Uncertainty-Driven Comovements in Booms and Busts: A Structural Interpretation*

Giovanni Caggiano Monash University University of Padova Efrem Castelnuovo University of Melbourne University of Padova

Giovanni Pellegrino University of Melbourne Please click here for the latest version

December 7, 2017

Abstract

We estimate a nonlinear VAR to quantify the responses of output, consumption, investment, and hours to a financial uncertainty shock in booms and busts in the post-WWII U.S. data. We find evidence of comovements both in expansions and in recessions, with stronger responses of all real activity indicators in the latter state. We interpret this state-dependent responses with a version of the Basu and Bundick (2017) model in which an uncertainty shock conceptually comparable to the one used in our VAR analysis generates comovements in real activity. A state-contingent estimation of this model conducted via Bayesian direct inference points to counter-cyclical risk aversion as the crucial ingredient to replicate the evidence produced with our nonlinear IVAR. An exercise focusing on the great recession suggests that the nonlinear DSGE model is able to replicate about 50% of the cumulative output loss in the 2009-2014 period, twice as much what the same model would predict if estimated conditional on a linear VAR.

Keywords: Uncertainty shock, comovement, nonlinear IVAR, nonlinear DSGE framework, minimum-distance estimation, great recession.

JEL codes: C22, E32, E52.

^{*} We thank Jesús Fernández-Villaverde, Georgios Georgiadis, Pablo Guerron-Quintana, Anastasios Karantounias, Johannes Pfeifer, Pedro Gomis-Porqueras, Alejandro Justiniano, Greg Kaplan, Thijs van Rens, Timo Teräsvirta, and participants to the WAMS (Canberra), Dynare conference (Tokyo), and "Frontiers in Macroeconomics and Macroeconometrics" Workshop (Tokyo) for their useful comments and suggestions. Financial support by the Australian Research Council via the Discovery Grant DP160102281 is gratefully acknowledged. Authors's contacts: giovanni.caggiano@monash.edu , efrem.castelnuovo@unimelb.edu.au , giovanni.pellegrino@unimelb.edu.au.

1 Introduction

The great recession has revamped the attention on the role played by fluctuations in uncertainty as a possible driver of the U.S. business cycle. Table 1 documents the correlations between financial volatility, a proxy for uncertainty widely employed in the literature, and four indicators of real activity, i.e., output, consumption, investment, and hours per capita, all expressed in quarterly growth rates. These correlations are negative, i.e., uncertainty is countercyclical, a well known fact (see, e.g., Bloom (2014)). Interestingly, such correlations are two- to six-time larger when one focuses on observations corresponding to NBER recessions as opposed to expansions, a fact robust to excluding the zero lower bound period from the sample.

This paper offers a structural interpretation to these correlations. We do so by proceeding in two steps. First, we show that the real effects of uncertainty shocks are stronger in recessions by estimating a nonlinear interacted vector autoregressive (IVAR) model with post-WWII U.S. quarterly data. A jump in uncertainty of the same size in the two phases of the business cycle is associated to a peak response of output in recessions 50% larger than in expansions, and to a response of investment and hours twice as large. Second, we estimate a version of the Basu and Bundick (2017) model with the Bayesian minimum-distance direct inference approach developed by Christiano et al. (2011) by considering as facts the impulse responses produced with our nonlinear VAR. This strategy, which allows us to estimate the nonlinear DSGE framework in a state-dependent fashion, is designed to identify the crucial parameter instabilities one needs to allow to replicate the IVAR asymmetric responses of real activity to an uncertainty shock. We find the estimated DSGE model to be able to replicate the documented facts in both states of the business cycle. While our strategy allow us to estimate a large vector of structural parameters, counter-cyclical risk aversion arises as the necessary and sufficient a model needs to possess to match the facts. This finding supports the idea of working with a state-specific calibration of the risk aversion parameter to correctly quantify the role played by uncertainty shocks in explaining the fluctuations of U.S. real activity.

We then push our investigation further and scrutinize how the "risk-aversion only" story fares when it comes to replicating an extreme event in our sample such as the great recession. Exploiting the flexibility of our nonlinear IVAR framework, we first move from states to dates and document the time-varying response of output, consumption, investment, and hours to an uncertainty shock. The global peak response of all four

real activity indicators is found to occur during the great recession, with values that are between two and three times as large as those estimated with a linear VAR framework. We then re-estimate the nonlinear DSGE framework conditional on the IVAR responses produced when focusing on the effects of an uncertainty shock in 2008Q4, which is the quarter associated to the highest realization of the uncertainty shock in our sample. We find a combination of a high degree of risk aversion and a moderately inertial, weak policy response to inflation to be sufficient for replicating our IVAR responses. We then use our estimated DSGE model to compute the contribution of an uncertainty shock in 2008Q4 to the loss of output registered during and after the great recession. We find such model to be able to explain about 1/2 of the total output loss in 2008-2014. By contrast, the very same DSGE model estimated conditional on impulse responses produced with a standard linear VAR explains just about 1/4 of the total output loss. This result clearly speaks in favor of using correctly calibrated structural frameworks for replicating the facts and, ultimately, for conducting policy analysis.

Our paper relates to different but interconnected strands of the literature. The identification of uncertainty shocks is achieved by focusing on financial uncertainty, which has previously been singled out as a possible driver of the US business cycle (Bloom (2009), Ludvigson, Ma, and Ng (2017)). Methodologically, we use a nonlinear IVAR model to establish novel facts regarding the different intensity of comovements along the business cycle. IVAR model have increasingly been exploited to study the nonlinear effects of macroeconomic shocks other than uncertainty shocks (Towbin and Weber (2013), Sá, Towbin, and Wieladek (2014), Aastveit, Natvik, and Sola (2017)). Caggiano, Castelnuovo, and Pellegrino (2017) employ IVARs to investigate the link between uncertainty shocks and the stance of systematic policy. With respect to them, we focus on the stance of the business cycle and document the nonlinear effects of uncertainty shocks in recessions and expansions. In computing our impulse responses, we follow Pellegrino (2017a,b) and Caggiano, Castelnuovo, and Pellegrino (2017) and allow both uncertainty and real activity - i.e., the interaction terms in our nonlinear VAR - to endogenously evolve after an uncertainty shock. This modeling feature is highly desirable in light of the well-documented interactions between uncertainty and the business cycle that can occur after an uncertainty shock (Jurado, Ludvigson, and Ng (2015), Ludvigson, Ma, and Ng (2017)). Our IVAR-related findings, which point to more severe consequences for output, investment, consumption, and hours when uncertainty shocks hit in recessions, complement those documented with alternative nonlinear frameworks and related to unemployment (Caggiano, Castelnuovo, and Groshenny

(2014), Caggiano, Castelnuovo, and Figueres (2017)) and industrial production and employment (Caggiano, Castelnuovo, and Nodari (2017)), or obtained with indicators correlated with the business cycle like financial stress (Alessandri and Mumtaz (2014)).

Turning to the DSGE-based part of the analysis, nonlinear frameworks have recently been shown to be able to generate comovements in real activity in response to uncertainty shocks. Fernández-Villaverde, Guerrón-Quintana, Rubio-Ramírez, and Uribe (2011) and Born and Pfeifer (2014b) investigate the real effects of a second moment shock to the world real interest rate for Argentina, Brazil, Ecuador, and Venezuela. Turning to the U.S., Fernández-Villaverde, Guerrón-Quintana, Kuester, and Rubio-Ramírez (2015) study the role of fiscal policy uncertainty; Born and Pfeifer (2014a) quantify the relevance of policy risk; Born and Pfeifer (2017) investigate the role of second moment shocks to technology and fiscal spending; Drautzburg, Fernández-Villaverde, and Guerrón-Quintana (2017) investigate the role of political distribution risks; Basu and Bundick (2017) show that a demand uncertainty shock which can be interpreted as a negative financial uncertainty shock triggers a negative response of real activity indicators. Our decision of working with a version of the last framework is justified by the presence, in such model, of a formally precise definition of implied financial volatility which can be meaningfully matched to the one we employ in our IVAR analysis. We exploit this match to estimate the third-order approximation of the DSGE model via the Bayesian impulse response function matching proposed by Christiano, Trabandt, and Walentin (2011). This novel empirical strategy enables us to estimate the nonlinear DSGE model we work with in a state-dependent fashion, something we do in order to unveil instabilities at a structural level which are needed to track the dynamic response of real activity in booms and busts. Our approach represents an alternative to the GMM/SMM-based methodology recently developed by Andreasen, Fernández-Villaverde, and Rubio-Ramírez (2017), who rely on moments simulated with a pruned version of the third-order approximation of a DSGE model. The common characteristic of these two approaches is that they solve the nonlinear DSGE framework by appealing to perturbation, developed by Schmitt-Grohe and Uribe (2004) and Andreasen (2012) and shown to be particularly efficient against alternatives - such

¹Castelnuovo and Pellegrino (2017) perform a similar exercise by working with Bayesian impulse response function matching in order to assess the performance of some medium-scale DSGE frameworks in presence of high/low uncertainty. Crucially, they work with linearized models, while this paper estimates a truly nonlinear DSGE framework in a state-dependent fashion. At least a third-order approximation of the model is required to study the effects of a time-varying uncertainty shock in a structural framework.

as value function iteration or projection - by Caldara, Fernández-Villaverde, Rubio-Ramírez, and Yao (2012). Methodologically, the closest approach to ours is probably the one by Ruge-Murcia (2017), who estimates a small-scale third-order approximated DSGE model with an impulse-response matching procedure based on a class of non-linear VAR models as an auxiliary model for the purpose of indirect inference via a classical minimum distance estimator. In doing so, he imposes the perturbation solution on the nonlinear DSGE model on the nonlinear VAR framework to approximate as closely as possible the DSGE-related policy functions. His approach, which is extremely neat, becomes difficult to implement when one works with models with several states. Our novel estimation strategy easily accommodates large state spaces.

The paper develops as follows. Section 2 presents the non-linear VAR model employed and presents results on the business cycle-dependent consequences of uncertainty shocks from this relatively unrestricted framework. Section 3 briefly presents the Basu and Bundick (2011) model, describes the econometric strategy adopted to estimate the DSGE model and discusses the regime-dependent estimation results found with a focus on the sources of the different transmission mechanism of uncertainty shocks between recessions and expansions. Section 4 quantifies the performance of our empirical framework as regards the negative effects of the large uncertainty shock happened at the onset of the great recession. Section 5 concludes. An Appendix available upon request reports additional results, the algorithm for the computation of the generalized impulse response functions and the relevant derivation for a version of the Basu and Bundick model extended to consider external habits in consumption.

2 Uncertainty-driven comovements: Empirical evidence

2.1 Nonlinear empirical methodology

The IVAR is a nonlinear framework which augments a standard linear VAR model with interaction terms to determine how the effects of a shock to a variable depend on the level of another variable. Following Pellegrino (2017a,b) and Caggiano, Castelnuovo, and Pellegrino (2017), we focus on a parsimonious IVAR to maximize the available degrees of freedom while capturing the nonlinearity of interest.

Our IVAR is the following:

$$\mathbf{Y}_{t} = \boldsymbol{\alpha} + \sum_{j=1}^{L} \mathbf{A}_{j} \mathbf{Y}_{t-j} + \left[\sum_{j=1}^{L} \mathbf{c}_{j} \ln VXO_{t-j} \times \Delta \ln GDP_{t-j} \right] + \mathbf{u}_{t}$$
 (1)

where \mathbf{Y}_t is the $(n \times 1)$ vector of the endogenous variables, $\boldsymbol{\alpha}$ is the $(n \times 1)$ vector of constant terms, \mathbf{A}_i are $(n \times n)$ matrices of coefficients, and \mathbf{u}_t is the $(n \times 1)$ vector of error terms whose variance-covariance (VCV) matrix is Ω . The interaction term in brackets makes an otherwise standard VAR a non-linear IVAR model. Per each lag, such interaction term includes a $(n \times 1)$ vector of coefficients \mathbf{c}_j , a measure of uncertainty $\ln VXO_t$, and an indicator of the business cycle $\Delta \ln GDP_{t-j}$, which is the quarter-onquarter growth rate of real GDP. The interaction term $\ln VXO_{t-i} \times \Delta \ln GDP_{t-i}$ enables us to capture the potentially state-contingent effects of a shock to $\ln VXO_{t-i}$ (i.e., an uncertainty shock) conditional on the state of the business cycle, which is proxied by the growth rate of real GDP. Alternatives to IVAR frameworks are available to capture the nonlinear effects of macroeconomic shocks (for a recent survey, see Teräsvirta (2017)). We prefer to employ the above formalized IVAR framework for three reasons. First, it closely resembles the approximated nonlinear policy functions of nonlinear DSGE frameworks we work with.² Second, it naturally focuses on nonlinearities related to the business cycle, which is the research question under scrutiny. Third, it does not feature nuisance parameters, which are often difficult to estimate in nonlinear frameworks.³

We model the vector $\mathbf{Y}_t = [\ln VXO, \ln GDP, \ln C, \ln I, \ln H, \ln P, FFR]'$, where VXO denotes the stock market S&P 100 implied volatility index, GDP per capita GDP, C per capita consumption, I per capita investment, H per capita hours worked, P the price level, and FFR the federal funds rate. Uncertainty shocks are identified via a Cholesky decomposition of the reduced-form VCV matrix Ω , with the VXO ordered first. This assumption implies that the VXO does not contemporaneously respond to first moment shocks like, e.g., technology and preference shocks. Importantly, this assumption is in line with the predictions of Basu and Bundick's model. In fact, while being technically endogenous, the VXO in their model is almost exclusively explained

²Such nonlinear policy functions typically feature different, higher order interaction terms. We focus on terms featuring uncertainty and the real GDP growth because of our interest in the nonlinear effects to uncertainty shocks along the business cycle. Simulations conducted with higher order terms deliver even stronger empirical results in favor of such nonlinear effects. Appendix A documents the state-dependent impulse responses obtained with a framework involving a higher number of terms.

³Notice that IVARs featuring interactions terms resemble approximated Smooth Transition VAR frameworks (Teräsvirta, Tjøstheim, and Granger (2010)).

⁴The vector closely resembles the one used by Basu and Bundick (2017) in their linear VAR analysis, which also features the presence of money. Adding money implies no changes in our empirical results. The definition and construction of the variables is exactly the same as in Basu and Bundick (2017).

by second moment preference shocks, i.e., uncertainty shocks. Non surprisingly, an exercise conducted by simulating data with the Basu and Bundick model and estimating a VAR framework on such simulated data with shocks identified as described above returns VAR impulse responses which perfectly replicate those produced by the DSGE model. Appendix B documents this result, which enables use to estimate the parameters of the Basu and Bundick model with a direct inference approach (explained below). Appendix B also documents the correlation between second moment preference shocks and the model consistent VXO in the Basu and Bundick model, which confirms that the latter is clearly driven by the former.

We estimate an IVAR model with four lags over the 1962Q3-2017Q2 sample. Given that the VXO is unavailable before 1986, we follow Bloom (2009) and splice it with the within-month volatility of S&P500 daily returns, which has displayed an extremely high correlation with the VXO since 1986. The sample includes the zero lower bound period experienced by the Federal Reserve during the period 2008Q4-2015Q4. Appendix C shows that our results are robust to the employment of the shadow rate constructed by Wu and Xia (2016).

A standard likelihood-ratio test favors our IVAR specification against the Basu and Bundick's (2016) linear VAR model (which is nested in our IVAR model in case of the overall exclusion of the interaction terms from model (1)). In particular, the LR test suggests a value for the test statistic $\chi_{28} = 60.16$, which allows us to reject the null hypothesis of linearity at any conventional statistical level in favor of the alternative of our I-VAR model.

The interaction term of our IVAR is treated as an endogenous object. We compute GIRFs à la Koop et al. (1996) to account for both the endogenous reaction of the growth rate of per capita GDP, i.e. our conditioning variable, to the uncertainty shock and the feedback this reaction can imply on the dynamics of the economy. GIRFs acknowledge the fact that, in a fully nonlinear model, responses depend on the sign of the shock, the size of the shock, and initial conditions. Theoretically, the GIRF at horizon h of the vector \mathbf{Y}_t to a shock of size δ computed conditional on an initial history $\boldsymbol{\varpi}_{t-1} = \{\mathbf{Y}_{t-1}, ..., \mathbf{Y}_{t-L}\}$ is given by the following difference of conditional expectations:

$$GIRF_{Y,t}(h, \delta_t, \varpi_{t-1}) = E\left[\mathbf{Y}_{t+h} \mid \delta, \varpi_{t-1}\right] - E\left[\mathbf{Y}_{t+h} \mid \varpi_{t-1}\right].$$

Along with this history-conditional GIRF, thanks to which we can recover one empirical response for each quarter in our sample, one can also define some states so that to recover state-conditional GIRFs summarizing the average evidence for a given state.

We will use this latter approach with the purpose of comparing IVAR-based responses with DSGE-based responses, something clarified later on. In computing our GIRFs, we focus on "extreme events", i.e., deep recessions (strong expansions) characterized by initial conditions associated to realizations of the real GDP growth rate belonging to the first (tenth) decile of the empirical distribution. We do so in order to minimize the risk of confounding recessions and expansions, a risk one incurs when considering initial conditions which are too close to the sample mean of the conditioning variable (which is, the growth rate of real GDP) (Ferrara (2003)). Papers adopting this logic when studying the nonlinear effects of fiscal spending shocks, see Auerbach and Gorodnichenko (2012) and Caggiano, Castelnuovo, Colombo, and Nodari (2015), while Caggiano, Castelnuovo, and Groshenny (2014) follow the same strategy to study the response of unemployment to uncertainty shocks in recessions. The GIRF conditional on a given state is computed by simulating the system starting from the average initial condition across the histories linked to the state, i.e., starting from the unconditional mean of each state. Conditional on being in a regime, this method is consistent with the way responses are usually computed for a nonlinear DSGE model, i.e. by simulating the model starting from the ergodic mean. Appendix D describes the algorithm used to compute the GIRFs.

2.2 Empirical results

Figure 1 plots the generalized impulse responses computed with our IVAR approach. A few facts stand out. First, there is evidence in favor of uncertainty-driven comovements in both states. Hence, one needs to use DSGE models able to generate comovement in response to an uncertainty shock no matter what the state of the economic system is. Second, the response of real activity is larger in recessions than in expansions, a fact implying that the statistical evidence in favor of the use of a nonlinear VAR implies an economically relevant difference when it comes to quantifying the reaction of real activity along the business cycle to an equally sized uncertainty shock. To fix ideas about this point, Table 2 collects figures regarding the peak (i.e., maximum, in absolute value) responses produced with our nonlinear VAR. The peak response of output in recessions is about 50% larger than in expansions. The same holds as regards consumption, whose peak reaction is 36% larger in contractions, and even more so for investment and hours, whose peak responses is more than twice as large. Third, the larger strength of the response in recessions regards the entire path of the short-run response of real activity

after an uncertainty shock, and not only its peak reaction. Fourth, the synchronization of real activity indicators in response to an uncertainty shock differs between the two phases of the business cycle, with investment and hours reacting - in relative terms with respect to output - about 40% and 50% more in recessions than in expansions. Finally, the response of the policy rate is negative and persistent in both states of the business cycle, while that of the price level is not, i.e., the price level persistently decreases in recessions but increases in expansions.

Are these responses actually different from a statistical standpoint? Figure 2 shows the one standard deviation confidence bands of differences between the reactions in recessions and in expansions.⁵ As evident from the figure, the responses of output, investment, and hours are significantly larger in recessions, an evidence which offers statistical support to the comovement-related facts motivated above. The reaction of consumption is only borderline significant, with the mass of the distribution which however hints to a larger response in recessions. Finally, also the response of the price level and the nominal interest rate is found to be significantly different in recessions.⁶

Overall, these results point to an economically and significantly stronger response of real activity to an uncertainty shock. This fact calls for the use of a structural model, something we do in the next Section.

3 Uncertainty-driven comovements: A structural interpretation

3.1 DSGE model: Description

The Basu and Bundick (2017) model extends an otherwise standard New Keynesian model to consider an ex-ante second moment shock in the preference shock process, which can be interpreted as a demand-side uncertainty shock. Households work, con-

⁵Per each variable, the figure is based on the distribution stemming from 1,000 differences between responses in recessions and responses in expansions. Such responses are generated from 1,000 samples obtained via the standard residual-based bootstrap, computing - per each of such samples - the corresponding state-conditional GIRFs in recessions and expansions, and taking the difference between the latter and the former. The 16th and 84th percentiles of each density are then reported. The construction of the test statistic takes into account the correlation between the estimated impulse responses. Given the equal size of the shock in the two states per each given draw, the differences take - by construction - an on-impact zero value.

⁶These results are robust to the use of a recursive-design wild bootstrap (based on a Rademacher distribution) which is robust to possible conditional heteroskedasticity of unknown form (see Goncalves and Kilian (2004) for the proposal, and Kilian (2009) and Mertens and Ravn (2014) as examples of applications).

sume, and invest in equity shares and one-period risk-free bonds. Households are all similar and are indexed with $j \in [0,1]$. They feature Epstein-Zin preferences over streams of consumption and leisure, formalized as follows:

$$V_t(j) = \left[a_t(\left(C_t(j) - H_t^C(j)\right)^{\eta} (1 - N_t(j))^{(1-\eta)}\right)^{(1-\sigma)/\theta_V} + \beta \left(E_t V_{t+1}^{1-\sigma}(j)\right)^{1/\theta_V}\right]^{\theta_V/(1-\sigma)}$$

where $C_t(j)$ is household j's consumption, $H_t^C(j)$ is the habit level for consumption which is assumed to be external and to follow the lagged aggregate consumption level in the form $H_t^C(j) = bC_{t-1}$, $N_t(j)$ is household j's hours worked, β is the discount factor, σ measures the degree of risk aversion, ψ is the intertemporal elasticity of substitution, $\theta_V \equiv (1-\sigma)/(1-\psi^{-1})^{-1}$ captures households' preferences for the resolution of uncertainty, η weights consumption and labor in households' happiness function, and a_t is a stochastic shifter influencing the relevance of today's realizations of consumption and labor vs. those expected to occur during the next period. The stochastic process followed by this preference shock is:

$$a_t = (1 - \rho_a)a + \rho_a a_{t-1} + \sigma_{t-1}^a \varepsilon_t^a$$

$$\sigma_t^a = (1 - \rho_{\sigma^a})\sigma^a + \rho_{\sigma^a}\sigma_{t-1}^a + \sigma^{\sigma^a}\varepsilon_t^{\sigma^a}$$

where ε_t^a is the first-moment preference shock, and $\varepsilon_t^{\sigma^a}$ is a second-moment uncertainty shock to the preference process which loads the law of motion regulating the evolution of the time-varying second moment σ_t^a relative to the distribution of ε_t^a . The original framework by Basu and Bundick (2017) is modified to allow for (external) habit formation in consumption. We do so to capture the hump-shaped response of consumption in the data. Andreasen, Fernández-Villaverde, and Rubio-Ramírez (2017) also jointly model Epstein-Zin preferences and habits in consumption in order to match macro-finance facts (see Binsbergen, Fernández-Villaverde, Koijen, and Rubio-Ramírez

⁷Epstein-Zin preferences written in this way imply a direct impact of the uncertainty shock on the current utility level only. This way of writing such preferences is not uncommon, see, on top of Basu and Bundick (2017), Albuquerque, Eichenbaum, Luo, and Rebelo (2016), Andreasen, Fernández-Villaverde, and Rubio-Ramírez (2017). A note recently circulated by de Groot, Richter, and Throckmorton (2017) shows that these preferences imply a response of real activity indicators to an uncertainty shock which is discontinuous over the set of values the intertemporal elasticity of substitution ψ can take. When $\psi = 1$, the reaction of real activity is not defined, while values of ψ below (above) one are consistent with a negative (positive) response of real activity. While acknowledging the intellectually stimulating point made by de Groot, Richter, and Throckmorton (2017), we stress here that our paper is an empirical contribution whose aim is that of replicating the negative response of real activity to an uncertainty shock with an estimated model. In this sense, we see no clash with the findings in de Groot, Richter, and Throckmorton (2017).

(2012) and Andreasen, Fernández-Villaverde, and Rubio-Ramírez (2017), respectively, for the importance of Epstein-Zin preferences and habits in consumption in this regard in the context of New Keynesian models).

Intermediate goods producers rent labor from households, pay wages, and produce intermediate goods in a monopolistically competitive framework. They own capital and choose its utilization rate, issue equity shares and one-period riskless bonds, and invest in physical capital to maximize the discounted stream of their profits. In doing so, they face quadratic costs of adjusting nominal prices à la Rotemberg (1982), capital adjustment costs à la Jermann (1998) and capital utilization costs influencing the capital depreciation rate.⁸ All intermediate firms have the same Cobb-Douglas production function, and are subject to a fixed cost of production and stationary technology shocks. Intermediate goods are packed by a representative final goods producer operating in a perfectly competitive market. The model is closed by assuming that the central bank follows a standard Taylor rule.

As anticipated above, the model features a well-defined implied volatility concept. This is due to the fact that intermediate firms issue equity shares on top of one-period riskless bonds. Each equity share has a price P_t^E and pays dividends D_t^E , implying a one-period return $R_{t+1}^E = \left(P_{t+1}^E + D_{t+1}^E\right)/P_t^E$. The model-implied financial uncertainty index V_t^M is computed as the annualized expected volatility of equity returns, i.e., $V_t^M = 100 \cdot \sqrt{4 \cdot VAR_t\left(R_{t+1}^E\right)}$, where $VAR_t\left(R_{t+1}^E\right)$ is the quarterly conditional variance of the return on equity R_{t+1}^E . Equity returns are endogenous in the model, which makes V_t^M endogenous too. However, it can be shown that, in this model, V_t^M is almost entirely driven by second-moment preference shocks. This enables us to treat the uncertainty shock as a financial uncertainty shock proxied by V_t^M , which has a clear empirical counterpart and which enable us to exploit the facts established in the previous Section to estimate the model.

In this framework, an uncertainty shock propagates to the economy via precautionary savings and precautionary labor supply. The former effect reduces current consump-

⁸Given that adjustment costs are convex, this model does not imply a "wait-and-see" effect after an uncertainty shock. To solve the model, we use perturbation methods which require policy functions to be differentiable, a feature which is not possessed by threshold policy functions arising in presence of real option effects.

 $^{^9}$ Basu and Bundick (2017) assume that firms finance a share ν of their capital stock each period with one-period riskless bonds. Given that the Modigliani-Miller theorem holds in their model, leverage does neither influence firms' value nor firms' optimal decisions. Firms' leverage only influences the first two unconditional moments of financial-related quantities (e.g., the average level and unconditional volatility of the model-implied VXO and the equity premium), but it does not influences impulse responses to an uncertainty shock.

tion in response to an increase in uncertainty, while the latter increases labor supply, which drives real wages and firms' marginal costs down. Given that prices are sticky, the price mark up increases. Output, which is demand-driven in this model, falls due to the drop in consumption, and labor demand contracts driving hours down. Given the lower return on capital, investment falls too. Hence, in equilibrium, an increase in uncertainty causes a drop in all four real activity indicators, i.e., output, consumption, investment, and hours, which is what we observe in the data.

We work with a third-order approximation of the nonlinear framework, which we solve via perturbation techniques (Schmitt-Grohe and Uribe (2004)). The third order approximation of agents' decision rules feature an independent role for uncertainty, whose independent effect on the equilibrium values of the endogenous variables of the framework can therefore be studied (Andreasen (2012)). Perturbation represents an accurate and fast way to find a solution also working with frameworks featuring recursive preferences (Caldara, Fernández-Villaverde, Rubio-Ramírez, and Yao (2012)).

3.2 Minimum-distance estimation strategy

We estimate Basu and Bundick's (2017) model via an impulse response function-matching approach, i.e., we choose the values of the structural parameters of the DSGE model which minimize a measure of the distance between our IVAR impulse responses, which are interpreted as "data", and the DSGE model-based ones. Following Christiano, Trabandt, and Walentin (2011), we employ a Bayesian approach via which we impose economically sensible prior densities on the structural parameters while asking the data to shape the posterior density of the estimated model. With respect to Christiano et al. (2011), who focus on a linearized DSGE framework and a linear VAR as auxiliary model, we estimate a nonlinear DSGE framework approximated at a third order with moments produced with an Interacted VAR.¹⁰

The state-dependent Bayesian minimum distance estimator works as follows. Denote by $\widehat{\psi}^i$ the vector in which we stack the I-VAR estimated generalized impulse responses over a 20-quarter horizon to an uncertainty shock for regime i = 1, 2 (for us the responses

¹⁰One way of interpreting this exercise is to think of a regime-switching time of estimation in which we allow the parameters of the nonlinear DSGE model to be state-dependent. Bianchi and Melosi (2017) formally model policy-related uncertainty with a regime-switching approach which allows agent to formulate a prediction over future regime switches in an empirical framework where the DSGE model is a linearized framework within each state. A challenge for future research is how to conduct such an exercise with a nonlinear DSGE model like the one we work with.

displayed in figure 2).¹¹ When the number of observations per each regime n^i is large, standard asymptotic theory suggests that

$$\widehat{\boldsymbol{\psi}^{i}} \stackrel{a}{\sim} N(\boldsymbol{\psi}\left(\boldsymbol{\zeta}_{0}^{i}\right), \mathbf{V}^{i}(\boldsymbol{\zeta}_{0}^{i}, n^{i})), \text{ for } i = 1, 2$$
(2)

where ζ_0^i denotes the true vector of structural parameters that we estimate (i = 1, 2) and $\psi(\zeta^i)$ denotes the model-implied mapping from a vector of parameters to the analog impulse responses in $\widehat{\psi}^i$.

As already explained earlier, the IVAR GIRFs $\widehat{\psi}^i$ conditional on a given state i are computed by iterating forward the system starting from the average initial condition across the histories linked to the state, i.e., starting from the unconditional mean of each state. As regards the DSGE model-related responses per each given set of parameter values $\psi\left(\zeta^i\right)$, we compute them by iterating forward the approximated solution of the DSGE model starting from the (state-specific in our case) simulated stochastic steady state. Both DSGE-based and VAR-based impulse responses are interpreted as percent deviations of variables induced by an uncertainty shock, with the exception – in our case – of the interest rate response which would be measured in percentage points terms as implied from the VAR specification.

To compute the posterior density for ζ^i given $\widehat{\psi}^i$ using Bayes' rule, we first need to compute the likelihood of $\widehat{\psi}^i$ conditional on ζ^i . Given (2), the approximate likelihood of $\widehat{\psi}^i$ as a function of ζ^i reads as follows:

$$f(\widehat{\boldsymbol{\psi}^{i}}|\boldsymbol{\zeta}^{i}) = \left(\frac{1}{2\pi}\right)^{\frac{N^{i}}{2}} \left| \mathbf{V}^{i}(\boldsymbol{\zeta}_{0}^{i}, n^{i}) \right|^{-\frac{1}{2}} \times \exp\left[-\frac{1}{2} \left(\widehat{\boldsymbol{\psi}^{i}} - \boldsymbol{\psi}\left(\boldsymbol{\zeta}^{i}\right) \right)^{'} \mathbf{V}^{i}(\boldsymbol{\zeta}_{0}^{i}, n^{i})^{-1} \left(\widehat{\boldsymbol{\psi}^{i}} - \boldsymbol{\psi}\left(\boldsymbol{\zeta}^{i}\right) \right) \right]$$

$$(3)$$

¹¹For a paper proposing information criteria to select the responses that produce consistent estimates of the true but unknown structural parameters and those that are most informative about DSGE model parameters, see Hall, Inoue, Nason, and Rossi (2012).

¹²Following Basu and Bundick (2017), we set the value of the exogenous processes to zero and iterate forward until the model converges to its stochastic steady state. Then, we hit the model with a one standard deviation uncertainty shock and compute impulse responses as the percent deviation between the stochastic path followed by the endogenous variables and their stochastic steady state. Given that no future shocks are considered, this way of computing GIRFs does not line up with Koop, Pesaran and Potter's (1996) algorithm. We do so to avoid simulating the model several times and then integrate across all simulations, a procedure which would be very time consuming, above all when combined with the MCMC algorithm we adopt for our Bayesian estimation. Basu and Bundick (2017) show that the differences between these two ways of computing GIRFs are negligible with a framework like theirs. We also verified that our IVAR GIRFs remained unchanged when future shocks are not taken into account, something which augments the comparability between IVAR and DSGE GIRFs. Analytical expressions for GIRFs produced with nonlinear models are available in Andreasen, Fernández-Villaverde, and Rubio-Ramírez (2017).

where N^i denotes the number of elements in $\widehat{\psi}^i$ and $\mathbf{V}^i(\zeta_0^i,n^i)$ is treated as a fixed value. We use a consistent estimator of \mathbf{V}^i . Because of small sample-related considerations, such estimator features only diagonal elements (see Christiano, Trabandt, and Walentin (2011) and Guerron-Quintana, Inoue, and Kilian (2017)). In our case, \mathbf{V}^i is a regime-dependent diagonal matrix with the variances of the $\widehat{\psi}^i$'s along the main diagonal. This choice is widely adopted in the literature and allows one to put more weight in replicating VAR-based responses with relatively smaller confidence bands. Treating eq. (3) as the likelihood function of $\widehat{\psi}^i$, it follows that the Bayesian posterior of $\widehat{\zeta}^i$ conditional on $\widehat{\psi}^i$ and \mathbf{V}^i is:

$$f(\zeta^{i}|\widehat{\psi^{i}}) = \frac{f(\widehat{\psi^{i}}|\zeta^{i})p(\zeta^{i})}{f(\widehat{\psi^{i}})},$$
(4)

where $p(\zeta^i)$ denotes the priors on ζ^i and $f(\widehat{\psi}^i)$ is the marginal density of $\widehat{\psi}^i$. The mode of the posterior distribution of ζ^i is computed by maximizing the value of the numerator in 4. The posterior distribution of ζ^i is computed using a standard Markov Chain Monte Carlo (MCMC) algorithm.

We estimate 8 structural parameters, i.e. $\zeta^i = \left[\rho_{\sigma^a}, \sigma, b, \phi_K, \phi_P, \rho_R, \rho_\pi, \rho_y \right]$. These parameters are the persistence of the second moment preference shock ρ_{σ^a} , the household risk aversion parameter σ , the consumption habit formation parameter b, the parameter regulating investment adjustment costs ϕ_K , the parameter regulating price adjustment costs ϕ_P , and the parameters of the Taylor rule ρ_R, ρ_π, ρ_y which - respectively - capture the degree of interest rate smoothing and the systematic response to inflation and output growth. Our priors are described in columns 3-4 of Table 3. We calibrate our prior means with the parameters used in Basu and Bundick's (2017) analysis, and we use diffuse priors. For the habit formation parameter and the parameters

¹³Guerron-Quintana, Inoue, and Kilian (2017) studies the asymptotic theory for VAR-based impulse response matching estimators of the structural parameters of linearized DSGE models when the number of impulse responses exceeds the number of linear VAR model parameters. The number of impulse responses in our analysis (140) is lower than the number of estimated coefficients of the VAR (251, constants excluded). We are aware of no contributions studying the asymptotic theory for this estimator when nonlinear frameworks are employed.

¹⁴Denoting by $\hat{\mathbf{W}}^i$ the bootstrapped variance-covariance matrix of VAR-based impulse responses $\hat{\boldsymbol{\psi}}^i$ for regime i, i.e., $\frac{1}{M}\sum_{j=1}^{M}(\boldsymbol{\psi}^i_j-\bar{\boldsymbol{\psi}}^i)(\boldsymbol{\psi}^i_j-\bar{\boldsymbol{\psi}}^i)'$ (where $\boldsymbol{\psi}^i_j$ denotes the realization of $\hat{\boldsymbol{\psi}}^i$ in the j^{th} (out of M=1000) bootstrap replication and $\bar{\boldsymbol{\psi}}^i$ denotes the mean of $\boldsymbol{\psi}^i_j$), \mathbf{V}^i is based on the diagonal of this matrix. Notice that \mathbf{V}^i contains the same variances that will be used to plot the confidence intervals for the I-VAR responses in next Section. This is the same approach used in Altig, Christiano, Eichenbaum, and Lindé (2011).

of the Taylor rule, we use the same priors employed by Christiano et al. (2011).¹⁵ The remaining parameters of the model are calibrated as in Basu and Bundick (2017). We confine a discussion on the calibration of these parameters to Appendix E for the sake of brevity.

3.3 Regime-specific estimation results

Our state-conditional model-based responses are reported in Figures 3 and 4 along with the IVAR-based bootstrapped confidence bands. 16 The model captures remarkably well the unrestricted dynamics of the economy in both regimes. Most of the DSGE impulse responses lie within the 68% confidence bands of the IVAR impulse responses. The model is able to replicate the stronger responses of real variables during contractions as well as the fact that their responses are longer-lived than responses in expansions. While working well for output, consumption, and investment, a note on the response of hours in recessions produced by the DSGE model is warranted. The model is able to generate a persistent contractionary response in hours worked whose shape is the same as the one produced by the IVAR framework. However, it falls short in replicating the facts as regards the magnitude of the response, which is clearly underestimated. This issue, which we share with Basu and Bundick (2017), may be due to various reasons. First, as pointed out by Basu and Bundick (2017), the dynamics of hours worked in the data during recessions is substantially influenced by low-productive types, which tend to quickly exit the labor market and whose dynamics drive hours worked on aggregate. By contrast, the model features a representative agent. Hence, by construction, it is ill-suited to represent aggregate dynamics driven by heterogeneities in the labor market. Second, the model predicts an expansions in precautionary labor supply which is contrasted by the contraction of labor demand due to the weakened demand for goods. A strong effect on hours by the expansion in labor supply makes the life of the model hard when it comes to generating a contraction in the equilibrium level of hours. Third,

¹⁵Canova and Sala (2009) show that the use of priors can hide identification issues even in population when it comes to estimating linearized DSGE frameworks. Given that we use priors common to the two regimes we focus on, lack of identification would work against finding state-dependent parameter values. We anticipate that our results point to differences in the parameter estimates between regimes. An exercise dealing with identification issues in the estimation of nonlinear DSGE frameworks is material for future research.

 $^{^{16}}$ Our bootstrapped confidence bands are based over 1,000 bootstrapped realizations for the impulse responses, which are used to compute the bootstrapped estimate of the standard errors of the impulse response functions. As in Altig, Christiano, Eichenbaum, and Lindé (2011), the confidence bands are constructed by considering the point estimates of the impulse response ± 1.64 times the bootstrapped estimate of the standard errors.

the labor market model in this framework features no relevant rigidities. Leduc and Liu (2016) show that search frictions work in favor of magnifying the effects of uncertainty shocks on labor market indicators. Our choice of working with a flexible labor market makes our results more directly comparable to those in Basu and Bundick (2017). Moreover, while falling short from a quantitative standpoint, the model is clearly able to generate comovements involving also hours worked in recessions.

The overall good performance of the model can also be appreciated by looking at Table 2, which compares the peak responses produced by the DSGE framework with the data. The model is clearly able to generate a relatively strong response of all real activity indicators in recessions with respect to expansions.

Turning to the nominal side, the performance of the model is admittedly less successful. The response of prices is, in general, not well captured by the DSGE model. The reason is that this model features an upward pricing bias (also present in other frameworks, see Born and Pfeifer (2014a), Fernández-Villaverde, Guerrón-Quintana, Kuester, and Rubio-Ramírez (2015), and Mumtaz and Theodoridis (2016)). This upward pricing bias, which relates to the uncertainty over future profits faced by entrepreneurs when setting their prices, contrasts the price effects of the contraction in real activity. Consequently, the model response of the federal funds rate is also milder than in the data. However, the lack of adherence of the model with the facts does not prevent the DSGE framework to replicate the generalized fall in real activity after an uncertainty shock in the two states of interest, which is what we focus on in this paper.

Table 3 (last two columns) collects the estimated parameters of the DSGE model for both regimes. In spite of sharing the same priors, some of the estimated parameters are clearly state-dependent.¹⁷ In particular, household's risk aversion is estimated to be larger in recessions; prices are found to be stickier during recessions. On the other hand, the persistence of the second moment preference shock ρ_{σ^a} is estimated to be the same between states. This implies that the different effects of uncertainty shocks in model-based responses are fully due to a different propagation mechanism which is only explained by differences in deep structural parameters. The degree of habits in consumption b and the parameter regulating investment adjustment costs ϕ_K are estimated to be basically the same in the two states. Given the difference between the prior means on the parameters and their posterior means, as well as the smaller posterior

¹⁷Appendix F reports priors and posterior densities of the estimated parameters. While not being a sufficient condition for identification, it is interesting to notice that priors and posteriors are clearly different. Appendix G reports detailed convergence diagnostics for the MCMC estimation.

standard deviation with respect to prior standard deviations, this result does not seem to be driven by an identification issue. Finally, the estimated policy rule signals a mildly stronger response to inflation and output in expansions, where monetary policy is also found to be more inertial. However, the differences between estimated policy parameters appear to be small.

Going back to the parameter instabilities detected by our econometric exercise, one may wonder what is the contribution of each single parameter to the different responses produced by our DSGE model in recessions and expansions. We then check the impact of each parameter on the impulse responses produced by the DSGE model as follows. Conditional on the set of estimates in expansions, we replace the value of each parameter with the corresponding estimated value in recessions. To be sure, the way in which the exercise is designed is such that, if we replaced all estimated parameters contemporaneously, by construction we would replicate the impulse responses produced by the DSGE in recessions. Appendix H documents the outcome of this exercise. Among the unstable parameters of the model, the dominant role is played by risk aversion. In line of this finding, we re-estimate the DSGE model by allowing only the degree of risk aversion to be state-dependent, while the other parameters take the estimated values conditional on the impulse responses produced with the linear version of the VAR. The estimated parameters conditional on the linear VAR are presented in Table 4 (fourth column). When we allow only the risk aversion parameter to vary we get estimates of 150.36 and 83.90 for recessions and expansions, respectively. Figures 3 and 4 plots the impulse responses produced by the version of the DSGE model in which risk aversion is the only parameter free to adjust between states over the responses produced by allowing all parameters to adjust. Evidently, risk aversion, which is estimated to be countercyclical, does the job by itself.

Wrapping up, our empirical investigation reveals that a DSGE model estimated by matching facts produced with a linear VAR does an excellent job in replicating the impulse responses produced with a nonlinear VAR framework and related to recessions and expansions. However, such excellent job occurs if risk aversion is actually allowed to change between states. In particular, risk aversion is estimated to be larger in recessions. Interestingly, a countercyclical risk aversion has recently been advocated by Cochrane (2017) as a feature macro-finance models should possess to match the data. Cohn, Engelmann, Fehr, and André Maréchal (2015) provide experimental evidence suggesting that financial market professionals are more risk averse during a financial bust than a boom. This evidence suggests that fear may play an important role in explain-

ing countercyclical risk aversion. The same conclusion is reached by Guiso, Sapienza, and Zingales (2017), who provide experimental evidence in favor of a fear model in which agents experience higher risk aversion in periods of crisis. Kim (2014) estimates a consumption-based capital asset pricing model with time-varying risk aversion based on the Epstein–Zin recursive utility, and finds strong support for the countercyclicality of the risk aversion parameter. Our paper lines up with these contributions by identifying countercyclical risk aversion as crucial to replicate the asymmetric response of real activity to uncertainty shocks along the business cycle. As conjectured by Cohn, Engelmann, Fehr, and André Maréchal (2015), we find that risk aversion amplifies economic dynamics in response to a shock.¹⁸

The ability of the estimated model to replicate the empirical facts produced with our nonlinear VAR depends exclusively from the instability in the risk aversion parameter. In fact, the role played by initial conditions in the nonlinear DSGE model we work with is empirically negligible. Appendix I shows this by computing the DSGE-based (state-) conditional GIRFs as defined in Andreasen, Fernández-Villaverde, and Rubio-Ramírez (2017). Conditional GIRFs relative to uncertainty shocks are found to be quantitatively insensitive to different initial conditions. Hence, to replicate the different real effects of uncertainty shocks, the estimated DSGE model has to rely on parameter instability. The message in this paper is that instability in risk aversion is empirically found to be necessary and sufficient for the DSGE model to replicate the facts.

4 The great recession

So far, our analysis has shown that a nonlinear DSGE model estimated with an auxiliary nonlinear VAR framework goes a long way in replicating the asymmetric reaction of real activity indicators to an uncertainty shock. In performing this exercise, the reference facts we considered regarded the different responses of real activity in recessions and expansions in general. However, the recent great recession is clearly a rare event which hardly falls under the "standard recessions" category. A way to lend support to this statement is to exploit the flexibility of our nonlinear VAR, which allows us to move

¹⁸Andreasen, Fernández-Villaverde, and Rubio-Ramírez (2017) show that, in a model featuring a portfolio allocation problem related to short- and long-term bonds plus a systematic response of the central bank to the term spread, uncertainty shocks to households' preferences generate moments consistent with the data even in presence of moderate values of risk aversion. The moments studied by Andreasen, Fernández-Villaverde, and Rubio-Ramírez (2017) are, however, unconditional moments, i.e., they are not state-specific.

from an analysis between states to an analysis across dates. This is possible because of the dependence of impulse responses to initial conditions in nonlinear models (Koop, Pesaran, and Potter (1996)). We then work out the temporal evolution of the peak response of GDP, consumption, investment and hours worked over a five-year horizon to an equally sized uncertainty shock by associating to each quarter an uncertainty shock occurring in each initial quarter of our sample.¹⁹

Figure 5 displays the outcome of this exercise. The peak responses are much higher (in absolute terms) in recessions, a finding in line with the empirical facts documented in Section 2. Moreover, the peaks point to a particularly strong response of real activity in three recessions, i.e., 1974-1975, 1981-1982, and 2007-2009. Clearly, the great recession is an outlier, i.e., all four real activity indicators display their maximum responses (in absolute terms) during that extreme event. When compared to the peaks predicted by a linear VAR model, the nonlinear framework returns figures regarding the great recession which are between two and three times larger.

Can our structural DSGE model replicate the drop in real activity occurred during the great recession? While to replicate the average effects of uncertainty shocks in recession an adjustment of the calibration of the risk aversion parameter may be enough, for this particular recession some other characteristic of the model may also need to adjust. In fact, the beginning of the great recession called for a dramatic cut of the policy rate, which moved from 5.25% in July 2008 to basically zero in just five months. Hence, one would think that also the Taylor rule parameters should adjust to have the model replicate the spectacular contraction experienced by the U.S. economy. To investigate this issue, we then focus on 2008Q4, the quarter associated to the largest realization of the uncertainty shock according to our IVAR model, and estimate our DSGE model conditional on the IVAR responses for this quarter.

Table 4 (last three columns) collects the estimates relative to different versions of the estimated model, i.e., one which just allows risk aversion to change; one which allows risk aversion and the policy parameters to adjust; and one which allows all parameters to adjust. The information reported in the table confirms that changes in the estimated values of risk aversion and the policy parameters are needed to replicate the great recession. This is confirmed by the next exercise.

Equipped with this estimated framework, we conduct an exercise which aims at quantifying the contribution of uncertainty shocks for the cumulative output loss recorded

¹⁹The figure with the cumulative responses of real activity over a five-year horizon delivers the very same qualitative message.

by the U.S. economy during and after the great recession. To quantify the relative contribution of the uncertainty shock for the cumulative output loss experienced by the U.S. economy, we follow Basu and Bundick (2017) and take as an external reference the CBO output gap, which is a "detrended" measure of output.²⁰ The exercise goes as follows. Assume output to be at its stochastic steady state before the advent of the large uncertainty shock in 2008Q4. Then, the real world is hit by the large uncertainty shock in 2008Q4, as well as a variety of other shocks. We then set the output gap to zero in 2008Q3, and compute the cumulative output loss during the period is which this re-scaled measure of the output gap is negative, i.e., 2008Q4-2014Q2. Finally, we produce the response of output with our estimated DSGE model in which risk aversion and the Taylor rule parameters are free to adjust. To appreciate the role of nonlinearities for the calibration of the DSGE model, we also produce the response of output to an equally sized uncertainty shock with a version of the DSGE model estimated conditional on the impulse responses produced with a linear VAR. We consider responses to a 4.5 standard deviation uncertainty shock, which is the one estimated by our VAR for the 2008Q4.

Figure 6 compares the responses produced with our favorite DSGE models, i.e., the one estimated on the nonlinear VAR, a version of the DSGE model estimated conditional on a linear VAR, and the evolution of the output gap (normalized to zero in 2008Q3, i.e., before the shock hits). The cumulative loss of output in the period considered here is equal to 53% (with respect to the trend). Our nonlinear DSGE model estimates an uncertainty shock in 2008Q4 to be associated to a cumulative output loss equal to 24%, which is almost 1/2 of the total loss. This is twice as much the loss predicted by a DSGE model estimated conditional on a linear VAR, which is, 13%. Allowing for the risk aversion only parameter to adjust would still imply a good prediction of the peak response of output after the shock. However, it would also imply a too quick return of predicted output to the steady state, which would lead to an underestimation of the output loss caused by the uncertainty shock.

5 Conclusion

This paper employs a nonlinear VAR framework to document that financial uncertainty shocks exert a stronger effect on real activity in recessions. A nonlinear structural DSGE

²⁰The evolution of the CBO output gap is quantitatively the same as the one obtained by filtering log real GDP per capita with a standard Hodrick-Prescott filter (smoothing weight: 1,600).

model is fitted to the data relative to booms and busts to interpret this evidence. The estimation of the DSGE model is conducted by working with a Bayesian direct inference approach, which is applied in a novel manner to a nonlinear DSGE structure. Counter-cyclical risk aversion is identified as the key element that enables the DSGE model to replicate the empirical facts. Focusing on the great recession, we show that a combination of high risk aversion and weak, low-inertial response to inflation is sufficient for the structural model to replicate the response of real activity in 2008Q4 produced by the nonlinear VAR. When estimated targeting such response, the model assigns about 50% of the output loss materialized in 2008-2014 to a big financial uncertainty shock occurred at the end of 2008. The same DSGE model estimated by targeting the impulse responses produced by a linear VAR is shown to substantially downplay the role played by such shock.

This paper offers solid support to Federal Reserve's former Chairman Alan Greenspan view on macroeconomic modeling: "[...] it is apparent that a prominent shortcoming of our structural models is that, for ease in parameter estimation, not only are economic responses presumed fixed through time, but they are generally assumed to be linear. An assumption of linearity may be adequate for estimating average relationships, but few expect that an economy will respond linearly to every aberration." Our results stress the importance of using nonlinear frameworks for correctly quantifying the effects of uncertainty shocks - and, more generally, macroeconomic shocks - for the U.S. business cycle. Nonlinear VARs can fruitfully be used to establish facts which serve as a reference to build and evaluate structural frameworks used to conduct policy analysis. Correctly calibrating such frameworks is of paramount importance to conduct informative historical and policy analysis.

Our paper shows that a state-of-the-art nonlinear DSGE model is able to explain the state-dependent response of real activity to an uncertainty shock via the instability of the estimated risk aversion parameter. We see this result as informative for the construction of theoretical models featuring endogenous mechanisms able to replicate the nonlinear effects of uncertainty shocks.

References

AASTVEIT, K. A., G. J. NATVIK, AND S. SOLA (2017): "Economic Uncertainty and the Influence of Monetary Policy," *Journal of International Money and Finance*, 76, 50–67.

- Albuquerque, R., M. Eichenbaum, V. X. Luo, and S. Rebelo (2016): "Valuation Risk and Asset Pricing," *Journal of Finance*, LXXI(6), 2861–2903.
- ALESSANDRI, P., AND H. MUMTAZ (2014): "Financial Regimes and Uncertainty Shocks," Queen Mary University of London Working Paper No. 729.
- ALTIG, D., L. J. CHRISTIANO, M. EICHENBAUM, AND J. LINDÉ (2011): "Firm-Specific Capital, Nominal Rigidities and the Business Cycle," *Review of Economic Dynamics*, 14(2), 225–247.
- Andreasen, M. M. (2012): "On the Effects of Rare Disasters and Uncertainty Shocks for Risk Premia in Non-Linear DSGE Models," *Review of Economic Dynamics*, 15(3), 295–316.
- Andreasen, M. M., J. Fernández-Villaverde, and J. F. Rubio-Ramírez (2017): "The Pruned State-Space System for Non-Linear DSGE Models: Theory and Empirical Applications," *Review of Economic Studies*, forthcoming.
- AUERBACH, A., AND Y. GORODNICHENKO (2012): "Measuring the Output Responses to Fiscal Policy," American Economic Journal: Economic Policy, 4(2), 1–27.
- Basu, S., and B. Bundick (2017): "Uncertainty Shocks in a Model of Effective Demand," *Econometrica*, 85(3), 937–958.
- BIANCHI, F., AND L. MELOSI (2017): "Escaping the Great Recession," American Economic Review, 107(4), 1030–58.
- BINSBERGEN, J. H. V., J. FERNÁNDEZ-VILLAVERDE, R. S. KOIJEN, AND J. RUBIO-RAMÍREZ (2012): "The term structure of interest rates in a DSGE model with recursive preferences," *Journal of Monetary Economics*, 59(7), 634Ű648.
- BLOOM, N. (2009): "The Impact of Uncertainty Shocks," *Econometrica*, 77(3), 623–685.
- ———— (2014): "Fluctuations in Uncertainty," Journal of Economic Perspectives, 28(2), 153–176.
- BORN, B., AND J. PFEIFER (2014a): "Policy Risk and the Business Cycle," *Journal of Monetary Economics*, 68, 68–85.

- ———— (2014b): "Risk Matters: The Real Effects of Volatility Shocks: Comment," American Economic Review, 104(12), 4231–4239.
- ———— (2017): "Uncertainty-driven business cycles: assessing the markup channel," University of Bonn, mimeo.
- CAGGIANO, G., E. CASTELNUOVO, V. COLOMBO, AND G. NODARI (2015): "Estimating Fiscal Multipliers: News From a Nonlinear World," *Economic Journal*, 125(584), 746–776.
- CAGGIANO, G., E. CASTELNUOVO, AND J. M. FIGUERES (2017): "Economic Policy Uncertainty and Unemployment in the United States: A Nonlinear Approach," *Economics Letters*, 151, 31–34.
- CAGGIANO, G., E. CASTELNUOVO, AND N. GROSHENNY (2014): "Uncertainty Shocks and Unemployment Dynamics: An Analysis of Post-WWII U.S. Recessions," *Journal of Monetary Economics*, 67, 78–92.
- CAGGIANO, G., E. CASTELNUOVO, AND G. NODARI (2017): "Uncertainty and Monetary Policy in Good and Bad Times," Melbourne Institute Working Paper No. 9/17.
- CAGGIANO, G., E. CASTELNUOVO, AND G. PELLEGRINO (2017): "Estimating the Real Effects of Uncertainty Shocks at the Zero Lower Bound," *European Economic Review*, 100, 257–272.
- Caldara, D., J. Fernández-Villaverde, J. F. Rubio-Ramírez, and W. Yao (2012): "Computing DSGE Models with Recursive Preferences and Stochastic Volatility," *Review of Economic Dynamics*, 15, 188–206.
- Canova, F., and L. Sala (2009): "Back to Square One: Identification Issues in DSGE Models," *Journal of Monetary Economics*, 56(4), 431–449.
- Castelnuovo, E., and G. Pellegrino (2017): "Uncertainty-dependent Effects of Monetary Policy Shocks: A New Keynesian Interpretation," mimeo.
- Christiano, L., M. Trabandt, and K. Walentin (2011): "DSGE Models for Monetary Policy Analysis," in: B. M. Friedman and M. Woodford (Eds.): Handbook of Monetary Economics, Volume 3a, 285–367.

- Christiano, L. J., M. Eichenbaum, and C. Evans (1999): "Monetary Policy Shocks: What Have We Learned and to What End?," In: J.B. Taylor and M. Woodford (eds.): Handbook of Macroeconomics, Elsevier Science, 65–148.
- COCHRANE, J. (2017): "Macro-Finance," Review of Finance, 21(3), 945–985.
- Cohn, A., J. Engelmann, E. Fehr, and M. A. Maréchal (2015): "Evidence for countercyclical risk aversion: an experiment with financial professionals," *The American Economic Review*, 105(2), 860–885.
- DE GROOT, O., A. W. RICHTER, AND N. A. THROCKMORTON (2017): "Uncertainty Shocks in a Model of Effective Demand: Comment," Federal Reserve Bank of Dallas Research Department Working Paper 1706.
- Drautzburg, T., J. Fernández-Villaverde, and P. Guerrón-Quintana (2017): "Political Distribution Risk and Aggregate Fluctuations,".
- Fernández-Villaverde, J., P. Guerrón-Quintana, K. Kuester, and J. F. Rubio-Ramírez (2015): "Fiscal Volatility Shocks and Economic Activity," *American Economic Review*, 105(11), 3352–3384.
- FERNÁNDEZ-VILLAVERDE, J., P. GUERRÓN-QUINTANA, J. F. RUBIO-RAMÍREZ, AND M. URIBE (2011): "Risk Matters: The Real Effects of Volatility Shocks," *American Economic Review*, 101, 2530–2561.
- FERRARA, L. (2003): "A three-regime real-time indicator for the US economy," *Economics Letters*, 81(3), 373–378.
- Geweke, J. (1992): "Evaluating the Accuracy of Sampling-Based Approaches to the Calculation of Posterior Moments," in *IN BAYESIAN STATISTICS*.
- Gonçalves, S., and L. Kilian (2004): "Bootstrapping autoregressions with conditional heteroskedasticity of unknown form," *Journal of Econometrics*, 123(1), 89–120.
- Greenspan, A. (2003): "Monetary Policy under Uncertainty," Remarks at a symposium sponsored by the Federal Reserve Bank of Kansas City, Jackson Hole, Wyoming, August 29.
- Guerron-Quintana, P., A. Inoue, and L. Kilian (2017): "Impulse Response Matching Estimators for DSGE Models," *Journal of Econometrics*, 196, 144–155.

- Guiso, L., P. Sapienza, and L. Zingales (2017): "Time Varying Risk Aversion," Journal of Financial Economics, forthcoming.
- Hall, A., A. Inoue, J. Nason, and B. Rossi (2012): "Information Criteria for Impulse Response Function Matching Estimation of DSGE Models," *Journal of Econometrics*, 170(2), 499–518.
- JERMANN, U. (1998): "Asset Pricing in Production Economies," *Journal of Monetary Economics*, 41, 257–275.
- Jurado, K., S. C. Ludvigson, and S. Ng (2015): "Measuring Uncertainty," American Economic Review, 105(3), 1177–1216.
- KILIAN, L. (2009): "Not all oil price shocks are alike: Disentangling demand and supply shocks in the crude oil market," *American Economic Review*, 99(3), 1053–69.
- KILIAN, L., AND R. VIGFUSSON (2011): "Are the Responses of the U.S. Economy Asymmetric in Energy Price Increases and Decreases?," *Quantitative Economics*, 2, 419–453.
- Kim, K. H. (2014): "Counter-cyclical risk aversion," Journal of Empirical Finance, 29, 384–401.
- KOOP, G., M. PESARAN, AND S. POTTER (1996): "Impulse response analysis in nonlinear multivariate models," *Journal of Econometrics*, 74(1), 119–147.
- Leduc, S., and Z. Liu (2016): "Uncertainty Shocks are Aggregate Demand Shocks," Journal of Monetary Economics, 82, 20–35.
- Ludvigson, S. C., S. Ma, and S. Ng (2017): "Uncertainty and Business Cycles: Exogenous Impulse or Endogenous Response?," New York University and Columbia University, mimeo.
- MERTENS, K., AND M. O. RAVN (2014): "A reconciliation of SVAR and narrative estimates of tax multipliers," *Journal of Monetary Economics*, 68, S1–S19.
- Mumtaz, H., and K. Theodoridis (2016): "The changing transmission of uncertainty shocks in the US: An empirical analysis," *Journal of Business and Economic Statistics*, forthcoming.

- Pellegrino, G. (2017a): "Uncertainty and Monetary Policy in the US: A Journey into Non-Linear Territory," Melbourne Institute Working Paper No. 6/17.
- ——— (2017b): "Uncertainty and the Real Effects of Monetary Policy Shocks in the Euro Area," *Economics Letters*, forthcoming.
- ROTEMBERG, J. J. (1982): "Monopolistic Price Adjustment and Aggregate Output," Review of Economic Studies, 49, 517–531.
- RUGE-MURCIA, F. (2017): "Indirect Inference Estimation of Nonlinear Dynamic General Equilibrium Models: With an Application to Asset Pricing under Skewness Risk," McGill University, mimeo.
- SCHMITT-GROHE, S., AND M. URIBE (2004): "Solving Dynamic General Equilibrium Models Using a Second-Order Approximation to the Policy Function," *Journal of Economic Dynamics and Control*, 28, 755–775.
- SÁ, F., P. TOWBIN, AND T. WIELADEK (2014): "Capital Inflows, Financial Structure and Housing Booms," Journal of the European Economic Association, 12(2), 522–546.
- TERÄSVIRTA, T. (2017): "Nonlinear models in macroeconometrics," CREATES Research Paper 2017-32.
- TERÄSVIRTA, T., D. TJØSTHEIM, AND C. W. GRANGER (2010): "Modeling Nonlinear Economic Time Series," Oxford University Press, Oxford.
- TOWBIN, P., AND S. WEBER (2013): "Limits of floating exchange rates: The role of foreign currency debt and import structure," *Journal of Development Economics*, 101(1), 179–101.
- Wu, J. C., and F. D. Xia (2016): "Measuring the Macroeconomic Impact of Monetary Policy at the Zero Lower Bound," *Journal of Money, Credit, and Banking*, 48(2-3), 253–291.

\overline{Sample}	Regime	Y	C	I	Н
Full sample	Linear	-0.31	-0.31	-0.40	-0.40
	Recessions	-0.32	-0.31	-0.42	-0.39
	Expansions	-0.05	-0.14	-0.14	-0.16
Pre-ZLB	Linear	-0.30	-0.30	-0.38	-0.35
	Recessions	-0.31	-0.26	-0.37	-0.27
	Expansions	-0.06	-0.14	-0.17	-0.15

Table 1: **Uncertainty-real activity correlations.** Real GDP, consumption, investment, and hours considered in per capita terms and quarterly growth rates. Full sample: 1962Q3-2017Q2, pre-ZLB: 2008Q4-2008Q4. Uncertainty proxied by the VXO (available since 1986) spliced with the within-quarter volatility of the SP500 returns (available before 1986) as in Bloom (2009).

	Peak re	esponses rela	ative to s	trong expansions
	Deep c	ontractions	Stro	ng expansions
Variable	VAR	DSGE	VAR	DSGE
Y	1.49	1.67	1	1
\mathbf{C}	1.36	1.46	1	1
I	2.08	1.82	1	1
Η	2.28	1.79	1	1
	Р	eak response	es relativ	e to output
	Deep c	ontractions	Stro	ng expansions
Variable	VAR	DSGE	VAR	DSGE
Y	1	1	1	1
\mathbf{C}	0.55	0.42	0.60	0.48
I	3.42	2.67	2.46	2.46
H	1.45	0.64	0.95	0.60

Table 2: **IVAR responses: Relative moments.** Sample: 1962Q3-2017Q2. VAR estimated with four lags.

		P	Priors	Poste	Posteriors
				Deep contractions	Strong expansions
		Distribution	Distribution Mean, std.dev.	Mean, std.dev.	Mean, std.dev.
Parameter	Parameter Interpretation	[spunoq]		[5% and $95%]$	[5% and 95%]
ρ_{σ^a}	Pers. unc. shock	Beta	0.74, 0.10	0.67 , 0.02	0.67, 0.03
		[0,1]		[0.63, 0.71]	[0.63, 0.72]
σ	Risk aversion	Gamma	80, 60	148.09 , 20.43	86.02 , 22.13
		[0,8]		[112.34, 182.62]	[54.81, 113.28]
q	Habit form.	\mathbf{Beta}	0.75, 0.15	0.75 , 0.06	0.70 , 0.11
		[0,1]		[0.65, 0.84]	[0.54, 0.85]
ϕ_K	Inv. adj. costs	Gamma	2.09, 2	1.25 , 0.27	1.43 , 0.30
		[0,8]		[0.76, 1.74]	[0.91, 1.86]
ϕ_P	Pr. adj. costs	Gamma	100, 60	280.07 , 65.54	104.37 , 58.39
		[0,8]		[167.36, 392.02]	[37.10, 166.69]
$ ho_R$	TR par., smooth.	Beta	0.800, 0.10	0.70 , 0.09	0.80 , 0.10
		[0,1]		[0.54,0.85]	[0.66, 0.95]
$ ho_{\pi}$	TR par., infl.	Gamma	1.600, 0.15	1.50, 0.15	1.61 , 0.15
		[1.01, 4]		[1.26, 1.71]	[1.37, 1.84]
$ ho_y$	TR par., out. gr.	Gamma	0.200, 0.15	0.30 , 0.06	0.33 , 0.08
S		[0,2]		[0.19, 0.41]	$[\ 0.19,\ 0.48]$

Table 3: Regime-dependent estimated parameter values. Posteriors computed via MCMC with a random walk metropolis algorithm. 60 000 draws, 20 percent for burn-in. Acceptance rates: 27 percent for times of deep contractions, $26\ \mathrm{percent}$ for times of strong contractions.

		Priors	Posteriors			
			Linear VAR	Great Recession	nc	
			All param.	only σ	σ and TR	All param.
Parameter	Interpretation	D(mean,std)	Mean,std	Mode,std	Mode,std	Mode,std
σ_{σ^a}	Unc.shock, size	IG(0.0025,0.0025)	0.005,0.001	ı	ı	ı
$ ho_{\sigma^a}$	Unc.shock,pers.	B(0.74,0.10)	0.67,0.03	ı	ı	0.68,0.019
δ	Risk aversion	G(80,60)	137.79 ,27.49	200.85 , 9.70	202.02 , 12.20	246.55 ,22.90
q	Habits	B(0.75,15)	0.71,0.06	ı	ı	0.81 ,0.03
ϕ_K	Inv. adj. costs	G(2.09,2)	1.57,0.46	ı	ı	2.64 ,0.36
ϕ_P	Price adj. costs	G(100,60)	216.23,37.23	ı	ı	182.11 ,28.64
$ ho_R$	TR par., smooth.	B(0.800,0.10)	0.75,0.09	ı	0.35,0.09	0.28,0.07
$ ho_{\pi}$	TR par., inflat.	G(1.600,0.15)	$\boldsymbol{1.53}, 0.15$	1	1.14,0.02	1.14,0.02
ρ_y	TR par., out. gr.	G(0.200,0.15)	0.36,0.1	1	0.28,0.01	0.41,0.05

Table 4: **DSGE model: Average evidence vs. Great recession.** Values estimated conditional on both the linear VAR impulse responses in 2008Q4.

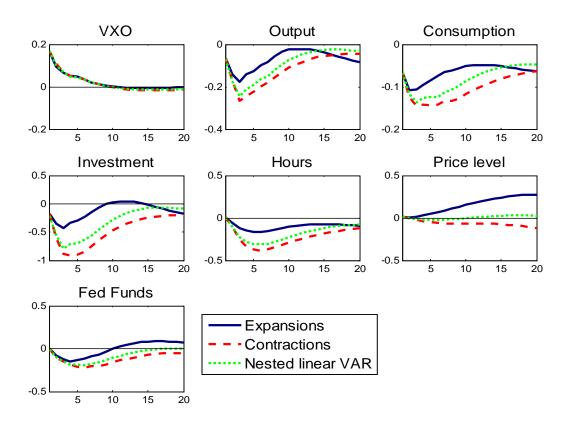


Figure 1: **IVAR** impulse responses to an uncertainty shock in recessions and **expansions.** Red line: Deep recessions. Blue line: Strong expansions. Green line: Responses associated to the nested linear VAR. Sample: 1962Q3-2017Q2. VAR estimated with four lags.

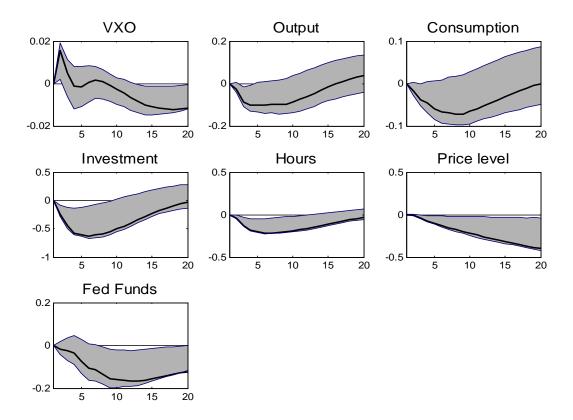


Figure 2: Differences of the IVAR impulse responses to an uncertainty shock: Recessions vs. expansions. Solid black lines: difference between the point estimated state-conditional GIRFs in recessions and expansions (taking the difference between the latter and the former). Grey areas: 68 percent confidence bands of the difference (from its distribution constructed with 2,000 bootstrap draws). Sample: 1962Q3-2017Q2. VAR estimated with four lags.

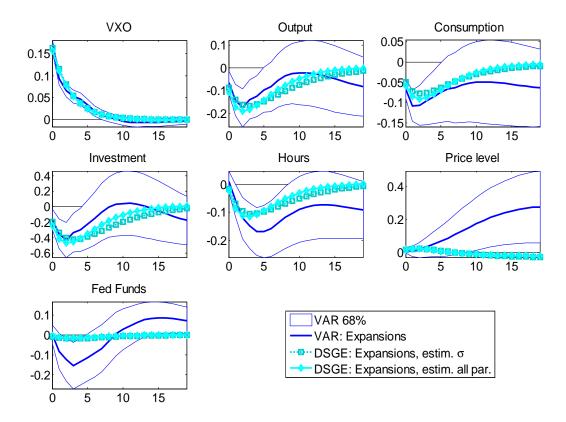


Figure 3: **DSGE vs. IVAR impulse responses to an uncertainty shock: Expansions.** Solid lines with squares: Responses of the DSGE model estimated by allowing only risk aversion to adjust between recessions and expansions. Solid lines with diamonds: Responses of the DSGE model estimated by allowing all parameters to adjust between recessions and expansions. Areas identified by blue lines: 68% confidence interval produced with the IVAR. Sample: 1962Q3-2017Q2. VAR estimated with four lags.

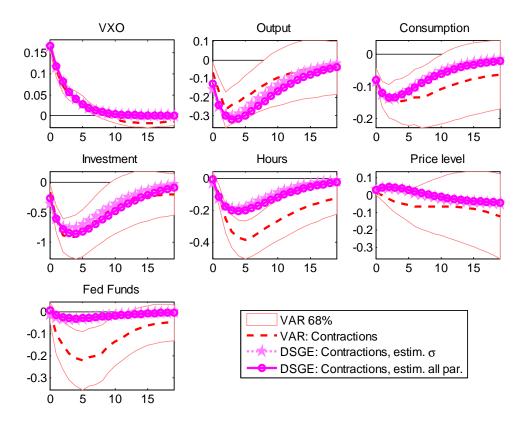


Figure 4: **DSGE vs. IVAR impulse responses to an uncertainty shock: Recessions.** Solid lines with squares: Responses of the DSGE model estimated by allowing only risk aversion to adjust between recessions and expansions. Solid lines with diamonds: Responses of the DSGE model estimated by allowing all parameters to adjust between recessions and expansions. Areas identified by blue lines: 68% confidence interval produced with the IVAR. Sample: 1962Q3-2017Q2. VAR estimated with four lags.

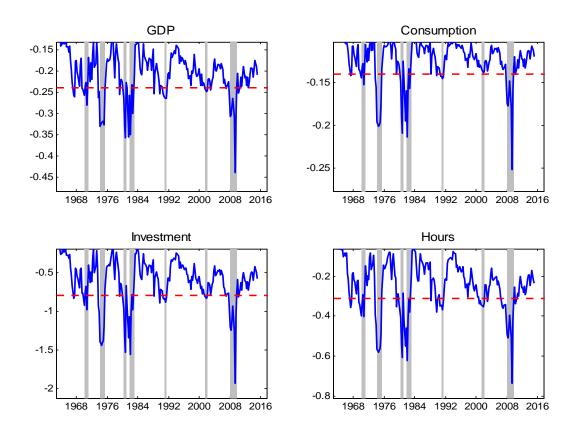


Figure 5: **IVAR time-varying impulse responses to an uncertainty shock.** Blue lines: Peak responses over a five-year horizon. Red lines: Peak responses as computed by a linear VAR. Sample: 1962Q3-2017Q2. VARs estimated with four lags.

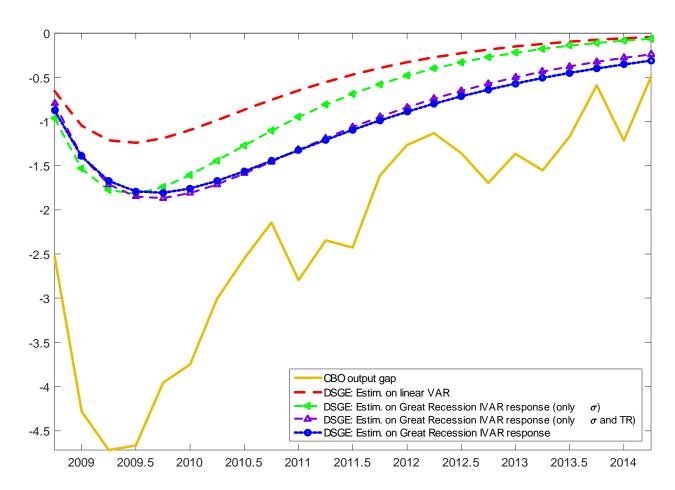


Figure 6: **Great Recession: Contribution of uncertainty shocks.** Output gap normalized to zero in 2008Q3. DSGE: Estim on Great Recession IVAR (only sigma) and (only sigma and TR): Non-estimated parameters calibrated at the value found when estimating the model with a linear VAR.

Part II

Appendix

A: Parsimonious vs. extended IVAR

The IVAR model employed in the paper is a parsimonious version of a more sophisticated IVAR which we estimated to check the robustness of our results. Thinking of the third-order approximation of the DSGE model we work with, it is natural to extent our baseline IVAR framework to add extra interaction terms involving quadratic terms as follows:

$$\mathbf{Y}_{t} = \boldsymbol{\alpha} + \sum_{j=1}^{L} \mathbf{A}_{j} \mathbf{Y}_{t-j} + \begin{bmatrix} \sum_{j=1}^{L} \mathbf{c}_{j} \ln VXO_{t-j} \times \Delta \ln GDP_{t-j} \\ + \sum_{j=1}^{L} \mathbf{c}_{j} (\ln VXO_{t-j})^{2} \times \Delta \ln GDP_{t-j} \\ + \sum_{j=1}^{L} \mathbf{c}_{j} \ln VXO_{t-j} \times (\Delta \ln GDP_{t-j})^{2} \end{bmatrix} + \mathbf{u}_{t}$$

Cubic terms $((\ln VXO_{t-j})^3, (\Delta \ln GDP_{t-j})^3)$ are omitted to minimize the risk of explosiveness.

Figure A1 contrasts the impulse responses obtained with our baseline model with those produced with the enriched framework. If anything, the reactions produced by this framework speak even more clearly in favor of nonlinearities in the data.

B: Match between VAR and DSGE model

The Basu and Bundick (2017) has a structure which is de facto consistent with the assumptions undertaken in Section 2 which regard the identification of uncertainty shocks. In our recursively identified nonlinear VAR model the VXO is ordered first and hence it is assumed that, while uncertainty shocks can contemporaneously affect all variables in the VAR, the VXO cannot be contemporaneously affected by other shocks. The Basu and Bundick (2017) model features an endogenous measure of financial uncertainty (a model-consistent VXO) which responds to three shocks, i.e., a first-moment technology shock, a first-moment preference shock, and a second-moment preference shock, this last one being the uncertainty shock. Conditional on Basu and Bundick's (2017) calibration, however, the uncertainty shock and the model-consistent VXO move hand-in-hand, i.e., the VXO reacts very little to shocks other than the uncertainty one.

A Monte Carlo exercise with artificial data simulated with the Basu and Bundick (2017) framework confirms this statement. We conduct a population analysis and simulate a sample of 50,000 observations with the model calibrated as in Basu and Bundick

(2017). ²¹ We then estimate a linear VAR and produce impulse responses to an uncertainty shock identified with a Cholesky decomposition of the reduced-form variance-covariance matrix. In the VAR, the VXO is ordered first. We focus on a population analysis and on a linear VAR to make sure that our result is not driven by any small-sample issue or fancy nonlinear reduced-form framework.

Figure A2 documents the performance of the Cholesky-VAR in replicating the DSGE-model consistent impulse responses. The ability of the VAR to correctly capture the responses of the DSGE model is impressive. This result justifies the use of a direct impulse-response function matching approach when we estimate the DSGE framework. Figure A3 adds evidence on the "quasi-exogeneity" of the model consistent VXO process by plotting the volatility of the preference shock against that of the VXO. The two series clearly comove, and their correlation is equal to 0.95.

C: GIRFs in presence of the shadow rate

Figure A4 shows that our results are practically the same if the Wu and Xia (2016) shadow rate is used in place of the federal funds rate for the period of zero lower bound. As shown by the authors the shadow rate, and its meaningful variations, can be used to proxy unconventional monetary policy at the zero lower bound.

D: Computation of the Generalized Impulse Response Functions

The algorithm for the computation of the Generalized Impulse Response Functions follows the steps suggested by Koop, Pesaran, and Potter (1996), and it is designed to simulate the effects of an orthogonal structural shock as in Kilian and Vigfusson (2011). The idea is to compute the empirical counterpart of the theoretical $GIRF_{\mathbf{y}}(h, \delta, \boldsymbol{\omega}_{t-1})$ of the vector of endogenous variables \mathbf{y}_t , h periods ahead, for a given initial condition $\boldsymbol{\omega}_{t-1} = \{\mathbf{y}_{t-1}, ..., \mathbf{y}_{t-k}\}$, k is the number of VAR lags, and δ is the structural shock hitting at time t. Following Koop, Pesaran, and Potter (1996), such GIRF can be expressed as follows:

$$GIRF_{\mathbf{y}}(h, \delta, \boldsymbol{\omega}_{t-1}) = E[\mathbf{y}_{t+h} | \delta, \boldsymbol{\omega}_{t-1}] - E[\mathbf{y}_{t+h} | \boldsymbol{\omega}_{t-1}]$$

²¹Notice that nothwistanding the fact that we use a DSGE model with three shock to simulate data on which to estimate a seven variables VAR model, we do not suffer from stochastic singularity given that the DSGE model at the basis of the simulation is nonlinearized, and hence there is no linear combination of variables that is perfectly collinear to others.

where $E[\cdot]$ is the expectation operator, and h = 0, 1, ..., H indicates the horizons from 0 to H for which the computation of the GIRF is performed.

Given our model (1), we compute our GIRFs as follows:

- 1. we pick an initial condition ω_{t-1} . Notice that, given that uncertainty and the policy rate are modeled in the VAR, such set includes the values of the interaction terms $(\ln VXO \times \Delta \ln GDP)_{t-j}$, j=1,...,k;
- 2. conditional on ω_{t-1} and the structure of the model (1), we simulate the path $[\mathbf{y}_{t+h} | \omega_{t-1}]^r$, h = [0, 1, ..., 19] (which is, realizations up to 20-step ahead) by loading our VAR with a sequence of randomly extracted (with repetition) residuals $\widetilde{\mathbf{u}}_{t+h}^r \sim d(0, \widehat{\Omega})$, h = 0, 1, ..., H, where $\widehat{\Omega}$ is the estimated VCV matrix, $d(\cdot)$ is the empirical distribution of the residuals, and r indicates the particular sequence of residuals extracted;
- 3. conditional on ω_{t-1} and the structure of the model (1), we simulate the path $[\mathbf{y}_{t+h} | \delta, \boldsymbol{\omega}_{t-1}]^r$, h = [0, 1, ..., 19] by loading our VAR with a perturbation of the randomly extracted residuals $\widetilde{\mathbf{u}}_{t+h}^r \sim d(0, \widehat{\Omega})$ obtained in step 2. In particular, we Cholesky-decompose $\widehat{\Omega} = \widehat{\mathbf{C}}\widehat{\mathbf{C}}'$, where $\widehat{\mathbf{C}}$ is a lower-triangular matrix. Hence, we recover the orthogonalized elements (shocks) $\widetilde{\boldsymbol{\varepsilon}}_t^r = \widehat{\mathbf{C}}^{-1}\widetilde{\mathbf{u}}_t^r$. We then add a quantity $\delta > 0$ to the $\widetilde{\boldsymbol{\varepsilon}}_{unc,t}^r$, where $\widetilde{\boldsymbol{\varepsilon}}_{unc,t}^r$ is the scalar stochastic element loading the uncertainty equation in the VAR. This enable us to obtain $\widetilde{\boldsymbol{\varepsilon}}_t^r$, which is the vector of perturbed orthogonalized elements embedding $\widetilde{\boldsymbol{\varepsilon}}_{unc,t}^r$. We then move from perturbed shocks to perturbed residuals as follows: $\widetilde{\mathbf{u}}_t^r = \widehat{\mathbf{C}}\widetilde{\boldsymbol{\varepsilon}}_t^r$. These are the perturbed residuals that we use to simulate $[\mathbf{y}_{t+h} | \delta, \boldsymbol{\omega}_{t-1}]^r$;
- 4. we compute the difference between paths for each simulated variable at each simulated horizon $[\mathbf{y}_{t+h} | \delta, \boldsymbol{\omega}_{t-1}]^r [\mathbf{y}_{t+h} | \boldsymbol{\omega}_{t-1}]^r$, h = [0, 1, ..., 19];
- 5. we repeat steps 2-4 a number of times equal to R = 500. We then store the horizon-wise average realization across repetitions r. In doing so, we obtain a consistent estimate of the GIRF per each given initial quarter of our sample, i.e., an history-dependent GIRF, $\widehat{GIRF}_{\mathbf{y}}(h, \delta_t, \boldsymbol{\omega}_{t-1}) = \widehat{E}[\mathbf{y}_{t+h} | \delta, \boldsymbol{\omega}_{t-1}] \widehat{E}[\mathbf{y}_{t+h} | \boldsymbol{\omega}_{t-1}],$ h = [0, 1, ..., 19]. If a given initial condition $\boldsymbol{\omega}_{t-1}$ leads to an explosive response (namely if this is explosive for most of the R sequences of residuals $\widetilde{\mathbf{u}}_{t+h}^r$, in the sense that the response of the shocked variable diverges instead than reverting to

- zero), then such initial condition is discarded (i.e., they are not considered for the computation of state-dependent GIRFs in step 6);²²
- 6. in order to produce our state-dependent GIRFs for recessions and expansions, we first split previous initial conditions into two subsets of interest. To do so, an initial condition $\varpi_{t-1} = \{\mathbf{Y}_{t-1}, ..., \mathbf{Y}_{t-L}\}$ is classified to belong to the "deep contractions" state if $\Delta \ln GDP_{t-1}$ is in the bottom decile of the quarter-on-quarter GDP growth rate empirical distribution and to the "strong expansions" state if $\Delta \ln GDP_{t-1}$ is in the top decile of the quarter-on-quarter GDP growth rate empirical distribution. Out of these two sets of initial conditions we take the within-set average to obtain, for each state, the average initial condition across the histories linked to the state, i.e. $\bar{\varpi}_{t-1}^{rec}$ and $\bar{\varpi}_{t-1}^{exp}$, which work as a sort of unconditional mean for each state. Then, to produce our state-dependent GIRFs for recessions and expansions, $\widehat{GIRF}_{Y,t}(\delta_t, \bar{\varpi}_{t-1}^{rec})$ and $\widehat{GIRF}_{Y,t}(\delta_t, \bar{\varpi}_{t-1}^{exp})$, we adopt the same steps 1-5 above for $\bar{\varpi}_{t-1}^{rec}$ and $\bar{\varpi}_{t-1}^{exp}$ as initial conditions.
- 7. confidence bands surrounding the point estimates obtained in step 6 are computed via a bootstrap procedure. In particular, we simulate S=1,000 samples of size equivalent to the one of actual data. Then, per each dataset, we i) estimate our nonlinear VAR model; ii) implement step $6.^{23}$ In implementing this procedure the initial conditions and VCV matrix used for our computations now depend on the particular dataset s used, i.e., ω_{t-1}^s and Ω_t^s . Confidence bands are the constructed by considering the 84th and 16th percentiles of the resulting distribution of state-conditional GIRFs.

E: Model calibration

Some parameters of the model are calibrated as in Basu and Bundick (2017) for comparability reasons. Table A1 collects all the calibrated parameters. We do not estimate these parameters for several reasons. We follow a long tradition in macroeconomics and calibrate the capital's share in production α , the household discount factor β and the steady state depreciation rate δ to values that are standard in the literature. The first-order utilization parameter δ_1 and the consumption weight in the period utility function

²²This never happens for our responses estimated on actual data. We verified that it happens quite rarely as regards our bootstrapped responses.

²³The bootstrap used is similar to the one used by Christiano, Eichenbaum, and Evans (1999) (see their footnote 23). The code discards the explosive artificial draws to be sure that exactly 1,000 draws are used. In our simulations, this happens a negligible fraction of times.

 η cannot be estimated, because the first is determined endogenously by a steady state relationship (involving δ and β) and the second is fixed in order to imply a Frisch elasticity equal to 2. The steady state inflation rate Π cannot be estimated by a impulse response functions matching procedure that focuses on out-of-steady state dynamics, i.e., deviations from the (stochastic) steady state. The firm leverage parameter ν does not influence impulse responses in the absence of financial frictions and hence is not identified. As regards the parameters of the stochastic shock processes, we calibrate the volatility of the second moment preference shock σ_{σ^a} by appealing to the estimated responses of our nested linear VAR model. The parameters governing the processes of the preference and technological shocks, i.e. ρ^a , σ^a , ρ^Z and σ^Z are calibrated by borrowing values from Basu and Bundick (2017). In spite of our focus on the effects of the uncertainty shocks, we calibrate also these parameters because these stochastic processes can in principle influence (even on-impact) the response of the model-consistent VXO to an uncertainty shock. We also do not estimate the second-order utilization parameter δ_2 , the elasticity of substitution between intermediate goods θ_{μ} , and the IES ψ to not further increase the computational burden of the estimation procedure.

F: Priors vs. posterior densities in the DSGE model estimation

Figure A5 displays the prior and posterior densities of our estimated parameters. The evidence points to the information in our sample as able to shift and modify the prior densities.

G: DSGE model estimation convergence diagnostics

Table A2 shows the results of the Geweke (1992)-convergence diagnostics test that compares the means of the first 20% retained draws with that of the last 50%. As indicated by the p-values of the χ^2 -test for the equality of the means, all MCMC chains converge to their stationary distribution. Figures A7 and A8 show the corresponding MCMC chains and the evolution of their means over time.

H: Counterfactuals to identify relevant parameter instabilities

We conduct counterfactual exercises per each version of our DSGE model to identify the relevant parameters affecting the impulse responses of the variables of our interest to an uncertainty shock. As regards our analysis of recessions and expansions, we check the impact of each parameter on the impulse responses produced by the DSGE model as follows. Conditional on the set of estimates in expansions, we replace the value of each parameter with the corresponding estimated value in recessions. To be sure, the way in which the exercise is designed is such that, if we replaced all estimated parameters contemporaneously, by construction we would replicate the impulse responses produced by the DSGE in recessions. Figures A8 and A9 display the outcome of this exercise. Bottom line: The parameter which leads to a substantial change of the impulse responses is clearly the risk aversion parameter.

I: DSGE model state-conditional GIRFs

This Section investigates whether the initial conditions in the nonlinear DSGE model we employ play a role for the dynamics of the system after an uncertainty shock. Andreasen, Fernández-Villaverde, and Rubio-Ramírez (2017) show that the initial values of the states are potentially very important for the effects of the macroeconomic shocks they study. The computation of the GIRFs in our paper follows Basu and Bundick (2017) and do not properly take into account the role of initial conditions. Hence, this possible omitted factor could be behind the evidence of countercyclical risk aversion we find.²⁴ It is therefore important to provide a check on the relevance of initial conditions in the model we work with.

Figure A10 compares DSGE-related unconditional GIRFs computed at the ergodic mean of the states with those computed in a state-conditional manner, i.e., by taking initial conditions corresponding to deep contractions and strong expansions.²⁵ To control for the role of parameter instability, all responses displayed in Figure 10 are computed conditional on the estimates we obtained with the facts established by the linear VAR. To ease comparison, we also plot our baseline responses à la Basu and Bundick (2017) based on the same parameters values. The two states/regimes of deep contractions and strong expansions are defined consistently with the definition adopted for the GIRFs computed with our IVAR model.²⁶

Two comments are in order. First, the Basu and Bundick (2017) way of comput-

²⁴As explained in the main text, we compute responses in the model starting from the regime-specific stochastic steady state implied by the estimated set of parameters.

²⁵Unconditional and conditional GIRFs are computed based on the replication codes of Andreasen, Fernández-Villaverde, and Rubio-Ramírez (2017). Consistently with their definition, these responses have to be interpreted as deviations from the (deterministic) steady state of the model.

²⁶Consistently with Andreasen, Fernández-Villaverde, and Rubio-Ramírez (2017), we split the initial values of the state variables between the two regimes on the basis of the first and last deciles of the distribution of the GDP growth rate obtained from a simulated sample path. 500 draws in each regimes are selected.

ing responses produces results very similar to the ones produced by the Andreasen, Fernández-Villaverde, and Rubio-Ramírez (2017) method.²⁷ Second, the initial conditions in the DSGE model do not materially influence the computed GIRFs to an uncertainty shock. The evidence documented in Figure 10 points to the role of initial conditions for the computation of the state-consistent GIRFs as negligible.²⁸

²⁷One difference is observed in the response of investment. This appears due to a different computation of GIRFs between the two approaches. The adoption of the Basu and Bundick method in our work is justified by two reasons. First, this choice enhance comparability with their empirical results. Second, Basu and Bundick (2017) show that their methodology produces impulse responses that are very similar to the unconditional *simulation-based* GIRFs à la Koop, Pesaran and Potter (1996).

²⁸Consistently with Andreasen, Fernández-Villaverde, and Rubio-Ramírez (2017), we find that initial conditions are particularly relevant for first moments shocks, like the preference shock in our model (results available upon request). Our intuition is that initial conditions are less relevant for the propagation of second moments shocks because such shocks propagate only via a third-order approximated part of the solution.

References

- AASTVEIT, K. A., G. J. NATVIK, AND S. SOLA (2017): "Economic Uncertainty and the Influence of Monetary Policy," *Journal of International Money and Finance*, 76, 50–67.
- Albuquerque, R., M. Eichenbaum, V. X. Luo, and S. Rebelo (2016): "Valuation Risk and Asset Pricing," *Journal of Finance*, LXXI(6), 2861–2903.
- ALESSANDRI, P., AND H. MUMTAZ (2014): "Financial Regimes and Uncertainty Shocks," Queen Mary University of London Working Paper No. 729.
- ALTIG, D., L. J. CHRISTIANO, M. EICHENBAUM, AND J. LINDÉ (2011): "Firm-Specific Capital, Nominal Rigidities and the Business Cycle," *Review of Economic Dynamics*, 14(2), 225–247.
- Andreasen, M. M. (2012): "On the Effects of Rare Disasters and Uncertainty Shocks for Risk Premia in Non-Linear DSGE Models," *Review of Economic Dynamics*, 15(3), 295–316.
- Andreasen, M. M., J. Fernández-Villaverde, and J. F. Rubio-Ramírez (2017): "The Pruned State-Space System for Non-Linear DSGE Models: Theory and Empirical Applications," *Review of Economic Studies*, forthcoming.
- AUERBACH, A., AND Y. GORODNICHENKO (2012): "Measuring the Output Responses to Fiscal Policy," American Economic Journal: Economic Policy, 4(2), 1–27.
- Basu, S., and B. Bundick (2017): "Uncertainty Shocks in a Model of Effective Demand," *Econometrica*, 85(3), 937–958.
- BIANCHI, F., AND L. MELOSI (2017): "Escaping the Great Recession," *American Economic Review*, 107(4), 1030–58.
- BINSBERGEN, J. H. V., J. FERNÁNDEZ-VILLAVERDE, R. S. KOIJEN, AND J. RUBIO-RAMÍREZ (2012): "The term structure of interest rates in a DSGE model with recursive preferences," *Journal of Monetary Economics*, 59(7), 634Ű648.
- BLOOM, N. (2009): "The Impact of Uncertainty Shocks," *Econometrica*, 77(3), 623–685.

- ——— (2014): "Fluctuations in Uncertainty," Journal of Economic Perspectives, 28(2), 153–176.
- BORN, B., AND J. PFEIFER (2014a): "Policy Risk and the Business Cycle," *Journal of Monetary Economics*, 68, 68–85.
- ——— (2014b): "Risk Matters: The Real Effects of Volatility Shocks: Comment," American Economic Review, 104(12), 4231–4239.
- ———— (2017): "Uncertainty-driven business cycles: assessing the markup channel," University of Bonn, mimeo.
- CAGGIANO, G., E. CASTELNUOVO, V. COLOMBO, AND G. NODARI (2015): "Estimating Fiscal Multipliers: News From a Nonlinear World," *Economic Journal*, 125(584), 746–776.
- CAGGIANO, G., E. CASTELNUOVO, AND J. M. FIGUERES (2017): "Economic Policy Uncertainty and Unemployment in the United States: A Nonlinear Approach," *Economics Letters*, 151, 31–34.
- CAGGIANO, G., E. CASTELNUOVO, AND N. GROSHENNY (2014): "Uncertainty Shocks and Unemployment Dynamics: An Analysis of Post-WWII U.S. Recessions," *Journal of Monetary Economics*, 67, 78–92.
- CAGGIANO, G., E. CASTELNUOVO, AND G. NODARI (2017): "Uncertainty and Monetary Policy in Good and Bad Times," Melbourne Institute Working Paper No. 9/17.
- CAGGIANO, G., E. CASTELNUOVO, AND G. PELLEGRINO (2017): "Estimating the Real Effects of Uncertainty Shocks at the Zero Lower Bound," *European Economic Review*, 100, 257–272.
- Caldara, D., J. Fernández-Villaverde, J. F. Rubio-Ramírez, and W. Yao (2012): "Computing DSGE Models with Recursive Preferences and Stochastic Volatility," *Review of Economic Dynamics*, 15, 188–206.
- Canova, F., and L. Sala (2009): "Back to Square One: Identification Issues in DSGE Models," *Journal of Monetary Economics*, 56(4), 431–449.
- Castelnuovo, E., and G. Pellegrino (2017): "Uncertainty-dependent Effects of Monetary Policy Shocks: A New Keynesian Interpretation," mimeo.

- Christiano, L., M. Trabandt, and K. Walentin (2011): "DSGE Models for Monetary Policy Analysis," in: B. M. Friedman and M. Woodford (Eds.): Handbook of Monetary Economics, Volume 3a, 285–367.
- Christiano, L. J., M. Eichenbaum, and C. Evans (1999): "Monetary Policy Shocks: What Have We Learned and to What End?," In: J.B. Taylor and M. Woodford (eds.): Handbook of Macroeconomics, Elsevier Science, 65–148.
- COCHRANE, J. (2017): "Macro-Finance," Review of Finance, 21(3), 945–985.
- Cohn, A., J. Engelmann, E. Fehr, and M. A. Maréchal (2015): "Evidence for countercyclical risk aversion: an experiment with financial professionals," *The American Economic Review*, 105(2), 860–885.
- DE GROOT, O., A. W. RICHTER, AND N. A. THROCKMORTON (2017): "Uncertainty Shocks in a Model of Effective Demand: Comment," Federal Reserve Bank of Dallas Research Department Working Paper 1706.
- Drautzburg, T., J. Fernández-Villaverde, and P. Guerrón-Quintana (2017): "Political Distribution Risk and Aggregate Fluctuations,".
- Fernández-Villaverde, J., P. Guerrón-Quintana, K. Kuester, and J. F. Rubio-Ramírez (2015): "Fiscal Volatility Shocks and Economic Activity," *American Economic Review*, 105(11), 3352–3384.
- FERNÁNDEZ-VILLAVERDE, J., P. GUERRÓN-QUINTANA, J. F. RUBIO-RAMÍREZ, AND M. URIBE (2011): "Risk Matters: The Real Effects of Volatility Shocks," *American Economic Review*, 101, 2530–2561.
- FERRARA, L. (2003): "A three-regime real-time indicator for the US economy," *Economics Letters*, 81(3), 373–378.
- Geweke, J. (1992): "Evaluating the Accuracy of Sampling-Based Approaches to the Calculation of Posterior Moments," in *IN BAYESIAN STATISTICS*.
- Gonçalves, S., and L. Kilian (2004): "Bootstrapping autoregressions with conditional heteroskedasticity of unknown form," *Journal of Econometrics*, 123(1), 89–120.
- Greenspan, A. (2003): "Monetary Policy under Uncertainty," Remarks at a symposium sponsored by the Federal Reserve Bank of Kansas City, Jackson Hole, Wyoming, August 29.

- Guerron-Quintana, P., A. Inoue, and L. Kilian (2017): "Impulse Response Matching Estimators for DSGE Models," *Journal of Econometrics*, 196, 144–155.
- Guiso, L., P. Sapienza, and L. Zingales (2017): "Time Varying Risk Aversion," Journal of Financial Economics, forthcoming.
- Hall, A., A. Inoue, J. Nason, and B. Rossi (2012): "Information Criteria for Impulse Response Function Matching Estimation of DSGE Models," *Journal of Econometrics*, 170(2), 499–518.
- JERMANN, U. (1998): "Asset Pricing in Production Economies," *Journal of Monetary Economics*, 41, 257–275.
- Jurado, K., S. C. Ludvigson, and S. Ng (2015): "Measuring Uncertainty," *American Economic Review*, 105(3), 1177–1216.
- KILIAN, L. (2009): "Not all oil price shocks are alike: Disentangling demand and supply shocks in the crude oil market," *American Economic Review*, 99(3), 1053–69.
- KILIAN, L., AND R. VIGFUSSON (2011): "Are the Responses of the U.S. Economy Asymmetric in Energy Price Increases and Decreases?," *Quantitative Economics*, 2, 419–453.
- Kim, K. H. (2014): "Counter-cyclical risk aversion," *Journal of Empirical Finance*, 29, 384–401.
- KOOP, G., M. PESARAN, AND S. POTTER (1996): "Impulse response analysis in nonlinear multivariate models," *Journal of Econometrics*, 74(1), 119–147.
- Leduc, S., and Z. Liu (2016): "Uncertainty Shocks are Aggregate Demand Shocks," Journal of Monetary Economics, 82, 20–35.
- Ludvigson, S. C., S. Ma, and S. Ng (2017): "Uncertainty and Business Cycles: Exogenous Impulse or Endogenous Response?," New York University and Columbia University, mimeo.
- MERTENS, K., AND M. O. RAVN (2014): "A reconciliation of SVAR and narrative estimates of tax multipliers," *Journal of Monetary Economics*, 68, S1–S19.

- Mumtaz, H., and K. Theodoridis (2016): "The changing transmission of uncertainty shocks in the US: An empirical analysis," *Journal of Business and Economic Statistics*, forthcoming.
- Pellegrino, G. (2017a): "Uncertainty and Monetary Policy in the US: A Journey into Non-Linear Territory," Melbourne Institute Working Paper No. 6/17.
- ———— (2017b): "Uncertainty and the Real Effects of Monetary Policy Shocks in the Euro Area," *Economics Letters*, forthcoming.
- ROTEMBERG, J. J. (1982): "Monopolistic Price Adjustment and Aggregate Output," Review of Economic Studies, 49, 517–531.
- RUGE-MURCIA, F. (2017): "Indirect Inference Estimation of Nonlinear Dynamic General Equilibrium Models: With an Application to Asset Pricing under Skewness Risk," McGill University, mimeo.
- SCHMITT-GROHE, S., AND M. URIBE (2004): "Solving Dynamic General Equilibrium Models Using a Second-Order Approximation to the Policy Function," *Journal of Economic Dynamics and Control*, 28, 755–775.
- SÁ, F., P. TOWBIN, AND T. WIELADEK (2014): "Capital Inflows, Financial Structure and Housing Booms," Journal of the European Economic Association, 12(2), 522–546.
- TERÄSVIRTA, T. (2017): "Nonlinear models in macroeconometrics," CREATES Research Paper 2017-32.
- TERÄSVIRTA, T., D. TJØSTHEIM, AND C. W. GRANGER (2010): "Modeling Nonlinear Economic Time Series," Oxford University Press, Oxford.
- TOWBIN, P., AND S. WEBER (2013): "Limits of floating exchange rates: The role of foreign currency debt and import structure," *Journal of Development Economics*, 101(1), 179–101.
- Wu, J. C., and F. D. Xia (2016): "Measuring the Macroeconomic Impact of Monetary Policy at the Zero Lower Bound," *Journal of Money, Credit, and Banking*, 48(2-3), 253–291.

Par.	Description	Value	Source
σ_{σ^a}	volatility of the uncertainty shock	0.005	linear VAR
ρ^a	persistence of the preference shock	0.94	BB (2017)
σ^a	volatility of the preference shock	0.003	BB (2017)
ρ^Z	persistence of the technology shock	0.99	BB (2017)
σ^Z	volatility of the technology shock	0.001	BB (2017)
σ	capital's share in production	0.333	BB (2017)
β	household discount factor	0.994	BB (2017)
δ	steady state depreciation rate	0.025	BB (2017)
δ_1	first-order utilization parameter	0.03	BB (2017)
μ	consumption weight in the period utility function	0.35	BB (2017)
П	steady state inflation rate	1.005	BB (2017)
\mathcal{V}	firm leverage parameter	0.0	BB (2017)
δ_2	second-order utilization parameter	0.0003	BB (2017)
θ_{μ}	elasticity of subst. between intermediate goods	0.9	BB (2017)
Ę	intertemporal elasticity of substitution	0.95	BB (2017)

Table A1: DSGE model: Calibrated parameters. BB (2017) stands for Basu and Bundick (2017).

Par.	Interpretation	4% taper	8% taper	15% taper	4% taper	8% taper	15% taper
		De	ep contracti	ons	Str	ong expansi	lons
$ ho_{\sigma^a}$	Pers. unc. shock	0.081	990.0	0.029	0.113	0.105	0.076
О	Risk aversion	0.760	0.731	0.682	0.408	0.387	0.400
9	Habit form.	0.168	0.190	0.136	0.764	0.755	0.757
ϕ_K	Inv. adj. costs	0.266	0.267	0.269	0.460	0.456	0.405
ϕ_P	Pr. adj. costs	0.081	0.099	0.112	0.176	0.184	0.175
$ ho_R$	TR par., smooth.	0.735	0.747	0.753	0.416	0.400	0.388
$ ho_{\pi}$	TR par., infl.	0.012	0.013	0.004	0.101	0.117	0.123
ρ_{y}	TR par., out. gr.	0.749	0.747	0.745	0.241	0.269	0.274

Table A2: **DSGE model estimation:** Geweke (1992) Convergence Diagnostics. Numbers are p-values of the Chi-squared test for equality of means of the first 20draws (after the first 12000 draws are discarded as burn-in).

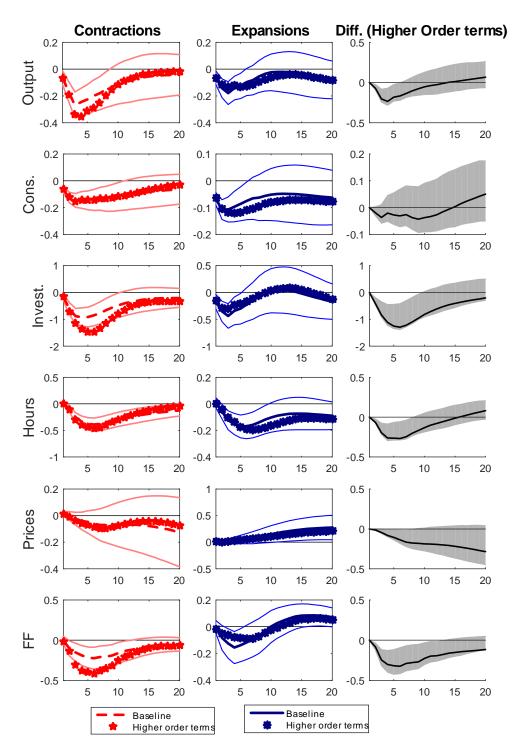


Figure A1: **IVAR impulse responses: Role of higher order terms.** Solid lines in the first and second columns: Impulse responses and 68% confidence bands produced with the baseline, parsimonious IVAR. Lines with stars (first and second columns): Impulse responses produced with the expanded IVAR featuring extra-interaction terms. Densities of the differences between recessions and expansions (68% bands) plotted in the third column.

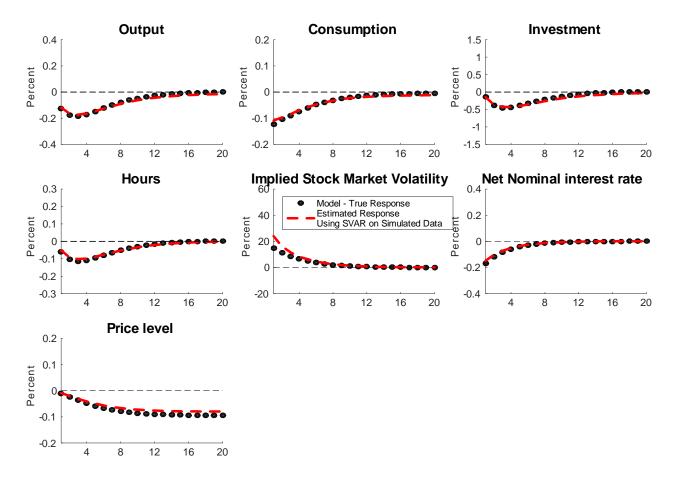


Figure A2: Monte Carlo simulation: DSGE model vs. VAR responses to an uncertainty shock. Calibration of the DSGE model as in Basu and Bundick (2017). Size of the simulated sample: 50,000 observations. Uncertainty shock in the VAR framework identified by assuming a recursive structure of the economic system with the VXO ordered first.

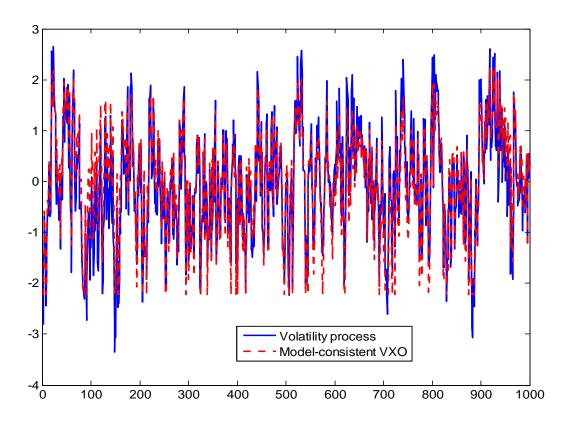


Figure A3: **DSGE-consistent processes: Volatility vs. VXO**. Series produced with the Basu and Bundick (2017) model. Simulated series: 100,000 observations, 99,000 used as a burn in. Both series are standardized to ease readibility. Correlation coefficient: 0.95.

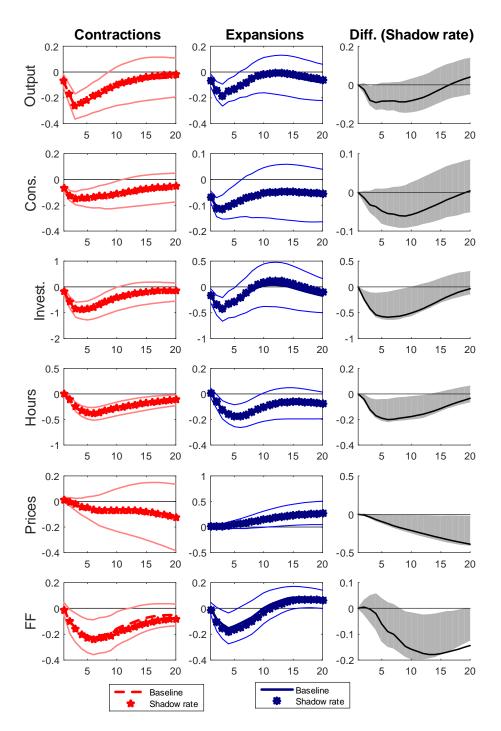


Figure A4: **IVAR impulse responses: check with shadow rate.** Solid lines in the first and second columns: Impulse responses and 68% confidence bands produced with the baseline IVAR with the FFR on all the sample. Lines with stars (first and second columns): Impulse responses produced with the IVAR featuring the Wu and Xia (2016) shadow rate for the period of ZLB. Densities of the differences between recessions and expansions (68% bands) plotted in the third column.

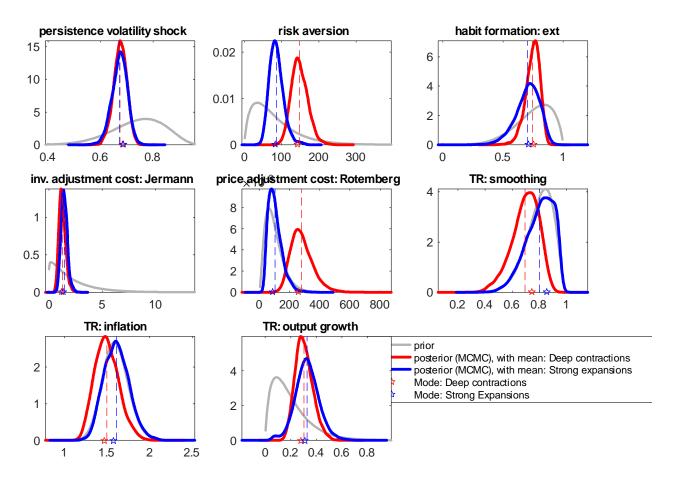


Figure A5: Estimated parameters: Priors vs. posterior densities. Priors in table 3 of the paper. Posteriors computed via MCMC with a random walk metropolis algorithm. 60 000 draws, 20 percent for burn-in. Acceptance rates: 27 percent for times of deep contractions, 26 percent for times of strong contractions.

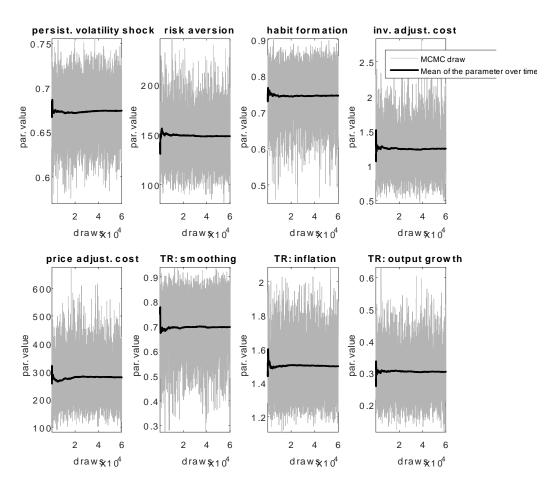


Figure A6: Evolution of the MCMC sampler over time. Grey line: MCMC evolution for a particular parameter (60,000 draws). Black line: Expanding-window mean of of the chain over time.

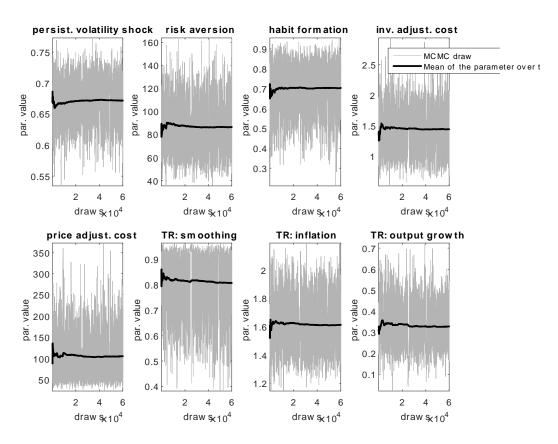


Figure A7: Evolution of the MCMC sampler over time. Grey line: MCMC evolution for a particular parameter (60,000 draws). Black line: Expanding-window mean of of the chain over time.

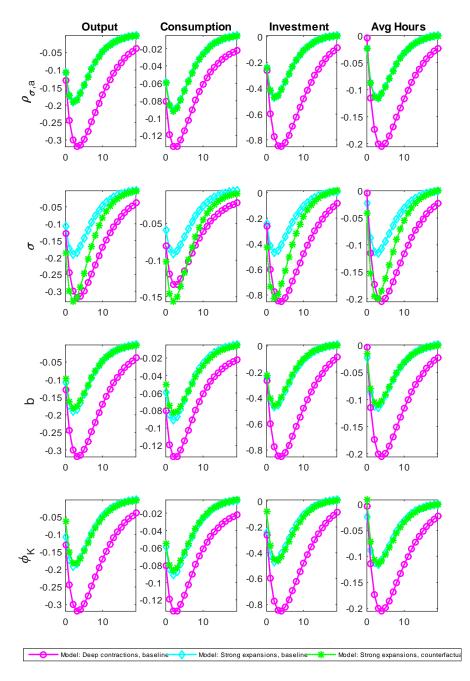


Figure A8: Role of structural parameters for the state-contingent IRFs produced by the DSGE model: First set of parameters.

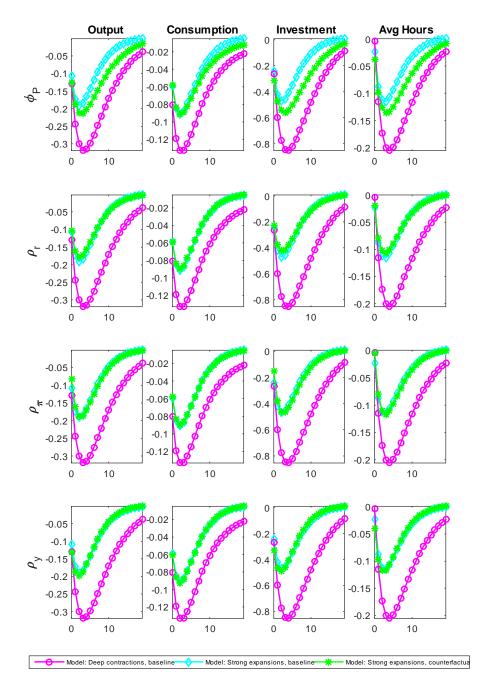


Figure A9: Role of structural parameters for the state-contingent IRFs produced by the DSGE model: Second set of parameters.

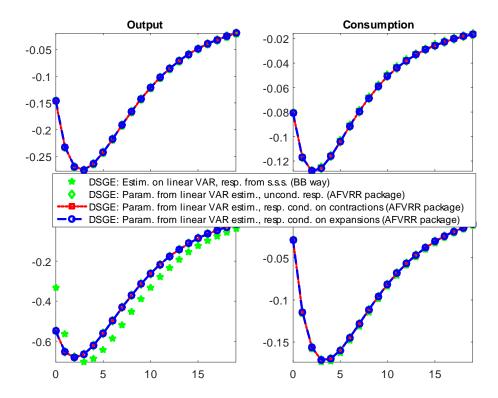


Figure A10: Role of initial conditions for DSGE impulse responses: unconditional GIRFs versus conditional GIRFs à la Andreasen, Fernández-Villaverde, and Rubio-Ramírez (AFVRR, 2017). Green stars: response of the DSGE estimated on the linear VAR model with computation à la Basu and Bundick (2017). Green diamonds: unconditional response of the DSGE for the same set of parameters values with computation à la AFVRR (2017). Red squares: response of the DSGE conditional on deep contractions for the same set of parameters values with computation à la AFVRR (2017). Blue circles: response of the DSGE conditional on strong expansions for the same set of parameters values with computation à la AFVRR (2017).