Expecting the unexpected: Economic growth under stress

Gloria González-Rivera*
Department of Economics
University of California, Riverside

Vladimir Rodríguez-Caballero
Department of Statistics, ITAM (Mexico)
and CREATES, Aarhus University (Denmark)

Esther Ruiz
Department of Statistics
Universidad Carlos III de Madrid

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Abstract

We construct the US conditional growth densities under stressed factor scenarios and propose a vulnerability index, Growth-in-Stress (GiS), to assess the level of exposure of the economy to small probability but potentially catastrophic events. The choice of severe yet plausible stress scenarios is based on the joint probability distribution of the driving factors of growth. The factors are extracted with a multi-level Dynamic Factor Model (DFM) from a wide set of local and worldwide macroeconomic and financial variables. All together, we provide a risk management tool that allows for a complete visualization of growth dynamics under average and probabilistic stressed scenarios where warning signals are coming from the quantiles in the left tails of the average and stressed growth densities. We show that GiS is a useful and complementary tool to Growth-at-Risk (GaR) for policy makers wishing to carry out a multi-dimensional scenario analysis and illustrate their implementation in the context of the COVID19 pandemic.

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*Corresponding author. E-mail: gloria.gonzalez@ucr.edu

1 Introduction

In hindsight, the COVID-19 induced decline in GDP growth across the world economies had at least three common features. First, the decline was almost synchronous and worldwide; second, the magnitude of the decline was extraordinary by historical standards; and third, it was unpredictable. Given this historical experience and its lack of predictability, it seems natural to ask macroeconomists and policy makers for the development of new tools to recreate extreme scenarios and provide warning signals of what to expect under the possibility of extreme events. Properly speaking, we cannot characterize this exercise as forecasting but we can have the foresight of creating a virtual future that will teach us how resilient the present economic systems are.

The increasingly growing literature on macroeconomic risk measurement is even more relevant now precisely because of the havoc generated by the COVID pandemic. It is desirable for policy makers to be prepared for extreme and unexpected shocks that will generate severe recessions as well as for the implementation of corrective measures to minimize their risks; see Kilian and Manganelli (2008) and Alessi et al. (2014) for the importance of having appropriate measures of risk for policy makers and central banks, respectively. The high costs of recessions underscore the need to strengthen the resilience of the economies, notably by assessing early on potential vulnerabilities that can lead to such costly events; see the discussion in Rohn et al. (2015), who describe more than 70 vulnerability indicators that could be monitored to assess country risks in OECD economies, and Ludvigson et al. (2021), who discuss the economic costs of the COVID19 pandemic in US.

Recent popular macroeconomic risk indexes are based on estimates of the full probability distribution of growth; see Ravazzolo and Rothman (2016) for an early contribution proposing a recession index indicator for US GDP based on the growth density modelled as a function of oil prices, and De Nicoló and Luccetta (2017), who propose using factor-augmented quantile regressions for industrial production growth and employment growth. More recently, Adrian et al. (2019) propose the Growth at Risk (GaR) index, which is a lower quantile of the growth density modelled as a function of "local" underlying financial factors. GaR has been adopted by the IMF as the main quantitative criterion to gauge global financial stability risk; see Prasad et al. (2019) and Adrian et al. (2020) for descriptions of the use of GaR at the International Monetary Fund (IMF).

Conceptually, GaR mimics the spirit of the popular Value-at-Risk (VaR) measure in finance and, consequently, it also shares its caveats. In particular, it is well known that VaR is not designed to measure financial risk under stressed conditions; see Borio et al. (2014) and Flood and Korenko (2015) for very interesting discussions on stress testing and Wang and Ziegel (2021) for stress scenarios in the area of finance. When dealing with economic growth, stress refers to the analysis of the conditional distribution of GDP growth, when exposed to extreme financial and/or real shocks, which are rare and large in magnitude relative to those shocks expected during tranquil times. As it happens with VaR, GaR is not designed to measure growth densities in stressed conditions and, consequently, it could not be adequate to measure the exposure of growth to potential extreme risks; see, for example, Plagborg-Möller et al. (2020), who show that GaR did not yield useful advanced warnings of tail risk during the COVID19 pandemic.

This paper adds to the important literature on growth vulnerability by proposing an index that measures the response of the economy when there are large and unexpected shocks to the underlying growth factors, which could be either macroeconomic or financial and local or international. We implement the proposed index to the US economy. First, we obtain the factor structure that drives the distribution of growth by estimating a novel multi-level dynamic factor model (DFM) that considers a vast array of worldwide and local macroeconomic and financial variables. The multi-level factor model is an extension of that proposed by Breitung and Eickmeier (2015), which allows for overlapping blocks of factors. In the application to US GDP growth, we find a first pervasive factor common to all variables in the system, a second semi-pervasive factor common to the worldwide variables, and three additional non-pervasive factors that are common to three different subsets of variables (worldwide financial, local macroeconomic, and worldwide macroeconomic variables). We also find that the local financial factors (key to model GaR) do not provide any additional information to explain US GDP growth risk once worldwide financial and macroeconomic factors are taking into account. With the estimated factor structure and similarly to Adrian et al. (2019), we proceed to estimate factor-augmented quantile regressions to obtain the one-step-ahead (and multi-step) conditional probability distribution of GDP growth.

The conditional distribution of GDP growth delivers any quantile of interest under normal circumstances, that is, when the factors are around their average values. Lower quantiles, like the 5% or 1% tails, provide an estimation of potentially large but expected decline in growth (GaR). However, under unexpected and rare circumstances, the factors underlying the distribution of growth are also under stress and thus, far from their average values. We quantify stress in the factors in a probabilistic way by considering the multivariate distribution of the factors and focusing on the values in the tails of their multivariate distribution.[1] These values are the probabilistic stress scenarios to evaluate growth risk. We analyse the distribution of growth under different stressed-factors scenarios by implementing the Growth-in-Stress (GiS) methodology proposed by González-Rivera et al. (2019). For 2020Q2, US growth risk estimated by the 5%-quantile GaR was -15.29% (annualized quarter-over-quarter growth) and by the 5% quantile GiS with 95% stress in the factors was -29.13%. The observed growth decline was -31.20% according to the IMF. The warning provided by GaR was rather conservative not only in magnitude but also by considering a partial set of local financial factors when in the COVID crisis the extreme shock was mainly affecting the worldwide macroeconomic factors. We show that popular growth risk measures as the GaR need to be complemented with scenario analysis. The warning provided by GiS is helpful to policy makers to evaluate the trade-off between building greater resilience in normal times and reduce downside risk in highly stressed periods; see Adrian and Liang (2018) for a discussion of this trade-off.

The rest of the paper is organized as follows. In Section 2, we describe the methodology to obtain GiS. In particular, we describe how to estimate a multi-level DFM to extract the relevant factors. We also describe the construction of the conditional densities of growth in a “normal” scenario as well as in “stressed” scenarios for different levels of stress of the underlying factors. In Section 3, we obtain the US conditional distribution of growth based on factors extracted from local and worldwide financial and macroeconomic variables. In Section 4, we estimate the probability distribution of US GDP growth under different stressed-factor scenarios. In Section 5, we conclude and offer some final considerations.

[1]The multivariate probability distribution of the factors is based on the asymptotic distribution of Principal Components (PC) factors corrected by a subsampling procedure to account for parameter uncertainty.

2 Growth in Stress

We describe the methodology to estimate the probability distribution of growth based on factor-augmented quantile regressions and the computation of GiS and GaR measures of growth risk.

2.1 The conditional distribution of growth

Let GDP_t be the Gross Domestic Product (GDP) observed quarterly at time t, for $t = 1, ..., T$. Define the annualized quarter-over-quarter growth as $y_t = 400 \times \triangle log(GDP_t)$. We obtain the h-step ahead conditional distribution of growth by estimating factor-augmented quantile regressions for several quantiles τ^*

$$q_{\tau^*}\left(y_{t+h}|y_t, F_t\right) = \mu(\tau^*, h) + \phi(\tau^*, h)y_t + \sum_{k=1}^{r} \beta_k(\tau^*, h)F_{kt}, \tag{1}$$

where $\mu(\tau^*, h), \phi(\tau^*, h)$ and $\beta_k(\tau^*, h)$ are parameters and $F_t = (F_{1t}, ..., F_{rt})'$ is the $r \times 1$ vector of underlying unobserved factors at time t, which embed the information contained in a large number of potential predictors of the quantiles of growth, X_t, which is a set of N macroeconomic and/or financial variables.

The factor-augmented quantile-regression model in (1) is appropriate for representing the potentially asymmetric and non-linear relationship between economic growth and the underlying factors; see, for instance, Plagborg-Möller et al. (2020) for evidence about asymmetries in economic growth fluctuations. Factor-augmented quantile regressions is a standard approach in modelling growth quantiles; see, Manzan (2005), Giglio et al. (2016), Adrian et al. (2019), González-Rivera et al. (2019), Adrian et al. (2019), and Adrian et al. (2022), among others.[2] In practice, the underlying factors in (1) are replaced by estimated factors, $\hat{F}$.

The parameters in equation (1) are estimated using the algorithm by Koenker and d'Orey (1987), which implements the estimator proposed by Koenker and Bassett (1978); see Ando and Tsay (2011) and Giglio et al. (2016) for its asymptotic properties. For a given quantile τ^*, and horizon h, the goodness of fit of the estimated factor-augmented

[2]De Nicoló and Luccetta (2017) also fit factor-augmented quantile regressions to measure the tail risk of industrial production and employment in US.

quantile regressions is estimated by $R^1 = 1 - \frac{\sum_{t=2}^{T} \hat{\nu}_t[\tau^* I(\hat{\nu}_t \geq 0) + (\tau^* - 1) I(\hat{\nu}_t < 0)]}{\sum_{t=1}^{T} y_t[\tau^*(I(y_t \geq \bar{y}) + (\tau^* - 1) I(y_t < \bar{y})]}$, where $\hat{\nu}_t = y_t - \hat{\mu}(\tau^*, h) - \hat{\phi}(\tau^*, h) y_{t-h} - \sum_{k=1}^{r} \hat{\beta}_k(\tau^*, h) F_{kt-h}$ and $\bar{y}$ is the sample mean of y_t; see Koenker and Machado (1999). Note that R^1 is the natural analogue of the R^2 coefficient in a regression model.

After estimating (1) for different quantiles $\tau^* = \{5\%, 10\%, ...\}$, we obtain the conditional distribution of growth by fitting the Skewed-t distribution of Azzalini and Capitanio (2003) to the estimated quantiles, $\hat{q}_{\tau^*}(y_{t+h}|y_t, F_t)$. At each moment of time t, we estimate the four parameters that defined the Skewed-t by minimizing the squared distance between the estimated quantiles and the corresponding quantiles of the Skewed-t distribution. Denote this density by $\hat{k}_0(y_{t+h})$.[3]

2.2 The conditional distribution of growth under stress

Adrian et al. (2019) propose measuring the h-step ahead growth risk at time t by GaR, which is defined as the τ quantile, most popular $\tau = 0.05$, of the estimated conditional distribution of growth, $\hat{k}(y_{t+h})$. GaR is an extreme left quantile of the distribution of growth estimated as a function of the underlying estimated factors. Note that GaR is computed under "non-stressed" conditions, i.e., when the underlying factors are fixed at their estimated averages, $\hat{F}_t$. Consequently, GaR measures the vulnerability of the economy in the current scenario. However, if an extreme event were to shock the economy, it would be of interest to analyze the probabilistic distribution of growth under unusual extreme circumstances. The extreme conditions will be reflected in the behavior of the factors that drive growth. Considering the factors themselves under stress, González-Rivera et al. (2019) propose GiS as an additional measure of vulnerability.

Consider the factor-augmented quantile regression in (1). For a fixed quantile τ^*, define the minimum value of the τ^*-quantile of growth when the underlying factors are

[3]Finding the uncertainty of the quantiles of this smoothed estimate of the conditional density of growth is a challenging and interesting problem. It could be possible to obtain confidence intervals for the quantiles of the smoothed conditional density by bootstrapping the factor-augmented quantile predictive regressions in (1); see Gregory et al. (2018) and Gonçalves et al. (2017) for bootstrapping in the context of quantile regressions and factor-augmented regressions, respectively. For each bootstrap replicate of the quantiles, one can obtain the smoothed density of growth. Based on a large number of bootstrap replicates of these smoothed densities, we could construct confidence intervals for the risk measures GaR and GiS. The computational burden involved in these simulations can be alleviated by using the fast bootstrap procedures proposed by Chernozhukov et al. (2022) in the context of quantile regressions. However, the statistical properties of this bootstrap procedure are unexplored.

subject to α-probability stressed scenarios, as follows,

$$\min_{F_t} q_{\tau^*}(y_{t+h}|y_t, F_t) \tag{2}$$

$$s.t. \quad g(F_t, \alpha) = 0,$$

where $g(F_t, \alpha) = 0$ is the α-probability contour, an ellipsoid that contains the true factor vector, F_t, with probability α. The values of F_t on the boundary of the ellipsoid $g(F_t, \alpha) = 0$ are considered the extreme events of the factors.

In general, the constrained optimization (2) requires the estimation of the iso-quantile surfaces, i.e., the combination of factors that generates the same value of the τ^*-quantile, as well as of the tangency point between these surfaces and the α-ellipsoid of the factors. When the number of underlying factors is larger than two, the constrained minimization is solved by using the simple binary mesh algorithm proposed by Flood and Korenko (2015).[4]

The optimization exercise in (2) is repeated for different τ^*-quantiles (keeping the α-level of stress fixed). After fitting a Skewed-t density to the minimal growths corresponding to different estimated τ^*-quantiles, the conditional "stressed" density of growth is obtained. Denote this stressed density as $\hat{k}_\alpha(y_{t+h})$. Finally, for an α-level of stress of the factors, the h-step-ahead GiS is given by the τ-quantile of this stressed density as follows

$$GiS_{t+h} = inf\left\{y_{t+h} \mid \int_{-\infty}^{y_{t+h}} \hat{k}_\alpha(u)du \geq \tau\right\}. \tag{3}$$

To illustrate the computation of GiS, let us examine the following example.[5] We consider the following quantile regression model for $\tau^* = 0.05$ in which growth depends on two factors, F_{1t} and F_{2t},

$$q_{0.05}\left(y_{t+1}|F_t\right) = -3.5 - 0.7F_{1t} + 1.5F_{2t}. \tag{4}$$

[4]Software is available in https://cran.r-project.org/web/packages/SySc Selection/index.html. In a spaced grid or mesh on the ellipsoid, the fineness parameter determines the number of points iterated along each dimension until the optimal combination of points is found. We choose a fineness parameter of 8. We have experimented with several values of the fineness parameter and our results are very robust to this choice.

[5]In this example, we are not smoothing the densities but considering the quantiles as directly obtained from the factor-augmented quantile predictive regressions.

Figure 1 plots combinations of F_{1t} and F_{2t} for which we obtain the same value of the 5% quantile of growth; these are the iso-5%-quantile lines in red. Suppose that, at time t, the estimated factors are $\hat{F}_{1t} = 5$ and $\hat{F}_{2t} = 2$, which implies a GaR of -4. In Figure 1, we also plot the α-probability ellipses of the factors for $\alpha = 0.7$ and 0.95, in highlighted blue. GiS is the tangency point between the α-ellipse and the iso-5%-quantile line. Thus, for $\alpha = 0.7$, the GiS is -8.5, and for $\alpha = 0.95$, the GiS is -11.5. These are big differences with GaR = -4. The reason being that GaR is calculated under "normal" circumstances, that is, when the factors are fixed at their estimated averages, which correspond to the central point of the ellipse in Figure 1.

The GiS measures the risk exposure of the economy to extreme movements in the underlying factors that drive growth. The policy maker could choose different α-levels of stress and generate the corresponding stressed densities of growth and GiS values.[6] For policy makers, knowledge of the growth density under stressed factors is a tool to assess whether the economy is too exposed to any of the factors and, if so, how to act to reduce exposure. In this sense, GiS underscores the arguments in Breuer et al. (2009), who argue that measures based on historical experience, as GaR, may risk to ignore plausible but harmful scenarios, as those we currently observe as a result of the COVID19 pandemic. The probability contours of the underlying factors provide a benchmark for plausibility and severity of the stressed factors. GiS captures plausibility by specifying how much stress to exercise into the tails of the factors' distribution, while severity is maximized by systematically searching for the worst growth case in the factor region determined by the chosen level of stress; see also Flood and Korenko (2015) and Breuer et al. (2009) for discussions on the trade-off between plausibility and severity of stress scenarios.

2.3 Stressing the factors

Calculating GiS in (3) requires to estimate not only the factor-augmented quantile regression in (1) and the corresponding conditional densities of growth but also the multivariate

[6] In this set up, the α-level of stress is chosen by the decision maker. It might be possible to choose α in an optimal way if the decision maker were to have a loss function that depends on GiS somehow. However, this is a different research question that may fit within the context of a very recent paper, Manski (2021), who proposes the use of confidence sets for decision problems. The discussions by Granger and Machina (2006), Elliot and Timmermann (2016) and Watson and Holmes (2016) may also be relevant.

probability density of the factors. From that density, it is possible to construct probability contours of the factors $g(F_t, \alpha) = 0$ at a desired probability level α, say $\alpha = 95\%$, so that the contour is an ellipsoid that contains 95% of the values of F_t, with the most extreme 5% of events outside of the ellipsoid.[7]

For completeness, we briefly describe the procedure to construct the probability ellipsoids for the factors. Consider the following static DFM for the variables in X_t

$$X_t = PF_t + \varepsilon_t, \tag{5}$$

where P is the $N \times r$ matrix of factor loadings and ε_t is the $N \times 1$ vector of idiosyncratic components, which are allowed to be weakly cross-sectionally correlated but uncorrelated with the factors.[8] Furthermore, to uniquely identify the factors and loadings, we assume as usual in this literature that $\frac{F'F}{T} = I_r$, where $F = (F_1, ..., F_T)$ is an $r \times T$ matrix and $P'P$ is diagonal with its elements ordered from largest to smallest.

After determining the number of factors, r, they are extracted by a Principal Components (PC)-based procedure from X_t. Define $X = (X_1, ..., X_T)$. The PC factors, $\widehat{F}_t$, are given by $\sqrt{T}$ times the eigenvectors corresponding to the r largest eigenvalues of XX' arranged in decreasing order while $\widehat{P}' = \frac{1}{T}\widehat{F}'Y$. For example, Adrian et al. (2019) consider $r = 1$ factor extracted from a set of local financial variables.[9] In another application, González-Rivera et al. (2019) model the distribution of growth after extracting $r = 3$ factors from a set of international GDPs. Bai and Ng (2008) show that, as far as $\frac{T^{5/8}}{N} \to 0$ for $N, T \to \infty$, the PC estimated factors can be plugged into the factor-augmented quantile regression in (1) as if they were observed. However, in applications for which N and/or T are not too large, we need to consider the uncertainty associated with the estimated factors; see Aastveit et al. (2016), Jackson et al. (2016) and Thorsrud (2020) for the importance of taking into account factor uncertainty in empirical applications.

To stress the factors, one needs first to construct the density of the PC factors.

[7]This proposal is closely related to that of Haugh and Ruiz Lacedelli (2020), who carry out scenario analysis for derivative portfolios via DFMs expressed as state space models (SSMs) by computing and simulating from the distribution of unstressed risk factors conditional on a given scenario.

[8]Note that after imposing the adequate restrictions on the matrix of loadings, P, the static DFM in (5) can be written as a multi-level DFM.

[9]In particular, they consider the Chicago Fed's National Conditions Index (NFCI), which provides a weekly update on US financial conditions in money markets, debt and equity markets and the traditional and "shadow" banking systems.

González-Rivera et al. (2019) propose using the subsampling correction of the asymptotic distribution of the underlying factors of Maldonado and Ruiz (2021). In particular, at each moment of time, the PC factor vector can be written as a linear filter of the original observations as follows

$$\widehat{F}_t = \left(\widehat{P}'\widehat{P}\right)^{-1}\widehat{P}'X_t. \tag{6}$$

Under general conditions, Bai (2003) shows that, if $\frac{F'F}{T} = I_r$ and $\frac{\sqrt{N}}{T} \to 0$ when $N, T \to \infty$, at each moment of time, t, the asymptotic distribution of $\widehat{F}$ is given by

$$\sqrt{N}\left(\widehat{F}_t - F_t\right) \xrightarrow{d} N\left(0, \Sigma_P^{-1}\Gamma_t\Sigma_P^{-1}\right), \tag{7}$$

where $\Sigma_P = lim_{N\to\infty}\frac{P'P}{N}$ and $\Gamma_t = lim_{N\to\infty}\sum_{i=1}^{N}\sum_{j=1}^{N} p_i p_j' E(\varepsilon_{it}\varepsilon_{jt})$ with p_i' being the $1 \times r$ i'th row of $\widehat{P}$ and ε_{it} the idiosyncratic component corresponding to the i'th variable in X_t. The finite sample approximation of the asymptotic covariance matrix of $\widehat{F}_t$ can be estimated as follows

$$MSE_t = \left(\frac{\hat{P}'\hat{P}}{N}\right)^{-1}\frac{\hat{\Gamma}_t}{N}\left(\frac{\hat{P}'\hat{P}}{N}\right)^{-1}, \tag{8}$$

where $\hat{\Gamma}_t$ is an estimate of Γ. Under the assumption of cross-sectionally uncorrelated idiosyncratic errors, Bai and Ng (2006) propose estimating it as follows

$$\widehat{\Gamma}_t = \frac{1}{N}\sum_{i=1}^{N}\hat{p}_i\hat{p}_i'\hat{\varepsilon}_{it}^2, \tag{9}$$

where $\hat{\varepsilon}_{it} = x_{it} - \hat{p}_i'\hat{F}_t$ are the residuals from the DFM model.

Poncela and Ruiz (2016) show that the uncertainty of the factors is underestimated when based on their asymptotic distribution. To obtain more accurate confidence regions, Maldonado and Ruiz (2021) propose a correction to the asymptotic covariance matrix of the factors in (8). The correction is based on subsampling subsets of size N^* of series in the cross-sectional space, with each series containing all temporal observations. For each subsample, the loadings and factors are estimated as explained above, obtaining $\hat{F}_t^{*(b)}$ and $\hat{P}^{*(b)}$, for $b = 1, ..., B$. The subsampling analogue of the MSE, due to parameter uncertainty associated with the factor loadings $MSE(\hat{F}-F)$, is estimated as follows

$$\frac{1}{B}\sum_{b=1}^{B}\left(\left(\hat{F}_t^{*(b)}-\hat{F}_t\right)\left(\hat{F}_t^{*(b)}-\hat{F}_t\right)'\right). \tag{10}$$

Finally, the finite sample MSE of $\hat{F}$ is estimated as

$$MSE_t^* = \frac{1}{N}\left(\frac{\hat{P}'\hat{P}}{N}\right)^{-1}\hat{\Gamma}_t\left(\frac{\hat{P}'\hat{P}}{N}\right)^{-1}+\frac{N^*}{NB}\sum_{b=1}^{B}\left(\left(\hat{F}_t^{*(b)}-\hat{F}_t\right)\left(\hat{F}_t^{*(b)}-\hat{F}_t\right)'\right); \tag{11}$$

see Maldonado and Ruiz (2021) for the good properties of this MSE when used to construct confidence ellipsoids for the underlying factors.

By choosing different values of α in the constraint $g(F_t,\alpha)=0$, i.e. different levels of stress in the factors, GiS provides an analysis of growth under different scenarios.[10] In our context, the scenarios are different values and combinations of the estimated factors; see Wang and Ziegel (2021) for a similar proposal in the context of scenarios for financial risk. By working with the probability contours of the underlying factors, the policy maker can understand those scenarios where severe but plausible factor values may substantially affect economic growth.

3 The US conditional distribution of growth

In this section, we obtain the conditional distribution of US growth based on factors extracted from a multi-level DFM, which considers a large system of macroeconomic and financial variables, some of which are local to the US and some are worldwide.

3.1 Data

We consider annualized quarter-over-quarter real US GDP growth observed from 2005Q3 to 2021Q1. The in-sample period spans from 2005Q3 to 2020Q1 while the observations

[10]Scenario analysis is rather popular in the context of financial markets; see Glasserman et al. (2015), who identify sensible combinations of stresses to multiple factors to assess financial risk; Hagfors et al. (2016) for scenario analysis of electricity prices in the context of quantile regressions; European Central Bank (2006) for the importance of scenario analysis in the context of stress testing in the financial sector, Rebonato (2019) for financial stress testing based on Bayesian nets, and Haugh and Ruiz Lacedelli (2020) who carry out scenario analysis for derivative portfolios via DFMs. Finally, it is important to remark that the Basel Committee on Banking Supervision (2005) recommends choosing scenarios that are plausible and severe.

from 2020Q2 to 2021Q1 are reserved for out-of-sample exercises.

To construct quarterly predictive distributions of real GDP growth, we use conditioning information available at the moment the prediction is made. In particular, the factors underlying the conditional distribution of growth are extracted from a large set of variables observed quarterly from 2005Q3 to 2020Q1 ($T = 59$ observations). First, we consider the same local financial variables underlying the construction of the Chicago Fed's National Conditions Index (NFCI); see, Brave and Butters (2012) for a description of the NFCI. This subset of variables, denoted as X_{1t}, has cross-sectional dimension of $N_1 = 105$ variables.[11] After standardization, we analyze outliers using the procedure in Kristensen (2014). We find one outlier in the variable "T-note futures Euro/Dollar market depth" in 2008Q4.

Given the increasing globalization of the economy, we also consider the potential effect of worldwide financial factors on US growth; see, for example, Arregui et al. (2018), who show that the rapid speed at which foreign shocks affect domestic financial conditions may make it difficult to react in a timely and effective manner, if deemed necessary. The worldwide financial variables considered are the same as those in Arregui et al. (2018). They are denoted as X_{2t} and have cross-sectional dimension of $N_2 = 208$. As before, the worldwide financial variables are standardized and corrected for outliers. An outlier is found in Hungary 2015Q2, which may be due to the brokerage scandals in that year. A second outlier is found in Venezuela in 2018Q4, which may be attributed to large inflation and its repercussions in the stock market.

An important strand of the literature claims that macroeconomic variables are better suited than financial variables to explain the growth distribution. Consequently, we also consider the effect of local and worldwide macroeconomic factors on the conditional distribution of US growth. With this goal, we consider the popular database of McCracken and Ng (2016) with $N = 248$ variables; De Nicoló and Luccetta (2017) and Plagborg-Möller et al. (2020) also use this dataset to extract factors to estimate factor-augmented quantile regressions. This subset of variables is denoted as X_{3t}.

[11] The NFCI is constructed on a weekly basis. We average weekly observations within each quarter to obtain observations with a quarterly frequency. For the attribution of weeks to overlapping quarters, we follow the same criteria as Adrian et al. (2019). Weeks that start in one quarter and end in the next one are fully assigned to the latter quarter.

Finally, we also consider a set of annualized quarterly GDP growths of $N = 63$ countries. The GDPs have been obtained from the IMF with the sample of countries chosen to maximize the amount of common data among them. Note that in González-Rivera et al. (2019), the factors are extracted from a panel of annual growths corresponding to 83 countries obtained from the World Bank database. The countries considered, which are listed in the online appendix, represent 91.62% of the world GDP in 2019, according to data by the World Bank. We also look for outliers using the procedure described by Kristensen (2014). We find two outliers in Thailand 2011Q4 and 2012Q1. These outliers may be due to the severe flooding occurred during the 2011 monsoon season, which caused the fourth costliest economic disaster according to the World Bank; see Tanonue et al. (2020). China 2020Q1 and Ireland 2015Q1 are also outliers. We think that the main reason for the outlier in China is that the COVID19 affected China one quarter earlier than the rest of the world. With respect to the large Irish GDP growth, it could be due to the relocation of intellectual property of a number of large multinational corporations, which was triggered by the Irish low corporate tax rates. Given the size of these companies, the boost to GDP growth was correspondingly large. The subset of worldwide growths is denoted as X_{4t} with $N_4 = 63$ variables.

In summary, we denote $X_t^* = (X_{1t}, X_{2t}, X_{3t}, X_{4t})'$ the entire set of local and worldwide financial and macroeconomic variables from which a set of common factors will be extracted. The cross-sectional dimension of the full system X_t^* is $N = 624$ variables.

It is important to note that the US real GDP as well as all the variables X_t^* to extract the factors are final records at the time of writing. However, in most countries, national accounts are recorded quarterly and published late (often more than one month after the close of the quarter), and are subsequently revised. On the other hand, the variables published at a higher frequency than growth (monthly or even weekly), are known in advance. Consequently, the accuracy and timeliness of the estimated growth densities can be improved by augmenting the quarterly information with the available high frequency information. This is the proposal of Ferrara, Mogliani and Sahuc (2021).[12]

[12] An interesting issue to investigate is the possibility of implementing the GiS methodology to construct a "nowcasting" measure of growth vulnerability in different scenarios.

3.2 Multi-level dynamic factor models

A strand of the literature analyzes the conditional distribution of growth by focusing on factors extracted only from local financial variables. Adrian et al. (2019) estimate US growth densities as functions of a local financial (LF) factor, in particular, the NFCI. Further works considering the local financial factor are De Nicoló and Luccetta (2017), Adams et al. (2021), Catania et al. (2021), Ferrara et al. (2021) and Adrian et al. (2022), among many others.[13] The popularity of LF factors may be a consequence of the strong influence of local financial conditions in US during the 2008 Great Recession; see, for example, Dovern and van Roye (2014). The main argument for the link between financial factors and growth is based on the premise that financial prices incorporate market expectations of future price and output developments and, consequently, bear timely information on future economic conditions. However, other authors considering macroeconomic in addition to financial variables argue that the latter do not contribute much to distributional forecasts of growth; see, for example, Plagborg-Möller et al. (2020), Carriero et al. (2020), Reichlin et al. (2020) and Çakmakli et al. (2022). Beyond the debate about whether financial and/or macroeconomic factors should be considered when modelling the conditional distribution of growth, other authors debate whether only local factors should be considered when assessing growth risk; see, for example, Mishkin (2011) and Breitung and Eickmeier (2015) for a discussion on the global character of some crisis, and Cerutti et al. (2019) on global financial factors. In general, they argue that forecasting growth risk based on only "local" factors could be misleading in the current globalized world. In this direction, Djogbenou (2020) propose a two-level DFM with two specific developed and emerging economy activity factors in addition to a world economic factor.[14]

Our proposal is to jointly consider factors extracted from a rich set of variables that include local and worldwide macroeconomic and financial variables, and analyze their

[13]The ability of financial factors to predict future real economic activity has been discussed by Hatzius et al. (2010), Matheson (2012), Giglio et al. (2016), De Nicoló and Luccetta (2017), Menden and Proaño (2017), Arrigoni et al. (2020) and Boyarchenko et al. (2020), among others. The link between economic and financial conditions has experienced a revival after the 2008 Great Recession; see, for example, Dovern and van Roye (2014). As pointed out by Ng and Wright (2013), using US data from 1960 to 2012, all the post-1982 recessions have originated in financial markets, and these recessions are different from recessions where financial markets play a passive role.

[14]There are other proposals with world and local financial factors. However, as far as we know, these factors have not been linked with economic growth; see Amiti et al. (2019) for a recent contribution.

joint effect on the US economic growth.[15] To achieve this goal, we follow Rodríguez-Caballero and Caporin (2019) and propose a novel multi-level DFM that decompose the factor structure into different levels such that some factors are associated with the full cross-section of variables (pervasive factors) while some others either impact a specific subset of variables (non-pervasive factors) or several subsets of variables (semi-pervasive factors).

As proposed by Hallin and Liska (2011), we determine the factor structure of the multi-level DFM by analyzing the pairwise correlations between the factors extracted from each subset of variables separately.[16] Accordingly, the final multi-level DFM is specified with seven factors as follows

$$X_t^* = \begin{bmatrix} X_{1t} \\ X_{2t} \\ X_{3t} \\ X_{4t} \end{bmatrix} = \begin{bmatrix} \lambda_{11} & 0 & \lambda_{13} & \lambda_{14} & 0 & 0 & 0 \\ \lambda_{21} & \lambda_{22} & \lambda_{23} & 0 & \lambda_{25} & 0 & 0 \\ \lambda_{31} & 0 & 0 & \lambda_{34} & 0 & \lambda_{36} & 0 \\ \lambda_{41} & \lambda_{42} & 0 & 0 & 0 & 0 & \lambda_{47} \end{bmatrix} \begin{bmatrix} F_{1t}^* \\ F_{2t}^* \\ F_{3t}^* \\ F_{4t}^* \\ F_{5t}^* \\ F_{6t}^* \\ F_{7t}^* \end{bmatrix} + \varepsilon_t^*, \tag{12}$$

where F_{1t}^* is a pervasive factor that loads in all the variables in the system, F_{2t}^*, F_{3t}^* and F_{4t}^* are semi-pervasive factors with loadings in the worldwide (financial and macroeconomic), financial (local and worldwide), and local (financial and macroeconomic) variables, respectively. Finally, F_{5t}^*, F_{6t}^* and F_{7t}^* are non-pervasive factors that load on the worldwide financial, local macroeconomic, and worldwide macroeconomic variables, respectively. This factor structure explains the relation between the financial cycle and the business cycle, though both cycles have different characteristics; see Claessens et al. (2012), who, in a different context, has already pointed out that macroeconomic and financial dynamics could be driven by the same global and regional factors.

[15] Busetti et al. (2021) also consider domestic and worldwide financial and real variables when modeling the distribution of Italian GDP. However, they do not pursue a factor extraction as they focus on some individual variables.

[16] In the on-line appendix, we describe the factors extracted separately from each subset of variables and report the pairwise correlations, the pairwise scatter plots, and the smooth histograms of each of the estimated factors.

Examining the structure of the multi-level DFM in (12), we note that the local financial variables, X_{1t}, load on the factor F^*_{3t}, which corresponds to the financial variables, and on the factor F^*_{4t}, which corresponds to the local variables. However, there is not a non-pervasive separate factor for the local financial variables alone. Consequently, the information contained in the underlying local financial factors is already contained in the worldwide financial and local macroeconomic variables. This result is in agreement with Reichlin et al. (2020), who conclude that the NFCI contains little advanced information on growth beyond what is already contained in the real economic indicators. Plagborg-Möller et al. (2020), estimating US growth risk, also conclude that the performance of a model with both a macroeconomic factor and a financial factor is indistinguishable from a model with only a macroeconomic factor.[17] They show that financial variables contribute little to distributional forecasts of growth, beyond the information contained in real indicators. In the same vein, Carriero et al. (2020) find limited improvements in accuracy when using financial indicators in addition to macroeconomic indicators. Consequently, we simplify the model by considering only the variables in $X_t = (X_{2t}, X_{3t}, X_{4t})'$. Following the same methodology described above, we select the following final multi-level DFM

$$X_t = \begin{bmatrix} X_{2t} \\ X_{3t} \\ X_{4t} \end{bmatrix} = \begin{bmatrix} \lambda_{11} & \lambda_{12} & \lambda_{13} & 0 & 0 \\ \lambda_{21} & 0 & 0 & \lambda_{24} & 0 \\ \lambda_{31} & \lambda_{32} & 0 & 0 & \lambda_{35} \end{bmatrix} \begin{bmatrix} F_{1t} \\ F_{2t} \\ F_{3t} \\ F_{4t} \\ F_{5t} \end{bmatrix} + \varepsilon_t, \tag{13}$$

where F_{1t} and F_{2t} are the pervasive and semi-pervasive factors that load in all variables and in the world (financial and macroeconomic) variables of the system, respectively. The other three factors in (13) correspond to non-pervasive factors that only load in the worldwide financial (F_{3t}), and local (F_{4t}), and worldwide macroeconomic (F_{5t}) variables.

Estimation of model (13) is based on the procedure proposed in Breitung and Eickmeier (2015) and Rodríguez-Caballero and Caporin (2019).[18] Figure 2 plots the five

[17]Indeed, Plagborg-Möller et al. (2020) conclude that no predictors provide robust and precise advanced warnings about any features of GDP growth distribution other than the mean.

[18]See also Aastveit et al. (2016), who propose an alternative estimation procedure for multi-level DFMs and a bootstrap procedure to construct confidence bounds for the factors.

factors extracted from the multi-level DFM in (13) together with their 95% confidence intervals obtained by the subsampling procedure explained above. Note that the world-wide financial factor, F_{3t}, increases during the crisis periods. Positive values of this factor indicate tighter financial conditions than average, while negative values indicate looser financial conditions than average. Neither the pervasive F_{1t} nor the non-pervasive F_{3t} and F_{4t} factors warn about the plausibility of a forthcoming big decline in growth due to the COVID19 pandemic. However, the warnings coming from the semi-pervasive world factor F_{2t} was strong and from the non-pervasive world macroeconomic F_{5t} factor was indeed very strong. It is this last factor that truly captures a sharp decline in the world macroeconomy.

3.3 Factor-augmented predictive quantile regressions

After extracting the underlying factors from the multi-level DFM in (13), we estimate the corresponding factor-augmented quantile regression models in (1) for horizons $h = 1, 2, 3$ and 4 and for quantiles of growth τ^* from 0.05 to 0.95 at intervals of 0.01. The estimated parameters are plotted in Figure 3 together with their corresponding 95% confidence intervals for $h = 1$ and 4.[19] Table 1 reports the estimated parameters for $h = 1, 2, 3$ and 4 and $\tau^* = 0.05$, 0.5 and 0.95 together with their corresponding p-values and the analogue coefficient of determination R^1 . Several interesting insights on the conditional density of growth are obtained from Table 1 and Figure 3.

First, the fit of the factor-augmented quantile regressions based on the multi-level factors is much higher at the extreme quantiles than the fit of the quantile regressions with factors extracted from separate DFM based on subsets of variables. These latter results are reported in the online appendix. In Table 1, we also observe that the fit of the factor-augmented quantile regressions is rather large in the extreme quantiles with R^1 ranging, depending on h, from 39 to 49% for the 5% quantile and from 32 to 36% for the 95% quantile. For the median quantile, the fit is much lower, between 11 and 16%. The larger fit is the result of the significant effect of the five factors in the extreme 5% and 95% quantiles, which are more vunlerable than quantiles in the center of the distribution

[19] The covariance matrix of the estimators has been obtained as proposed by Koenker and Bassett (1978) assuming i.i.d. errors.

to economic and financial conditions. For the median quantile, the factors do not seem to be significant variables, with only a very small effect of F_3 in the short run ($h = 1$). Figure 3 confirms that the overall five factors are the most significant variables either in the extreme left tails or in the extreme right tails of the distribution of growth but their significance fades to zero in the median and neighboring quantiles. The most remarkable feature of Figure 3 is the strong effect of F_2, F_3, and F_5 in the extreme 5% and neighboring quantiles indicating that growth in recessions is mainly driven by worldwide macro and financial variables but in expansions (95% and neighboring quantiles), it is mainly the worldwide financial variables (F_3) that drive growth. The joint effects of different factors with their different magnitude in the extreme left and right tails of the growth distribution generate the asymmetry of this distribution, which is in agreement with the findings in several current works; see, for instance, Adams et al. (2021), Baker et al. (2020), Bloom (2014), Jurado et al. (2015), Ludvigson et al. (2021), and Plagborg-Möller et al. (2020).

4 The US conditional growth densities: GaR and GiS

We construct the conditional one-step-ahead growth densities for the US under average factor scenarios and under stressed scenarios and calculate the GaR and GiS risk measures respectively.

Smooth estimates of the growth distribution under average factor scenarios is obtained by fitting the Skewed-t distribution to the estimated quantiles of growth for $\tau^* = 0.05, 0.25, 0.75$ and 0.95 from the factor-augmented predictive quantile regressions. The GaR measure is the 5%-quantile of the smoothed density.

Under stressed scenarios, first we need to construct the joint α%-confidence regions to obtain plausible stress scenarios for the five factors extracted from the DFM.[20] Next, we minimize the τ^*-quantile growth subject to a fixed ellipsoid with α-coverage as in

[20]Even though there is not yet a formal result on the asymptotic distribution of the factors extracted from multi-level models, we construct these regions based on the asymptotic distribution derived by Choi et al. (2018) for the pervasive factor, which is extracted in the first step and has the same asymptotic distribution derived by Bai (2003) and described in Section 2. For the rest of the factors, which are extracted based on the residuals from the previous step, we also assume asymptotic normality. Since they are based on residuals, their asymptotic MSE will be affected by parameter estimation uncertainty but this problem should be mitigated by extending the subsampling procedure of Maldonado and Ruiz (2021) to the multi-level DFM framework.

(2). The minimization exercise takes place for different $\tau^* = 0.05, 0.25, 0.75$ and 0.95. The α-stressed conditional distributions of growth are obtained by fitting the Skewed-t distribution to the optimal estimated τ^*-quantiles. The GiS measure is the 5%-quantile of the smoothed α-stressed density.

In Figure 4, we plot the US one-step-ahead growth densities through time from 2005Q4 to 2020Q1. In the left panel, we show the smoothed densities when the factors are centered at their average values and, in the right panel, the smoothed α-stressed densities with stressed factors at the $\alpha = 95\%$ level. Note that the location, scale and shape of the conditional growth densities change over time and, as expected, the stressed densities are located to the left of the non-stressed densities. It is interesting to see that by stressing the factors, the stressed densities tend to show increased uncertainty and asymmetry. In Figure 5, we offer a close-up of these densities in three specific quarters: in 2008Q4 just after the 2008 Great Recession, in 2017Q1 in a quarter of low uncertainty, and in 2020Q2 during the beginning of the COVID-19 crisis. In 2017Q1, the two densities are closer to each other and are approximately symmetric with low dispersion. However, in crisis periods like 2008Q1 and 2020Q2, both densities tend to move to the left showing increased uncertainty and pronounced asymmetry with a long left tail. Both features are more acute in the COVID period than in the 2008 Great Recession. In Figure 5, we also plot the GaR value (5% quantile of the non-stressed density) and the GiS value (5% quantile of the stressed density). The distance between these two values depends on the level of stress chosen by the policy maker. In this plot, we have chosen a most severe stress scenario for the five factors ($\alpha = 95\%$).

In Figure 6, we provide a different way to visualize the non-stressed and stressed densities. We plot the US actual quarterly growth over the sample period 2005Q4 to 2021Q1. The dashed lines are the estimated one-step ahead 5% (GaR) and 95% quantiles, which for the most part of the sample envelop the actual growth. We plot the 5% and 95% quantiles of growth (light red) and the 25% and 75% quantiles (grey), when the factors are stressed at the 95% level. As before, the stressed density falls below the non-stress density and provides a complete assessment of the vulnerability of the economy in very different scenarios.

The densities in Figures 4, 5, and 6 summarize our proposed tool for risk assessment.

The policy maker has a complete visualization of growth dynamics under average and α-stressed scenarios of her choice with warning signals coming from the quantiles in the left tail of the densities. An additional piece of information that the GiS methodology provided are the values of the factors in the α-stressed scenario that give rise to the GiS warning. As an example, in 2020Q1, the values of the stressed factors in the 95% scenario were -1.26 (F_1), -5.74 (F_2), 1.94 (F_3), -0.19 (F_4) and -7.52 (F_5). We observe that the main factors contributing to the vulnerability of US growth at the time of the COVID pandemic were coming from the worldwide factor, F_2, and from the worldwide macroeconomic factor, F_5. Neither local information nor financial information *per se* were so influential.

In Table 2, we provide numerical information regarding GaR and GiS for four quarters ahead ($h = 1, 2, 3, 4$), for three quantiles (τ = 5, 50, 95%), and for three different levels of stress (α = 70, 95, 99%). With information up to 2020Q1, the GaR warning for the following quarter 2020Q2 (beginning of the pandemic) was -15.29% decline in growth, the GiS (95%) warning was -29.13%, and the observed decline was -31.20%. GaR was rather conservative compared to GiS. Note that the 95% level of stress for the factors reflects that the COVID19 has been a truly exceptional event. Finally, note that, in the following quarters, the economy substantially improved due to all the fiscal and monetary stimuli pumped up into it. Since GiS and GaR are warnings with fixed information up to 2020Q1, they could not realistically capture the positive growth in the following quarters. From a policy maker point of view, the reading of GaR and GiS warnings several quarters into the future should inform about where the economy would go if no remedial measures were imposed at the outset. They show the path of no action. The GaR warning pointed out to a potential recovery in the four quarters ahead (τ = 5%, h = 4, GaR = 2.55%) and GiS (70%) pointed out to a mild improvement but still negative growth if the factors were kept at the chosen 70% stress level (τ = 5%, h = 4, GiS = -5.24%).

5 Final considerations

We propose a set of statistical tools to dynamically monitor the vulnerability of the economy. We are directly answering to the sentiment expressed by policy makers such as the former Chairman of the Federal Reserve Alan Greenspan: "Policymakers often have to act [...] even though [they] may not fully understand the full range of possible outcomes, [...]. As a result, [...] policymakers have needed to reach to broader, though less mathematically precise, hypotheses about how the world works ..." (quoted in Frydman and Goldberg (2007) and Kwiatkowski and Rebonato (2011)), and Governor Brainard: "Policymakers tend to distinguish the most likely path, which I will refer to as the "modal" outlook, from risks around that path –events that are not the most likely to happen, but that have some probability of happening and that, if they do materialize, would have a one-sided effect" (Speech March 7, 2019, https://www.federalreserve.gov/newsevents/speech/brainard20190307a.htm). Aickman et al. (2021) also argue about the relevance for policy makers of "what if" exercises as those carried out in this paper.

These comments refer to the rare or extreme event that even with a small probability of occurrence could bring catastrophic losses to the economy. We show how to select rare events in a probabilistic sense with the construction of plausible but stressful scenarios and we summarize their potential effect in the economy with GiS as a measure of risk or vulnerability index. We have defined GiS as the 5% quantile of the stressed conditional growth distribution. To achieve this end, first, we have assumed that any quantile of the growth distribution is a function of a set of factors, extracted with a multi-level DFM from a wide set of macroeconomic and financial variables collected at the country and worldwide levels. Secondly, we have chosen severe and yet plausible stress scenarios based on the joint probability distribution of the underlying factors. This methodology allows the policy maker to choose the severity of the stress on the factors and construct the density of growth under different scenarios. The macro-financial scenarios considered by the policy-maker should be severe if she wants to be prepared for a large decline in growth as that observed during the COVID19 pandemic. In summary, for the policy-maker, we provide a risk management tool that allows for a complete visualization of growth dynamics under average and α-stressed scenarios of her choice with warning signals coming from the quantiles in the left tail of the densities. We see GiS as a complementary measure to GaR. Applied systematically, GiS is an useful tool for policy makers wishing to carry out a multi-dimensional scenario analysis.

Finally, the proposed methodology to measure growth vulnerability could be implemented to measure risk in other variables of interest such as inflation or unemployment; see Adams et al. (2021) for a very recent contribution considering risk in these two variables in addition to growth.

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Table 1: Estimates of the parameters of the factor-augmented regression models for the 5%, 50% and 95% quantiles of the US growth distribution. β_1, β_2, β_3, β_4 and β_5 are the corresponding parameters of the total factor F_1, worldwide factor F_2, worldwide financial factor F_3, local macroeconomic factor F_4, and worldwide macroeconomic factor F_5, respectively. Estimation sample from 2005Q3 up to 2020Q1. p-values in parenthesis.

	μ	ϕ	β_1	β_2	β_3	β_4	β_5	R^1
$\tau = 0.05$								
$h = 1$	**-2.62**	0.15	0.68	**2.19**	**-1.20**	**-1.21**	**3.44**	0.49
	(0.00)	(0.54)	(0.26)	(0.00)	(0.01)	(0.03)	(0.00)	
$h = 2$	**-2.03**	**-0.39**	**2.27**	**1.83**	**-1.65**	**-0.59**	**3.61**	0.46
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	
$h = 3$	**-4.55**	**0.91**	**2.61**	**1.35**	**-1.24**	**-2.00**	**-1.09**	0.40
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.02)	
$h = 4$	-0.73	**-1.08**	0.15	-1.03	**-3.12**	-0.83	-0.73	0.39
	(0.39)	(0.01)	(0.87)	(0.22)	(0.00)	(0.31)	(0.36)	
$\tau = 0.5$								
$h = 1$	**2.04**	-0.19	0.45	-0.01	**-0.87**	0.48	0.58	0.16
	(0.00)	(0.37)	(0.38)	(0.99)	(0.02)	(0.28)	(0.19)	
$h = 2$	**2.39**	-0.15	0.09	-0.30	-0.58	0.34	0.03	0.11
	(0.00)	(0.40)	(0.83)	(0.40)	(0.07)	(0.36)	(0.94)	
$h = 3$	**1.95**	0.13	-0.14	-0.21	-0.56	-0.22	-0.34	0.11
	(0.00)	(0.50)	(0.76)	(0.60)	(0.11)	(0.59)	(0.39)	
$h = 4$	**2.57**	**-0.31**	0.19	-0.18	-0.31	0.26	0.28	0.16
	(0.00)	(0.02)	(0.54)	(0.52)	(0.19)	(0.34)	(0.30)	
$\tau = 0.95$								
$h = 1$	**4.33**	**-0.24**	**1.30**	**0.62**	**-1.06**	0.23	**-0.59**	0.36
	(0.00)	(0.01)	(0.00)	(0.00)	(0.00)	(0.21)	(0.00)	
$h = 2$	**4.52**	**-0.34**	**0.77**	0.23	-0.28	**0.86**	-0.32	0.22
	(0.00)	(0.00)	(0.01)	(0.33)	(0.17)	(0.00)	(0.18)	
$h = 3$	**3.45**	**0.18**	-0.26	0.14	**-0.82**	-0.31	**-0.67**	0.35
	(0.00)	(0.05)	(0.25)	(0.49)	(0.00)	(0.12)	(0.00)	
$h = 4$	**4.73**	**-0.56**	0.31	**0.53**	**-1.08**	-0.15	-0.14	0.32
	(0.00)	(0.00)	(0.23)	(0.02)	(0.00)	(0.51)	(0.51)	

Table 2: US growth risk (in annualized percentage over previous quarter). The table reports h-step-ahead forecasts of 5%, 50% and 95% quantiles of growth with information up to 2020Q1 and computed by GaR (without stressing the underlying factors) and by GiS (with factors stressed at 70%, 95% and 99%).

	$h = 1$ 2020Q2	$h = 2$ 2020Q3	$h = 3$ 2020Q4	$h = 4$ 2021Q1
Observed	-31.20	33.89	4.50	6.30
$\tau = 0.05$				
GaR	-15.29	-18.07	-1.04	2.55
GiS(70%)	-25.49	-29.01	-9.70	-5.24
GiS(95%)	-29.13	-32.84	-12.53	-7.99
GiS(99%)	-31.48	-35.39	-14.58	-9.74
$\tau = 0.50$				
GaR	-4.94	-3.94	2.36	3.10
GiS(70%)	-12.29	-10.63	-3.06	-1.78
GiS(95%)	-14.83	-12.96	-4.96	-3.51
GiS(99%)	-16.49	-14.51	-6.18	-4.62
$\tau = 0.95$				
GaR	0.98	3.00	4.34	3.40
GiS(70%)	-6.04	-3.22	0.24	0.02
GiS(95%)	-8.42	-5.25	-1.23	-1.20
GiS(99%)	-9.96	-6.56	-2.17	-2.00

Figure 1: Illustration of GiS and GaR. The red dash-dot lines are the iso-5% quantile growth lines for values of the 5% quantile of growth corresponding to $q_{0.05}\left(y_{t+1}|F_t\right) = 3.5, -4, -8.5$ and -11.5. The blue ellipses are the contours of the bivariate probability density function of the factors. The highlighted blue ellipses are the 70% and 95% probability contours. The GaR is the value of the iso-5% quantile line at the center of the ellipse, which are the averages of the factors (GaR=-4). The GiS is the value of the iso-5% quantile line that is tangent to the 70% probability contour (GiS = -8.5) or 95% probability contour (GiS = -11.5).

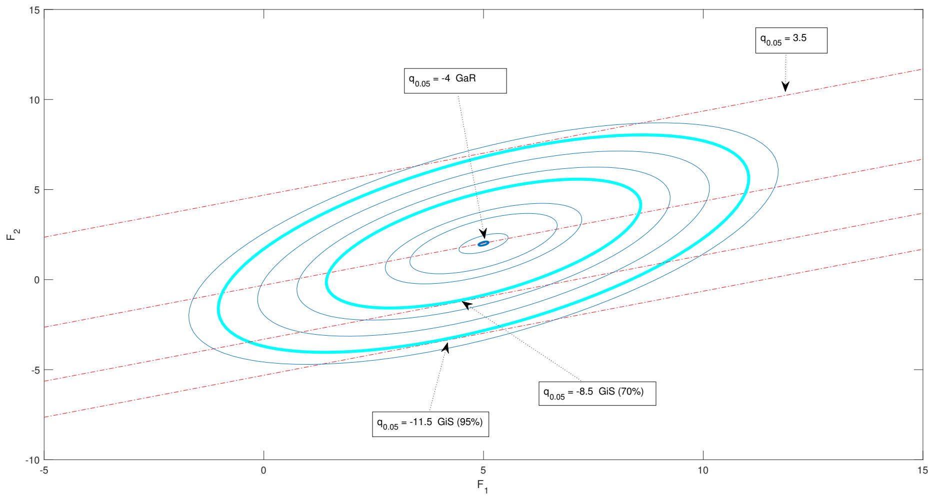
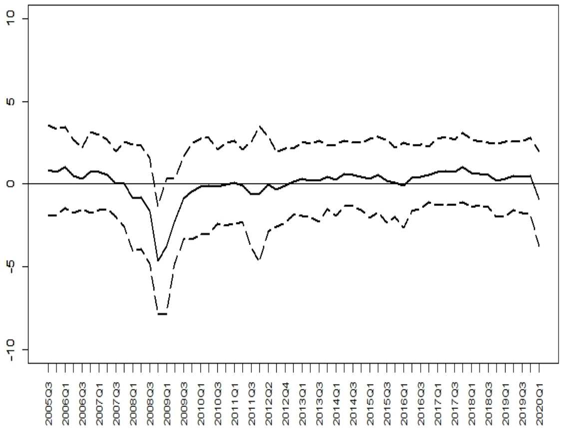
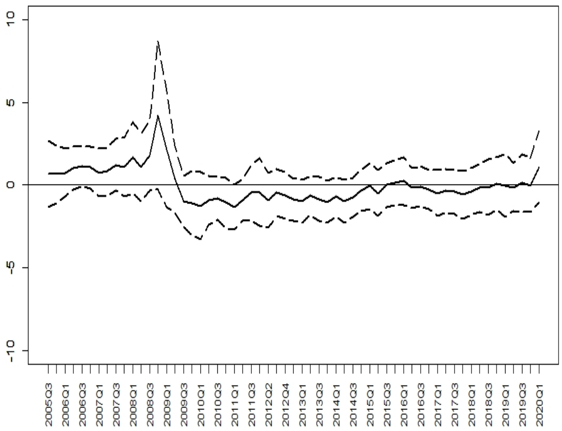


Figure 2: Estimated factors from multi-level DFM with their pointwise 95% confidence bounds. Total factor F_1, worldwide factor F_2, worldwide financial factor F_3, local macroeconomic factor F_4, and worldwide macroeconomic factor F_5. Estimation sample from 2005Q3 up to 2020Q1.

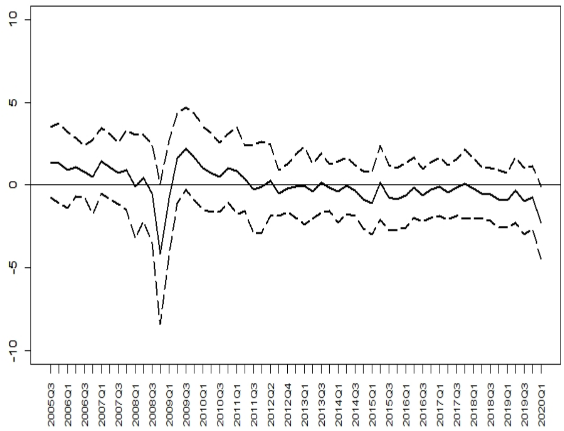


(a) F_1

(b) F_2



(c) F_3



(d) F_4

(e) F_5

Figure 3: Estimated parameters of the factor-augmented predictive quantile regressions for each quantile of the growth distribution ranging from $\tau^* = 0.5$ to $\tau^* = 0.95$ and for horizons $h = 1$ (black) and 4 (red) lines. The shade areas represent the 95% confidence pointwise intervals for the parameters (blue for $h = 1$ and light red for $h = 4$).

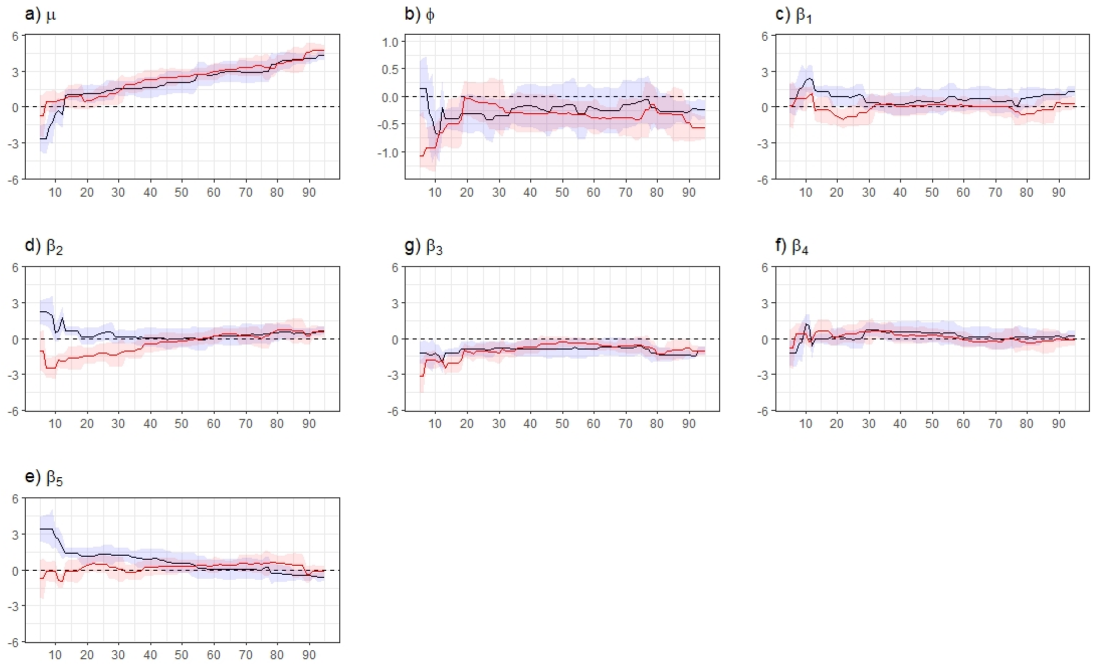


Figure 4: One-step-ahead US growth densities estimated from the factor-augmented quantile regression model with multi-level factors. The densities are calculated when factors are centered at their means (left panel), and when they are stressed at the 95% level (right panel).

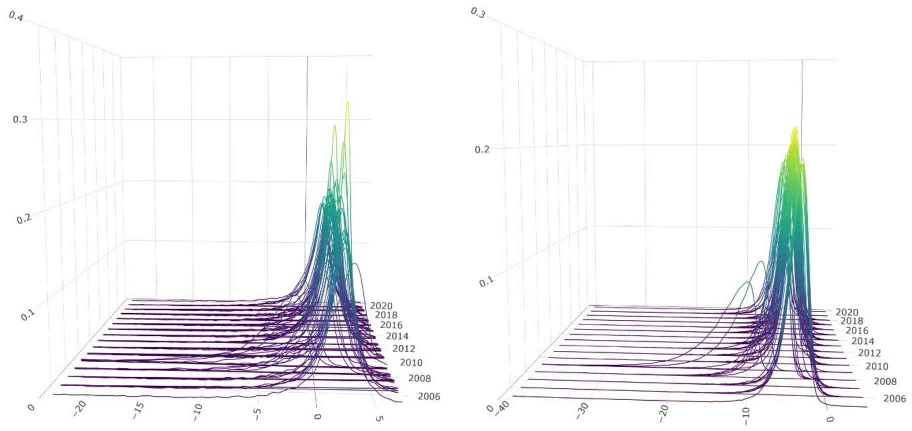


Figure 5: One-step-ahead US conditional growth densities in 2008Q4 (after the 2008 Great Recession), 2017Q1 (low uncertainty), and 2020Q2 (COVID-crisis). Densities calculated when factors are centered at their means (black) and when they are stressed at the 95% level (blue). The vertical dashed lines in black (GaR) and blue (GiS) correspond to the values of the 5% quantile of their respective densities.

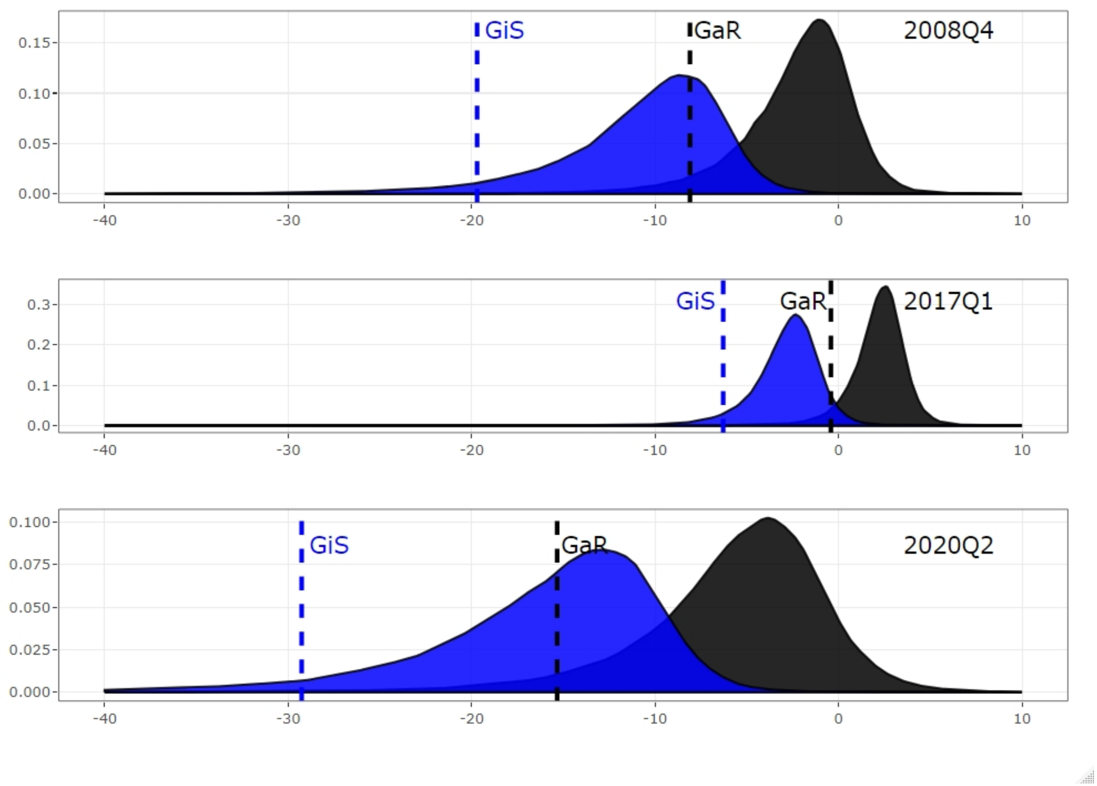


Figure 6: US quarterly growth (annualized rates, black line). 5% (GaR) and 95% quantiles (dashed lines) of the conditional one-step-ahead distribution of growth. In shades of light red (grey) the 5% (GiS) and 95% (25% and 75%) quantiles of the conditional α-stressed density with $\alpha = 95\%$.