another robust correlation pattern – between consumption and GDP growth, which is stable around 0.7 and does not change at all with the end of the Bretton Woods System. The other correlations, including that between RER and net exports, as well as for GDP (consumption) growth between countries, are generally not very strong and not particularly stable over time, suggesting only weak patterns of change across the two monetary regimes. The correlation between the real exchange rate and net exports switches from negative under the peg to positive under the float, but in both cases it is close to zero for most countries in our sample.

Finally, both GDP growth and consumption growth are uncorrelated or mildly negatively correlated across countries (with the exception of Canada) before 1970s and since then become sizably positively correlated, especially the GDP growth rates – a surprising pattern emphasized by Kollmann (2005). Figure A4 reveals that this is, however, largely driven by the high correlation of growth rates across countries in the late 1970s, a period of large global oil price shocks.

⁹This observation is consistent with the findings in Colacito and Croce (2013) that both Backus-Smith and UIP conditions held better under the pegged exchange rates, as well as in Devereux and Hnatkovska (2014) that the Backus-Smith condition holds better across regions within countries, in contrast with its cross-country violations. Another pattern emphasized by Berka, Devereux, and Engel (2012) is a substantially greater role of the non-tradable (Balassa-Samuelson) component in the RER variation under a nominal peg.

3 Conventional Models

Conventional models are defined by the following property: if prices are flexible, a switch in the monetary regime does not affect the behavior of real variables, including the real exchange rate q_t , consumption c_t and output y_t . Therefore, we only consider the case of sticky prices in this section, as flexible-price version of these models is falsified by the changing property of the real exchange rate (Figure 1). We first describe the model setup and then prove a negative result that even the sticky-price version of these models is falsified by the extended set of Mussa facts documented above.

3.1 Model setup

We build on a standard New Keynesian open-economy model (NKOE) with productivity shocks, PCP Calvo price stickiness, flexible wages and no capital. In our quantitative exploration in Section 5, we generalize this environment by introducing intermediate goods, capital, sticky wages and considering alternative variants of price stickiness including LCP and DCP, as well as monetary shocks. The model features home bias in consumption and exogeneous shocks to international risk sharing. We allow for various degrees of financial market (in)completeness, including complete markets, bonds-only and financial autarky.

There are two mostly symmetric countries – home (Europe) and foreign (US, denoted with a *). Each country has its nominal unit of account in which the local prices are quoted: for example, the home wage rate is W_t euros and the foreign wage rate is W_t^* dollars. The nominal exchange rate \mathcal{E}_t is the price of dollars in terms of euros, hence an increase in \mathcal{E}_t signifies a nominal devaluation of the euro (the home currency).

The monetary policy is conducted according to a conventional Taylor rule targeting inflation or nominal exchange rate – depending on the monetary regime. In particular, the foreign country (US) always targets inflation, while the home country (Europe) switches from an exchange rate peg (‘peg’) to an inflation targeting (‘float’). We consider the limiting case where the monetary authorities have the ability to fully stabilize prices or the exchange rate, depending on the regime.

Households A representative home household maximizes the discounted expected utility over consumption and labor:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{1}{1-\sigma} C_t^{1-\sigma} - \frac{1}{1+\varphi} L_t^{1+\varphi} \right), \quad (4)$$

where σ is the relative risk aversion parameter and φ is the inverse Frisch elasticity of labor supply. The flow budget constraint is given by:

$$P_t C_t + \sum_{j \in J_t} \Theta_t^j B_{t+1}^j \leq W_t L_t + \sum_{j \in J_{t-1}} e^{-\zeta_t^j} D_t^j B_t^j + \Pi_t + T_t, \quad (5)$$

where P_t is the consumer price index and W_t is the nominal wage rate, Π_t are profits of home firms, T_t are lump-sum transfers from the government, and B_{t+1}^j is quantity of asset $j \in J_t$ purchased at time t

at price Θ_t^j and paying out a state-contingent dividend D_{t+1}^j at $t + 1$ taxed at rate ζ_t^j (which we think of as [Chari, Kehoe, and McGrattan 2007](#) wedges).¹⁰

Household optimization results in standard optimality conditions for labor supply:

$$C_t^\sigma L_t^\varphi = \frac{W_t}{P_t}, \quad (6)$$

Euler equations for state-contingent bonds:

$$\Theta_t^j = \beta \mathbb{E}_t \left\{ \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{P_t}{P_{t+1}} e^{-\zeta_{t+1}^j} D_{t+1}^j \right\} \quad \text{for all } j \in J_t. \quad (7)$$

The foreign households are symmetric, having access to a set J_t^* of state contingent assets with dividends taxed at country-specific tax rate ζ_t^{j*} . The assets $j \in J_t \cap J_t^*$ can be purchased by households of both countries at a common price Θ_t^j in units of home currency.¹¹

Expenditure and demand The domestic households allocate their within-period consumption expenditure between home and foreign varieties of the goods, $P_t C_t = \int_0^1 [P_{Ht}(i)C_{Ht}(i) + P_{Ft}(i)C_{Ft}(i)] di$ to maximize the CES consumption aggregator:

$$C_t = \left[\int_0^1 \left([(1-\gamma)e^{-\gamma\xi_t}]^{1/\theta} C_{Ht}(i)^{\frac{\theta-1}{\theta}} + [\gamma e^{(1-\gamma)\xi_t}]^{1/\theta} C_{Ft}(i)^{\frac{\theta-1}{\theta}} \right) di \right]^{\frac{\theta}{\theta-1}}, \quad (8)$$

with parameter $\gamma \in [0, 1/2)$ capturing the level of the *home bias*, which can be due to a combination of home bias in preferences, trade costs and non-tradable goods (see [Obstfeld and Rogoff 2001](#)) and ξ_t denoting the relative demand shock for the foreign good or other sources of time-varying home bias (see [Pavlova and Rigobon 2007](#)).¹² The solution to the optimal expenditure allocation results in the conventional constant-elasticity demand schedules:

$$C_{Ht}(i) = (1-\gamma)e^{-\gamma\xi_t} \left(\frac{P_{Ht}(i)}{P_t} \right)^{-\theta} C_t \quad \text{and} \quad C_{Ft}(j) = \gamma e^{(1-\gamma)\xi_t} \left(\frac{P_{Ft}(j)}{P_t} \right)^{-\theta} C_t, \quad (9)$$

where the price index is given by $P_t = \left[\int_0^1 \left((1-\gamma)e^{-\gamma\xi_t} P_{Ht}(i)^{1-\theta} + \gamma e^{(1-\gamma)\xi_t} P_{Ft}(i)^{1-\theta} \right) di \right]^{1/(1-\theta)}$.

¹⁰When set J_t contains a home-currency risk-free bond B_{t+1}^f , its price is one over gross nominal interest rate $\Theta_t^f = 1/R_t$ and it pays out one unit of home currency in every state of the world next period, $D_{t+1}^f \equiv 1$; when it contains a foreign-currency risk-free bond B_{t+1}^{f*} , its price is \mathcal{E}_t/R_t^* and its dividend is $D_{t+1}^{f*} = \mathcal{E}_{t+1}$.

¹¹From the point of view of foreign households, the foreign currency price of asset j is Θ_t^j/\mathcal{E}_t in units of foreign currency, and the Euler equation is correspondingly $\frac{\Theta_t^j}{\mathcal{E}_t} = \beta \mathbb{E}_t \left\{ \left(\frac{C_{t+1}^*}{C_t^*} \right)^{-\sigma} \frac{P_t^*}{P_{t+1}^*} e^{-\zeta_{t+1}^{j*}} \frac{D_{t+1}^j}{\mathcal{E}_{t+1}} \right\}$ for $j \in J_t^*$.

¹²The particular way in which we introduce the foreign-good demand shock ξ_t in (8) ensures that changes in ξ_t shift the relative demand between home and foreign goods without having a first-order effect on the price level. The aggregate implications of the model do not depend on whether the home bias emerges on the extensive margin due to non-tradable goods or on the intensive margin due to trade costs and preferences, and therefore we do not explicitly introduce the non-tradables.

The expenditure allocation of the foreign households is symmetrically given by:

$$C_{Ht}^*(i) = \gamma e^{(1-\gamma)\xi_t^*} \left(\frac{P_{Ht}^*(i)}{P_t^*} \right)^{-\theta} C_t^* \quad \text{and} \quad C_{Ft}^*(j) = (1-\gamma) e^{-\gamma\xi_t^*} \left(\frac{P_{Ft}^*(j)}{P_t^*} \right)^{-\theta} C_t^*, \quad (10)$$

where ξ_t^* is the foreign demand shock for home goods, $P_{Ht}^*(i)$ and $P_{Ft}^*(j)$ are the foreign-currency prices of the home and foreign goods in the foreign market, and P_t^* is the foreign price level. The *real exchange rate* is the relative consumer price level in the two countries:

$$Q_t \equiv \frac{P_t^* \mathcal{E}_t}{P_t}, \quad (11)$$

with an increase in Q_t corresponding to a real depreciation, that is a decrease in the relative price of the home consumption basket (note that (1) is the log version of (11)).

Production and price setting Home output is produced by a given pool of symmetric firms according to a linear technology in labor:

$$Y_t(i) = e^{a_t} L_t(i), \quad (12)$$

where a_t is the aggregate productivity shock, which implies that the marginal cost of production is:

$$MC_t = e^{-a_t} W_t, \quad (13)$$

identical for all firms.

The firms adjust prices infrequently à la Calvo with a constant per-period hazard rate λ of price nonadjustment, that is $P_{Ht}(i) = P_{H,t-1}(i)$ with probability λ and with complementary probability $(1-\lambda)$ the firm resets its price to $\bar{P}_{Ht}(i)$. For concreteness, we assume producer-currency pricing (PCP), which implies that the law of one price holds:

$$P_{Ht}(i)^* = P_{Ht}(i) / \mathcal{E}_t. \quad (14)$$

The per-period profit of the firm is given by

$$\Pi_t(i) = (P_{Ht}(i) - MC_t)(C_{Ht}(i) + C_{Ht}^*(i)) \quad (15)$$

where consumer demand $C_{Ht}(i)$ and $C_{Ht}^*(i)$ satisfies (9) and (10). The aggregate profits of the domestic firms, $\Pi_t = \int_0^1 \Pi_t(i) di$, are distributed to the domestic households.

The firms set prices $\bar{P}_{Ht}(i)$ as the solution of the following optimization problem:

$$\bar{P}_{Ht}(i) = \arg \max \mathbb{E}_t \sum_{k=0}^{\infty} (\beta\lambda)^k \left(\frac{C_{t+k}}{C_t} \right)^{-\sigma} \frac{P_t}{P_{t+k}} \Pi_{t+k}(i), \quad (16)$$

where future profits are discounted using the SDF of the households and the probability of the reset price $\bar{P}_{Ht}(i)$ staying in effect k periods after it has been set. Since firms are symmetric (conditional on the last date they adjusted prices), they all set the price at the same reset level, $\bar{P}_{Ht}(i) = \bar{P}_{Ht}$ for all i .

Government The fiscal authority collects taxes on financial dividends and returns them lump-sum to the households:

$$T_t = \sum_{j \in J_{t-1}} (1 - e^{-\zeta_t^j}) D_t^j B_t^j. \quad (17)$$

The monetary policy is implemented by means of a Taylor rule:

$$i_t = \rho_m i_{t-1} + (1 - \rho_m) [\phi_\pi \pi_t + \phi_e (e_t - \bar{e})] + \sigma_m \varepsilon_t^m, \quad (18)$$

where $i_t = \log R_t$ is the log nominal interest rate, $\pi_t = \Delta \log P_t$ is the inflation rate, $\varepsilon_t^m \sim iid(0, 1)$ is the monetary policy shock with volatility parameter $\sigma_m \geq 0$, and the parameter ρ_m characterizes the persistence of the monetary policy rule. The coefficients $\phi_\pi > 1$ and ϕ_e are the Taylor rule parameters which weight the two nominal objectives of the monetary policy – inflation and exchange rate stabilization. We assume that the monetary authority can fully achieve the chosen goal (by increasing ϕ_π or ϕ_e unboundedly) – that is, either $\pi_t \equiv 0$ or $e_t \equiv \bar{e}$, depending on the regime. We further assume that the foreign country (the US) only has the inflation objective, so that $\phi_e^* = 0$, and $\pi_t^* \equiv 0$. We study the differential behavior of the macro variables across the two monetary regimes of the home country.

Equilibrium conditions The labor market clearing requires that the labor supplied by households according to (6) equals the aggregate labor demand of the home firm, $L_t = e^{-a_t} \int_0^1 Y_t(i) di$, where we used the production function (12). The goods market clearing for each firm i requires $Y_t(i) = C_{Ht}(i) + C_{Ht}^*(i)$, defined in (9)–(10), and given prices $P_{Ht}(i)$. Symmetric market clearing conditions hold in the foreign country. All assets B_{t+1}^j are in zero net supply, and for $j \in J_t \cap J_t^*$, we have $B_{t+1}^j + B_{t+1}^{j*} = 0$ given a common home-currency price Θ_t^j .

We focus here on two equilibrium conditions – the country budget constraint and the equilibrium in the financial market. The latter set of conditions can be written using (7) and their foreign counterparts as follows:

$$\mathbb{E}_t \left\{ e^{-\zeta_{t+1}^j} D_{t+1}^j \frac{P_t}{P_{t+1}} \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} - \left(\frac{C_{t+1}^*}{C_t^*} \right)^{-\sigma} \frac{Q_t}{Q_{t+1}} e^{\tilde{\zeta}_{t+1}^j} \right] \right\} = 0, \quad \forall j \in J_t \cap J_t^*, \quad (19)$$

where by convention we denote $\tilde{\zeta}_{t+1}^j \equiv \zeta_{t+1}^j - \zeta_{t+1}^{j*}$ the relative wedge across countries. This condition simply states that home and foreign households agree on the price Θ_t^j of all assets j that they can mutually trade. This allows the household to equalize the stochastic discount factors across the two countries in the best possible way given the available set of internationally-traded assets.

The country budget constraint derives from substituting firm profits (15) and government transfers (17) into the household budget constraint (5):

$$\mathcal{B}_{t+1} - \mathcal{R}_t \mathcal{B}_t = NX_t, \quad (20)$$

where the right-hand side is net exports $NX_t = \int_0^1 P_{Ht}(i) C_{Ht}^*(i) di - \mathcal{E}_t \int_0^1 P_{Ft}^*(i) C_{Ft}(i) di$ and the left hand-side is the evolution of net foreign assets $\mathcal{B}_{t+1} \equiv \sum_{j \in J_t} \Theta_t^j B_{t+1}^j$ with the cumulative realized

return $\mathcal{R}_t \equiv \frac{\sum_{j \in J_{t-1}} D_t^j B_t^j}{\sum_{j \in J_{t-1}} \Theta_{t-1}^j B_t^j}$. The foreign budget constraint is redundant by Walras Law. Substituting the demand schedules (9)–(10) into the expression for net exports and combining prices into CES price indexes of exports P_{Ht} and imports P_{Ft}^* , we can rewrite the country budget constraint (20) as:

$$\mathcal{B}_{t+1} - \mathcal{R}_t \mathcal{B}_t = \gamma e^{(1-\gamma)\xi_t} (\mathcal{E}_t P_{Ft}^*)^{1-\theta} P_t^\theta C_t \left[e^{-(1-\gamma)\tilde{\xi}_t} \mathcal{S}_t^{\theta-1} \mathcal{Q}_t^\theta \frac{C_t^*}{C_t} - 1 \right], \quad (21)$$

where again $\tilde{\xi}_t \equiv \xi_t - \xi_t^*$, \mathcal{Q}_t is the real exchange rate, and $\mathcal{S}_t \equiv \frac{\mathcal{E}_t P_{Ft}^*}{P_{Ht}}$ is the terms of trade, which in light of PCP Calvo price setting and the implied law of one price (14) satisfies:

$$\mathcal{S}_t \equiv \frac{\mathcal{E}_t P_{Ft}^*}{P_{Ht}} = \mathcal{Q}_t \left[\frac{(1-\gamma)e^{-\gamma\xi_t^*} + \gamma e^{(1-\gamma)\xi_t^*} \mathcal{S}_t^{-(1-\theta)}}{(1-\gamma)e^{-\gamma\xi_t} + \gamma e^{(1-\gamma)\xi_t} \mathcal{S}_t^{1-\theta}} \right]^{\frac{1}{1-\theta}} \Rightarrow \mathcal{S}_t \approx \mathcal{Q}_t^{\frac{1}{1-2\gamma}}, \quad (22)$$

where the approximation is a log-linearization around symmetric equilibrium with $\mathcal{Q}_t = \mathcal{S}_t = 1$ and $\xi_t = \xi_t^* = 0$. Conditions (19) and (21) allow to study a variety of cases with different degree of financial openness of the economies.

3.2 Cointegration relationship between consumption and exchange rate

We now show that under a variety of circumstances, there exists a constant ς such that the equilibrium process for $\varsigma(\Delta c_t - \Delta c_t^*) - \Delta q_t$ is independent of the monetary regime. In other words, a change in the statistical process for the real exchange rate Δq_t should result in a change in the statistical process for the relative consumption growth $\Delta c_t - \Delta c_t^*$.

3.2.1 Limiting cases: financial autarky and complete markets

We start with the two extreme cases of international financial integration – financial autarky and complete markets. We show that our main result obtains immediately in these two cases, independently of the structure of the rest of the model. We then proceed with the more interesting intermediate case.

Financial autarky Consider first the special case when $J_t \cap J_t^* = \emptyset$ at all t , so that net foreign assets $\mathcal{B}_t \equiv 0$, and therefore $NX_t \equiv 0$ in every state and period. Equation (21) then implies, $C_t/C_t^* = e^{-(1-\gamma)\tilde{\xi}_t} \mathcal{S}_t^{\theta-1} \mathcal{Q}_t^\theta$, and using (22), we have:

$$c_t - c_t^* = -(1-\gamma)\tilde{\xi}_t + \left[\theta + \frac{\theta-1}{1-2\gamma} \right] q_t. \quad (23)$$

If the process for taste shocks (home bias) for goods $\tilde{\xi}_t$ is independent from the monetary regime, it must be that the process for $\varsigma(\Delta c_t - \Delta c_t^*) - \Delta q_t$ with $\varsigma = \frac{1-2\gamma}{2(1-\gamma)\theta-1}$ is also independent from the monetary regime. Note that for $\theta > 1$, $\varsigma < 1$ and is independent from relative risk aversion σ .

Complete markets Now consider a situation where there exists a $j \in J_t \cap J_t^*$ for every state of the world (Arrow securities), so that (19) holds as equality not just in expectation, but also state-by-state,

$\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma} \equiv \left(\frac{C_{t+1}^*}{C_t^*}\right)^{-\sigma} \frac{Q_t}{Q_{t+1}} e^{\tilde{\zeta}_{t+1}}$, where $\tilde{\zeta}_{t+1}$ is the state-specific realization of the relative wedge, or equivalently:

$$\sigma(\Delta c_t - \Delta c_t^*) = \Delta q_t + \tilde{\zeta}_t. \quad (24)$$

This corresponds to the first difference of (2) in the introduction. Assuming that wedges $\tilde{\zeta}_t$ do not change with the monetary regime, it must be that $\sigma(\Delta c_t - \Delta c_t^*) - \Delta q_t$ does not change as well. Of course, this nests the case of perfect international risk-sharing with $\tilde{\zeta}_t \equiv 0$.

Cole and Obstfeld (1991) case This is a special case with $\sigma = \theta = 1$, which is commonly used in the literature, as in this case the equilibrium allocation does not depend on the degree of asset market completeness. Assume additionally that $\tilde{\zeta}_t^j = \tilde{\xi}_t \equiv 0$. Under these circumstances, balanced trade $NX_t \equiv 0$ implies $C_t/C_t^* = Q_t$, which ensures financial market equilibrium (19) independently of what assets j are in the set $J_t \cap J_t^*$. Therefore, this case is equivalent to the two cases considered above, but the logic of the Cole-Obstfeld case is more general, as we show below.

3.2.2 General asset market structure

Consider a case where $J_t \cap J_t^*$ contains at least one asset, namely the foreign-currency (US dollar) risk-free bond. Considering the risk-sharing condition (19) for this bond:

$$\mathbb{E}_t \left\{ \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{Q_{t+1}}{Q_t} e^{-\tilde{\zeta}_{t+1}} - \left(\frac{C_{t+1}^*}{C_t^*} \right)^{-\sigma} \right\} = 0, \quad (25)$$

where we made use of the assumption that foreign monetary policy ensures $\pi_{t+1}^* \equiv 0$, and we additionally assumed for concreteness $\zeta_{t+1}^* \equiv 0$, that is foreigners do not face a wedge when trading the foreign-currency bond.

To make further progress, we rely on a log-linear solution to the equilibrium system. We do it primarily for tractability reasons, but this is also justified on the following grounds: (a) the log-linear solution concept is typical in the literature, including the conventional RBC and New Keynesian models; (b) even when higher order terms are included in the numerical solutions of these models, the results barely change, as the aggregate macro variables are not volatile (see Figure 3) and therefore the omitted risk premia terms are small. While doing so, we nonetheless allow for risk-sharing wedges $\tilde{\zeta}_t$, which may proxy for risk premia shocks. In particular, note that $\psi_t = -\mathbb{E}_t \tilde{\zeta}_{t+1}$ represent a UIP shock. Indeed, assume that home households can additionally trade a home-currency bond with each other (without facing a wedge), then we can derive the UIP condition from combining the home Euler equations for home- and foreign-currency risk-free bonds (see footnote 10):

$$\mathbb{E}_t \left\{ \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{P_t}{P_{t+1}} \left[R_t - e^{-\tilde{\zeta}_{t+1}} R_t^* \frac{\mathcal{E}_{t+1}}{\mathcal{E}_t} \right] \right\} = 0,$$

which upon log-linearization yeilds:

$$i_t - i_t^* - \mathbb{E}_t \Delta e_{t+1} = \psi_t. \quad (26)$$

In our earlier work (Itskhoki and Mukhin 2019), we have argued that such UIP shocks ψ_t are essential to explain the exchange rate disconnect behavior under the floating regime. Here, however, we show that they are not enough on their own to also explain the broad set of Mussa facts, which involves a change in the exchange rate regime.¹³

For concreteness, we focus on two types of shocks – a relative productivity shock $\tilde{a}_t = a_t - a_t^*$ and a risk-sharing wedge $\psi_t = -\mathbb{E}_t \tilde{\zeta}_{t+1}$ – which we assume both follow independent exogenous AR(1) processes with common persistence parameter $\rho \in (0, 1)$ and innovations $\sigma_a \varepsilon_t^a$ and $\sigma_\psi \varepsilon_t$ respectively. Our results generalize to the case with additional shocks, including monetary shocks and foreign-good demand shocks ξ_t , and we incorporate them in our quantitative analysis in Section 5.

The log-linear approximation to (25) is given by:

$$\mathbb{E}_t \{ \sigma (\Delta c_{t+1} - \Delta c_{t+1}^*) - \Delta q_{t+1} \} = \psi_t. \quad (27)$$

Any solution to (27) can be written in the following form:

$$\sigma (c_t - c_t^*) - q_t = -\frac{\psi_t}{1 - \rho} + m_t, \quad (28)$$

where m_t is a martingale, that is $m_{t+1} = m_t + u_{t+1}$ with $\mathbb{E}_t u_{t+1} = 0$. Any fundamental solution to (27) in addition has u_{t+1} equal some linear combination of the innovations of the exogenous shock processes, $\sigma_a \varepsilon_{t+1}^a$ and $\sigma_\psi \varepsilon_{t+1}$. In what follows, we solve for u_{t+1} and prove that under various circumstances it is independent of the monetary policy regime, and thus so is $\sigma (c_t - c_t^*) - q_t$.

The fundamental solution for u_{t+1} must satisfy the intertemporal budget constraint. We log-linearize the flow budget constraint (21), using the definition of terms of trade (22), to obtain:

$$\beta b_{t+1} - b_t = \gamma \left[\frac{2(1 - \gamma)\theta - 1}{1 - 2\gamma} q_t + c_t^* - c_t \right], \quad (29)$$

where $b_t = \frac{d(\mathcal{R}_t \mathcal{B}_t)}{P_H Y / \beta}$ is the deviation of net foreign assets from zero (symmetric steady state NFA) in units of home nominal GDP. Solve forward, imposing the NPGC, to arrive at the intertemporal budget constraint:

$$b_t + \gamma \sum_{k=0}^{\infty} \beta^k \left[\frac{2(1 - \gamma)\theta - 1}{1 - 2\gamma} q_{t+k} - (c_{t+k} - c_{t+k}^*) \right] = 0. \quad (30)$$

Generalized Cole-Obstfeld case Consider the case with the following parameter restriction:

$$\sigma = \frac{1 - 2\gamma}{2(1 - \gamma)\theta - 1}. \quad (31)$$

¹³As we show in Section 4, explaining the Mussa puzzle requires a refinement of the theory of ψ_t shocks.

This nests the Cole-Obstfeld case above as $\sigma = \theta = 1$ is a special case of (31).

When (31) is satisfied, we can use (28) and (30) to solve for the equilibrium martingale innovation.¹⁴

$$\Delta m_t = u_t = \frac{1 - \beta}{(1 - \rho)(1 - \beta\rho)} \sigma_\psi \varepsilon_t,$$

which does not depend on the monetary policy rule. Substituting into the first difference of (28), we obtain:

$$\sigma(\Delta c_{t+1} - \Delta c_{t+1}^*) - \Delta q_{t+1} = \psi_t - \frac{\beta}{1 - \beta\rho} \sigma_\psi \varepsilon_{t+1}, \quad (32)$$

confirming that the statistical process for $\sigma(\Delta c_{t+1} - \Delta c_{t+1}^*) - \Delta q_{t+1}$ does not depend on the monetary regime. In particular, $\sigma(\Delta c_{t+1} - \Delta c_{t+1}^*) - \Delta q_{t+1}$ does not respond to productivity shocks under either regime and responds to the risk-premium/risk-sharing shock ψ_t in the same way independently of the monetary regime.

3.2.3 Equilibrium price dynamics

Outside the generalized Cole-Obstfeld case, the solution cannot be obtained independent of the equilibrium price dynamics, which is clearly sensitive to changes in monetary policy, and therefore constitutes our key focus of interest. To characterize the equilibrium dynamics in this case, we derive the final dynamic equation of the model, namely the Phillips curve for prices.

In Appendix A.3.2, we log-linearize the price setting equation (16) and combine it with labor and product market clearing conditions, to arrive at the following open-economy Phillips curve:

$$(1 - \beta L^{-1})[\pi_t - \pi_t^* - 2\gamma \Delta e_t] = \kappa[(c_t - c_t^*) + \gamma \kappa_q q_t - \kappa_a \tilde{a}_t], \quad (33)$$

where L is the lag operator and $L^{-1}z_t = \mathbb{E}_t z_{t+1}$, and $\kappa \equiv (1 - 2\gamma)[\sigma + (1 - 2\gamma)\varphi] \frac{(1-\lambda)(1-\beta\lambda)}{\lambda}$ is the slope coefficient of the Phillips curve, $\kappa_q \equiv \frac{2\theta(1-\gamma)\varphi+1}{\sigma+(1-2\gamma)\varphi} \frac{2}{1-2\gamma}$ and $\kappa_a \equiv \frac{1+\varphi}{\sigma+(1-2\gamma)\varphi}$. The term in the square brackets in (33) is the result of the goods market clearing condition under flexible prices, which may be violated when prices are sticky. In addition to the risk-sharing condition (27) and the budget constraint (29), the Phillips curve (33) provides the third equilibrium condition in the goods market, which ties together the equilibrium behavior of $(c_t - c_t^*)$ and q_t .

Our assumption on the conduct of monetary policy allows us to convert (33) into a sharp tool for characterizing the equilibrium behavior of the real exchange rate. We define:

$$k_R = \begin{cases} \kappa/\sigma, & R = \text{peg}, \\ \kappa/(2\gamma\sigma), & R = \text{float}, \end{cases}$$

¹⁴Using (28), the net present value of the innovation to the budget constraint (30) at time t , which must be zero, is given by

$$\frac{\gamma}{\sigma} \sum_{k=0}^{\infty} \beta^k \left[\frac{\rho^k \varepsilon_t}{1 - \rho} - u_t \right] = \frac{\gamma}{\sigma} \left[\frac{\varepsilon_t}{(1 - \rho)(1 - \beta\rho)} - \frac{u_t}{1 - \beta} \right] = 0.$$

This yields the solution in the text.

where under the ‘float’ the monetary authorities ensure $\pi_t = \pi_t^* \equiv 0$, while under the ‘peg’ $\Delta e_t = \pi_t^* = 0$. We then combine (33) with the definition of a real depreciation, $\Delta q_t = \Delta e_t + \pi_t^* - \pi_t$, to obtain:

Lemma 1 *The equilibrium dynamics of the real exchange rate satisfies:*

$$\Delta q_t = \beta \mathbb{E}_t \Delta q_{t+1} - \sigma k_R [(c_t - c_t^*) + \gamma \kappa_q q_t - \kappa_a \tilde{a}_t], \quad (34)$$

under both monetary regimes, $R \in \{\text{float}, \text{peg}\}$.

Equation (34) is a dynamic error correction model, where an overvalued real exchange rate from the point of view of flexible-price market clearing leads to a real depreciation, and vice versa. This general error correction structure is independent from the monetary regime, however, the speed of correction does depend on it – with a faster correction under the float, since $k_{\text{float}} > k_{\text{peg}}$ as $\gamma < 1/2$. Furthermore, a change from a peg to a float is equivalent to an increase in the slope of the Phillips curve κ , e.g. due to the increased flexibility of prices (lower λ) (cf. Engel 2018). Therefore, in general, the change in the monetary policy regime can affect the equilibrium comovement between consumption and the real exchange rate.¹⁵

To solve for the equilibrium cointegration between consumption and real exchange rate and alternative monetary regimes, we substitute (28) into (34) to obtain a second-order difference equation in q_t :

$$[1 + \beta + (1 + \gamma \sigma \kappa_q) k_R] q_t - q_{t-1} - \beta \mathbb{E}_t q_{t+1} = k_R z_t, \quad (35)$$

where $z_t \equiv \frac{\psi_t}{1-\rho} + \sigma \kappa_a \tilde{a}_t - m_t$. The unique non-explosive solution consistent with (35) is given by:

$$(1 - \delta_R L) q_t = k_R \delta_R \sum_{k=0}^{\infty} (\beta \delta_R)^k \mathbb{E}_t z_{t+k}, \quad (36)$$

where $\delta_R \in (0, 1)$ is the only such root of $\beta x^2 - [1 + \beta + (1 + \gamma \sigma \kappa_q) k_R] x + 1 = 0$, and is decreasing in k_R . Using the fact that ψ_t and \tilde{a}_t are AR(1) and m_t is random walk, we evaluate the right-hand side of (46), and then use the intertemporal budget constraint (30) to solve for the equilibrium innovation to m_t :¹⁶

$$\Delta m_t = u_t = \frac{1 - \beta}{1 - \beta \rho} \left[\frac{1 + \chi \frac{k_R \delta_R}{1 - \beta \rho \delta_R} \frac{1 - \rho}{1 - \beta \rho^2} \sigma \psi \varepsilon_t}{1 + \chi \frac{k_R \delta_R}{1 - \beta \delta_R} \frac{1 - \rho}{1 - \beta \rho}} + \frac{\chi \frac{k_R \delta_R}{1 - \beta \rho \delta_R} \frac{1 - \rho}{1 - \beta \rho^2} \sigma \kappa_a \sigma_a \tilde{\varepsilon}_t}{1 + \chi \frac{k_R \delta_R}{1 - \beta \delta_R} \frac{1 - \rho}{1 - \beta \rho}} \right], \quad (37)$$

where $\chi \equiv \sigma \frac{2(1-\gamma)\theta-1}{1-2\gamma} - 1$, which is equal to zero under the generalized Cole-Obstfeld parameter

¹⁵The two limiting cases of fully rigid or fully flexible prices are trivial exceptions, since in these cases $k_R \equiv \infty$ or $k_R \equiv 0$ respectively.

¹⁶Substituting (28) into (30) and imposing a zero innovation to budget constraint, implies:

$$\frac{\sigma \psi \varepsilon_t}{(1-\rho)(1-\beta\rho)} - \frac{u_t}{1-\beta} + \chi \sum_{k=0}^{\infty} \beta^k IRF_k^q = 0,$$

where IRF_k^q is the impulse response function of the equilibrium RER process (46), which we use to solve for u_t in (37). The solution for q_t is given by (for further derivation see Appendix A.3.3):

$$q_t = \delta_R q_{t-1} + k_R \delta_R \left[\frac{\psi_t}{(1-\rho)(1-\beta\rho\delta_R)} + \frac{\sigma \kappa_a \tilde{a}_t}{1-\beta\rho\delta_R} - \frac{m_t}{1-\beta\delta_R} \right].$$

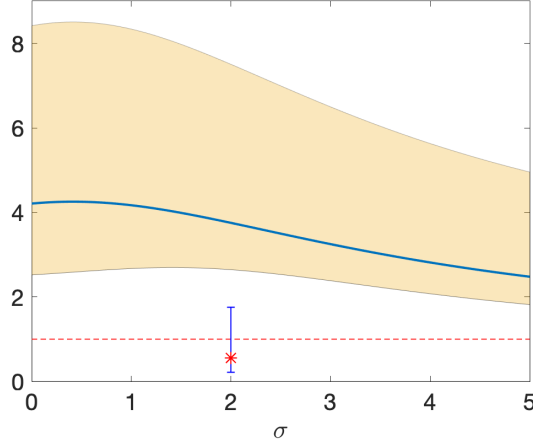


Figure 7: Ratio of $\text{std}(\tilde{\zeta}_t)$ after/during the Bretton Woods System, where $\tilde{\zeta}_t \equiv \sigma \Delta \tilde{c}_t - \Delta q_t$

Note: $\text{std}(\sigma(\Delta c_t - \Delta c_t^*) - \Delta q_t)$ is computed for 1960–72 and 1973–89 for the RoW vs the U.S. for different value of σ . The dashed red line at 1 illustrates the prediction of Proposition 1 for the ratio of the standard deviation across the regimes. The red asterisk (and the simulated 90% blue confidence interval) correspond to the calibrated quantitative model, which is not nested as a special case of Proposition 1.

restriction (31). Note that, in general, u_t depends on the monetary regime via k_R and δ_R terms in (37), however, it is not the case under any of the following circumstances: (a) $\chi = 0$; (b) $k_R \equiv 0$ or $\delta_R \equiv 0$; (c) $\beta \rightarrow 1$; or (d) $\rho \rightarrow 1$, as we summarize in Proposition 1 below.

3.3 Empirical Falsification

The general characterization above suggests that the cointegration relationship (28) between consumption and the real exchange rate is in general endogenous to the monetary regime, as is evident from the presence of k_R and δ_R terms in (37). Nonetheless, the result below emphasizes a number of limiting cases when it does not depend on the monetary regime:

Proposition 1 *The cointegration relationship between relative consumption $c_t - c_t^*$ and the real exchange rate q_t does not depend on the exchange rate regime (peg vs float) under any of the following circumstances:*

1. *international financial autarky;*
2. *complete international asset markets (with or without international risk-sharing wedges);*
3. *generalized Cole-Obstfeld case, $\sigma = \frac{1-2\gamma}{2(1-\gamma)\theta-1}$, with or without risk-sharing wedges;*
4. *in the limit of both fully fixed and fully flexible prices;*
5. *in the limit of perfect patience, $\beta \rightarrow 1$, independently of the persistence of shocks $\rho < 1$;*
6. *in the limit of persistent shocks, $\rho \rightarrow 1$, independently of the discount factor $\beta < 1$.*

In all these case, the process for $\sigma(c_t - c_t^) - q_t$ is independent of the exchange rate regime, and in particular $\text{var}(\sigma(\Delta c_t - \Delta c_t^*) - \Delta q_t)$ remains unchanged as the economy switches between a peg and a float.*

The first three cases are clear from the discussion above, while the remaining two cases require an additional explanation. The case of $\beta \rightarrow 1$ is intuitive, as prices become flexible in the medium run,

and hence the slope of the Phillips curve is becoming inessential from the point of view of a perfectly patient economy. As $\rho \rightarrow 1$, the ψ_t shock starts to dominate the volatility of the real exchange rate...

Proposition 1 is clearly falsified by the data, as we illustrate in Figure 7, as well as in Figure 5c and Table 1. While Proposition 1 suggests that the relationship between relative consumption and real exchange rate should not be affected by monetary policy, the data suggests a dramatic change in the volatility of $\sigma(\Delta c_t - \Delta c_t^*) - \Delta q_t$, falsifying the models nested by the proposition. Such models are of course special limiting cases, however, quantitatively the prediction of Proposition 1 is accurate even outside these special cases — as we illustrate in Figure 5 and discuss further in the quantitative Section 5.

4 An Alternative Model of Non-neutrality

We now present an alternative explanation to the broad set of Mussa facts documented in Section 2. Specifically, we propose a model with monetary non-neutrality emerging due to financial market segmentation, rather than as a result of goods-market price stickiness. We build on the modeling environment of Section 3, but to emphasize the point assume away price stickiness (set $\lambda = 0$). The only other change to the environment is the modeling of the international financial market, as we describe next.

4.1 Segmented Financial Market

There are three types of agents participating in the financial market: households, noise traders and professional intermediaries. The home and foreign households trade local currency bonds only. In particular, the home households demand at time t a quantity B_{t+1} of the home-currency bonds. Similarly, foreign households demand a quantity B_{t+1}^* of the foreign-currency bonds. Both B_{t+1} and B_{t+1}^* can take positive or negative values, depending on whether the households save or borrow respectively. Therefore, in the notation of Section 3, $|J_t| = |J_t^*| = 1$ and $J_t \cap J_t^* = \emptyset$.

The trades of the households are intermediated by risk-averse intermediaries, or market makers. There are m symmetric intermediaries, who adopt a zero-capital *carry trade* strategy, that is take a long position in the foreign-currency bonds and a short position of equal value in the home-currency bonds, or vice versa. The return on this carry trade is therefore:

$$\tilde{R}_{t+1}^* = R_t^* - R_t \frac{\mathcal{E}_t}{\mathcal{E}_{t+1}} \quad (38)$$

per one dollar invested in the foreign-currency bond and \mathcal{E}_t euros sold of home-currency bonds at time t . We denote the size of their position by d_{t+1}^* , which may take positive or negative values depending on whether they are long or short in the foreign-currency bond, and assume that intermediaries maximize the CARA utility of the real return on their investment in units of the foreign consumption good:

$$\max_{d_{t+1}^*} \mathbb{E}_t \left\{ -\frac{1}{\omega} \exp \left(-\omega \frac{\tilde{R}_{t+1}^*}{P_{t+1}^*} d_{t+1}^* \right) \right\}, \quad (39)$$

where $\omega \geq 0$ is the risk aversion parameter.¹⁷ In aggregate, all m intermediaries invest $\frac{D_{t+1}^*}{R_t^*} = m d_{t+1}^*$ dollars in foreign-currency bonds, as one dollar at time t affords a quantity R_t^* of dollar bonds D_{t+1}^* , each paying out one dollar at $t+1$. The intermediaries also take an offsetting position of $\frac{D_{t+1}}{R_t} = -\mathcal{E}_t \frac{D_{t+1}^*}{R_t^*}$ euros in home-currency bonds, resulting in a zero-value portfolio at time t .

Finally, there are n symmetric noise (or liquidity) traders, who have an exogenously-evolving demand for the foreign currency.¹⁸ Like intermediaries, noise traders take a zero-capital position long in the foreign currency and short equal value in the home currency, or vice versa if they have an excess demand for the home currency. The overall position of the noise traders is

$$\frac{N_{t+1}^*}{R_t^*} = n \left(e^{\psi_t} - 1 \right) \quad (40)$$

in foreign-currency bonds and respectively $\frac{N_{t+1}}{R_t} = -\mathcal{E}_t \frac{N_{t+1}^*}{R_t^*}$ in home-currency bonds. We refer to the noise trader shock ψ_t as the *financial shock*, and assume it follows an AR(1) process:

$$\psi_t = \rho \psi_{t-1} + \sigma_\psi \varepsilon_t, \quad \varepsilon_t \sim iid(0, 1), \quad (41)$$

where $\rho \in [0, 1]$ is the persistence of the financial shock and $\sigma_\psi \geq 0$ is its volatility. The incomes (and losses) of both intermediaries and noise traders are returned in the end of each trading period to the foreign households as lump sum payments together with the dividends of the foreign firms, Π_{t+1}^* .¹⁹

Both currency bonds are in zero net supply, and therefore financial market clearing requires that the positions of the households, intermediaries and the noise traders balance out:

$$B_{t+1} + N_{t+1} + D_{t+1} = 0 \quad \text{and} \quad B_{t+1}^* + N_{t+1}^* + D_{t+1}^* = 0. \quad (42)$$

As both noise traders and intermediaries hold zero-capital positions, financial market clearing (42) implies a balanced position for the home and foreign households combined, $\frac{B_{t+1}}{R_t} + \mathcal{E}_t \frac{B_{t+1}^*}{R_t^*} = 0$. In other words, the financial market merely intermediates the intertemporal borrowing between home and foreign households.

In equilibrium, the intermediaries absorb the demand for home and foreign currency of both households and noise traders. If intermediaries were risk neutral, $\omega = 0$, they would be happy to do so without risk premia, resulting in the *uncovered interest parity* (UIP), or equivalently a zero expected return on the carry trade, $\mathbb{E}_t \frac{\tilde{R}_{t+1}^*}{P_{t+1}^*} = 0$. However, under risk aversion, $\omega > 0$, the intermediaries are not willing

¹⁷A property of the portfolio choice under CARA utility is that the solution does not depend on the level of wealth of the intermediaries, thus avoiding the need to carry it as an additional state variable, which would be the case under CRRA utility.

¹⁸The noise traders demand a certain position in home and foreign currency independently of the expected return on this portfolio, $\mathbb{E}_t \tilde{R}_{t+1}^*$, and of the other macroeconomic fundamentals reflected in the state variables of the home and foreign economies. Their demand for currency can be motivated as a liquidity demand, or alternatively as emerging from biased expectations about the exchange rate, $\mathbb{E}_t^n \mathcal{E}_{t+1} \neq \mathbb{E}_t \mathcal{E}_{t+1}$, as in [Jeanne and Rose \(2002\)](#).

¹⁹This generates an additional income of $\frac{D_{t+1}^* + N_{t+1}^*}{R_t^*} \tilde{R}_{t+1}^*$ dollars for the foreign households. As a result of this transfer, the foreign country budget constraint becomes the same as the home country budget constraint (20), despite that foreign households face a generally different rate of return $R_t^* \neq R_t$. See Appendix ?? for details, where we also show that this transfer is second order and hence does not affect the first order dynamics of the equilibrium system.

to take a risky carry trade position without an appropriate compensation, resulting in equilibrium risk premia and deviations from the UIP. We characterize the equilibrium in the financial market in:

Lemma 2 *The equilibrium condition in the financial market, log-linearized around a symmetric steady state with $\bar{B} = \bar{B}^* = 0$, $\bar{R} = \bar{R}^* = 1/\beta$ and assuming that $\omega\sigma_e^2$ is asymptotically finite and non-zero, is:*

$$i_t - i_t^* - \mathbb{E}_t \Delta e_{t+1} = \chi_1 \psi_t - \chi_2 b_{t+1}, \quad (43)$$

where $i_t \equiv \log R_t$, $i_t^* \equiv \log R_t^*$, $b_{t+1} \equiv \frac{1}{P_{HY}} B_{t+1} = -\frac{\bar{\varepsilon}}{P_{HY}} B_{t+1}^*$, the coefficients $\chi_1 \equiv \frac{n}{m} \omega \sigma_e^2$ and $\chi_2 \equiv \frac{\bar{P}_{HY}}{m} \beta \omega \sigma_e^2$, and $\sigma_e^2 \equiv \text{var}_t(\Delta e_{t+1})$ is the conditional volatility of the nominal exchange rate.

The equilibrium condition (3) is the *modified UIP* in our model with imperfect financial intermediation, where the right hand side corresponds to the departures from the UIP. Condition (3) arises from the combination of the financial market clearing (42) with the solution to the portfolio choice problem of the intermediaries (39), as we formally show in Appendix A.3.4. Intuitively, the optimal portfolio of the intermediaries d_{t+1}^* is proportional to the expected log return on the carry trade, $i_t - i_t^* - \mathbb{E}_t \Delta e_{t+1}$, scaled by the risk absorption capacity of the intermediaries, $\omega\sigma_e^2$, i.e. the product of their risk aversion and the volatility of the carry trade return (namely, the exchange rate risk). As $\omega\sigma_e^2 \rightarrow 0$, the risk absorption capacity of the intermediaries increases, and the UIP deviations disappear in the limit as $\chi_1, \chi_2 \rightarrow 0$. With $\omega\sigma_e^2 > 0$, the UIP deviations remain first order and hence affect the first-order equilibrium dynamics. The noise-trader shocks ψ_t create exogenous UIP deviations, while all other shocks result in endogenous UIP deviations by means of their effect on the external imbalances b_{t+1} , which need to be intermediated by the financial sector. Note that both $\psi_t > 0$ and $b_{t+1} < 0$ correspond to the excess demand for the foreign-currency bond – by the noise traders and households, respectively – and hence result in a negative expected return on the foreign currency bond.²⁰

4.2 Mussa puzzle: limiting resolution

As in the baseline model from Section 3, dynamics of the real exchange rate is determined by three equilibrium conditions: the country's budget constraint (29), the modified risk-sharing condition (43), and the goods market clearing condition (33) in the limit of flexible prices $\kappa \rightarrow \infty$. The latter can be used together with the household Euler equations for local bonds to substitute out interest rates from the modified UIP condition:

$$c_t - c_t^* = \kappa_a \tilde{a}_t - \gamma \kappa_q q_t, \quad (44)$$

$$r_t - r_t^* = \sigma \kappa_a \mathbb{E}_t \Delta \tilde{a}_{t+1} - \gamma \kappa_q \mathbb{E}_t \Delta q_{t+1}, \quad (45)$$

where $r_t = i_t - \mathbb{E}_t \pi_{t+1} = \sigma \mathbb{E}_t \Delta c_{t+1}$ is the real interest rate. The general solution for the real exchange rate is then given by (see the proof in Appendix A.3.5):

²⁰Imperfect risk absorption capacity of the intermediaries results in the expected deviations from the UIP and thus expected profits in the financial market, which are returned lump sum to the foreign households. While the resulting UIP wedge is first order, the expected profits from the carry trade are second order (as the optimal portfolio is proportional to the expected UIP deviation), and hence it is negligible from the point of view of the linearized budget constraint of the foreign country.

Lemma 3 *The equilibrium always exists under both floating and peg regimes and the real exchange rate follows ARMA(2,1) process:*

$$(1 - \delta L)q_t = \frac{1}{1 + \gamma\sigma\kappa_q} \frac{\beta\delta}{1 - \beta\rho\delta} \left[(1 - \beta^{-1}L)\chi_1\psi_t \right. \quad (46)$$

$$\left. + \left(\frac{1}{\beta\delta} \frac{1 - \beta\delta}{1 + \frac{\chi}{1 + \gamma\sigma\kappa_q}} (1 - \rho\delta L) + (1 - \rho)(1 - \beta^{-1}L) \right) \sigma\kappa_a \tilde{a}_t \right],$$

where (i) $\delta \in (0, 1)$ under float with $\text{var}_t(\Delta e_{t+1}) > 0$, and (ii) $\delta = 1$ under peg with $\text{var}_t(\Delta e_{t+1}) = 0$.

In line with the empirical evidence, the real exchange rate exhibits near-random-walk properties under the float when $\beta\rho \approx 1$: the autocorrelation of changes in exchange rate, $\text{corr}(\Delta e_t, \Delta e_{t-1})$, is arbitrary close to zero and the contribution of the predictable component $\mathbb{E}_t \Delta e_{t+j}$ to the variance of Δe_{t+j} is negligible. Intuitively, persistent shocks with $\rho \approx 1$ generate prolonged depreciation or appreciation in the future, which under incomplete asset markets, have to be compensated by a jump in exchange rate on impact to ensure that the country's budget constraint holds. These unexpected initial jumps dominate in exchange rate dynamics in the limit $\beta\rho \rightarrow 1$ (see [Itskhoki and Mukhin 2017](#)).

While the ARMA process (46) nests both monetary regimes, there is an important difference between the two. Under the peg, the monetary policy fully eliminates exchange rate risk associated with the carry trade, allowing arbitrageurs to take large positions that fully offset the demand of the noise traders, i.e. $\chi_1 = 0$ and shocks ψ_t have no effect on the real exchange rate. In contrast, under floating regime, $\chi_1 > 0$, arbitrageurs are reluctant to take large gross positions and noise-trader shocks ψ_t result in UIP deviations. Thus, given that shocks ψ_t matter under the float, but not under the peg, the model can generate arbitrary large differences in the volatility of the real exchange rate across two regimes. In addition, because the nominal exchange rate is by assumption fully stable under the peg and mirrors dynamics of the real exchange rate under the float, the model also predicts a discontinuous change in the volatility of the *nominal* exchange rate. Notice, however, that the real exchange rate responds to productivity shocks under both regimes, consistent with the non-negligible volatility of q_t (relative to that of e_t) under the peg (see [Figure A1](#)). In the limiting case of $\rho \approx 1$, the volatility of the real exchange rate that is due to productivity shocks is independent from monetary policy and we obtain the following result:

Proposition 2 *A change in the monetary policy rule from peg to float results in a sharp increase in the volatility of both nominal and real exchange rates — arbitrary large when $\rho \approx 1$ — with the change in the behavior of the other real macro variables vanishingly small when $\gamma \approx 0$.*

The first part of the proposition rationalizes the classical Mussa fact about the discontinuity in the volatility of exchange rates across float and peg regimes, while the second part shows that the model is also consistent with the additional facts about no change in the volatility of other macro aggregates. The intuition for the latter can be clearly seen from the market clearing condition (44), which characterizes the equilibrium relationship between relative consumption, real exchange rate and relative productivity in the two countries. With flexible prices, the only channel through which

monetary policy affects relative consumption is through the volatility of the real exchange rate. This effect is, however, proportional to the openness of the economy – in the autarky limit with $\gamma \approx 0$ movements in relative prices across countries are irrelevant for allocation as domestic consumption are determined solely by local productivity shocks. The same logic applies to output and all other real outcomes. As a result, large changes in the volatility of the real exchange rate are consistent with only minor changes in the volatility of the real macro variables, if the equilibrium behavior of the latter is mostly driven by the productivity shocks, even in the presence of substantial exchange rate volatility. In particular, this is the case when countries are sufficiently home biased in the goods market.

The case of the nominal macro variables is somewhat more involved. Consider the consumer price inflation, π_t . The volatility of prices changes from zero under the float to a positive number which is proportional to the volatility of the productivity shock. Under the peg, when $\rho \rightarrow 1$, the volatility of inflation (and hence the real exchange rate, since $\Delta q_t = -\pi_t$) is given by:

$$\text{std}(\pi_t) = \text{std}(\Delta q_t) = \frac{\sigma \kappa_a}{1 + \gamma \sigma \kappa_q + \chi} \text{std}(\Delta \tilde{a}_t).$$

Therefore, $\text{std}(\pi_t)$ is arbitrary smaller than $\text{std}(\Delta q_t)$ under the float, and it is equal to $\text{std}(\Delta q_t)$ under the peg, consistent with empirical patterns in Figure 3. The next section relaxes the extreme assumption of full price/nominal exchange rate stabilization under the two regimes, and shows that under conventional Taylor rule the volatility of inflation changes little across the two regimes.

4.3 Additional evidence

So far, our analysis has focused mostly on the volatilities of exchange rates and macro variables across the two monetary regimes. This choice of moments is driven mostly by robust discontinuities, or the lack thereof, around the end of the Bretton Woods in the data. As Table 2 makes clear, the patterns are less obvious for other second moments, namely the correlations. Nevertheless, the change in the empirical correlations is an important overidentification test of the theoretical mechanism and is consistent with the predictions of the model with segmented financial market.

In particular, the key prediction of our model is that financial shocks are central to the exchange rate dynamics under the floating regime, and become significantly less important under the peg. It follows that the main drivers of the real exchange rate under the peg are ‘fundamental’ shocks, such as productivity shocks. Given a conventional transmission of these shocks, the model predicts that most exchange rate puzzles that emerge under a floating regime should disappear under a peg. This is true in particular for the forward premium puzzle (Fama 1984), the Backus-Smith puzzle (Backus and Smith 1993, Kollmann 1995), and the Balassa-Samuelson effect (Balassa 1964, Samuelson 1964).²¹

Proposition 3 *A change in the monetary policy rule from peg to float results in the emergence of (a) the forward premium puzzle, (b) the Backus-Smith puzzle, and (c) a weaker Balassa-Samuelson effect.*

²¹Note that the Meese and Rogoff (1983) puzzle trivially disappears under the peg given that nominal exchange rate becomes perfectly predictable. The case of the PPP puzzle – the surprising combination of high volatility and high persistence of the real exchange rate (Rogoff 1996) – is more involved: in line with the empirical evidence, while the peg decreases the volatility of the real exchange rate, it does not necessarily change its persistence (see Lemma 3).

Consider first the forward premium puzzle. Clearly, this anomaly cannot emerge when risk premium is zero and therefore, one would expect smaller deviations from the UIP under the peg. The empirical evidence is largely consistent with this prediction of the model: using historical data for the U.K and the U.S., [Colacito and Croce \(2013\)](#) show that the estimated UIP coefficient was close to one during most of the Bretton-Woods periods and became negative afterwards. Note also that the model is consistent with the CIP holding well in the data both before and after the end of the Bretton Woods, as documented in [Frenkel and Levich \(1975\)](#).

Similarly, the model predicts that the Backus-Smith condition should hold – at least conditionally in expected terms – in the absence of risk-premium shocks in the financial markets. Table 2 shows that the correlation between the real exchange rate and the relative consumption is indeed higher under the peg than under the float for every single country in our sample, and flips the sign to negative under the float in all cases but one. This is also consistent with the findings of [Colacito and Croce \(2013\)](#) based on longer historical series for the U.S. and the U.K. In addition, [Devereux and Hnatkovska \(2014\)](#) provide empirical evidence using the alternative quasi-experiment, namely the formation of the Eurozone. In particular, they show that the Backus-Smith risk-sharing condition holds much better for members of the currency union than for both the same countries before the formation of the Eurozone and between the countries with different currencies.

Finally, in a straightforward extension of the baseline model with two sectors, the real exchange rates should appreciate according to the Balassa-Samuelson effect when country’s productivity in the tradable sector goes up relative to the productivity in the non-tradable sector. While true under either monetary regime, this correlation is harder to identify under the floating regime because of the relatively small overall contribution of productivity shocks in the exchange rate dynamics. Again, we find empirical evidence in line with the model’s predictions: while the Balassa-Samuelson effect has almost no explanatory power under the float ([Engel 1999](#)), the recent literature has shown that the effect holds surprisingly well for the Eurozone countries with a fixed exchange rate ([Berka, Devereux, and Engel 2012, 2018](#)).

5 Quantitative Exploration

This section shows that both negative results from Propositions 1 and positive results from Proposition 2 and 3 remain robust in a quantitative version of the model. We compare three classes of models – without UIP shocks, with exogenous UIP shocks, and finally with UIP shocks endogenous to the monetary policy regime, consistent with the model of the segmented financial market in Section 4. We show that *only* the latter class of models is consistent with the umbrella of Mussa facts documented in Section 2, while nominal rigidities are neither necessary, nor sufficient to explain the puzzle.

The quantitative version of the model is from [Itskhoki and Mukhin \(2019\)](#) and is described in Appendix. In particular, we augment the baseline model from Section 2 with capital investment with adjustment costs, pricing-to-market, intermediate goods, Calvo sticky wages and LCP sticky prices, and a conventional Taylor rule with a weight on the nominal exchange rate. The regime change corresponds to the change in this weight that the monetary authority puts on the nominal exchange rate.

5.1 Calibration

For most parameters we use standard values from the literature (see Appendix Tables A1–A2). In particular, we calibrate the openness of the economy to $\gamma = 0.035$ to match the average import-to-GDP ratio of 0.07 for the U.S. for the period from 1960–1990. Following a common practice in the literature (see Schmitt-Grohé and Uribe 2003), and consistent with our own model of the financial sector, we make sure that the model is stationary by assuming a coefficient $\chi_2 = 0.001$ in front of b_{t+1} in the UIP condition (43) under a floating regime, allowing it to change with the monetary policy in the model with endogenous risk premium. We consider three versions of each model: a flexible-price model with productivity shocks (IRBC), a sticky-price model with productivity shocks (NKOE-1), and a sticky-price model with monetary shocks (NKOE-2). In all specifications, we also allow for the financial shock ψ_t and taste shocks ξ_t between foreign and home goods. The baseline specifications with sticky prices assume local currency pricing (LCP), but results are also similar under the dominant currency pricing (DCP).

In all specifications of the model, the correlation of productivity (monetary) shocks across countries is calibrated to match $\text{corr}(\Delta gdp_t, \Delta gdp_t^*) = 0.3$ under the floating regime (see Table A2). We target the volatility of the nominal exchange rate $\text{std}(\Delta e_t)$ under the float to calibrate the volatility of the shocks. The relative volatility of productivity (monetary) and trade shocks is calibrated to match respectively $\text{corr}(\Delta q_t, \Delta c_t - \Delta c_t^*) = -0.4$ under floating regime and $\text{corr}(\Delta q_t, \Delta n x_t) = -0.1$ under the peg.²² The costs of capital adjustment are calibrated to match $\frac{\text{std}(\Delta z_t)}{\text{std}(\Delta gdp_t)} = 2.5$ under the floating regime, where z_t is real investment expenditure. The weight of NER in the Taylor rule of Foreign country is always zero, while the weight for Home country is zero under the floating regime and is calibrated under the peg to match an eight-fold decrease in $\text{std}(\Delta e_t)$. We keep all other coefficients constant across regimes. In the model with the endogenous UIP shocks, we change the coefficients χ_1 and χ_2 in the UIP condition (43) in proportion with the change in σ_e^2 across the two monetary regimes.²³

5.2 Results

Table 3 reports the volatilities of exchange rates and macro variables under the two monetary regimes for the alternative versions of the model. Consider first the three models (IRBC, NKOE-1 and 2) without the UIP shock ψ_t . Calibrated to match the volatility of nominal exchange rates, all three specifications also feature a high volatility in the real exchange rate. This, however, comes at a cost: in the absence of financial shocks, the models require large productivity or monetary shocks to generate volatile exchange rates, which in turn results in counterfactually volatile macro variables. Indeed, the volatility of consumption and GDP is 5-10 times higher than in the data and is of the same order of magnitude as of the exchange rates. Thus, consistent with the findings in Itskhoki and Mukhin (2019), the models without financial shocks are inconsistent with the exchange rate disconnect under the floating regime.

²²In the version of the model with exogenous risk-premium shocks, we use the same calibration as the one with endogenous financial shocks.

²³All moments are calculated by simulating the model for $T = 100,000$ quarters – all standard errors are below 0.01 in this case. The results remain almost the same if one uses medians from several simulations of $T = 120$ quarters except for the autocorrelations and the UIP coefficient, which are severely biased downwards in small samples.

Table 3: Quantitative models

Panel A: standard deviations															
	Δe_t			Δq_t			π_t			Δc_t			Δgdp_t		
	peg	float	ratio	peg	float	ratio	peg	float	ratio	peg	float	ratio	peg	float	ratio
Models w/o UIP shock:															
IRBC	1.5	12.0	8.0	15.4	15.4	1.0	12.7	3.2	0.2	9.1	9.1	1.0	15.0	15.0	1.0
NKOE-1	1.5	12.0	8.0	4.2	12.8	3.0	3.1	1.8	0.6	7.1	6.8	1.0	17.7	11.7	0.7
NKOE-2	1.5	12.0	8.0	1.5	11.5	7.4	1.3	1.3	1.0	5.0	5.2	1.0	8.1	8.4	1.0
Models w/ exogenous UIP shock:															
IRBC	1.5	12.0	8.0	11.0	11.0	1.0	10.2	0.9	0.1	1.8	1.8	1.0	2.5	2.5	1.0
NKOE-1	1.5	12.0	8.0	2.2	11.9	5.3	1.4	0.4	0.3	5.8	1.3	0.2	14.5	2.1	0.1
NKOE-2	1.5	12.0	8.0	2.1	11.8	5.7	1.3	0.3	0.2	5.8	1.1	0.2	8.6	1.8	0.2
Models w/ endogenous UIP shock:															
IRBC	1.5	12.0	8.0	3.0	11.0	3.6	1.4	0.9	0.7	1.6	1.8	1.1	2.5	2.5	1.0
NKOE-1	1.5	12.0	8.0	1.7	11.9	6.9	0.4	0.4	1.0	1.1	1.3	1.1	1.9	2.1	1.1
NKOE-2	1.5	12.0	8.0	1.4	11.8	8.2	0.2	0.3	1.5	0.9	1.1	1.2	1.5	1.8	1.2

Panel B: correlations															
	$\Delta q_t, \Delta e_t$		$\Delta q_t, \Delta c_t - \Delta c_t^*$		$\Delta q_t, \Delta nx_t$		$\Delta gdp_t, \Delta gdp_t^*$		$\Delta c_t, \Delta c_t^*$		$\Delta c_t, \Delta gdp_t$		β^{UIP}		
	peg	float	peg	float	peg	float	peg	float	peg	float	peg	float	peg	float	
Models w/o UIP shock:															
IRBC	0.86	0.99	0.91	0.91	-0.10	-0.10	0.30	0.30	0.34	0.34	0.99	0.99	0.8	0.9	
NKOE-1	0.67	0.99	0.28	0.70	-0.10	-0.49	0.38	0.31	0.65	0.41	0.91	0.97	0.3	1.0	
NKOE-2	0.96	0.99	0.49	0.99	-0.10	0.05	0.95	0.30	0.97	0.33	1.00	1.00	1.0	1.0	
Models w/ exogenous UIP shock:															
IRBC	0.86	0.99	-0.40	-0.40	0.93	0.93	0.30	0.30	0.15	0.15	0.88	0.88	0.0	-1.3	
NKOE-1	0.81	1.00	-0.88	-0.40	0.89	0.93	0.60	0.30	-0.06	0.32	0.99	0.84	-0.1	-1.6	
NKOE-2	0.82	1.00	-0.89	-0.40	0.92	0.97	0.51	0.30	-0.10	0.26	1.00	0.79	-0.1	-2.2	
Models w/ endogenous UIP shock:															
IRBC	0.98	1.00	0.92	-0.40	-0.10	0.93	0.30	0.30	0.39	0.16	0.99	0.88	1.0	-1.4	
NKOE-1	0.98	1.00	0.84	-0.40	-0.10	0.93	0.44	0.30	0.54	0.32	0.96	0.84	1.0	-1.6	
NKOE-2	1.00	1.00	0.94	-0.40	-0.10	0.97	0.66	0.30	0.70	0.26	0.99	0.79	1.0	-2.3	

Note: see the text and notes to Appendix Table A2.

The next three specifications of the model allow for the exogenous UIP shocks with the same volatility under the two monetary regimes. All three specifications are now much more successful in addressing the disconnect puzzle – under the floating regime, the volatility of exchange rates is an order of magnitude higher than the volatility of consumption, GDP and inflation. At the same time, the models struggle to match the Mussa facts. Expectedly, the flexible-price IRBC model produces no change in the behaviour of real variables across monetary regimes and is inconsistent with the original Mussa puzzle about structural break in the dynamics of the real exchange rate in 1973. The specifications with nominal rigidities, on the other hand, perform much better in terms of the exchange rate moments – the sluggish adjustment of prices ensures that the real exchange rate follows closely the nominal one and is about 5 times more volatile under the float relative to the peg. The sticky-price models, however, have counterfactual predictions for other macro variables. In contrast to almost no differences in the volatilities of inflation, consumption and GDP across two monetary regimes in the data, the models feature a 5-10 times increase in standard deviations of macro variables. This generalizes

Table 4: Variance decomposition

	peg			float		
	ψ	ξ	a or m	ψ	ξ	a or m
Real exchange rate, $\text{var}(\Delta q_t)$						
IRBC	1	23	76	92	3	5
NKOE-1	1	22	77	97	1	2
NKOE-2	1	4	95	97	1	2
Consumption, $\text{var}(\Delta c_t)$						
IRBC	0	1	99	15	1	84
NKOE-1	0	1	99	10	0	90
NKOE-2	0	1	99	13	0	87

Note: variance decomposition of RER and consumption in model specifications with the endogenous UIP shock.

the insight from Proposition 1 by showing that models with exogenous UIP shocks are inconsistent with the broader set of Mussa puzzles not only in the special limiting cases, but also in a realistically calibrated quantitative models.

Finally, the last three rows of Table 3 allow for an endogenous UIP shock, as in the model of the segmented financial market in Section 4. Under the float, the model specifications are naturally isomorphic to the ones with the exogenous UIP shocks, and produce a high volatility in exchange rates relative to the other macro variables, consistent with exchange rate disconnect. In contrast to the previous three specifications, however, the models also match well the data under the peg – the volatility of the real exchange rate is 4-to-8 times lower than under the float and the volatility of other macro variables changes only modestly by about 10%, as is the case in the data. Importantly, the results are similar across specifications and do not depend strongly on the type of shocks (productivity vs monetary) or the presence of nominal rigidities. In this sense, nominal rigidities are neither necessary, nor sufficient to match the Mussa puzzle. On the margin, however, sticky prices do slightly improve the fit of the model.

To see the intuition behind these results, consider the variance decomposition in the last set of model specifications with endogenous UIP shocks. From Table 4, more than 90% of real exchange rate volatility is driven by financial shocks. A switch in monetary policy to the peg eliminates most of the carry trade risk and almost fully eliminates financial shocks, which results in a drastic fall in the volatility of the real exchange rate. At the same time, the dynamics of other variables does not change much for two reasons. On the one hand, because of low openness of the economies, financial shocks account only for a modest share of volatility in macro variables even under the float (namely, 10–15% of consumption volatility). As a result, the decreasing importance of the UIP shocks under the peg has only minor implications for macro aggregates. Nonetheless, a change in the monetary policy *per se* could significantly change the behavior of inflation (under flexible prices) and real variables (under sticky prices). This does not happen, however, due to small changes in the *equilibrium* monetary policy. Indeed, with financial shocks gone, the policy does not need to change much to ensure a stable nominal

exchange rate. In other words, the government commitment to peg already goes a long way towards stabilizing exchange rates and does not require large interventions on the equilibrium path.

Lastly, the lower panel of Table 3 complements the analysis with several correlations. Consistent with Proposition 3 and the empirical evidence from Section 2, it shows that the UIP puzzle and the Backus-Smith puzzle are much more pronounced under the floating regime, while the moments are closer to the predictions of standard IRBC and NKOE models under the peg regime. Indeed, both anomalies are due to financial shocks and disappear when monetary policy eliminates risk premium in the international financial markets. At the same time, most other business cycle correlations – e.g. between consumption and output within countries and between home and foreign GDP across countries – do not change much, which is in line with empirical evidence in Section 2.

6 Conclusion

A Appendix

A.1 Additional Tables and Figures

Table A1: Calibrated parameters

β	discount factor	0.99
σ	inverse of the IES	2
γ	openness of economy	0.035
φ	inverse of Frisch elasticity	1
ϕ	intermediate share in production	0.5
ϑ	capital share	0.3
δ	capital depreciation rate	0.02
θ	elasticity of substitution between H and F goods	1.5
ϵ	elasticity of substitution between different types of labor	4
λ_w	Calvo parameter for wages	0.85
λ_p	Calvo parameter for prices	0.75
ρ	autocorrelation of shocks	0.97
ρ_r	Taylor rule: persistence of interest rates	0.95
ϕ_π	Taylor rule: reaction to inflation	2.15

Table A2: Estimated parameters

	σ_ψ	σ_ξ	σ_a	σ_m	ρ_{a,a^*}	κ	ϕ_e
Models w/o UIP shock:							
IRBC	0.00	13.8	8.1	–	0.28	11	13.0
NKOE-1	0.00	5.71	10.6	–	0.30	7	1.8
NKOE-2	0.00	4.38	–	0.77	0.30	22	5.3
Models w/ exogenous UIP shock:							
IRBC	0.61	3.37	1.41	–	0.30	15	14.5
NKOE-1	0.59	2.80	1.01	–	0.35	7.5	3.7
NKOE-2	0.59	1.23	–	0.15	0.42	20	3.6
Models w/ endogenous UIP shock:							
IRBC	0.61	3.37	1.41	–	0.30	15	0.25
NKOE-1	0.59	2.80	1.01	–	0.35	7.5	0.03
NKOE-2	0.59	1.23	–	0.15	0.42	20	0.08

Note: in all calibrations, shocks are normalized to obtain $\text{std}(\Delta e_t) = 12\%$. Parameter ϕ_e in the Taylor rule is calibrated to generate eightfold reduction in $\text{std}(\Delta e_t)$ from float to peg. Relative volatilities of shocks are calibrated to match $\text{corr}(\Delta q_t, \Delta c_t - \Delta c_t^*) = -0.4$ under the float and $\text{corr}(\Delta q_t, \Delta n x_t) = -0.1$ under the peg. The cross-country correlation of productivity/monetary shocks matches $\text{corr}(\Delta gdp_t, \Delta gdp_t^*) = 0.3$ under the float. Capital adjustment parameter ensures that $\frac{\text{std}(\Delta z_t)}{\text{std}(\Delta gdp_t)} = 2.5$ under the float. The moments are calculated by simulating the model for $T = 100,000$ quarters.

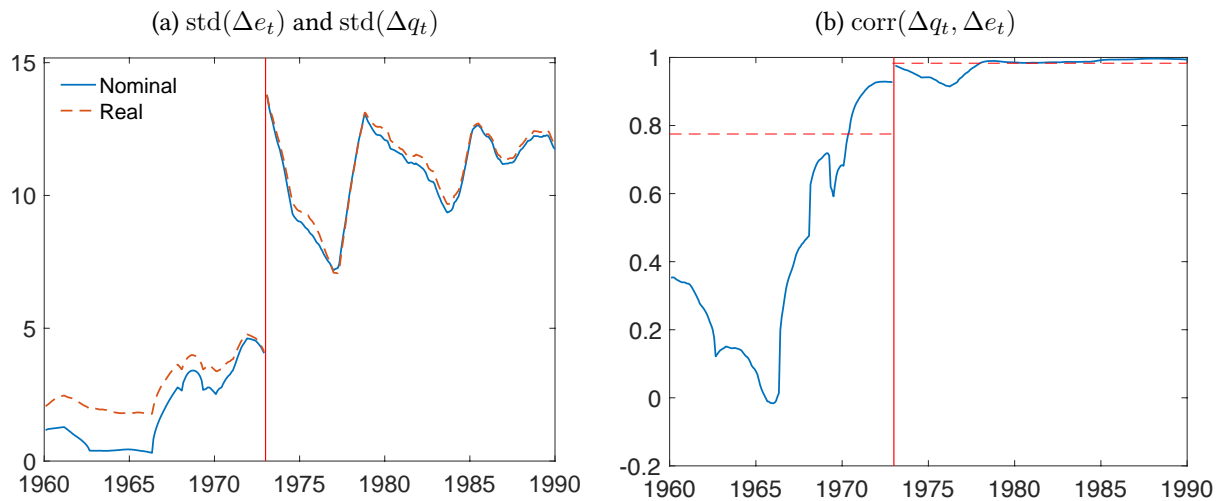


Figure A1: Volatility and correlation of nominal and real exchange rates over time

Note: The left panel plots standard deviations and the right panel plots correlation of Δe_t and Δq_t over time. Series from Figure 1; triangular moving averages with a window over 18 months before and after the date, treating 1973:01 as the end point for the two regimes (see Figure 3 and Appendix Figure A4 for other variables).

A.2 Data

Additional details for Section 2:

1. CPI data for Canada in 1960 is from OECD, but is downloaded from FRED and made consistent with the rest of the series.
2. **Outliers:**
 - (a) civil unrests in France in May–June 1968 led to a more than 20% fall in production; France also had abnormally volatile production index during the whole 1960s;
 - (b) earthquake and tsunami in Japan in March–April 2011 led to 17% fall in production. Since these observations are required for aggregation across non-U.S. countries, I replaced them with extrapolations using the values before and after the episodes.
 - (c) The same applies to Germany production index in 1984:06, which constitutes a clear measurement error, and Spain production index in 1960, which is missing.

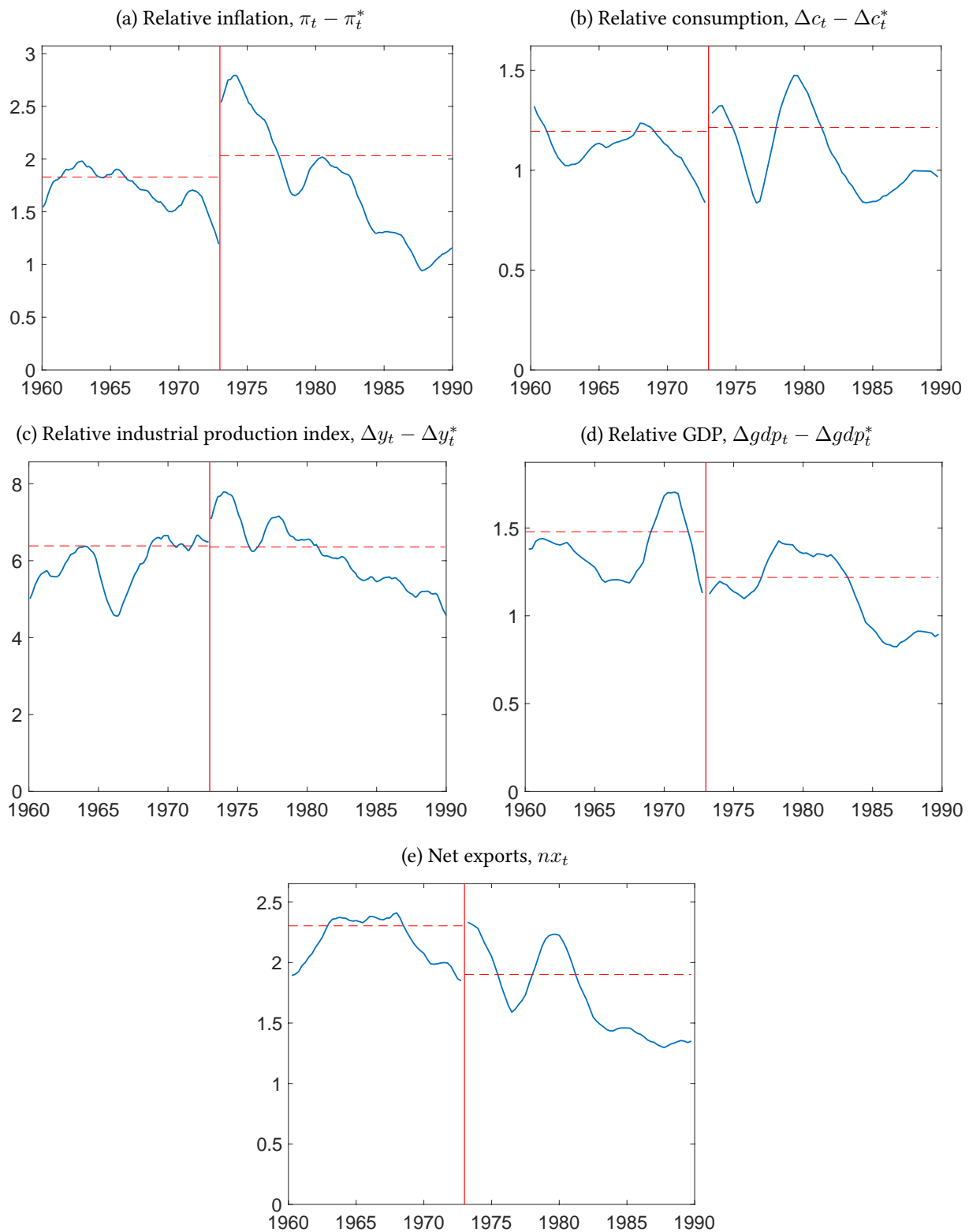


Figure A2: Volatility of macroeconomic variables over time

Note: The top four panels zoom in on the panels b, d, e and f in Figure 3. The bottom panel plots the standard deviations of nx_t defined as the ratio of export minus imports to the sum of exports and imports, for the US against the rest of the world.



Figure A3: Volatility over time: alternative break at 1971:08

Note: Like Appendix Figure A2, but with an alternative break date in 1971:08 (1971:Q3). **DIFFERENT DEFINITION**

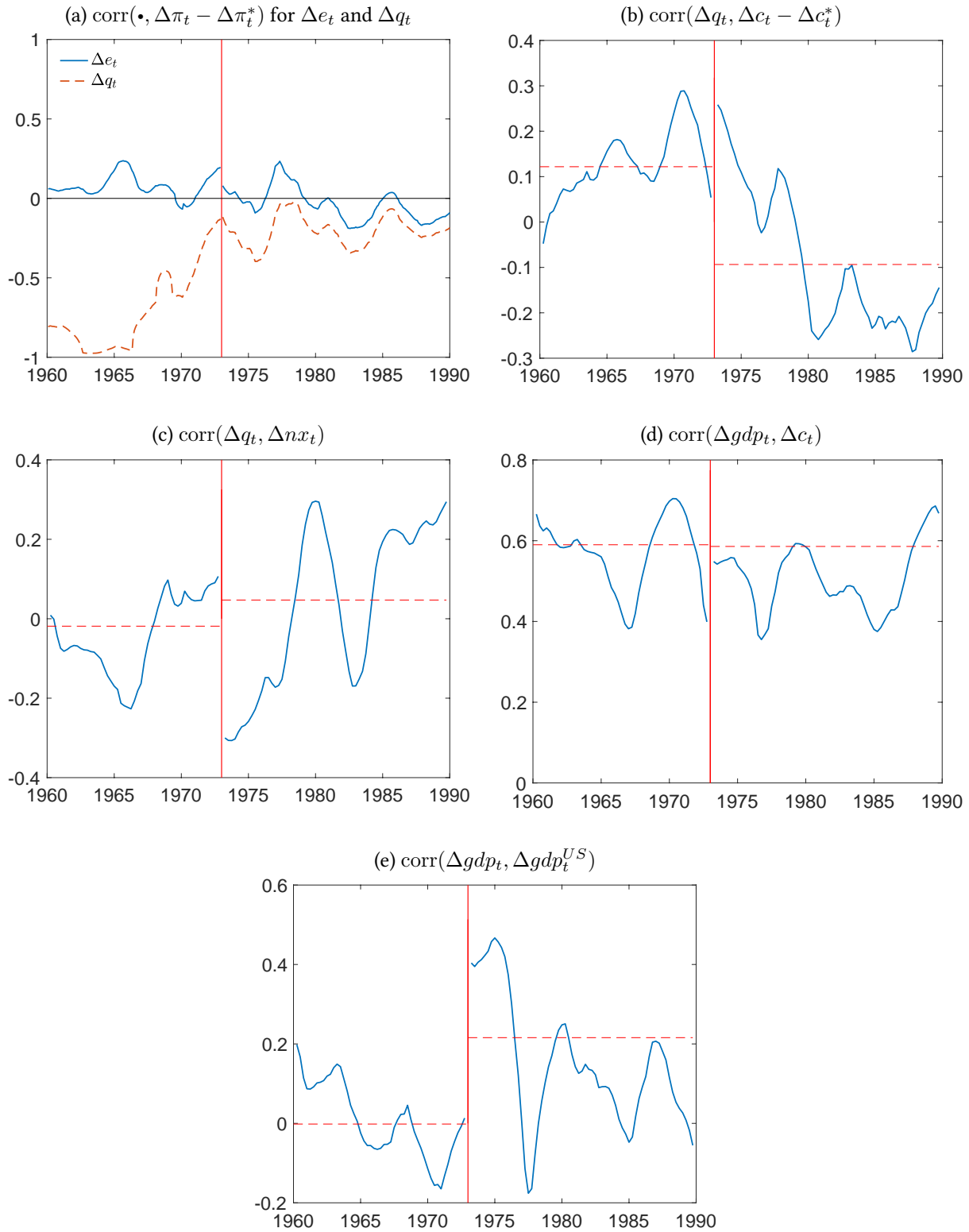
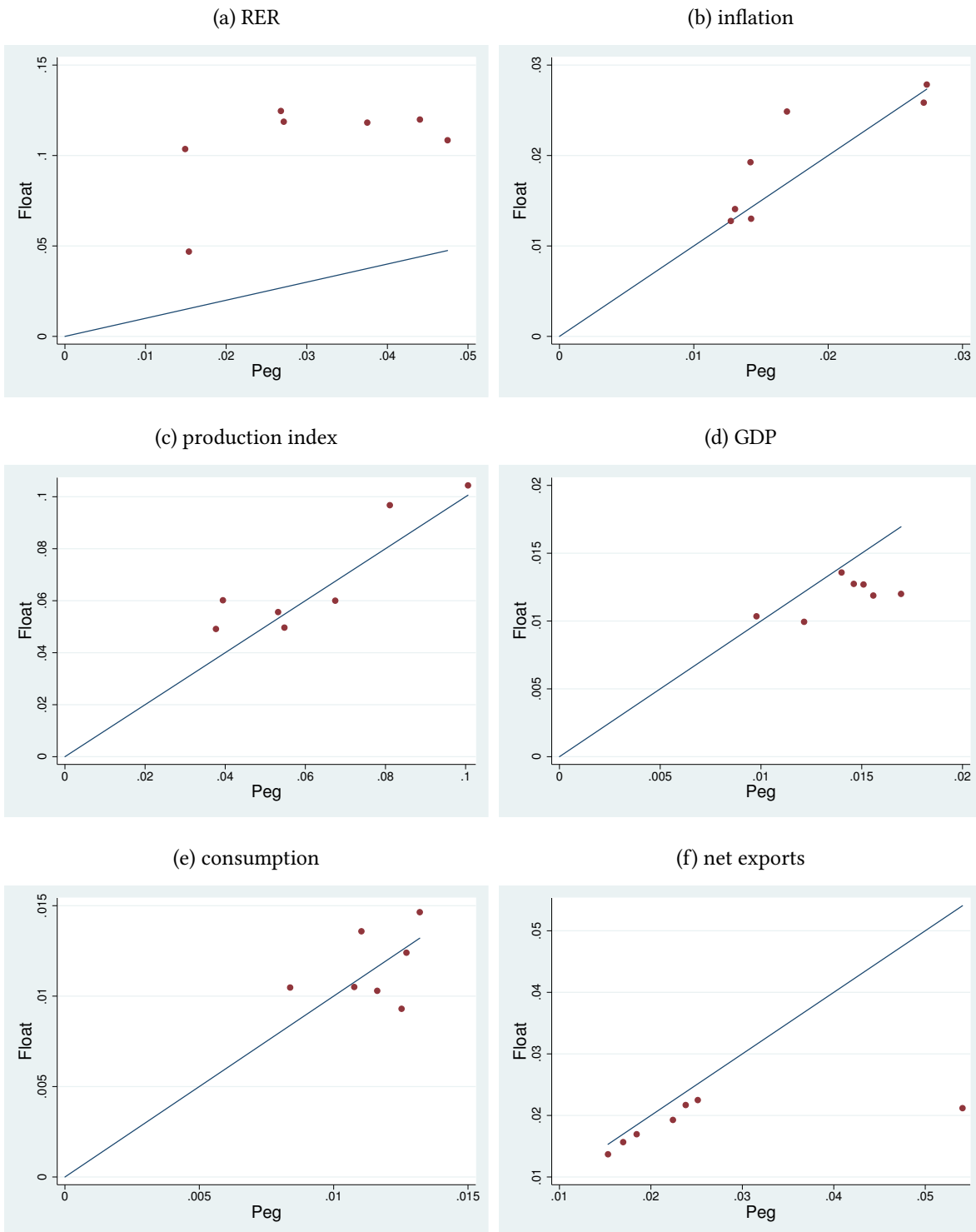


Figure A4: Correlations over time

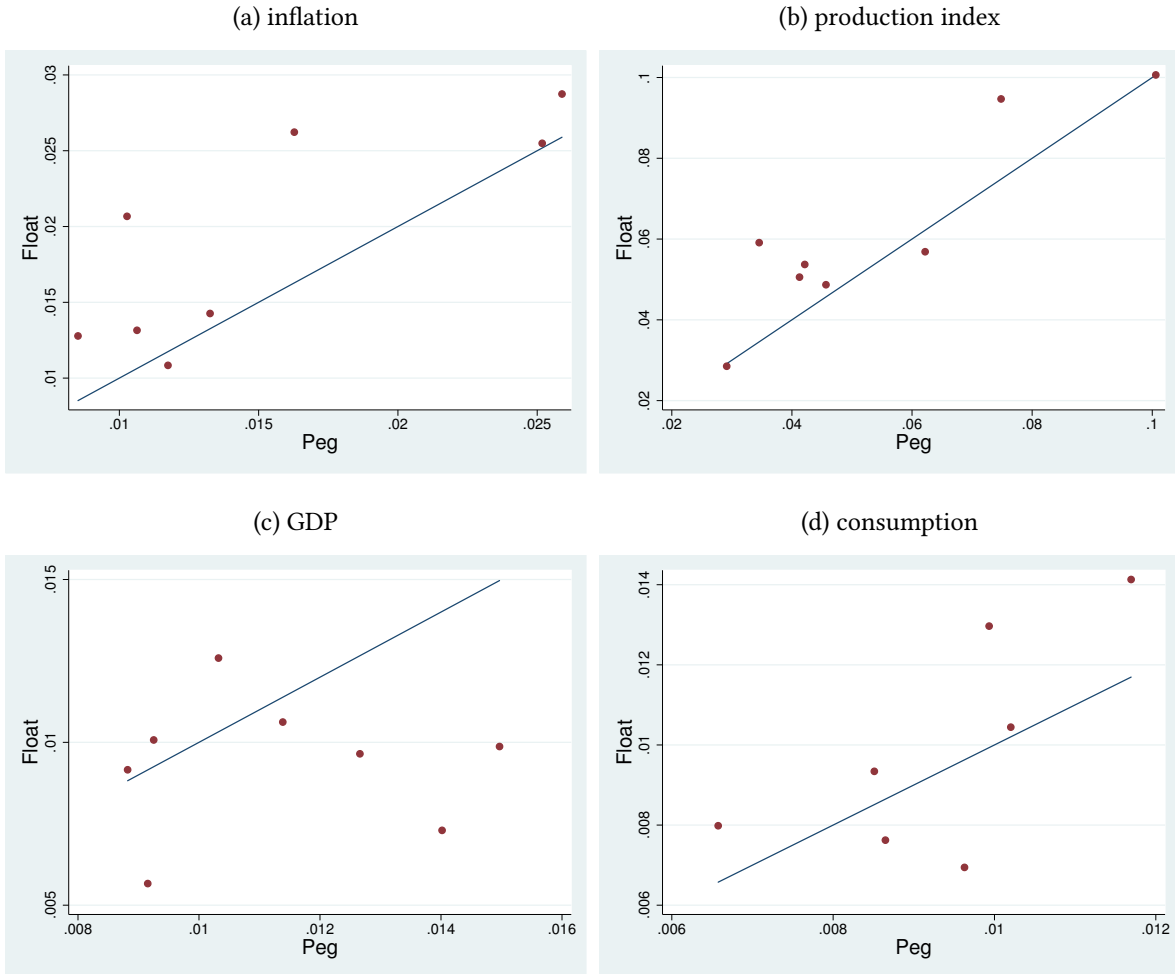
Note: the pictures shows the correlations for the RoW estimated separately before and after January 1973. The rolling window is up to 5 years for quarterly series – shorter at the corners – with linearly decreasing weights.

Figure A5: Scatter plots: volatility before and after the end of the Bretton-Woods System



Note: the plot shows standard deviations of different variables for 1960-71:07 vs. 1973-1989 across individual countries. Canada is the outlier in terms of float-RER volatility and Spain is the outlier in terms of peg-NX volatility.

Figure A6: Scatter plots: country-level instead of cross-differences



Note: the plot shows standard deviations of different variables for 1960-71:07 vs. 1973-1989 across individual countries. Canada is the outlier in terms of float-RER volatility and Spain is the outlier in terms of peg-NX volatility.

The scatter plots from Figures A5 show that the volatility of the RER is higher under the float than under the peg for every single country in our sample. Interestingly, the floating regime results in almost equal volatility of exchange rates across countries except for Canada, which retained partial peg to the dollar during 1970-80s. At the same time, the countries concentrate tightly along the 45-degree line for other macroeconomic variables indicating small changes in their volatilities across the regimes. (The only exception is Spain with an abnormally high volatility of net exports in 1960s.) Interestingly, there is more variation for country-level series instead of the cross-country differences (see Figure A6), but again we find no systematic differences for fundamentals between two regimes.

A.3 Derivations

A.3.1 Equilibrium system, steady state, log-linearization

A.3.2 Phillips curve

The log-linearized price setting equation (16) is given by:

$$\bar{p}_{Ht} = (1 - \beta\lambda) \sum_{k=0}^{\infty} (\beta\lambda)^k \mathbb{E}_t \{w_{t+k} - a_{t+k}\},$$

that is a discounted weighted average of future marginal costs (13) conditional on the price staying in effect. We rewrite this condition as:

$$\bar{p}_{Ht} - p_{t-1} = (1 - \beta\lambda) \sum_{k=0}^{\infty} (\beta\lambda)^k \mathbb{E}_t \{w_{t+k} - a_{t+k} - p_{t+k}\} + \sum_{k=0}^{\infty} (\beta\lambda)^k \mathbb{E}_t \pi_{t+k}$$

or equivalently

$$(1 - \beta\lambda L^{-1})(\bar{p}_{Ht} - p_{H,t-1}) = (1 - \beta\lambda)(w_t - a_t - p_{Ht}) + \pi_{Ht}. \quad (\text{A1})$$

Next, due to the Calvo pricing structure, the evolution of the home-good inflation is then given by:

$$\pi_{Ht} = (1 - \lambda)(\bar{p}_{Ht} - p_{H,t-1}), \quad (\text{A2})$$

and thus we can combine the previous two equation to obtain the Phillips curve for π_{Ht} :

$$\pi_{Ht} = \beta \mathbb{E}_t \pi_{H,t+1} + \hat{\kappa}(w_t - a_t - p_{Ht}), \quad \hat{\kappa} \equiv \frac{(1 - \lambda)(1 - \beta\lambda)}{\lambda}. \quad (\text{A3})$$

Using the price index and the law of one price (14), we have:

$$p_t = (1 - \gamma)p_{Ht} + \gamma(p_{Ft}^* + e_t),$$

or equivalently:

$$p_t - p_{Ht} = \gamma s_t = \frac{\gamma}{1 - 2\gamma} q_t.$$

Substituting this into (A3):

$$\pi_t - \frac{\gamma}{1 - 2\gamma} \Delta q_t = \beta \mathbb{E}_t \left\{ \pi_{t+1} - \frac{\gamma}{1 - 2\gamma} \Delta q_{t+1} \right\} + \hat{\kappa} \left(\sigma c_t + \varphi y_t - (1 + \varphi) a_t + \frac{\gamma}{1 - 2\gamma} q_t \right),$$

where we have used (6) and (12) to establish that:

$$w_t - p_t - a_t = \sigma c_t + \varphi \ell_t - a_t = \sigma c_t + \varphi y_t - (1 + \varphi) a_t.$$

Next, taking the difference between π_t and the equivalent Phillips curve for π_t^* , we arrive at:

$$z_t = \beta \mathbb{E}_t z_{t+1} + \hat{\kappa}(1 - 2\gamma)x_t,$$

where

$$z_t \equiv \frac{1}{1-2\gamma} \left[\pi_t - \pi_t^* - \frac{2\gamma}{1-2\gamma} \Delta q_t \right] = \pi_t - \pi_t^* - 2\gamma \Delta e_t,$$

$$x_t \equiv \sigma(c_t - c_t^*) + \varphi(y_t - y_t^*) - (1 + \varphi) \tilde{a}_t + \frac{2\gamma}{1-2\gamma} q_t.$$

The last step is to solve out $y_t - y_t^*$ using the market clearing condition:

$$y_t = (1 - \gamma) \left[-\theta(p_{Ht} - p_t) + c_t \right] + \gamma \left[-\theta(p_{Ht} - e_t - p_t^*) + c_t^* \right]$$

$$= (1 - \gamma)c_t + \gamma c_t^* + 2\theta \frac{\gamma(1 - \gamma)}{1 - 2\gamma} q_t,$$

so that

$$y_t - y_t^* = (1 - 2\gamma)(c_t - c_t^*) + 4\theta \frac{\gamma(1 - \gamma)}{1 - 2\gamma} q_t,$$

and we can rewrite:

$$x_t = \left[\sigma + (1 - 2\gamma)\varphi \right] \left[(c_t - c_t^*) + \frac{2\gamma}{1 - 2\gamma} \frac{2\theta(1 - \gamma)\varphi + 1}{\sigma + (1 - 2\gamma)\varphi} q_t - \frac{1 + \varphi}{\sigma + (1 - 2\gamma)\varphi} \tilde{a}_t \right].$$

Note that $x_t = 0$ is goods market clearing under flexible prices (Backus-Smith resolution in disconnect).

A.3.3 Cointegration

See footnote 16. The impulse response of $(1 - \delta_R L)q_t$ is given by:

$$IRF_k^{(1-\delta_R L)q_t} = \frac{k_R \delta_R}{1 + \gamma \sigma \kappa_q} \left[\frac{\rho^k}{1 - \beta \rho \delta_R} \left(\frac{\sigma_\psi \varepsilon_t}{1 - \rho} + \sigma \kappa_a \sigma_a \tilde{\varepsilon}_t^a \right) - \frac{u_t}{1 - \beta \delta_R} \right]$$

And therefore the impulse response of q_t is:

$$IRF_k^{q_t} = \sum_{j=0}^k \rho^{k-j} IRF_k^{(1-\delta_R L)q_t}$$

We calculate:

$$\sum_{k=0}^{\infty} \beta^k IRF_k^{q_t} = \frac{k_R \delta_R}{1 + \gamma \sigma \kappa_q} \sum_{k=0}^{\infty} \sum_{j=0}^k \beta^k \rho^{k-j} \left[\frac{\rho^k}{1 - \beta \rho \delta_R} \left(\frac{\sigma_\psi \varepsilon_t}{1 - \rho} + \sigma \kappa_a \sigma_a \tilde{\varepsilon}_t^a \right) - \frac{u_t}{1 - \beta \delta_R} \right]$$

$$= \frac{k_R \delta_R}{1 + \gamma \sigma \kappa_q} \left[\frac{1}{1 - \beta \rho \delta_R} \left(\frac{\sigma_\psi \varepsilon_t}{1 - \rho} + \sigma \kappa_a \sigma_a \tilde{\varepsilon}_t^a \right) \underbrace{\sum_{k=0}^{\infty} (\beta \rho)^k \frac{1 - \rho^{k+1}}{1 - \rho}}_{=\frac{1-\rho}{(1-\beta\rho)(1-\beta\rho^2)}} - \frac{u_t}{1 - \beta \delta_R} \underbrace{\sum_{k=0}^{\infty} \beta^k \frac{1 - \rho^{k+1}}{1 - \rho}}_{=\frac{1-\rho}{(1-\beta)(1-\beta\rho^2)}} \right]$$

Substituting this back into the first expression in footnote 16 results in the solution (37) in the text.

Limiting case $\rho \rightarrow 1$

$$\Delta m_t = u_t = \frac{\sigma_\psi \varepsilon_t}{1 - \rho},$$

$$\Delta m_t - \frac{\Delta \psi_t}{1 - \rho} = \frac{\sigma_\psi \varepsilon_t}{1 - \rho} - \frac{(\rho - 1)\psi_{t-1} + \sigma_\psi \varepsilon_t}{1 - \rho} = \psi_{t-1}$$

$$\sigma(\Delta c_t - \Delta c_t^*) - \Delta q_t = \psi_{t-1}$$

$$(1 - \delta_R L)\Delta q_t = -\frac{k_R \delta_R}{1 - \beta \delta_R} \frac{1}{1 + \gamma \sigma \kappa_q} \psi_{t-1}.$$

A.3.4 Segmented financial market

Lemma A4 *The solution to the portfolio choice problem (39) when the time periods are short is given by:*

$$d_{t+1}^* = -\frac{i_t - i_t^* - \mathbb{E}_t \Delta e_{t+1} + \frac{1}{2} \sigma_e^2 + \sigma_{e\pi^*}}{\omega \sigma_e^2}, \quad (\text{A4})$$

where $i_t \equiv \log R_t$, $i_t^* \equiv \log R_t^*$, $\sigma_e^2 \equiv \text{var}_t(\Delta e_{t+1})$ and $\sigma_{e\pi^*} = \text{cov}_t(\Delta e_{t+1}, \Delta p_{t+1}^*)$.

Proof: See [Campbell and Viceira \(2002, Chapter 3 and Appendix 2.1.1\)](#). ■

Note the extra terms in the numerator of (A4), which correspond to the Jensen inequality corrections to the expected log return on the carry trade.

Proof of Lemma 2 Consider the market clearing for the foreign-currency bond in (42) and substitute in (40) and (A4) to obtain after a few algebraical manipulations:

$$\frac{B_{t+1}^*}{R_t^*} + n(e^{\psi_t} - 1) = m \frac{i_t - i_t^* - \mathbb{E}_t \Delta e_{t+1} + \frac{1}{2} \sigma_e^2 + \sigma_{e\pi^*}}{\omega \sigma_e^2}.$$

The market clearing conditions in (42) together with the fact that both intermediaries and noise traders take zero capital positions, that is $\frac{D_{t+1} + N_{t+1}}{R_t} = -\mathcal{E}_t \frac{D_{t+1}^* + N_{t+1}^*}{R_t^*}$, results in the equilibrium balance between home and foreign household asset positions:

$$\frac{B_{t+1}}{R_t} = -\mathcal{E}_t \frac{B_{t+1}^*}{R_t^*}.$$

Therefore, we rewrite the equilibrium condition in the financial market as:

$$m \frac{i_t - i_t^* - \mathbb{E}_t \Delta e_{t+1} + \frac{1}{2} \sigma_e^2 + \sigma_{e\pi^*}}{\omega \sigma_e^2} = n(e^{\psi_t} - 1) - \frac{B_{t+1}}{\mathcal{E}_t R_t}.$$

Next we log-linearize this condition around a symmetric equilibrium with $\bar{R} = \bar{R}^* = 1/\beta$ and $\bar{B} = \bar{B}^* = 0$. Furthermore, we assume that shocks are small, resulting in σ_e^2 and $\sigma_{e\pi^*}$ being second order, however we adopt the asymptotics in which as σ_e^2 shrinks ω increases proportionally leaving $\omega \sigma_e^2$ constant, finite and nonzero. This results in the following log linearization:

$$\frac{m}{\omega \sigma_e^2} (i_t - i_t^* - \mathbb{E}_t \Delta e_{t+1}) = n \psi_t - \frac{\bar{P}_H \bar{Y}}{\bar{\mathcal{E}} \bar{R}} b_{t+1},$$

where $b_{t+1} = \frac{1}{\bar{P}_H \bar{Y}} B_{t+1}$, or equivalently

$$i_t - i_t^* - \mathbb{E}_t \Delta e_{t+1} = \underbrace{\frac{n}{m} \omega \sigma_e^2}_{\equiv \chi_1} \cdot \psi_t - \underbrace{\frac{\beta \bar{P}_H \bar{Y}}{m} \omega \sigma_e^2}_{\equiv \chi_2} \cdot b_{t+1}.$$

This completes the proof of the lemma. ■

DISCUSS FINITE $\omega \sigma_e^2$ AND CONSTANT σ_e^2 ... Cite [Hansen and Sargent \(2011\)](#)

Profits and losses in the financial market Consider the profit and losses of the non-households participants in the financial market – the intermediaries and the noise traders:

$$\frac{D_{t+1}^* + N_{t+1}^*}{R_t^*} \tilde{R}_{t+1}^* = \left(m d_{t+1}^* + n(e^{\psi_t} - 1) \right) \left[1 - e^{i_t - i_t^* - \Delta e_{t+1}} \right] R_t^*,$$

where we used the definition of \tilde{R}_{t+1}^* in (38). Using Lemma A4, the lower-order terms of these profits are:

$$\begin{aligned} - \left(-m \frac{i_t - i_t^* - \mathbb{E}_t \Delta e_{t+1}}{\omega \sigma_e^2} + n \psi_t \right) (i_t - i_t^* - \Delta e_{t+1}) \bar{R}^* &= -\frac{n}{\beta} x_{t+1} \left(-\frac{1}{\chi_1} \mathbb{E}_t x_{t+1} + \psi_t \right) \\ &= \frac{n \chi_2}{\beta \chi_1} x_{t+1} b_{t+1} = \bar{P}_H \bar{Y} \cdot b_{t+1} x_{t+1}, \end{aligned}$$

where $x_{t+1} \equiv i_t - i_t^* - \Delta e_{t+1}$, and Lemma 2 implies $\mathbb{E}_t x_{t+1} = \chi_1 \psi_t - \chi_2 b_{t+1}$, where $\chi_1 = \frac{n}{m} \omega \sigma_e^2$. Therefore, while the UIP deviations (realized x_{t+1} and expected $\mathbb{E}_t x_{t+1}$) are first order, the profits and losses in the financial markets are also first order, since $B_{t+1} = \bar{P}_H \bar{Y} \cdot b_{t+1}$ is first order around $\bar{B} = 0$. Intuitively, the profits and losses in the financial market are equal to the realized UIP deviation times the gross portfolio position of the households; while both are first order, their product is second order, and hence negligible from the point of view of the country budget constraint.

Covered interest parity Consider the extension of the portfolio choice problem (39) of the intermediaries with the additional option to invest in the CIP deviations:

$$\max_{d_{t+1}^*, d_{t+1}^{F*}} \mathbb{E}_t \left\{ -\frac{1}{\omega} \exp \left(-\omega \left[\frac{\tilde{R}_{t+1}^*}{P_{t+1}^*} d_{t+1}^* + \frac{R_t^{F*}}{P_{t+1}^*} d_{t+1}^{F*} + \frac{R_t^*}{P_{t+1}^*} \mathcal{W}_t^* \right] \right) \right\},$$

where the return on one dollar invested in the CIP deviation (long home-currency bond plus a forward and short foreign currency bond) is:

$$R_t^{F*} = \frac{\mathcal{E}_t}{\mathcal{F}_t} R_t - R_t^*,$$

since 1 dollar at t buys $\mathcal{E}_t R_t$ units of home-currency bonds, and $d_{t+1}^{F*} \geq 0$ is the period- t dollar size of this position. Note that we also allowed for non-zero wealth \mathcal{W}_t^* of the intermediaries, which is by default invested into the ‘riskless’ foreign-currency bond. Both CIP investment and wealth investment are subject to the foreign inflation risk only, but no risk of nominal return, unlike the carry trade d_{t+1}^* . Note that the CIP investment, just like the carry trade, requires no capital at time t .

The first order optimality condition with respect to the CIP investment is:

$$R_t^{F^*} \cdot \mathbb{E}_t \left\{ \frac{1}{P_{t+1}^*} \exp \left(-\omega \left[\frac{\tilde{R}_{t+1}^*}{P_{t+1}^*} d_{t+1}^* + \frac{R_t^{F^*}}{P_{t+1}^*} d_{t+1}^{F^*} + \frac{R_t^*}{P_{t+1}^*} \mathcal{W}_t^* \right] \right) \right\} = 0.$$

However, since the expectation term is strictly positive for any $d_{t+1}^{F^*} \in (-\infty, \infty)$, this condition can be satisfied only if $R_t^{F^*} = 0$. If $R_t^{F^*} > 0$, then the intermediaries will take an unbounded position in the CIP trade, $d_{t+1}^{F^*} = \infty$, and vice versa.

A.3.5 Real exchange rate

Proof of Lemma 3

Equations (43) and (29) form a system of dynamic equations:

$$\begin{aligned} -(1 + \gamma\sigma\kappa_q)\mathbb{E}_t\Delta q_{t+1} &= \chi_1\psi_t - \chi_2b_{t+1}, \\ \beta b_{t+1} - b_t &= \gamma \left[\frac{2(1-\gamma)\theta - 1}{1-2\gamma}q_t + \gamma\kappa_q q_t - \kappa_a\tilde{a}_t \right] \end{aligned}$$

where we used the fact that $\pi_t = \pi_t^* = 0$ under the floating regime and also that

$$\begin{aligned} c_t - c_t^* &= \kappa_a\tilde{a}_t - \gamma\kappa_q q_t, \\ i_t - i_t^* &= \mathbb{E}_t\{\sigma(\Delta c_{t+1} - \Delta c_{t+1}^*) + (\pi_t - \pi_t^*)\} = \sigma\kappa_a\mathbb{E}_t\Delta\tilde{a}_{t+1} - \gamma\sigma\kappa_q\mathbb{E}_t\Delta q_{t+1}, \end{aligned}$$

where the first equation is the result of market clearing (see derivations in Appendix A.3.2) and the latter equation is the manipulation of the household Euler equations for local bonds (see footnote 10).

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