

# Monetary Policy According to HANK<sup>\*</sup>

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## Abstract

We revisit the transmission mechanism of monetary policy for household consumption in a Heterogeneous Agent New Keynesian (HANK) model. The model yields empirically realistic distributions of wealth and marginal propensities to consume because of two features: uninsurable income shocks and multiple assets with different degrees of liquidity and different returns. In this environment, the indirect effects of an unexpected cut in interest rates, which operate through a general equilibrium increase in labor demand, far outweigh direct effects such as intertemporal substitution. This finding is in stark contrast to small- and medium-scale Representative Agent New Keynesian (RANK) economies, where the substitution channel drives virtually all of the transmission from interest rates to consumption. Failure of Ricardian equivalence implies that, in HANK models, the fiscal reaction to the monetary expansion is a key determinant of the overall size of the macroeconomic response.

**JEL Codes:** D14, D31, E21, E52.

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# 1 Introduction

A prerequisite for the successful conduct of monetary policy is a satisfactory understanding of the monetary transmission mechanism – the ensemble of economic forces that link the monetary policy instrument to the aggregate performance of the economy. This paper follows the tradition of treating the short-term nominal interest rate as the primary policy instrument and is concerned with its transmission to the largest component of GDP, household consumption.

Changes in interest rates affect household consumption through both direct and indirect effects. Direct effects are those that operate even in the absence of any change in household disposable labor income. The most important direct effect is intertemporal substitution: when real rates fall, households save less or borrow more, and therefore increase their demand for consumption. In general equilibrium, additional indirect effects on consumption arise from the expansion in labor demand, and thus in labor income, that emanates from the direct impact of the original interest rate cut.

A full grasp of the monetary transmission mechanism requires an assessment of the importance of direct and indirect effects. The relative magnitudes of each channel are determined by how strongly household consumption responds to changes in real interest rates, given income; and to changes in disposable income, given the real rate.

Our first result concerns Representative Agent New Keynesian (RANK) models. In these commonly used benchmark economies, the aggregate consumption response to a change in interest rates is driven entirely by the Euler equation of the representative household. Therefore, for any reasonable parameterization, monetary policy in RANK models works almost exclusively through intertemporal substitution. As a consequence, direct effects account for the full impact of interest rate changes on the macroeconomy, and indirect effects are negligible.

The strong response of aggregate consumption to movements in real rates that accounts for the large direct effects in RANK is questionable in light of empirical evidence. Macroeconometric analysis of aggregate time-series data finds a small sensitivity of consumption to changes in the interest rate after controlling for income ([Campbell and Mankiw, 1989](#); [Yogo, 2004](#); [Canzoneri et al., 2007](#)). Crucially, this finding does not necessarily imply that the individual intertemporal elasticity of substitution is small, as other offsetting direct effects can be powerful. First, micro survey data on household portfolios show that a sizable fraction of households (between 1/4 and 1/3) hold close to zero liquid wealth and face high borrowing costs ([Kaplan et al., 2014](#)). Since these households are at a kink in their budget set, they are insensitive to small changes in interest rates (consistent with evidence in [Vissing-Jorgensen, 2002](#), that non asset-holders do not react to interest rate cuts). Moreover, the possibility of being at a

kink in the future, effectively shortens the time horizon and dampens the substitution effect even for those households with positive holdings of liquid wealth. Second, simple consumption theory implies that an interest rate cut has negative income effects on the consumption of rich households. Third, these same survey data reveal vast inequality in wealth holdings and composition across households (Diaz-Gimenez et al., 2011). Some households may react to a short-term rate cut by rebalancing their asset portfolio, rather than by saving less and consuming more.

The small indirect effects in RANK models follow from the property that the representative agent is, in essence, a permanent income consumer and so is not responsive to transitory income changes. This type of consumption behavior is at odds with a vast macro and micro empirical literature (Jappelli and Pistaferri, 2010). The most convincing corroboration of this behavior is the quasi-experimental evidence that uncovers (i) an aggregate quarterly marginal propensity to consume (MPC) out of small transitory government transfers of around 25 percent (Johnson et al., 2006; Parker et al., 2013) and (ii) a vast heterogeneity in consumption responses across the population which is largely driven by the level of liquid wealth and by the composition of household balance sheets (Misra and Surico, 2014; Cloyne and Surico, 2014; Broda and Parker, 2014).<sup>1</sup>

In light of this empirical evidence, we argue that the relative strength of the direct and indirect channels of monetary policy can be properly gauged only within a framework that offers a better representation of household consumption and household finances than RANK. To this end, we develop a quantitative Heterogeneous Agent New Keynesian (HANK) model that combines two leading workhorses of modern macroeconomics. On the supply side, we follow closely the standard New Keynesian model by assuming that prices are set by monopolistically competitive producers who face nominal rigidities and that monetary policy follows a Taylor rule. On the household side, we build on the standard “Aiyagari-Huggett-İmrohoroglu” incomplete market model, with one important modification: as in Kaplan and Violante (2014), households can save in two assets, a low-return liquid asset and a high-return illiquid asset that is subject to a transaction cost. This extended model has the ability to be consistent with both the joint distribution of earnings, liquid and illiquid wealth, and with the sizable aggregate MPC out of small windfalls.

Our main finding is that, in stark contrast to RANK economies, in our HANK model the direct effects of interest rate shocks are always small, while the indirect effects can be substantial. Monetary policy is effective only to the extent that it generates a general equilibrium response in household disposable income. In our framework, by

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<sup>1</sup>A recent body of work estimating the marginal propensity to consume out of changes in housing net worth also documents consumption responses that are very heterogeneous and heavily dependent on portfolio composition (e.g., Mian et al., 2013).

virtue of this indirect channel, overall consumption responses can be large, even though the strength of the direct channel is modest.

The sharply different consumption behavior between RANK and HANK lies at the heart of these results. Uninsurable risk, combined with the co-existence of liquid and illiquid assets in financial portfolios leads to the presence of a sizable fraction of poor and wealthy hand-to-mouth households, as in the data. These households are highly sensitive to labor income shocks but are not responsive to interest rate changes. Moreover, the vast inequality in liquid wealth implies that, even for non hand-to-mouth households, a cut in liquid rates leads to strong offsetting income effects on consumption. Finally, with this multiple asset structure, to the extent that the spread between asset returns widens after a monetary expansion, household portfolios adjust away from liquid holdings and towards more lucrative assets, rather than towards higher consumption expenditures. All these economic forces counteract the intertemporal substitution effect and lower the direct channel of monetary policy in HANK.

A second important finding is that in HANK, because of a failure of Ricardian equivalence, the consequences of monetary policy are intertwined with the fiscal side of the economy. Since the government is a major issuer of liquid obligations, a change in the interest rate necessarily affects the intertemporal government budget constraint, and generates some form of fiscal response that affects household disposable income. Unlike in RANK models, the details of this response, both in terms of the timing and the distributional burden across households, matter a great deal for the overall macroeconomic impact of a monetary shock and for its split between direct and indirect channels.<sup>2</sup>

We study the implications of these results for two key trade-offs policymakers face in the conduct of monetary policy. First, when attempting to stimulate the macroeconomy, the monetary authority faces a choice between large but transitory versus small but persistent nominal rate cuts. In RANK models, transitory rate cuts and persistent rate cuts are equally powerful, as long as the cumulative interest rate deviations are the same. Instead, in HANK a more transitory, but larger, interest rate cut can be more effective at expanding aggregate consumption because such a policy leads to a more immediate reduction in interest payments on government debt that can translate into an additional fiscal stimulus. Second, we analyzed the inflation-activity trade-off in RANK and HANK. Our experiments suggest that the slope of this relationship is quite similar in the two economies. Intuitively, it is the New-Keynesian side, common

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<sup>2</sup>The importance of government debt for the monetary transmission mechanism is also emphasized by [Sterk and Tenreiro \(2015\)](#) in a model with flexible prices and heterogeneous households where open market operations have distributional wealth effects, and by [?](#) in a model in which Ricardian equivalence fails because of imperfect knowledge.

across models, that largely pins down that relationship. However, in HANK the slope depends on the type of fiscal adjustment: more passive adjustment rules, where more or less government debt absorbs the change in interest payments, are associated to a more favorable trade-off for the monetary authority.

We are not the first to integrate incomplete markets and nominal rigidities, and there is a burgeoning literature on this topic.<sup>3</sup> Relative to this literature, our paper adds an empirically realistic model of the consumption side of the economy by exploiting state-of-the art ideas for modeling household consumption and the joint distribution of income and wealth. As explained, the combination uninsurable earnings risk and a two-asset structure is at the root of our finding that most of the monetary transmission is due to indirect general equilibrium effects. In the paper, we show that the one-asset model explored by the whole literature up to this point faces a daunting challenge when used to study monetary policy. If calibrated to match total wealth in the economy, it implies a very small MPC (similar to the one in RANK) and enormous income effects on consumption because all wealth is liquid. If calibrated to match only *liquid* wealth, it features a large aggregate MPC out of transitory income and reasonable income effects. However, because such calibration misses over 95 pct of the wealth in the economy, the model must completely abstract from some key sources of indirect effects of monetary policy, such as those originating from firm investment and from movements in the price of capital.

Additionally, the focus of our paper differs from that of earlier papers studying monetary policy in the presence of incomplete markets (Gornemann et al., 2014; McKay et al., 2016) in that we inspect the transmission mechanism of monetary policy and decompose it into direct and indirect general equilibrium effects. Our emphasis on general equilibrium effects is shared by Werning (2015) who develops a useful theoretical benchmark where direct and indirect channels exactly offset so that the overall effect of interest rate changes on consumption is unchanged relative to the RANK benchmark. Werning’s assumptions do not hold in our economy, which explains why the total effect of monetary policy, and not just its decomposition, is affected by the presence of heterogeneity and incomplete markets. Conceptually, our decomposition is similar to the one proposed by Auclert (2016).

Our paper is also related to the literature that studies New Keynesian models with limited heterogeneity, building on the spender-saver model of Campbell and Mankiw (1989).<sup>4</sup> The “spenders” in these models consume their entire income every period and

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<sup>3</sup>See Guerrieri and Lorenzoni (2011), Oh and Reis (2012), Ravn and Sterk (2012), McKay and Reis (2016), Gornemann et al. (2014), Auclert (2016), McKay et al. (2016), Den Haan et al. (2015), Bayer et al. (2015), Luetticke (2015), and Werning (2015).

<sup>4</sup>See, e.g., Iacoviello (2005), Gali et al. (2007), Bilbiie (2008) and Challe et al. (2015).

therefore share some similarities with our hand-to-mouth households in that they do not respond to interest rate changes. However, these Two-Agent New-Keynesian (TANK) models also feature “savers” who substitute intertemporally and are highly responsive to interest rate changes. In contrast, in our model even high liquid-wealth households do not increase consumption much in response to an interest rate cut mainly because the risk of receiving negative income shocks and of binding liquidity constraints in the future truncates their effective time horizon. We show that TANK models, when the fraction of spenders reflects the share of hand-to-mouth households in the data, still feature a monetary transmission mechanism with a large role for direct effects. Our accent on indirect channels is shared by [Caballero and Farhi \(2014\)](#) which proposes an alternative framework where the transmission of monetary policy works through its general equilibrium impact on asset values.

Finally, we solve the model in continuous time building on [Achdou et al. \(2014\)](#). In addition to imparting some notable computational advantages, continuous time provides a natural and parsimonious approach to modeling a leptokurtic individual earnings process, as recently documented by [Guvenen et al. \(2015\)](#): random (Poisson) arrival of normally distributed jumps generates kurtosis in data observed at discrete time intervals. This process, estimated by matching targets from Social Security Administration data, may prove useful in other contexts where an empirically realistic representation of household earning dynamics is vital.

The rest of the paper proceeds as follows. [Section 2](#) introduces the idea of decomposing the monetary transmission mechanism into direct and indirect effects, and applies it to small- and medium-scale RANK models and spender-saver models. [Section 3](#) lays out our HANK framework and [Section 4](#) describes how we take it to the data. [Section 5](#) contains our quantitative analysis of monetary policy in HANK, and [Section 6](#) examines some implications of our findings for some key trade-offs faced by policymakers in the conduct of monetary policy. [Section 7](#) concludes.

## 2 Monetary Policy in Benchmark New-Keynesian Models

In this section, we introduce a formal decomposition of the overall consumption response to an interest rate change into direct and indirect effects.<sup>5</sup> Since this decomposition is instrumental to our analysis of the transmission of monetary policy in our larger quantitative model, we begin by applying it to a series of stylized models of monetary policy. We first demonstrate that, in representative agent economies,

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<sup>5</sup>This section benefitted greatly from detailed comments by Emmanuel Farhi and some of the results directly reflect those comments.

conventional monetary policy works almost exclusively through direct intertemporal substitution, and that indirect general equilibrium effects are unimportant. Next, we illustrate how the monetary transmission mechanism is affected by the presence of non-Ricardian hand-to-mouth households: (i) introducing hand-to-mouth households increases the relative importance of indirect general equilibrium effects; (ii) the overall effect of monetary policy now depends on the fiscal response that necessarily arises because monetary policy affects the government budget constraint. Finally, we show that these insights carry over to richer representative agent economies, such as typical medium-scale monetary DSGE models. Appendix A contains proofs of all the results in this section.

## 2.1 Representative Agent Model

**Setup** A representative household has CRRA utility from consumption  $C_t$  with parameter  $\gamma > 0$ , and discounts the future at rate  $\rho \geq 0$ . A representative firm produces output using only labor, according to the production function  $Y = N$ . Both the wage and final goods price are perfectly rigid and normalized to one. The household commits to supplying any amount of labor demanded at the prevailing wage so that its labor income equals  $Y_t$  in every instant. The household receives (pays) lump-sum government transfers (taxes)  $\{T_t\}_{t \geq 0}$  and can borrow and save in a riskless government bond at rate  $r_t$ . Its initial bond holdings are  $B_0$ . In the absence of aggregate uncertainty, household optimization implies that the time path of consumption satisfies the Euler equation  $\dot{C}_t/C_t = \frac{1}{\gamma}(r_t - \rho)$ . The government sets the path of taxes in a way that satisfies its intertemporal budget constraint.

Since prices are perfectly rigid, the real interest rate  $r_t$  also equals the nominal interest rate, so we assume that the monetary authority sets an exogenous time path for real rates  $\{r_t\}_{t \geq 0}$ . We restrict attention to interest rate paths with the property that  $r_t \rightarrow \rho$  as  $t \rightarrow \infty$  so that the economy converges to an interior steady state. Our results place no additional restrictions on the path of interest rates. However, clean and intuitive formulae can be obtained for the special case

$$r_t = \rho + e^{-\eta t}(r_0 - \rho), \quad t \geq 0 \tag{1}$$

whereby the interest rate unexpectedly jumps at  $t = 0$  and then mean reverts at rate  $\eta > 0$ . In equilibrium, the goods market clears  $C_t(\{r_t, Y_t, T_t\}_{t \geq 0}) = Y_t$ , where  $C_t(\{r_t, Y_t, T_t\}_{t \geq 0})$  is the optimal consumption function for the household. We assume that the economy returns to its steady state level in the long-run,  $C_t \rightarrow \bar{C} = \bar{Y}$  as



$t \rightarrow \infty$ .<sup>6 7</sup>

**Overall effect of monetary policy** We can analyze the effects of a change in the path of interest rates on consumption using only two conditions: the household Euler equation, and our assumption that consumption returns back to its steady state level. It therefore follows that  $C_t = \bar{C} \exp\left(-\frac{1}{\gamma} \int_t^\infty (r_s - \rho) ds\right)$ . When the path of interest rates satisfies (1), this formula collapses to a simple expression for the elasticity of initial consumption to the initial change in the interest rate

$$\frac{d \log C_0}{dr_0} = -\frac{1}{\gamma \eta}. \quad (2)$$

The response of consumption is large if the elasticity of substitution  $1/\gamma$  is high, and if the monetary expansion is persistent ( $\eta$  is low).

Note that if initial government debt is positive  $B_0 > 0$ , then a drop in interest rates necessarily triggers a fiscal response. This is because the time path of taxes must satisfy the government budget constraint, and therefore depends on the path of interest rates:  $T_t = T_t(\{r_s\}_{s \geq 0})$ . The government pays less interest on its debt and so will eventually rebate this income gain to households. However, Ricardian equivalence implies that *when* the government chooses to do this does *not* affect the consumption response to monetary policy. In present value terms, the government's gain from lower interest payments is exactly offset by the household's loss from lower interest receipts.

**Decomposition into direct and indirect effects** We begin with the case of zero government debt,  $B_t = 0$  (and  $T_t = 0$ ) for all  $t$ . We use a perturbation argument around the steady state. Assume that initially  $r_t = \rho$  for all  $t$  so that  $Y_t = \bar{Y}$  for all  $t$ . Now consider a small change to the path of interest rates  $\{dr_t\}_{t \geq 0}$ , while holding the path of income  $\{Y_t\}_{t \geq 0}$  constant. The effect of this change in interest rates on consumption is the *direct effect*. In equilibrium, the consumption change induces changes in labor income  $\{dY_t\}_{t \geq 0}$  which lead to further changes in consumption. This is the *indirect effect*. Formally, these two effects are defined by totally differentiating

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<sup>6</sup>There are multiple equilibria in this economy. We select an equilibrium by anchoring the economy in the long run and focusing only on paths for which  $Y_t \rightarrow \bar{Y}$  as  $t \rightarrow \infty$  for some fixed  $0 < \bar{Y} < \infty$ . For any value of steady state output  $\bar{Y}$ , the equilibrium is then unique. Since we are only concerned with deviations of consumption and output from steady state, the level of  $\bar{Y}$  is not important for any of our results.

<sup>7</sup>Rather than assuming that wages and prices are perfectly rigid, our equilibrium could be viewed as a “demand-side equilibrium” as in [Werning \(2015\)](#). In this interpretation, we characterize the set of time paths  $\{r_t, Y_t\}_{t \geq 0}$  that are consistent with optimization on the demand (household) side of the economy without specifying the supply (firm) side. Our results thus apply in richer environments such as the textbook three-equation New Keynesian model.



the initial consumption function  $C_0(\{r_t, Y_t\}_{t \geq 0})$ :

$$dC_0 = \underbrace{\int_0^\infty \frac{\partial C_0}{\partial r_t} dr_t dt}_{\text{direct response to } r} + \underbrace{\int_0^\infty \frac{\partial C_0}{\partial Y_t} dY_t dt}_{\text{indirect effects due to } Y} . \quad (3)$$

The income innovations  $\{dY_t\}_{t \geq 0}$  are equilibrium outcomes induced by the changes in interest rates, which satisfy  $d \log Y_t = -\frac{1}{\gamma} \int_t^\infty dr_s ds$ .<sup>8</sup>

The key objects in the decomposition (3) are the partial derivatives of the consumption function  $\partial C_0 / \partial r_t$  and  $\partial C_0 / \partial Y_t$ , i.e. the household's responses to interest rate and income changes. In this simple model, these two derivatives can be computed analytically which leads to the main result of this section.<sup>9</sup>

**Proposition 1** *Consider small deviations  $dr_t$  of the interest rate from steady state. The overall effect on initial consumption  $d \log C_0 = -\frac{1}{\gamma} \int_0^\infty dr_s ds$  can be decomposed as*

$$d \log C_0 = -\underbrace{\frac{1}{\gamma} \int_0^\infty e^{-\rho t} dr_t dt}_{\text{direct response to } r} - \underbrace{\frac{\rho}{\gamma} \int_0^\infty e^{-\rho t} \int_t^\infty dr_s ds dt}_{\text{indirect effects due to } Y} . \quad (4)$$

*The decomposition is additive, i.e. the two components sum to the overall effect.*

This decomposition of the initial consumption response holds for *any* time path of interest rate changes  $\{dr_t\}_{t \geq 0}$ . The relative importance of the direct effect *does not* depend on the intertemporal elasticity of substitution  $1/\gamma$ .

When the interest rate path follows (1), the decomposition becomes:

$$-\frac{d \log C_0}{dr_0} = \frac{1}{\gamma \eta} \left[ \underbrace{\frac{\eta}{\rho + \eta}}_{\text{direct response to } r} + \underbrace{\frac{\rho}{\rho + \eta}}_{\text{indirect effects due to } Y} \right] . \quad (5)$$

The split between direct and indirect effect depends only on the discount rate  $\rho$  and the rate of mean reversion  $\eta$ . A higher discount rate implies a smaller direct effect and a larger indirect general equilibrium effect. This reflects the fact that (i) in this model, the marginal propensity to consume out of current income is equal to the discount rate; and (ii) the lower is  $\eta$  the larger is the impact of the interest rate change on

<sup>8</sup>Adjustments in income  $dY_t$  can themselves be further decomposed into direct effects and indirect general equilibrium effects. We nevertheless find this version of the decomposition especially useful. In particular, it allows us to distinguish whether, following a change in interest rates, individual households primarily respond through intertemporal substitution in and of itself or to changes in their labor income.

<sup>9</sup>See Theorem 3 in [Auclert \(2016\)](#) for a related decomposition.

the permanent component of labor income. One important implication of equation (5) is that, for any reasonable parameterization, the indirect effect is very small, and monetary policy works almost exclusively through the direct channel. For example in a representative agent model, a quarterly steady state interest rate of 0.5% (2% annually, as we assume in our quantitative analysis later in the paper) implies  $\rho = 0.5\%$ . Suppose the monetary policy shock mean reverts at rate  $\eta = 0.5$ , i.e. a quarterly autocorrelation of  $e^{-\eta} = 0.61$ , then the direct effect accounts for  $\eta/(\rho + \eta) = 99\%$  of the overall effect.<sup>10</sup> Note that, even with a quarterly autocorrelation of 0.95 ( $\eta = 0.05$ ), i.e. with an implausibly persistent monetary shock, from an empirical standpoint, the contribution of the direct effect would still be above 90 pct.<sup>11</sup>

These results extend to the case where government debt is non-zero,  $B_0 > 0$ . When the government issues debt, in equilibrium a monetary expansion necessarily triggers a fiscal response  $T_t = T_t(\{r_s\}_{s \geq 0})$  in order to satisfy the government budget constraint. This equilibrium feedback from fiscal policy affects household consumption which now depends on taxes/transfers  $C_t(\{r_t, Y_t, T_t\}_{t \geq 0})$ . In this case, the direct-indirect decomposition becomes:

$$dC_0 = \underbrace{\int_0^\infty \frac{\partial C_0}{\partial r_t} dr_t dt}_{\text{direct response to } r} + \underbrace{\int_0^\infty \left( \frac{\partial C_0}{\partial Y_t} dY_t + \frac{\partial C_0}{\partial T_t} dT_t \right) dt}_{\text{indirect effects}}. \quad (6)$$

Thus, in the special case (1) where interest rates mean-revert at rate  $\eta$ , we have:

$$-\frac{d \log C_0}{dr_0} = \frac{1}{\gamma \eta} \left[ \underbrace{\frac{\eta}{\rho + \eta} \left( 1 - \rho \gamma \frac{B_0}{\bar{Y}} \right)}_{\text{direct response to } r} + \underbrace{\frac{\rho}{\rho + \eta}}_{\text{indirect effects due to } Y} + \underbrace{\frac{\eta}{\rho + \eta} \rho \gamma \frac{B_0}{\bar{Y}}}_{\text{indirect effects due to } T} \right]. \quad (7)$$

As already noted, due to Ricardian equivalence, the overall effect of monetary policy is not impacted. Relative to (5), the presence of government debt reduces the direct effect. This is because households now own some wealth and hence experience a negative (capital) income effect following an interest rate cut. Ricardian equivalence manifests itself in the fact that the reduction in the direct effect is exactly offset by an additional indirect effect due to changes in transfers. The split between these two components depends on the debt-to-GDP ratio  $B_0/\bar{Y}$ . In principle, with large enough government

<sup>10</sup>This value implies the shock is fully reabsorbed after around 6 quarters. This speed of mean-reversion is consistent with the dynamics of a shock to the federal fund rate commonly estimated by VARs. See, e.g., Christiano et al. (2005) and Gertler and Karadi (2015).

<sup>11</sup>As suggested by John Cochrane <http://johnhcochrane.blogspot.com/2015/08/whither-inflation.html> a better name for the standard New Keynesian model may therefore be the “sticky-price intertemporal substitution model.”

debt, direct effects can be small even in RANK. However, for plausible debt levels, the decomposition is hardly affected relative to (5). For instance, with a quarterly debt-to-GDP ratio  $B_0/\bar{Y} = 1$  (the number used in our calibration) and log-utility  $\gamma = 1$ , the direct effect accounts for  $\frac{\eta}{\rho+\eta} (1 - \rho\gamma\frac{B_0}{\bar{Y}}) = 98\%$  of the overall effect.

## 2.2 Non-Ricardian Hand-to-Mouth Households

We now introduce “rule-of-thumb” households as in [Campbell and Mankiw \(1989, 1991\)](#) and [Bilbiie \(2008, 2017\)](#). The setup is identical, except that we assume that a fraction  $\Lambda$  of households consume their entire current income, i.e. per-capita consumption of these “spenders” is given by  $C_t^{sp} = Y_t + T_t^{sp}$  where  $T_t^{sp}$  is a lump-sum transfer to spenders. Spenders therefore have a marginal propensity to consume out of labor income and transfers equal to one. The remaining fraction  $1 - \Lambda$  of households optimize as before, yielding a consumption function for these “savers”  $C_t^{sa}(\{r_t, Y_t, T_t^{sa}\}_{t \geq 0})$ . Aggregate consumption is given by  $C_t = \Lambda C_t^{sp} + (1 - \Lambda)C_t^{sa}$ . In equilibrium  $C_t = Y_t$ .

The results from RANK extend in a straightforward fashion to this Two-Agent New-Keynesian (TANK) economy. Consider first the case in which  $B_t = 0$  for all  $t$ . For brevity, we again only analyze the generalization of (5):

$$-\frac{d \log C_0}{dr_0} = \frac{1}{\gamma\eta} \left[ \underbrace{(1 - \Lambda)\frac{\eta}{\rho + \eta}}_{\text{direct response to } r} + \underbrace{\left( (1 - \Lambda)\frac{\rho}{\rho + \eta} + \Lambda \right)}_{\text{indirect effects due to } Y} \right]. \quad (8)$$

Note first that the total aggregate effect of monetary policy is exactly as in RANK. The contribution of the direct effect and the indirect effect are each a weighted average of the corresponding quantities for spenders and savers, with the weights equal to each group’s population share. Since the direct effect for spenders is zero and the indirect effect is one, the overall share of the indirect effect approximately equals the population fraction of spenders  $\Lambda$ . A reasonable estimate for the proportion of hand-to-mouth households in the U.S. is 0.3 ([Kaplan et al., 2014](#)), thus in TANK the share of direct effects is roughly 0.7.

The result that the overall effect in TANK is the same as in RANK is due to the fact that the smaller direct effects due to the presence of hand-to-mouth households are exactly offset by larger indirect effects. To see this, note that aggregate consumption is given by  $C_t = Y_t = \Lambda Y_t + (1 - \Lambda)C_t^{sa}$  where consumption of savers is pinned down from the time path of interest rates only  $C_t^{sa} = \bar{C} \exp\left(-\frac{1}{\gamma} \int_t^\infty (r_s - \rho) ds\right)$ . Equivalently,  $C_t = M \times (1 - \Lambda)C_t^{sa}$  where  $M = \frac{1}{1 - \Lambda} > 1$  is a multiplier. The presence of hand-to-mouth households hence scales down direct effects by a factor  $1 - \Lambda$ , but these then

get scaled up again by an exactly offsetting factor  $\frac{1}{1-\Lambda}$ . This is the same logic that lies behind a result of [Werning \(2015\)](#) who showed that, in a particular sticky price economy with heterogeneous agents and incomplete markets, direct and indirect channels exactly offset so that the overall effect of interest rate changes on consumption is unchanged relative to the representative agent complete markets benchmark. In Werning's economy, as well as in our toy model, labor is demand-determined and, therefore, labor supply plays no role. [Bilbiie \(2008, 2017\)](#) studied the monetary transmission mechanism in a TANK model with endogenous labor supply. His analysis implies that this 'as if' result holds only in the knife-edge case of infinite elasticity. In general, the aggregate effect of a monetary policy shock in TANK can be smaller or larger than in RANK.<sup>12</sup>

Now consider the case where the government issues debt  $B_0 > 0$ . As in [Section 2.1](#), a change in the path of interest rates affects the government budget constraint and induces a fiscal response. Because Ricardian equivalence need not hold in the spender-saver economy, the effect of monetary policy depends crucially on the specifics of this fiscal response. In particular, as long as the fiscal response entails increasing transfers to the hand-to-mouth households, then this will increase the overall response of aggregate consumption to monetary policy. This mechanism can be seen most clearly in the case of the exponentially decaying interest rate path [\(1\)](#). If we assume that the government keeps debt constant at its initial level,  $B_t = B_0$  for all  $t$ , and transfers a fraction  $\Lambda^T$  of the income gains from lower interest payments to spenders (and the residual fraction to savers) so that  $\Lambda T_t^{sp}(\{r_s\}_{s \geq 0}) = -(r_t - \rho)\Lambda^T B_0$ , then initial consumption is<sup>13</sup>

$$-\frac{d \log C_0}{dr_0} = \frac{\Lambda^T}{1-\Lambda} \frac{B_0}{\bar{Y}} + \frac{1}{\gamma\eta}, \quad (9)$$

Note the presence of the term  $\Lambda^T(B_0/Y)$ : the overall effect of monetary policy differs from RANK only if there is *both* a debt-issuing government ( $B_0 > 0$ ) *and* Non-Ricardian households who receive a positive share of the transfers ( $\Lambda^T > 0$ ). It is only under this scenario that the indirect component of the transmission mechanism could be much larger in TANK, compared to RANK models (for the decomposition corresponding to [\(9\)](#) see equation [\(62\)](#) in the Appendix). As will become clear in [Section 5](#),

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<sup>12</sup>The equivalence result between TANK and RANK derived in [\(8\)](#) also depends on the identity  $C_t = Y_t$  and hence on the fact that this model does not feature capital and investment. In the presence of investment, the introduction of hand-to-mouth households has ambiguous effects on the elasticity of aggregate consumption. In particular, we would have  $C_t + I_t = Y_t = M \times ((1-\Lambda)C_t^{sa} + I_t)$  and hence  $C_t = C_t^{sa} + \frac{\Lambda}{1-\Lambda}I_t$  and hence the elasticity may be larger or smaller depending on  $\Lambda/(1-\Lambda)$  as well as other factors determining the size of the investment response.

<sup>13</sup>This is equivalent to assuming that the government maintains budget balance by adjusting lump sum transfers, which is the baseline assumption we make in our full quantitative model.

|                    | RANK           |                |            |                   | TANK           |                |                   |
|--------------------|----------------|----------------|------------|-------------------|----------------|----------------|-------------------|
|                    | $B = 0$<br>(1) | $B > 0$<br>(2) | S-W<br>(3) | $B, K > 0$<br>(4) | $B = 0$<br>(5) | $B > 0$<br>(6) | $B, K > 0$<br>(7) |
| Elasticity of $C$  | -2.00          | -2.00          | -0.74      | -2.07             | -2.00          | -2.43          | -2.77             |
| P.E. elast. of $C$ | -1.98          | -1.96          | -0.73      | -1.95             | -1.38          | -1.39          | -1.39             |
| Direct effects     | 99%            | 98%            | 99%        | 94%               | 69%            | 57%            | 50%               |

Table 1: Elasticity of aggregate consumption and share of direct effects in several versions of the RANK and TANK models.

Note:  $B = 0$  denotes the simple models of Section 2 with wealth in zero net supply.  $B > 0$  denotes the extension of these models with government bonds in positive net supply. In RANK, we set  $\gamma = 1, \eta = 0.5, \rho = 0.005$ , and  $B_0/Y = 1$ . In addition, in TANK we set  $\Lambda = \Lambda^T = 0.3$ . ‘S-W’ is the medium-scale version of the RANK model described in Appendix A.4 based on Smets-Wouters. ‘ $B, K > 0$ ’ denotes the richer version of the representative-agent and spender-saver New Keynesian model featuring a two-asset structure, as in HANK. See Appendix A.5 for a detailed description of this model and its calibration. In all cases lump-sum transfers adjust to balance the government budget constraint in the economies with bonds in positive supply. ‘P.E. elast of  $C$ ’ is the partial equilibrium (or direct) elasticity computed as total elasticity times the share of direct effects.

in richer HANK models, the indirect effect is the larger component of monetary policy even when the fiscal response following a monetary shock does not increase transfers to hand-to-mouth households.

## 2.3 Richer RANK and TANK Models

Is our finding that conventional monetary policy works almost exclusively through direct intertemporal substitution special to these simple models? Compared to typical medium-scale DSGE models used in the literature, the RANK model in the present section is extremely stylized. For instance, state-of-the-art medium-scale DSGE models typically feature investment subject to adjustment costs, variable capital utilization, habit formation, and prices and wages that are partially sticky as opposed to perfectly rigid. We therefore conducted a decomposition exactly analogous to that in (4) in one such state-of-the-art framework, the Smets and Wouters (2007) model (see the appendix for details). The result confirms our earlier findings: 99 percent of the consumption response to an expansionary monetary policy shock is accounted for by direct intertemporal substitution effects. The reason is that none of the additional features of this richer model change the property that the consumption of the representative agent is insensitive to the transitory income changes resulting from monetary shocks.<sup>14</sup>

<sup>14</sup>With Smets and Wouters’ baseline parameterization, the total elasticity for consumption at impact is  $-0.74$ , so substantially smaller than that of our stylized models. The key reason is that their model features habit formation in consumption which mutes the consumption response at impact. We conducted a number of robustness checks, particularly with respect to the habit formation parameter

We have also solved more complex versions of RANK and TANK models which, like the HANK model that follows, have both government debt and capital in positive supply, and a New Keynesian production side with Rotemberg-style price adjustment costs. These models, which are fully described in Appendix A.5, are designed to be as close as possible to HANK, except for the nature of household heterogeneity. Comparing column (4) of Table 1 with columns (1) and (2) (for RANK), and comparing column (7) with columns (5) and (6) (for TANK) illustrates that the simple models of Sections 2.1 and 2.2 approximate well these richer economies both in terms of the size of the total consumption response and decomposition into direct and indirect share.

### 3 HANK: A Framework for Monetary Policy Analysis

We now turn to our paper’s main contribution: the development and analysis of our Heterogeneous Agent New Keynesian (HANK) model. Our main innovation is a rich representation of household consumption and saving behavior. Households face uninsurable idiosyncratic income risk which they can self-insure through two savings instruments with different degrees of liquidity. The model’s supply side is kept purposefully simple, and we borrow a number of assumptions from the New Keynesian literature: there is price stickiness and a monetary authority that operates a Taylor rule, and we analyze the economy’s response to an innovation to this Taylor rule. For simplicity, we consider a deterministic transition following a one-time zero-probability shock.

#### 3.1 The Model

**Households** The economy is populated by a continuum of households indexed by their holdings of liquid assets  $b$ , illiquid assets  $a$ , and their idiosyncratic labor productivity  $z$ . Labor productivity follows an exogenous Markov process that we describe in detail in Section 4.2.2. Time is continuous. At each instant in time  $t$ , the state of the economy is the joint distribution  $\mu_t(da, db, dz)$ . Households die with an exogenous Poisson intensity  $\zeta$ , and upon death give birth to an offspring with zero wealth and labor productivity equal to a random draw from its ergodic distribution.<sup>15</sup> There are perfect annuity markets so that the estates of the deceased are redistributed to other individuals in proportion to their asset holdings.<sup>16</sup>

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which directly enters the representative agent’s Euler equation, and found that the share due to direct effects never drops below 90 percent.

<sup>15</sup>We allow for stochastic death to help in generating a sufficient number of households with zero illiquid wealth relative to the data. This is not a technical assumption that is needed to guarantee the existence of a stationary distribution, which exists even in the case  $\zeta = 0$ .

<sup>16</sup>The assumption of perfect annuity markets is implemented by making the appropriate adjustment to the asset returns faced by surviving households. To ease notation, we fold this adjustment directly

Households receive a utility flow  $u$  from consuming  $c_t \geq 0$  and a disutility flow from supplying labor  $\ell_t$ , where  $\ell_t \in [0, 1]$  are hours worked as a fraction of the time endowment, normalized to one. The function  $u$  is strictly increasing and strictly concave in consumption, and strictly decreasing and strictly convex in hours worked. Preferences are time-separable and, conditional on surviving, the future is discounted at rate  $\rho \geq 0$ :

$$\mathbb{E}_0 \int_0^\infty e^{-(\rho+\zeta)t} u(c_t, \ell_t) dt, \quad (10)$$

where the expectation is taken over realizations of idiosyncratic productivity shocks. Because of the law of large numbers, and the absence of aggregate shocks, there is no economy-wide uncertainty.

Households can borrow in liquid assets  $b$  up to an exogenous limit  $\underline{b}$  at an interest rate of  $r_t^{b-} = r_t^b + \kappa$ , where  $\kappa > 0$  is an exogenous wedge between borrowing and lending rates. With a slight abuse of notation,  $r_t^b(b_t)$  summarizes the full interest rate schedule. Short positions in illiquid assets are not allowed.

Assets of type  $a$  are illiquid in the sense that households need to pay a cost for depositing into or withdrawing from their illiquid account. We use  $d_t$  to denote a household's deposit rate (with  $d_t < 0$  corresponding to withdrawals) and  $\chi(d_t, a_t)$  to denote the flow cost of depositing at a rate  $d_t$  for a household with illiquid holdings  $a_t$ . As a consequence of this transaction cost, in equilibrium the illiquid asset pays a higher return than the liquid asset, i.e.  $r_t^a > r_t^b$ .

A household's asset holdings evolve according to

$$\dot{b}_t = (1 - \tau_t)w_t z_t \ell_t + r_t^b(b_t)b_t + T_t - d_t - \chi(d_t, a_t) - c_t \quad (11)$$

$$\dot{a}_t = r_t^a a_t + d_t \quad (12)$$

$$b_t \geq -\underline{b}, \quad a_t \geq 0. \quad (13)$$

Savings in liquid assets  $\dot{b}_t$  equal the household's income stream (composed of labor earnings taxed at rate  $\tau_t$ , interest payments on liquid assets, and government transfers  $T_t$ ) net of deposits into or withdrawals from the illiquid account  $d_t$ , transaction costs  $\chi(d_t, a_t)$ , and consumption expenditures  $c_t$ . Net savings in illiquid assets  $\dot{a}_t$  equal interest payments on illiquid assets plus net deposits from the liquid account  $d_t$ . Note that while we distinguish between liquid and illiquid assets, we follow much of the incomplete-markets literature and net out assets and liabilities within the two asset classes. That is, ours is not a model of gross positions.

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into the rates of return, which should therefore be interpreted as including the return from the annuity.



The functional form for the transaction cost  $\chi(d, a)$  is given by

$$\chi(d, a) = \chi_0 |d| + \chi_1 \left| \frac{d}{a} \right|^{\chi_2} a. \quad (14)$$

This transaction cost has two components that play distinct roles. The linear component generates an inaction region in households' optimal deposit policies because for some households the marginal gain from depositing or withdrawing the first dollar is smaller than the marginal cost of transacting  $\chi_0 > 0$ . The convex component ( $\chi_1 > 0, \chi_2 > 1$ ) ensures that deposit rates are finite,  $|d_t| < \infty$  and hence household's holdings of assets never jump. Finally, scaling the convex term by illiquid assets  $a$  delivers the desirable property that marginal costs  $\chi_d(d, a)$  are homogeneous of degree zero in the deposit rate  $d/a$  so that the marginal cost of transacting depends on the fraction of illiquid assets transacted, rather than the raw size of the transaction.<sup>17</sup>

Households maximize (10) subject to (11)–(14). They take as given equilibrium paths for the real wage  $\{w_t\}_{t \geq 0}$ , the real return to liquid assets  $\{r_t^b\}_{t \geq 0}$ , the real return to illiquid assets  $\{r_t^a\}_{t \geq 0}$ , and taxes and transfers  $\{\tau_t, T_t\}_{t \geq 0}$ . As we explain below,  $\{r_t^b\}_{t \geq 0}$  will be determined by monetary policy and a Fisher equation, and  $\{w_t\}_{t \geq 0}$  and  $\{r_t^a\}_{t \geq 0}$  will be determined by market clearing conditions for capital and labor. In Appendix B.1 we describe the household's problem recursively with a Hamilton-Jacobi-Bellman equation. In steady state, the recursive solution to this problem consists of decision rules for consumption  $c(a, b, z; \Gamma)$ , deposits  $d(a, b, z; \Gamma)$ , and labor supply  $\ell(a, b, z; \Gamma)$ , with  $\Gamma \equiv (r^b, r^a, w, \tau, T)$ .<sup>18</sup> These decision rules imply optimal drifts for liquid and illiquid assets and, together with a stochastic process for  $z$ , they induce a stationary joint distribution of illiquid assets, liquid assets, and labor income  $\mu(da, db, dz; \Gamma)$ . In the appendix, we also describe the Kolmogorov forward equation that characterizes this distribution. Outside of steady state, each of these objects is time-varying and depends on the time path of prices and policies  $\{\Gamma_t\}_{t \geq 0} \equiv \{r_t^b, r_t^a, w_t, \tau_t, T_t\}_{t \geq 0}$ .

**Final-goods producers** A competitive representative final-good producer aggregates a continuum of intermediate inputs indexed by  $j \in [0, 1]$

$$Y_t = \left( \int_0^1 y_{j,t}^{\frac{\varepsilon-1}{\varepsilon}} dj \right)^{\frac{\varepsilon}{\varepsilon-1}}$$

<sup>17</sup>Because the transaction cost at  $a = 0$  is infinite, in computations we replace the term  $a$  with  $\max\{a, \underline{a}\}$ , where the threshold  $\underline{a} > 0$  is a small value (always corresponding to less than \$500 in all calibrations) that guarantees costs remain finite even for households with  $a = 0$ .

<sup>18</sup>In what follows, when this does not lead to confusion, we suppress the explicit dependence of decision rules on the vector of prices and policies  $\Gamma$ .

where  $\varepsilon > 0$  is the elasticity of substitution across goods. Cost minimization implies that demand for intermediate good  $j$  is

$$y_{j,t}(p_{j,t}) = \left(\frac{p_{j,t}}{P_t}\right)^{-\varepsilon} Y_t, \quad \text{where} \quad P_t = \left(\int_0^1 p_{j,t}^{1-\varepsilon} dj\right)^{\frac{1}{1-\varepsilon}}.$$

**Intermediate goods producers** Each intermediate good  $j$  is produced by a monopolistically competitive producer using effective units of capital  $k_{j,t}$  and effective units of labor  $n_{j,t}$  according to the production function

$$y_{j,t} = k_{j,t}^\alpha n_{j,t}^{1-\alpha}. \quad (15)$$

Intermediate producers rent capital at rate  $r_t^k$  in a competitive capital market and hire labor at wage  $w_t$  in a competitive labor market. Cost minimization implies that the marginal cost is common across all producers and given by

$$m_t = \left(\frac{r_t^k}{\alpha}\right)^\alpha \left(\frac{w_t}{1-\alpha}\right)^{1-\alpha}, \quad (16)$$

where factor prices equal their respective marginal revenue products.

Each intermediate producer chooses its price to maximize profits subject to price adjustment costs as in [Rotemberg \(1982\)](#). These adjustment costs are quadratic in the rate of price change  $\dot{p}_t/p_t$  and expressed as a fraction of produced output  $Y_t$

$$\Theta_t \left(\frac{\dot{p}_t}{p_t}\right) = \frac{\theta}{2} \left(\frac{\dot{p}_t}{p_t}\right)^2 Y_t, \quad (17)$$

where  $\theta > 0$ . Suppressing notational dependence on  $j$ , each intermediate producer chooses  $\{p_t\}_{t \geq 0}$  to maximize

$$\int_0^\infty e^{-\int_0^t r_s^a ds} \left\{ \tilde{\Pi}_t(p_t) - \Theta_t \left(\frac{\dot{p}_t}{p_t}\right) \right\} dt,$$

where

$$\tilde{\Pi}_t(p_t) = \left(\frac{p_t}{P_t} - m_t\right) \left(\frac{p_t}{P_t}\right)^{-\varepsilon} Y_t \quad (18)$$

are per-period profits. The choice of  $r_t^a$  for the rate at which firms discount future profits is justified by a no-arbitrage condition that we explain below.

Lemma 1, proved in Appendix B.2, characterizes the solution to the pricing problem and derives the exact New Keynesian Phillips curve in our environment. The combina-

tion of a continuous-time formulation of the problem and quadratic price adjustment costs allows us to derive a simple equation characterizing the evolution of inflation without the need for log-linearization.

**Lemma 1** *The aggregate inflation rate  $\pi_t = \dot{P}_t/P_t$  is determined by the New Keynesian Phillips curve*

$$\left(r_t^a - \frac{\dot{Y}_t}{Y_t}\right) \pi_t = \frac{\varepsilon}{\theta} (m_t - m^*) + \dot{\pi}_t, \quad m^* = \frac{\varepsilon - 1}{\varepsilon}. \quad (19)$$

The expression in (19) can be usefully written in present-value form as:

$$\pi_t = \frac{\varepsilon}{\theta} \int_t^\infty e^{-\int_t^s r_\tau^a d\tau} \frac{Y_s}{Y_t} (m_s - m^*) ds. \quad (20)$$

Note that the marginal payoff to a firm from increasing its price at time  $s$  is  $\Pi'_s(p_s) = \varepsilon Y_s (m_s - m^*)$ . Firms raise prices when their markup  $\mathcal{M}_s = 1/m_s$  is below the flexible price optimum  $\mathcal{M}^* = 1/m^* = \frac{\varepsilon}{\varepsilon - 1}$ . Inflation in (20) is the rate of price changes that equates the discounted sum of all future marginal payoffs from changing prices this period to its marginal cost  $\theta \pi_t Y_t$  obtained from (17).

**Composition of illiquid wealth** Illiquid savings can be invested in two assets: (i) capital  $k_t$ , and (ii) shares in the equity of the aggregate portfolio of intermediate firms, which we denote by  $s_t$ . This equity represents a claim on the entire future stream of monopoly profits net of price adjustment costs,  $\Pi_t \equiv \tilde{\Pi}_t - \frac{\theta}{2} \pi_t^2 Y_t$ . Let  $q_t$  denote the share price. An individual's illiquid assets can thus be expressed as  $a_t = k_t + q_t s_t$ . The dynamics of capital and equity satisfy

$$\dot{k}_t + q_t \dot{s}_t = (r_t^k - \delta) k_t + \tilde{\Pi}_t s_t + d_t. \quad (21)$$

We assume that within the illiquid account, resources can be costlessly shifted between capital and shares. Hence a no-arbitrage condition must hold for the two assets, implying that the return on equity equals the return on capital:

$$\frac{\Pi_t + \dot{q}_t}{q_t} = r_t^k - \delta \equiv r_t^a. \quad (22)$$

We can therefore reduce the dimensionality of the illiquid asset space and consider only the combined illiquid asset  $a$  with rate of return  $r^a$  given by any of the two

returns in (22) and with law of motion as in (12).<sup>19</sup> Finally, note that (22) implies that  $q_t = \int_t^\infty e^{-\int_t^\tau r_s^a ds} \tilde{\Pi}_\tau d\tau$  which justifies the use of  $r_t^a$  as the rate at which future profits are discounted by the intermediate firms and, thus, as the discount rate appearing in the Phillips curve.

**Monetary Authority** The monetary authority sets the nominal interest rate on liquid assets  $i_t$  according to a Taylor rule

$$i_t = \bar{r}^b + \phi\pi_t + \epsilon_t \quad (23)$$

where  $\phi > 1$  and  $\epsilon_t = 0$  in steady state. Our main experiment studies the economy's adjustment after an unexpected temporary monetary shock  $\epsilon_t$ .<sup>20</sup>

Given inflation and the nominal interest rate, the real return on the liquid asset is determined by the Fisher equation  $r_t^b = i_t - \pi_t$ . The real liquid return  $r_t^b$  needs also to be consistent with equilibrium in the bond market, which we describe in Section 3.2.

**Government** The government faces exogenous government expenditures  $G_t$  and administers a progressive tax and transfer scheme on household labor income  $w_t z \ell_t$  that consists of a lump-sum transfer  $T_t$  and a proportional tax rate  $\tau_t$ , with  $\tau_t, T_t > 0$ . The government is the sole issuer of liquid assets in the economy, which are real bonds of infinitesimal maturity  $B_t^g$ , with negative values denoting government debt. Its intertemporal budget constraint is

$$\dot{B}_t^g + G_t + T_t = \tau_t \int w_t z \ell_t(a, b, z) d\mu_t + r_t^b B_t^g \quad (24)$$

Outside of steady state, the fiscal instrument that adjusts to balance the budget can be either  $\tau_t$ ,  $T_t$ , or  $G_t$ . In our experiments, we consider various alternative possibilities for the time path of these adjustments.

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<sup>19</sup>The no-arbitrage condition that allows us to reduce the illiquid portfolio to a single state variable, holds only in the absence of jumps in share price  $q$ , for example in steady state. In this case each individual's illiquid asset portfolio composition between capital and equity is indeterminate, even though the aggregate composition is determined.

<sup>20</sup>We assume that the monetary authority responds only to inflation. Generalizing the Taylor rule (91) to also respond to output gaps is straightforward and does not substantially affect our conclusions. Since our focus is on understanding the transmission mechanism of conventional monetary policy in normal times, we do not consider cases in which the zero-lower bound on nominal interest rates becomes binding.

## 3.2 Equilibrium

An equilibrium in this economy is defined as paths for individual household and firm decisions  $\{a_t, b_t, c_t, d_t, \ell_t, n_t, k_t\}_{t \geq 0}$ , input prices  $\{w_t, r_t^k\}_{t \geq 0}$ , returns on liquid and illiquid assets,  $\{r_t^b, r_t^a\}_{t \geq 0}$ , the share price  $\{q_t\}_{t \geq 0}$ , the inflation rate  $\{\pi_t\}_{t \geq 0}$ , fiscal variables  $\{\tau_t, T_t, G_t, B_t\}_{t \geq 0}$ , measures  $\{\mu_t\}_{t \geq 0}$ , and aggregate quantities such that, at every  $t$ , (i) households and firms maximize their objective functions taking as given equilibrium prices, taxes, and transfers, (ii) the sequence of distributions satisfies aggregate consistency conditions, (iii) the government budget constraint holds, and (iv) all markets clear. There are five markets in our economy: the liquid asset (bond) market, markets for capital and shares of the intermediate firms (that can be folded into a single illiquid asset), the labor market, and the goods market.

The liquid asset market clears when

$$B_t^h + B_t^g = 0, \quad (25)$$

where  $B_t^g$  is the stock of outstanding government debt and  $B_t^h = \int b d\mu_t$  are total household holdings of liquid bonds. The illiquid asset market clears when physical capital  $K_t$  plus the equity value of monopolistic producers  $q_t$  (with the total number of shares normalized to one) equals households' holdings of illiquid assets  $A_t = \int a d\mu_t$ ,

$$K_t + q_t = A_t, \quad (26)$$

The labor market clears when

$$N_t = \int z \ell_t(a, b, z) d\mu_t. \quad (27)$$

Finally, the goods market clearing condition is:

$$Y_t = C_t + I_t + G_t + \Theta_t + \chi_t + \kappa \int \max\{-b, 0\} d\mu_t. \quad (28)$$

Here,  $Y_t$  is aggregate output,  $C_t$  is total consumption expenditures,  $I_t$  is gross additions to the capital stock  $K_t$ ,  $G_t$  is government spending,  $\Theta_t$  are total price adjustment costs, and the last two terms reflects transaction and borrowing costs (to be interpreted as financial services).

### 3.3 Monetary Transmission in HANK

We are interested in analyzing the response of the economy to a one-time unexpected expansionary monetary shock. We assume that the economy is initially in steady state with monetary policy following the Taylor rule (91) with  $\epsilon_t = 0$ . Then at time  $t = 0$  there is an innovation to the Taylor rule  $\epsilon_0 < 0$  with some deterministic decay back to zero. To examine the economy's response to this shock, we generalize the methodology proposed in Section 2 to decompose the total effect of a monetary shock into direct/partial equilibrium and indirect/general equilibrium effects. Our focus is on the transmission mechanism of the shock on the dynamics of aggregate consumption at impact, but it is clear that our decomposition can be extended to any other aggregate variable at any horizon.

Let us begin by writing aggregate consumption  $C_t$  explicitly as a function of the sequence of equilibrium prices, taxes and transfers  $\{\Gamma_t\}_{t \geq 0}$ , with  $\Gamma_t = \{r_t^b, r_t^a, w_t, \tau_t, T_t\}$ , induced by the path of the monetary shock  $\{\epsilon_t\}_{t \geq 0}$  from its initial innovation until its full reversal back to zero:

$$C_t(\{\Gamma_t\}_{t \geq 0}) = \int c_t(a, b, z; \{\Gamma_t\}_{t \geq 0}) d\mu_t. \quad (29)$$

Here  $c_t(a, b, z; \{\Gamma_t\}_{t \geq 0})$  is the household consumption policy function and  $\mu_t(da, db, dz; \{\Gamma_t\}_{t \geq 0})$  is the joint distribution of liquid and illiquid assets and idiosyncratic income.<sup>21</sup>

Totally differentiating (29), we decompose the consumption response at  $t = 0$  as

$$dC_0 = \underbrace{\int_0^\infty \frac{\partial C_0}{\partial r_t^b} dr_t^b dt}_{\text{direct effect}} + \underbrace{\int_0^\infty \left( \frac{\partial C_0}{\partial w_t} dw_t + \frac{\partial C_0}{\partial r_t^a} dr_t^a + \frac{\partial C_0}{\partial \tau_t} d\tau_t + \frac{\partial C_0}{\partial T_t} dT_t \right) dt}_{\text{indirect effects}}. \quad (30)$$

The first term in the decomposition reflects direct effects of a change in the path of the liquid return, holding the wage, the illiquid return, and fiscal policy constant.<sup>22</sup> Since the path of liquid rates enters the budget constraint (11), households respond

<sup>21</sup>Strictly speaking, because households are forward-looking the consumption policy function at time  $t$  is only a function of the sequence of prices from time  $t$  onwards  $\{\Gamma_s\}_{s \geq t}$ . Similarly, the distribution is backward-looking and is only a function of the sequence of prices up to time  $t$ ,  $\{\Gamma_s\}_{s < t}$ . We chose the somewhat less precise notation above for simplicity.

<sup>22</sup>We define the direct effect of a monetary policy with respect to changes in  $r_t^b$  because this is the relevant price from the point of view of households. Alternatively, we could define it “even more directly” with respect to the monetary policy shock  $\epsilon_t$ . With this alternative decomposition, the direct effect in (30) would be split further into a direct due to  $\epsilon_t$  and an indirect effect due to inflation  $\pi_t$ . This follows because  $r_t^b = \bar{r}^b + (\phi - 1)\pi_t + \epsilon_t$  from the Taylor rule and the Fisher equation. Figure 3(a) in our quantitative analysis shows that the drops in  $r_t^b$  and  $\epsilon_t$  are almost equal so that the two decompositions are quantitatively similar.

directly to interest rate changes. This direct effect itself consists of both intertemporal substitution and income effects. The latter arise because aggregate liquid assets are unequally distributed in the cross section and in positive net supply.

The remaining terms in the decomposition reflect the indirect effects of changes in wages, the illiquid return, and the government budget constraint that arise in general equilibrium. There are three separate indirect channels at work in response to an expansionary monetary policy shock. First, when the liquid return falls, intertemporal substitution causes non hand-to-mouth households to increase consumption. In order to meet this additional demand for goods, intermediate firms increase their demand for labor, which pushes up the wage. Households respond to the increase in labor income by further increasing their consumption expenditures.

Second, when the illiquid return changes in response to the change in the liquid return, consumption may be further affected as households choose to rebalance their asset portfolio with deposits into or withdrawals from the illiquid account.<sup>23</sup>

Third, there is a fiscal response to changes in the liquid rate through the government budget constraint. A fall in  $r^b$  reduces government's interest payments on its debt and results in higher tax revenues because of the additional labor income from the economic expansion. Both forces loosen the government budget constraint, and lead to an adjustment in one of the fiscal instruments. As will become clear from our numerical experiments, both the total size of the macroeconomic effects of monetary policy and the split between direct and indirect components depends on the type of fiscal response, a consequence of the non-Ricardian nature of this class of HANK economies.

In practice, we need to compute each of these components numerically. For example, the formal definition of the first term in (30), the direct effect of changes in the liquid return  $\{r_t^b\}_{t \geq 0}$ , is

$$\int_0^\infty \frac{\partial C_0}{\partial r_t^b} dr_t^b dt = \int_0^\infty \left( \int \frac{\partial c_0(a, b, z; \{r_t^b, \bar{r}^a, \bar{w}, \bar{\tau}, \bar{T}\}_{t \geq 0})}{\partial r_t^b} d\mu_0^{r^b} \right) dr_t^b dt \quad (31)$$

where  $\mu_0^{r^b} = \mu_0(da, db, dz; \{r_t^b, \bar{r}^a, \bar{w}, \bar{\tau}, \bar{T}\}_{t \geq 0})$ . That is, this term is the aggregate *partial-equilibrium* consumption response of a continuum of households that face a time-varying interest rate path  $\{r_t^b\}_{t \geq 0}$  but paths for illiquid asset return  $\bar{r}^a$ , wage  $\bar{w}$ ,

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<sup>23</sup>At impact, the share price  $q_0$  jumps (reflecting the change in expected future profits) inducing a revaluation of illiquid wealth. With a slight abuse of notation, the derivative of  $C_0$  with respect to  $r_0^a$  (29) embeds this effect (and all the results we report later in the paper take this initial instantaneous jump into account). For  $t > 0$ , changes in  $q_t$  are already embedded in  $r_t^a$  through the no-arbitrage condition (22). In order to quantify the effect of this price movement on the value of households illiquid assets, we need to make an assumption about the portfolio composition between shares and capital. We simply assume that every agent has the same portfolio composition as the aggregate.



and taxes and transfers  $(\bar{\tau}, \bar{T})$  constant at their steady-state values. We calculate this term from the model by feeding these time paths into the households' optimization problem, computing  $c_0$  for each household, and aggregating across households using the corresponding distribution.

The other terms in the decomposition are computed in a similar fashion. There are similarities between our decomposition and one proposed by Auclert (2016). We discuss these in more detail in Section 5.3.

## 4 Taking the Model to the Data

In this section, we explain how we map our framework to the data. We begin by presenting a small extension of HANK that allows the model to generate more volatile and pro-cyclical investment in response to a monetary shock. Next, we describe our calibration strategy and illustrate our parameterization. Finally, we demonstrate that the model offers a realistic representation of microeconomic consumption behavior.

### 4.1 Distribution of Monopoly Profits

In models of monopolistic competition with price rigidities only, countercyclical mark-ups are at the heart of economic fluctuations. Our framework is no exception. Because prices are sticky but nominal marginal costs are not, expansionary monetary shocks shrink mark-ups, causing firm profits to fall. In RANK models these fluctuations in profits are typically borne lump-sum by the representative household. But in HANK models, additional assumptions are needed about how profits are distributed across households, and in two-asset HANK models further assumptions are needed about how profits are distributed between liquid and illiquid assets. These assumptions, which are neutral in RANK models, can have a large effect on the volatility and cyclicity of investment.

For example, in our baseline HANK model of section 3.1 the fluctuations in profits manifest as movements in the share price  $q_t$ . Since equity is a component of *illiquid* assets, rather than *liquid* assets, the fall in profits associated with an expansionary monetary shock creates a downward pull on investment at a time when output is expanding.<sup>24</sup> This feature is in stark contrast with the data where, quantitatively, investment is the most volatile and procyclical component of output.

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<sup>24</sup>Aggregating (21) across households and using the relevant market clearing conditions we have  $\dot{K}_t = (r_t^k - \delta)K_t + \Pi_t + D_t$  or equivalently that aggregate investment is  $I_t = r_t^k K_t + \Pi_t + D_t$  where  $\Pi_t$  is the profit flow and  $D_t$  are aggregate net deposits. When net deposits  $D_t$  do not move much over the business cycle as is the case in practice, countercyclical fluctuations in profits  $\Pi_t$  have a large effect on investment  $I_t$ .

To avoid this counterfactual implication of sticky prices and restore an empirically realistic co-movement between output, consumption and investment, we make a simple modification to the baseline HANK model: we add one parameter,  $\omega \in [0, 1]$ , that controls the fraction of profits reinvested directly into the illiquid account. Then, if we aggregate the total illiquid income flow across all households, we obtain

$$(r_t^k - \delta)K_t + \omega\Pi_t = \alpha m_t Y_t + \omega(1 - m_t)Y_t. \quad (32)$$

The profit distribution scheme that fully sterilizes the impact of fluctuating mark-ups corresponds to  $\omega = \alpha$ , the capital share of output. In this case, the total income flow accruing into the illiquid account becomes  $\alpha Y_t$ , which is independent of  $m_t$  and is always procyclical.

We assume that the residual share of profits  $(1 - \omega)\Pi_t$  is paid in liquid form to every individual  $i$  as a lump-sum transfer in proportion to household productivity, i.e.  $\pi_{it}^b = \frac{z_{it}}{\bar{z}}(1 - \omega)\Pi_t$ , where  $\bar{z}$  is average productivity. We interpret this additional income as the profit-sharing component of worker compensation from bonuses, commissions, and gains from exercising stock options, and in what follows the term labor income should be interpreted as the sum of wage payments  $w_t z_{it} \ell_{it}$  and bonuses  $\pi_{it}^b$ , whenever  $\omega < 1$ . Appendix B.4 contains more details of this extension of the model.<sup>25</sup>

## 4.2 Calibration Strategy

We have four broad goals in choosing parameters for the model. First, we need to develop a mapping between our aggregated two-asset (liquid-illiquid) structure and data on the complex balance sheet of the U.S. household sector. Second, we seek a calibration of the exogenous stochastic process for labor earnings, which is the ultimate source of inequality in the model. Third, in order to obtain quantitatively realistic consumption behavior at the microeconomic level, our model must generate realistic distributions of liquid and illiquid assets. Of particular importance is the skewness of liquid wealth holdings: matching the fraction of households with low liquid wealth bears directly on the sensitivity of consumption to income changes, whereas matching the top of the liquid wealth distribution is key to generate plausible redistributive effects of interest rate changes. Finally, since the production side of the model is essentially a textbook New Keynesian model, we want to remain as close as possible

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<sup>25</sup>The importance of countercyclical profits has recently been highlighted by [Broer et al. \(2016\)](#) who argue that the resulting income effects on labor supply greatly amplify the New Keynesian monetary transmission mechanism in RANK. As we discuss in the next section, the parameter  $\omega$  also allows us to discipline the strength of such income effects which, under standard balanced-growth preferences, in our model are present whenever  $\omega < 1$ .

|              | Liquid ( $B^h$ )        |       | Illiquid ( $A = K + q$ ) |      | Total |
|--------------|-------------------------|-------|--------------------------|------|-------|
|              | Revolving consumer debt | -0.03 | Net housing              | 1.09 |       |
|              | Deposits                | 0.23  | Net durables             | 0.22 |       |
|              | Corporate bonds         | 0.04  | Corporate equity         | 1.02 |       |
|              | Government bonds        | 0.02  | Private equity           | 0.59 |       |
| <b>Total</b> |                         | 0.26  |                          | 2.92 | 3.18  |

Table 2: Summary of taxonomy of assets

Notes: Categorization of assets into liquid versus illiquid. Values are expressed as a multiple of 2004 GDP(\$12,300B). See Appendix C for details of all calculations.

to the parameterization that is well accepted in that literature.

#### 4.2.1 Categorization of Assets into Liquid and Illiquid

Mapping the model to data requires classifying assets held by US households as liquid versus illiquid. We label an asset as liquid or illiquid based on the extent to which buying or selling the asset involves transaction costs. We define net liquid assets  $B^h$  as all deposits in financial institutions (checking, saving, call, and money market accounts), government bonds, and corporate bonds net of revolving consumer credit. We define illiquid assets  $A$  as real estate wealth net of mortgage debt, consumer durables net of non-revolving consumer credit, plus equity in the corporate and non-corporate business sectors. We have chosen to include equity among illiquid assets, because nearly 3/4 of total equity is either indirectly held (in tax-deferred retirement accounts) or held in the form of private businesses. Both of these assets are significantly less liquid than all the other asset classes included in our definition of  $B^h$ .<sup>26</sup>

We measure the aggregate size of each category of assets and liabilities using data from the Flow of Funds (FoF) and the Survey of Consumer Finances (SCF). We use data from 2004, since this is the last SCF survey year before the Great Recession. In Appendix C, we undertake a comprehensive comparison between these two data sources for each component of the balance sheet. Based on this analysis, we choose to use FoF measures for all assets and liabilities except for the three main categories of liquid assets – deposits, government bonds and corporate bonds – for which we use estimates from the SCF. Table 2 summarizes our preferred estimate, expressed as

<sup>26</sup>In a former version of the paper (Kaplan et al., 2016a), we also separated illiquid assets between productive (equity) and non-productive (housing and durables). In that version of the model, the productive assets served as input into production and paid the rate of return  $r^a$ , whereas the non-productive ones yielded a utility flow to households. Our key quantitative findings are largely invariant to adopting this richer classification, once the model is properly recalibrated.

fractions of annual 2004 GDP (\$12,300B). The total quantity of net liquid assets  $B^h$  amounts to \$2,700B (26% of annual GDP). The total quantity of net illiquid assets  $A$  amounts to \$36,000B (2.92 times annual GDP).

#### 4.2.2 Continuous Time Earnings Dynamics

When households have a choice between saving in assets with different degrees of liquidity, as in our model, the frequency of earnings shocks is a crucial input for determining the relative holdings of the two assets. Households who face small, but frequent, shocks have a strong incentive to hold low-return liquid assets to smooth consumption, while households who face large infrequent shocks would prefer to hold high-return illiquid assets that can be accessed at a cost in the unlikely event of a sizable windfall or a severe income loss.

In standard discrete-time error component models (e.g., the classic persistent-transitory model), the frequency of arrival of earnings shocks is dictated by the assumed time period. In continuous-time models, the frequency at which shocks arrive is a property of the stochastic process, and must be estimated alongside the size and persistence of shocks. Empirically, the challenge in estimating the frequency of earnings shocks is that almost all high quality panel earnings data are available only at an annual (or lower) frequency. It is thus challenging to learn about the dynamics of earnings at any higher frequency. Our strategy to overcome this challenge is to infer high frequency earnings dynamics from the *high-order moments of annual earnings changes*. To understand why this identification strategy has promise, consider two possible distributions of annual earnings changes, each with the same mean and variance, but with different degrees of kurtosis. The more leptokurtic distribution (i.e. the distribution with more mass concentrated around the mean and in the tails) is likely to have been generated by an earnings process that is dominated by large infrequent shocks; the more platykurtic distribution (i.e. the distribution with more mass in the shoulders) by a process that is dominated by small frequent shocks.

Motivated by these observations, we model log-earnings as the sum of two independent components

$$\log z_{it} = z_{1,it} + z_{2,it} \tag{33}$$

where each component  $z_{j,it}$  evolves according to a “jump-drift” process. Jumps arrive at a Poisson rate  $\lambda_j$ . Conditional on a jump, a new log-earnings state  $z'_{j,it}$  is drawn from a normal distribution with mean zero and variance  $\sigma_j^2$ ,  $z'_{j,it} \sim \mathcal{N}(0, \sigma_j^2)$ . Between

| Moment                     | Data | Model |
|----------------------------|------|-------|
| Variance: annual log earns | 0.70 | 0.70  |
| Variance: 1yr change       | 0.23 | 0.23  |
| Variance: 5yr change       | 0.46 | 0.46  |
| Kurtosis: 1yr change       | 17.8 | 16.5  |
| Kurtosis: 5yr change       | 11.6 | 12.1  |
| Frac 1yr change < 10%      | 0.54 | 0.56  |
| Frac 1yr change < 20%      | 0.71 | 0.67  |
| Frac 1yr change < 50%      | 0.86 | 0.85  |

Table 3: Earnings Process Estimation Fit

jumps, the process drifts toward zero at rate  $\beta_j$ . Formally, the process for  $z_{j,it}$  is

$$dz_{j,it} = -\beta_j z_{j,it} dt + dJ_{j,it}, \quad (34)$$

where  $dJ_{j,it}$  captures jumps in the process.<sup>27</sup>

The process for each component is closely related to a discrete time AR(1) process.<sup>28</sup> The key difference is that in our continuous time formulation, the arrival of each innovation is stochastic, and hence each process has an additional parameter,  $\lambda_j$ , which captures the frequency of arrival.<sup>29</sup>

**Estimation with male earnings data** We estimate the earnings process in (33)-(34) by Simulated Method of Moments using Social Security Administration (SSA) data on male earnings from Guvenen et al. (2015).<sup>30</sup> These authors report eight key moments that we target in the estimation (see Table 3).<sup>31</sup> Moments of the distribution

<sup>27</sup>Even more formally, the infinitesimal generators  $\mathcal{A}_j f(z) := \lim_{t \downarrow 0} \frac{\mathbb{E}[f(z_t)] - f(z)}{t}$  of the two components  $j = 1, 2$  are given by  $\mathcal{A}_j f(z) = -\beta_j z f'(z) + \lambda_j \int_{-\infty}^{\infty} (f(x) - f(z)) \phi_j(x) dx$  where  $\phi_j$  is the density of a Normal distribution with mean zero and variance  $\sigma_j^2$ .

<sup>28</sup>In particular, if the earnings innovations always arrived at regular intervals (say, annually), rather than stochastically at rate  $\lambda_j$ , then each component would follow an AR(1) process. The drift parameter  $\beta_j$  would correspond to (one minus) the discrete time auto-regressive parameter and the innovation variance  $\sigma_j^2$  would describe the size of innovations. In this sense, the model is only a minimal departure from the familiar persistent-transitory process used to model discrete time earnings data.

<sup>29</sup>Schmidt (2015) models earnings dynamics as a discrete-time compound Poisson process, using a similar logic.

<sup>30</sup>The main benefits of targeting moments from administrative earnings data such as the SSA are that they are based on a very large sample and so are less prone to measurement error than survey data, and that they are not top-coded. Both features are important: the sample size and absence of measurement error allows a precise estimate of higher-order moments, and the absence of top-coding allows for an accurate portrayal of the right-tail of the income distribution, which is important for capturing the skewness in wealth holdings.

<sup>31</sup>We restrict attention to a symmetric process since Guvenen et al. (2015) find only a small amount of negative skewness in 1-year and 5-year annual changes. It is possible to generate skewness in annual changes by allowing the drift parameters  $\beta_j$  to differ based on the sign of  $z_{j,it}$ .

| Parameter                    |             | Component | Component |
|------------------------------|-------------|-----------|-----------|
|                              |             | $j = 1$   | $j = 2$   |
| Arrival rate                 | $\lambda_j$ | 0.080     | 0.007     |
| Mean reversion               | $\beta_j$   | 0.761     | 0.009     |
| St. Deviation of innovations | $\sigma_j$  | 1.74      | 1.53      |

Table 4: Earnings Process Parameter Estimates. Rates expressed as quarterly values.

of earnings changes at multiple durations are needed to separately identify the two components. Since these data refer to annual earnings, we simulate earnings from the model at a high frequency, aggregate to annual earnings and compare moments from model and data.

The fitted earnings process matches the eight targeted moments well. The estimated parameter values, reported in Table 4, are consistent with the existence of a transitory and a persistent component in earnings. The transitory component ( $j = 1$ ) arrives on average once every three years and has a half-life of around one quarter. The persistent component ( $j = 2$ ) arrives on average once every 38 years and has a half-life of around 18 years. Both components are subject to relatively large, similar sized innovations. In the context of an infinite horizon model, the estimated process thus has the natural interpretation of a large and persistent “career” or “health” shock that is perturbed by periodic temporary shocks. Note that relative to a discrete-time model, our estimated transitory shock is both less frequent, and more temporary than an IID annual shock.

In our model, flow earnings are given by  $y_{it} \equiv w_t z_{it} \ell_{it}$  and are thus determined by both the realization of productivity shocks  $z_{it}$  and the choice of labor supply  $\ell_{it}$ . In Appendix D.1 we explain how we convert the estimated process for individual male earnings to a discrete-state process for idiosyncratic productivity that is consistent with our assumption of a household as the unit of observation.<sup>32</sup> Relative to typical earnings process calibrations based on survey data, and consistent with the cross-sectional earnings distribution in SSA data, the resulting earnings process features a large amount of right-tail inequality. The top 10, 1, and 0.1 percent shares of gross household labor earnings in the steady state are 32%, 7% and 2% respectively. This skewed earnings distribution is an important factor in the model’s ability to generate skewed distributions of liquid and illiquid assets. However, unlike much of the existing literature that has generated wealth concentration at the top of the distribution from ad-hoc skewed earnings distributions, here both inequality and dynamics of earnings

<sup>32</sup>Our discrete approximation to the estimated productivity process uses 11 points for the persistent component and 3 points for the transitory component. In Appendix D.1 we describe the discretization process in detail and report further statistics from the discretized distribution, including plots of the Lorenz curves for the ergodic distributions from the continuous and discretized processes.

|                                | Data | Model |                  | Liquid Wealth |       | Illiquid Wealth |       |
|--------------------------------|------|-------|------------------|---------------|-------|-----------------|-------|
|                                |      |       | Moment           | Data          | Model | Data            | Model |
| Mean illiquid assets           | 2.92 | 2.92  | Top 0.1% share   | 17%           | 2.3%  | 12%             | 7%    |
| Mean liquid assets             | 0.26 | 0.23  | Top 1% share     | 47%           | 18%   | 33%             | 40%   |
| Frac. with $b = 0$ and $a = 0$ | 0.10 | 0.10  | Top 10% share    | 86%           | 75%   | 70%             | 88%   |
| Frac. with $b = 0$ and $a > 0$ | 0.20 | 0.19  | Bottom 50% share | -4%           | -3%   | 3%              | 0.1%  |
| Frac. with $b < 0$             | 0.15 | 0.15  | Bottom 25% share | -5%           | -3%   | 0%              | 0%    |
|                                |      |       | Gini coefficient | 0.98          | 0.86  | 0.81            | 0.82  |

Table 5: Left panel: Moments targeted in calibration and reproduced by the model. Means are expressed as ratios to annual output. Right panel: Statistics for the top and bottom of the wealth distribution not targeted in the calibration. Source: SCF 2004.

are disciplined directly by high quality data.<sup>33</sup>

#### 4.2.3 Adjustment Cost Function and Wealth Distribution

We set the steady-state real return on liquid assets  $\bar{r}^b$  at 2% per annum and steady-state inflation to zero. Given values for the capital share, demand elasticity and depreciation rate (all set externally as described in Section 4.2.4) and for the unsecured borrowing limit — our target for the illiquid assets of 2.9 times output yields a steady-state return to illiquid assets  $r^a$  of 5.7% per annum.

Given these returns, and the exogenous process for idiosyncratic labor income, the key parameters that determine the incentives for households to accumulate liquid and illiquid assets are the borrowing limit  $\underline{b}$ , the discount rate  $\rho$ , the intermediation wedge  $\kappa$ , and the three parameters of the adjustment cost function  $\chi_0, \chi_1$  and  $\chi_2$ .

Borrowing in the model should be interpreted as unsecured credit, so we set the borrowing limit  $\underline{b}$  exogenously at 1 times quarterly average labor income.<sup>34</sup> We then choose the remaining five parameters  $(\rho, \kappa, \chi_0, \chi_1, \chi_2)$  to match five moments of the distribution of household wealth from the SCF 2004: (i)-(ii) the mean of the illiquid and liquid wealth distributions; (iii)-(iv) the fraction of poor and wealthy hand-to-mouth households, since these are the most important moments of the liquid wealth distribution for determining household consumption responses to income shocks; and (v) the fraction of households with negative net liquid assets, which serves to identify

<sup>33</sup>The existing literature reverse-engineers a process for earnings risk in order to match data on wealth inequality. This approach typically requires an implausibly extreme characterization of risk, with a top income state around 500 times as large as the median, and a high probability of a dramatic fall in earnings once the top state is reached. See Benhabib and Bisin (2016) and De Nardi et al. (2016) for a discussion of this issue. In our discretized process, instead, the highest productivity realization is around 13 times as large as the median, and is realized by only 0.03% of the population.

<sup>34</sup>In the steady state ergodic distribution only 0.02% of households are at the limit.



the borrowing wedge.<sup>35</sup>

The calibrated annual discount rate  $\rho$  is 5.1%, and the annual wedge  $\kappa$  is 6.0% (implying an annual borrowing rate of 8.0%). The calibrated transaction cost function is displayed in Figure D.3 in Appendix D. In the resulting ergodic distribution, roughly 80% of households are adjusting at any point in time. Conditional on making a deposit or withdrawal, the mean absolute quarterly transaction as a fraction of the stock of illiquid assets is 1.7%. The quarterly transaction cost for a transaction this size is 23% of the transaction. In steady-state the equilibrium aggregate transaction costs, which one can interpret as financial services, amount to less than 4% of GDP.

The model replicates the five targeted moments well (left panel of Table 5).<sup>36</sup> Figure 1 displays the distributions of liquid and illiquid wealth in the model. Despite only targeting a handful of moments of each distribution, the model successfully matches the distributions of liquid and illiquid wealth up to the very top percentiles, as is clear from the right-panel of Table 5 which reports top wealth shares from the model and data. Both Gini coefficients in the model are close to their data counterparts. The reason for this success is a combination of the realistically skewed earnings distribution and the heterogeneity in effective returns on wealth because of the two-asset structure: a fraction of households end up spending a long time in high earnings states, hold high-return illiquid assets, and accumulate a lot of wealth. These households populate the upper tail of the wealth distribution.<sup>37</sup>

#### 4.2.4 Remaining Model Parameters

**Demographics** We set the quarterly death rate  $\zeta$  to 1/180 so that the average lifespan of a household is 45 years.

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<sup>35</sup>We define hand-to-mouth households in the model as those with zero liquid wealth. The targets of 10% and 20% are chosen to replicate the fraction of households with net liquid wealth  $\in [-\$1,000, \$1,000]$  with zero and positive illiquid assets, respectively. These targets are similar to estimates in Kaplan et al. (2014). The target of 15% of households with negative liquid wealth reproduces the fraction of households with net liquid wealth  $< -\$1,000$  in the data.

<sup>36</sup>Besides matching the share of hand-to-mouth agents in the population, the model also does well with respect to their relative importance in terms of consumption share (20 pct vs. 25 pct in the PSID)) and wealth share (2.5 pct vs. 4.4 pct in the SCF).

<sup>37</sup>To put this result in the context of the literature, see the survey by Benhabib and Bisin (2016) on the sources of skewed wealth distributions in macroeconomic models. Note that our model is not able to match the extreme right tail of the liquid wealth distribution and also does not feature a Pareto tail for the distribution of total wealth as arguably observed in the data. It is notoriously challenging to match the extreme right tail of wealth distributions with labor income risk alone and our model is no exception.

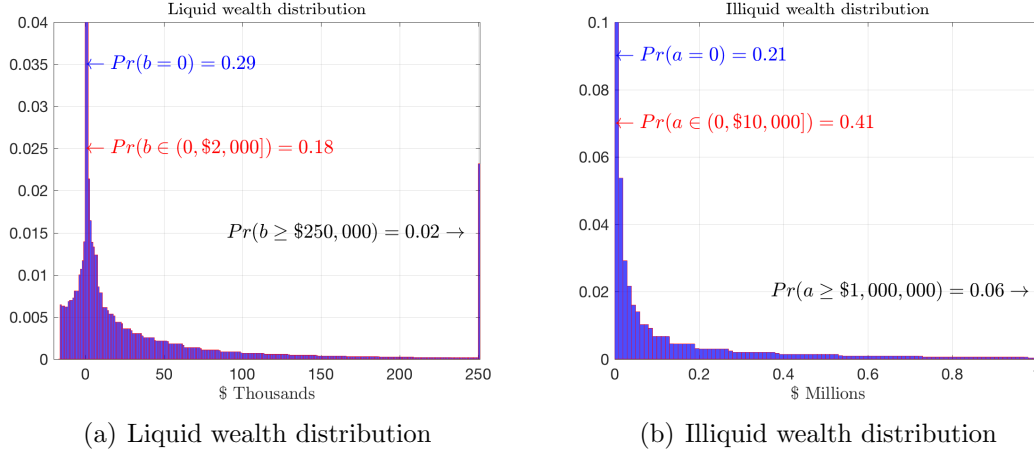


Figure 1: Distributions of liquid and illiquid wealth.

**Preferences** Households have instantaneous utility that is separable over consumption and hours worked:

$$u(c, \ell) = \frac{c^{1-\gamma}}{1-\gamma} - \varphi \frac{\ell^{1+\nu}}{1+\nu} \quad (35)$$

with  $\gamma \geq 0$  and  $\nu \geq 0$ . We set  $1/\gamma$ , the intertemporal elasticity of substitution (IES), to one and  $1/\nu$ , the Frisch elasticity of labor supply at the household level, to one.<sup>38</sup> The weight on the labor supply component of utility,  $\varphi$ , is set so that average hours worked are equal to 1/3 of the time endowment in steady-state.

**Production** The elasticity of substitution for final goods producers is set to  $\varepsilon = 10$ , implying a steady state markup  $1/(\varepsilon - 1)$  of 11%. Intermediate goods producers have a weight on capital of  $\alpha = 0.33$ , which yields a capital share of 30%, a labor share of 60%, and a profit share of 10%. We set the constant  $\theta$  in the price adjustment cost function to 100, so that the slope of the Phillips curve in (19) is  $\varepsilon/\theta = 0.1$ .<sup>39</sup> The fraction  $\omega$  of aggregate profits reinvested into the illiquid accounts is set to  $\alpha$ , as explained in Section 4.1. This choice which, as explained, neutralizes the countercyclicality of mark ups, happens to be also roughly in line with the data. In 2004, the sum of undistributed corporate profits (the empirical counterpart of profits reinvested in the illiquid account

<sup>38</sup>This number is slightly higher than the 0.82 identified by Chetty et al. (2011) as the representative estimate from existing studies of the micro elasticity at the individual level, accounting for intensive and extensive margins of adjustment. At the household level though, the marginally attached worker is often the wife and a Frisch labor supply elasticity of one is in line with the estimates of Blundell et al. (2016) for married women.

<sup>39</sup>See e.g. Schorfheide (2008) who survey many studies using the labor share as a proxy to measure marginal costs, an approach suggested by Gali and Gertler (1999).

in the model) and dividend income (the counterpart of profits paid to households) was \$946B. Of these, undistributed profits amounted to \$384B, thus about 40 pct of the total.<sup>40</sup>

**Government policy** We set proportional labor income tax rate  $\tau$  to 0.30 and the lump-sum transfer  $T$  to be 6% of output (equivalent to around \$7,000 per year). In steady state just over 9% of households receive a net transfer from the government. In our model, the government is the only provider of liquid assets. Given our calibration of household liquid holdings, government debt amounts to 23.3% of annual GDP. Government expenditures are then determined residually from the government budget constraint (24).

**Monetary Policy** We set the Taylor rule coefficient  $\phi$  to 1.25, which is in the middle of the range commonly used for New Keynesian models.

Table D.2 in the Appendix summarizes our parameter values. In Section 5.2, we verify the robustness of our results with a series of sensitivity analyses.

### 4.3 Micro Consumption Behavior

How successful is the calibrated model at generating empirically realistic distributions of household responses to changes in labor income? Some of the most convincing empirical evidence on marginal propensities to consume (MPCs) comes from household consumption responses to the tax rebates of 2001 and fiscal stimulus payments of 2008 (see e.g. Johnson et al., 2006; Parker et al., 2013; Misra and Surico, 2014; Broda and Parker, 2014). While the estimates are often imprecise because of the small sample size, this collective quasi-experimental evidence concludes that households spend approximately 15-25 percent of these payments (which average between \$500 and \$1,000 depending on the episode) on nondurables in the quarter that they are received.

Let  $\text{MPC}_\tau^x(a, b, z)$  be the MPC over a period of length  $\tau$  quarters out of one-time inflow of  $x$  additional dollars of liquid wealth. This is the notion of an MPC that is comparable to the empirical evidence cited above (as opposed to the slope of the consumption function with respect to liquid wealth). In Appendix B.3 we state the formal definition and show how to compute it directly from households' consumption policy functions using the Feynman-Kac formula.

The average quarterly MPC out of a \$500 transfer is 16% in the model, which is within the range of typical empirical estimates, and an order of magnitude larger than

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<sup>40</sup>See NIPA Tables 2.1 and 5.1. Also over the period 1990-2016, for example, this fraction fluctuates around 40 pct.

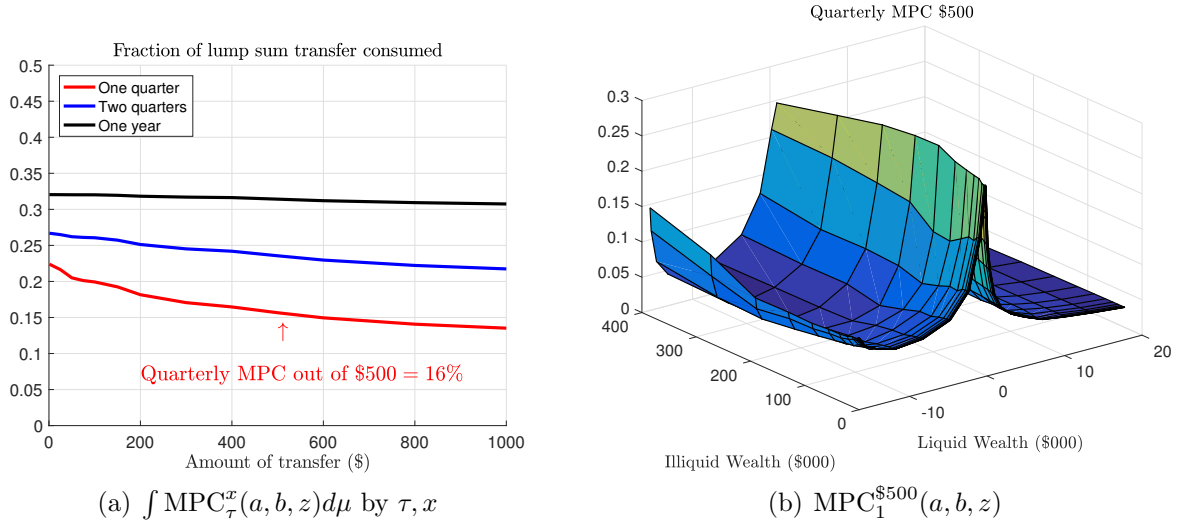


Figure 2: MPC Heterogeneity

its counterpart in one-asset incomplete-market models (Kaplan and Violante, 2014). As seen in Figure 2(a) the fraction consumed decreases with the size of the transfer, and increases sharply as the horizon increases.

The average MPCs in Figure 2(a) mask important heterogeneity across the population. This heterogeneity can be seen in Figure 2(b), which plots the function  $MPC_{\tau}^x(a, b, z)$  for a  $x = \$500$  payment over one quarter as a function of liquid and illiquid assets, averaged across labor productivity  $z$ . The figure illustrates the strong source of bi-modality in the distribution of consumption responses in the population. In the model, the average response of 16% is composed of a group of households with positive net liquid wealth and very low consumption responses, and another group of hand-to-mouth households with no liquid wealth who display strong consumption responses. Of these hand-to-mouth households, roughly two-third are have positive illiquid wealth.

Several recent empirical papers have documented patterns of the distribution of MPCs that are consistent with Figure 2. Broda and Parker (2014) find much stronger consumption responses to the 2008 fiscal stimulus payments among households with low easily accessible liquid funds. Misra and Surico (2014) use quantile regression techniques to study the consumption responses in the tax rebate episodes of 2001 and 2008 and document the presence of high-income households both in the low MPC and the high MPC group, a fact consistent with the presence of wealthy HtM households. Kaplan et al. (2014) use the method proposed by Blundell et al. (2016) to estimate the consumption response to transitory income shocks on PSID data for three groups —non hand-to-mouth (HtM), poor HtM and wealthy HtM— and uncover much higher MPCs

for both types of HtM households. [Baker \(2016\)](#) finds that households with similar net asset positions behave differently with respect to income shocks if they hold varying shares of liquid and illiquid wealth. Finally, [Fagereng et al. \(2016\)](#) examine MPCs out of lottery prizes using Norwegian administrative data. They find that MPCs vary with the amount of households' liquid assets, and that households with close to zero liquid assets have high MPCs even if they are wealthy in terms of their illiquid asset positions.

This striking heterogeneity in MPCs underlines the importance of obtaining a realistic distribution of both wealth components. With such distributions in hand, we now turn to the monetary transmission mechanism.

## 5 Monetary Transmission: Quantitative Results

Our main results concern the response of the economy to a one-time unexpected monetary shock. We consider an experiment in which at time  $t = 0$ , there is a quarterly innovation to the Taylor rule (91) of  $\epsilon_0 = -0.25\%$  (i.e.  $-1\%$  annually) that mean-reverts at rate  $\eta$ , i.e.  $\epsilon_t = e^{-\eta t} \epsilon_0$ . We set  $\eta = 0.5$ , corresponding to a quarterly autocorrelation of  $e^{-\eta} = 0.61$ , a value consistent with the VAR-based empirical evidence, as argued in [Section 2.1](#).

In our baseline specification, lump-sum transfers  $T_t$  adjust so as to keep the budget balanced, with government consumption and debt fixed at their steady-state level, as we did in [Section 2.2](#). In [Section 5.2](#) we provide results under alternative assumptions, including allowing government expenditure or government debt to adjust in the wake of an unexpected shock.

### 5.1 Impulse Response to a Monetary Shock

[Figure 3\(a\)](#) displays the exogenous time path for the innovation  $\epsilon$  and the implied changes in the liquid interest rate and rate of inflation. [Figure 3\(b\)](#) displays the corresponding impulse responses for aggregate quantities.

In response to an expansionary monetary policy shock, the real return on liquid assets  $r_t^b$  falls, which stimulates consumption and investment, and leads to an increase in both output and inflation. The magnitudes of these responses are, at least qualitatively, consistent with empirical evidence from VARs: consumption increases by less than output and by much less than investment, with an elasticity to the change in  $r^b$  over the first year after the shock equal to  $-2.9$ .<sup>41</sup>

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<sup>41</sup>See e.g. [Figure 1](#) in [Christiano et al. \(2005\)](#). Our model cannot generate hump-shaped impulse responses since we abstract from the modeling ingredients in typical medium-scale DSGE models that

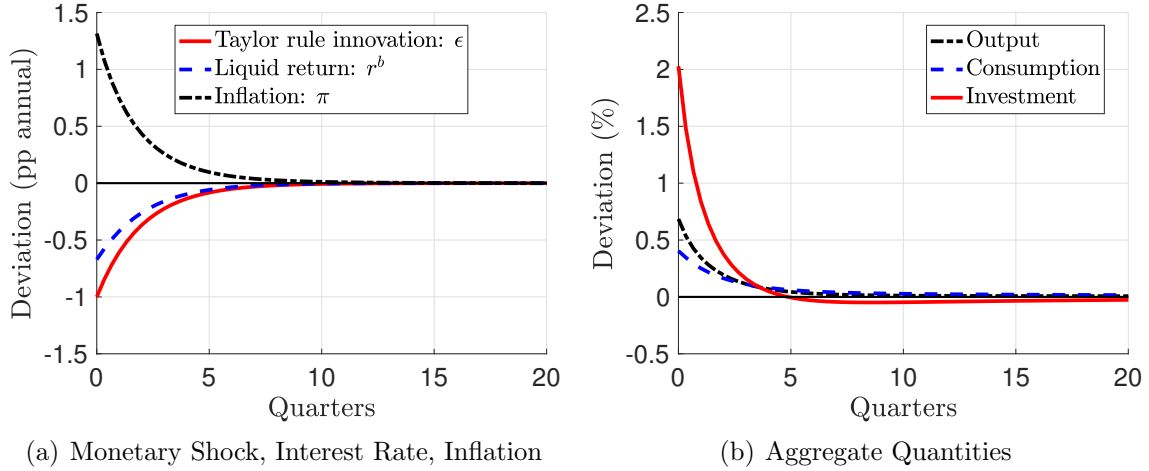


Figure 3: Impulse responses to a monetary policy shock (a surprise, mean-reverting innovation to the Taylor Rule)

How does this magnitude compare to the corresponding response in the RANK models analyzed in Section 2.1? Table 1 shows that, across RANK models, the total elasticity is always around  $-2$ . Thus, in our baseline specification of HANK (with lump-sum transfers adjusting) the elasticity is almost 50 pct higher than in RANK. Notably, this discrepancy implies that the ‘as if’ result of [Werning \(2015\)](#) does not hold in our framework.<sup>42</sup>

In the next section, we decompose this total effect of the monetary shock on aggregate consumption into direct and indirect components through the lens of our methodology developed in Section 3.3.

## 5.2 The Size of Direct and Indirect Effects

The equilibrium time paths for prices and government transfers induced by the monetary shock that we feed into the household problem to compute each element of the direct and indirect effects are displayed in Figure 4(a), alongside the resulting decomposition in Figure 4(b). In the bottom panel of Table 6 we explicitly report the contribution of each component to the overall consumption response over the first year following the shock.<sup>43</sup>

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generate these dynamics, such as external habits and investment-rate adjustment costs.

<sup>42</sup>[Werning \(2015\)](#) studies deviations from his benchmark incomplete markets economy and argues that, in plausible cases, consumption becomes more sensitive than in RANK to current and future interest rate changes, as we find.

<sup>43</sup>In principle, the contribution of the components need not add to 100%, since the exact decomposition holds only for infinitesimal changes in prices, as in Proposition 1 for the stylized model of Section 2. In practice, though, they almost exactly do.

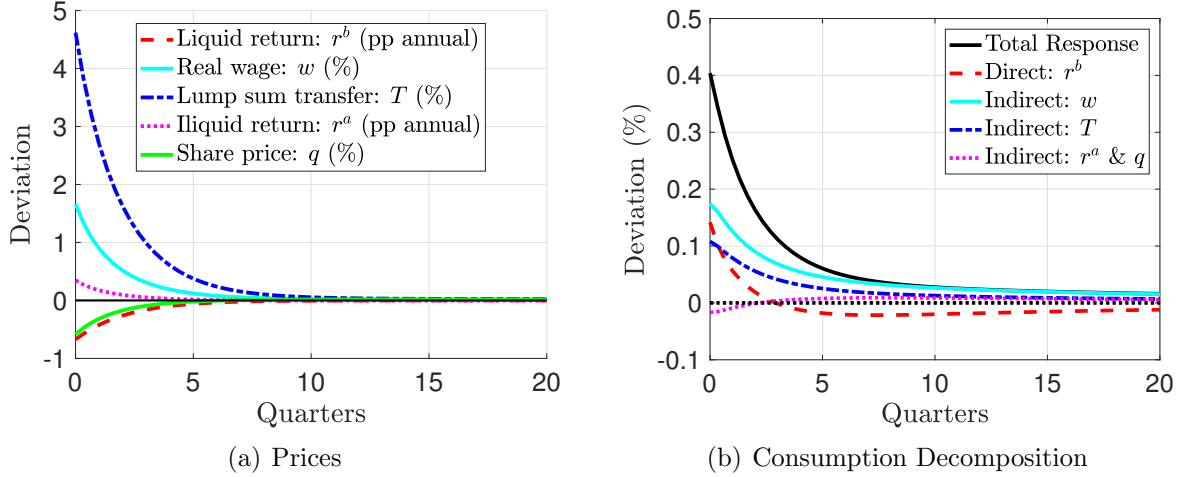


Figure 4: Direct and Indirect Effects of Monetary Policy in HANK

Notes: Returns are shown as annual percentage point deviations from steady state. Real wage and lump sum transfers are shown as log deviations from steady state.

The decomposition reveals the first novel quantitative insight into the monetary transmission mechanism. The combined indirect effects are much larger than the direct effect. In our HANK model, the indirect component account for 80 percent of the consumption response while the direct component accounts for only 20 percent of the response. This is in stark contrast to typical RANK models, as argued in Section 2.

This finding is very robust, as evident from the remaining columns of Table 6 that report analogous results from alternative model specifications. In the baseline model we allocate a fraction  $\omega = \alpha$  of profits to illiquid equity, in order to neutralize the effect of counter-cyclical profits on investment as explained in Section 4.1. Columns (2) and (3) show that this assumption is important for generating pro-cyclical investment and a positive output response. When all profits are allocated to equity in the illiquid account ( $\omega = 1$ ), the fall in profits following the monetary shock substantially dampens the response of investment and thus reduces the total consumption response (line 4 in the table) because with lower investment there is a smaller increase in labor demand, and hence in wages. However, the decomposition between direct and indirect effects is barely affected. When profits are nearly all paid as dividends to households ( $\omega = 0.1$ ), consumption responds very aggressively but the decomposition between direct and indirect effects is virtually unchanged.<sup>44</sup>

Columns (4) and (5) of Table 6 show that the two key parameters that determine the strength of the New Keynesian elements in the model – the Taylor rule coefficient

<sup>44</sup>Within indirect effects, the role of the illiquid return component is now much more important. We come back to the reasons in Section 5.3.2.



|                                    | Baseline<br>(1) | $\omega = 1$<br>(2) | $\omega = 0.1$<br>(3) | $\frac{\varepsilon}{\theta} = 0.2$<br>(4) | $\phi = 2.0$<br>(5) | $\frac{1}{\nu} = 0.5$<br>(6) |
|------------------------------------|-----------------|---------------------|-----------------------|---|---------------------|------------------------------|
| Change in $r^b$ (pp)               | -0.28%          | -0.34%              | -0.16%                | -0.21%                                    | -0.14%              | -0.25%                       |
| Elasticity of $Y$                  | -3.96           | -0.13               | -24.9                 | -4.11                                     | -3.94               | -4.30                        |
| Elasticity of $I$                  | -9.43           | 7.83                | -105                  | -9.47                                     | -9.72               | -9.79                        |
| Elasticity of $C$                  | -2.93           | -2.06               | -6.50                 | -2.96                                     | -3.00               | -2.87                        |
| Partial Eq. Elast. of $C$          | -0.55           | -0.45               | -0.99                 | -0.57                                     | -0.59               | -0.62                        |
| Component of change in $C$ due to: |                 |                     |                       |   |                     |                              |
| Direct effect: $r^b$               | 19%             | 22%                 | 15%                   | 19%                                       | 20%                 | 22%                          |
| Indirect effect: $w$               | 51%             | 56%                 | 51%                   | 51%                                       | 51%                 | 38%                          |
| Indirect effect: $T$               | 32%             | 38%                 | 19%                   | 31%                                       | 31%                 | 45%                          |
| Indirect effect: $r^a$ & $q$       | -2%             | -16%                | 15%                   | -2%                                       | -2%                 | -4%                          |

Table 6: Decomposition of the effect of monetary shock on aggregate consumption

Notes: Average responses over the first year. Column (1) is the baseline specification. In column (2) profits are all reinvested into the illiquid account. In column (3) 10% of profits are reinvested in the illiquid account and 90% of profits are paid as a labor subsidy into the liquid account. In column (4) we reduce the stickiness of prices by lowering the cost of price adjustment  $\theta$ . In column (5) we increase  $\phi$ , which governs the responsiveness of the monetary policy rule to inflation. In column (6) we lower the Frisch elasticity of labor supply from 1 to 0.5.

$\phi$  and the degree of price stickiness  $\theta$  – do not substantially affect either the overall size of the consumption elasticity or its decomposition between direct and indirect effects. Rather, these elements primarily affect the inflation response for a given monetary shock, and hence the extent of movements in the real interest rate.<sup>45</sup>

In column (6) of Table 6, we set the Frisch elasticity to 0.5, half its baseline value. The decomposition between direct and indirect effects is unaffected by this change. The effect of lowering the Frisch elasticity is to shift the composition of the indirect effects away from the wage component towards the transfer component.

In a previous version of this paper (Kaplan et al., 2016a), we showed that replacing the separable preference specification with GHH utility (Greenwood et al., 1988) yields an elasticity of aggregate consumption  $C$  to  $r^b$  that is almost twice as large as in the baseline.<sup>46</sup> Moreover, the indirect general equilibrium effects account for over 90 percent of the total effect. The key feature of GHH utility that drives these results

<sup>45</sup>Table 6 reports results from a more aggressive monetary policy rule and lower price stickiness; a less aggressive policy rule or higher price stickiness have similarly sized opposite effects.

<sup>46</sup>The intra-period utility function is specified as  $\log(c - \psi \frac{h^{1+\nu}}{1+\nu})$ , with  $\nu = 1$  and  $\psi$  set to replicate average hours equal to 1/3 of time endowment.

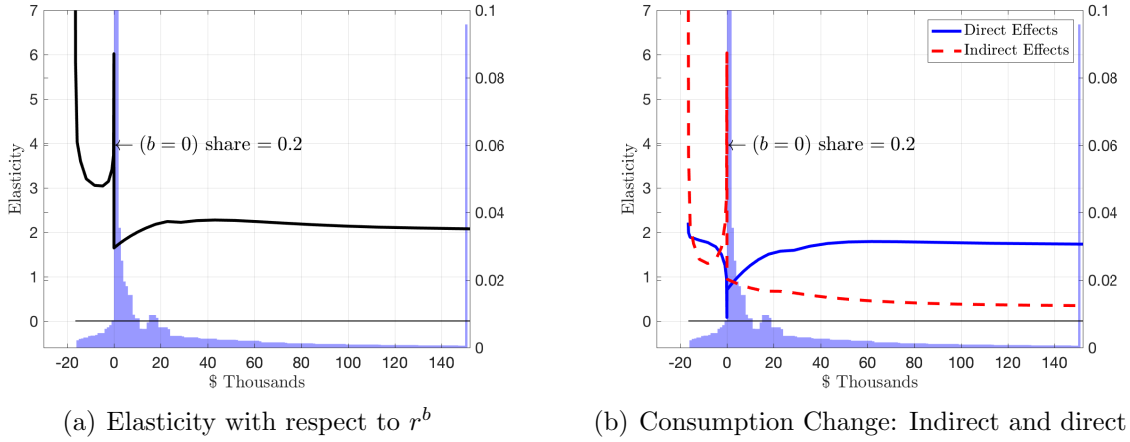


Figure 5: Consumption Responses by Liquid Wealth Position

is the strong complementarity between hours worked and consumption. As aggregate demand, and hence the wage rate, increases, households increase their labor supply. Because of the complementarity, this leads to an increase in desired consumption for all households, even for non hand-to-mouth households with low marginal propensities to consume.

In Tables E.3 and E.4 in Appendix E we report results from a comprehensive robustness analysis of the key parameters that govern behavior in the “heterogeneous agent block” of the model. We show that changes in the tightness of borrowing limits, the cost of borrowing and the adjustment cost function can have large effects on the level of liquid wealth holdings and the fraction of poor hand-to-mouth and wealth hand-to-mouth households. However, in all cases the decompositions of the monetary policy shocks into direct and indirect effects remain essentially unchanged from the findings reported in Table 6.

These experiments (including those done under alternative fiscal adjustment that we discuss below) reveal another, related, robust feature of HANK models. The partial-equilibrium (or direct) elasticity of aggregate consumption —the consumption response to changes in the liquid rate, keeping all other prices and taxes/transfers unchanged— never deviates too much from 0.55, its baseline value, even across configurations where the total elasticity differs greatly.<sup>47</sup> This magnitude is considerably lower than in all the versions of RANK, where the direct elasticity is always above 1.9 (see Table 1).

<sup>47</sup>We here purposely exclude the results in column (3) of Table 6 for  $\omega = 0.1$  where the aggregate partial equilibrium elasticity is 0.99 because these are from an extremely unrealistic parameterization with an investment elasticity of 105 and an output elasticity of 25.

### 5.3 The Distribution of the Monetary Transmission

To better understand why the direct effects of an unexpected reduction in interest rates are small, and the indirect effects are large, in HANK relative to RANK, it is instructive to inspect the consumption response to the monetary policy shock across the entire distribution of liquid wealth holdings.

Figure 5(a) shows the elasticity of average consumption of households with a given liquid wealth level to the change in the interest rate at each point in the liquid wealth distribution (black line, left axis), along with the corresponding consumption shares of each liquid wealth type (light blue histogram, right axis).<sup>48</sup> The distribution of consumption responses features big spikes at the borrowing constraint  $b = \underline{b}$  and at  $b = 0$ . Our model features few households at the borrowing limit, but those with zero liquid wealth account for 20 percent of total consumption and have an elasticity of around 6. Because many of these households have moderate income and own illiquid assets, i.e. they are wealthy hand-to-mouth, their consumption share is much larger than in models where hand-to-mouth are income- and wealth-poor. All other households with positive liquid assets, representing around 80 percent of total consumption expenditures, contribute an elasticity that is around 2.0. A back of the envelope calculation yields  $0.2 \times 6 + 0.8 \times 2 = 2.8$ , which is roughly the overall impact elasticity.

Figure 5(b) separates the total elasticity into the direct and indirect elasticities. These two additive components measure the strength of the direct and general-equilibrium channels of monetary policy. We now examine each of them separately.

#### 5.3.1 Why are Direct Effects Small?

Figure 5(b) reveals that the direct effects are highest for households close to the borrowing constraint, then decline to zero for households with no liquid wealth. As liquid wealth grows, the direct effects also increase until the direct elasticity peaks just below a value of two. After a sufficiently high level of liquid wealth, direct effects start to slowly decline. Most of the population and consumption distribution is between 0 and \$20,000 of liquid wealth, and in that range the direct elasticity is quite small. This explains why the aggregate partial-equilibrium elasticity is only about 0.55 (see column (1) of Table 6).

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<sup>48</sup>Note that the figure reports the elasticity of consumption on impact of the monetary policy shock, in contrast to the numbers in Table 6 which report elasticities over the first year. This is because the impact elasticities are considerably easier to compute. Integrating the elasticities in the figure weighted by the consumption shares yields (the negative of) the overall impact elasticity to the monetary shock, which is  $-2.81$ . The average consumption of households with a given liquid wealth level  $b$  is defined as  $C_t(b) = \int c_t(a, b, z) \mu_t(da, b, dz)$  so that aggregate consumption satisfies  $C_t = \int_b^\infty C_t(b) db$ . Therefore the overall elasticity is a consumption weighted average of the elasticities at each level of liquid wealth.

To better understand these patterns, we further decompose the direct elasticity into a substitution effect and an income effect.<sup>49</sup> Our decomposition is conceptually identical to the one in Auclert (2016), but we build on recent work by Olivi (2017) who substantially generalizes Auclert’s approach to allow for persistent price changes and a more general stochastic process for idiosyncratic risk. We here only briefly lay out the relevant result. The derivations as well as some additional results can be found in Kaplan et al. (2017). The time-zero consumption response  $d \log c_0(a, b, z)$  to small interest rate deviations  $\{dr_t^b\}_{t \geq 0}$  of an individual with asset portfolio  $(a, b)$  and labor income  $z$  can be decomposed into substitution and income effects as:

$$d \log c_0(a, b, z) = \underbrace{-\frac{1}{\gamma} \mathbb{E}_0 \left[ \int_0^\infty e^{-\int_0^t \varrho_s ds} M_t dr_t^b dt \right]}_{\text{substitution effect}} + \underbrace{\mathbb{E}_0 \left[ \int_0^\infty e^{-\int_0^t \varrho_s ds} M_t \left( \frac{\partial_b c_t}{c_t} \right) b_t dr_t^b dt \right]}_{\text{income effect}}$$

where  $\varrho_t := \partial_b c_t + (1 + \chi_d(d_t, a_t)) \partial_b d_t - \partial_a d_t$  and  $M_t := \frac{u'(c_t)}{u'(c_0)} e^{-\int_0^t (\rho - r_s^b) ds}$  (36)

where the expectations are over sample paths of  $(a_t, b_t, z_t)$  starting from  $(a_0, b_0, z_0) = (a, b, z)$ , and where  $\partial_b c_t$  is short-hand notation for  $\partial_b c_t(a_t, b_t, z_t)$ , i.e. the (instantaneous) marginal propensity to consume (MPC) out of liquid wealth, and similarly for  $\partial_b d_t$  and  $\partial_a d_t$ .

The first term in (36) is the substitution effect, which is negative and scales with the intertemporal elasticity of substitution (IES)  $1/\gamma$ . The second term is the income effect, which depends on the time paths of both liquid wealth  $b_t$  and the (instantaneous) MPC  $\partial_b c_t$ . To better understand (36) it is useful to consider the corresponding formula in a one-asset model. In this case the formula remains unchanged, except that deposits are zero ( $d_t \equiv 0$ ) so that the effective discount rate simplifies to  $\varrho_t = \partial_b c_t$ . Because  $\partial_b c_t$  is always weakly higher than  $\rho$ , the MPC in RANK, the substitution effect is a dampened version of its counterpart in RANK.<sup>50</sup>

The first term in (36) is therefore a natural dynamic generalization of Auclert’s expression for the substitution effect in response to a transitory one-period interest rate change which he writes as  $-\text{IES} \times (1 - \text{MPC})$ . It also captures in a transparent fashion the intuition of McKay et al. (2016) that a high likelihood of being constrained in the future is equivalent to a shorter planning horizon.

<sup>49</sup>Or more precisely the combination of different types of income effects, in particular a classic income effect and a wealth/endowment effect.

<sup>50</sup>Specifically, in RANK the substitution effect can be written as  $-\frac{1}{\gamma} \int_0^\infty e^{-\rho t} \mathbb{E}_0(M_t) dr_t^b dt$ . In the one asset model, it can be written as  $-\frac{1}{\gamma} \int_0^\infty \left[ \mathbb{E}_0(e^{-\int_0^t \partial_b c_s ds}) \mathbb{E}_0(M_t) + \text{cov}(e^{-\int_0^t \partial_b c_s ds}, M_t) \right] dr_t^b dt$ . The substitution effect is higher in RANK because  $\partial_b c_t \geq \rho$  and the covariance term is negative by concavity of the consumption function.

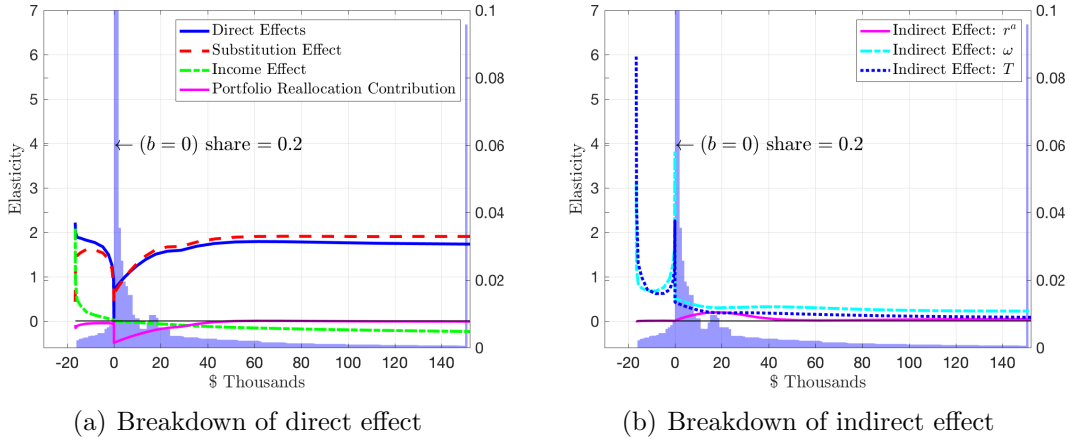


Figure 6: Consumption Responses by Liquid Wealth Position

In our two-asset model, the effective discount rate  $\rho_t$  differs from that in a one-asset model by the term  $(1 + \chi_d(d_t, a_t))\partial_b d_t - \partial_a d_t$ . In our computations this term is always strictly positive and hence the effective discount rate is strictly larger than that in a one-asset model, which further dampens the intertemporal substitution effect. This additional effect is linked to portfolio rebalancing between liquid and illiquid asset. In a one-asset model a fall in the liquid rate is an incentive for households to reduce savings and consume more. In contrast, in a two-asset model, households also have the option to shift funds from their liquid to illiquid accounts. If the gap between illiquid and liquid rates becomes sufficiently large, then households respond to the fall in  $r^b$  by rebalancing their portfolios rather than by increasing their consumption. Intuitively, the fact that individuals may rebalance their portfolios in the future further shortens their effective time horizon. The symmetric logic explains how the income effect, the second term in term in (36) is affected by portfolio reallocation.

To summarize, in the two-asset model an interest rate change induces income and substitution effects, just as in a one-asset model. But the size of both effects differ from those in a one-asset model because the effect discount rate  $\rho$  accounts for future portfolio rebalancing in addition to future MPCs. Under plausible calibrations, the income and substitution effects are larger in one-asset models than in the two-asset model both because of the absence of portfolio rebalancing effect, and because MPCs are lower. We return to this point in Section 5.5.

Figure 6(a) implements the decomposition in (36). The solid blue line plots the direct effect, i.e. the same line as in Figure 5(b). The Figure then breaks down this direct effect into the substitution effect (dashed red line) and the income effect (dash-dotted green line), as defined in (36). Households with zero holdings of liquid

wealth do not substitute intertemporally because they are at a kink in their budget constraint. For households with positive holdings of liquid wealth, the substitution effect gradually increases. It levels off at 1.95 for households with high holdings of liquid wealth, which is the size of the substitution effect for a household who is fully insured against idiosyncratic income risk, as in RANK.

The income effect is a monotonically decreasing function of liquid wealth. It is positive for borrowers since lower interest payments on their debt translate into higher consumption and is negative for lenders with positive liquid wealth.<sup>51</sup> For households with sufficiently high holdings of liquid wealth (outside the range plotted in the graph), the income effect becomes so strong that the direct elasticity becomes negative.

As noted above, the direct response of households with positive, but moderate, amounts of liquid wealth (\$1,000 to \$20,000) is small and this is what accounts for most of the small aggregate direct elasticity. Why do these low MPC households not respond more strongly to the reduction in interest rates? The decomposition in 6(a) suggests that the reason is twofold. First, portfolio rebalancing (solid pink line) has a considerably negative effect exactly for households with positive but moderate amounts of liquid wealth.<sup>52</sup> Second, although these households currently have low MPCs, they face the possibility of having higher MPCs in the future.

**Relationship to Auclert (2016).** As already noted, our decomposition is closely related to the one proposed by Auclert (2016, Theorems 1 and 3). Auclert’s Theorem 1 is analogous to our decomposition and breaks down households’ micro consumption response into income and substitution effects. He refers to the income effect of the interest rate change as “unhedged interest rate exposure.” Auclert’s Theorem 3 then aggregates the micro decomposition into an aggregate decomposition as we do in equation (30). The combination of indirect effects from wages and fiscal policy can be interpreted as his “earnings heterogeneity channel” expanded to include income from the government besides labor income. At the same time there are also some important differences between the two decompositions. First, Auclert emphasizes heterogeneity in asset maturities, whereas our emphasis is on asset liquidity. Second, our model does not feature his Fisher channel because all assets in our model are real. Third, building on Olivi (2017), our decomposition can handle persistent dynamics of the economy.

<sup>51</sup>Di Maggio et al. (2014), Flodén et al. (2016) study borrowers with adjustable rate mortgages who faced changes in monthly interest payments, and find evidence of a positive consumption response to a drop in monthly payments. In addition, Cloyne et al. (2015) offer supporting evidence that the direct channel is small relative to indirect effects occurring through changes in household labor income.

<sup>52</sup>Figure 6(a) isolates the contribution of portfolio reallocation at different points of the liquid wealth distribution. The line plots the difference between the income and substitution effects in the figure and the analogous quantities computed as in a one-asset model (by setting the effective discount rate equal to  $\varrho_t = \partial_b c_t$ ).

This is important because in models like ours and Auclert’s even purely transitory one-time shocks typically lead to endogenous persistence through movements in the wealth distribution, a case to which Auclert’s decomposition does not apply. Finally, his decomposition relies heavily on being able to collapse period budget constraints into a single present-value constraint: as a result, his approach cannot handle binding borrowing limits, a wedge between borrowing and savings rates or illiquid assets – features that are at the heart of our model. For further details see [Kaplan et al. \(2017\)](#).

### 5.3.2 Why are Indirect Effects Large?

Figure 5(b) reveals that the indirect effects are very large for households with zero liquid wealth. The presence of hand-to-mouth households is thus a key determinant of the transmission mechanism of monetary policy on the macroeconomy.

Figure 6(b), which offers a breakdown of the indirect effect among its three components, shows that these households respond sharply to the change in both labor income and government transfers that occur in equilibrium in the wake of a monetary shock. The rise in labor income is a consequence of an expansionary monetary shock that increases demand for final goods. Transfers rise because the interest payments on government debt fall and because the rise in aggregate income increases tax revenues. This mechanism shares similarities with TANK models with government debt where, like in HANK, the presence of non-Ricardian households means that the fiscal response can play an important role in the indirect effects of monetary policy.

Finally, the combined indirect effect due to changes in  $r^a$  and  $q$  is slightly negative, but very small, everywhere in the distribution. Our model inherits the typical feature of the standard New Keynesian model that markups and profits falls in a monetary expansion. Since the stock price  $q$  is the present discounted value of future profits,  $q$  drops as well. A sizable literature examines the response of equity prices to monetary policy shocks and finds positive, but only weakly significant, responses of stock prices to expansionary monetary policy shocks.<sup>53</sup> There are a number of potential strategies for generating procyclical profits, and hence stock prices, in New Keynesian models that would also apply in HANK. Chief among these is the introduction of sticky wages.<sup>54</sup>

While capital gains on equity are always countercyclical, our model is capable of generating both procyclical and countercyclical returns through their dividend component depending on the assumption made on the fraction of profits  $\omega$  reinvested in

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<sup>53</sup>See [Rigobon and Sack \(2004\)](#), [Bernanke and Kuttner \(2005\)](#) and [Gürkaynak et al. \(2005\)](#). In the words of [Rigobon and Sack \(2004\)](#) the literature has been “somewhat inconclusive about the significance of the response of stock prices to monetary policy actions”.

<sup>54</sup>Indeed, this is the route taken by [Challe and Giannitsarou \(2014\)](#) in the context of a RANK model.



illiquid accounts. As shown in columns (2) and (3) of Table 6, this parameter changes the share of the indirect effect due to the illiquid return  $r^a$ , but the direct effects always account for a small fraction of the total consumption response.<sup>55</sup> More clearcut empirical evidence on the response of various asset prices to monetary policy shocks as well as the design of a HANK model that is consistent with this evidence should be a priority for future research.<sup>56</sup>

## 5.4 The Role of the Fiscal Response to a Monetary Shock

We now discuss some important implications of Ricardian non-neutrality in HANK. In Table 7 we report the overall response and decomposition for alternative assumptions about how the government satisfies its intertemporal budget constraint after a monetary shock.

Column (1) contains the baseline case, in which government expenditures and debt are held constant, and transfers adjust in every instant. When, instead, government expenditures adjust, the overall impact of monetary policy on aggregate output is stronger (column (2)). This is because when transfers adjust, only high MPC households increase consumption, and by less than one-for-one with the transfer; when government expenditures adjust, the reduced interest payments on debt translate one-for-one into an increase in aggregate demand, which contributes directly to an increase in output. The elasticity of private consumption is similar to the baseline, but the bulk of the indirect effects are accounted for by higher labor income rather than a combination of labor income and transfers.

In column (3), we let the tax rate adjust. Compared to the case where transfers rise there are offsetting forces: on the one hand, a lower tax rate expands labor supply across the board, whereas the higher transfers have a small negative impact on hours worked; on the other hand, lowering taxes is less redistributive than more generous lump-sum transfers and, thus, spurs a smaller demand for private consumption. Overall, results are similar to the baseline.

The remaining alternative is to let government debt absorb the majority of the fiscal imbalance in the short run. In the economies of columns (1) and (2), a sizable fraction of the overall effect of monetary policy is due to additional government transfers

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<sup>55</sup>Some readers may argue that the the indirect effects due to illiquid returns  $r^a$  should be counted as direct effects because all effects working through changes in asset returns are intimately linked due to arbitrage considerations. Note that even in the case of column (3) in Table 6 where the indirect effects due to  $r^a$  are large, the combined effect due to  $r^a$  and  $r^b$  are still only 30% of the overall effect.

<sup>56</sup>That correctly modeling asset price movements is potentially important is also consistent with a result in [Werning \(2015\)](#) who shows that the consumption response to monetary policy depends on the cyclical of asset prices.



|                                    | $T$ adjusts<br>(1) | $G$ adjusts<br>(2) | $\tau$ adjusts<br>(3) | $B^g$ adjusts<br>(4) |
|------------------------------------|--------------------|--------------------|-----------------------|----------------------|
| Change in $r^b$ (pp)               | -0.28%             | -0.23%             | -0.33%                | -0.34%               |
| Elasticity of $Y$                  | -3.96              | -7.74              | -3.55                 | -2.17                |
| Elasticity of $I$                  | -9.43              | -14.44             | -8.80                 | -5.07                |
| Elasticity of $C$                  | -2.93              | -2.80              | -2.75                 | -1.68                |
| Partial Eq. Elast. of $C$          | -0.55              | -0.60              | -0.56                 | -0.71                |
| Component of Change in $C$ due to: |                    |                    |                       |                      |
| Direct effect: $r^b$               | 19%                | 21%                | 20%                   | 42%                  |
| Indirect effect: $w$               | 51%                | 81%                | 62%                   | 49%                  |
| Indirect effect: $T$               | 32%                | -                  | -                     | 9%                   |
| Indirect effect: $\tau$            | -                  | -                  | 18%                   | -                    |
| Indirect effect: $r^a$ & $q$       | -2%                | -2%                | 0%                    | 0%                   |

Table 7: Importance of Fiscal Response to Monetary Shock

Notes: Average responses over the first year. Column (1) is the baseline specification in which transfers  $T$  adjust to balance the government budget constraint. In column (2) government expenditure  $G$  adjusts, and in column (3) the labor income tax  $\tau$  adjusts. In column (4) government debt adjusts, as described in main text.

or expenditures from reduced debt payments. Without this additional stimulus to aggregate demand, labor income does not increase as much and indirect effects account for a smaller share of the total (60 pct compared to 80 pct in the other two scenarios, but still an order of magnitude larger than in RANK). As a result, when government debt absorbs the slack, the monetary shock has a much smaller impact on the economy, roughly half of the baseline value.<sup>57</sup>

To sum up, our second quantitative insight into the transmission mechanism of monetary policy is that the type of fiscal adjustment following the shock matters for the effectiveness of monetary policy. This result represents another important deviation from RANK and from versions of the HANK model, such as the one developed by [Werning \(2015\)](#), where the overall effectiveness of monetary policy does not depend on liquidity constraints and incomplete markets.

<sup>57</sup>In this experiment, we assume that lump-sum transfers jump by a very small amount on impact and then decay back to their steady state level at a slow exogenous rate. Given the assumed rate of decay, the initial jump is chosen so that the government's budget constraint holds in present value terms. In Column (4) of Table 7 the transfer decays at a quarterly rate of 0.02. We experimented with smaller and bigger decay rates and our main conclusions are unchanged.

## 5.5 The Role of Two Assets and Micro Heterogeneity

At this stage of our analysis, two questions naturally arise: (i) What do we gain from the two-asset version of HANK relative to the one-asset versions that have been studied in the existing literature? and (ii) What do we gain from a realistic model of household heterogeneity relative to the simpler spender-saver structure of TANK models?

### 5.5.1 Two-Asset vs. One-asset HANK Models

In this section we compare our model to a one-asset HANK model. We choose a version of HANK as in [McKay et al. \(2016\)](#) in which all wealth is held as liquid government bonds, and we let transfers adjust to balance the government budget constraint following the monetary shock.<sup>58</sup>

Recall that in our calibrated two-asset HANK model the wealth-to-output ratio was over 3 (Table 2) and the average quarterly MPC out of \$500 was 0.16 (Figure 2). In one-asset HANK models, however, there is a well-known tension between matching the high observed aggregate wealth-to-output ratios and generating a large average MPC. Figure 7, which plots aggregate wealth and the average quarterly MPC out of \$500 in the one-asset HANK model for values of the discount rates  $\rho$  between 2.5% p.a. to 7.5% p.a., illustrates this tension: the one-asset model can generate high average wealth, or a high MPC, but not both simultaneously.<sup>59</sup>

Notwithstanding this failure of the one-asset model, Figure 7(b) plots the direct and total elasticities of aggregate consumption following a monetary policy shock (the analogues for the two-asset model are in Table 6). For low discount rates, in which there is a large amount of liquid wealth in the economy, the direct elasticity becomes negative because of the strong wealth effects that pull down the direct channel. For high discount rates, the direct elasticity becomes larger but is always a small share of the overall elasticity. This is because even though wealth effects are now modest due to the smaller amount of wealth in the economy, there is a larger fraction of hand-to-mouth households and so the intertemporal substitution channel is muted. The total elasticity is hump-shaped with respect to  $\rho$  because of two offsetting forces: as the discount rate increases, MPCs rise (panel (a)) resulting in larger indirect effects; at the same time, the amount of liquid wealth (all of which is government debt) decreases,

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<sup>58</sup>The model can be thought of as the limit of our two-asset model as the capital share  $\alpha$  goes to zero, implying that the marginal product of capital is zero. Because we assume that a fraction  $\omega = \alpha$  of firm profits are reinvested directly into illiquid assets (see Section section 4.1), this also implies that all profits are paid out in liquid form. As a result, the illiquid return  $r_t^a$  goes to zero and all wealth is held as liquid assets.

<sup>59</sup>The rest of the calibration is identical to the one in our baseline model, except for  $\alpha = \omega = 0$ .

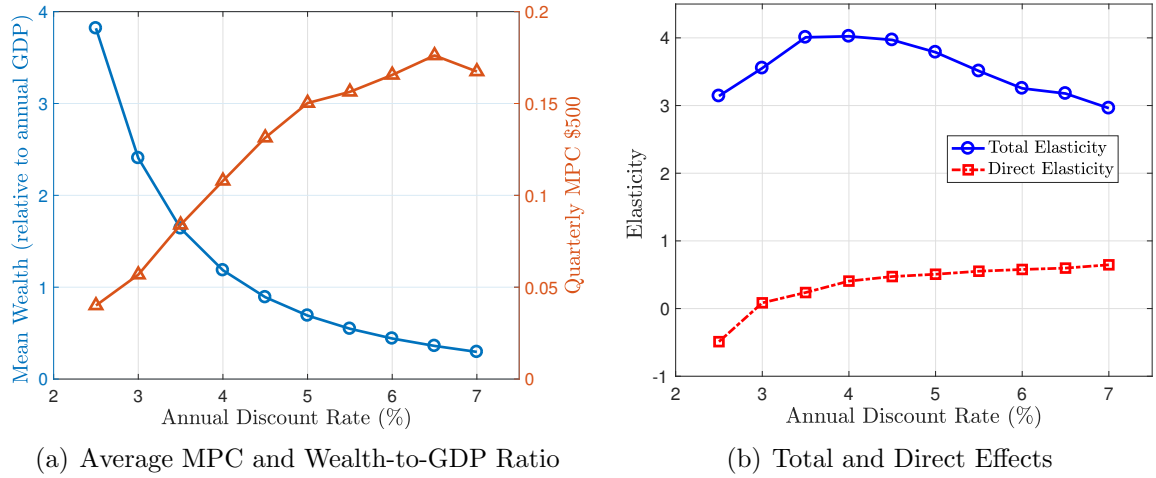


Figure 7: Key Features of One-Asset Model for Different Calibrations

resulting in a weaker fiscal response to the monetary shock, and hence smaller indirect effects.

Perhaps surprisingly, one calibration of the one-asset model replicates many features of our two-asset model closely: with a discount rate of 7%, the average MPC is 0.16, the direct elasticity is just above 0.5, and the overall elasticity is just below 3. The one moment of the data that this calibration misses completely is the total amount of wealth in the economy (0.25 versus 3). But if one interprets wealth as liquid wealth only, this calibration performs well in that dimension (a value of 0.25 just as in Table 2). This calibration can therefore be thought of as the “liquid-wealth-only calibration,” advocated by [Carroll et al. \(2014\)](#).

This result then raises the question of what is to be gained from studying monetary policy through the lens of a two-asset HANK model, rather than a one-asset HANK model calibrated only to liquid wealth? The answer is that this latter model completely abstracts from capital, and the responses of quantity and price of capital greatly matter for the monetary transmission, through the indirect channel. To illustrate this point, columns (2) and (3) of Table 6 report results of our experiments in the two-asset model for alternative values of the parameter  $\omega$  that disciplines what fraction of firm profits get reinvested in illiquid assets (our baseline results in column (1) are for  $\omega = \alpha = 1/3$ ). When  $\omega = 0.1$ , strong procyclical movements in the illiquid rate of return  $r^a$ , stock price  $q$  and investment  $I$  result in the total consumption response more than doubling relative to the baseline. Instead, when  $\omega = 1$ , the illiquid price falls sharply in the wake of a monetary expansion and the total consumption elasticity shrinks to 2/3 of the baseline. While these two alternative calibrations are counterfactual in some dimensions (in particular, with regard to the extreme investment responses),

they illustrate qualitatively an important point: the effects of monetary policy depend strongly on how investment and equity returns move in equilibrium. It is hard to see how the liquid-wealth-only calibration of the one-asset model could ever accommodate these effects.

### 5.5.2 Micro-Heterogeneity vs. Spender-Saver structure

In this section, we compare our model to the TANK models analyzed in Section 2.2. The total consumption elasticity in TANK models with wealth in positive supply (columns (6) and (7) of Table 1) is somewhat smaller than that in HANK, and the share of direct effects is roughly three times larger than in HANK. As a result, the direct elasticity of aggregate consumption in TANK is always around 1.38, which is two and a half times higher than in HANK.

There are three main reasons that account for this discrepancy between the simple TANK model without wealth (column (5) in Table 1) and HANK. The first is a dampening of the substitution effect, even for low MPC households, because the prospect of having a high MPC in the future shortens their planning horizon. The second is income effects. The third is portfolio rebalancing. The fact that this direct elasticity is the same also in the version with wealth (where income effects are present, see column (6)) and in the two-asset version (where portfolio rebalancing is present, see column (7)) suggests that the last two forces are small in TANK. In particular, income effects are negligible in TANK because the MPC of the savers is very small (equal to  $\rho$ , the discount rate).

We conclude that the key reason for why direct effects are small in HANK is the one associated with occasional periods of high MPC behavior, arising from the interaction between the two-asset structure and the wedge between the interest rates on liquid borrowing and liquid savings. This result underscores the importance of modeling heterogeneity through uninsurable earnings shocks as opposed to via built-in differences in preferences.

## 6 Monetary Policy Tradeoffs in HANK

We have thus far emphasized two main results. First, in our HANK model the indirect effects of monetary policy on aggregate consumption far outweigh the direct effects that are dominant in RANK models. Second, the overall response of aggregate consumption to a cut in interest rates may be larger or smaller than in RANK models, depending on a number of factors that are neutral in RANK, in particular the fiscal reaction to the monetary expansion.

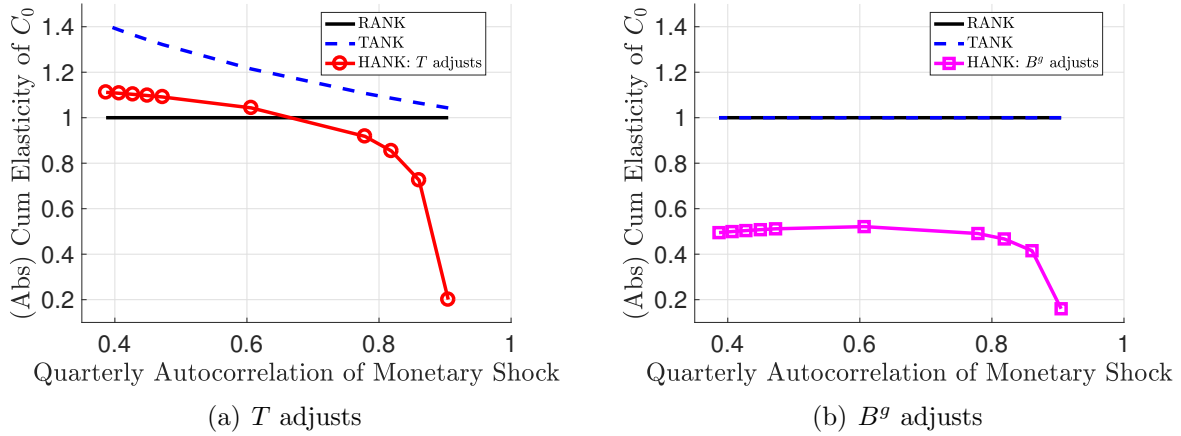


Figure 8: Cumulative Elasticity of Aggregate Consumption by Persistence of the Shock

We now highlight two implications of these differences for some key tradeoffs policymakers face in the conduct of monetary policy. First, we study the choice between sharper but more transitory versus smaller but more persistent interest rate cuts. Second, we analyze the most classical tradeoff of monetary policy: the one between inflation and real activity.

## 6.1 Tradeoff between Size and Persistence of Monetary Shocks

Our aim is to compare, within RANK and HANK models, a transitory drop in the interest rate of a given size and persistence with a smaller but more persistent drop.

Recall, from Section 2, that the aggregate Euler equation in RANK implies:

$$C_0 = \bar{C} \exp \left( -\frac{1}{\gamma} \int_0^\infty (r_s - \rho) ds \right). \quad (37)$$

The integral, which we thereafter denote as  $R_0$ , is the cumulative deviation of the real interest rate from the natural rate  $\rho$ . The elasticity of aggregate consumption at impact with respect to  $R_0$ ,  $-d \log C_0 / dR_0$ , is always equal to  $1/\gamma$  and equal to one under our calibrated value for the IES. Crucially, this *cumulative elasticity* is independent of the particular path of the real rate. More or less persistent paths with the same cumulative deviation  $R_0$  have the same impact on aggregate consumption. Put differently, RANK models feature a neutrality property with respect to the timing of monetary policy and do not feature a size-persistence tradeoff.

This neutrality property of RANK models does not hold in HANK. Figure 8(a) plots the cumulative elasticity of aggregate consumption at impact with respect to different values for the persistence of the innovation  $e^{-\eta}$  for our baseline fiscal policy

scenario in which transfers adjust. Intuitively, persistence is irrelevant for the non hand-to-mouth households, as in RANK, but it does affect the response of hand-to-mouth households. When shocks are persistent, a large portion of the interest rate cut, and the associated relaxation of the government budget constraint, occurs in the future. Hence the hand-to-mouth households receive a smaller increase in transfers upon the impact of the shock, and so their consumption response is weaker. As a result, the cumulative elasticity – which is invariant to persistence in RANK – declines sharply with persistence in HANK. The failure of Ricardian equivalence implies that not only the timing of fiscal policy matters but also that of monetary policy. For comparison, we also plot the cumulative elasticity  $-d \log C_0 / dR_0$  for the simple TANK model of Section 2.2. As in HANK the timing of monetary policy matters. This is again due to the failure of Ricardian equivalence. However, the difference is much smaller and, in contrast to HANK, the consumption response in TANK is always weakly larger than that in RANK.

Figure 8(b) repeats the exercise for the case where government debt adjusts. As explained in the context of Table 7, in this case the consumption response in HANK is considerably diminished because of the lack of transfers accruing to hand-to-mouth households. Even in the absence of transfers, highly persistent interest rate cuts are considerably less potent than sharp but transitory ones. The reasons are related to our discussion of why the direct effects are so much smaller in HANK, compared to RANK (Section 5.3.1). First, households recognize that their MPCs may be larger in the future, which effectively reduces the horizon over which the substitution effects is active. Second, portfolio reallocation is more potent in response to persistent changes in the relative returns (because more durable changes in returns justify paying the transaction cost).

Summarizing, in RANK models, transitory and persistent interest rate cuts are equally powerful as long as the time average of the interest rate deviations is the same. Instead, in HANK a more transitory but sharper interest rate cut is more effective at stimulating aggregate consumption.

## 6.2 Inflation-Activity Tradeoff

In New Keynesian models, any desired increase in aggregate output can be achieved by an appropriate choice of the size of the monetary innovation. A relevant question is about the cost of such monetary stimulus in terms of inflation. That is, the proper conduct of monetary policy requires knowledge of the tradeoff between inflation and real activity.

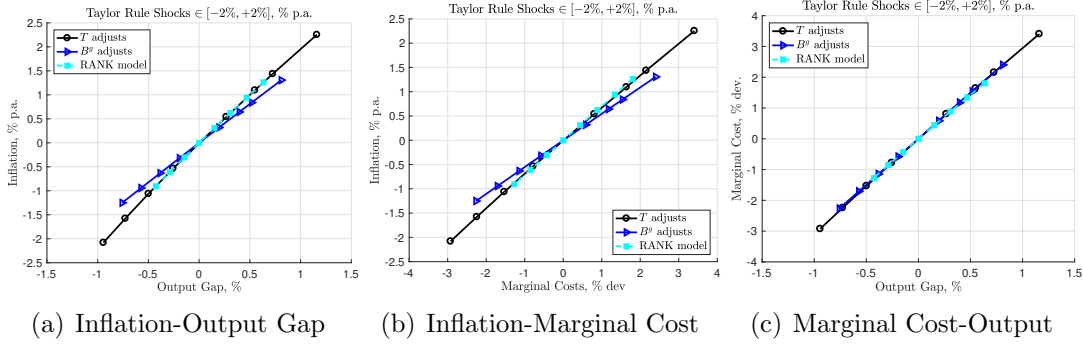


Figure 9: Analysis of Inflation-Activity Tradeoff

Figure 9 graphically examines this tradeoff in RANK and HANK.<sup>60</sup> Panel (a) plots the inflation-output tradeoff, panel (b) the inflation-marginal cost relationship, and panel (c) the marginal cost-output relationship. For each model, we feed monetary policy shocks  $\varepsilon_0$  ranging from  $-2\%$  to  $+2\%$  annually into the Taylor rule.

Let's begin by comparing the T-adjust case in HANK, our baseline, with RANK. The main result, visible from panel (a), is that the inflation-activity tradeoff is similar between RANK and HANK. Panels (b) and (c) illustrate that the way movements in marginal costs induced by policy shocks translate into movements in inflation and output is basically identical across models. The reason is that the inflation-activity relationship is largely determined by the New Keynesian side (summarized by the Phillips curve and Taylor rule), which is the same in RANK and HANK.

However, although the slopes of inflation-activity trade-off is the same across the two models, the length of the lines in panel (a) differs sharply. This is a reflection of the different elasticities of economic activity to the monetary shock in the two models. As explained in Section 5, the elasticity of  $C$  in HANK under the T-adjust case is higher than in RANK. As a consequence, the same expansionary policy shock generates more inflation and a larger output gap in HANK.

An examination of the inflation-activity tradeoff across different types of fiscal adjustments in HANK (T-adjusts vs. B-adjusts) reveals an additional finding. As opposed to RANK, where Ricardian neutrality implies that fiscal adjustment is irrelevant, in HANK it matters for the slope of this tradeoff. Panel (a) implies that a more passive fiscal adjustment rule, where debt absorbs the change in interest payments following the monetary shock, is associated to a more favorable tradeoff, i.e. a flatter line. The reason, which can be seen in panel (b), is the different marginal cost-inflation equilibrium relationship: in the B-adjust case, changes in marginal costs are spread out

<sup>60</sup>For this set of results, we use the richer RANK model outlined in Section 2.3 and in Appendix A.5.

over a longer period of time and, as a result, inflation reacts less at impact.

Finally, note that in our version of HANK the mapping from the output gap to marginal costs is the same as in RANK (see panel (c)). An interesting avenue for future research is to examine whether this is true more generally. In principle, this mapping may depend on the heterogeneity in the economy, which would lead to a different inflation-output tradeoff (panel (a)) between HANK and RANK.

## 7 Conclusion

In our Heterogeneous Agent New Keynesian (HANK) framework, monetary policy affects aggregate consumption primarily through indirect effects that arise from a general equilibrium increase in labor demand. This finding is in stark contrast to Representative Agent New Keynesian (RANK) economies, where intertemporal substitution drives virtually all of the transmission from interest rates to consumption. Moreover, in HANK, the way that fiscal policy responds to an interest rate change profoundly affects the overall effectiveness of monetary policy – a result that is also at odds with the Ricardian nature of standard RANK economies.

These differences between HANK and RANK matter for the conduct of monetary policy. First, they imply different trade-offs in the choice between a large but transitory interest rate change versus a small but persistent change. Second, although the inflation-output gap trade-off is not too different in RANK and HANK (because the New Keynesian block is common across the two models), in HANK the slope of this relationship depends on the type of fiscal adjustment to movements in the nominal rate.

Third, and perhaps most importantly, when direct effects are dominant as in a RANK model, for the monetary authority to boost aggregate consumption it is sufficient to influence real rates: intertemporal substitution ensures that consumption will respond. In contrast, when this direct transmission mechanism is small, as in HANK, the monetary authority must rely on general equilibrium feedbacks that boost household income in order to influence aggregate consumption. Reliance on these indirect channels implies that the overall effect of monetary policy may be more difficult to fine-tune by simply manipulating the nominal rate. The precise functioning of complex institutions, such as labor and financial markets, and the degree of coordination with the fiscal authority play an essential role in mediating the way that the monetary impulse affects aggregate consumption.

Our model’s ability to match the cross-section of household portfolios, wealth distribution, and microeconomic consumption behavior lies at the heart of this set of results.



Nonetheless, the household side of the model could be improved in a number of dimensions. The model lacks a distinction between net and gross positions, which would be necessary to assess the affects of household leverage on monetary transmission. The model also lacks a distinction between real and nominal assets, which is a consequence of all assets in our economy being infinitely short duration. This distinction would be necessary to study re-valuation (or Fischer) effects of monetary policy. Together these two abstractions mean that our model cannot generate a commonly observed household portfolio: illiquid housing assets together with long-term nominal mortgage debt with either fixed or variable nominal coupon payments. Such a balance-sheet configuration brings additional channels of monetary transmission; recent progress in this area has been made, for example, by [Garriga et al. \(2015\)](#) and [Wong \(2016\)](#).

There are several other open areas for the next generation of HANK models to address. First, in our version of HANK, the price of illiquid assets (which can be interpreted as stock and house prices) co-moves slightly negatively with a monetary shock. The empirical evidence on this co-movement is inconclusive, but if anything points to a positive correlation. Getting this co-movement right is important for the size of the portfolio rebalancing behavior that mutes the intertemporal substitution channel in HANK.

Second, we have only studied deterministic transitional dynamics of the economy following one-time monetary shocks. The computational method recently developed by [Ahn et al. \(2017\)](#) will allow future HANK models to directly incorporate aggregate fluctuations into the economic environment.

Third, we have focused on the macroeconomic effects of conventional monetary policy, i.e. shocks to the Taylor rule, in economies that are far from the zero lower bound on nominal interest rates. When the lower bound is binding, the relevant monetary instrument switches from short term rates to forward guidance and asset purchases. Our experiments on monetary shocks with different levels of persistence suggest that in HANK models, forward guidance may be less effective than conventional monetary policy, providing a possible solution to the forward guidance puzzle ([Giannoni et al., 2012](#)). In [Kaplan et al. \(2016b\)](#) we fully articulate this point following the lead of [McKay et al. \(2016\)](#) and [Werning \(2015\)](#). The presence of assets with different degrees of liquidity also makes the framework a natural one to analyze the macroeconomic effects of large-scale asset purchases (quantitative easing).

Finally, in RANK models there is a clear pecking order between monetary and fiscal policies: in economies that are away from the zero lower bound, monetary policy can by itself restore the first-best equilibrium allocation (what [Blanchard and Galí, 2007](#), have termed the “divine coincidence”). An important question that remains

unanswered is the design of optimal policy in HANK economies where the presence of incomplete markets and distributional concerns, in addition to nominal rigidities, breaks such “divine coincidence”.

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# Appendix: For Online Publication

## A Proofs and Additional Details for Section 2

This Appendix spells out in more detail the simple RANK and TANK models in Section 2 and proves the results stated there.

### A.1 Details for Section 2.1

A representative household has preferences over utility from consumption  $C_t$  discounted at rate  $\rho \geq 0$

$$\int_0^\infty e^{-\rho t} U(C_t) dt, \quad U(C) = \frac{C^{1-\gamma}}{1-\gamma}, \quad \gamma > 0. \quad (38)$$

There is a representative firm that produces output using only labor according to the production function  $Y = N$ . Both the wage and final goods price are perfectly rigid and normalized to one. The household commits to supplying any amount of labor demanded at the prevailing wage so that its labor income equals  $Y_t$  in every instant. The household receives (pays) lump-sum government transfers (taxes)  $\{T_t\}_{t \geq 0}$  and can borrow and save in a riskless government bond at rate  $r_t$ . Its initial bond holdings are  $B_0$ . The household's budget constraint in present-value form is

$$\int_0^\infty e^{-\int_0^t r_s ds} C_t dt = \int_0^\infty e^{-\int_0^t r_s ds} (Y_t + T_t) dt + B_0. \quad (39)$$

The government sets the path of taxes/transfers in a way that satisfies its budget constraint

$$\int_0^\infty e^{-\int_0^t r_s ds} T_t dt + B_0 = 0. \quad (40)$$

As described in Section 2, the monetary authority sets an exogenous time path for real rates  $\{r_t\}_{t \geq 0}$ .

An equilibrium in this economy is a time path for income  $\{Y_t\}_{t \geq 0}$  such that (i) the household maximizes (38) subject to (39) taking as given  $\{r_t, Y_t, T_t\}_{t \geq 0}$ , (ii) the government budget constraint (40) holds, and (iii) the goods market clears

$$C_t(\{r_t, Y_t, T_t\}_{t \geq 0}) = Y_t, \quad (41)$$

where  $C_t(\{r_t, Y_t, T_t\}_{t \geq 0})$  is the optimal consumption function for the household.

The overall effect of a change in the path of interest rates on consumption is determined from only two conditions. First, household optimization implies that the time path of consumption satisfies the Euler equation  $\dot{C}_t/C_t = \frac{1}{\gamma}(r_t - \rho)$ . Second, by

assumption, consumption returns back to its steady state level  $C_t \rightarrow \bar{C} = \bar{Y}$  as  $t \rightarrow \infty$ . Therefore, we have

$$C_t = \bar{C} \exp \left( -\frac{1}{\gamma} \int_t^\infty (r_s - \rho) ds \right) \Leftrightarrow d \log C_t = -\frac{1}{\gamma} \int_t^\infty dr_s ds. \quad (42)$$

## A.2 Proof of Proposition 1

The proof covers both the case  $B_0 = 0$  as in Proposition 1 and the case  $B_0 > 0$  as in (7). A key virtue of the simple model we consider is that it admits a closed-form solution for the household's optimal consumption function.

**Lemma A.2** *For any time paths  $\{r_t, Y_t, T_t\}_{t \geq 0}$ , initial consumption is given by*

$$C_0(\{r_t, Y_t, T_t\}_{t \geq 0}) = \frac{1}{\chi} \left( \int_0^\infty e^{-\int_0^t r_s ds} (Y_t + T_t) dt + B_0 \right), \quad (43)$$

$$\chi = \int_0^\infty e^{-\frac{\gamma-1}{\gamma} \int_0^t r_s ds - \frac{1}{\gamma} \rho t} dt. \quad (44)$$

The derivatives of the consumption function evaluated at  $(r_t, Y_t, T_t) = (\rho, \bar{Y}, \bar{T})$  are.<sup>61</sup>

$$\frac{\partial C_0}{\partial r_t} = -\frac{1}{\gamma} \bar{Y} e^{-\rho t} + \rho B_0 e^{-\rho t} \quad \frac{\partial C_0}{\partial Y_t} = \frac{\partial C_0}{\partial T_t} = \rho e^{-\rho t}. \quad (45)$$

**Proof of Lemma A.2** Integrating the Euler equation forward in time, we have

$$\log C_t - \log C_0 = \frac{1}{\gamma} \int_0^t (r_s - \rho) ds \Rightarrow C_t = C_0 \exp \left( \frac{1}{\gamma} \int_0^t (r_s - \rho) ds \right)$$

Substituting into the budget constraint (39):

$$C_0 \int_0^\infty e^{-\int_0^t r_s ds + \frac{1}{\gamma} \int_0^t (r_s - \rho) ds} dt = \int_0^\infty e^{-\int_0^\tau r_s ds} (Y_\tau + T_\tau) d\tau + B_0,$$

or, equivalently, (43) with  $\chi$  defined in (44).

Next, consider the derivatives  $\partial C_0 / \partial r_t$ ,  $\partial C_0 / \partial Y_t$  and  $\partial C_0 / \partial T_t$ . Differentiating  $C_0$  in (43) with respect to  $Y_t$  yields  $\partial C_0 / \partial Y_t = \frac{1}{\chi} e^{-\int_0^t r_s ds}$ . Evaluating at the steady state, we have

$$\frac{\partial C_0}{\partial Y_t} = \rho e^{-\rho t}. \quad (46)$$

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<sup>61</sup>In our continuous-time model the interest rate  $r_t$  and income  $Y_t$  are *functions* of time. Strictly speaking, the consumption function  $C_0(\{r_t, Y_t, T_t\}_{t \geq 0})$  is therefore a functional (i.e. a “function of a function”). The derivatives  $\partial C_0 / \partial r_t$ ,  $\partial C_0 / \partial Y_t$  and  $\partial C_0 / \partial T_t$  are therefore so-called functional derivatives rather than partial derivatives.



The derivative with respect to  $T_t$  is clearly identical.

Next consider  $\partial C_0/\partial r_t$ . Write (43) as

$$\begin{aligned} C_0 &= \frac{1}{\chi} (Y^{PDV} + T^{PDV} + B_0), \\ Y^{PDV} &= \int_0^\infty e^{-\int_0^\tau r_s ds} Y_\tau d\tau, \quad T^{PDV} = \int_0^\infty e^{-\int_0^\tau r_s ds} T_\tau d\tau. \end{aligned} \quad (47)$$

We have

$$\frac{\partial C_0}{\partial r_t} = \frac{1}{\chi} \left( \frac{\partial Y^{PDV}}{\partial r_t} + \frac{\partial T^{PDV}}{\partial r_t} \right) - \frac{1}{\chi^2} \frac{\partial \chi}{\partial r_t} (Y^{PDV} + T^{PDV} + B_0). \quad (48)$$

We calculate the different components in turn. From (47)

$$\frac{\partial Y^{PDV}}{\partial r_t} = \frac{\partial}{\partial r_t} \int_0^\infty e^{-\int_0^\tau r_s ds} Y_\tau d\tau = \frac{\partial}{\partial r_t} \int_t^\infty e^{-\int_0^\tau r_s ds} Y_\tau d\tau \quad (49)$$

where we used that  $e^{-\int_0^\tau r_s ds} Y_\tau$  does not depend on  $r_t$  for  $\tau < t$ . Next, note that for  $\tau > t$

$$\frac{\partial}{\partial r_t} e^{-\int_0^\tau r_s ds} = -e^{-\int_0^\tau r_s ds} \frac{\partial}{\partial r_t} \int_0^\tau r_s ds = -e^{-\int_0^\tau r_s ds}$$

where the second equality uses  $\frac{\partial}{\partial r_t} \int_0^\tau r_s ds = 1$  for  $t < \tau$ . Substituting into (49), we have

$$\frac{\partial Y^{PDV}}{\partial r_t} = - \int_t^\infty e^{-\int_0^\tau r_s ds} Y_\tau d\tau.$$

Similarly

$$\frac{\partial T^{PDV}}{\partial r_t} = - \int_t^\infty e^{-\int_0^\tau r_s ds} T_\tau d\tau, \quad (50)$$

and

$$\frac{\partial \chi}{\partial r_t} = \frac{\partial}{\partial r_t} \int_t^\infty e^{-\frac{\gamma-1}{\gamma} \int_0^\tau r_s ds - \frac{1}{\gamma} \rho \tau} d\tau = -\frac{\gamma-1}{\gamma} \int_t^\infty e^{-\frac{\gamma-1}{\gamma} \int_0^\tau r_s ds - \frac{1}{\gamma} \rho \tau} d\tau.$$

Plugging these into (48)

$$\frac{\partial C_0}{\partial r_t} = -\frac{1}{\chi} \int_t^\infty e^{-\int_0^\tau r_s ds} (Y_\tau + T_\tau) d\tau + \frac{1}{\chi^2} \frac{\gamma-1}{\gamma} \int_t^\infty e^{-\frac{\gamma-1}{\gamma} \int_0^\tau r_s ds - \frac{1}{\gamma} \rho \tau} d\tau (Y^{PDV} + T^{PDV} + B_0).$$

Evaluating at the steady state and using  $\bar{\chi} = 1/\rho$ ,  $Y^{PDV} = \bar{Y}/\rho$ ,  $T^{PDV} = \bar{T}/\rho$  and  $\int_t^\infty e^{-\rho \tau} d\tau = e^{-\rho t}/\rho$ :

$$\frac{\partial C_0}{\partial r_t} = -(\bar{Y} + \bar{T})e^{-\rho t} + \frac{\gamma-1}{\gamma} e^{-\rho t} (\bar{Y} + \bar{T} + \rho B_0). \quad (51)$$

The government budget constraint is  $T^{PDV} + B_0 = 0$ , so that in steady state  $\bar{T} = -\rho B_0$  and hence (51) reduces to the expression in (45).  $\square$

**Conclusion of Proof** Plugging (45) into (6), we have

$$dC_0 = \left( -\frac{1}{\gamma} \bar{Y} + \rho B_0 \right) \int_0^\infty e^{-\rho t} dr_t dt + \rho \int_0^\infty e^{-\rho t} dY_t dt + \rho \int_0^\infty e^{-\rho t} dT_t dt. \quad (52)$$

It remains to characterize  $dY_t$  and  $dT_t$  and to plug in. First, from (42) in equilibrium

$$d \log Y_t = -\frac{1}{\gamma} \int_t^\infty dr_s ds. \quad (53)$$

Next, totally differentiate the government budget constraint

$$\int_0^\infty \frac{\partial}{\partial r_t} \left( \int_0^\infty e^{-\int_0^\tau r_s ds} T_\tau d\tau \right) dr_t dt + \int_0^\infty e^{-\int_0^\tau r_s ds} dT_\tau d\tau = 0.$$

Using (50) and evaluating at the steady state  $-\frac{1}{\rho} \int_0^\infty \bar{T} e^{-\rho t} dr_t dt + \int_0^\infty e^{-\rho t} dT_\tau d\tau$ . Using that  $\bar{T} = -\rho B_0$ ,

$$\int_0^\infty e^{-\rho t} dT_\tau d\tau = -B_0 \int_0^\infty e^{-\rho t} dr_t dt \quad (54)$$

Plugging (53) and (54) into (52), we have

$$d \log C_0 = \underbrace{\left( -\frac{1}{\gamma} + \rho \frac{B_0}{\bar{Y}} \right) \int_0^\infty e^{-\rho t} dr_t dt}_{\text{direct response to } r} - \underbrace{\frac{\rho}{\gamma} \int_0^\infty e^{-\rho t} \int_t^\infty dr_s ds dt}_{\text{indirect effects due to } Y} - \underbrace{\rho \frac{B_0}{\bar{Y}} \int_0^\infty e^{-\rho t} dr_t dt}_{\text{indirect effects due to } T}. \quad (55)$$

Equation (4) in Proposition 1 is the special case with  $B_0 = 0$ .

To see that this decomposition is additive, consider the second term in (55) and integrate by parts:

$$\begin{aligned} \frac{\rho}{\gamma} \int_0^\infty e^{-\rho t} \int_t^\infty dr_s ds dt &= -\frac{\rho}{\gamma} \int_t^\infty e^{-\rho s} ds \int_t^\infty dr_s ds \Big|_0^\infty - \frac{\rho}{\gamma} \int_0^\infty \int_t^\infty e^{-\rho s} ds dr_t dt \\ &= -\frac{\rho}{\gamma} \frac{1}{\rho} e^{-\rho t} \int_t^\infty dr_s ds \Big|_0^\infty - \frac{\rho}{\gamma} \frac{1}{\rho} \int_0^\infty e^{-\rho t} dr_t dt \\ &= \frac{1}{\gamma} \int_0^\infty dr_s ds - \frac{1}{\gamma} \int_0^\infty e^{-\rho t} dr_t dt. \end{aligned}$$

Therefore it is easy to see that the first, second and third terms in (55) sum to  $-\frac{1}{\gamma} \int_0^\infty dr_s ds$ .  $\square$

**Remark:** The fact that second term in (4) scales with  $1/\gamma$  —and therefore the result that with  $B_0 = 0$  the split between direct and indirect effects is independent of  $1/\gamma$ — is an equilibrium outcome. In particular, without imposing equilibrium, the decomposition with  $B_0 = 0$  (4) is

$$d \log C_0 = - \underbrace{\frac{1}{\gamma} \int_0^\infty e^{-\rho t} dr_t dt}_{\text{direct response to } r} + \underbrace{\rho \int_0^\infty e^{-\rho t} d \log Y_t dt}_{\text{GE effects due to } Y}.$$

But in equilibrium  $d \log Y_t = -\frac{1}{\gamma} \int_t^\infty dr_s ds$  which scales with  $1/\gamma$ . Also see footnote 8.

**Derivation of (5):** In the special case (1), we have  $dr_t = e^{-\eta t} dr_0$ . Hence  $\int_0^\infty e^{-\rho t} dr_t dt = \int_0^\infty e^{-(\rho+\eta)t} dt dr_0 = \frac{1}{\rho+\eta} dr_0$ . Similarly  $\int_0^\infty e^{-\rho t} \int_t^\infty dr_s ds dt = \int_0^\infty e^{-\rho t} \int_t^\infty e^{-\eta s} ds dt dr_0 = \frac{1}{\eta} \int_0^\infty e^{-(\rho+\eta)t} dt dr_0 = \frac{1}{\eta} \frac{1}{\rho+\eta} dr_0$ . Plugging these into (4) yields (5).

### A.3 Details for Section 2.2

In the environment described in Section 2.2, aggregate consumption is given by

$$C_t = \Lambda C_t^{sp} + (1 - \Lambda) C_t^{sa}. \quad (56)$$

Savers face the present-value budget constraint

$$\int_0^\infty e^{-\int_0^t r_s ds} C_t^{sa} dt = \int_0^\infty e^{-\int_0^t r_s ds} (Y_t + T_t^{sa}) dt + B_0^{sa},$$

The government budget constraint is

$$\int_0^\infty e^{-\int_0^t r_s ds} (\Lambda T_t^{sp} + (1 - \Lambda) T_t^{sa}) dt + B_0 = 0, \quad (57)$$

where  $B_t$  is government debt. The market clearing condition for government debt is

$$B_t = (1 - \Lambda) B_t^{sa}. \quad (58)$$

We additionally assume that the economy starts at a steady state in which  $C_t^{sp} = C_t^{sa} = \bar{C} = \bar{Y}$  (and hence  $\bar{T}^{sp} = 0$ ). As before, we also assume that the economy ends up in the same steady state (and hence in particular  $T_t^{sp} \rightarrow \bar{T}^{sp} = 0$  as  $t \rightarrow \infty$ ).

We now show how to derive the results of Section 2.2. First, consider the overall effect of interest rate changes on aggregate consumption. As before, the consumption response of savers is given by  $C_t^{sa} = \bar{C} \exp\left(-\frac{1}{\gamma} \int_t^\infty (r_s - \rho) ds\right)$ . From (56) and because

spender consumption equals  $C_t^{sp} = Y_t + T_t^{sp}$ , therefore

$$C_t = \Lambda(Y_t + T_t^{sp}) + (1 - \Lambda)\bar{C} \exp\left(-\frac{1}{\gamma} \int_t^\infty (r_s - \rho) ds\right).$$

Using that in equilibrium  $C_t = Y_t$ :

$$C_t = \frac{\Lambda}{1 - \Lambda} T_t^{sp} + \bar{C} \exp\left(-\frac{1}{\gamma} \int_t^\infty (r_s - \rho) ds\right) \quad (59)$$

We first show how (9) is derived. The government budget constraint (57) can be written in flow terms as  $\dot{B}_t = r_t B_t + \Lambda T_t^{sp} + (1 - \Lambda) T_t^{sa}$ . Under the assumption that the government keeps debt constant at its initial level,  $B_t = B_0$ , we need

$$\Lambda(T_t^{sp} - \bar{T}^{sp}) + (1 - \Lambda)(T_t^{sa} - \bar{T}^{sa}) + (r_t - \rho)B_0 = 0$$

Alternatively, denoting by  $\Lambda^T$  the fraction of income gains that is rebated to spenders and using the assumption that  $\bar{T}^{sp} = 0$ :

$$\Lambda T_t^{sp} = -\Lambda^T (r_t - \rho) B_0$$

Substituting into (59) and totally differentiating

$$d \log C_t = -\frac{\Lambda^T}{1 - \Lambda} \frac{B_0}{\bar{Y}} dr_t - \frac{1}{\gamma} \int_t^\infty dr_s ds \quad (60)$$

Equation (9) is obtained by specializing to the interest rate time path (1). When  $B_0 = 0$ , the total response of aggregate consumption and income in this simple TANK model is therefore identical to that in the RANK version above.

Finally, we show how equation (8) is derived. Because the savers in our TANK model solve the same problem as the representative agent in the RANK model above, their consumption satisfies the analogue of (52):

$$dC_0^{sa} = \left(-\frac{1}{\gamma} \bar{Y} + \rho B_0^{sa}\right) \int_0^\infty e^{-\rho t} dr_t dt + \rho \int_0^\infty e^{-\rho t} dY_t dt + \rho \int_0^\infty e^{-\rho t} dT_t^{sa} dt$$

From (60) and using  $Y_t = C_t$  and (57), their income satisfies  $d \log Y_t = -\Lambda^T \frac{B_0^{sa}}{\bar{Y}} dr_t - \frac{1}{\gamma} \int_t^\infty dr_s ds$ . Since spenders receive a fraction  $\Lambda^T$  of the government's income gains from expansionary monetary policy, savers receive the rest and hence  $(1 - \Lambda) T_t^{sa} = -(1 - \Lambda^T)(r_t - \rho) B_0$  or from (57)  $T_t^{sa} = -(1 - \Lambda^T)(r_t - \rho) B_0^{sa}$  and hence  $dT_t^{sa} =$

$-(1 - \Lambda^T)B_0^{sa}dr_t$ . Therefore

$$\int_0^\infty e^{-\rho t} dT_t^{sa} dt = -(1 - \Lambda^T)B_0^{sa} \int_0^\infty e^{-\rho t} dr_t dt$$

Substituting these expressions into the one for saver consumption:

$$\begin{aligned} d \log C_0^{sa} = & \left( -\frac{1}{\gamma} + \rho \frac{B_0^{sa}}{\bar{Y}} \right) \int_0^\infty e^{-\rho t} dr_t dt - \rho \int_0^\infty e^{-\rho t} \left( \frac{1}{\gamma} \int_t^\infty dr_s ds + \Lambda^T \frac{B_0^{sa}}{\bar{Y}} dr_t \right) dt \\ & - \rho(1 - \Lambda^T) \frac{B_0^{sa}}{\bar{Y}} \int_0^\infty e^{-\rho t} dr_t dt \end{aligned}$$

Next, characterize spenders' consumption response

$$\begin{aligned} \frac{dC_0^{sp}}{C_0} = \frac{dY_0 + dT_0^{sp}}{Y_0} &= -\frac{\Lambda^T}{1 - \Lambda} \frac{B_0}{\bar{Y}} dr_0 - \frac{1}{\gamma} \int_0^\infty dr_t dt - \frac{\Lambda^T}{\Lambda} \frac{B_0}{\bar{Y}} dr_0 \\ &= -\frac{\Lambda^T}{\Lambda(1 - \Lambda)} \frac{B_0}{\bar{Y}} dr_0 - \frac{1}{\gamma} \int_0^\infty dr_t dt \end{aligned}$$

From (56)  $d \log C_0 = (1 - \Lambda) d \log C_0^{sp} + \Lambda \frac{dY_0 + dT_0^{sp}}{Y_0}$ . Therefore, the analogue of Proposition 1 is

$$\begin{aligned} d \log C_0 = & \left( -\frac{1 - \Lambda}{\gamma} + \rho \frac{B_0}{\bar{Y}} \right) \int_0^\infty e^{-\rho t} dr_t dt \\ & - \frac{\rho(1 - \Lambda)}{\gamma} \int_0^\infty e^{-\rho t} \int_t^\infty dr_s ds dt - \rho \Lambda^T \frac{B_0}{\bar{Y}} \int_0^\infty e^{-\rho t} dr_t dt - \frac{\Lambda \Lambda^T}{1 - \Lambda} \frac{B_0}{\bar{Y}} dr_0 \quad (61) \\ & - \rho(1 - \Lambda^T) \frac{B_0}{\bar{Y}} \int_0^\infty e^{-\rho t} dr_t dt - \frac{\Lambda}{\gamma} \int_0^\infty dr_t dt - \Lambda^T \frac{B_0}{\bar{Y}} dr_0 \end{aligned}$$

The first line is the direct response to  $r$ , the second line are indirect effects due to  $Y$ , and the third line are indirect effects due to  $T$ . An instructive special case is the one without government debt,  $B_t = 0$  for all  $t$ . In that case

$$d \log C_0 = \underbrace{-\frac{1 - \Lambda}{\gamma} \int_0^\infty e^{-\rho t} dr_t dt}_{\text{direct response to } r} - \underbrace{\frac{\rho(1 - \Lambda)}{\gamma} \int_0^\infty e^{-\rho t} \int_t^\infty dr_s ds dt - \frac{\Lambda}{\gamma} \int_t^\infty dr_s ds}_{\text{indirect effects due to } Y}.$$

Equation (8) then follows from the fact that in the special case (1),  $dr_t = e^{-\eta t} dr_0$ .

For completeness, we also derive the split between direct and indirect effects for our analytic example  $dr_t = e^{-\eta t} dr_0$  in the case with both hand-to-mouth agents  $\Lambda > 0$  and government debt  $B_0 > 0$ . Collecting some of the indirect effects on the second and

third lines of (61) and specializing to  $dr_t = e^{-\eta t} dr_0$ , we have

$$-\frac{d \log C_0}{dr_0} = \frac{1}{\gamma \eta} \left[ \underbrace{\frac{\eta}{\rho + \eta} \left( 1 - \Lambda - \rho \gamma \frac{B_0}{\bar{Y}} \right)}_{\text{direct response to } r} + (1 - \Lambda) \frac{\rho}{\rho + \eta} + \Lambda + \frac{\eta}{\rho + \eta} \rho \gamma \frac{B_0}{\bar{Y}} \right] + \frac{\Lambda^T}{1 - \Lambda} \frac{B_0}{\bar{Y}}. \quad (62)$$

#### A.4 Details on Medium-Scale DSGE Model (Section 2.3)

The Smets-Wouters model is a typical medium-scale DSGE RANK model with a variety of shocks and frictions. The introduction of Smets and Wouters (2007) provides a useful overview and a detailed description of the model can be found in the paper’s online Appendix.<sup>62</sup> We here only outline the ingredients of the model that are important for the purpose of our decomposition exercise (reported in Table 1) as well as some details on the implementation of this exercise.

An important difference relative to the stylized model of Section 2.1 is that the representative household’s utility function features external habit formation:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{1}{1 - \sigma_c} (C_t(j) - h C_{t-1})^{1 - \sigma_c} \exp \left( \frac{\sigma_c - 1}{1 + \sigma_l} L_t(j)^{1 + \sigma_l} \right) \quad (63)$$

where  $C_t(j)$  is consumption of one of a continuum of individual households and  $C_t$  is aggregate consumption (in equilibrium the two are equal). The parameter  $h \in [0, 1]$  disciplines the degree of external habit formation. As mentioned in the main text, the model also features investment with investment adjustment costs and capital utilization, as well as partially sticky prices and wages.

Our starting point for the decomposition are the impulse response functions (IRFs) to an expansionary monetary policy shock in a log-linearized, estimated version of the model. We set each of the model’s parameters to the mode of the corresponding posterior distribution (see Table 1 in Smets and Wouters (2007) for the parameter values). The IRFs are computed in Dynare using an updated version of the replication file of the published paper.<sup>63</sup> For our purposes, the relevant IRFs are the sequences  $\{C_t, R_t, Y_t, I_t, G_t, UC_t, L_t\}_{t=0}^{\infty}$  for consumption  $C_t$ , interest rates  $R_t$ , labor income  $Y_t$ , investment  $I_t$ , government spending  $G_t$ , capital utilization costs  $UC_t = a(Z_t)K_{t-1}$  and labour supply  $L_t$ . We further denote consumption at the initial steady state by  $\bar{C}$ .

Given these IRFs, we decompose the overall consumption response to an expansionary monetary policy shock into direct and indirect effects as follows. Suppressing

<sup>62</sup> Available at [https://www.aeaweb.org/aer/data/june07/20041254\\_app.pdf](https://www.aeaweb.org/aer/data/june07/20041254_app.pdf)

<sup>63</sup> Available at <http://www.dynare.org/phpBB3/viewtopic.php?f=1&t=3750>.

$j$ -indices for individual households, the budget constraint of households is

$$C_t + \frac{B_t}{R_t P_t} + T_t \leq \frac{B_{t-1}}{P_t} + M_t \quad (64)$$

$$M_t = \frac{W_t^h L_t}{P_t} + \frac{R_t^h K_{t-1} Z_t}{P_t} - a(Z_t) K_{t-1} + \frac{Div_t}{P_t} + \frac{\Pi_t}{P_t} - I_t \quad (65)$$

where the reader should refer to the online Appendix of [Smets and Wouters \(2007\)](#) for an explanation of each term (the budget constraint is their equation (9)).<sup>64</sup> In present-value form

$$\sum_{t=0}^{\infty} \frac{1}{\Pi_{k=0}^{t-1} \tilde{R}_k} C_t = \sum_{t=0}^{\infty} \frac{1}{\Pi_{k=0}^{t-1} \tilde{R}_k} (M_t - T_t)$$

where  $\tilde{R}_t = \frac{R_t}{\Pi_t}$  denotes the real interest rate. Households maximize (63) subject to this budget constraint. For any price sequences, initial consumption  $C_0$  then satisfies:

$$\begin{aligned} C_0 &= \frac{1}{\chi} \left( X + \frac{B_{-1}}{P_0} + \sum_{t=0}^{\infty} \frac{1}{\Pi_{k=0}^{t-1} \tilde{R}_k} (M_t + T_t) \right) \\ \chi &= \sum_{t=0}^{\infty} \frac{1}{\Pi_{k=1}^{t-1} \tilde{R}_k} \left( \sum_{k=0}^t x_{t-k} \left( \frac{h}{g} \right)^k \right) \\ X &= \sum_{t=0}^{\infty} \frac{1}{\Pi_{k=1}^{t-1} \tilde{R}_k} \sum_{k=0}^{t-1} x_{t-k} \left( \frac{h}{g} \right)^{k+1} \bar{C} \\ x_s &= \left( \bar{\beta}^s \Pi_{k=0}^{s-1} \tilde{R}_k \right)^{1/\sigma_c} \exp \left( \frac{\sigma_c - 1}{\sigma_c (1 + \sigma_l)} (L_s - L_0) \right) \end{aligned} \quad (66)$$

where  $\bar{\beta} = \frac{\beta}{g^{\sigma_c}}$  and  $g$  is the gross growth rate of the economy. The direct effect of consumption to interest rate changes is then computed from (66) by feeding in the equilibrium sequence of real interest rates  $\{\tilde{R}_t\}_{t=0}^{\infty}$  while holding  $\{M_t, T_t, L_t\}_{t=0}^{\infty}$  at their steady state values. When computing this direct effect in practice, we simplify the right-hand side of (66) further taking advantage of the fact that most terms are independent of the sequence of real interest rates  $\{\tilde{R}_t\}_{t=0}^{\infty}$ . In particular, in equilibrium, profits and labor union dividends are  $\Pi_t = P_t Y_t - W_t L_t - R_t^h Z_t K_{t-1}$  and  $Div_t = (W_t - W_t^h) L_t$  and therefore, substituting into (65)

$$M_t = Y_t - a(Z_t) K_{t-1} - I_t. \quad (67)$$

---

<sup>64</sup>Note that Smets and Wouters' budget constraint features some typos: it does not include dividends from firm ownership  $\Pi_t$  and there is a "minus" in front of  $T_t$  suggesting it is a transfer even though it enters as a tax in the government budget constraint (equation (24) in their online Appendix).

Further, the government budget constraint in present-value form is

$$\sum_{t=0}^{\infty} \frac{1}{\prod_{k=0}^t \tilde{R}_k} T_t = \sum_{t=0}^{\infty} \frac{1}{\prod_{k=0}^t \tilde{R}_k} G_t. \quad (68)$$

Substituting (67) and (68) into (66), we have

$$C_0 = \frac{1}{\chi} (X + Y^{PDV} - I^{PDV} - G^{PDV} - UC^{PDV}) \quad (69)$$

where  $Y^{PDV}, I^{PDV}, G^{PDV}$  and  $UC^{PDV}$  are the present values of  $\{Y_t, I_t, G_t, UC_t\}_{t=0}^{\infty}$  discounted at  $\{\tilde{R}_t\}_{t=0}^{\infty}$ .

Note that although the series  $\{C_t, \tilde{R}_t, Y_t, I_t, G_t, UC_t, L_t\}_{t=0}^{\infty}$  are generated using a log-linearized approximation around the trend, our decomposition uses a non-linear solution. In particular, both the overall and direct elasticities of consumption to interest rate changes in Table 1 are computed using the exact non-linear Euler equation but evaluated at the equilibrium prices from the linearized models – see the formula (69). The fraction due to direct effects is the ratio of this direct elasticity to the overall elasticity, with both numerator and denominator computed in this non-linear fashion. In our baseline exercise in Table 1 this fraction equals 99 percent. For small shocks the overall elasticity of consumption computed with the exact formula is very close to the elasticity computed using the linearized output from Dynare. For larger shocks, the two can differ somewhat. We have also recomputed the share of direct effects as the ratio of the direct elasticity computed in a non-linear fashion and the overall elasticity computed in a linear fashion. For the baseline exercise in Table 1, this yields a share of direct effects of 91%.

As already stated in the main text, our main result is that – at the estimated parameter values of Smets and Wouters (2007) – the direct effect amounts for 99 percent of the total response of initial consumption to an expansionary monetary policy shock. We have conducted a number of robustness checks with respect to various parameter values, and in particular with respect to the habit formation parameter  $h$ . The results are robust. In the case without habit formation  $h = 0$ , 95.1 percent of the overall effect are due to direct intertemporal substitution effects. Finally, note that a difference between (63) and the specification of preferences in textbook versions of the New Keynesian model is the non-separability between consumption and labor supply. We have conducted an analogous decomposition exercise with a separable version of (63). The decomposition is hardly affected.



## A.5 Details on the Two-Asset RANK and TANK Models

### A.5.1 Model

We begin by outlining the two-agent, spender-saver version of the model (TANK). The representative agent is a special case with the fraction of spenders equal to zero. The model is written and solved in discrete time.

**Households.** A fraction  $\Lambda$  of households are spenders indexed by “sp” and a fraction  $1 - \Lambda$  are savers indexed by “sa”.

*Savers.* Savers derive utility from consuming  $c_t^{sa}$  and have disutility from supplying labor  $\ell_t^{sa}$ . Savers are able to borrow and save in a liquid government bond at rate  $r_t^b$ . They also have access to an illiquid asset  $a_t$  with rate of return  $r_t^a$ . Assets of type  $a$  are illiquid in the sense that households need to pay a cost for depositing into or withdrawing from their illiquid account. Let  $d_t$  denote the deposit decision and  $\chi(d_t)$  the cost of depositing  $d_t$ . The saver’s problem in its sequential formulation is therefore given by

$$\max_{\{c_t^{sa}, \ell_t^{sa}, d_t, b_{t+1}, a_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t u(c_t^{sa}, \ell_t^{sa})$$

$$\text{S.t.} \quad c_t^{sa} + b_{t+1} + d_t + \chi(d_t, a_t) = (1 - \tau)(w_t \ell_t^{sa} + \Gamma_t^{sa}) + T_t^{sa} + (1 + r_t^b) b_t \quad (\lambda)$$

$$a_{t+1} = (1 + r_t^a) a_t + d_t \quad (\eta)$$

where

$$u(c, \ell) = \log c - \varphi \frac{\ell^{1+\nu}}{1+\nu}$$

$$\chi(d) = \chi_1 |d|^{\chi_2}, \quad \chi_1 > 0, \chi_2 > 1.$$

The first-order conditions for the consumer's problem can be written as

$$1 = \left\{ \lambda_{t,t+1} (1 + r_{t+1}^b) \right\} \quad (70)$$

$$\eta_t = \left\{ \lambda_{t,t+1} \left[ \eta_{t+1} (1 + r_{t+1}^a) \right] \right\} \quad (71)$$

$$\eta_t = 1 + \text{sign}(d_t) \times \tilde{\chi} |d_t|^{\chi_2 - 1}, \quad \tilde{\chi} = \chi_1 \chi_2 \quad (72)$$

$$\varphi(\ell_t^{sa})^\nu c_t^{sa} = (1 - \tau) w_t \quad (73)$$

where

$$\lambda_{t,t+1} := \frac{\lambda_{t+1}}{\lambda_t} = \beta \left( \frac{c_{t+1}^{sa}}{c_t^{sa}} \right)^{-1}. \quad (74)$$

Note that, by combining (70) and (71), one obtains that in steady state  $r_b = r_a$ .

*Spenders.* Spenders are hand-to-mouth, i.e. consume their labor income every period. Their only margin of adjustment is labor supply  $\ell^{sp}$ . The spender's problem is

$$\begin{aligned} \max_{c_t^{sp}, \ell_t^{sp}} \quad & u(c_t^{sp}, \ell_t^{sp}) \quad \text{s.t.} \\ c_t^{sp} = & (1 - \tau)(w_t \ell_t^{sp} + \Gamma_t^{sp}) + T_t^{sp} \end{aligned}$$

with first-order conditions

$$c_t^{sp} = (1 - \tau)(w_t \ell_t^{sp} + \Gamma_t^{sp}) + T_t^{sp} \quad (75)$$

$$w_t = \frac{\varphi}{1 - \tau} (\ell_t^{sp})^\nu c_t^{sp}. \quad (76)$$

**Firms.** There is a continuum of intermediate-goods monopolistic firms, each producing a variety  $j$  using a constant returns to scale production function

$$y_t(j) = k_t(j)^\alpha n_t(j)^{1-\alpha}. \quad (77)$$

Each intermediate producer chooses its price  $p_t(j)$  and inputs  $k_t(j), n_t(j)$  to maximize

$$\frac{p_t(j)}{P_t} y_t(j) - w_t n_t(j) - r_t^k k_t(j) - \Theta \left( \frac{p_t(j)}{p_{t-1}(j)} \right) \quad (78)$$

taking into account that the demand for its product depends on the price  $p_t(j)$  charged. The function  $\Theta(\cdot)$  is a quadratic adjustment cost for the price change and is expressed

as a fraction of final good output  $Y_t$

$$\Theta_t \left( \frac{p_t}{p_{t-1}} \right) = \frac{\theta}{2} \left( \frac{p_t}{p_{t-1}} - 1 \right)^2 Y_t.$$

We divide the problem of the firm in two parts. First, the cost minimization problem of producing  $y$  units of variety  $j$  delivers the following optimality conditions

$$w_t = (1 - \alpha) m_t \frac{y}{n_t(j)} \quad (79)$$

$$r_t^k = \alpha m_t \frac{y}{k_t(j)} \quad (80)$$

where marginal cost  $m_t$  is the same across firms

$$m_t = \left( \frac{r_t^k}{\alpha} \right)^\alpha \left( \frac{w_t}{1 - \alpha} \right)^{1 - \alpha}.$$

Since all firms face the same marginal cost, we drop the  $j$  subscript from now onwards. Taking cost minimization decisions as given, each intermediate producer chooses  $\{p_t\}_{t=0}^\infty$  to maximize discounted profits

$$\max_{\{p_t\}_{t=0}^\infty} \sum_{t=0}^\infty (\lambda_t \eta_t) \left\{ \left( \frac{p_t}{P_t} - m_t \right) y_t - \Theta_t \left( \frac{p_t}{p_{t-1}} \right) \right\} \quad (81)$$

$$\text{s.t. } y_t = \left( \frac{p_t}{P_t} \right)^{-\varepsilon} Y_t \quad (82)$$

where the discount factor used by the firm reflects that dividends will accrue to the illiquid account of *savers*. In a symmetric equilibrium, all firms will choose the same price, which will be also the aggregate price  $P_t$ . That gives rise to the following Phillips curve relating aggregate inflation  $\pi_t = \frac{P_t}{P_{t-1}} - 1$  and marginal costs

$$1 - \theta \pi_t (1 + \pi_t) + \theta \left( \frac{\lambda_{t+1} \eta_{t+1}}{\lambda_t \eta_t} \right) \pi_{t+1} (1 + \pi_{t+1}) \frac{Y_{t+1}}{Y_t} = (1 - m_t) \varepsilon. \quad (83)$$

Note that, from equation (71), effectively firm discount at rate  $r_a$ , which is also the discounting that appears in the Phillips curve, exactly as in our HANK model.

Moreover, since in equilibrium all firms choose the same price, they all produce the same quantity and hire the same amount of input on factor markets. Hence we can

aggregate production function of each firm to get

$$y_t(j) = k_t(j)^\alpha n_t(j)^{1-\alpha} \Rightarrow Y_t = K_t^\alpha N_t^{1-\alpha}.$$

Finally, profits are then given by

$$\Pi_t = Y_t \left( (1 - m_t) - \frac{\theta}{2} \pi^2 \right). \quad (84)$$

**Illiquid Assets.** As in HANK, illiquid assets  $a_t$  consist of both capital holdings ( $k_t^{sa}$ ) and equity claims ( $s_t$ ) to a fraction  $\omega$  of profits. Since there is no aggregate uncertainty, no arbitrage dictates that the return to capital must be equal to the return on equity. We denote this return by  $r_t^a$

$$r_t^a \equiv \frac{\omega \Pi_t + (q_t - q_{t-1})}{q_{t-1}} = r_t^k - \delta \quad (85)$$

which restricts how asset prices  $q_t$  evolves over time:

$$q_t = \frac{1}{1 + r_{t+1}^a} (\omega \Pi_{t+1} + q_{t+1}).$$

In the event of an unexpected shock, however, the realized returns between capital and shares *do not need to be equalized* at the moment of impact. What no-arbitrage pricing requires, in such a circumstance, is for the stock price to jump so as to make the return from holding shares the same as the return from holding capital from that period onwards. Realized returns at impact, though, need not to be equalized, since asset positions are pre-determined. Hence, it is useful to write the law of motion of illiquid assets by keeping track of portfolio composition

$$\begin{aligned} a_{t+1} &\equiv k_{t+1}^{sa} + s_{t+1} q_t \\ &= (1 + r_t^a) a_t + d_t \\ &= (1 + r_t^k - \delta) k_t^{sa} + s_t (\omega \Pi_t + q_t) + d_t. \end{aligned} \quad (86)$$

By combining (85) and (86), it is easy to see that, in steady-state, savers withdraw from the illiquid account an amount  $d = -r^a a$ .

As in HANK, the remaining fraction  $(1 - \omega)$  of profits are distributed to households (both spenders and savers) as a direct transfer  $\Gamma_t$  to agents liquid budget constraint (since there is no difference in productivity between the two groups, they are distributed

lump-sum):

$$\Gamma_t = (1 - \omega)\Pi_t, \quad \Gamma_t^{sp} = \Gamma_t^{sa} = \Gamma_t. \quad (87)$$

We set  $\omega = \alpha$  so as to neutralize the role of countercyclical profits as explained in the main text.

**Government.** The government issues bonds denoted by  $B^g$ , with the convention that negative values denote government debt. Its budget constraint is therefore given by

$$B_{t+1}^g = (1 + r_t^b)B_t^g + \tau(w_t N_t + \Gamma_t) - T_t - G_t \quad (88)$$

with government transfers  $T_t$  given by

$$T_t = \Lambda T_t^{sp} - (1 - \Lambda)T_t^{sa} \quad (89)$$

$$\Lambda^T T_t = \Lambda T_t^{sp} \quad (90)$$

Note that we allow for  $\Lambda^T \neq \Lambda$ , i.e. spenders may receive bigger or smaller share of transfers than their population share.

**Monetary authority.** Monetary policy follows a Taylor rule for the nominal interest rate

$$i_t = \bar{r}_t^b + \phi\pi_t + \epsilon_t, \quad \epsilon_{t+1} = \rho\epsilon_t + u_{t+1}, \quad u_{t+1} \sim \mathcal{N}(0, \sigma_\epsilon^2) \quad (91)$$

Given inflation and the nominal interest rate, the real return realized on liquid assets hold by savers is given by

$$1 + r_t^b = \frac{1 + i_{t-1}}{1 + \pi_t}. \quad (92)$$

**Equilibrium.** To close the model, we state market clearing conditions

$$0 = (1 - \Lambda)b_{t+1}^{sa} + B_{t+1}^g \quad (93)$$

$$1 = (1 - \Lambda)s_{t+1} \quad (94)$$

$$K_{t+1} = (1 - \Lambda)k_{t+1}^{sa} \quad (95)$$

$$I_t = K_{t+1} - (1 - \delta)K_t \quad (96)$$

$$Y_t = C_t + I_t + G_t + (1 - \Lambda)\chi_t + \Theta_t \quad (97)$$

where

$$C_t = \Lambda c_t^{sp} + (1 - \Lambda) c_t^{sa} \quad (98)$$

$$N_t = \Lambda \ell_t^{sp} + (1 - \Lambda) \ell_t^{sa}. \quad (99)$$

### A.5.2 Parameterization

Some parameter values are set exactly as in the simple TANK models of Section 2. In particular, we set  $\Lambda = \Lambda^T = 0.3$ , risk aversion  $\gamma$  to 1, and the discount rate to 5% annually. In this model, the liquid real rate also equals 5% in steady state.

Other parameters (Frisch elasticity, transaction cost, demand elasticity, price adjustment cost, share of profits paid as dividends, government policy parameters, and Taylor rule coefficient) are set as in HANK. Only two parameters are calibrated internally. The disutility of labor is set so that on average 1/3 of the time endowment is spent working, and the depreciation rate is set to match the same illiquid wealth to GDP ratio as in HANK (13, quarterly). Table A.8 summarizes the parameterization.

### A.5.3 Simulations and decompositions

We use Dynare to solve for the model's steady state, its transitional dynamics and the decompositions. The model is solved globally (i.e. without local linearization) from the equilibrium system of nonlinear equations. We always analyze monetary shocks of the same size as in HANK, i.e., 25 basis points, with a quarterly persistence of 0.5. It is worth emphasizing that  $\eta = 0.5$  is the correct choice also for discrete time, if we wish to compare across models. To see this, consider that the cumulative deviation of the interest rate path from  $t = 0$  is  $\int_0^\infty (r_s - \rho) ds = \int_0^\infty \exp(-\eta s) ds = 1/\eta$ . In discrete time the cumulative deviation is  $\sum_{t=0}^\infty \rho^t = 1/(1 - \rho)$ . Thus, a proper comparison with a continuous time model where  $\eta = 0.5$  requires setting  $\rho = 0.5$ .

Figure A.10 reports the IRFs and decomposition in RANK and TANK for the baseline fiscal policy scenario (T-adjust case). It is the counterpart of Figure 4 for HANK. The elasticities and share of direct effects for the two-asset RANK and TANK reported in Table 1 are obtained from these experiments.

Table A.8: Parameter values (period length is one quarter)

|  |               |               |
|--|---------------|---------------|
| <i>Preferences</i>                       |               |               |
| Fraction of HtM                          | $\Lambda$     | 0.30          |
| Discount factor                          | $\beta$       | $1.0125^{-1}$ |
| Risk aversion                            | $\gamma$      | 1.00          |
| Disutility of labor                      | $\varphi$     |               |
| Frisch elasticity                        | $1/\nu$       | 1.00          |
| <i>Deposit costs</i>                     |               |               |
| Scaling                                  | $\chi_1$      | 0.956         |
| Curvature                                | $\chi_2$      | 1.402         |
| <i>Production</i>                        |               |               |
| Demand elasticity                        | $\varepsilon$ | 10            |
| Capital share                            | $\alpha$      | 0.33          |
| Depreciation rate                        | $\delta$      | 0.017         |
| Price adjustment cost                    | $\theta$      | 100           |
| Share of profit paid as liquid dividends | $\omega$      | 0.33          |
| <i>Government policy</i>                 |               |               |
| Proportional labor tax                   | $\tau$        | 0.25          |
| Lump sum transfer (rel GDP)              | $T$           | 0.06          |
| Fraction transfer to HtM                 | $\Lambda^T$   | 0.30          |
| Govt debt/annual GDP                     | $ B^g /(4Y)$  | 0.23          |
| <i>Monetary Policy</i>                   |               |               |
| Steady state real liquid return (pa)     | $r^b$         | 0.05          |
| Taylor rule coefficient                  | $\phi$        | 1.25          |
| Shock size at impact                     | $u_0$         | 0.0025        |
| Shock persistence                        | $\rho$        | 0.50          |

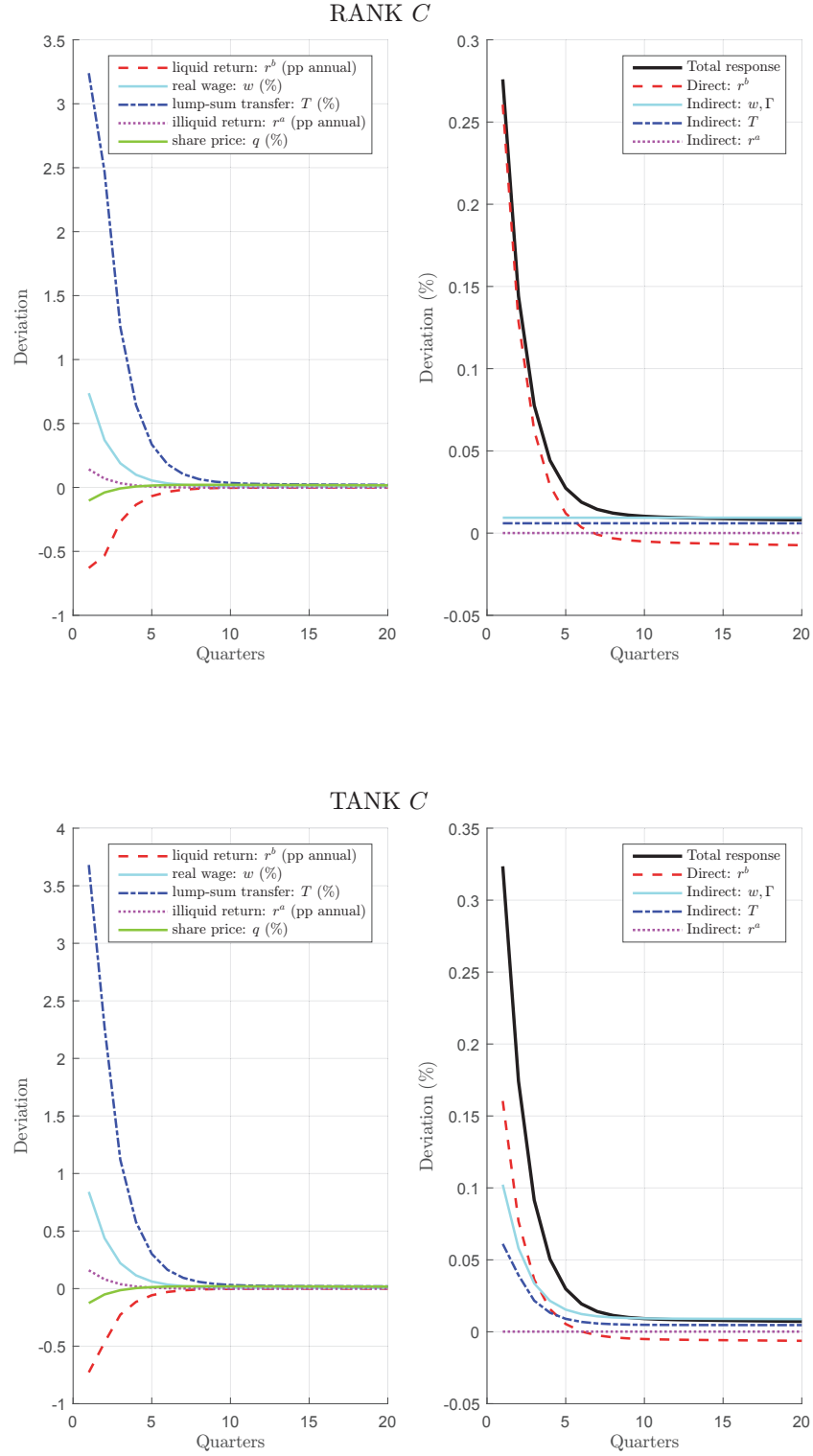


Figure A.10: Impulse Response Functions and Decompositions in RANK and TANK



## B Additional Details on the HANK Model

### B.1 HJB and Kolmogorov Forward Equations for Household's Problem

We here present the households' HJB equation, and the Kolmogorov forward equation for the evolution of the cross-sectional distribution  $\mu$ . We focus on the stationary versions of these equations under the assumption that the logarithm of income  $y_{it} = \log z_{it}$  follows a “jump-drift process”

$$dy_{it} = -\beta y_{it} dt + dJ_{it}.$$

Jumps arrive at a Poisson arrival rate  $\lambda$ . Conditional on a jump, a new log-earnings state  $y'_{it}$  is drawn from a normal distribution with mean zero and variance  $\sigma^2$ ,  $y'_{it} \sim \mathcal{N}(0, \sigma^2)$ . The stationary version of households' HJB equation is then given by

$$\begin{aligned} (\rho + \zeta)V(a, b, y) = & \max_{c, \ell, d} u(c, \ell) + V_b(a, b, y) [(1 - \tau)we^y \ell + r^b(b)b + T - d - \chi(d, a) - c] \\ & + V_a(a, b, y)(r^a a + d) \\ & + V_y(a, b, y)(-\beta y) + \lambda \int_{-\infty}^{\infty} (V(a, b, x) - V(a, b, y)) \phi(x) dx \end{aligned} \quad (100)$$

where  $\phi$  is the density of a normal distribution with variance  $\sigma^2$ .

Similarly, the evolution of the joint distribution of liquid wealth, illiquid wealth and income can be described by means of a Kolmogorov forward equation. To this end, denote by  $g(a, b, y, t)$  the density function corresponding to the distribution  $\mu_t(a, b, z)$ , but in terms of log productivity  $y = \log z$ . Furthermore, denote by  $s^b(a, b, y)$  and  $s^a(a, b, y)$  the optimal liquid and illiquid asset saving policy functions, i.e. the optimal drifts in the HJB equation (100). Then the stationary density satisfies the Kolmogorov forward equation

$$\begin{aligned} 0 = & -\partial_a(s^a(a, b, y)g(a, b, y)) - \partial_b(s^b(a, b, y)g(a, b, y)) \\ & - \partial_y(-\beta yg(a, b, y)) - \lambda g(a, b, y) + \lambda \int_{-\infty}^{\infty} g(a, b, x)\phi(x)dx. \end{aligned} \quad (101)$$

Achdou et al. (2014) explain in detail how to solve (100) and (101), including how to handle the state constraints, using a finite difference method.

## B.2 Proof of Lemma 1 (Derivation of Phillips Curve)

The firm's problem in recursive form is

$$r^a(t)J(p, t) = \max_{\pi} \left( \frac{p}{P(t)} - m(t) \right) \left( \frac{p}{P(t)} \right)^{-\varepsilon} Y(t) - \frac{\theta}{2} \pi^2 Y(t) + J_p(p, t) p \pi + J_t(p, t)$$

where  $J(p, t)$  is the real value of a firm with price  $p$ . The first order and envelope conditions for the firm are

$$\begin{aligned} J_p(p, t) p &= \theta \pi Y \\ (r^a - \pi) J_p(p, t) &= - \left( \frac{p}{P} - m \right) \varepsilon \left( \frac{p}{P} \right)^{-\varepsilon-1} \frac{Y}{P} + \left( \frac{p}{P} \right)^{-\varepsilon} \frac{Y}{P} + J_{pp}(p, t) p \pi + J_{tp}(p, t). \end{aligned}$$

In a symmetric equilibrium we will have  $p = P$ , and hence

$$J_p(p, t) = \frac{\theta \pi Y}{p} \quad (102)$$

$$(r^a - \pi) J_p(p, t) = - (1 - m) \varepsilon \frac{Y}{p} + \frac{Y}{p} + J_{pp}(p, t) p \pi + J_{tp}(p, t). \quad (103)$$

Differentiating (102) with respect to time gives

$$J_{pp}(p, t) \dot{p} + J_{pt}(p, t) = \frac{\theta Y \dot{\pi}}{p} + \frac{\theta \dot{Y} \pi}{p} - \frac{\theta Y \dot{p}}{p^2}.$$

Substituting into the envelope condition (103) and dividing by  $\theta Y/p$  gives

$$\left( r^a - \frac{\dot{Y}}{Y} \right) \pi = \frac{1}{\theta} (-(1 - m) \varepsilon + 1) + \dot{\pi}.$$

Rearranging, we obtain equation (19) in the main text.  $\square$

## B.3 Computation of Marginal Propensities to Consume

We begin by stating a notion of an MPC in our model that is directly comparable to the empirical evidence:

**Definition 1** The *Marginal Propensity to Consume over a period  $\tau$*  for an individual

with state vector  $(a, b, z)$  is given by

$$\text{MPC}_\tau(a, b, z) = \frac{\partial C_\tau(a, b, z)}{\partial b}, \quad \text{where} \quad (104)$$

$$C_\tau(a, b, z) = \mathbb{E} \left[ \int_0^\tau c(a_t, b_t, z_t) dt | a_0 = a, b_0 = b, z_0 = z \right]. \quad (105)$$

Similarly, the *fraction consumed out of  $x$  additional units of liquid wealth over a period  $\tau$*  is given by

$$\text{MPC}_\tau^x(a, b, z) = \frac{C_\tau(a, b + x, z) - C_\tau(a, b, z)}{x}. \quad (106)$$

The conditional expectation  $C_\tau(a, b, z)$  in (105) and, therefore, the MPCs in Definition 1 can be conveniently computed using the Feynman-Kac formula. This formula establishes a link between conditional expectations of stochastic processes and solutions to partial differential equations. Applying the formula, we have  $C_\tau(a, b, z) = \Gamma(a, b, y, 0)$ , with  $y = \log z$ , where  $\Gamma(a, b, y, t)$  satisfies the partial differential equation

$$\begin{aligned} 0 = & c(a, b, y) + \Gamma_b(a, b, y, t) s^b(a, b, y) + \Gamma_a(a, b, y, t) s^a(a, b, y) \\ & + \Gamma_y(a, b, y, t) (-\beta y) + \lambda \int_{-\infty}^{\infty} [\Gamma(a, b, x, t) - \Gamma(a, b, y, t)] \phi(x) dx \end{aligned}$$

on  $[0, \infty) \times [\underline{b}, \infty) \times [y_{\min}, y_{\max}] \times (0, \tau)$ , with terminal condition  $\Gamma(a, b, y, \tau) = 0$ , and where  $c, s^b$  and  $s^a$  are the consumption and saving policy functions that solve (100).

## B.4 Extension with firms' profits allocated to both $a$ and $b$

Under this extension, a fraction  $\omega$  of aggregate profits is paid into the illiquid accounts proportionately to the shares owned by each household and the remaining  $1 - \omega$  fraction is paid in liquid form to every individual  $i$  as a lump-sum rescaled by household productivity, i.e.,  $\pi_t^b(z_{it}) = \frac{z_{it}}{\bar{z}} (1 - \omega) \Pi_t$  where  $\bar{z}$  is average productivity. As explained in the main text, we interpret  $\pi_t^b(z_t)$  as bonuses and commissions and  $w_t z_t \ell_t + \pi_t^b(z_t)$  as total compensation. Labor income taxes are levied on total compensation.

Therefore, omitting the subscript  $i$  to ease notation, a household's holdings of liquid assets  $b_t$  evolve according to

$$\dot{b}_t = (1 - \tau_t) [w_t z_t \ell_t + \pi_t^b(z_t)] + r_t^b(b_t) b_t + T_t - d_t - \chi(d_t, a_t) - c_t \quad (107)$$

The dynamics of illiquid assets are still given by (11).

To solve their optimization problem, households take also as given  $\Pi_t^b = \frac{(1-\omega)}{\bar{z}} \Pi_t$ , the rescaled fraction of aggregate profits that is paid out proportionally to individual

productivity. It is useful to define  $W_t = (w_t, \Pi_t^b)$ , the vector of aggregates that characterizes worker's compensation. Then, in the vector  $\Gamma_t$ ,  $w_t$  should be replaced by  $W_t$ . Similarly, in our decomposition, the term capturing the indirect effects from changes in labor income induced by the monetary shock —the third term of equation (30)— becomes:

$$\int_0^\infty \left( \frac{\partial C_0}{\partial W_t} \right)' dW_t dt = \int_0^\infty \left( \frac{\partial C_0}{\partial w_t} dw_t + \frac{\partial C_0}{\partial \Pi_t^b} d\Pi_t^b \right) dt. \quad (108)$$

Finally, the arbitrage condition between shares of the intermediate producers and capital and the government budget constraint become

$$\frac{\omega \Pi_t + \dot{q}_t}{q_t} = r_t^k - \delta. \quad (109)$$

and the government budget constraint reads

$$\dot{B}_t^g + G_t + T_t = \tau_t \int (w_t z \ell_t(a, b, z) + \pi_t^b(z)) d\mu_t + r_t^b B_t^g \quad (110)$$

## C Details on 2004 SCF and FoF data

Our starting point is the balance sheet for U.S. households in 2004 (Flow of Funds Tables B.100, and B100e for the value of market equity). An abridged version of this table that aggregates minor categories into major groups of assets and liabilities is reproduced in Table C.1 (columns labelled FoF).

The columns labelled SCF in Table C.1 report the corresponding magnitudes, for each asset class, when we aggregate across all households in the 2004 Survey of Consumer Finances (SCF). The comparison between these two data sources is, in many respects, reassuring. For example, aggregate net worth is \$43B in the FoF and \$49B in the SCF, and the FoF ranking (and order of magnitude) of each of these major categories is preserved by the SCF data.<sup>65</sup> Nevertheless, well known discrepancies exist across the two data sources.<sup>66</sup>

On the liabilities side, credit card debt in FoF data is roughly half as large as in SCF data. The reason is that SCF measures outstanding consumer debt, whereas the FoF measures consumer credit, which includes current balances, whether or not they

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<sup>65</sup>This is remarkable, since the underlying data sources are entirely different. The SCF is a household survey. The macro-level estimates of U.S. household sector net worth in the FoF are obtained as a residual with respect to all the other sectors of the economy, whose assets and liabilities are measured based on administrative data derived from aggregate government reports, regulatory filings as well as data obtained from private vendors and agencies such as the Bureau of Economic Analysis (BEA), the Census Bureau, and the Internal Revenue Service (IRS).

<sup>66</sup>For systematic comparisons, see Antoniewicz (2000) and Henriques and Hsu (2013).

| <b>Assets</b>           |               |               |        | <b>Liabilities</b>     |              |               |        |
|-------------------------|---------------|---------------|--------|------------------------|--------------|---------------|--------|
|                         | FoF           | SCF           | Liquid |                        | FoF          | SCF           | Liquid |
| Real estate             | 21,000        | 27,700        | N      | Mortgage Debt          | 7,600        | 8,500         | N      |
| Consumer Durables       | 4,100         | 2,700         | N      | Nonrev. Cons. Credit   | 1,400        | 1,200         | N      |
|                         |               |               |        | Revolving Cons. Credit | 800          | 400           | Y      |
| Deposits                | 5,800         | 2,800         | Y      |                        |              |               |        |
| Treasury Bonds          | 700           | 200           | Y      |                        |              |               |        |
| Corporate Bonds         | 900           | 500           | Y      |                        |              |               |        |
| Corporate Equity        | 12,600        | 14,200        | N      |                        |              |               |        |
| Equity in Noncorp. Bus. | 7,300         | 11,100        | N      |                        |              |               |        |
| <b>Total</b>            | <b>52,400</b> | <b>59,200</b> |        | <b>Total</b>           | <b>9,800</b> | <b>10,100</b> |        |

Table C.1: Balance sheet of US households for the year 2004.

Sources: Flow of Funds (FoF) and Survey of Consumer Finances (SCF). Values are in Billions of 2004 US\$. Y/N stands for Yes/No in the categorization of that asset class as liquid.

get paid in full. Thus, the SCF estimate seems more appropriate, given that a negative value of  $b$  in the model means the household is a net borrower.

On the asset side, real estate wealth in the SCF is 30 pct higher than in the FoF. The SCF collects self-reported values that reflects respondents' subjective valuations, whereas the FoF combines self-reported house values, from the American Housing Survey (AHS) with national housing price index from CoreLogic and net investment from the BEA. However, during the house-price boom, AHS owner-reported values were deemed unreliable and a lot more weight was put on actual house price indexes, an indication that SCF values of owner-occupied housing may be artificially inflated by households' optimistic expectations.

The valuation of private equity wealth is also much higher in the SCF, by a factor exceeding 1.5. Once again, the FoF estimates appear more reliable, as it relies on administrative intermediary sources such as SEC filings of private financial businesses (security brokers and dealers) and IRS data on business income reported on tax returns, whereas, as with owner-occupied housing, the SCF asks non-corporate business owners how much they believe their business would sell for today.<sup>67</sup>

Finally, deposits and bonds are more than twice as large in the FoF.<sup>68</sup> Antoniewicz (2000) and Henriques and Hsu (2013) attribute this discrepancy to the fact that the FoF "household sector" also includes churches, charitable organizations and personal trusts (that are more likely to hold wealth in safe instruments) and hedge-funds (that

<sup>67</sup>According to Henriques and Hsu (2013), another reason why the SCF data on private business values is problematic is the combination of a very skewed distribution and the small sample size of the survey that make the aggregate value obtained in the SCF very volatile.

<sup>68</sup>The SCF does not contain questions on household currency holdings, but SCF data summarized above contain an imputation for cash. See Kaplan and Violante (2014) for details.

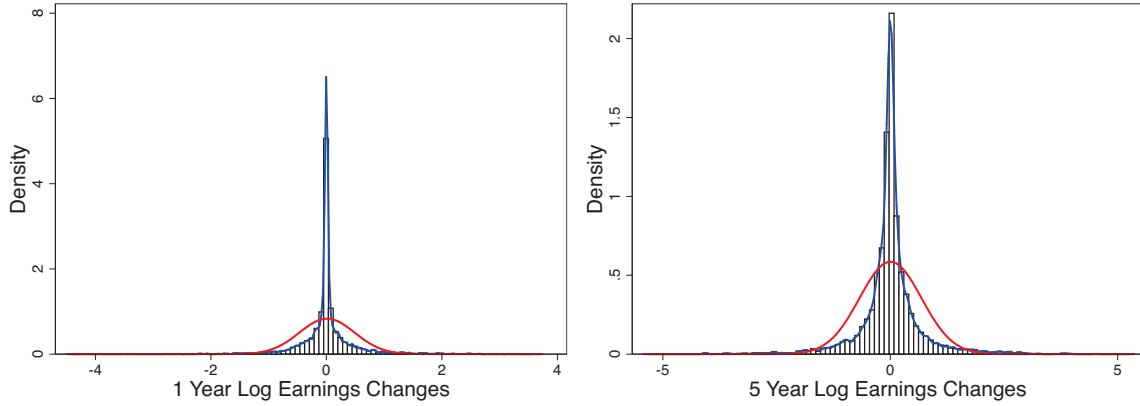


Figure D.1: Growth Rate Distribution of Estimated Earnings Process

may hold large amount of cash to timely exploit market-arbitrage opportunities).

## D Further Details on Calibration

### D.1 Earnings Process

Figure D.1 displays histograms of one- and five-year log earnings changes generated by our estimated earnings process (33)-(34), overlaid with Normal distributions with the same means and variances. The leptokurtosis of annual income changes is clearly evident from these figures. For a comparison with the analogous figures from SSA male earnings data, we refer readers to Figure 1 in Guvenen et al. (2015).

In order to translate the estimated earnings processes (33)-(34) to a form that can be used in the households' consumption-saving problem (100), we take the following steps.

First, we approximate the estimated continuous-time continuous-state processes with continuous-time discrete-state processes. For each of the two components ( $j = 1, 2$ ) we construct a grid for  $z_j$ . We use 11 grid points for the persistent component and 3 grid points for the transitory component. We then construct the associated continuous time transition matrix based on a finite difference approximation of the processes in (33)-(34), evaluated at the estimated parameters. We choose the grid widths and spacing so that the annual moments produced by simulating the combined discrete-state process are as close as possible to the annual earnings moments from the combined continuous-state process. These moments are reported in Table D.1. The Lorenz curves for the ergodic distributions associated with the continuous and discretized process are shown by the black dashed line and the green dash-dot line in Figure D.2, respectively. The two Lorenz curves are very close, as are the moments of

| Moment                     | Data | Model     | Model       |
|----------------------------|------|-----------|-------------|
|                            |      | Estimated | Discretized |
| Variance: annual log earns | 0.70 | 0.70      | 0.74        |
| Variance: 1yr change       | 0.23 | 0.23      | 0.21        |
| Variance: 5yr change       | 0.46 | 0.46      | 0.49        |
| Kurtosis: 1yr change       | 17.8 | 16.5      | 15.5        |
| Kurtosis: 5yr change       | 11.6 | 12.1      | 13.2        |
| Frac 1yr change < 10%      | 0.54 | 0.56      | 0.63        |
| Frac 1yr change < 20%      | 0.71 | 0.67      | 0.71        |
| Frac 1yr change < 50%      | 0.86 | 0.85      | 0.83        |

Table D.1: Earnings Process Estimation Fit

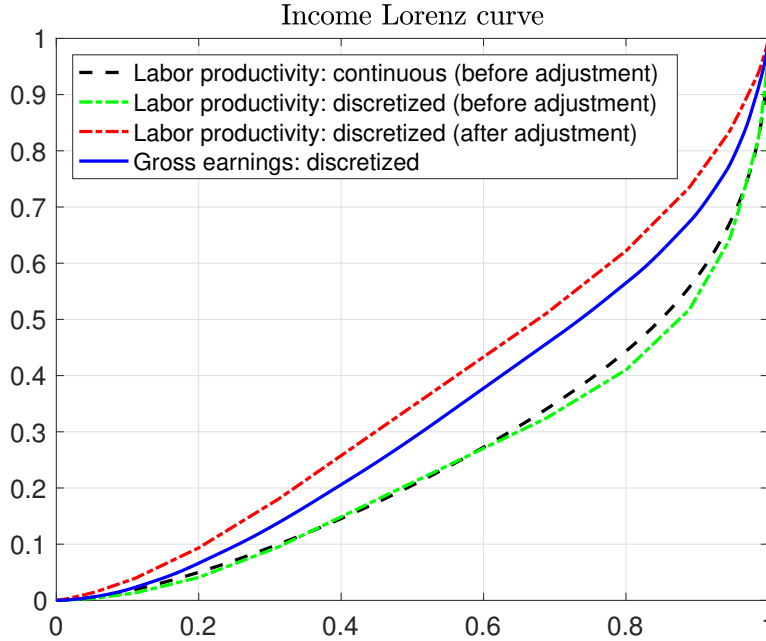


Figure D.2: Earnings Lorenz Curve

earnings changes, which demonstrates that the discrete approximations are accurate.

Second, since earnings in the model are determined by both idiosyncratic productivity  $z_{it}$  and endogenous labor supply decisions  $\ell_{it}$ , we make an adjustment to the productivity grid so that the resulting cross-sectional distribution of earnings  $y_{it} = w_t z_{it} \ell_{it}$  is as dispersed as in the data. This adjustment is necessary because with our preference specification, optimal labor supply decisions  $\ell_{it}$  are positively related to individual productivity  $z_{it}$ . Hence earnings inequality in the model with labor supply is larger than productivity inequality. To bring earnings inequality in line with the data we shrink the log productivity grid by a factor  $1 + \zeta \frac{1}{\nu}$ , where  $\frac{1}{\nu}$  is the Frisch elasticity of labor supply. We set the constant  $\zeta$  equal to 0.85, which generates a standard deviation

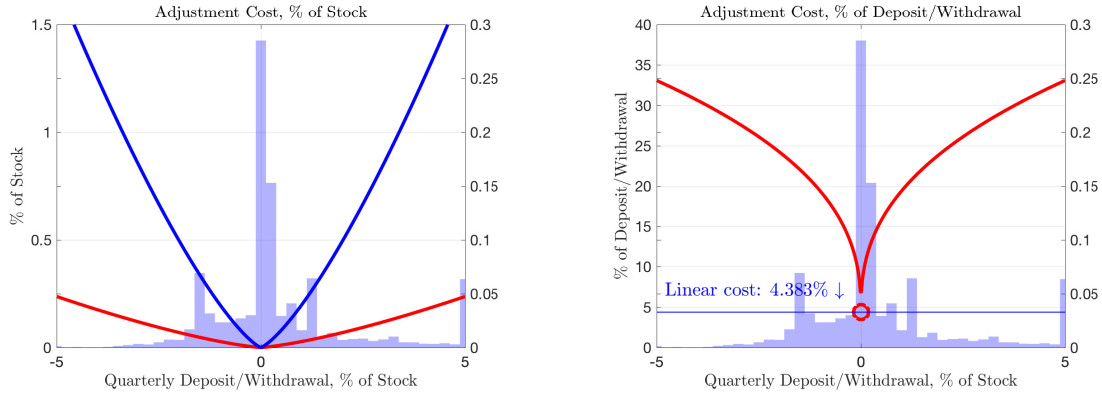


Figure D.3: Calibrated Adjustment Cost Function

of log earnings in the model  $\log y_{it}$  equal to the standard deviation of log household earnings in the data. To estimate the standard deviation of log household earnings implied by the SSA data (which we cannot observe directly), we rescale the standard deviation of log male earnings in the SSA data by the ratio of the standard deviation of log household earnings to the standard deviation of log male earnings in the Panel Study of Income Dynamics from 2002 to 2006.

The red dash-dot line in Figure D.2 shows the Lorenz curve for the productivity distribution once it has been re-scaled in this way. Note that it is less dispersed than the raw productivity process (green dash-dot line). The blue solid line shows the Lorenz curve for gross labor income, taking into account the optimal labor supply decisions of households. Note that gross labor income is more dispersed than the adjusted productivity process (because of the substitution effect), but is less dispersed than the raw productivity process (because of the distinction between household earnings and individual male earnings).

## D.2 Adjustment Cost Function

The calibrated transaction cost function is shown in Figure D.3. Consider first panel (a). The horizontal axis shows the quarterly transaction expressed as a fraction of a household's existing stock of illiquid assets,  $d/a$ . The vertical axis shows the cost of withdrawing or depositing this amount in a single quarter expressed as a fraction of the stock of illiquid assets,  $\chi(d, a)/a$ . The red line plots the adjustment cost function at the median wealth level. The light-blue histogram displays the stationary distribution of adjustments  $d/a$ . Roughly 55% percent of households are inactive and neither deposit nor withdraw. Of the remaining households, some deposit and some withdraw. The histogram shows that, on average, households in the stationary distribution withdraw



taking advantage of the fact that the income generated by the high return of illiquid assets replenishes the illiquid account. The blue line plots the adjustment cost function for all wealth levels above the threshold  $\underline{a}$  – from (14) for  $a > \underline{a}$ ,  $\chi(d, a)/a = \chi_0|d/a| + \chi_1|d/a|^{\chi_2}$ , i.e. the adjustment cost as a function of  $d/a$  is the same for all levels of  $a$ .

Panel (b) provides an alternative view of the adjustment cost function. The horizontal axis shows the quarterly transaction expressed as a fraction of illiquid assets,  $d/a$ , as in panel (a). The vertical axis shows the cost of withdrawing or depositing expressed as a fraction of the amount being transacted,  $\chi(d, a)/d$ , i.e. the “fee” for each transaction. The overlaid histogram is the same as in panel (a). The interpretation of the blue and red lines is as before.

These two panels illustrate that, for the most common transaction sizes, the cost is at most 25 percent of the value of the transaction. Or, put differently, at most 1 percent of the stock of illiquid wealth.

|                 | Description                           | Value    | Target / Source                                  |
|-----------------|---------------------------------------|----------|--|
|                 | <u>Preferences</u>                    |          |  |
| $\zeta$         | Death rate                            | 1/180    | Avg. lifespan 45 years                           |
| $1/\gamma$      | Intertemporal elasticity of subst.    | 1        |  |
| $1/\nu$         | Frisch elasticity of labor supply     | 1        |  |
| $\varphi$       | Disutility of labor                   | 27       | Avg. hours worked equal to 1/3                   |
| $\rho$          | Discount rate (pa)                    | 4.8%     | Internally calibrated                            |
|                 | <u>Production</u>                     |          |  |
| $\varepsilon$   | Demand elasticity                     | 10       | Profit share of 10 %                             |
| $\theta$        | Price adjustment cost                 | 100      | Slope of Phillips curve, $\epsilon/\theta = 0.1$ |
| $\alpha$        | Capital share                         | 0.33     |  |
| $\bar{\delta}$  | Steady state depreciation rate (p.a.) | 7%       |  |
| $\delta^u$      | Elasticity of capital utilization     | 1.2      |  |
|                 | <u>Government</u>                     |          |  |
| $\tau$          | Proportional labor tax                | 0.25     |  |
| $T$             | Lump sum transfer (rel GDP)           | 0.06     | 40% hh with net govt. transfer                   |
|                 | <u>Monetary Policy</u>                |          |  |
| $\phi$          | Taylor rule coefficient               | 1.25     |  |
| $\bar{r}^b$     | Steady state real liquid return (pa)  | 2%       |  |
|                 | <u>Unsecured borrowing</u>            |          |  |
| $r^{borr}$      | Borrowing rate (pa)                   | 7.9%     | Internally calibrated                            |
| $\underline{b}$ | Borrowing limit                       | \$16,500 | $1 \times$ quarterly labor income                |
|                 | <u>Adjustment cost function</u>       |          |  |
| $\chi_0$        | Linear component                      | 0.0438   | Internally calibrated                            |
| $\chi_1$        | Convex component                      | 0.956    | Internally calibrated                            |
| $\chi_2$        | Convex component                      | 1.402    | Internally calibrated                            |
| $\underline{a}$ | Min a in denominator                  | \$360    |  |

Table D.2: List of Calibrated Parameter Values

## E Additional Monetary Policy Experiments

In this appendix we report results on the overall effectiveness of monetary policy and its decomposition between direct and indirect effects, when we vary the key parameters that govern the “heterogenous agent block” of the model. These features include the borrowing limit, the cost of borrowing and the parameters of the adjustment cost function. Unlike the robustness experiments conducted in the main text, changing these parameters affects the level and distribution of wealth in the steady state. Hence to maintain comparability across experiments, in each case we re-calibrate the discount rate  $\rho$  so as to keep the mean of the illiquid asset distribution (and hence the equilibrium  $\frac{K}{Y}$  ratio, wage rate  $w$ , interest rates  $(r, r^a)$ , and output  $Y$  constant. The distribution of illiquid wealth, as well as the mean and distribution of liquid wealth, necessarily differ across the experiments, hence we report these features of the alternative economies alongside the results of the monetary policy shock

Table E.3 reports robustness analyses on the borrowing environment —the tightness of the borrowing limit and the wedge between the interest rates on borrowing and saving in the liquid assets. Reasonable changes in these features of the model have a significant effect on the level and distribution of liquid wealth holdings, but none of the main findings about the size and decomposition of the monetary policy shock are affected by these changes.

Table E.4 reports robustness analyses on the adjustment cost function. As with the borrowing environment, changes in the adjustment cost function affect the level and distribution of liquid wealth holdings but none of the main findings about the size and decomposition of the monetary policy shock are affected.

|                                    | Baseline | Loose<br>Borrow<br>Limit<br>$\underline{b} = 4 \times Y^{qu}$ | Low<br>Borrow<br>Costs<br>$\kappa = 4\% \text{ pa}$ | High<br>Borrow<br>Costs<br>$\kappa = 8\% \text{ pa}$ | Very High<br>Borrow<br>Costs<br>$\kappa = 20\% \text{ pa}$ |
|------------------------------------|----------|---|---|--|--|
|                                    | (1)      | (2)   | (3)   | (4)  | (5)  |
| Mean $b$ (rel. to GDP)             | 0.23     | 0.21  | 0.22  | 0.24   | 0.25   |
| Frac with $b = 0, a = 0$           | 0.10     | 0.10  | 0.07  | 0.12   | 0.09   |
| Frac with $b = 0, a > 0$           | 0.19     | 0.17  | 0.19  | 0.18   | 0.15   |
| Frac with $b < 0$                  | 0.15     | 0.17  | 0.25  | 0.09   | 0.04   |
| Quarterly \$500 MPC                | 0.16     | 0.16  | 0.14  | 0.16   | 0.15   |
| Change in $r^b$ (pp)               | -0.28 %  | -0.29 %   | -0.28 %   | -0.27 %  | -0.27 %  |
| Elasticity of $Y$                  | -3.96    | -3.75   | -3.72   | -4.17  | -4.17  |
| Elasticity of $I$                  | -9.43    | -9.05   | -8.94   | -9.89  | -9.94  |
| Elasticity of $C$                  | -2.93    | -2.81   | -2.75   | -3.08  | -3.08  |
| Partial Eq. Elast. of $C$          | -0.55    | -0.64   | -0.62   | -0.52  | -0.52  |
| Component of change in $C$ due to: |          |   |   |  |  |
| Direct effect: $r^b$               | 19 %     | 23 %  | 23 %  | 17 %   | 17 %   |
| Indirect effect: $w$               | 51 %     | 50 %  | 49 %  | 53 %   | 53 %   |
| Indirect effect: $T$               | 32 %     | 29 %  | 30 %  | 33 %   | 33 %   |
| Indirect effect: $r^a \text{ } q$  | -2 %     | -1 %  | -2 %  | -2 %   | -2 %   |

Table E.3: Robustness: borrowing environment

Notes: Average responses over the first year. Column (1) is the baseline specification. Column (2) loosens the borrowing limit from 1 times quarterly GDP to 4 times quarterly GDP. Column (3) lowers the wedge between the liquid borrowing and liquid savings rates from 6%pa to 4%pa. Column (4) raises the wedge between the liquid borrowing and liquid savings rates from 6%pa to 8%pa. Column (5) raises the wedge between the liquid borrowing and liquid savings rates from 6%pa to 20%pa. All experiments re-calibrate the discount rate  $\rho$  so that mean illiquid assets relative to GDP is held constant.

|                                    | Baseline | No<br>Linear<br>Cost<br>$\chi_0 = 0$ | High<br>Linear<br>Cost<br>$\chi_0 = 0.10$ | Low<br>Convex<br>Cost<br>$\chi_2 = 0.10$ | High<br>Convex<br>Cost<br>$\chi_2 = 1$ |
|------------------------------------|----------|--------------------------------------|---|--|--|
|                                    | (1)      | (2)                                  | (3)                                       | (4)                                      | (5)                                    |
| Mean $b$ (rel. to GDP)             | 0.23     | 0.22                                 | 0.26                                      | 0.44                                     | 0.11                                   |
| Frac with $b = 0, a = 0$           | 0.10     | 0.09                                 | 0.12                                      | 0.22                                     | 0.01                                   |
| Frac with $b = 0, a > 0$           | 0.19     | 0.19                                 | 0.18                                      | 0.05                                     | 0.19                                   |
| Frac with $b < 0$                  | 0.15     | 0.15                                 | 0.14                                      | 0.12                                     | 0.13                                   |
| Quarterly \$500 MPC                | 0.16     | 0.16                                 | 0.15                                      | 0.11                                     | 0.14                                   |
| Change in $r^b$ (pp)               | -0.28 %  | -0.28 %                              | -0.27 %                                   | -0.27 %                                  | -0.26 %                                |
| Elasticity of $Y$                  | -3.96    | -3.99                                | -3.94                                     | -3.64                                    | -5.52                                  |
| Elasticity of $I$                  | -9.43    | -9.68                                | -9.20                                     | -7.36                                    | -18.64                                 |
| Elasticity of $C$                  | -2.93    | -2.88                                | -3.02                                     | -3.47                                    | -2.59                                  |
| Partial Eq. Elast. of $C$          | -0.55    | -0.54                                | -0.55                                     | -0.56                                    | -0.43                                  |
| Component of change in $C$ due to: |          |                                      |   |  |  |
| Direct effect: $r^b$               | 19 %     | 19 %                                 | 18 %                                      | 16 %                                     | 17 %                                   |
| Indirect effect: $w$               | 51 %     | 52 %                                 | 51 %                                      | 43 %                                     | 62 %                                   |
| Indirect effect: $T$               | 32 %     | 31 %                                 | 33 %                                      | 42 %                                     | 26 %                                   |
| Indirect effect: $r^a q$           | -1 %     | -2 %                                 | -2 %                                      | 0 %                                      | -4 %                                   |

Table E.4: Robustness: adjustment cost function

Notes: Average responses over the first year. Column (1) is the baseline specification. Column (2) sets the linear component of the adjustment cost function to zero. Column (3) increases the linear component of adjustment cost function from 4.4% to 10%. Column (4) reduces the exponent on the convex component of the adjustment cost function from 0.4 to 0.1. Column (5) increases the exponent on the convex component of the adjustment cost function from 0.4 to 1. All experiments re-calibrate the discount rate  $\rho$  so that mean illiquid assets relative to GDP is held constant.

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