

Fractional Cointegration and EMU Government Bond Market Integration

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Abstract

It is commonly found that the markets for long-term government bonds of EMU countries were highly integrated prior to the subprime mortgage and EMU debt crisis. In contrast to this, we show that there were periods of integration and disintegration that coincide with bull- and bear-market periods in the stock market. This finding is based on the interrelation between market integration and fractional cointegration in the context of the common currency area and confirmed by a wide array of suitable semiparametric tests. A simple econometric argument about the spectral behavior of long-memory time series leads to the conclusion that there is a stronger differentiation between bonds with different default risks during periods of disintegration, so that the dynamics of the yields implied the possibility of macroeconomic and fiscal divergence between the EMU countries long before the crisis periods.

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1 Introduction

We show that even though the yields on long-term government bonds of the major EMU countries were largely co-moving prior to the crisis, the degree of market integration exhibited considerable variation over time. This time variation is related to the stock market sentiment. During bear-market periods, there was no equilibrium mechanism between the yields that would have ensured the subsistence of a stable relationship.

In contrast to our findings, it is nearly universally accepted in the literature on the integration of EMU bond markets that the introduction of the Euro led to essentially complete integration of EMU bond markets that ended with the advent of the sub-prime mortgage crisis. This was found empirically by contributions such as [Ehrmann et al. \(2011\)](#), [Baele et al. \(2004\)](#), [Pozzi and Wolswijk \(2012\)](#), [Christiansen \(2014\)](#), and [Ehrmann and Fratzscher \(2017\)](#) and is also implicitly assumed by studies on the determinants of yield spreads between government bonds in the Eurozone, such as [Beber et al. \(2008\)](#), who treat the yield spreads as stationary variables.

The difference between these studies and ours is rooted in the fact that we take a very different perspective from previous contributions to the literature. Instead of focusing on the shock transmission among the spreads or the relative importance of global and local factors, we test for the existence of an equilibrium among the interest rates themselves. Our study adopts a definition of market integration that is widely used in other areas such as the analysis of commodity markets. This definition is directly based on the law of one price and closely connected to the existence of a (fractional) cointegrating relationship. Using it enables us to draw conclusions about market equilibria by applying a wide set of modern methods for the analysis of fractionally cointegrated systems.

Utilizing this direct correspondence between economic theories and statistical concepts allows us to make several major contributions. First, we establish that the EMU bond markets were integrated during bull markets but disintegrated in bear markets. This is achieved directly by testing for pairwise fractional cointegration among the yields and indirectly by considering the persistence of the yield spreads. The yield spreads

between the countries are the cointegrating residuals obtained by imposing the cointegrating vector $(1, -1)'$ on the yields. The persistence of the spreads is therefore directly related to the existence of an equilibrium relationship among the yields. Further insights into the dynamics of integration and disintegration in the EMU bond markets are therefore obtained from a rolling window analysis of the memory of the spreads.

The second contribution is to provide insights into the sources of the time-varying persistence in the spreads. To this end, the estimated degree of persistence is regressed on a set of variables that proxy for market sentiment, risk, and risk aversion. The analysis not only confirms the relationship between integration and bull and bear markets, but also shows that the degree of market integration is driven by market risk.

Finally, the third contribution is to provide insights into the possible economic origins of the observed time variation in market integration. Here, we make use of the fact that the yields are the sum of the risk-free rate, the default risk premium, and the liquidity risk premium of the respective country. Due to the special situation in the EMU where (due to the common currency area) the risk-free rate is the same for all countries and Germany is typically assumed to be risk-free, the spreads relative to Germany are solely determined by the default risk premium and the liquidity risk premium. Standard results on the properties of linear combinations of long-memory time series from [Chambers \(1998\)](#) then give rise to two possible mechanisms that can generate the observed time variation in the persistence of the spreads. The first one is that markets expect economic and fiscal divergence within the EMU area in bear markets, whereas they are optimistic about convergence within the Eurozone in bull markets. The second possible explanation is that markets always assume that divergence is a possibility, but the default risk premium exhibits so little variation in good times that the persistence of the spreads is dominated by the liquidity premium. In contrast to that, in bad times, when risk and risk aversion are high, the persistence of the spreads is dominated by the default risk premium, due to its increased variability.

Both of these arguments lead to the conclusion that (at least in crisis times) the

pricing of EMU government bonds implied the possibility of macroeconomic and fiscal divergence between the EMU countries, long prior to the EMU debt crisis. Also, differences between the core and periphery countries are already visible during previous bear-market periods.

The rest of the paper is structured as follows. Section 2 provides a discussion of market integration and a discussion of fractional integration and cointegration. Subsequently, Section 3 describes the data set and discusses the definition of bull and bear markets. Section 4 contains the empirical analysis including formal tests for market integration separately for bull and bear markets, rolling window estimates of the persistence of the spreads, and an analysis of the drivers of the degree of market integration. Finally, Section 5 concludes.

2 Market Integration, Fractional Integration, and Fractional Cointegration

In international finance, measures for market integration are typically based on factor models for the returns. The most widely adopted approaches in recent years are those of [Bekaert and Harvey \(1995\)](#) and [Pukthuanthong and Roll \(2009\)](#). [Bekaert and Harvey \(1995\)](#) consider two markets to be integrated if their movement is completely determined by global factors, whereas local factors (that are specific to individual countries) are not priced. Similarly, [Pukthuanthong and Roll \(2009\)](#) consider the explanatory power of a multifactor model as a measure for market integration. While both of these measures are intuitive for asset returns, they lack a rigorous foundation in economic theory and they are not readily applicable to bond yields that are typically found to have unit roots.

Here, we therefore consider a different definition that is commonly used for the analysis of commodity markets. According to this definition markets for different goods that are close substitutes, or markets for the same good that are spatially separated are considered to be (economically) integrated with each other if the law of one price (LOP)

applies. In the strict sense, the LOP requires that there is a correction mechanism (such as arbitrage) in place that enforces the stability of an equilibrium relationship, and that the form of this equilibrium is such that prices in both markets are exactly the same. The weaker definition of partial market integration only requires the existence of a stable equilibrium relationship but not exact equality between the prices.

For non-stationary prices, this definition is often tied to the concept of cointegration (cf. [Ravallion \(1986\)](#), [Ardeni \(1989\)](#)), since cointegration implies the existence of an equilibrium relationship between unit root processes. In the classical $I(0)/I(1)$ framework, deviations from this equilibrium have to be weakly persistent in the sense that they are stationary and have short memory. This, however, is an unnecessary restriction, since an equilibrium relationship only requires deviations from the mean to be transitory in the sense that they are mean reverting.

We therefore allow for fractional cointegration when testing for (partial) market integration and consider a bivariate system of the form

$$X_{1t} = c_1 + \xi_1 Y_t + \Delta^{-(d-b_1)} u_{1t} \mathbb{1}_{\{t>0\}} \quad (1)$$

$$X_{2t} = c_2 + \xi_2 Y_t + \Delta^{-(d-b_2)} u_{2t} \mathbb{1}_{\{t>0\}} \quad (2)$$

$$Y_t = \Delta^{-d} e_t \mathbb{1}_{\{t>0\}}, \quad (3)$$

where the coefficients c_1 , c_2 , ξ_1 , and ξ_2 are finite, $0 \leq b_1, b_2 \leq d$, L is the lag-operator, the fractional differences $\Delta^d Y_t = (1-L)^d Y_t$ are defined in terms of generalized binomial coefficients such that

$$(1-L)^d = \sum_{k=0}^{\infty} \binom{d}{k} (-1)^k L^k = \sum_{k=0}^{\infty} \pi_k L^k,$$

with $\binom{d}{k} = \frac{d(d-1)(d-2)\dots(d-(k-1))}{k!},$

and e_t and $u_t = (u_{1t}, u_{2t})'$ are martingale difference sequences. The memory of both X_{1t} and X_{2t} is determined by Y_t so that they are integrated of the same order d , denoted

by $X_t \sim I(d)$, where the memory parameter is restricted to $d \in (0, 1]$ and $X_t = (X_{1t}, X_{2t})'$. Since it is assumed that $u_{1t} = u_{2t} = e_t = 0$ for all $t \leq 0$, the processes under consideration are fractionally integrated of type-II. For a detailed discussion of type-I and type-II processes confer [Marinucci and Robinson \(1999\)](#). The spectral density of X_t can be approximated by

$$f_X(\lambda) \sim \Lambda_j(d) G \overline{\Lambda_j(d)}, \quad \text{as } \lambda \rightarrow 0^+, \quad (4)$$

where G is a real, symmetric, finite, and positive definite matrix, $\Lambda_j(d) = \text{diag}(\lambda^{-d} e^{i\pi d/2}, \lambda^{-d} e^{i\pi d/2})$ is a 2×2 diagonal matrix and $\overline{\Lambda_j(d)}$ is its complex conjugate transpose. The periodogram of a process X_t is defined through the discrete Fourier transform $w_X(\lambda_j) = \frac{1}{\sqrt{2\pi T}} \sum_{t=1}^T X_t e^{i\lambda_j t}$ as $I_X(\lambda_j) = w_X(\lambda_j) \overline{w_X(\lambda_j)}$, with Fourier frequencies $\lambda_j = 2\pi j/T$ for $j = 1, \dots, \lfloor T/2 \rfloor$, where the operator $\lfloor \cdot \rfloor$ returns the integer part of its argument.

The two series X_{1t} and X_{2t} are said to be fractionally cointegrated, if there exists a linear combination

$$\beta' X_t = v_t,$$

so that the cointegrating residuals v_t are fractionally integrated of order $I(d-b)$ for some $0 < b \leq d$. Obviously, for the model in equations (1) to (3), this is the case for every multiple of the vector $(1, -\frac{\xi_1}{\xi_2})'$ and $b = \min(b_1, b_2)$.

Here, we conclude that markets for EMU government bonds that could be considered as close substitutes are (partially) economically integrated if the yields are fractionally cointegrated with each other. From the definition above, this is the case if there exists an equilibrium relationship between the yields (X_{1t} and X_{2t}) so that the persistence of deviations from the equilibrium denoted by v_t is reduced compared to that of the individual series.¹

¹A similar approach that uses fractional cointegration to test for market integration was recently adopted by [García-Enrriquez et al. \(2014\)](#).

In the following, we will test this hypothesis in two different ways. First, we apply a number of tests for the null hypothesis of no fractional cointegration among the yields of long-term EMU government bonds. The methods used are semiparametric and do not impose any assumptions on the short-run behavior of the series, apart from mild regularity conditions. This approach has the advantage that we can avoid spurious findings that might arise due to misspecifications. Research on semiparametric tests for fractional cointegration has been an active field in recent years and there is a variety of competing approaches.

The first group of tests is based on the fact that the rank of the matrix G in (4) is reduced for fractionally cointegrated systems. This property is used by the rank estimation criterion of [Nielsen and Shimotsu \(2007\)](#) that extends the approach of [Robinson and Yajima \(2002\)](#) to nonstationary processes, the spectral regression approach of [Souza et al. \(2017\)](#), and the Hausman-type test of [Robinson \(2008\)](#). [Robinson and Yajima \(2002\)](#) and [Nielsen and Shimotsu \(2007\)](#) use the singularity of the G matrix in case of cointegration to propose an information criterion that is based on the eigenvalues of an estimate \hat{G} .

[Souza et al. \(2017\)](#) use the fractionally differenced process $\Delta^d X_t$ and the fact that the determinant $D_{\Delta^d}(\lambda)$ of $f_{\Delta^d X}(\lambda)$ is of the form $D_{\Delta^d}(\lambda) \sim \tilde{G}|1 - e^{-i\lambda}|^{2b}$, where \tilde{G} is a scalar constant and $0 < \tilde{G} < \infty$. An estimate of b can therefore be obtained via a log-periodogram regression and the hypothesis that $b = 0$ can be tested based on the resulting estimate.

The test of [Robinson \(2008\)](#) is based on the fact that univariate estimates of d for the component series X_{1t} and X_{2t} are consistent both in the absence and in the presence of fractional cointegration. In contrast to that, the objective function of multivariate local Whittle estimates for the memory in X_t depends on the inverse of G , so that the estimator is inconsistent under fractional cointegration. On the other hand, the estimator is more efficient in absence of fractional cointegration, due to its multivariate nature. This provides the basis for a Hausman-type test.

A second group of tests is residual-based, since the cointegrating residuals v_t have reduced memory of order $d - b$ instead of d if a fractional cointegrating relationship exists. [Chen and Hurvich \(2006\)](#) and [Wang et al. \(2015\)](#) provide tests that rely on this property.

The test of [Wang et al. \(2015\)](#) is based on the sum over the fractionally differenced process $\Delta^{\hat{d}_v} X_{2t}$, where \hat{d}_v is an estimate of the memory from the cointegrating residuals obtained using a consistent estimator for the cointegrating vector β such as the narrow-band least squares estimator of [Robinson \(1994\)](#), [Robinson and Marinucci \(2003\)](#), and [Christensen and Nielsen \(2006\)](#), among others. In contrast to that, the test of [Chen and Hurvich \(2006\)](#) is directly based on \hat{d}_v , but the cointegrating space is estimated by the eigenvectors of the averaged and tapered periodogram matrix local to the origin.

A third group of tests proposed by [Marmol and Velasco \(2004\)](#) and [Hualde and Velasco \(2008\)](#) relies on the behavior of pairs of estimators for the cointegrating vector β . These pairs include one estimator that is only consistent under the null hypothesis of no fractional cointegration and one estimator that is only consistent under fractional cointegration. While the test of [Marmol and Velasco \(2004\)](#) has a non-standard distribution, the test of [Hualde and Velasco \(2008\)](#) utilizes the GLS estimates of [Robinson and Hualde \(2003\)](#) and has a chi-square distribution.

Finally, [Nielsen \(2010\)](#) suggests a variance ratio test. The test statistic is based on the sum of the eigenvalues of the variance-covariance matrix of the series multiplied with the inverse of the variance-covariance matrix of the fractionally differenced series. This is because the eigenvalues associated with eigenvectors that are in a cointegrating direction are $O_P(1)$, whereas the eigenvalues corresponding to eigenvectors in non-cointegrating directions are $o_P(1)$, for $d - b < 1/2$.

If a cointegrating relationship is found with one of these procedures, the degree of (market) integration corresponds to b — the strength of the relationship. This is because b determines the speed of adjustment towards the equilibrium. The higher b , the stronger the degree of integration and the faster is the adjustment after shocks that

cause deviations from the equilibrium. In the cases of [Nielsen and Shimotsu \(2007\)](#) and [Robinson \(2008\)](#), where the methods themselves do not produce an estimate of the cointegrating strength, we estimate it by the difference between the memory of the yields and the memory of the spread. This is because the spreads are the cointegrating residuals obtained by imposing the cointegrating vector $(1, -1)'$, as discussed in detail below.

Using domain specific knowledge about the behavior of the yields in the common currency area also allows us to adopt a second approach and test for cointegration based on simple estimations of the memory parameters in the yield spreads. We denote the interest rate yield on bonds of country i in period t by y_{it} for $i = 1, \dots, N$ and $t = 1, \dots, T$. The spreads s_{it} are usually formed relative to the yield of the German bonds

$$s_{it} = y_{it} - y_t^{GER}. \quad (5)$$

It is commonly assumed that the interest rates of country i can be decomposed into

$$y_{it} = r_t^f + \delta_{it} + l_{it}, \quad (6)$$

where r_t^f is the risk-free interest rate, and δ_{it} and l_{it} are the risk premiums for the default risk and liquidity risk of country i . The risk-free rate is the same across countries due to the common currency area. If Germany — the benchmark country — is assumed to have no default risk and no liquidity risk, so that $y_t^{GER} = r_t^f$, it follows that

$$s_{it} = \delta_{it} + l_{it}. \quad (7)$$

Therefore, the spreads are the risk premiums associated with the liquidity and default risk of the respective country. If Germany is not assumed to be risk-free, δ_{it} and l_{it} are interpreted as risk premium differentials between the respective country and Germany. However, if the risk of Germany and its variation are low compared to that of the

respective country, the behavior of the differentials will still be dominated by the risk premiums of the country. We therefore maintain the assumption that Germany is risk-free to simplify the verbal description of the results.

The risk-free interest rate r_t^f in (6) is driven by expected macroeconomic factors such as GDP-growth, inflation rates, and interest rates, and it is widely found to be $I(1)$ (cf. for example [Stock and Watson \(1988\)](#), [Mishkin \(1992\)](#), [Chen and Hurvich \(2003\)](#) and [Nielsen \(2010\)](#)). That means y_{it} and y_t^{GER} can only be cointegrated if r_t^f is removed from the linear combination $\beta'(y_{it}, y_t^{GER})'$, as it is the case in the spreads in (7). Forming the spreads according to (5) therefore means to impose the cointegrating vector $\beta = (1, -1)'$ on the yields, which is the only possible cointegrating direction according to the theoretical arguments outlined above. The spreads are therefore the cointegrating residuals. Since in this case the cointegrating residuals are not affected by estimation error, we can apply a simple test for the null hypothesis that the memory $d(s_{it})$ of the spread s_{it} of country i at time t is equal to one to test for the null hypothesis of no fractional cointegration among the yields. Formally, we test

$$\begin{aligned}
 H_0 : & \quad d(s_{it}) = 1 \\
 \text{versus} \quad H_1 : & \quad d(s_{it}) < 1,
 \end{aligned}$$

for all i and t . If this hypothesis can be rejected, this is statistical evidence for market integration.

To gain a deeper economic understanding of the mechanisms driving market integration and disintegration, reconsider the decomposition of the spreads in equation (7). Since the spreads are the cointegrating residuals between the yields, their persistence determines whether there is an equilibrium or not. According to equation (7), the spreads consist of two components — the liquidity risk premium l_{it} and the default risk premium δ_{it} . Since credit default swap data is not available for most of the time period before the subprime mortgage crisis, we cannot use this information to disentangle the default and liquidity risk premiums as for example in [Longstaff et al. \(2005\)](#).

We can, however, draw some conclusions based on properties of long-memory processes. Denote the memory of the default risk premium for country i at time t by $d(\delta_{it})$ and let $d(l_{it})$ denote the memory of the liquidity risk premium. To see how the persistence of the aggregate s_{it} relates to the components δ_{it} and l_{it} , the properties of linear combinations of long-memory time series have to be considered. With constant unconditional mean and variance of the component series, it was shown by [Chambers \(1998\)](#) that the memory of a linear combination of long-memory processes is determined by the most persistent series in the combination. For two long-memory series a_t and b_t with memory parameters d_a and d_b this means that $c_t = a_t + b_t$ has long memory of order $d_c = \max\{d_a, d_b\}$. The memory of the spreads s_{it} is therefore either $d(\delta_{it})$, or $d(l_{it})$, according to which is larger.

The reasoning behind this result of [Chambers \(1998\)](#) is as follows. If a_t and b_t are mutually independent, the spectral density of c_t local to the origin is given by

$$f_c(\lambda) \sim G_a |\lambda|^{-2d_a} + G_b |\lambda|^{-2d_b},$$

as $\lambda \rightarrow 0$. Here, G_a and G_b denote the long-run variance of the short-memory components in the respective series. Obviously, both of the components on the right-hand side generate poles and the smaller one is dominated by the larger one.²

These results are based on the assumption that G_a and G_b are fixed, finite, and positive. In practice, however, there could arise situations in which one of the components is very small compared to the other one. A more fitting theoretical framework for such a situation would be to assume that $G_a/G_b \rightarrow 0$, as $T \rightarrow \infty$. In this case, the ratio of the long-run variances of the short-memory components depends on the sample size and goes to zero. More formally, let $c_t = a_t + b_t$, with $d_a > d_b$ and $G_a(T)/G_b(T) \rightarrow 0$, as $T \rightarrow \infty$, then $d_c = d_b$. This implies that in practice the estimated degree of persistence in the spreads s_{it} will be a convex combination of $d(l_{it})$ and $d(\delta_{it})$ that depends on the relative

²If a_t and b_t are dependent, there is also an interaction term in $f_c(\lambda)$, but the mechanism remains the same.

	Begin	Index	End
Bull 1	01/01/1999	313.92	03/05/2000
Bear 1	03/06/2000	466.24	03/11/2003
Bull 2	03/12/2003	165.43	05/31/2007
Bear 2	06/01/2007	442.87	03/08/2009
Crisis	03/09/2009	169.38	08/08/2017

Table 1: Definition of bull- and bear-market periods.

The Table shows the exact timing of bull and bear markets according to our definition, along with the values of the index at the beginning of the period.

scale of the variation of the two risk premiums.

Most importantly, if the persistence of the spreads is high and that of the liquidity premium is low, than the behavior of the default premium δ_{it} has to be the main driver of the spreads.

3 Data and Definition of Bull and Bear Markets

Our analysis is based on the daily interest rates on 10-year maturity benchmark government bonds of eleven EMU countries. As is customary in the literature, we refer to Spain, Italy, Portugal, Ireland, and Greece as the periphery countries. Belgium, Austria, Finland, the Netherlands, and France are called the core countries. The data set contains daily (bid) yields on benchmark bonds for these ten countries and for Germany as well as a range of explanatory variables. All series are obtained from Thomson Reuters Eikon and observed between January 1, 1999 and August 8, 2017.

As discussed in the introduction, one of the main objectives of this paper is to show that the degree of EMU bond market integration differs between bull and bear markets. To do so, we need to define which periods are regarded as bull markets and which ones are regarded as bear markets. Since there is no universally accepted definition of bull and bear markets, we simply rely on a visual inspection of the trajectory of the Eurostoxx index. Every bull-market period begins with a local minimum and every bear-market period begins with a local maximum. The timing of these local extrema

	ES	IT	PT	IE	GR	BE	AT	FI	NL	FR	GER	(s.e.)
Bull 1	1.02	1.02	1.07	1.03	0.95	1.03	1.02	0.98	1.04	0.98	1.05	(0.07)
Bear 1	0.94	0.94	0.93	0.94	0.95	0.93	0.93	0.94	0.93	0.92	0.95	(0.05)
Bull 2	1.05	1.06	1.04	1.06	1.05	1.05	1.04	1.06	1.06	1.05	1.06	(0.04)
Bear 2	1.00	0.91	0.93	1.00	0.91	0.94	0.89	0.98	0.99	0.99	1.04	(0.06)
Crisis	0.89	0.92	0.97	1.02	0.95	0.96	0.99	1.00	0.99	0.99	0.98	(0.03)
Full sample	1.04	1.01	0.96	0.99	0.93	0.95	1.03	1.10	0.98	0.97	1.00	(0.02)

Table 2: Memory estimates of the yields for different subperiods.

In the Bull 2 period the standard error of the estimate for Ireland is 0.05.³ The estimates of d are obtained using the exact local Whittle estimator of Shimotsu and Philips (2005) with a bandwidth of $m = \lfloor T \rfloor^{0.7}$. The exact definition of the market phases can be found in Table 1.

along with the index values at the starting date of the respective series is given in Table 1. The first two periods are determined by the Dot-com bubble and the subsequent crash starting on March 6, 2000. The recovery and boom thereafter lasted from March 12, 2003, until May 31, 2007, when the subprime mortgage crisis began. This bear market lasted until March 8, 2009. In the recovery after that, it could be argued that there were several shorter bull- and bear-market periods. However, it can be expected that the mechanisms driving the pricing of EMU government bonds changed permanently with the onset of the EMU debt crisis in October 2009, when the Greek government revised its deficit figures. This is also confirmed empirically by previous studies such as Pozzi and Wolswijk (2012), Christiansen (2014), and Ehrmann and Fratzscher (2017). We therefore focus on the previous bull and bear markets and refer to the post-2009 period as the crisis period.

Estimates of the memory parameters of the yields in each subsample are given in Table 2. Here and hereafter, all memory parameters are estimated using the exact local Whittle estimator of Shimotsu and Philips (2005) and a bandwidth of $m = \lfloor T^{0.7} \rfloor$. The estimator is given by

$$\hat{d}_{ELW} = \arg \min_{-1 < d < 3.5} \left\{ \log \hat{G}_m(d) - d \left(\frac{2}{m} \sum_{j=1}^m \log \lambda_j \right) \right\},$$

³The standard error of IE differs from the others due to a number of missing observations that reduce the sample size.

where $\lambda_j = 2\pi j/T$, $\hat{G}_m(d) = m^{-1} \sum_{j=1}^m I_{\Delta^d x}(\lambda_j)$, and $I_{\Delta^d x}(\lambda)$ denotes the periodogram of the fractionally differenced process $(1-L)^d X_t$. Under mild regularity conditions [Shimotsu and Philips \(2005\)](#) show that

$$\sqrt{m}(\hat{d}_{ELW} - d) \xrightarrow{d} N(0, 1/4).$$

As can be seen in [Table 2](#), the estimated memory parameters are statistically indistinguishable from one, so that it is reasonable to assume that the interest rates follow a stochastic trend. This is also supported by formal tests.

4 Empirical Analysis

Using the definition of bull and bear markets from the previous section, we now analyze the dynamics of integration and disintegration in EMU government bond markets using several approaches. First, we test for fractional cointegration among the yields, separately for bull and bear markets. Second, to determine the robustness of our findings to the definition of the subperiods, we use the second approach and test in a rolling window whether the order of integration in the spreads is equal to one, so that we do not impose any restrictions on the timing of periods of integration and disintegration. Finally, we conduct a regression analysis to gain further insights into the forces driving these results.

4.1 Testing for Market Integration Among the Yields

As discussed in [Section 2](#), integration in the market for EMU government bonds requires the yields to be pairwise fractionally cointegrated. Since the German government bonds are considered to be the most liquid and essentially risk free, it is customary to use Germany as the base country and to analyze the pairwise relationship of each country with Germany. We therefore adopt this approach and start our analysis by applying tests for the null hypothesis of no fractional cointegration on these pairs in each of the

		ES	IT	PT	IE	GR	BE	AT	FI	NL	FR
Bull 1	NS07	0.62	0.38	0.48	0.49		0.60	0.55	0.77	0.69	0.64
	SRF16	0.55	0.33	0.54	0.54		0.49	0.63	0.86	0.67	0.79
	MV04	0.62	0.38	0.53	0.56		0.62	0.71	0.77	0.71	0.71
	WWC15	0.62	0.38	0.53	0.57	0.07	0.62	0.71	0.77	0.71	0.71
	CH06	0.60	0.38	0.53	0.54		0.62	0.69	0.76	0.71	0.71
	R08	0.62	0.38	0.48	0.49		0.60	0.55	0.77	0.69	0.64
	HV08	0.62	0.38	0.53	0.57		0.62	0.71	0.77	0.71	0.71
	N10			0.53	0.56		0.57	0.71	0.63	0.70	0.71
Bear 1	NS07	0.14	0.05	0.10	0.12	0.08	0.14	0.13	0.32	0.20	0.33
	SRF16	0.29		0.25	0.24			0.27	0.39	0.27	0.36
	MV04				0.12				0.33	0.16	0.30
	WWC15	0.10			0.12		0.08		0.33	0.16	0.30
	CH06	0.13						0.12	0.33	0.20	0.33
	R08								0.32		0.33
	HV08								0.33		0.30
	N10										
Bull 2	NS07	0.49	0.13	0.36	0.44	0.39	0.22	0.45	0.43	0.46	0.32
	SRF16	0.44		0.28	0.39	0.34	0.18	0.36		0.29	0.27
	MV04	0.49		0.36	0.45	0.38	0.21	0.44	0.48	0.47	0.32
	WWC15	0.49		0.36	0.45	0.38	0.21	0.44	0.48	0.47	0.32
	CH06	0.48	0.13	0.36	0.45	0.39	0.21	0.43	0.46	0.46	0.32
	R08	0.49		0.36	0.44	0.39		0.45	0.43	0.46	0.32
	HV08	0.49				0.38		0.44	0.48	0.47	
	N10	0.49			0.44			0.45	0.44	0.46	0.32
Bear 2	NS07	0.07	0.08	-0.00			-0.02	0.01	0.18	0.11	0.15
	SRF16		0.28					0.35	0.41	0.30	0.36
	MV04		0.15				0.10	0.15	0.23	0.18	0.25
	WWC15		0.15					0.15	0.23	0.18	0.25
	CH06		0.17						0.25	0.19	0.26
	R08										0.15
	HV08										
	N10										
Crisis	NS07							0.11	0.13	0.19	0.11
	SRF16								0.18	0.16	
	MV04										
	WWC15										
	CH06								0.14	0.21	
	R08	0.06					0.10		0.13	0.19	
	HV08										
	N10										

Table 3: Strength of the fractional cointegration relationship between the yields of bonds of the respective country and Germany.

Empty fields indicate the absence of a significant fractional cointegrating relationship at the 5%-level. Non-empty fields give an estimate of b — the strength of the cointegrating relationship. Larger values of b indicate a stronger equilibrium relationship. The exact definition of the market phases can be found in Table 1.

subsamples. The results of this exercise are given in Table 3.

Since the methods employed are based on very different properties of fractionally cointegrated systems, it is not surprising that there is some variation in the findings. However, overall the results show that the majority of interest rates were indeed cointegrated with the German rate during the bull-market periods but not during the bear-market periods. A notable exception is Greece in the first bull market, since it only joined the EMU in 2001, which is during our first bear-market period. When comparing the bull-market periods and bear-market periods, it is immediately noticeable that the tests reject the null hypothesis less often during the bear markets than during the bull markets. Evidence for the existence of an equilibrium relationship during the bear-market periods is mainly found for the core countries. Furthermore, when comparing the strength of the cointegrating relationships that persist during bull and bear markets, we can observe that the strength declines in bear-market periods.

If we consider Finland, for example, deviations from the equilibrium have a memory of approximately $1 - b_8 = 0.25$ in the first bull market. This increases to nearly 0.7 in the first bear market, before dropping to 0.5 in the second bull market, and rising again to about 0.8 in the second bear market.

When we consider the results for the EMU crisis period, we find that there is no evidence for the existence of an equilibrium relationship for the periphery countries anymore. Among the core countries some weak evidence is found, but mostly for the Netherlands and Finland. The overwhelming majority of the tests are unable to detect any evidence for market integration during this period.

To gain further insights into the dynamics of market integration between all possible country pairs, we repeat the same analysis using the method of [Chen and Hurvich \(2006\)](#). The results are presented in heatmaps in [Figure 1](#). Here, a darker color indicates a strong equilibrium relationship. Clearly, there is much more evidence for pairwise market integration between the countries during the bull-market periods, which are shown on the left-hand side, than during the bear-market periods depicted on the right-hand side.

We observe that, during the bull markets, there is a larger number of cointegrating

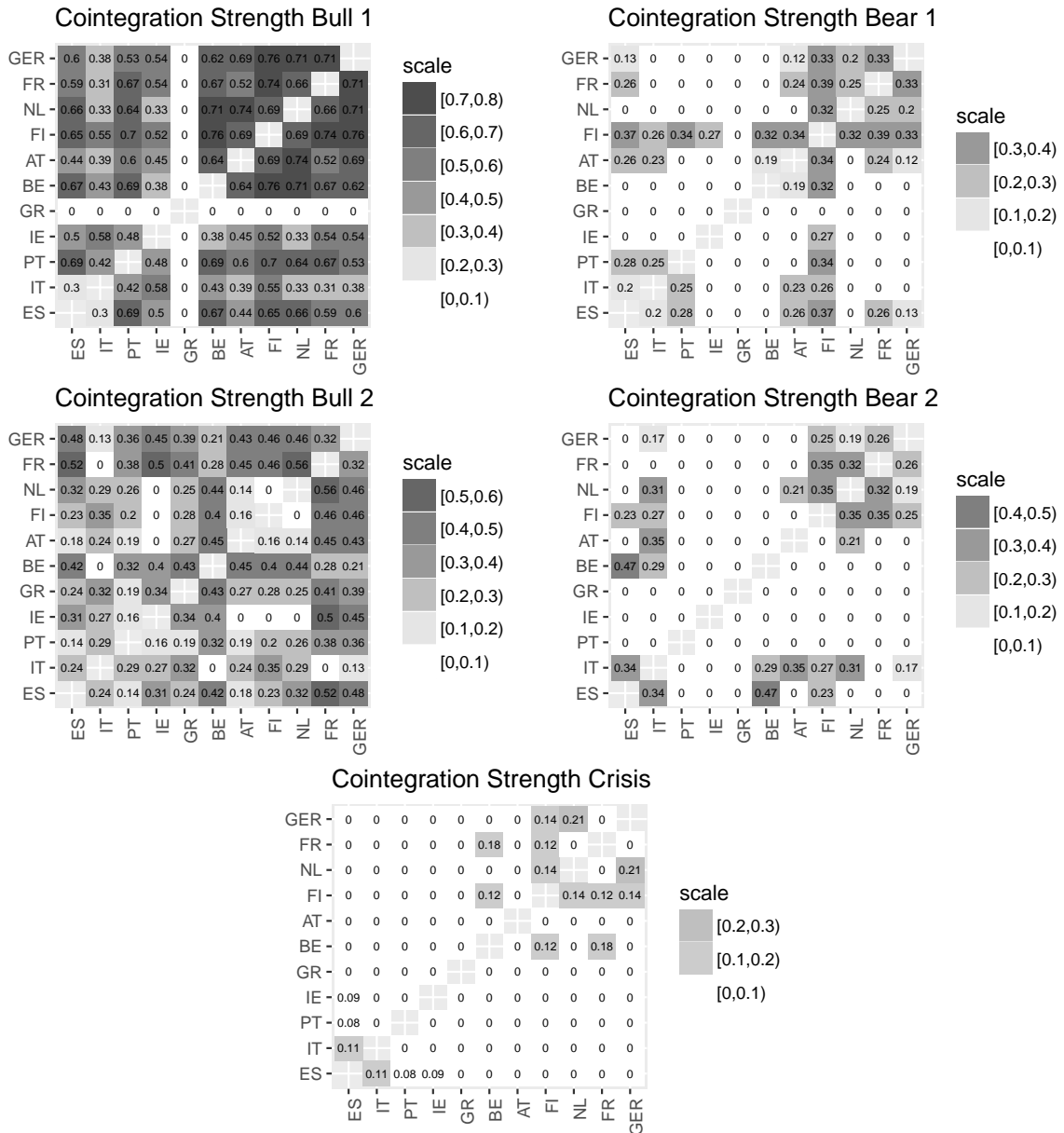


Figure 1: Heatmaps for the strength of all pairwise cointegration relationships. The test for the existence of a cointegrating relationship and the estimation of their strength are carried out for different subperiods using the method of [Chen and Hurvich \(2006\)](#). Dark fields indicate a strong equilibrium relationship between the countries. The exact definition of the market phases can be found in Table 1.

relationships among the core countries than there is among the periphery countries. During the first bear market, Finland is a notable exception, since it appears to be fractionally cointegrated with all of the core countries and with all of the periphery countries, except for Greece. In the second bear market Italy is an exception, since it is in equilibrium with the majority of core countries. We can also observe that there is a

tendency of Portugal, Italy, and Spain to remain in equilibrium with each other during the bear markets. Finally, we observe a clear distinction between periphery countries and core countries during the crisis period. Here, the core countries tend to remain (weakly) integrated with each other, whereas the periphery countries disintegrate completely.

Taken together, we find that there are periods of integration and periods of disintegration associated with bull and bear markets. We can observe that there is stronger market integration between the core countries than between the core and the periphery during bear markets. Finally, we observe a disintegration of the yields for all countries during the crisis. Considering the behavior of the Eurostoxx, the EMU crisis could be regarded as a bull-market period, which usually is a period of integration. The cyclical relationship with periods of integration and disintegration therefore breaks down with the advent of the EMU debt crisis.

4.2 Testing for Market Integration among the Yield Spreads

As discussed in Section 2, a second approach to test for fractional cointegration is to consider the persistence of the spreads directly. Figure 2 shows the spreads for the bull- and bear-market subperiods. Visually, the spreads appear to be less persistent during bull markets than during bear markets. This is also confirmed by the memory estimates in Table 4. These findings clearly support those from the previous section. Furthermore, it can be seen that there is little evidence for market integration if the whole sample period is considered.

However, in this context we no longer need to impose specific time periods that are defined to be bull or bear markets. We can therefore gain further insights into the dynamics of economic integration and disintegration among the interest rates in the Eurozone, by adopting a semiparametric approach and testing for $d(s_{it}) = 1$ in a rolling window. The window size is set to 250 observations, which corresponds to one year, and provides a good tradeoff between bias and sampling variation of the estimate.

The results of this exercise are shown in Figure 3 for the core countries and in Figure

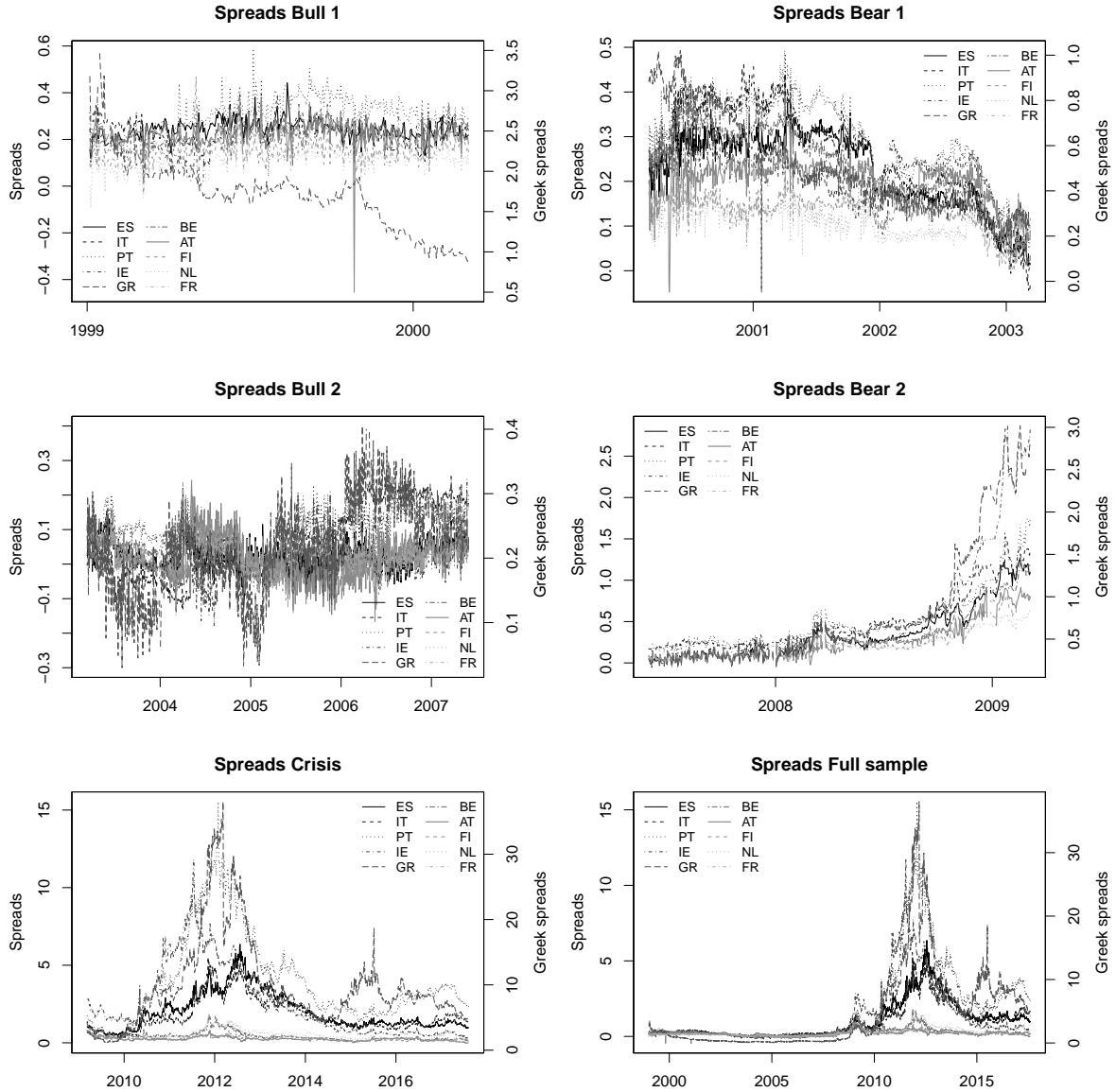


Figure 2: Interest rate yield spreads s_{it} relative to Germany.

The exact definition of the market phases can be found in Table 1.

4 for the periphery countries. Each point represents the estimated memory parameter $\hat{d}(s_{it})$ from the window that ends on this date. The horizontal dashed lines represent a 95% confidence band centered around $d(s_{it}) = 1$, based on $1.96 / \left(2 \sqrt{\sum_{j=1}^m v_j^2} \right)$, where $v_j = \log \lambda_j - m^{-1} \sum_{j=1}^m \log \lambda_j$ and $\lambda_j = 2\pi j / 250$. This is the typical finite sample correction for the variance of the estimator that is based on its Hessian (cf. Hurvich and Beltrao (1994), Lemma 1). It is well known that these tests remain liberal even despite the correction. We therefore might reject the hypothesis of no fractional cointegration too

	ES	IT	PT	IE	GR	BE	AT	FI	NL	FR	(s.e.)
Bull 1	0.41	0.64	0.58	0.55	0.93	0.44	0.47	0.24	0.35	0.37	(0.07)
Bear 1	0.81	0.89	0.84	0.83	0.88	0.81	0.82	0.62	0.74	0.61	(0.05)
Bull 2	0.56	0.92	0.68	0.62	0.67	0.84	0.59	0.62	0.60	0.73	(0.04)
Bear 2	0.95	0.90	0.99	1.06	1.01	1.01	0.95	0.82	0.90	0.87	(0.06)
Crisis	0.88	0.91	0.96	0.96	0.96	0.87	0.88	0.86	0.80	0.88	(0.03)
Full sample	0.93	0.99	0.94	0.95	0.93	0.81	0.86	1.07	0.80	0.96	(0.02)

Table 4: Memory estimates of the spreads s_{it} relative to Germany for different subperiods.

In the Bull 2 period the standard error of the estimate for Ireland is 0.05 and in the full sample the standard error of the estimate for Greece is 0.03. The estimates of d are obtained using the exact local Whittle estimator of Shimotsu and Philips (2005) with a bandwidth of $m = \lfloor T \rfloor^{0.7}$. The exact definition of the market phases can be found in Table 1. Values significantly smaller than one indicate the presence of an equilibrium relationship.

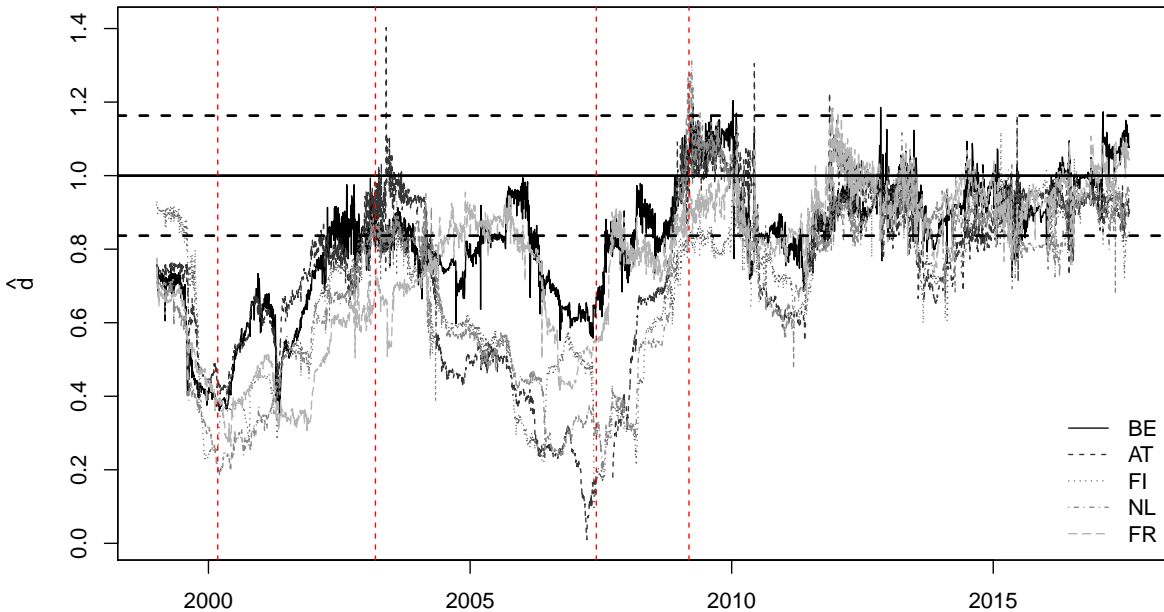


Figure 3: Rolling window estimates of the memory $d(s_{it})$ in the spreads of the core countries.

The estimates of d are obtained using the exact local Whittle estimator of Shimotsu and Philips (2005) with a bandwidth of $m = \lfloor T \rfloor^{0.7}$ in a rolling window of 250 observations. Every value represents the estimated memory parameter from the sample ending on the respective day. Vertical dashed lines indicate the timing of bull and bear markets as defined in Table 1 and horizontal dashed lines mark pointwise critical values for a test of $H_0 : d(s_{it}) = 1$.

often. As before, the vertical dashed lines mark the start and endpoints of the bull- and bear-market periods defined as before.

Considering the results for the core countries in Figure 3, we can make several ob-

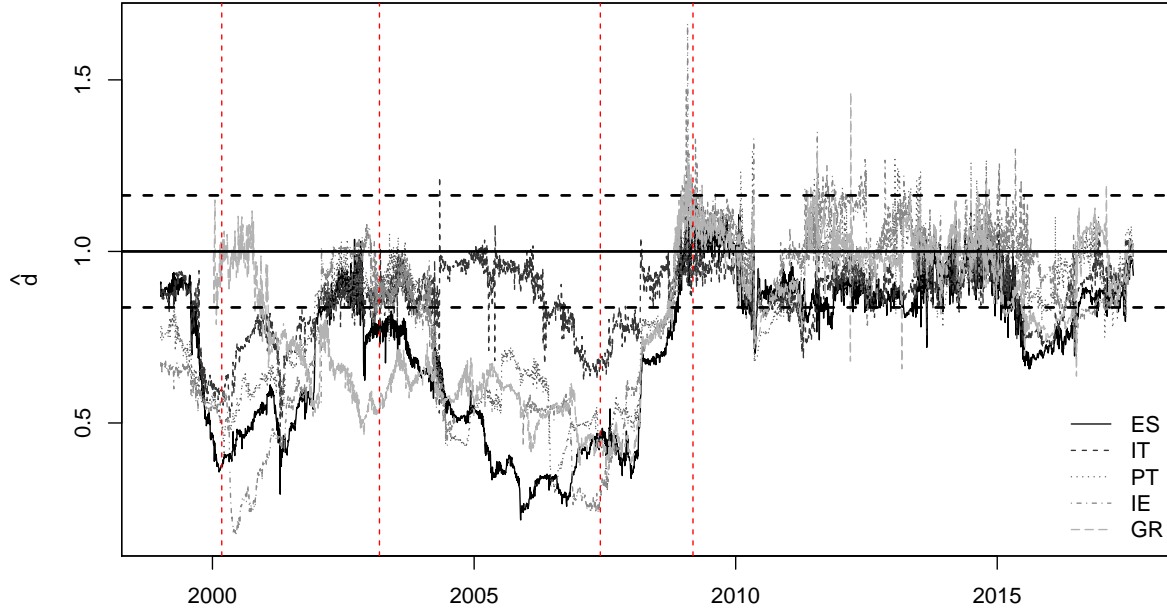


Figure 4: Rolling window estimates of the memory $d(s_{it})$ in the spreads of the periphery countries.

The estimates of d are obtained using the exact local Whittle estimator of [Shimotsu and Philips \(2005\)](#) with a bandwidth of $m = \lfloor T \rfloor^{0.7}$ in a rolling window of 250 observations. Every value represents the estimated memory parameter from the sample ending on the respective day. Vertical dashed lines indicate the timing of bull and bear markets as defined in [Table 1](#) and horizontal dashed lines mark pointwise critical values for a test of $H_0 : d(s_{it}) = 1$.

servations. When we move from a bull-market period to a bear-market period, the estimated memory parameter increases as new observations enter the estimation window. Conversely, when we enter a bull market after a bear market, the new observations entering the estimation window tend to decrease the estimated memory parameter. A similar pattern can be observed for the periphery countries in [Figure 4](#), although they are a bit less homogenous.

Around the end of the first bear market in 2003, there is an extended period during which the estimated memory parameters indicate the absence of a fractional cointegrating relationship and thus no evidence for market integration.

In both groups there are some deviations from the general pattern. Among the core countries the persistence of the Belgian and French spreads keeps increasing in the initial phase of the second bull market. Similarly, the persistence of the Greek and Italian spreads remains high in the same period. Finally, Ireland shows a somewhat

different behavior during the first bull and bear market.

After the second bear market — with the advent of the EMU debt crisis — the relationship breaks down. The estimates of the $d(s_{it})$ are close to 1, and well within the confidence bands, indicating that there is no equilibrium relationship. A notable exception is a short dip in the level of the persistence after April, 2010 when the European Financial Stability Facility (EFSF) was first established. Here, the estimated memory parameters are close to the lower confidence band. However, this period ended quickly thereafter, which implies that the EFSF as a policy measure was not sufficient to effectively calm the market and re-establish an equilibrium.

Overall, the results are clearly in line with those in the previous section that show that there are periods of integration and periods of disintegration that are related to bull markets and bear markets.

4.3 Drivers of Market Integration and Disintegration

To gain further insights into the determinants of EMU bond market integration, we conduct a regression analysis of the sources of variation in the estimated memory parameters from Section 4.2. The main objective of this analysis is to determine whether the observed time variation in the persistence of the spreads can be explained by factors such as market risk or risk aversion that might also drive bull and bear markets.

Typical measures for market risk or "uncertainty" include realized and implied volatility. Let there be N intraday returns r_{it} observed at trading day t , then the realized variance is given by

$$RV_t = \sum_{i=1}^N r_{it}^2,$$

which provides a consistent estimate of the quadratic variation of the respective asset as $N \rightarrow \infty$. We therefore consider the realized volatility of the Eurostoxx index as a measure of current market risk. The implied volatility measured by the VIX and its

European equivalent, the VSTOXX, is a forward-looking measure that extracts the expected average volatility over the next 22 trading days from a panel of option prices, assuming that market participants are risk-neutral. As discussed in detail in [Chernov \(2007\)](#), the VSTOXX can therefore be decomposed into the expected average volatility over the next month and a risk premium according to

$$VSTOXX_t = E_t[RV_{t+22}^{(22)}] + VP_t,$$

where $RV_t^{(H)} = H^{-1} \sum_{h=0}^{H-1} RV_{t-h}$. Under the assumption of rational expectations, we can obtain an ex post estimate of VP_t via

$$VP_t = VSTOXX_t - RV_{t+22}^{(22)}.$$

It is typically found that the VSTOXX has explanatory power for the flight-to-quality effect (cf. for example [Connolly et al. \(2005\)](#)). Due to the persistence of RV_t and the relationships discussed above, it is unclear whether this explanatory power is due to the current level of market risk RV_t , the expected change in the average market risk over the next month

$$\Delta RV_t = RV_{t+22}^{(22)} - RV_t,$$

or the variance premium VP_t . The variance premium VP_t has received a lot of attention in the recent literature, since it is related to the degree of risk aversion. [Bollerslev et al. \(2009\)](#) and [Bollerslev et al. \(2013\)](#), for example, show theoretically and empirically that it has some explanatory power for future stock returns, and [Bekaert and Hoerova \(2014\)](#) show that it improves forecasts of future realized volatility.

Instead of including the VSTOXX itself, we therefore consider RV_t , ΔRV_t , and VP_t separately so that it is possible to distinguish the effect of current market risk from that of (expected) future risk and that of changes in risk pricing.

To formally test the hypothesis that the existence and strength of equilibrium relationships between the bonds of the respective country and Germany are driven by bull- and bear-market periods, we include the bull-market indicator ($\mathbb{1}_{bull,t}$) that corresponds to the market phases defined in Section 3. Due to the special interest in this variable, we include interaction terms between the bull-market indicator and all market-uncertainty measures.

As additional control variables, the daily return of the Eurostoxx (r_t), the spread between BBB-rated US corporate bonds and AAA-rated US government bonds (BBB_t), and the 3-month Euribor rate ($Euribor_t$) are included.

For a better approximation by the normal distribution, we consider the log of RV_t and VP_t . Furthermore, due to the different levels of persistence among these variables, the regressors RV_t , ΔRV_t , VP_t , BBB_t and $Euribor_t$ are fractionally differenced to achieve balanced regressions. Finally, the regressors are standardized to have zero mean and unit variance to facilitate the interpretation of the regression coefficients. This leads to the regression equation

$$\begin{aligned} \hat{d}_{t+125}(s_{it}) = & \beta_0 + \beta_1 \mathbb{1}_{bull,t} + \beta_2 RV_t + \beta_3 \Delta RV_t + \beta_4 VP_t + \beta_5 r_t + \beta_6 BBB_t \\ & + \beta_7 Euribor_t + \beta_8 \mathbb{1}_{bull,t} \times RV_t + \beta_9 \mathbb{1}_{bull,t} \times \Delta RV_t + \beta_{10} \mathbb{1}_{bull,t} \times VP_t + v_t, \end{aligned} \quad (8)$$

where v_t is the innovation term. To achieve the best possible estimation of the respective memory parameters, the dependent variable at time t is the rolling window estimate from period $t+125$ so that the day of interest is in the middle of the estimation window. We observed in the previous sections that the relationship between market sentiment and persistence of the spreads breaks down in the EMU crisis period. Here, the spreads remain persistent despite the bullish environment due to investors' concerns about sovereign default risks. Our estimation period is therefore restricted to the period up to March 8, 2009 — the end of the second bear market.

An econometric complication lies in the fact that our dependent variable itself is estimated in a rolling window of 250 observations, which induces a long autocorrelation

structure. This issue is similar to the problems incurred in long-horizon regressions that test stock return predictability for overlapping time periods. However, in our case the plausible dependence structure is more general than that of asset returns. The dependent variable is not directly observable, and the overlap concerns also past variables. Typical approaches to address this problem, such as those of [Hansen and Hodrick \(1980\)](#) and its extensions by [Richardson and Smith \(1991\)](#) and [Hodrick \(1992\)](#), can therefore not be applied in our setup.

Another common approach is to use HAC estimators with a long lag structure, as for example in [Bekaert and Hoerova \(2014\)](#). This is also the approach we follow here. To account for the autocorrelation caused by the rolling window estimation of the dependent variable, we use a Newey-West estimator with 500 lags. Since this number of lags is relatively large in proportion to the sample size, we cannot resort to standard asymptotics when conducting hypothesis tests. Instead we use so-called fixed- b asymptotics introduced by [Kiefer and Vogelsang \(2005\)](#). Denote the standard HAC estimator based on the first B autocovariances by \widehat{V}_{HAC} . Standard asymptotic theory is based on the assumption that $B/T \rightarrow 0$, as $T \rightarrow \infty$, so that \widehat{V}_{HAC} is consistent for the true variance V . In contrast to this, [Kiefer and Vogelsang \(2005\)](#) assume that $B/T \rightarrow b_{HAC}$, where $b_{HAC} \in (0, 1]$ is a fixed non-zero constant. In this case \widehat{V}_{HAC} is no longer consistent, but converges to the true variance V multiplied by a functional of a Brownian bridge process $Q(k, b_{HAC})$. The corresponding t -statistic t_{FB} has a non-standard limiting distribution that depends on both the kernel k used by the HAC estimator and b_{HAC} . Here, $t_{FB} \Rightarrow \frac{W(1)}{\sqrt{Q(k, b_{HAC})}}$, where $W(r)$ is a standard Brownian motion on $r \in [0, 1]$ and for the *Bartlett* kernel $Q(k, b_{HAC})$ is given by

$$Q(k, b_{HAC}) = \frac{2}{b_{HAC}} \left(\int_0^1 \widetilde{W}(r)^2 dr - \int_0^{1-b_{HAC}} \widetilde{W}(r+b_{HAC}) \widetilde{W}(r) dr \right),$$

with $\widetilde{W}(r) = W(r) - rW(1)$ denoting a standard Brownian bridge. This approach typically provides better size control in persistent time series and can be particularly useful in our setup, where the number of lags employed by the Newey-West estimator is very large.

	ES	IT	PT	IE	GR	BE	AT	FI	NL	FR
const	0.73 **	0.85 **	0.82 **	0.79 **	0.75 **	0.80 **	0.78 **	0.63 **	0.64 **	0.62 **
$\mathbb{1}_{bull,t}$	-0.27 **	-0.02	-0.24 **	-0.29 **	-0.14	-0.08	-0.32 **	-0.15 **	-0.18 *	0.05
RV_t	0.15	0.09	0.11	0.28	0.11	0.19*	0.20 **	0.15	0.28*	0.25 **
ΔRV_t	0.15	0.08	0.10	0.27	0.11	0.19*	0.20 **	0.15	0.28*	0.24 **
VP_t	-0.00	-0.00	-0.00	-0.00	-0.01*	-0.00	-0.00	-0.00	-0.00	-0.00
r_t	-0.01	-0.01	-0.01	-0.01	-0.00	-0.01	-0.01	-0.01	-0.01*	-0.01
BBB_t	-0.00	-0.00	-0.00	-0.00	0.00	-0.00	-0.00	-0.00	-0.00	-0.00
$Euribor_t$	-0.01	-0.00	-0.01	-0.00	-0.00	-0.01	-0.01*	-0.01 **	-0.01	-0.00
$\mathbb{1}_{bull,t} \times RV_t$	-0.41 *	-0.05	-0.17	-0.45*	-0.27	-0.00	-0.38 **	-0.31 **	-0.34*	-0.19
$\mathbb{1}_{bull,t} \times \Delta RV_t$	-0.41 *	-0.05	-0.17	-0.45*	-0.27	0.00	-0.38 **	-0.30 **	-0.34*	-0.19
$\mathbb{1}_{bull,t} \times VP_t$	-0.02	-0.00	-0.01 **	-0.01	0.00	-0.00	-0.02 **	-0.01	-0.02 **	-0.00
R_{adj}^2	0.42	0.02	0.41	0.33	0.14	0.13	0.45	0.18	0.20	0.03
b_{HAC}	0.20	0.20	0.20	0.23	0.21	0.20	0.20	0.20	0.20	0.20
$crit_{0.975}$	2.55	2.55	2.55	2.66	2.59	2.55	2.58	2.55	2.55	2.55
$crit_{0.95}$	2.08	2.08	2.08	2.16	2.11	2.08	2.10	2.08	2.08	2.08

Table 5: Dependence between bond market integration and ex post determined stock market sentiment.

The regression equation in (8) relates the rolling window estimate $\hat{d}(s_{it})$ to the bull-market dummy $\mathbb{1}_{bull,t}$ and a number of control variables. The estimation is carried out for the period 01/01/1999–03/08/2009. The symbols * and ** indicate significance at the 10% level and 5% level, respectively.

The results of this exercise are shown in Table 5. It can be seen that the estimated memory parameters are indeed significantly lower in bull markets. The estimated coefficients of the bull-market indicator are negative for all countries but France and significant in 6 out of 10 cases. The reduction in memory in these cases ranges from -0.15 for Finland to -0.32 for Austria. We also find a significant impact of current risk and future risk changes for the core countries, as well as a number of significant interaction terms between the bull-market dummies and the risk variables. In bear markets a one standard deviation increase in the fractionally differenced realized volatility leads to an increase of the memory parameter of about 0.2, whereas the effect is offset or even reversed in bull markets where the interaction terms come into effect. The variance risk premium does not generally have a significant effect, and where it does, the size of the effect is not economically meaningful. With regard to the quality of the models, the R_{adj}^2 is around 0.4 for Spain, Portugal, and Austria and it is between 0.18 and 0.33 for Ireland, the Netherlands, and Finland. For Belgium and Greece the explanatory power is lower and the model fails to explain the time variation in the persistence of the spreads of France

	ES	IT	PT	IE	GR	BE	AT	FI	NL	FR	(s.e.)
$\hat{d}(s_{it})$	0.92	0.86	0.93	1.02	0.98	0.92	0.92	0.81	0.83	0.83	(0.03)
$\hat{d}(ba_{it})$	0.27	0.29	0.06	0.55	0.24	0.09	0.41	0.24	0.13	0.26	(0.04)

Table 6: Memory estimates for the yield spreads s_{it} and the bid-ask spreads ba_{it} . The estimation is carried out for the period from 12/01/2001–03/08/2009. The standard error of the estimate for the bid-ask spread of Ireland is 0.05.

and Italy.

Overall, however, we find that the time variation in the estimated memory parameters is well explained by a bull-market indicator and the evolution of current and future risk.

As discussed in Section 2, the persistence of the spreads may be driven by that of the default risk premium or that of the liquidity risk premium. Unfortunately, since there were no credit default swaps during the period of interest, we cannot draw any direct conclusions about the memory of the default risk premium. We can, however, consider the bid-ask spreads of the benchmark bonds (ba_{it}) as a proxy for liquidity. Estimates of their memory parameters are provided in Table 6, along with estimates of the memory in the yield spreads for the same period. It can be observed that the level of persistence in the bid-ask spreads is much lower than that in the yield spreads. From the theoretical results on the memory of linear combinations discussed above, the persistence of the spreads and thus the periods of integration and disintegration therefore could not have been caused by changes in the persistence of the liquidity risk premium. This would require the persistence of the bid-ask spreads to be as high as that of the spreads. Instead, it has to be caused by changes of the persistence or relative variability of the default risk premium.

Based on these results, it seems reasonable to assume that $d(\delta_{it}) \geq d(l_{it})$ for all $i = 1, \dots, N$ and $t = 1, \dots, T$. Hence, the theoretical arguments discussed above give rise to two mechanisms that generate the observed time variation in the memory of the spreads that tends to be one in bear markets but much lower in bull markets: (i) breaks in $d(\delta_{it})$ from $d(\delta_{it}) < 1$ to $d(\delta_{it}) = 1$ and vice versa, or (ii) $d(\delta_{it}) = 1$, for all t , but the relative scale

of variations in δ_{it} compared to l_{it} differs for bull and bear markets.

Since the default risk is driven by the macroeconomic and fiscal conditions in the respective country, mean reverting default risk premiums imply the existence of a stable equilibrium relationship between the countries' default risk and the default risk of the benchmark country (Germany). In contrast to that, integrated default risk premiums imply the possibility of divergence between the respective country and Germany, since the variance of integrated series grows linearly with time.

The conclusion in situation (i) would therefore be that market participants considered the possibility of economic and fiscal divergence within the EMU area in bear markets, whereas they expected economic convergence within the currency area in bull markets. In situation (ii), market participants would permanently anticipate the possibility of economic and fiscal divergence between the EMU countries, but the level and variability of the default risk premium is so low during bull markets that the memory properties are dominated by those of the less persistent liquidity risk premium. Conversely, during bear markets risk and risk aversion are high so that the variability of the default risk premium increases relative to that of the liquidity risk premium and the persistence of the spreads is dominated by that of the default risk premium.

These findings provide clear support for the assertion that the persistence of the spreads can be attributed to time variation in either the persistence of the default risk premium or its variability. Both of these arguments ((i) and (ii)) lead to the conclusion that (at least in crisis times) the pricing of EMU government bonds implied the possibility of macroeconomic and fiscal divergence between the EMU countries.

5 Conclusion

The analysis in this paper is based on the application of a wide array of modern methods for the analysis of fractionally cointegrated time series, coupled with a careful consideration of the interrelations between the dynamics driving long-term interest rates and spreads, the persistence of these series, and the implications of the relationships for the

existence or non-existence of equilibria in the EMU government bond market.

Contrary to previous results in the literature, we find that EMU government bond markets are not continually integrated prior to the EMU debt crisis. Even though the level of the spreads was very small compared to that of the yields, we establish that there were periods during which the spreads became unit root processes so that there was no correction mechanism that would drive the yields back to their equilibrium relationship. This is a critical component of the law of one price, which was therefore not fulfilled. These periods of disintegration tended to coincide with bear-market periods, whereas EMU bond markets tended to be economically integrated if stock markets were bullish. Furthermore, the integration among the core countries used to be more intense than that among the periphery countries and especially the degree of integration between the core and the periphery countries was already low in periods prior to the EMU debt crisis.

Altogether, these results imply that investors do not only shift their portfolios from (comparatively) risky stocks to safer bonds in bear markets as described by flight-to-quality effects, there is also a stronger differentiation between sovereign default risks during these periods. As discussed in the previous section, the nature of this differentiation between the default risks of the different countries implies that at least in bear markets investors did consider the possibility of macroeconomic and fiscal divergence between the EMU countries, even though the low magnitude of the spreads shows that this was considered very unlikely.

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