Real Effects of Financial Distress: The Role of Heterogeneity^{*}

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Abstract

How severe are the real consequences of financial distress caused by the sovereign debt crisis? What are the channels through which sovereign debt crisis affects banks and firms? Does firm heterogeneity matter? If yes, what are the important dimensions of heterogeneity? Using micro data from Portugal during the sovereign debt crisis, starting in 2010, we address these questions. We make use of the Bank of Portugal's detailed credit registry database together with bank and firm balance sheets and income statements to conduct this analysis.

We first study the direct effect of the sovereign debt crisis on bank balance sheets by analyzing the differential impact on firms that had relations with banks who were more exposed to the sovereign (pre-crisis). We find that more fragile firms that had relations with more exposed banks contracted more than their counterparts. Specifically we find leverage and maturity structure of debt to be important dimensions of heterogeneity determining a firm's fragility. Highly leveraged firms and those that had a larger share of short term debt contracted more during the sovereign debt crisis. We analyze firm performance on the basis of growth rate of employment, assets, liabilities and usage of intermediate commodities.

We then document the spillover effects across firms that are mediated through the banking sector. To do this, we focus on the set of firms that were in good standing on all their loans through the crisis, i.e., the set of performing firms. We find that the performing firms that had relations with banks whose corporate loan portfolio deteriorated by more were more affected by the crisis. Again, leverage and the maturity structure of debt appear to be important dimensions of form heterogeneity. Lastly, we show that our findings are consistent with a simple model of leverage and maturity choice under credit shocks.

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1 Introduction

Does financial distress lead to real consequences? Can the financial distress be caused by shocks to the real sector? In other words, is there a connection between health of banks on Wall Street and health of firms on Main Street, and vice versa? If yes, how strong are these two-way feedbacks? Are there potentially interesting dimensions of heterogeneity in the data that one may want to study? Such questions have come to the forefront of the policy debate in the aftermath of the financial crisis. In this paper, we study the real effects of the financial distress experienced by banks in Portugal in the spring of 2010. We address two main questions: (i) What are the channels of transmission of shocks from the financial to the real sector? (ii) Were some firms affected more than others, i.e. what does the data tell us about the heterogeneous impact of the shock on the non-financial firms.

Regarding the first question, we document two channels of transmission of shocks, namely, the sovereign channel and the spillover channel. Regarding the second question, we find that ex ante highly leveraged firms and firms who had a substantial amount of short-term debt on their balance sheets contracted significantly more that their counterparts.

The main reasons for choosing Portugal as a laboratory for this analysis are twofold: (i) Portugal is a country that has exceptionally rich micro data which enables us to link the universe of banks to the universe of firms i.e. one can clearly observe the financial-real sector linkages and (ii) Portugal is a country that has arguably suffered large financial shocks as the sovereign debt crisis was unfolding in Europe.

The natural experiment literature on the real effects of bank balance sheet shocks at the firm level is an emerging strand and this is indeed a challenging task as it requires data on firm-bank relationships as well as information on firms and banks. For this analysis, we use data on the universe of firms and banks in Portugal, including all the firm-bank credit relationships. Our main contributions to the literature are the following. First, we highlight two channels of transmission of financial shocks to the real economy: (i) the sovereign channel and (ii) the spillover channel. The sovereign channel operates through banks' holdings of risky sovereign debt. The exposure to distressed government bonds inhibits the ability of banks to raise funds in capital markets markets, leading to a transmission of this increased borrowing costs to the interest rates paid by non-financial corporations (Committee on the Global Financial System, 2011). The spillover channel investigates whether performing firms were adversely affected owing to the accumulation of non-performing loans on the balance sheets of their lenders.¹ Next, we identify potentially important dimensions of heterogeneity among firms. We find that ex ante highly leveraged firms and firms with a more shorter maturity structure of debt contracted more in the aftermath of the shock. Finally, we look at multiple dimensions of firms outcome variables to perform a comprehensive analysis of the real effects. We consider growth rates of firm employment, fixed assets,

¹Performing firms are defined as those who did not have any loans overdue for 90 days or more during 2009 and 2010.

firm liabilities, and the usage of intermediate commodities.

We present a simple model of investment and debt maturity under credit shocks to interpret our empirical results. The model highlights the role of leverage and debt maturity in determining the sensitivity of firms' investment to a credit shock. In addition, the model provides a simple theory that sheds light on the determinants of the maturity of a firm's debt vis-a-vis our empirical results. We show that when differences in the maturity of debt is driven by heterogeneity in the maturity of investment projects, then a higher quantity of short term debt is not associated with higher sensitivity of investment to credit shocks. On the contrary, when differences in the maturity of debt is driven by heterogeneity in the term premium faced by firm, then a higher quantity of short term debt is associated with a higher sensitivity of investment to shocks.

Our theoretical results, together with our empirical analysis, suggest that it is important to model frictions to the issuance of long term debt to account for the effects of financial crisis on firms' investment outcomes.

Literature Review The literature on the causes/effects of the financial crisis is vast. There has been extensive theoretical work done on the household as well the firm side. On the household side, Mian and Sufi (2010) show that pre crisis household leverage was a powerful statistical predictor of the severity of the 2007-09 recession across U.S. counties. High household leveraged counties from 2002 to 2006 showed a sharp relative decline in durable consumption. These empirical findings are captured by means of a dynamic macro model developed in Justiniano et al. (2016). Guerrieri and Lorenzoni (2009) show that when some agents are liquidity constrained then the aggregate impact of shocks is greatly amplified. Eggertson and Krugman (2012) present a new Keynesian model where debt overhang on some agents, who are forced into rapid deleveraging, depresses aggregate demand. On the firm side, one could think about shocks propagating by means of the financial accelerator mechanism (Bernanke et al. (1999), Gertler and Kiyotaki (2009)), worse reallocation effects (Buera and Moll (2015), Gilchrist et al. (2013)) or through non-financial linkages across various sectors of the economy (Shourideh and Zeltin-Jones (2016)).

The research on the empirical side is relatively scarce as it requires detailed data on firm bank relationships as well as information on firms and banks. Regarding the recent 2008-09 financial crisis in the US, Chodorow-Reich (2014) uses the DealScan database and employment data from the U.S. Bureau of Labor Statistics Longitudinal Database to show that firms that had pre-crisis relationships with banks that struggled during the crisis reduced employment more than firms that had relationships with healthier lenders. This paper uses the collapse of Lehman Brothers in the fall of 2008 as the event around which the analysis is constructed. Similarly, Bentolila et al. (2013) matches employment data from the Iberian Balance Sheet Analysis System and loan information obtained from the Bank of Spain's Central Credit Register to document that during the recent financial crisis, Spanish firms that had relationships with relationships with healthy

banks. Bofondi et al. (2013) looks at the aggregate credit supply effects of the sovereign debt crisis using data from the Italian credit register. Cingano et al. (2013) also uses the Bank of Italy's credit register to provide evidence that firms which borrowed from banks with a higher exposure to the interbank market experienced a larger drop in investment and employment levels in the aftermath of the recent financial crisis. Bottero et al. (2015) also uses data from the Italian Credit Register to show that the exogenous shock to sovereign securities held by financial intermediaries, which was triggered by the Greek bailout (2010), was passed on to firms through a contraction of credit supply. Finally, Acharya et al. (2015) explores the impact of the European sovereign debt crisis and the resulting credit crunch on the corporate policies of firms, using data from Amadeus, SNL, Bankscope and other sources, however they only look at the syndicated loan market.

2 An overview of the macroeconomic events

Until late 2009 or early 2010 the viability of sovereign debt was not a concern for the markets. For over a decade, the yields of bonds issued by European countries had been low and stable. However, in the spring 2010 when the Greek government requested an EU/IMF bailout package to cover its financial needs for the remaining part of the year, markets started to doubt the sustainability of sovereign debt. Soon after Standard & Poors downgraded Greece's sovereign debt rating to BB+ ("junk bond") leading investors to be concerned with the solvency and liquidity of the public debt issued by other peripheral Eurozone countries like Ireland and Portugal.

In May 2010, following the Greek bailout request, the CDS spreads on Portuguese sovereign bonds increased dramatically (Figure 1, top left panel) and suddenly the Portuguese banks lost access to international debt markets (Figure 1, top right panel). They could not obtain funding in medium and long term wholesale debt markets and this had been an important source of their funding until then. This sudden stop scenario could be attributed mainly to investors concerns on contagion from the sovereign crisis in Greece. The sudden rise in Portuguese CDS spreads meant that the banks who were more exposed to the public sector saw the risk in their balance sheets going up.²

The top-left panel of Figure 1 plots the sovereign credit default swap spreads for Portugal and the average of Italy, Ireland and Spain. We also plot Germany as a benchmark. The vertical line depicts May 2010. In the top right panel of Figure 1, we plot the funding obtained through securities (market funding).³ The two events combined i.e. the sudden decline in the value of assets and the increase in funding costs led to a pass-through into the lending rates paid by firms. Specifically we observed a rise in the short term interest rates. The bottom left panel of

²Brunnermeier et al. (2011) argues that the sudden panics and the spike in sovereign bond yields in Portugal and elsewhere were the consequence of the close inter-linkages between banks and sovereigns. Fears about the solvency of the sovereign can put the solvency of banks in a particular country at risk, since banks typically hold a substantial fraction of their assets in the form of sovereign debt of the respective country. The situation was no different in Portugal.

³Source: Alves et al. (2016).

Figure 1 shows the evolution of the spread between the average lending rates by banks at one year maturity relative to the return of a 1-year German sovereign bond. The two panels on the left lend credence to the fact that the sovereign and lending rates are extremely closely related. We call this channel of transmission of shock as the *sovereign channel*.

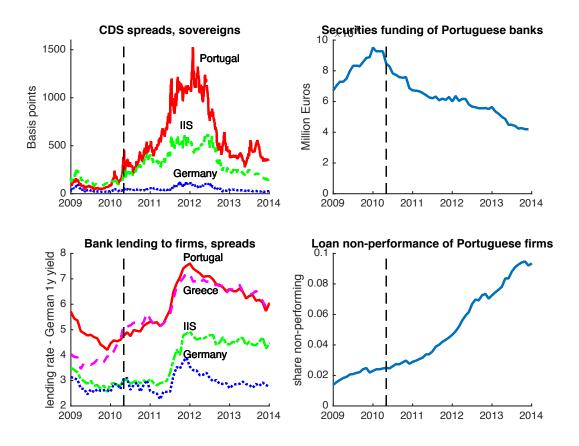


Figure 1: Evolution of sovereign CDS spreads, market funding to Portuguese banks, bank lending spreads to Portuguese non-financial corporations, and the share of non-performing loans to Portuguese firms during the sovereign european crisis.

Another stylized fact that we observe in the data is the rapid accumulation of non-performing loans on the banks' balance sheets. In the bottom right panel of Figure 1, we present the non-performing loans as a fraction of total loans of banks in Portugal.⁴ This motivates us to think of other potential channels of transmission of financial distress onto the real sector. To elaborate further, we are interested in studying if a firm, conditional on not having any loans in default (overdue>90 days) in 2009 or 2010, was affected adversely because its lenders were accumulating non-performing assets on their balance sheets. This is what we call the *spillover channel*.

To sum up, the aggregate economic environment in Portugal during this period was adverse

⁴Our analysis will however be strictly cross-sectional and we do not provide explanations for the spike in NPLs over time.

and all the agents of the economy were under stress.⁵ The banks were hit particularly hard as they were the center of the capital flows and in 2010 accounted for approximately half of the net foreign debt of Portugal. Arguably the trigger for these events was the bailout request by Greece in April 2010. This bailout request prompted a complete reassessment of the default risk of a number of countries of the European Economic and Monetary Union (especially the peripheral European countries) and can be considered as the first, unprecedented, and unanticipated episode that challenged the notion of risk-less sovereign debt in the euro area since the adoption of the Euro.

3 The empirical analysis

3.1 Our data

For this analysis we build a unique dataset for the Portuguese economy. We mainly use three separate datasets, which can be merged using the firm and bank identification codes. The datasets used were the Central Credit Register (CRC), the Central Balance Sheet Database (CBSD) and the Monetary and Financial Statistics. The CRC is managed by Bank of Portugal and contains information reported by the participants (the institutions which extend credit) concerning credit granted to individuals and non-financial corporations and the situation of all such credits extended. Any loan amounting to 50 euros or more is recorded in the credit register. For this analysis, we only consider credit extended to non-financial corporations and exclude the household sector. Further, we will only consider the total committed credit between the firm and a bank.⁶ The CBSD is based on accounting data of individual firms. Since 2006, annual CBSD data has improved significantly and has been based on mandatory financial statements reported in fulfillment of firms statutory obligations under the Informação Empresarial Simplificada (Simplified Corporate Information, Portuguese acronym: IES). The MFS data provides us with information on the bank balance sheet components. Variables such as bank size, capital ratio, and liquidity ratio are obtained from this database. The CRC and the CBSD can be merged using the firm identifier and then using the bank identifier, we merge it with the MFS to get our comprehensive dataset.

In tables 1 & 2 we provide an overview of the datastet we use. Table 1 reports aggregate statistics on firms while table 2 reports bank level characteristics. The first column of table 1 represents all firms from the CBSD i.e. all firms who file taxes in Portugal, the second column includes firms

⁵For a further detailed description we refer the reader to Reis (2013) who documents the events as they happened in the aftermath of the sovereign debt crisis in Greece. The yields on 10 year Portuguese bonds rose from 3.9% to 6.5% during 2010. Public spending also rose markedly, partly because of the automatic stabilizers, and partly because the government implemented a campaign promise of raising public sector wages after years of zero increases. The sudden stop in capital inflows affected, especially, the non-tradable sector and brought about a sharp decline in output, a phenomenon that has also been observed in many Latin American countries.

⁶We ignore items such as renegotiated or written off credit that also appears in the database owing to data quality issues.

who have obtained credit from a financial institution while the last column only shows firms who have multiple banking relationships. To further improve the quality of the analysis, we drop the micro firms i.e. we only consider firms who had an outstanding loan amount equal to or greater than ten thousand euros as of the last quarter of 2009. All figures reported are for 2009:Q4. Our final sample of firms is quite representative of the Portuguese economy overall. To provide some insight, the sample represents 71% of total loans granted as of December 2009. They further represent 70.51% of aggregate employment in Portugal, 76.41% by turnover, and 77.07% by assets. Further we check if the labor share of each sector in the population closely matches the labor share of each sector in the sample. The correlation coefficient stands at 0.98 with the three biggest sectors by employment being manufacturing, wholesale/retail services, and construction. Table 2 reports data from the financial institutions operating in Portugal. We group the individual financial institutions into 33 banking groups and work at this level of consolidation. For confidentiality reasons we are not able to provide further information on the identity of firms or banks used in this analysis.

3.2 Regression specifications

For the empirical analysis, all growth rates were constructed following Davis and Haltiwanger (1992), i.e.,

$$g_t^E = \frac{E_t - E_{t-1}}{x_t},$$

where g_t^E is the growth rate of variable '*E*' at time '*t*' and *x*_t is the mean, of the variable, over the current and the last period i.e.,

$$x_t = 0.5 * (E_t + E_{t-1})$$

This measure of net growth is bounded between +2 and -2 and symmetric around zero. A value of +2 corresponds to entrants while a value of -2 corresponds to firms who exit the market. This method of computing the growth rates helps us account for both the intensive and the extensive margin and also helps us minimize the effects of outliers. This method of computing the growth rate is monotonically related to the conventional measure and the two are equal for smaller growth rates. It can be shown that if $G_t^{E'}$ is the conventional growth rate measure i.e. the change in a variable normalized by the lagged value of that variable, then $G_t^E = 2g_t^E / (2 - g_t^E)$.

We now proceed in two steps. First we document the effects on credit supply during the crisis. Second, we document the real effects of the sovereign debt crisis. For this analysis, we construct a weighted sovereign exposure measure for each firm. To elaborate, we note all the bank-firm relationships in the fourth quarter of 2009 and the banks' respective sovereign holdings as a fraction of their total assets. Using the relative shares of each bank in a firms loan portfolio, we can construct our sovereign exposure measure for each firm. For the rest of the analysis, we keep the shares and therefore exposures constant. In other words, a firms exposure to the sovereign through its lenders are predetermined in our model. To be precise, our firm level sovereign

exposure measure is calculated as:

$$sov_{j,Q4:2009} = \sum_{b \in B_j} s_{j,b} * sovshare_b,$$
(1)

where $s_{j,b}$ is the share of bank 'b' in the total borrowing of firm 'j' and *sovshare*_b is the total Portuguese sovereign bond holdings of bank 'b' normalized by total assets. A similar exposure measure has been used in Bottero et al. (2015). Figure 2 presents the distribution of the weighted sovereign exposures of the firms in the fourth quarter of 2009. The important implicit assumption is that the banks transmit shocks to the real sector, proportional to their pre-crisis lending relationships. To verify the validity of this assumption, we document the fact that firm-bank relationships are extremely persistent in Portugal. The probability of a firm-bank relationship continuing in the next period, conditional on it existing in the current period, is around 0.87. The probability that a bank remains a firms lead lender next period, conditional on it being the lead lender in the current period, is 0.80. Table 14 in the appendix reports these results.

The real variables we use in our analysis are employment, fixed assets, firm liabilities/total debt, and the usage of intermediate commodities. To construct the growth rates, we use stocks in the fourth quarter of 2009 and 2010. Other robustness measures like taking 2 year averages on either side of the sovereign shock were also conducted and the results were consistent with the ones reported here.

To document the effects on lending on the intensive margin we take recourse to the methodology developed in Khwaja & Mian (2008). In Portugal, on average, around half of the firms have multiple banking relationships and we exploit this fact to identify if there were any adverse effects on lending, on the intensive margin. The baseline regression model we estimate is the following:

$$\%\Delta L_{i,j,Q4:10-Q4:09} = \alpha_0 + \alpha_1 sov_{j,Q4:09} + B_{j,Q4:09} + \alpha_i + \epsilon_j, \tag{2}$$

where $\&\Delta L_{i,j,Q4:10-Q4:09}$ is the growth rate of total committed credit between a firm-bank pair between Q4:09 and Q4:10. $sov_{j,Q4:09}$ is the sovereign share of bank 'j' in Q4:09 and α_i is a vector of firm fixed effects, that help us control for demand side factors. We later augment the above equation to include interaction terms with high leverage and high short-term debt dummies to identify such heterogeneities in the data.

The results are presented in Table 4. Column (1) reports a bank level regression of loan growth on sovereign exposures. Columns (2) - (6) represent regression results for firms having multiple banking relationships while columns (7) & (8) include firms having single relationships as well, for sake of completeness. Column (2) presents the baseline case without interactions and we observe no statistically significant average effect of bank sovereign exposures on lending. However, when we include interaction terms with a high leverage dummy and a dummy that captures high short-term debt share, we observe quite different results. We find that there was

an overall statistically significant reduction of lending to firms who were highly leveraged and those that had a significant share of short-term debt on their balance sheets. In terms of economic magnitudes, these effects are quite substantial as well. For the highly leveraged firms, the bank with a sovereign exposure in the 90th percentile reduces lending by 3.5% more than a bank in the 10th percentile, to the same firm. The same figure stands at 4.7% for firms who had a high share of short term debt on their balance sheets. To put these magnitudes in perspective, aggregate bank lending to the non-financial sector grew, although sluggishly, at 0.04% during the same time period.

We now turn to analyzing the effects on the real variables. The baseline regression we estimate is the following:

$$\%\Delta V_{j,Q4:10-Q4:09} = \alpha_0 + \alpha_1 sov_{j,Q4:09} + \Gamma_j^1 F_j + \Gamma_j^2 B_j + \beta_1^{ind} + \beta_2^{loc} + \epsilon_j,$$
(3)

where the variable 'V' represents employment, fixed assets, firm liabilities, and intermediate commodities, F_j is a set of firm specific controls and in this vector we use measures of profitability, age, size, leverage and maturity structure of debt. B_j is a vector of weighted bank controls and the variables we use here are the bank size, average interest rate on loans, capital ratio, and the liquidity ratio. We also include industry and location fixed effects in our regressions.

The results are presented in Table 5. On average we do not find statistically significant effects of the shock after controlling for bank and firm specific characteristics. However, we are interested in exploring potentially interesting dimensions of heterogeneity and we learn from the corporate finance literature that firm leverage and maturity structure of debt are two of the most important financial variables. Having this overall broad idea in mind, we proceed to estimate regressions that address more specific questions. The first question we ask is, are the firms that are highly leveraged more adversely affected than their lower leveraged counterparts? To answer this question, we modify equation (2) as follows:

Here we include a dummy for firms having pre-crisis leverage of higher than 47% and also the interaction of the dummy with the sovereign exposure measure.⁷ The leverage here is defined as all interest bearing liabilities normalized by total assets. We performed robustness analysis by considering pure bank leverage as well and our results were robust to this alternative measure as well. The results are reported in Table 6. The coefficient on the sovereign share variable captures the impact for the low leveraged firms where we do not find a statistically significant effect. The total real effect of the crisis on the highly levered firms can be obtained by taking the sum of the coefficients on the sovereign exposure term and the interaction term. For the sub category of

⁷The number 47% corresponds to the 75th percentile of the leverage distribution.

the highly levered firms, we do find significant negative effects of the crisis. The employment, capital, firm liabilities and intermediate commodities, all, show a sizable decline. In other words, firms that were highly leveraged prior to the onset of the sovereign debt crisis, appear to contract more than the ones who were less leveraged (better capitalized). The economic magnitudes are also quite significant. For the highly leveraged firms, moving from the 10th percentile of the distribution of weighted sovereign exposures to the 90th percentile, we observe a decline of 1.7% in terms of employment, relative to their low leveraged counterparts. During the same period the aggregate employment for all firms in our sample contracted by 4.4%. Similarly, the contraction in terms of assets, total debt, and intermediate commodities were 7.2%, 13.8%, and 3.9% respectively. For all the firms in our sample, in the same time period, the assets contracted by 1.3%, total debt contracted by 14%, and the usage of intermediate commodities was reduced by around 1%.

The next potentially interesting dimension of firm heterogeneity that we study is along the maturity structure of debt. The following regression that we estimate aims to answer the question if firms that had a significant share of short term debt on their balance sheets were more adversely affected by the sovereign debt crisis. The intuition is that the firms that have a longer maturity structure will not need to refinance in the height of the crisis and therefore would be relatively hedged. We conduct this analysis by using a dummy that is set equal to 1 for firms having a pre-crisis share of short term debt greater than 53%, which corresponds to the 75th percentile of the maturity distribution in 2009.

The results are presented in Table 7. Just as in the previous case, we find statistically significant negative effects on the firms who have a larger share of short term debt on their balance sheets. These results are robust across all our real variables. Once again, these magnitudes are economically significant as well. For a firm with a higher share of short term debt, moving from the 10th to the 90th percentile, of weighted sovereign exposures, brings about a decline of 1.2% in terms of employment, 2.3% in terms of assets, 2.5% in terms of total debt, and 1.9% in terms of intermediate commodity usage.

In Tables 5 and 6, we also report p values from the one sided t-test for the sum of the two coefficients of interest to be less than zero and we fail to reject the null hypotheses in all the cases considered. This is done to document the fact that the overall effect on the highly leveraged firms and the firms with a higher share of short term debt was indeed negative. A quick point must be made here regarding the rationale for including the total debt as one of our real variables. By means of estimating equation (2), we have documented that fragile firms experienced a decline in lending in the immediate aftermath of the sovereign debt crisis. A natural question that arises is were they not able to substitute the loss in funding by moving to other less exposed banks or

by taking recourse to other forms of funding like trade credit? This was indeed not the case. If it were, we would not observe a decline in total debt which is a comprehensive measure of all firm liabilities. Therefore, our total debt measure helps us document the fact that these fragile firms were not able to instantaneously seek funding elsewhere.

We have thus far documented that the overall level of debt and the maturity structure of debt were individually detrimental for real activity in the aftermath of the sovereign debt crisis. However, one may wish to see if either of the two variables dominate or are they both equally important? To answer this question, we include both the interaction terms in our baseline regression and the results are presented in table 8. We find persistently significant negative effects on the firms who were highly leveraged and those who had a significant share of short term debt. This makes us infer that both variables are equally important while analyzing the real effects of the crisis in Portugal.

4 The spillover channel

In the last section, we have documented the real effects of financial distress originating from the banks' holdings of (ex ante risk-free) sovereign bonds. In this section, we explore another novel channel of transmission of shocks from the financial to the real sector. The only difference is that in this section we look at the real effects on firms who did not have any non-performing loan in our sample period. The question we ask is were the firms, all of whose loans were in good standing, affected in any way by the aggregate shock to the economy. We perform the analysis in 3 steps.

1. We start by calculating the non-performing loans (NPL) of the firms, in 2009:Q4 and 2010:Q4, as a fraction of total outstanding loans. We define a dummy which takes a value of 1 if the firm has an NPL share bigger than 0. We then regress the NPL dummy in 2010 on the NPL dummy in 2009 and firm level controls in 2009. The predicted value from this regression is the probability that a particular firm will have positive NPLs in 2010 conditional on it having some non performing loans in 2009. We run the following regression and obtain the predicted values:

$$NPL_{j,Q4:2010} = NPL_{j,Q4:2009} + X_{j,Q4:2009} + \nu_j,$$
(6)

where $X_{j,Q4:2009}$ is a vector of firm level controls prior to the crisis. It includes variables like age, size, leverage, maturity structure of debt, location, and sector of operation. The results are reported in Table 15 in the appendix. The probability of having a non-performing loan in 2010 conditional on having some in 2009 was estimated to be in the interval 0.66-0.79, depending on the specification. We report results with the more optimistic estimate of 0.66

but we re-estimated all our regressions with the probability being 0.79 to ensure robustness of our analysis.⁸

2. The next step was to construct a proxy for risk on the banks' balance sheets. To this end, we use the predicted values from the last regression ($\widehat{NPL}_{j,Q4:2010}$). Our measure of ex ante bank risk is computed in a similar manner as we computed the weighted sovereign exposures. It is defined as follows:

$$Risk_{b,Q4:2009} = \sum_{j \in F_j} s_{j,b} * \widehat{NPL}_{j,Q4:2010},$$

where, $s_{j,b}$ is the share of bank b's loans going to firm 'j' in Q4:2009. To analyze spillover effects, we however need to look at firms who had all their loans in good standing in both the time periods under analysis. We perform this selection in step 3.

3. We take recourse to the central credit registry database once again. We have information on the status of all loans obtained by a firm. In the event that a loan is overdue, we have information on how long the loan has been overdue. We now apply our filtering criteria by dropping all the firms who had any of their loans overdue for 90 days or more. This is our subset of performing performing firms and our sample has about fifty three thousand of them. For these firms, we can now construct a weighted risk measure using the lending shares in Q4:2009 and the bank level risk measures from step 2 above. We can then use this as our main explanatory variable to see if these 'performing' firms experienced some real distress owing to the weakening of the balance sheets of their creditors.

The results are reported in Tables 11 and 12. The broad message emanating from these tables is quite similar to the sovereign channel analysis. Once again, we find that heterogeneity matters and particularly along the dimensions of leverage and the maturity structure of debt. Table 11 reports the results when we interact the weighted risk measure with the high leverage dummy. Economically, these results mean that for a highly leveraged performing firm, as we move from the 10th to the90th percentile of weighted bank risk, we experience a contraction of 1.02% in terms of employment, 1.77% in terms of assets, 3.06% in terms of total debt, and 0.99% in terms of intermediate commodity usage. The economic effects are bigger for the high short term debt regressions, as reported in Table 12. For a similar movement from the bottom to the top decile of bank risk, the firm experiences 1.7% decline in terms of employment, 3.9% in terms of assets, 9.2% in terms of total debt and 2.4% in terms of materials used.

The broad conclusion that we derive is that irrespective of the firm being in good standing or not, leverage and debt maturity structure are important determinants of a firms' access to credit

⁸In Table 14 we also report the sectoral coefficients to provide some insight about the NPL accumulation at an industry level. We observe that the major sectors like manufacturing, construction, and services all show a significant increase in NPLs while some sectors like healthcare and electricity show a decline.

and overall performance when the overall macroeconomic scenario is adverse. What is more important is the interaction of the shock with the borrower characteristics rather than the shock per se. We now present a simple model of leverage and debt maturity to gain further intuition for our results.

4.1 Discussion/Robustness

Thus far we have only considered the exposure of the banks to the Portuguese sovereign and arguably this was the most important source of risk for the Portuguese banks. However, one can argue that a broader measure of ex ante vulnerability could be constructed by allowing for the exposure to the sovereign debt of the GIIPS countries.⁹ To this effect, we now construct a firm level sovereign exposure variable, as before, allowing for the sovereign debt holdings for the GIIPS countries. Tables 9 and 10 highlight the fact that our previous results are robust to this alternative exposure measure. Similar checks were done with the banks' holding of Portuguese and Greek debt and Portuguese and Spanish debt. In all these cases, our results and conclusions remain unaltered. We also broadened our measure of risk on the banks' balance sheets by constructing a vulnerability index for the banks. This was simply the total amount of GIIPS bond holdings and the total amount of lending to the construction sector, as a fraction of total assets. Our results remain robust even to this broad vulnerability measure.

The next robustness check was done with respect to the selection of the time window. We compute growth rates between Q4:09 and Q4:10 and this is our main window of analysis. However, we also conducted our analysis for Q4:08 and Q4:11 and also by taking growth rates of the average values of Q4:08 and Q4:09 and Q4:10 and Q4:11. Once again our results and conclusions remain qualitatively unaltered. One of the principle reasons, for not including 2011 in the baseline analysis, is that 2011 was a very eventful year in terms of many significant events occurring simultaneously e.g. Portugal requested for the Eurosystem bailout, the EBA conducted the stress tests and the capital exercise and so on.

We also verify that our results are not driven by one particular sector. When one thinks about which sectors could be relatively more adversely affected by the sovereign debt crisis, construction seems to be the most natural candidate. Although we have sector fixed effects in all our regressions, we re-estimated our regressions excluding the construction sector and our results hold even in that sub-sample. In terms of estimation methodology, our robustness analysis included estimating weighted least square models. We used two different sets of weights, namely the importance of the firm in the credit market and the size of the firm.¹⁰

One could also argue that the Portuguese banking system consists of branches or subsidiaries of foreign banks which could be "bailed out" by the mother bank should they be in distress. It

⁹Greece, Ireland, Italy, Portugal, and Spain.

¹⁰For the first case, the weight a firm received was its' share of borrowing as a fraction of total borrowing by all firms in the sample. In the second case, the weight attributed was the firms' total assets as a fraction of aggregate assets.

must be mentioned here that the Portuguese loan market is dominated by Portuguese banks and hence the above concern is not a valid one in our analysis. Despite that, to convince the reader, we address this concern by re-estimating our regression models by excluding all foreign entities operating in Portugal and our results remain consistent to this specification as well.

Further analysis was conducted to make sure that our results are not driven by some particular way in which banks might be operating. For example, could it be the case that banks who were lending to riskier borrowers were also holding a high amount of "safe" sovereign debt? This could be justified as a case of diversification of the banks' portfolio. To verify that this was not the case, we constructed bank level risk measures (share of non performing loans in total loans), from the credit registry, and computed the correlations with sovereign holdings, exante. Figure 6 in the appendix lends credence to this statement. We report scatter plots and correlation coefficients in the four quarters prior to the sovereign shock. The correlations were found to be weak and insignificant. Despite this analysis, we augmented all our regressions with sector and location specific fixed effects because such (hypothetical) matching might take place if the firm and the bank are present in a particular sector or a particular location.

5 A Simple Model of Investment and Debt Maturity under Credit Shocks

We present a simple model to interpret our empirical results. The model highlights the role of leverage and debt maturity in determining the sensitivity of firm's investment decisions to interest rate shocks. In addition, it provides a simple theory of debt maturity that sheds light on the underlying drivers of debt maturity vis-a-vis our empirical results.

We study the problem of an entrepreneur that lives for three periods, owns a long term project, and has access to an additional risky investment in the interim period. The new investment, and the negative cash-flows associated with the long term investment, can be financed with short and long term debt issuance. The entrepreneur faces a credit shock in the interim period, i.e., the cost of credit in the interim period is uncertain. Consumption only takes place in the last period.

The entrepreneur starts the first period, t = 0, with a long term project with deterministic cash flows $\{y_t\}_{t=0}^2$. Cash-flows might include negative elements due to the initial investment or payments of previously issued debts. In the first period the entrepreneur chooses short (1-period) and long term (2-period) debt issuance d_0^1 and d_0^2 (bond purchases if negative) to finance a given amount of leverage d_0 ,

$$d_0^1 + d_0^2 = d_0 = -y_0$$

We denote by r_0^1 and r_0^2 the cost of short and long term credit in the first period, respectively.

At the beginning of the second period, t = 1, the cost of (short-term) credit $r_1^1 \in [\underline{r}, \overline{r}]$ is realized. In this interim period the entrepreneur has access to an investment opportunity k with

an uncertain return $z \in [0, \infty)$. She can issue new debt d_1^1 to roll-over the short term debt issued in the first period and/or finance the new investment,

$$k = y_1 - \left(1 + r_0^1\right) d_0^1 + d_1^1.$$

In the final period, t = 2, the last cash-flow of the long term project occurs, the return of the interim investment is realized, short and long term debts are repaid, and consumption takes place,

$$c_2 = y_2 + zk - \left(1 + r_1^1\right) d_1^1 - \left(1 + r_0^2\right) d_0^2.$$

The problem of the entrepreneur can be simplified as that of choosing the maturity of the debt in the initial period d_0^2 and the investment in the interim period *k* to maximize the expected utility of consumption in the final period

$$\begin{aligned} \max_{d_0^2,k} E_{r_1^1,z} \left[\log c_2 \right] \\ \text{s.t.} \\ c_2 &= \left(z - 1 - r_1^1 \right) k + y_2 + \left(1 + r_1^1 \right) \left(y_1 - \left(1 + r_0^1 \right) d_0 \right) \\ &+ \left(\left(1 + r_1^1 \right) \left(1 + r_0^1 \right) - \left(1 + r_0^2 \right) \right) d_0^2. \end{aligned}$$

We first discuss the investment choice in the interim period, given leverage d_0 and the maturity structure in the first period $d_0^1 = d_0 - d_0^2$ and d_0^2 , and then consider the maturity choice in the initial period.

5.1 Investment decision

The investment conditional on leverage, debt maturity, and the interest rate shock in the interim period,

$$\begin{split} &k\left(r_{1}^{1}\right) \\ &= \bar{k}\left(r_{1}^{1}\right) \cdot \left[y_{2} + \left(1 + r_{1}^{1}\right)\left(y_{1} - \left(1 + r_{0}^{1}\right)d_{0}\right) + \left(\left(1 + r_{1}^{1}\right)\left(1 + r_{0}^{1}\right) - \left(1 + r_{0}^{2}\right)\right)d_{0}^{2}\right] \\ &= \bar{k}\left(r_{1}^{1}\right) \cdot w\left(r_{1}^{1}\right) \end{split}$$

The first term is a decreasing function of the cost of credit in the interim period, $\partial \bar{k} (r_1^1) / \partial r_1 < 0$. It captures the pure effect of an interest rate shock on the net return of investment. The second term is the last period's value of the net worth of the entrepreneur conditional on the realization of the interest rate shock. These are the total resources available to invest. This term is independent of the interest rate shock provided $d_0^2 = d_0 - y_1 / (1 + r_0^1)$.

In our empirical analysis we study the sensitivity of investment to a credit shock, which we

show is associated with a rise in the cost of credit. Furthermore, we show that leverage and the fraction of short term debt amplify the effect of the credit shock. We now show that, taking the debt maturity decision as exogenous, this is a natural implication of the model.

In the analysis that follows we make two additional assumptions. To simplify the analysis, we assume that the present value of an entrepreneurs' cash-flows in the interim period, conditional on $d_0^2 = 0$, is positive for all realizations of r_1^1 :

Assumption 1 For all $r_1^1 \in [\underline{r}, \overline{r}]$

$$\frac{y_2}{1+r_1^1} + y_1 - \left(1+r_0^1\right)d_0 > 0.$$
⁽⁷⁾

In addition, we restrict long term debt positions to guarantee that investment is positive for all realizations of the interest rate in the interim period:

Assumption 2 We assume that

$$-\frac{y_2 + (1+\bar{r})\left(y_1 - (1+r_0^1)d_0\right)}{(1+\bar{r})\left(1+r_0^1\right) - (1+r_0^2)} < d_0^2 < \frac{y_2 + (1+\underline{r})\left(y_1 - (1+r_0^1)d_0\right)}{(1+r_0^2) - (1+\underline{r})\left(1+r_0^1\right)}$$
(8)

and

$$\left(1+r_0^1\right)\left(1+\underline{r}\right) - 1 < r_0^2 < \left(1+r_0^1\right)\left(1+\bar{r}\right) - 1.$$
(9)

Condition (8) will be satisfied when we endogenize the maturity structure, but will be required when analyzing the investment decision conditional on a given value of the maturity structure.

The investment in a high interest rate state relative to a low interest rate state, $k(r_h) / k(r_l)$, $r_h > r_l$, is decreasing in the total leverage provided the cash flow in the last period gross of long-term debt payments is positive, $y_2 - (1 + r_0^2) d_0^2 > 0$. Given (8), this condition will be satisfied provided the entrepreneur has a cash deficit in the intermediate period, $(1 + r_0^1) (d_0 - d_0^2) - y_1 \ge 0$. In other words, this occurs when the amount of long term debt is less than that needed to eliminate the interest rate risk, i.e., $d_0^2 \le d_0 - y_1 / (1 + r_0^1)$.

Proposition 1 *Assume* $y_2 - (1 + r_0^2) d_0^2 > 0$, then

$$rac{\partial \left(rac{k(r_h)}{k(r_l)}
ight)}{\partial d_0} < 0.$$

Related, the investment in a high interest rate state relative to a low interest rate state is increasing in the amount of long term debt provided $y_2 - (1 + r_0^2) \left(d_0 - \frac{y_1}{1 + r_0^1} \right) > 0$. This condition follows from assumptions (7) and (9).

Proposition 2 Assume $y_2 - (1 + r_0^2) \left(d_0 - \frac{y_1}{1 + r_0^1} \right) > 0$, then

$$\frac{\partial \left(\frac{k(r_h)}{\overline{k(r_l)}}\right)}{\partial d_0^2} = -\frac{\partial \left(\frac{k(r_h)}{\overline{k(r_l)}}\right)}{\partial d_0} + \left(1 + r_0^2\right) \frac{\overline{k}\left(r_h\right)}{\overline{k}\left(r_l\right)} \frac{w(r_h) - w(r_l)}{w(r_l)^2} > 0.$$

The condition in Proposition 2 is stronger than that in Proposition 1 when $d_0^2 < d_0 - y_1/(1+r_0^1)$. As we show next, this will be the relevant case when the term premium is strictly positive, i.e., $1 + r_0^2 > (1 + r_0^1) \mathbb{E} (1 + r_1^1)$. In this situation, the impact of an increase in leverage on the investment in a high relative to a low interest rate states is larger than that of a decline in the maturity of debt,

$$rac{\partial \left(rac{k(r_h)}{k(r_l)}
ight)}{\partial d_0} < -rac{\partial \left(rac{k(r_h)}{k(r_l)}
ight)}{\partial d_0^2},$$

as in this case the net worth is lower the higher the interest rate, $w(r_h) < w(r_l)$.

5.2 Maturity decision

The previous analysis takes as given the maturity structure of the debt in the initial period. We now study the optimal maturity choice and, therefore, how the maturity structure depends on the primitives of the model, e.g., the timing of the cash-flows of the long term investment, $\{y_t\}_{t=0}^2$, and the term premium, $1 + r_t^2$. This analysis guides us to interpret the variation of the debt maturity observed in the data and our empirical results.

We first consider the case in which the expectation hypothesis holds, i.e, $1 + r_0^2 = (1 + r_0^1) \mathbb{E} (1 + r_1^1)$. This is a very particular, though important benchmark. In this case, the debt maturity is chosen to fully offset the interest rate risk. As a consequence, the investment in a high interest rate state relative to a low interest rate state is independent of leverage and the maturity structure of the debt. The optimal debt choice

$$d_0^2 = d_0 - y_1 / \left(1 + r_0^1 \right)$$

Long term debt is chosen to finance all the initial leverage that cannot be paid back with the cash flows in the interim period. The variation in the amount of long term debt conditional on leverage is driven solely by the variation in the cash flow in the interim period y_1 .

Given the optimal maturity choice, the investment in the interim period

$$k(r_1^1) = \bar{k}(r_1^1) \cdot \left[y_2 + \frac{1+r_0^2}{1+r_0^1}y_1 - (1+r_0^2) d_0\right],$$

and the the investment in a high interest rate state relative to a low interest rate state is independent of the leverage and the maturity structure

$$\frac{k(r_h)}{k(r_l)} = \frac{\bar{k}(r_h)}{\bar{k}(r_l)}$$

Our empirical results do not correspond to a world where the expectation hypothesis holds. In this world, entrepreneurs who issue more short term debt conditional on leverage are those that expect to have a larger cash flow in the interim period. The larger cash flow exactly compensates the shorter maturity of the debt. Similarly, entrepreneurs with more leverage choose to issue more long term debt to fully insure against the interest rate risk.

We next consider a situation with a positive term premium. This is the empirically relevant case. Given Assumption (7), it is straightforward to show that

$$rac{\partial d_0^2}{\partial \left(1+r_0^2
ight)} < 0$$

When the term premium is positive entrepreneurs bear interest rate risk, i.e., when $1 + r_0^2 > (1 + r_0^1) \mathbb{E} (1 + r_1^1)$ we have $d_0^2 < d_0 - y_1 / (1 + r_0^1)$. As before, the quantity of long term debt is a decreasing function of the cash flow in the interim period, but now the effect is stronger

$$\frac{\partial d_0^2}{\partial y_1} < -\frac{1}{1+r_0^1} = \left. \frac{\partial d_0^2}{\partial y_1} \right|_{1+r_0^2 = \left(1+r_0^1\right) \mathbb{E}\left(1+r_1^1\right)}$$

The stronger effect is explained by the fact that the demand for interest rate insurance is a decreasing function of the net-worth when the utility function exhibits decreasing absolute risk aversion, e.g., as it is true in the case with log preferences.¹¹

This simple model suggests two important sources of variation of the maturity of debt, conditional on leverage. The first is given by the timing of the cash-flows of the long term investment, e.g., variation on y_1 or y_2 . The second one is given by variation across entrepreneurs in the term premium, r_0^2 . As the previous analysis shows, entrepreneurs whose projects matures early or face a higher term premium choose to issue more shorter term debt.

As the following proposition shows, these two sources of variation in the maturity of debt are associated with very different implications for the sensitivity of investment to interest rate shocks.

Proposition 3 *Assume* $1 + r_0^2 \ge (1 + r_0^1) \mathbb{E} (1 + r_1^1)$ *, then*

$$\frac{d\left(\frac{k(r_h^1)}{k(r_l^1)}\right)}{dy_1} = \frac{\partial\left(\frac{k(r_h^1)}{k(r_l^1)}\right)}{\partial y_1} + \frac{\partial\left(\frac{k(r_h^1)}{k(r_l^1)}\right)}{\partial d_0^2}\frac{\partial d_0^2}{\partial y_1} = 0$$
$$\frac{d\left(\frac{k(r_h^1)}{k(r_l^1)}\right)}{dy_2} = \frac{\partial\left(\frac{k(r_h^1)}{k(r_l^1)}\right)}{\partial y_2} + \frac{\partial\left(\frac{k(r_h^1)}{k(r_l^1)}\right)}{\partial d_0^2}\frac{\partial d_0^2}{\partial y_2} = 0$$

and

$$\frac{d\left(\frac{k(r_h^1)}{k(r_l^1)}\right)}{d\left(1+r_0^2\right)} = \frac{\partial\left(\frac{k(r_h^1)}{k(r_l^1)}\right)}{\partial\left(1+r_0^2\right)} + \frac{\partial\left(\frac{k(r_h^1)}{k(r_l^1)}\right)}{\partial d_0^2} \frac{\partial d_0^2}{\partial\left(1+r_0^2\right)} < 0.$$

¹¹Due to this effect, we also have that the amount of long term debt issued is a decreasing function of the cash flow in the last period. Similarly, the effect of initial leverage on the amount of long term debt issued is also stronger,

$$\frac{\partial d_0^2}{\partial y_1} = -\frac{1}{1+r_0^1} - \frac{1+r_0^2}{1+r_0^1} \frac{\partial d_0^2}{\partial y_2} = -\frac{1}{1+r_0^1} \frac{\partial d_0^2}{\partial d_0}$$

When the differences in the maturity structure of debt are driven by differences in the maturity of the long term project, i.e., y_1 and y_2 , the differential debt maturity is not associated with a differential sensitivity of investment to the interest rate shock. In this case, the longer debt maturity exactly compensates the fewer cash flows available in the interim period. On the contrary, when the differences in the maturity of debt are driven by differences in the term premium that the entrepreneur faces in the initial period, i.e., $1 + r_0^2$, the differential debt maturity is associated with a higher sensitivity of investment to interest rate shock.

These results, together with our empirical analysis, suggest that it is important to model frictions to the issuance of long term debt to account for the effects of financial crisis on firm's investment. In our simple model, frictions to the issuance of long term debt can be captured by an individual specific term premium.

5.3 Evidence from the data

Proposition 3 makes a powerful statement regarding why some firms might issue a sub-optimal amount of long-term debt and hence fail to insure themselves completely against any impending interest rate risk. The two reasons implied by the model are higher cash flows or higher borrowing costs. We take the theory back to the data by estimating a simple equation of the form,

$$(LT_debt_share)_{i,t} = f(X_{i,t}),$$

where the left hand side represents the long-term debt as a fraction of total debt for firm 'i' at time 't'. $X_{i,t}$ is a set of firm specific characteristics including variables like firm specific borrowing costs, cash flows, firm size, investment, and external finance dependence.¹² The goal here is not to make causal statements but explore which of the variables are most closely related with the long-term debt issuance of a firm, focusing mainly on cash flows and firm specific borrowing costs. We do no have information on firm specific interest rates at different levels of maturity and therefore we construct a proxy for firm borrowing costs. From the firm accounting database, we have information on total interest paid by firms and therefore, we construct our measure as total interest expenditure normalized by total debt. The cash flows are also normalized by total debt. Investment is defined as the growth rate of fixed assets and firm size is the log of total assets.

Table 13 presents the results. We use data from 2009-2014 except for the last column. The over time specifications are presented in columns 1 and 2 while column 3 reports results for the cross section 2009-2010. The results are in alignment with the theory as the interest rate and the cash flow variables show up with a negative sign and are statistically significant. However, we need to understand how important are each of these variables economically. In terms of economic magnitudes, a one standard deviation increase in cost of borrowing (interest) of the firm results

¹²Calculated at a sectoral level following the methodology developed in Rajan & Zingales (1998). It is defined as $(capital_expenditure - cash_flows)/capital_expenditure$

in a decline in the long-term debt by approximately 5 - 11 p.p., depending on the specification while a one standard deviation increase in cash flows leads to a reduction of long-term debt share by approximately 4 - 6 p.p. This suggests that the sub-optimal issuance of long-term debt might have been driven, predominantly, by borrowing costs rather than cash flows.

6 Conclusion

Using a novel loan level dataset from Portugal, we study two channels (sovereign and spillover) through which financial shocks may be transmitted to the real sector. The sovereign channel operates through the banks' holdings of risky public debt while the spillover channel operates through the accumulation of non-performing loans on the bank's balance sheets. We first analyze firms' access to credit and then proceed onto studying firm performance in terms of employment, assets, liabilities and usage of intermediate commodities, in the aftermath of the financial shock. Although we do not find significant effects on average, firm heterogeneity seems to matter significantly. Specifically, we show that ex ante highly leveraged firms and firms that had a shorter maturity structure of debt contracted significantly more than their counterparts. The overall amount of debt and the maturity structure, both seem to be important determinants of firm performance when the overall macroeconomic scenario is adverse. We also document that similar results hold also for firms who themselves did not have any loans in default but were indirectly affected because their lenders were in distress.

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A Tables

	CBSD			& CRC	>1 Relations		
Variables	Mean	SD	Mean	SD	Mean	SD	
Employment	13.66	120.345	14.81	126.864	18.89	150.535	
Fixed Assets	934068.3	2.98e+07	886924.3	2.92e+07	1190380	3.52e+07	
Tot. Liab	2848650	8.58e+07	2522380	8.69e+07	3404019	1.05e+08	
Int. Comm. Usage	203245.3	2.05e+06	214196.5	2.15e+06	278098.5	2.58e+06	
EBIT	80525.3	2684130	75880.12	2354905	103475.7	2845427	
ST debt share	0.51	0.39	0.52	0.39	0.50	0.38	
No. of firms	138211		106723		82561		

Figures are for 2009:Q4. CBSD is the firm balance sheet data, CRC is the central credit registry.

Table 1: Descriptive Statistics (Firms)

	All E	Banks	High So	ov Share	Low So	P Value	
Variables	Mean	SD	Mean	SD	Mean	SD	(t-test)
Total Assets	1.41e+10	2.83e+10	1.83e+10	3.52e+10	1.15e+10	2.14e+10	0.44
Capital Ratio	14.85	7.74	15.17	8.80	14.59	6.98	0.83
Liquidity Ratio	13.44	15.96	16.54	17.08	10.87	14.97	0.31
Overdue/total loans	2.72	2.62	2.91	2.86	2.57	2.51	0.71
Corp. Share	28.84	18.73	27.90	15.01	30.41	21.65	0.59
Hhs. Share	25.59	23.55	19.84	14.55	30.39	28.56	0.20
Funding (securities/assets)	6.32	9.74	7.05	10.62	4.91	8.70	0.45
Funding (inter-bank/assets)	24.46	19.78	25.00	21.54	24.01	18.28	0.88
Funding (central bank/assets)	7.49	13.98	9.71	16.27	6.65	11.92	0.41
Loan to deposit	2.22	2.24	1.88	1.59	2.50	2.68	0.43
No. of banking groups	33		15		18		

Table 2: Descriptive Statistics (Banks)

Note: Figures are for 2009:Q4. Consolidated for 33 main financial institutions. High-sov bank is one that had sovereign share>6%. Overdue/total loans is a measure of risk on the banks' balance sheet. We next report the share of bank lending going to the corporate and the household sectors. Funding from securities is a measure of market dependence of the bank. We also report funding obtained from the interbank market and the central bank, as fractions of total assets. The 15 high-sov banking groups comprise Portuguese and Spanish banks only. The 18 low-sov banking groups also contains mostly Portuguese banks. Other banks in this group have their origins in Spain, Germany, France, Brazil, and Angola. The last column reports the p-values from a simple two sided t-test for the equality of means between the high-sov and the low-sov banking groups. We fail to reject the null hypothesis: $H0 : \mu_{highsov} - \mu_{lowsov} = 0$.

	High Sov	⁷ Share	Low Sov	Share	
Variables	Mean	SD	Mean	SD	P Value
Age	19.24	4.73	18.79	5.01	0.79
Firmsize	15.32	0.78	15.68	0.91	0.24
ST debt share	0.27	0.09	0.23	0.09	0.21
Leverage	0.62	0.24	0.79	0.32	0.13
Profitability	0.01	0.01	0.01	0.05	0.75
NPL ratio	0.02	0.01	0.03	0.05	0.57
No. of banking groups	15		18		

Tab	le 3:	Ban	ks'	W	Veig	hted	Borrov	wer	Chara	acteristics	

Note: Figures are for 2009:Q4. Consolidated for 33 main financial institutions. High-sov bank is one that had sovereign share>6%. The figures above correspond to weighted average borrower characteristics of each bank. The weights are calculated using outstanding loans as of 2009:Q4. Firmsize is the log of total assets, ST debt share is short-term debt normalized by total debt, leverage is defined as all interest bearing liabilities normalized by total assets, profitability is earnings before interest and taxes normalized by total assets, and NPL ratio is the non-performing loans as a fraction of total loans. The last column reports the p-values from a simple two sided t-test for the equality of means between the high-sov and the low-sov banking groups. We fail to reject the null hypothesis: $H0 : \mu_{highsov} - \mu_{lowsov} = 0$.

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Bank Level	Baseline	Leverage	Leverage	ST Debt	ST Debt	Lev (All)	ST Debt (All)
-1.252***	0.094	0.135	0.353	0.206	0.442	0.280	0.391
(0.605)	0.409	(0.409)	(0.473)	(0.393)	(0.470)	(0.393)	(0.411)
		(0.146)	(0.155)			(0.140)	
							-0.560**
				(0.163)	. ,		(0.223)
							0.071
			. ,		. ,	. ,	(0.475)
							0.946
			(/		· · ·	· · · ·	(1.163)
							0.035**
			(0.017)		(0.017)	· · ·	(0.017)
						(0.010)	
							0.006
							(0.015)
-							-0.440**
(0.067)						(0.184)	(0.189)
N	v	v	v	v	v	N	Ν
	-	-	-	-	-		184,416
	Bank Level	Bank Level Baseline -1.252*** 0.094 (0.605) 0.409 0.172** 0.0172** (0.067) Y	Bank Level Baseline Leverage -1.252*** 0.094 0.135 (0.605) 0.409 (0.409) -0.412*** (0.146) 0.172** (0.067) N Y	Bank Level Baseline Leverage Leverage -1.252*** 0.094 0.135 0.353 (0.605) 0.409 (0.409) (0.473) -0.412*** (0.146) 0.192 (0.438) 1.108 1.124) 0.042** (0.017) 0.172** N Y Y	Bank Level Baseline Leverage Leverage ST Debt -1.252*** 0.094 0.135 0.353 0.206 (0.605) 0.409 (0.409) (0.473) (0.393) -0.412*** -0.360** (0.155) -0.537*** (0.146) (0.155) -0.537*** (0.163) 0.192 (0.438) 1.108 1.108 (1.124) 0.042** (0.017) -0.542** 0.172** (0.067) Y Y Y	Bank Level Baseline Leverage Leverage ST Debt ST Debt -1.252*** 0.094 0.135 0.353 0.206 0.442 (0.605) 0.409 (0.409) (0.473) (0.393) (0.470) -0.412*** -0.360** (0.163) (0.187) -0.556*** (0.163) 0.192 0.202 (0.438) 1.089 1.108 1.089 1.133) 0.042** (0.017) 0.172** (0.067) Y Y Y Y	Bank LevelBaselineLeverageLeverageST DebtST DebtLev (All)-1.252*** 0.094 0.135 0.353 0.206 0.442 0.280 (0.605) 0.409 (0.409) (0.473) (0.393) (0.470) (0.393) -0.412^{***} -0.360^{**} (0.155) (0.140) -0.279^{**} (0.146) (0.155) -0.537^{***} -0.556^{***} (0.140) 0.192 0.202 0.054 (0.438) (0.464) 1.108 1.089 0.973 (1.124) (1.133) (1.116) 0.042^{**} 0.042^{**} 0.043^{**} 0.033^{**} (0.010) 0.172^{**} (0.667) Y Y Y Y Y Y N Y Y Y Y Y Y Y Y

Table 4: Lending Effects

Note: The dependent variable is the loan growth rate at the bank-firm level. Column (1) reports a bank level regression of loan growth on sovereign exposures. Columns (2) - (6) represent regression results for firms having multiple banking relationships exante. Column (2) presents the baseline regression with no interaction terms. Columns (3) - (6) introduce interactions with the high leverage dummy and the high ST debt dummy. Columns (7) & (8) include firms having single relationships as well. Clustered standard errors (bank level) are reported in the parentheses. *** p<0.01, ** p<0.05, * p<0.1

	(1)	(2)	(3)	(4)
VARIABLES	Gr_emp	Gr_ast	Gr_liab	Gr_int
Wtd_sov_holding	-0.002	-0.427	-0.034	-0.048
-	(0.091)	(0.268)	(0.245)	(0.093)
Constant	0.166***	-0.453***	0.108***	0.093***
	(0.019)	(0.043)	(0.027)	(0.017)
Firm Controls	Y	Y	Y	Y
Wtd. Bank Controls	Y	Y	Y	Y
Sector FE	Y	Y	Y	Y
Observations	88,204	89,410	89,466	89,823

Clustered standard errors (bank level) are reported in the parentheses * p<0.1, ** p<0.05, *** p<0.01

Table 5: Average Effects

	(1)	(2)	(3)	(4)
VARIABLES	Gr_emp	Gr_ast	Gr_liab	Gr_int
Wtd_sov_holding (α_1)	0.030	-0.279	0.233	0.024
	(0.083)	(0.248)	(0.206)	(0.078)
Wtd_sov_holding*Highlev (α_2)	-0.199*	-0.834***	-1.605***	-0.450***
	(0.112)	(0.207)	(0.410)	(0.142)
Highlev	0.023***	-0.009	0.001	0.050
	(0.008)	(0.161)	(0.027)	(0.085)
Constant	0.168***	-0.422***	0.131***	0.096***
	(0.019)	(0.043)	(0.027)	(0.016)
Firm Controls	Y	Y	Y	Ŷ
	-	-	-	-
Wtd. Bank Controls	Y	Y	Y	Y
Sector FE	Y	Y	Y	Y
$P(\alpha_1 + \alpha_2 < 0)$	0.96	0.99	0.99	0.99
Observations	88,204	89,410	89,466	89,823

Table 6: Interaction with leverage

Note: The dependent variables are the growth rates of employment, fixed assets, bank liabilities and usage of intermediate commodities, respectively. The main independent variable is the weighted Portuguese sovereign bond holdings of firms in Sept. 09. Firm level controls include age, size, value added, and sector fixed effects. Weighted bank controls include capital ratio, liquidity ratio and average interest rates charged by the respective banks. Clustered standard errors (bank level) are reported in the parentheses. We also report the p-values from a one sided t-test with H0: $\alpha_1 + \alpha_2 < 0$. *** p<0.01, ** p<0.05, * p<0.1

	(1)	(2)	(3)	(4)
VARIABLES	Gr_emp	Gr_ast	Gr_liab	Gr_int
Wtd_sov_holding (α_1)	0.017	-0.392	0.097	-0.019
	(0.090)	(0.256)	(0.349)	(0.092)
Wtd_sov_holding* High_stdebt (α_2)	-0.140**	-0.265**	-0.289**	-0.218***
	(0.069)	(0.110)	(0.125)	(0.046)
High_stdebt	-0.023	-0.144	0.097***	0.000
	(0.017)	(0.160)	(0.036)	(0.044)
Constant	0.165***	-0.454***	0.142***	0.093***
	(0.019)	(0.042)	(0.033)	(0.017)
	•		• /	
Firm Controls	Y	Y	Y	Y
Wtd. Bank Controls	Y	Y	Y	Y
Sector FE	Y	Y	Y	Y
$P(\alpha_1 + \alpha_2 < 0)$	0.98	0.98	0.98	0.99
Observations	88,204	89,410	89,828	89,823

Table 7: Interaction with short term debt

Note: The dependent variables are the growth rates of employment, fixed assets, bank liabilities and usage of intermediate commodities, respectively. The main independent variable is the weighted Portuguese sovereign bond holdings of firms in Sept. 09. Firm level controls include age, size, value added, and sector fixed effects. Weighted bank controls include capital ratio, liquidity ratio and average interest rates charged by the respective banks. Clustered standard errors (bank level) are reported in the parentheses. We also report the p-values from a one sided t-test with H0: $\alpha_1 + \alpha_2 < 0$. *** p<0.01, ** p<0.05, * p<0.1

	(1)	(2)	(3)	(4)
VARIABLES	Gr_emp	Gr_ast	Gr_liab	Gr_int
Wtd_sov_holding	0.047	-0.250	0.876	0.050
-	(0.084)	(0.238)	(0.355)	(0.078)
Wtd_sov_holding * Highlev	-0.194*	-0.825***	-2.408***	-0.443***
	(0.111)	(0.206)	(0.519)	(0.142)
Wtd_sov_holding* High_stdebt	-0.131*	-0.229**	-0.163	-0.199***
	(0.067)	(0.107)	(0.110)	(0.045)
Highlev	0.024***	-0.008	-0.03	0.051
	(0.008)	(0.161)	(0.028)	(0.085)
High_stdebt	-0.025	-0.290	0.13	0.015
	(0.019)	(0.216)	(0.116)	(0.034)
Constant	0.168***	-0.422***	0.101**	0.096***
	(0.019)	(0.043)	(0.044)	(0.016)
Firm Controls	Y	Y	Y	Y
Wtd. Bank Controls	Y	Y	Y	Y
Sector FE	Y	Y	Y	Y
Observations	88,204	89,410	89,828	89,823

Table 8: Leverage and Short term debt

Note: The dependent variables are the growth rates of employment, fixed assets, bank liabilities and usage of intermediate commodities, respectively. The main independent variable is the weighted Portuguese sovereign bond holdings of firms in September 2009. The firm level controls used were firm age, firm size, value added, and fixed effects for the sector of operation. The weighted bank controls used were bank capital ratio, liquidity ratio and average loan interest rates charged by the respective banks. Clustered standard errors (bank level) are reported in the parentheses. *** p < 0.01, ** p < 0.05, * p < 0.1

	(1)	(2)	(3)	(4)
VARIABLES	Gr_emp	Gr_ast	Gr_liab	Gr_int
Wtd_GIIPS (α_1)	0.010	-0.159	0.292	0.031
	(0.065)	(0.214)	(0.121)	(0.060)
Wtd_GIIPS*Highlev (α_2)	-0.179*	-0.758***	-1.447***	-0.410***
	(0.105)	(0.172)	(0.338)	(0.122)
Highlev	0.023***	-0.010	0.000	0.050
	(0.008)	(0.162)	(0.027)	(0.085)
Constant	0.169***	-0.426***	0.126***	0.096***
	(0.019)	(0.043)	(0.026)	(0.016)
Firm Controls	Y	Y	Y	Y
Wtd. Bank Controls	Y	Y	Y	Y
Sector FE	Y	Y	Y	Y
$P(\alpha_1 + \alpha_2 < 0)$	0.95	0.99	0.99	0.99
Observations	88,204	89,410	89,466	89,823

Table 9: Interaction with leverage (GIIPS exposure)

Note: The dependent variables are the growth rates of employment, fixed assets, bank liabilities and usage of intermediate commodities, respectively. The main independent variable is the weighted GIIPS sovereign bond holdings of firms in September 2009. The firm level controls used were firm age, firm size, value added, and fixed effects for the sector of operation. The weighted bank controls used were bank capital ratio, liquidity ratio and average loan interest rates charged by the respective banks. Clustered standard errors (bank level) are reported in the parentheses. We also report the p-values from a one sided t-test with H0: $\alpha_1 + \alpha_2 < 0$. *** p<0.01, ** p<0.05, * p<0.1

	(1)	(2)	(3)	(4)
VARIABLES	Gr_emp	Gr_ast	Gr_liab	Gr_int
Wtd_GIIPS (α_1)	0.002	-0.244	0.155	-0.001
	(0.072)	(0.220)	(0.290)	(0.072)
Wtd_GIIPS * High_stdebt (α_2)	-0.129**	-0.242*	-0.269**	-0.204***
	(0.052)	(0.122)	(0.100)	(0.037)
High_stdebt	-0.023	-0.145	0.098***	0.000
	(0.017)	(0.160)	(0.036)	(0.044)
Constant	0.166***	-0.458***	0.139***	0.092***
	(0.019)	(0.043)	(0.032)	(0.017)
Firm Controls	Y	Y	Y	Y
Wtd. Bank Controls	Y	Y	Y	Y
Sector FE	Y	Y	Y	Y
$P(\alpha_1 + \alpha_2 < 0)$	0.99	0.97	0.99	0.99
Observations	88,204	89,410	89,828	89,823

Table 10: Interaction with ST Debt (GIIPS exposure)

Note: The dependent variables are the growth rates of employment, fixed assets, bank liabilities and usage of intermediate commodities, respectively. The main independent variable is the weighted GIIPS sovereign bond holdings of firms in September 2009. The firm level controls used were firm age, firm size, value added, and fixed effects for the sector of operation. The weighted bank controls used were bank capital ratio, liquidity ratio and average loan interest rates charged by the respective banks. Clustered standard errors (bank level) are reported in the parentheses. We also report the p-values from a one sided t-test with H0: $\alpha_1 + \alpha_2 < 0$. *** p<0.01, ** p<0.05, * p<0.1

	(1)	(2)	(3)	(4)
VARIABLES	Gr_emp	Gr_ast	Gr_liab	Gr_int
Wtd_ \widehat{NPL} (α_1)	-0.113	0.107	-0.425**	-0.133**
	(0.088)	(0.173)	(0.097)	(0.054)
Wtd_ \widehat{NPL} * Highlev (α_2)	-0.150***	-0.261***	-0.451***	-0.146***
C C	(0.030)	(0.051)	(0.027)	(0.033)
Highlev	0.002	-0.156***	0.24	-0.058***
	(0.008)	(0.010)	(0.012)	(0.010)
Constant	0.031**	0.350***	0.131	0.163***
	(0.015)	(0.023)	(0.018)	(0.023)
Firm Controls	Y	Y	Y	Y
Wtd. Bank Controls	Y	Y	Y	Y
Sector FE	Y	Y	Y	Y
$P(\alpha_1 + \alpha_2 < 0)$	0.99	0.99	1.00	0.99
Observations	53,780	53,528	54,425	54,444

Table 11: Spillover effects (Interaction with leverage)

Note: The firms included in this regression are the ones who did not have any loan overdue for 90 days or more in 2009:Q4 or 2010:Q4. The dependent variables are the growth rates of employment, fixed assets, bank liabilities and usage of intermediate commodities, respectively. The main independent variable is the weighted GIIPS sovereign bond holdings of firms in September 2009. The firm level controls used were firm age, firm size, value added, and fixed effects for the sector of operation. The weighted bank controls used were bank capital ratio, liquidity ratio and average loan interest rates charged by the respective banks. Clustered standard errors (bank level) are reported in the parentheses. We also report the p-values from a one sided t-test with H0: $\alpha_1 + \alpha_2 < 0$. *** p<0.01, ** p<0.05, * p<0.1

	(1)	(2)	(3)	(4)
VARIABLES	Gr_emp	Gr_ast	Gr_liab	Gr_int
Wtd_ \widehat{NPL} (α_1)	-0.076	0.203	-0.075	-0.067
	(0.089)	(0.180)	(0.119)	(0.053)
Wtd_ \widehat{NPL}^* High_stdebt (α_2)	-0.251***	-0.582***	-1.597***	-0.358***
Ū į	(0.031)	(0.087)	(0.127)	(0.040)
High_stdebt	-0.061	1.209*	-1.25	-0.063
	(0.287)	(0.615)	(0.687)	(0.366)
Constant	0.040**	0.344***	0.100	0.158***
	(0.016)	(0.023)	(0.016)	(0.021)
Firm Controls	Y	Y	Y	Y
Wtd. Bank Controls	Ŷ	Ŷ	Ŷ	Ŷ
Sector FE	Ŷ	Ŷ	Ŷ	Ŷ
$P(\alpha_1 + \alpha_2 < 0)$	1.00	0.99	1.00	1.00
Observations	53,780	53,528	54,445	54,444

Table 12: Spillover effects (Interaction with ST debt)

Note: The firms included in this regression are the ones who did not have any loan overdue for 90 days or more in 2009: Q4 or 2010:Q4. The dependent variables are the growth rates of employment, fixed assets, bank liabilities and usage of intermediate commodities, respectively. The main independent variable is the weighted GIIPS sovereign bond holdings of firms in September 2009. The firm level controls used were firm age, firm size, value added, and fixed effects for the sector of operation. The weighted bank controls used were bank capital ratio, liquidity ratio and average loan interest rates charged by the respective banks. Clustered standard errors (bank level) are reported in the parentheses. We also report the p-values from a one sided t-test with H0: $\alpha_1 + \alpha_2 < 0$. *** p<0.01, ** p<0.05, * p<0.1

	(1)	(2)	(3)
VARIABLES	Time FE	Macro controls	Cross section
Interest rate	-0.236***	-0.302***	-0.141***
	(0.008)	(0.008)	(0.011)
Cash flow	-0.026***	-0.030***	-0.034***
	(0.001)	(0.001)	(0.001)
Investment	0.016***	0.017***	0.015***
	(0.001)	(0.001)	(0.003)
Firm size	0.031***	-0.005**	0.031***
	(0.002)	(0.002)	(0.001)
Ext. dependence			0.024***
-			(0.009)
Constant	0.181***	0.569***	-0.099***
	(0.027)	(0.026)	(0.013)
Firm FE	Y	Y	Ν
Time FE	Y	Ν	Ν
Macro Controls	Ν	Y	Ν
Observations	514,663	514,663	70,016
R-squared	0.592	0.588	0.047

Table 13: LT debt, Cash flow, & Interest Rates

B Figures

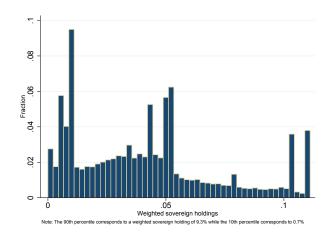


Figure 2: Firm weighted sovereign exposures

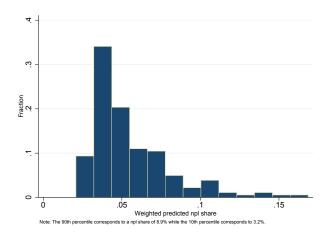


Figure 3: Firm weighted predicted NPL shares

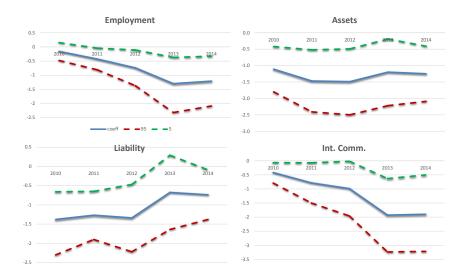


Figure 4: Effects over time: Leverage

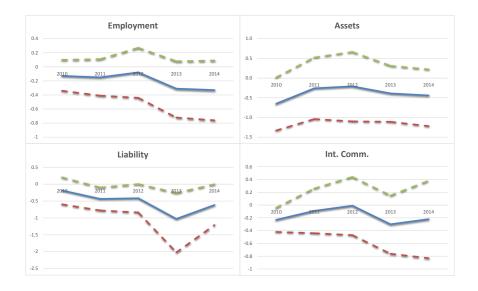
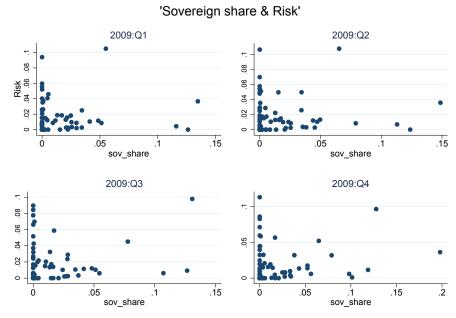


Figure 5: Effects over time: ST debt



Note: The respective correlations are -0.064, -0.067, -0.033 & -0.041 and none of them are statistically significant.

Figure 6: Bank sovereign shares vs. risk

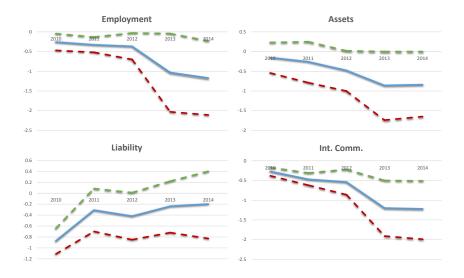


Figure 7: Spillover results: Effects over time (Leverage)

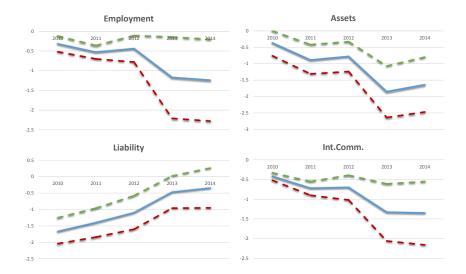


Figure 8: Spillover results: Effects over time (ST debt)

C Additional Figures and Tables

C.1 Persistence of relationships

The table below shows the persistence of bank-firm relationships in Portugal. In the first two columns we report the probability of a bank being a firms lead bank in period 't' conditional on it being the lead bank in period 't - 1'. In columns (3) and (4) we report the probability of a particular firm borrowing from a particular bank in period 't' conditional on it having borrowed in period 't - 1'. As we can observe, both the probabilities are in excess of 0.8 demonstrating the fact the relationships tend to be extremely persistent.

	$Y_t = lead_t$	$Y_t = lead_t$	$Y_t = any_t$	$Y_t = any_t$	
$Y_{t-1} = lead_{t-1}$	0.802*** [0.000]				
$Y_{t-1} = any_{t-1}$			0.867***		
			[0.000]		
$Y_{t-1} * 2006.year$		0.827***		0.876***	
		[0.000]		[0.000]	
$Y_{t-1} * 2007.year$		0.810***		0.856***	
		[0.000]		[0.000]	
$Y_{t-1} * 2008.year$		0.818***		0.859***	
-		[0.000]		[0.000]	
$Y_{t-1} * 2009.year$		0.760***		0.864***	
		[0.000]		[0.000]	
$Y_{t-1} * 2010.year$		0.795***		0.876***	
•		[0.000]		[0.000]	
$Y_{t-1} * 2011.year$		0.792***		0.864***	
•		[0.000]		[0.000]	
$Y_{t-1} * 2012.year$		0.810***		0.870***	
•		[0.000]		[0.000]	
Const	-0.001***	-0.001***	-0.001***	-0.001***	
	[0.000]	[0.000]	[0.000]	[0.000]	
Time Effects	Y	Y	Y	Y	
Number of obs.	84790059	84790059	84790059	84790059	
Robust standard errors in parentheses					
* 20 1 ** 20 05 *** 20 01					

* p<0.1, ** p<0.05, *** p<0.01

Table 14: Relationship Regression

C.2 Predicting the future NPLs

L.NPL	0.79***						0.66***
Constant	(0.002) 0.027***						(0.006) 0.020**
	(0.002)						(0.003)
Age							-0.00** (0.000)
Size							-0.001*
ST Debt							(0.00) 0.002*
Leverage							(0.001) 0.028**
			Industry Fiz	ked Effects			(0.002)
(Significantly Positive)	(Significantly Negative) (Significantly Positive)		(Significantly Negative)				
Transportation	0.025***	Engineering activities	-0.08***	Transportation	0.017***	Mineral extraction	-0.009**
Construction	(0.003) 0.038*** (0.002)	Tobacco	(0.003) -0.027*** (0.002)	Construction	(0.003) 0.021*** (0.002)	Tobacco	(0.003) -0.001* (0.002)
Telecommunications	(0.002) 0.034*** (0.012)	Coal extraction	-0.027*** (0.001)	Cork Industry	(0.002) 0.025*** (0.004)	Electricity/Gas	-0.015** (0.004)
Cork industry	(0.012) 0.033*** (0.004)	Defense	-0.026*** (0.002)	Printing	0.013*** (0.004)	Sewerage	-0.014** (0.002)
Leather	0.029*** (0.003)	Water	-0.026*** (0.011)	Manufacture (food)	0.019*** (0.006)	Legal Activities	-0.008** (0.002)
Civil Engineering	0.026*** (0.003)	Veterinary Activities	-0.022*** (0.004)	Manufacture (non-metallic goods)	0.007*** (0,003)	Veterinary Activities	-0.012** (0.004)
Real Estate Activities	0.020*** (0.003)	Mineral extraction	-0.019*** (0.007)	Manufacture (metallic goods)	0.012***	Healthcare	-0.006* (0.002)
Advertising	0.019*** (0.004)	Electricity/Gas	-0.017*** (0.005)	Manufacture (electrical goods)	0.013*** (0.006)		(,
Rental Activities	0.019*** (0.005)	Healthcare	-0.013*** (0.002)	Manufacture (transport equipment)	0.038*** (0.015)		
Manufacture (furniture)	0.017*** (0.005)	Research activities	-0.013* (0.008)	Manufacture (furniture)	0.022*** (0.004)		
Printing	0.016*** (0.004)	Manufacture (pharma)	-0.012* (0.007)	Civil Engineering	0.013*** (0.003)		
Lodging and Restaurants	0.016*** (0.003)	Legal Activities	-0.010*** (0.003)	Construction	0.012*** (0.002)		
Textiles	0.015*** (0.004)			Real Estate Activities	0.009*** (0.003)		
Repair	0.015*** (0.002)			Advertising	0.008** (0.004)		
Editing	0.015*** (0.006)			Rental Activities	0.008** ((0.004))		
Tourism Agencies	0.014*** (0.005)						
Management Consulting	0.012*** (0.003)						
Manufacture (metallic goods)	0.011*** (0.003)						
Manufacture (non-metallic goods)	0.012*** (0.004)						
Manufacture (transport equipment)	0.040*** (0.004)						
Observations R-squared	464,928 0.516						280,148 0.280

Robust standard errors in parentheses *** p<0.01, ** p<0.05, * p<0.1

Table 15: NPL Predictor

C.3 Exploring other dimensions of heterogeneity

We have analyzed firm heterogeneity along two main dimensions: leverage and maturity structure of debt. However, we also analyzed differences in terms of age, size, degree of external financing and profitability.¹³ We estimate equations similar to the ones in equations (4) and (5), i.e.,

$$\begin{split} \%\Delta V_{j,Q4:10-Q4:09} &= \alpha_0 + \alpha_1 sov_{j,Q4:09} + \alpha_2 sov_{j,Q4:09} * (high''x'') + \alpha_3 (high''x'') \\ &+ \Gamma_j^1 F_j + \Gamma_j^2 B_j + \beta_1^{ind} + \epsilon_j, \end{split}$$

where high"x" is a dummy and is equal to 1 for the top quartile of the respective variable, at the pre-crisis level and xe(size, age, external finance, profitability). $high_size = 1$ if the firm has assets more than 1 million euros, $high_age = 1$ if the firm is more than 18 years old, $high_extfin = 1$ if the firm finances more than 35% of it's capital expenditure through external financing, and $high_profit = 1$ if the firms profits as a ratio of total assets is bigger than 36%. Figure 9 plots $\alpha_1 + \alpha_2$ along with the 95% confidence intervals. As can be seen, we do not find statistically significant effects for any of the variables considered.

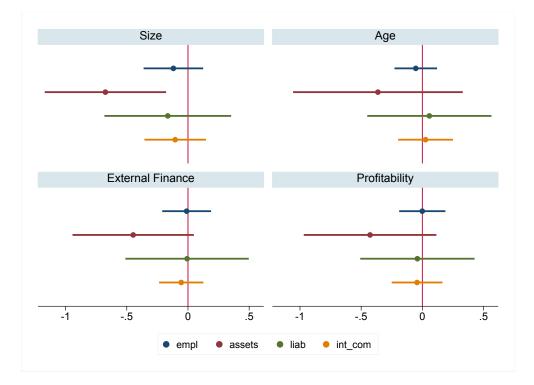


Figure 9: Exploring other dimensions of heterogeneity

¹³External finance = (capex-cash flows)/capex

C.4 Average effects over time?

As mentioned before, we do not find significant negative effects of the sovereign debt crisis, on average. In Figure 10 we re-estimate equation (3) to analyze if there were any significant average effects over time. We plot the point estimates of the weighted average sovereign debt exposures along with the 95% confidence intervals. The dependent variable changes as we analyze growth rates between 2009-10, 2010-11, 2011-12, 2012-13, and 2013-14. The sovereign exposure measure is held constant at the pre-crisis (2009) level. We control for appropriate bank, firm, geography, and sectoral characteristics. Overall, we do not observe any significant effects on average.

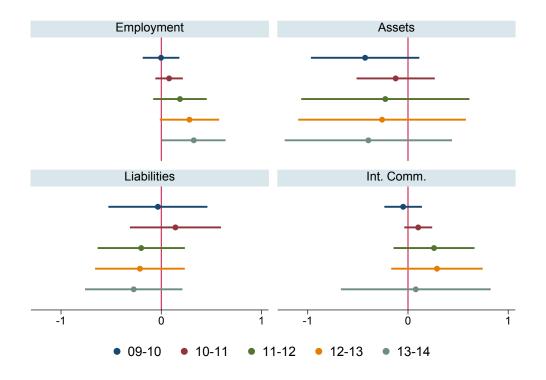


Figure 10: Average effects over time (Sovereign Channel)

We perform a similar analysis for the spillover channel to investigate the effects on average. The results are shown in Figure 11. The equation estimated is almost identical to equation (3) except that instead of the banks' sovereign exposures, the main independent variable is the estimated risk on the banks' balance sheet as explained in Section 4. Once again, we do not observe any significant pattern over time.

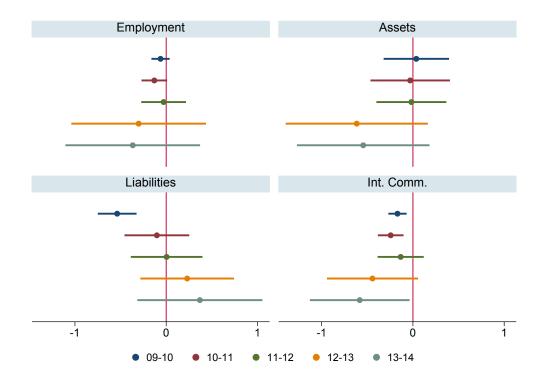


Figure 10: Average effects over time (Spillover Channel)

C.5 Placebo regressions

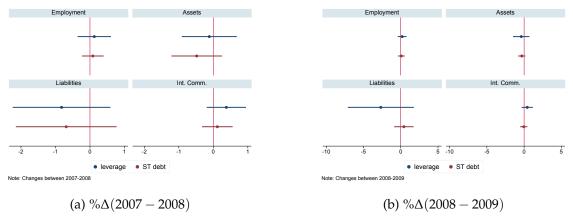


Figure 12: Placebo regressions

Figure 12 depicts results from our placebo regressions ran in the period preceding the sovereign debt crisis. We recreate Tables 5 and 6 but calculating the growth rates between 2007 and 2008 in panel (a) and between 2008-2009 in panel (b). The weighted sovereign shares, leverage, and short term debt shares were also made to be pre-determined in 2007 and 2008 respectively. We do not find any significant effects for the highly leveraged firms or the firms who had a higher share of short term debt thereby lending credence to the fact that the results are specific to the period under consideration.

C.6 Effects by quartiles of leverage and short term debt

In this sub-section, we report real effects of the sovereign channel on firms belonging to different quartiles of leverage and short term debt.

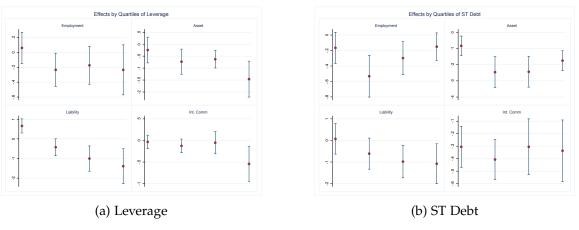


Figure 13: Effects by the respective quartiles

The results above correspond to the sovereign channel. We report the impact on our variables on interest for firms grouped into quartiles of leverage (panel (a)) and debt maturity (panel (b)). In the regression analysis presented in the paper, we compared the top quartile with the rest of the firms. This analysis breaks it down further to shed light on how firms in each of these quartiles perform in the immediate aftermath of the sovereign debt crisis. In line with the results obtained in our prior regression analysis, we observe that the as we move from the lowest to the highest quartile of leverage and debt maturity, the firms are more adversely affected.

D Theoretical Appendix

In this appendix we characterize the model presented in Section 5 and provide the proofs of the propositions stated in that section.

Given the total leverage d_0 and the quantity of long term debt d_0^2 , the investment decision in the interim period solves

$$\max_k \mathbb{E}_z \left\{ \log c_2 \right\}$$

where

$$c_{2} = y_{2} + \left(1 + r_{1}^{1}\right)y_{1} + \left(z - 1 - r_{1}^{1}\right)k$$
$$- \left(1 + r_{1}^{1}\right)\left(1 + r_{0}^{1}\right)\left(d_{0} - d_{0}^{2}\right) - \left(1 + r_{0}^{2}\right)d_{0}^{2}.$$

The first order condition

$$\mathbb{E}_z\left\{\frac{z-1-r_1^1}{c_2}\right\} = 0$$

The solution is given by

$$k\left(r_{1}^{1}\right) = \bar{k}\left(r_{1}^{1}\right)\left[\left(1+r_{1}^{1}\right)\left(y_{1}-\left(1+r_{0}^{1}\right)d_{0}\right)+y_{2}+\left(\left(1+r_{1}^{1}\right)\left(1+r_{0}^{1}\right)-\left(1+r_{0}^{2}\right)\right)d_{0}^{2}\right]$$

where $\bar{k}(r_1^1)$ solves

$$\mathbb{E}_{z}\left[\frac{1}{\bar{k}\left(r_{1}^{1}\right)+\frac{1}{z-1-r_{1}^{1}}}\right]=0$$

with

$$\frac{\partial \bar{k}\left(r_{1}^{1}\right)}{\partial r_{1}^{1}} = -\frac{\mathbb{E}_{z}\left[\frac{\frac{1}{\left(z-1-r_{1}^{1}\right)^{2}}}{\left(\bar{k}(r_{1}^{1})+\frac{1}{z-1-r_{1}^{1}}\right)^{2}}\right]}{\mathbb{E}_{z}\left[\frac{1}{\left(\bar{k}(r_{1}^{1})+\frac{1}{z-1-r_{1}^{1}}\right)^{2}}\right]} < 0.$$

The investment in a high interest rate state relative to a low interest rate state, $r_h > r_l$,

$$\frac{k(r_h)}{k(r_l)} = \frac{\bar{k}(r_h)}{\bar{k}(r_l)} - \frac{\bar{k}(r_h)}{\bar{k}(r_l)} - \frac{(1+r_h)(y_1 - (1+r_0^1)d_0) + y_2 + ((1+r_h)(1+r_0^1) - (1+r_0^2))d_0^2}{(1+r_l)(y_1 - (1+r_0^1)d_0) + y_2 + ((1+r_l)(1+r_0^1) - (1+r_0^2))d_0^2}.$$
(10)

The proof of propositions 1 and 2 follow from differentiating this expression with respect to leverage and the maturity of the debt in the first period.

Proof of Proposition 1: Differentiating (10) with respect to leverage

$$\begin{aligned} \frac{\partial \left(\frac{k(r_h)}{k(r_l)}\right)}{\partial d_0} &= -\frac{\bar{k}\left(r_h\right)}{\bar{k}\left(r_l\right)} \\ &\cdot \frac{\left(r_h - r_l\right)\left(1 + r_0^1\right)\left(y_2 - \left(1 + r_0^2\right)d_0^2\right)}{\left(y_2 + \left(1 + r_l\right)y_1 - \left(1 + r_l\right)\left(1 + r_0^1\right)d_0 + \left(\left(1 + r_l\right)\left(1 + r_0^1\right) - \left(1 + r_0^2\right)\right)d_0^2\right)^2} < 0.\Box \end{aligned}$$

Proof of Proposition 2: Differentiating (10) with respect to the maturity of the debt,

$$\begin{aligned} \frac{\partial \left(\frac{k(r_h)}{k(r_l)}\right)}{\partial d_0^2} &= \frac{\bar{k}\left(r_h\right)}{\bar{k}\left(r_l\right)} \\ &\cdot \frac{\left(r_h - r_l\right)\left(1 + r_0^1\right)\left(y_2 - \left(1 + r_0^2\right)\left(d_0 - \frac{y_1}{1 + r_0^1}\right)\right)}{\left(y_2 + \left(1 + r_l\right)y_1 - \left(1 + r_l\right)\left(1 + r_0^1\right)d_0 + \left(\left(1 + r_l\right)\left(1 + r_0^1\right) - \left(1 + r_0^2\right)\right)d_0^2\right)^2} > 0.\Box \end{aligned}$$

D.1 Maturity decision

Given the investment decision in the interim period, the optimal debt maturity solve

$$\max_{d^2} \mathbb{E}_{r_1^1} \log \left[y_2 + \left(1 + r_1^1 \right) \left(y_1 - \left(1 + r_0^1 \right) d_0 \right) + \left(\left(1 + r_1^1 \right) \left(1 + r_0^1 \right) - \left(1 + r_0^2 \right) \right) d_0^2 \right]$$

with first order condition

$$\mathbb{E}_{r_1^1} \frac{\left(1+r_1^1\right)\left(1+r_0^1\right)-\left(1+r_0^2\right)}{y_2+\left(1+r_1^1\right)\left(y_1-\left(1+r_0^1\right)d_0\right)+\left(\left(1+r_1^1\right)\left(1+r_0^1\right)-\left(1+r_0^2\right)\right)d_0^2}=0.$$
(11)

When the expectation hypothesis holds, i.e., $1 + r_0^2 = (1 + r_0^1) \mathbb{E} (1 + r_1^1)$, $d_0^2 = d_0 - y_1 / (1 + r_0^1)$ solves the first order condition,

$$\mathbb{E}_{r_1^1} \frac{(1+r_1^1)(1+r_0^1) - (1+r_0^2)}{y_2 + \frac{1+r_0^2}{1+r_0^1}y_1 - (1+r_0^2) d_0} = \frac{\mathbb{E}_{r_1^1}\left[(1+r_1^1)(1+r_0^1) - (1+r_0^2)\right]}{y_2 + \frac{1+r_0^2}{1+r_0^1}y_1 - (1+r_0^2) d_0} = 0.$$

Assumption (7) implies that the amount of long term debt is a decreasing function of the term premium

$$\frac{\partial d_0^2}{\partial (1+r_0^2)} = -\frac{\mathbb{E}_{r_1^1} \left\{ \frac{y_2 + (1+r_1^1) (y_1 - (1+r_0^1)d_0)}{[y_2 + (1+r_1^1) (y_1 - (1+r_0^1)d_0) + ((1+r_1^1) (1+r_0^1) - (1+r_0^2))d_0^2]^2} \right\}}{\mathbb{E}_{r_1^1} \left\{ \frac{((1+r_1^1) (1+r_0^1) - (1+r_0^2))^2}{[[y_2 + (1+r_1^1) (y_1 - (1+r_0^1)d_0) + ((1+r_1^1) (1+r_0^1) - (1+r_0^2))d_0^2]^2]^2} \right\}} < 0.$$

Next, we consider the comparative statics of long term debt when there is a strictly positive term premium $1 + r_0^2 > (1 + r_0^1) \mathbb{E} (1 + r_1^1)$.

As before, the amount of long term debt is a decreasesing function of the cash flow in the interim period

$$\begin{split} \frac{\partial d_{0}^{2}}{\partial y_{1}} &= -\frac{\mathbb{E}_{r_{1}^{1}} \left\{ \frac{(1+r_{1}^{1})((1+r_{1}^{1})(1+r_{0}^{1})-(1+r_{0}^{2}))}{\left[y_{2}+(1+r_{1}^{1})(y_{1}-(1+r_{0}^{1})d_{0})+((1+r_{1}^{1})(1+r_{0}^{1})-(1+r_{0}^{2}))d_{0}^{2}\right]^{2}}\right\}}{\mathbb{E}_{r_{1}^{1}} \left\{ \frac{((1+r_{1}^{1})(1+r_{0}^{1})-(1+r_{0}^{2}))^{2}}{\left[y_{2}+(1+r_{1}^{1})(y_{1}-(1+r_{0}^{1})d_{0})+((1+r_{1}^{1})(1+r_{0}^{1})-(1+r_{0}^{2}))d_{0}^{2}\right]^{2}}\right\}} \\ &= -\frac{1}{1+r_{0}^{1}} \frac{\mathbb{E}_{r_{1}^{1}} \left\{ \frac{((1+r_{1}^{1})(1+r_{0}^{1})-(1+r_{0}^{2}))^{2}+(1+r_{0}^{2})((1+r_{1}^{1})(1+r_{0}^{1})-(1+r_{0}^{2}))d_{0}^{2}\right]^{2}}{\left[y_{2}+(1+r_{1}^{1})(y_{1}-(1+r_{0}^{1})d_{0})+((1+r_{1}^{1})(1+r_{0}^{1})-(1+r_{0}^{2}))d_{0}^{2}\right]^{2}}\right\}} \\ &= -\frac{1}{1+r_{0}^{1}} \frac{\mathbb{E}_{r_{1}^{1}} \left\{ \frac{((1+r_{1}^{1})(1+r_{0}^{1})-(1+r_{0}^{2}))}{\left[y_{2}+(1+r_{1}^{1})(y_{1}-(1+r_{0}^{1})d_{0})+((1+r_{1}^{1})(1+r_{0}^{1})-(1+r_{0}^{2}))d_{0}^{2}\right]^{2}}\right\}}{-\frac{1+r_{0}^{2}}} \frac{\mathbb{E}_{r_{1}^{1}} \left\{ \frac{(1+r_{1}^{1})(y_{1}-(1+r_{0}^{1})d_{0})+((1+r_{1}^{1})(1+r_{0}^{1})-(1+r_{0}^{2}))d_{0}^{2}\right]^{2}}{\left[y_{2}+(1+r_{1}^{1})(y_{1}-(1+r_{0}^{1})d_{0})+((1+r_{1}^{1})(1+r_{0}^{1})-(1+r_{0}^{2}))d_{0}^{2}\right]^{2}}\right\}}{\mathbb{E}_{r_{1}^{1}} \left\{ \frac{(1+r_{1}^{1})(y_{1}-(1+r_{0}^{1})d_{0})+((1+r_{1}^{1})(1+r_{0}^{1})-(1+r_{0}^{2}))d_{0}^{2}\right]^{2}}\right\}}{\mathbb{E}_{r_{1}^{1}} \left\{ \frac{(1+r_{1}^{1})(y_{1}-(1+r_{0}^{1})d_{0})+((1+r_{0}^{1})(1+r_{0}^{1})-(1+r_{0}^{2}))d_{0}^{2}\right]^{2}}\right\}}{\mathbb{E}_{r_{1}^{1}} \left\{ \frac{(1+r_{0}^{1})(y_{1}-(1+r_{0}^{1})d_{0})+((1+r_{0}^{1})(1+r_{0}^{1})-(1+r_{0}^{2}))d_{0}^{2}\right\}}{\mathbb{E}_{r_{1}^{1}} \left\{ \frac{(1+r_{0}^{1})(y_{1}-(1+r_{0}^{1})d_{0})+((1+r_{0}^{1})(1+r_{0}^{1})-(1+r_{0}^{2}))d_{0}^{2}\right\}}\right\}}{\mathbb{E}_{r_{1}^{1}} \left\{ \frac{(1+r_{0}^{1})(y_{1}-(1+r_{0}^{1})d_{0})+((1+r_{0}^{1})(1+r_{0}^{1})-(1+r_{0}^{2}))d_{0}^{2}\right\}}\right\}}{\mathbb{E}_{r_{1}^{1}} \left\{ \frac{(1+r_{0}^{1})(y_{1}-(1+r_{0}^{1})(y_{1}-(1+r_{0}^{1})d_{0})+((1+r_{0}^{1})(1+r_{0}^{1})-(1+r_{0}^{2}))d_{0}^{2}\right\}}\right\}}$$

The first term equals the effects of y_1 on d_0^2 when the entrepreneur is not exposed to interest rate risk. As the cash flow in the iterim period increases, more of the initial leverage can be repaid in one period and, therefore, less long term debt needs to be issuaced. The sign of the second terms follows from (11) and the fact that when $d_0^2 < d_0 - y_1 / (1 + r_0^1)$ the net worth in the interim

period, $y_2 + (1 + r_1^1) (y_1 - (1 + r_0^1) (d_0 - d_0^2)) - (1 + r_0^2) d_0^2$, is a decreasing function of r_1^1 . The second term captures the effect of changes in the net-worth on the demand for insurance. In general, the sign of this term depends on the coefficient of risk aversion. In our log case, the coefficient of absolute risk aversion is a strictly decreasing function of networth. Therefore, the second term is negative.

All in all, when the term premium is positive the is a larger sensitivity of the long term issuance to the cash flow in the interim period

$$\frac{\partial d_0^2}{\partial y_1} < -\frac{1}{1+r_0^1} = \left. \frac{\partial d_0^2}{\partial y_1} \right|_{1+r_0^2 = (1+r_0^1)\mathbb{E}(1+r_1^1)}.$$

Related, the amount of long term debt is a decreasesing function of the cash flow in the last period y_2

$$\frac{\partial d_0^2}{\partial y_2} = -\frac{\mathbb{E}_{r_1^1} \left\{ \frac{\left(\left(1+r_1^1 \right) \left(1+r_0^1 \right) - \left(1+r_0^2 \right) \right)}{\left[y_2 + \left(1+r_1^1 \right) \left(y_1 - \left(1+r_0^1 \right) d_0 \right) + \left(\left(1+r_1^1 \right) \left(1+r_0^1 \right) - \left(1+r_0^2 \right) \right) d_0^2 \right]^2} \right\}}{\mathbb{E}_{r_1^1} \left\{ \frac{\left(\left(1+r_1^1 \right) \left(1+r_0^1 \right) - \left(1+r_0^2 \right) \right)^2}{\left[\left[y_2 + \left(1+r_1^1 \right) \left(y_1 - \left(1+r_0^1 \right) d_0 \right) + \left(\left(1+r_1^1 \right) \left(1+r_0^2 \right) - \left(1+r_0^2 \right) \right) d_0^2 \right]^2} \right\}} < 0$$

As was the case when considering the effect of the cash-flow in the interim period, as the coefficient of risk aversion is decreasing, the demand for insurance is a decreasing function of the cash flow in the last period.

We are now ready to prove Proposition 3.

Proof of Proposition 3: Totally differentiating (10) with respect to y_1

$$\begin{aligned} \frac{d\left(\frac{k(r_h)}{k(r_l)}\right)}{dy_1} &= \frac{\partial\left(\frac{k(r_h)}{k(r_l)}\right)}{\partial y_1} + \frac{\partial\left(\frac{k(r_h)}{k(r_l)}\right)}{\partial d_0^2} \frac{\partial d_0^2}{\partial y_1} \\ &= \frac{\bar{k}\left(r_h\right)}{\bar{k}\left(r_l\right)} \frac{(r_h - r_l)}{(y_2 + (1 + r_l) y_1 - (1 + r_l) (1 + r_0^1) d_0 + ((1 + r_l) (1 + r_0^1) - (1 + r_0^2)) d_0^2)^2} \\ &\left[\left(y_2 - (1 + r_0^2) d_0^2\right) \right] \\ &- \left(y_2 + (1 + r_0^2) \left(\frac{y_1}{1 + r_0^1} - d_0\right)\right) \frac{\mathbb{E}_{r_1^1} \left\{\frac{(1 + r_1^1)(1 + r_0^1)}{\left[\frac{y_2 + (1 + r_1^1)(1 + r_0^1) - (1 + r_0^2)}{\left((1 + r_1^1)(1 + r_0^1) - (1 + r_0^2)\right) + d_0^2\right]^2}\right\}} \\ &\left[\mathbb{E}_{r_1^1} \left\{\frac{1}{\left[\frac{y_2 + (1 + r_1^1)(1 + r_0^1) - (1 + r_0^2)}{\left((1 + r_1^1)(1 + r_0^1) - (1 + r_0^2)\right) + d_0^2\right]^2}\right\} \right] \end{aligned}$$

$$= \frac{\bar{k}(r_h)}{\bar{k}(r_l)} \frac{(r_h - r_l)}{\left(y_2 + (1 + r_l)y_1 - (1 + r_l)\left(1 + r_0^1\right)d_0 + \left((1 + r_l)\left(1 + r_0^1\right) - \left(1 + r_0^2\right)\right)d_0^2\right)^2}{(1 + r_1^1)(1 + r_0^1) + d_0^2\left((1 + r_1^1)\left(1 + r_0^1\right) - (1 + r_0^2)\right)}{\frac{(1 + r_1^1)(1 + r_0^1) - (1 + r_0^2)}{\left(\frac{1 + r_1^1}{(1 + r_1^1)(1 + r_0^1) - (1 + r_0^2)}\right)}} \left\{\frac{\frac{y_2 + \left(\frac{y_1}{1 + r_0^1} - d_0\right)\left(1 + r_1^1\right)\left(1 + r_0^1\right) + d_0^2\left((1 + r_1^1)(1 + r_0^1) - (1 + r_0^2)\right)}{\left(\frac{y_2 + (1 + r_1^1)y_1 - (1 + r_1^1)(1 + r_0^1)d_0}{\left((1 + r_1^1)(1 + r_0^1) - (1 + r_0^2)\right)} + d_0^2\right]^2}\right\}$$

$$= -\frac{\bar{k}(r_h)}{\bar{k}(r_l)} \frac{(r_h - r_l)(1 + r_0^2)}{(y_2 + (1 + r_l)y_1 - (1 + r_l)(1 + r_0^1)d_0 + ((1 + r_l)(1 + r_0^1) - (1 + r_0^2))d_0^2)^2} \\ \mathbb{E}_{r_1^1} \frac{((1 + r_1^1)(1 + r_0^1) - (1 + r_0^2))}{(y_2 + (1 + r_1^1)y_1 - (1 + r_1^1)(1 + r_0^1)d_0 + ((1 + r_1^1)(1 + r_0^1) - (1 + r_0^2))d_0^2} \\ = 0.$$

where the last equality follows from the first order condition in (11). Similarly, totally differentiating (10) with respect to y_2

$$\begin{split} \frac{d\left(\frac{k(r_{l})}{k(r_{l})}\right)}{dy_{2}} &= \frac{\partial\left(\frac{k(r_{l})}{k(r_{l})}\right)}{\partial y_{2}} + \frac{\partial\left(\frac{k(r_{l})}{k(r_{l})}\right)}{\partial d_{0}^{2}} \frac{\partial d_{0}^{2}}{\partial y_{2}} \\ &= \frac{\bar{k}\left(r_{h}\right)}{\bar{k}\left(r_{l}\right)} \frac{\left(r_{h} - r_{l}\right)}{\left(y_{2} + \left(1 + r_{l}\right)y_{1} - \left(1 + r_{l}\right)\left(1 + r_{0}^{1}\right)d_{0} + \left(\left(1 + r_{l}\right)\left(1 + r_{0}^{1}\right) - \left(1 + r_{0}^{2}\right)\right)d_{0}^{2}\right)^{2}} \\ &\left[-\left(y_{1} - \left(1 + r_{0}^{2}\right)\left(\frac{y_{1}}{1 + r_{0}^{1}} - d_{0}\right)\right) \frac{\mathbb{E}_{r_{1}^{1}}\frac{\left(\frac{\left(1 + r_{0}^{1}\right)}{\left(\left(1 + r_{1}^{1}\right)\left(1 + r_{0}^{1}\right) - \left(1 + r_{0}^{2}\right)\right)}^{2}}{\mathbb{E}_{r_{1}^{1}}\frac{\left[\frac{y_{2} + \left(1 + r_{1}^{2}\right)y_{1} - \left(1 + r_{0}^{1}\right)d_{0} + d_{0}^{2}\right]^{2}}{\left(1 + r_{0}^{1}\right)\left(1 + r_{0}^{1}\right)d_{0} + \left(1 + r_{l}\right)y_{1} - \left(1 + r_{0}^{1}\right)d_{0} + d_{0}^{2}\right)^{2}} \\ &= \frac{\bar{k}\left(r_{h}\right)}{\bar{k}\left(r_{l}\right)}\frac{\left(r_{h} - r_{l}\right)y_{1} - \left(1 + r_{l}\right)\left(1 + r_{0}^{1}\right)d_{0} + \left(\left(1 + r_{l}\right)\left(1 + r_{0}^{1}\right) - \left(1 + r_{0}^{2}\right)\right)d_{0}^{2}\right)^{2}}{\left(1 + r_{0}^{1}\right)\left(1 + r_{0}^{1}\right)d_{0} + \left(\left(1 + r_{l}\right)\left(1 + r_{0}^{1}\right) - \left(1 + r_{0}^{2}\right)\right)d_{0}^{2}\right)^{2}} \\ &\left(-1\right)\left(1 + r_{0}^{1}\right)\mathbb{E}_{r_{1}^{1}}\left\{\frac{\left(\frac{y_{2} + \left(1 + r_{1}^{1}\right)\left(y_{1} - \left(0 + r_{0}^{1}\right)\right) + \left(\left(1 + r_{1}^{1}\right)\left(1 + r_{0}^{1}\right)d_{0}}{\left(1 + r_{0}^{1}\right)\left(1 + r_{0}^{2}\right)d_{0}^{2}}\right)^{2}}\right\}$$

$$= -\frac{\bar{k}(r_h)}{\bar{k}(r_l)} \frac{(r_h - r_l)(1 + r_0^1)}{(y_2 + (1 + r_l)y_1 - (1 + r_l)(1 + r_0^1)d_0 + ((1 + r_l)(1 + r_0^1) - (1 + r_0^2))d_0^2)^2} \\ \mathbb{E}_{r_1^1} \frac{((1 + r_1^1)(1 + r_0^1) - (1 + r_0^2))}{(y_2 + (1 + r_1^1)y_1 - (1 + r_1^1)(1 + r_0^1)d_0 + ((1 + r_1^1)(1 + r_0^1) - (1 + r_0^2))d_0^2} \\ = 0.$$

where the last equality follows from the first order condition in (11). Finally, totally differentiating (10) with respect to $1 + r_0^2$

$$\begin{split} \frac{d\left(\frac{k(r_{h})}{k(r_{l})}\right)}{d\left(1+r_{0}^{2}\right)} &= \frac{\partial\left(\frac{k(r_{h})}{k(r_{l})}\right)}{\partial\left(1+r_{0}^{2}\right)} + \frac{\partial\left(\frac{k(r_{h})}{k(r_{l})}\right)}{\partial d_{0}^{2}} \frac{\partial d_{0}^{2}}{\partial\left(1+r_{0}^{2}\right)} \\ &= \frac{\bar{k}\left(r_{h}\right)}{\bar{k}\left(r_{l}\right)} \frac{\left(r_{h}-r_{l}\right)}{\left(y_{2}+\left(1+r_{l}\right)y_{1}-\left(1+r_{l}\right)\left(1+r_{0}^{1}\right)d_{0}+\left(\left(1+r_{l}\right)\left(1+r_{0}^{1}\right)-\left(1+r_{0}^{2}\right)\right)d_{0}^{2}\right)^{2}} \\ &\left[d_{0}^{2}\left(\frac{y_{1}}{1+r_{0}^{1}}-d_{0}+\left(1+r_{0}^{1}\right)d_{0}^{2}\right) \\ &- \left(y_{2}+\left(1+r_{0}^{2}\right)\left(\frac{y_{1}}{\left(1+r_{0}^{1}\right)}-d_{0}\right)\right) \frac{\mathbb{E}_{r_{1}^{1}}\frac{\frac{y_{2}+\left(1+r_{1}^{1}\right)y_{1}-\left(1+r_{1}^{1}\right)\left(1+r_{0}^{1}\right)d_{0}}{\left(\frac{y_{1}+r_{1}^{1}\right)\left(1+r_{0}^{1}\right)-\left(1+r_{0}^{2}\right)^{2}} \\ &= c_{0}, \end{split}$$

where the inequality follows from $y_1/(1+r_0^1) - d_0 + (1+r_0^1) d_0^2 \le 0$ (which is itself implied by $1 + r_0^2 \ge (1+r_0^1)\mathbb{E}(1+r_1^1)$), and conditions (7) and (9). \Box