

Payments, Credit and Asset Prices*

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Abstract

This paper studies a monetary economy with two layers of transactions. In enduser transactions, households and institutional investors pay for goods and securities with payment instruments provided by banks. Endusers' payment instructions generate interbank transactions that banks handle with reserves or interbank credit. The model links the payments system and securities markets so that beliefs about asset payoffs matter for the price level, and monetary policy matters for real asset values.

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1 Introduction

This paper studies the joint determination of payments, credit, and asset prices. The starting point is that, in modern economies, transactions occur in two layers. In the *enduser layer*, non-banks – for example, households, firms and institutional investors – trade goods and securities and pay for them using payment instruments supplied by banks. Payment instruments include not only short-term demandable assets such as deposits and money-market fund shares, but also credit lines that can be drawn on demand such as credit cards. Credit lines also play a key role in payment for securities. For example, institutional investors have sweep arrangements with their custodian banks. Participants in the triparty repo market obtain intraday credit from clearing banks.

A common denominator of different payment instruments is that banks commit to accept payment instructions from their clients. As a result of those payment instructions, transactions in the enduser layer generate interbank transactions in the *bank layer*. Perhaps the most obvious example are direct payments out of bank deposit accounts by check or wire transfer: payments between customers of different banks generate interbank transfers of funds. In many securities markets, transactions are cleared by specialized financial market utilities such as clearinghouses that provide some netting of transactions. Institutional investors then settle netted positions with those utilities through payment instructions to their banks.

Interbank payments are often made with reserves, but may also be handled through various forms of short-term credit. For example, in the United States, utilities like NSS and CHIPS allow for intraday netting of a share of interbank transactions, so only net positions are periodically settled with reserves. The central bank may also provide intraday overdraft credit to banks. Nevertheless, the bulk of interbank payments goes through gross settlement systems provided by central banks, such as Fedwire in the United States or Target in the Euro Area.

In the aftermath of recent financial crises, central banks have made unprecedented changes to the quantity as well as the price of reserves. Several central banks have dramatically increased the quantity of reserves relative to the value of transactions. These policy shifts have reduced the relative importance of interbank credit. For example, in the United States, the use of intraday overdraft as well as interbank overnight Fed funds borrowing have essentially disappeared. Moreover, a number of central banks have begun charging negative nominal interest rates for the use of reserves.

This paper proposes a stylized model of an economy with two layers of transactions. Endusers are households and institutional investors who must pay for some goods and securities with payment instruments – deposits or credit lines – supplied by a competitive banking sector. Banks handle endusers' payment instructions and must make some interbank payments with reserves supplied by the government. Both banks and the government incur costs of leverage that decline with the quantity and quality of available collateral, in particular securities and claims to future taxes.

The model determines asset prices, the nominal price level and agents' portfolios as a function of government policy and investor beliefs about asset payoffs. It also determines the share of resources used up as costs of leverage. An efficient payment system allocates collateral so as to minimize that share of resources and hence maximize consumption. Asset prices reflect

not only cash flow expectations and uncertainty premia, but also the collateral and liquidity benefits that assets provide to endusers and banks.

We use the model to think about links between securities markets and the payment system. The key properties that generate such links are illustrated by the quantity equation

$$PT = \bar{v}(D + L).$$

Here the total volume of transactions T includes institutional investors' securities purchases, not simply the value of goods traded. Moreover, the only medium of exchange in the enduser layer is "inside money" – deposits D and credit lines L – provided by banks who rely on securities as collateral. Outside money – that is, reserves – is only one input to the production of inside money, albeit a special one since it not only serves as collateral, but also provides liquidity for making interbank payments.

Consider an increase in uncertainty about asset payoffs that lowers asset values. As the value of collateral that banks can purchase declines, supplying payment instruments becomes more costly. A decline in inside money $D + L$ then puts downward pressure on the price level. At the same time, however, an increase in uncertainty also lowers institutional investors' demand for payment instruments, which has the opposite effect. The details of financial structure, including the use of payment instruments by institutional investors and the scope of netting arrangements – are thus important in order to assess the effects of asset market shocks on inflation.

In our model, some assets are priced exclusively by intermediaries. Segmentation of asset markets arises endogenously because banks receive collateral benefits from assets but households do not. Banks invest in assets to back payment instruments and thus bid up the prices of those assets. In particular, the real interest rate on short-term credit is so low in equilibrium that households choose not to lend short term. It therefore does not satisfy a consumption-based pricing equation. Instead, banks are the only marginal investors, and bank Euler equations say that interest rates are lower when bank leverage is higher and collateral is scarcer.

We also use the model to think about recent policy shifts, with a focus on two policy tools. First, the government can trade in securities markets to change the mix of collateral available to banks. Second, the government controls the real return on reserves. The central bank sets the nominal rate on reserves. Moreover, the inflation rate is given by the growth rate of nominal government liabilities. This result follows from the quantity equation and the fact that prices are flexible. Importantly, what matters is not the growth rate of reserves, but instead the growth rate of nominal payment instruments $D + L$, which in turn depends on nominal collateral available to banks.

The government can select one of two policy regimes. Reserves are *scarce* if banks do not always have sufficient reserves to handle all interbank payments but instead turn to the short-term credit market for liquidity. The liquidity benefit of reserves then generates a spread between the short-term interest rate and the interest rate on reserves. Reserves are scarce if the real return on reserves is sufficiently low, that is, the opportunity cost of holding reserves is high. Banks then choose higher leverage to maintain a high return on equity in spite of a higher effective tax on reserves.

As long as reserves are scarce, open-market purchases of short-term debt for reserves change the collateral mix towards more liquid bank assets. In our model, open-market purchases

permanently lower the real short-term interest rate. Indeed, when more liquid reserves are available, competition drives banks to produce more payment instruments, pushing the price level up. As a result, the real value of nominal collateral falls – banks become more levered and bid up the prices of all collateral including short bonds, a permanent “liquidity effect” on the real interest rate.

In the second policy regime, reserves are *abundant*: the quantity of reserves is sufficiently large relative to the volume of transactions that overnight borrowing is never needed. Once reserves lose their liquidity benefit, short-term loans and reserves become perfect substitutes and earn the same interest rate – the economy enters a “liquidity trap” where conventional open-market policy becomes ineffective. If the interest rate on reserves is zero, then reserves become abundant at the zero lower bound. More generally, however, reserves are abundant whenever the real return on reserves is sufficiently high, which can also happen with positive or negative interest on reserves.

The fact that payments occur in two layers has important implications for what it means to be in a “liquidity trap”. The textbook view is that equality of interest rates on outside money and short bonds implies that the medium of exchange and a safe store of value become perfect substitutes for all agents. In our model, this is true only for banks who are the only investors in both reserves and short bonds. In contrast, payment instruments for endusers require costly bank leverage and never become abundant. In particular, they retain their liquidity benefit even when reserves are abundant. At the same time, collateral remains scarce in the liquidity trap, so unconventional policy that exchanges reserves for lower quality collateral can still matter by changing the collateral mix.

Which regime is better depends on the relative leverage costs of banks versus the government. If the government can borrow more cheaply than banks, then it makes sense to move to abundant reserves, as several central banks have done recently. An extreme version would be narrow banking. In contrast, if the government has trouble to credibly commit to a path for nominal debt, then it is beneficial to have banks rely more on collateral other than government debt or reserves. Since the optimal system depends on the quality of collateral, it may make sense to switch between regimes over time in response to asset market events.

The availability of two separate policy tools implies that the stance of policy cannot be easily summarized by a single variable, such as the short-term nominal interest rate. For example, when reserves are scarce, the government can lower the nominal interest rate either through open-market purchases or by lowering the real return on reserves. However, the effect on real interest rates and inflation is generally different. The reason is that asset values reflect not only liquidity benefits – as in many monetary models – but also collateral benefits. Policy affects interest rates by altering both benefits.

Our model assumes that markets are competitive and all prices are perfectly flexible. Banks and other financial firms maximize shareholder value and operate under constant returns to scale. Moreover, they do not face adjustment costs to equity. The effects we highlight thus do not follow from a scarcity of bank capital. We think of our model as one of large banks that provide payment services in a world where credit markets are highly securitized. This perspective also motivates our leading example for a shock: a change in uncertainty that moves asset premia.

Financial frictions are formally introduced as follows. First, nominal payment instruments and reserves relax liquidity constraints in the enduser and bank layer, respectively. In this sense, those assets are more liquid than other assets. By assuming generalized cash-in-advance constraints for households and institutional investors, we abstract from effects of interest rates on the volume of transactions in units of goods and securities, respectively. While adding such effects is conceptually straightforward, our goal here is to provide a tractable setup that zeros in on novel effects for the demand and supply for money.

Second, banks and the government face an upward-sloping marginal cost of making commitments. This cost is smaller the more collateral the institution has available, that is, the larger and safer is a bank's asset portfolio or the larger the tax base, respectively. It can have either an ex post or an ex ante interpretation. For example, if more levered banks and governments are more likely to renege on certain promises, more labor may be required to ex post renegotiate those promises so that less labor is available for producing goods. Alternatively, more levered banks and governments may have to exert more effort ex ante to produce costly signals of their credibility.

Our analysis below starts with a baseline model in which the quantities of both real payment instruments provided by banks and securities on their balance sheets are fixed. In two extensions, we then introduce institutional investors whose demand for loans or payment instruments responds to changes in interest rates. We first consider *carry traders* who hold real assets and borrow against those assets using short-term credit supplied by banks. Carry traders have no demand for payment instruments, but supply collateral to banks in the form of short-term loans. An example are asset-management firms who finance securities holdings with repurchase agreements.

The new feature in an economy with carry traders is that the price level now depends on carry traders' demand for loans. For example, lower uncertainty increases the demand for loans and hence the quantity of collateral for banks, the supply of payment instruments and the price level. An asset-price boom can thus be accompanied by inflation even if the supply of reserves as well as the amount of goods transacted remains constant and banks hold no uncertain securities themselves. Moreover, monetary policy that lowers the real short rate lowers carry traders' borrowing costs and boosts the aggregate market by allowing more leverage.

The second extension introduces *active traders* who hold not only securities but also payment instruments, since they must occasionally rebalance their portfolio using cash payments. An example are asset management firms who sometimes want to exploit opportunities quickly before they can sell their current portfolio. Active traders' portfolio choice responds to the deposit interest rate offered by banks and the fee for credit lines they charge: if payment instruments are cheaper, active traders hold more of them, and the value of their transactions is higher. The strength of their response depends importantly on how much netting takes place among active traders through intraday credit systems.

The new feature in an economy with active traders is that the price level now depends on active traders' demand for payment instruments. For example, lower uncertainty increases their demand for deposits and credit lines. As more of payment instruments provided by banks are used in asset market transactions, fewer instruments are used in goods market transactions and the price level declines. During an asset price boom, we may thus see low inflation even if the

supply of reserves increases. Moreover, monetary policy that lowers the real short term interest rate lowers active traders' trading costs and further boosts the aggregate market.

Our model can be interpreted as describing the subset of worldwide transactions in a currency, rather than the closed economy of a country. The former interpretation is appropriate for economies like the United States that have banking systems and financial markets tightly integrated with those of other countries. We thus think of households in our models as agents who pay for goods out of dollar deposit accounts, while institutional investors may include foreign firms who obtain credit or payments from banks in terms of dollars. With this perspective in mind, the model can be used to think about how events in worldwide asset markets may affect nominal prices in the US.

The broad questions we are interested in are the subject of a large literature. The main new features of our model are that (i) transactions occur in layers, with payment instruments (inside money) used exclusively in the enduser layer and reserves (outside money) used exclusively in the bank layer, (ii) endusers include institutional investors, and (iii) both banks and the government face leverage costs. Relative to earlier work, these properties change answers to policy questions as well as asset pricing results, as explained in more detail in Section 6.

The paper is structured as follows. Section 2 presents a few facts about payments. Section 3 describes the model. Section 4 looks at the baseline model that features only households and banks. It shows how steady state equilibria can be studied graphically and considers different monetary policy tools. Section 5 introduces uncertainty and studies the link between the payment system and securities markets. It also extends the model to accommodate institutional investors as a second group of endusers. Finally, Section 6 discusses the related literature.

2 Facts on payments

This section presents a number of facts that motivate our model. We combine data from the BIS Payments Statistics, the Payments Risk Committee sponsored by the Federal Reserve Bank of New York, the Federal Reserve Board's Flow of Funds Accounts and Call Reports, as well as publications of individual clearinghouse companies.

Enduser transactions

Figure 1 gives an impression of enduser and interbank payments in US dollars. The left hand panel considers enduser payments, that is, payment instructions to various types of intermediaries. The blue area labeled "nonfinancial" adds up payments by cheque as well as various electronic means, notably Automated Clearinghouse (ACH) transfers as well as payments by credit card. While the area appears small in the figure, it does amount to several multiple of GDP. For example, in 2011, nonfinancial transactions were \$71trn whereas GDP was \$15trn. This is what one would expect given that there are multiple stages of production and commerce before goods reach the consumer. Moreover a share of trade in physical capital including real estate also is contained in this category.

Payment for securities in U.S. markets is organized by specialized financial market utilities who clear transactions and see them through to final settlement. A major player is the Depository Trust & Clearing Corporation (DTCC). One of its subsidiaries, the National Securities

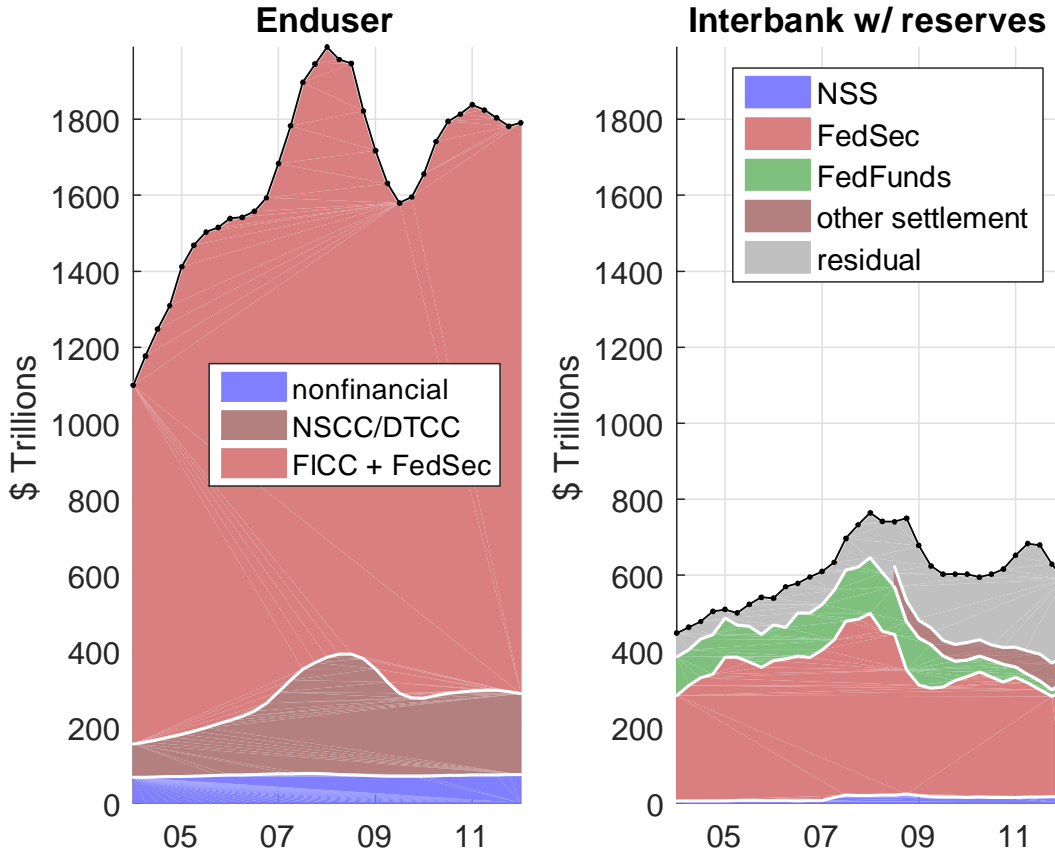


Figure 1: Selected US dollar transactions, quarterly at annual rates. *Left panel*: "nonfinancial" = cheque and electronic payments reported by banks, "NSCC/DTCC" = securities transactions cleared by NSCC, "FICC+FedSec" = securities transactions cleared by FICC and Fedwire Securities Service. *Right panel*: "NSS" = NSS settlement, "FedSec" = Fedwire Securities settlement, "FedFunds" = estimate of payments for Fed Funds borrowing by US banks, "other settlement" = estimate of payments to financial market utilities.

Clearing Corporation (NSCC) clears transactions on stock exchanges as well as over-the-counter trades in stocks, mutual fund shares and municipal and corporate bonds. NSCC cleared \$221trn worth of such trades in 2011. In the left hand panel of Figure 1, transactions cleared by NSCC are shaded in brown.

NSCC has a customer base ("membership") of large financial institutions, in particular brokers and dealers. When a buyer and a seller member agree on a trade – either in an exchange or in an over-the-counter market – the trade is reported to NSCC which then inserts itself as a counterparty to both buyer and seller. In the short run, members thus effectively pay for securities with credit from NSCC. To alleviate counterparty risk, members post collateral that limits their position relative to NSCC. Over time, NSCC nets opposite trades by the same member. Periodically, members settle net positions via payment instructions to members' bank which then make (receive) interbank payments to (from) DTCC. Netting implies that settlement payments amount to only a fraction of the dollar value of cleared transactions.

Another DTCC subsidiary, the Fixed Income Clearing Corporation (FICC) offers clearing for Treasury and agency securities. FICC payments are settled on the books of two "clearing

banks”, JP Morgan and Bank of New York Mellon. Interbank trades of Treasury and agency bonds can alternatively be made via the Fedwire Securities system offered by the Federal Reserve System to its member banks. The left hand panel of Figure 1 shows the sum of FICC and Fedwire Securities trades in red. This number is high partially because every repurchase agreement involves two separate security transactions (that is, the lender wires payment for a purchase to the borrower and the borrower wires payments back to the lender at maturity).

Figure 1 does not provide an exhaustive list of US dollar transactions. First, it leaves out financial market utilities handling derivatives and foreign exchange transactions. For example, the Continuous Linked Settlement (CLS) group is a clearinghouse for foreign exchange spot and swap transactions that handled trades worth \$1,440 trn in 2011. Netting in these markets is very efficient so that CLS payments after netting were only \$3Trn. Second, even for goods and securities covered, Figure 1 omits purchases made against credit from the seller that involves no payment instruction to a third party. This type of transaction includes trade credit arrangements. In securities markets, a share of bilateral repo trades between broker dealers and their clients is settled on the books of the broker dealers. Finally, the figure also leaves out transaction made with currency.

Even given these omissions, the message from the left panel of Figure is clear: transaction volume is large, and especially so in securities markets. The volume in securities markets also exhibits pronounced fluctuations in the recent boom-bust episode. We also emphasize that not all of these payment instructions are directly submitted to traditional banks. Financial market utilities that provide netting are also important. Moreover, customers of money market mutual funds may also pay by cheque or arrange ACH transfers. The payment instruction is then further relayed by the money market fund to its custodian bank.

Transactions in the bank layer

The right hand panel of Figure 1 shows transactions over two settlement systems provided by the Federal Reserve Banks. The blue area represents interbank payments via the National Settlement Service, which allows for multilateral netting of payments by cheque and ACH. To a first approximation, one can think of it as the counterpart of the blue area in the left hand panel, that is, non-securities payments after netting. All other areas in the right panel represent interbank payments over Fedwire, the real time gross settlement system of the Federal Reserve. Fedwire is accessed by participating banks who send reserves to each other.

The coloring of areas is designed to indicate roughly how the interbank payments were generated. The red area represents payments for Treasury and agency securities over Fedwire Securities. Since there is no netting involved, large securities transfers correspond to large transfers of reserves. For the years after 2008, the brown area is an estimate of payments made over Fedwire to settle positions with financial market utilities. The estimate includes not only NSCC and FICC, but also CHIPS, a private large value transfer system used by about 50 large banks. CHIPS uses a netting algorithm to simplify payments among its member banks; in 2011, it handled \$440trn worth of transactions.

The green area in the figure represents payments for interbank credit in the Fed Funds market, also sent over Fedwire. As for repo transactions, a relatively small amount of outstanding overnight credit can generate a large number for annual Fedwire transfers. The transition from a regime of scarce reserves to one with abundant reserves after the financial crisis is apparent

by the drop in Fed Funds transactions. The presence of government sponsored enterprises and Federal Home Loan banks implies that the Fed Funds market has not dried up completely.

The red and brown areas suggest that payment instructions generated by securities trading are responsible for a large share of interbank payments. This is true even though netting by financial market utilities reduced the cleared transactions from the left panel to much smaller numbers. At the same time, during times of scarce reserves, bank liquidity management via the Fed Funds market also generates a large chunk of payments. The figure also contains a gray area which we cannot assign to one of the payment types.

Payments by custodian banks

Figure 2 provides a closer look at the activities of banks who make a lot of payments. We consider 27 bank holding companies that are "systematically important" and hence report individual data on payments to regulators. Along the horizontal axis, we measure 2014 payments via large value transfer systems like Fedwire and CHIPS, normalized by bank assets. The 27 banks' joint payments are large: they account for over 75% of total payments over CHIPS and Fedwire.

Along the vertical axis, we measure securities held in custody, again normalized by assets. There is a strong positive relationship: banks who have more assets in custody also tend to make more payments. The fact that the relationship holds after normalization by assets suggests that it is not merely a scale effect. Moreover, the color of a dot indicates the size of the bank in terms of total assets. It is not obviously related to the two ratios measured along the axes.

In fact, the largest banks – JP Morgan, Citi, Wells Fargo and Bank of America all appear as bright pink dots – do not have the largest payments relative to assets. Instead, it is somewhat smaller banks specializing in the custodian business – State Street, Northern Trust and BoNY Mellon appear in the top right corner of the figure – who have the largest payments/assets ratios.

There are two plausible reasons why banks who are large custodians might be expected to make lots of payments. One is already apparent in Figure 1: there is simply a lot of "churn" in securities markets, in part due to frequent short term changes in positions. This possibility motivates the inclusion of liquidity constraints for securities traders together with a netting system, in our model below.

A second reason is that custodian banks hold the portfolios of money market funds, who in turn offer payment instruments to endusers. While the front office of the money market fund receives payment instruments, those instructions are still executed by banks who access large value transfer systems, in particular Fedwire. For the purposes of our model, our perspective on money market funds is thus to consolidate them with the banking system.

3 Model

Time is discrete, there is one good and there are no aggregate shocks. Households consume an endowment of goods as well as fruit from trees. The total amount of goods available for consumption is constant. Households also own competitive financial firms. For now, the only

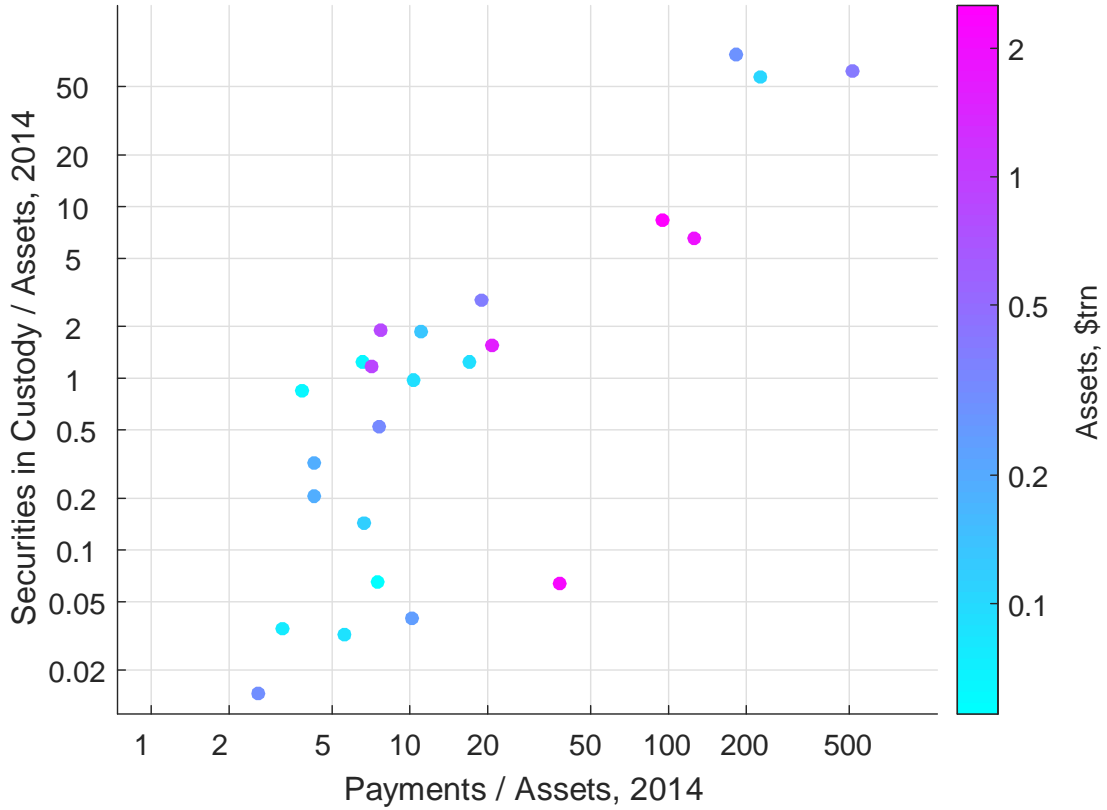


Figure 2: Payments / Assets vs Securities in Custody / Assets for systemically important bank holding companies. Flows for the year 2014, stocks for 2014 Q4. Color indicates assets in \$trn.

financial firms are banks who issue payment instruments. Below we introduce different types of asset management companies. All financial firms issue equity and participate in tree and credit markets along with households.

Layers and frictions

The model describes transactions and asset positions in both the enduser layer and the bank layer. In the enduser layer, households and nonbank financial firms trade goods and assets. In the bank layer, banks trade securities and also borrow from and lend to each other. A key connection between layers is that enduser transactions generate payment instructions to banks.

The model incorporates two frictions. First, to capture the costs of supplying payment instruments and debt as well as the benefits of safe collateral, we assume that there is a cost of leverage for banks as well as other financial firms. The government similarly faces a cost of leverage, with the tax base serving as collateral. Second, to capture why some assets provide liquidity benefits or costs, we impose liquidity constraints in both the enduser and bank layer.

In the enduser layer, liquidity constraints require payment instruments supplied by banks. We allow for both deposits held for one period and for credit lines arranged a period in advance. Those instruments are equivalent in equilibrium – both provide liquidity to endusers and both require costly leverage by banks. A binding enduser liquidity constraint thus generates a liquidity benefit of inside money.

Liquidity constraints in the bank layer require that interbank payments generated by customer withdrawals are paid with borrowed or unborrowed reserves. A binding bank liquidity constraint generates liquidity benefits for outside money (reserves) as well as interbank credit. It also generates a liquidity cost of providing payment instruments.

We assume that financial firms can be costlessly recapitalized every period and that their objective function exhibits constant returns to scale. As a result, a firm's history does not constrain its future portfolio and capital structure decisions. As in Lagos and Wright (2005), the distribution of heterogeneous agents' (here firms') histories thus plays essentially no role in the model.

Assets

There are four different asset classes. Reserves serve as numeraire; they are issued by the government, pay a nominal interest rate i^R and are held only by banks. Overnight credit pays an interest rate i ; it is less liquid than reserves as funds lent out at date t cannot be used to handle customer withdrawals at date $t + 1$.

Trees are infinitely lived assets that each pay an exogenous quantity of goods x_t in period t . Trees differ only in who is allowed to invest in them, as detailed below. For now, we index trees by types $j \in [0, 1]$. The nominal price of a type j tree is denoted Q_t^j , and the nominal value of its fruit is $P_t x_t$, where P_t is the nominal price level.

3.1 Households

Households have linear utility, discount the future at the rate $\delta = -\log \beta$ and receive an endowment Ω_t every period. Households enter the period with deposits D_t^h and outstanding credit lines L_t^h ; they buy consumption C_t at the nominal price P_t measured in units of reserves. Their liquidity constraint is

$$P_t C_t \leq \bar{v}(D_t^h + L_t^h),$$

where $\bar{v} \leq 1$ is a fixed parameter that determines the velocity of money available to households. A cash-in-advance approach helps zero in on the role of endogenous inside money.

The liquidity constraint allows for two types of nominal payment instruments. Deposits require investment one period in advance and earn the nominal interest rate i_{t-1}^D . Credit lines must be arranged one period in advance with a bank, but require no investment – they represent intraday credit extended by banks on demand. In exchange for the commitment to accept payment instructions, banks charge a fee $i_{t-1}^L L_t$ proportional to the (nominal) amount of credit.¹

¹We assume that an interest rate i^D is earned on deposits regardless of whether they are spent, and that the fee i^L is paid on credit lines regardless of whether they are drawn. These assumptions help simplify the algebra. More detailed modeling of the fee structure of different payment instruments is possibly interesting but not likely to be first order for the questions we address in this paper.

The household budget constraint is

$$P_t C_t + i_{t-1}^L L_t^h = P_t \Omega_t + D_t^h (1 + i_{t-1}^D) - D_{t+1}^h + \int_0^1 ((Q_t^j + P_t x_t) \theta_{t-1}^j - Q_t^j \theta_t^j) dj \quad (1)$$

+ dividends + fees + government transfers.

Expenditure on goods as well as fees paid for credit lines must be financed through either (i) the sale of endowment, (ii) changes in household asset positions in trees and deposits, or (iii) exogenous income from dividends, fees or government transfers, described in more detail below.

Households are allowed to invest in all trees $j \in [0, 1]$, but they cannot sell trees short, that is, we impose $\theta^j \geq 0$. We interpret the endowment as labor income (payoffs from human capital), while trees represent other long lived assets such as equity in nonfinancial firms or claims to housing services. Both trees and human capital are less liquid in the sense that they cannot be used to pay for consumption. The difference between them is that human capital must be held by households, whereas trees can also be held by financial firms or the government, and their ownership affects the production of payment instruments. A key equilibrium outcome is where in the economy trees are held.

We think of our model period as a short period such as a day, and we do not study hyperinflation periods, so that nominal and real rates of return are always small decimal numbers. We thus simplify formulas throughout by using the approximation $e^r = 1 + r$ for any small rate of return r , and setting any products of rates of return to zero. For example, with an inflation rate $\pi_t = \log P_t / P_{t-1}$, the real rate of return on deposits is $i_{t-1}^D - \pi_t$.

Household choices

We focus on equilibria in which the liquidity constraint is binding and households are indifferent between deposits and credit lines. We write households' marginal utility of wealth as $e^{-\gamma_t}$. The first order conditions for consumption and credit lines then imply

$$i_t^L = \bar{v} (e^{\gamma_{t+1}} - 1) = \bar{v} \gamma_{t+1},$$

where the second equality uses $\bar{v} \leq 1$ together with the fact that i_t^L is a small decimal number. Since utility is linear in consumption, the benefit to households of an additional unit of liquidity arranged for next period is measured by $\bar{v} \gamma_{t+1}$. If households take out credit lines, they equate this marginal benefit of liquidity to the cost of liquidity i_t^L .

From the first order condition for deposits, we have

$$\bar{v} \gamma_{t+1} = \hat{\delta}_t - (i_t^D - \pi_{t+1}); \quad \hat{\delta}_t := \delta - (\gamma_{t+1} - \gamma_t)$$

Here we have defined $\hat{\delta}_t$ as the log marginal rate of substitution between wealth at date t and $t + 1$, which serves as the effective discount rate for payoffs by the household. If households hold deposits, they equate the benefit of additional liquidity to the opportunity cost of deposits, that is, the difference between the effective discount rate and the real return on deposits. Since utility is linear in consumption, deviations of the log MRS from the discount rate δ are due only to variation in the cost of liquidity. In particular, if liquidity is more expensive next period, agents effectively discount the future at a lower rate.

The household first-order condition for type j trees is

$$Q_t^j \geq e^{-\hat{\delta}_t} (Q_{t+1}^j + P_{t+1}x_{t+1}) \frac{P_t}{P_{t+1}}.$$

The condition holds with equality if the household has a positive position in type j trees. The rate of return on trees held by the household is $\hat{\delta}_t$; such trees always exist in equilibrium since banks can hold only a subset of trees.

For the liquidity constraint to bind, prices must satisfy

$$\hat{\delta}_t - i_t^D - \pi_{t+1} = i_t^L > 0. \quad (2)$$

The equality implies that households are indifferent between the two payment instruments. Indeed, households who invest in deposits must provide funds a period in advance on which they receive the real return $i_t^D - \pi_{t+1}$. Households who arrange a credit line can instead invest the funds in trees that yield δ , but must then pay the fee i^L . The inequality says that payment instruments are costly. Households' optimal choice is then to hold as few payment instruments as necessary, that is, the liquidity constraint binds.

3.2 Banks

Households own many competitive banks. We describe the problem of a typical bank which maximizes shareholder value

$$\sum_{t=0}^{\infty} \exp\left(-\sum_{\tau=0}^{t-1} \hat{\delta}_{\tau}\right) y_t^b. \quad (3)$$

Here bank dividends y_t^b are discounted at the rate $\hat{\delta}_t$, as are payoffs from all trees owned by households. The dividends are positive when banks distribute profits or negative when banks recapitalize.

Liquidity management

The typical bank enters period t with deposits D_t , outstanding credit lines L_t and reserves M_t . We want to capture the fact that enduser payment instructions may lead to payments between banks. For example, a payment made by debiting a deposit account or drawing a credit line may be credited to an account holder at a different bank. We thus assume that the typical bank receives an idiosyncratic withdrawal shock: an amount $\tilde{\lambda}_t \bar{v}(D_t + L_t)$ must be sent to other banks, where $\tilde{\lambda}$ is iid across banks with mean zero and cdf G . We also assume that $\tilde{\lambda}$ is bounded above: the cdf G is increasing only up to a bound $\bar{\lambda}$ with $\bar{\lambda} \bar{v} < 1$.

In the cross section, some banks draw shocks $\tilde{\lambda}_t > 0$ and must make payments, while other banks draw shocks $\tilde{\lambda}_t < 0$ and thus receive payments. Since $E[\tilde{\lambda}_t] = 0$, any funds that leave one bank arrive at another bank; there is no aggregate flow into or out of the banking system. The scale of the required gross interbank payments depends on velocity in the enduser layer. In particular, if households almost never use their payment instruments, \bar{v} is close to zero and few interbank payments are needed. The distribution of $\tilde{\lambda}_t$ depends on the structure of the

banking system as well as the pattern of payment flows among endusers.²

Banks that need to make a transfer $\tilde{\lambda}_t \bar{v}(D_t + L_t) > 0$ can send reserves they have brought into the period, or they can borrow from other banks. The bank liquidity constraint is

$$\tilde{\lambda}_t \bar{v}(D_t + L_t) \leq (1 + \gamma) (M_t + F_{t+1}), \quad (4)$$

where $F_{t+1} \geq 0$ is overnight borrowing and $\gamma \geq 0$ is a parameter that captures the efficacy of netting arrangements among banks.

If $\gamma = 0$, there is no netting: all interbank transfers must be made using either reserves M_t or overnight credit F_{t+1} . More generally, $\gamma > 0$ allows banks to make more than one dollar of transfers per dollar of liquid funds $M_t + F_{t+1}$. We can think of those liquid funds as downpayments in an intraday credit system. Below we describe a timing protocol to capture this idea and derive implications for the observed volume of intraday and overnight credit.

Suppose the marginal cost of overnight credit is larger than other sources of funding available to the bank. In this case – which we consider below – it is always better to take out as little overnight credit as necessary. Banks thus optimally choose a threshold rule: do not borrow overnight unless $\tilde{\lambda}_t$ is so large that the withdrawal $\tilde{\lambda}_t \bar{v}(D_t + L_t)$ exhausts both reserves and the intraday credit limit γM_t available by just paying down reserves.

For a bank that enters the period with reserves M_t , deposits D_t and credit lines L_t , the liquidity constraint (4) implies a threshold shock

$$\lambda_t = \frac{(1 + \gamma) M_t}{\bar{v}(D_t + L_t)}. \quad (5)$$

We refer to λ_t as the *liquidity ratio* of a bank. It is inversely proportional to a money multiplier that relates the total value of payment instruments to the quantity of reserves.

If the liquidity constraint (4) binds, then reserves provide a liquidity benefit, measured by the multiplier on the constraint. A bank thus obtains a liquidity benefit from reserves if it draws a sufficiently large shock $\tilde{\lambda}_t > \lambda_t$. The bank then borrows overnight

$$F_{t+1} = \tilde{\lambda}_t \frac{\bar{v}(D_t + L_t)}{1 + \gamma} - M_t = \left(\frac{\tilde{\lambda}_t}{\lambda_t} - 1 \right) M_t. \quad (6)$$

Since $\tilde{\lambda}_t \leq \bar{\lambda}$, banks can choose a liquidity ratio high enough that they never have to borrow, but can handle even the largest shock just out of reserves.

Portfolio and capital structure choice

Banks adjust their portfolio and capital structure subject to leverage costs. They invest in reserves, overnight credit and trees while trading off returns, collateral values and liquidity benefits. They issue payment instruments and adjust equity capital, either through positive dividend payouts or negative recapitalizations y_t^b . Capital structure choices trade off returns, leverage costs and liquidity costs.

²The likelihood of payment shocks is the same across banks, regardless of bank size. Since the size distribution of banks is not determinate in equilibrium below, little is lost in thinking about equally sized banks. Alternatively, one may think about large banks consisting of small branches that cannot manage liquidity jointly but instead each must deal with their own shocks.

The bank budget constraint says that net payout to shareholders must be financed either through interest on credit lines or through changes in the bank's positions in reserves, deposits, overnight credit, or trees:

$$\begin{aligned}
P_t y_t^b &= i_{t-1}^L L_t + M_t (1 + i_{t-1}^R) - M_{t+1} - D_t (1 + i_{t-1}^D) + D_{t+1} \\
&+ (B_t - F_t) (1 + i_{t-1}) - (B_{t+1} - F_{t+1}) + \int_{\Theta^b} ((Q_t^j + P_t x_t) \theta_{t-1}^j - Q_t^j \theta_t^j) dj \\
&- e^{\pi_t} c_b (\ell_{t-1}) (\sigma (D_t + L_t) + F_t) - i_{t-1}^L L_t^b.
\end{aligned} \tag{7}$$

In the second line, $B \geq 0$ represents a positive position in overnight credit. The last line collects bank leverage costs and credit lines that banks use to pay those costs, both discussed in detail below.

The first line in (7) collects payoffs from payment instruments. Interest on reserves i_{t-1}^R accrues to the bank that held the reserves overnight, regardless of whether those reserves were used by the bank to make a payment. Similarly, deposit interest i_{t-1}^D is paid by the bank that issued the deposits in the previous period, regardless of whether the deposits were used by endusers to make a payment. Both conventions could be changed without changing the main points of the analysis, but at the cost of more cluttered notation.

In the second line of (7), banks' tree holdings must come from a subset Θ^b of all the trees available to households. One way to think about this restriction is as the result of unmodelled contracting frictions: we could interpret the fruit from trees x_t as housing services, so the trees represent all claims on housing, which consist of mortgage bonds, as well as – perhaps due to a commitment problem – housing equity. Banks and households both participate in the market for mortgage bonds, whereas only households own housing equity. Alternatively the restriction could be due to regulation, as banks cannot own stocks in some countries.

Leverage costs

If the last line in (7) were omitted, the Modigliani-Miller theorem would hold and bank capital structure would be indeterminate. We assume instead that the commitment to make future payments is costly. It takes resources to convince overnight lenders that debt will be repaid, as well as to convince customers that the bank will indeed accept and execute payment instructions. We also assume that convincing lenders and customers is cheaper if the bank owns more assets to back up the commitments, especially if those assets are safe.

To link the cost of commitment depends on banks' choices, we define the bank *leverage ratio*

$$\ell_t := \frac{\sigma (D_{t+1} + L_{t+1}) + F_{t+1}}{M_{t+1} + \rho \int_{\Theta^b} Q_t^j \theta_t^j dj + B_{t+1}}, \tag{8}$$

where σ and ρ are fixed parameters. A key feature of this ratio is that money-market mutual fund shares and outstanding credit lines are included in D_{t+1} and L_{t+1} , respectively. While both are off-balance sheet commitments of a bank, we include them in the numerator. Our perspective here is that payment instruments are valuable only if they are reliable, and the presence of collateral makes them more credible. The weight $\sigma \in [0, 1]$ allows a distinction between payment instruments and other debt.

Banks choose the leverage ratio ℓ_{t-1} at date $t - 1$ through their choice of nominal positions at that date. Banks then have to purchase real resources $c_b (\ell_{t-1}) (\sigma (D_t + L_t) + F_t) / P_{t-1}$ in the

goods market at date t to support that leverage ratio. The cost function c_b in (7) is smooth, nonnegative, strictly increasing and convex in bank leverage. We further assume that c_b slopes up sufficiently fast that banks always choose $\ell < 1$.

The denominator of the leverage ratio (8) introduces a collateral value for bank assets: the resources needed to convince customers about future commitments are smaller if the bank owns more assets. The weight ρ allows a distinction between safe assets (reserves and overnight lending) and trees, which we will later assume to be uncertain. Both the weights and the presence of outstanding credit lines L in the numerator implies that our leverage ratio (8) does not generally correspond to accounting measures of leverage.

Since leverage costs take up real resources, we need to address how banks pay for them. The details of this process are not essential and we choose an approach that simplifies formulas. Resources that support leverage chosen at date $t - 1$ are purchased by banks in the goods market at the price P_t . In order to pay for those goods, banks must arrange credit lines at other banks³. Banks thus face an additional liquidity constraint that is analogous to that for households:

$$e^{\pi_t} c_b(\ell_{t-1}) (\sigma(D_t + L_t) + F_t) \leq \bar{v} L_t^b. \quad (9)$$

Here the term e^{π_t} converts nominal debt at date $t - 1$ dollars into nominal expenditure on goods at date t . As long as the interest rate on credit lines is positive, the constraint binds in equilibrium: banks arrange for a line that is just large enough to cover the leverage costs that will accrue next period. We use the same velocity parameter \bar{v} as for the household liquidity constraint – in equilibrium this will lead to a quantity equation that ties the price level to total payment instruments and total output.

Bank optimal choices

The bank first-order conditions are derived in the appendix. Since the bank problem exhibits constant returns to scale, they only pin down the leverage ratio ℓ and the liquidity ratio λ . As long as the two ratios are chosen optimally, banks are indifferent between positions in all assets and liabilities that have effective rates of returns equal to the rate of return on equity $\hat{\delta}$. Effective rates of returns take into account not only future payoffs, but also the effects of the asset or liability position on leverage cost and the liquidity constraint.

In particular, the presence of leverage costs leads to a determinate optimal leverage ratio. Given households' indifference (2) between the payment instruments, all instruments provide a liquidity benefit to endusers and therefore require a lower pecuniary benefit. From the bank side, providing payment instruments thus taps a source of funds that is “cheap” in pecuniary terms: both issuing deposits and issuing equity together with extending credit lines is cheaper than just issuing equity. For small leverage, it thus always makes sense to provide payment instruments. Banks increase provision until the marginal leverage is high enough that further leverage is not profitable.

³Equivalently we could assume that banks must hold deposits at other banks.

3.3 Government & equilibrium

We treat the government as a single entity that comprises the central bank and the fiscal authority. The government issues reserves M_t , borrows B_t^g in the overnight market and chooses the reserve rate i_t^R . The government also makes lump sum transfers to households so that its budget constraint is satisfied every period. Below we further consider particular policies that target endogenous variables such as the overnight interest rate. Such policies are still implemented using the basic tools M_t , B_t^g and i_t^R .

Just like financial firms, the government incurs a cost of issuing debt, above and beyond the pecuniary cost. The government differs from firms in that it has the power to tax and hence the (implicit) collateral that is available to it. We define government leverage as $\ell_t^G = (M_{t+1} + B_{t+1}^g) / P_{t+1} \Omega_{t+1}$ and denote the date t government leverage cost as $e^{\pi_t} c_g(\ell_{t-1}^G) (M_t + B_t^g)$ where c_g is increasing and convex, as is the bank leverage cost function c_b . The more real debt $(M_t + B_t^g) / P_t$ the government issues relative to the labor income tax base Ω_t , the more resources it must spend to convince lenders that it will repay. In order to pay leverage cost, the government is required to arrange a credit line from banks, denoted L_t^g .

Market clearing

Equilibrium requires that markets clear at the optimal choices of banks and households, taking into account government policy. Tree market clearing requires that banks or households hold all trees. The overnight credit market clears if borrowing by banks F_t plus government borrowing B_t^g equals aggregate bank lending B_t . Banks must hold all reserves. Since the cross sectional distribution of bank portfolios is indeterminate, we now use the symbols M_t , L_t , D_t etc to denote *aggregate* bank positions.

The goods market clears if households consume the endowment and all fruit from trees, net of any resources spent by banks and the government as leverage costs. We denote the total quantity of goods sold at date t by T_t . In nominal terms, goods market clearing means

$$P_t T_t = P_t C_t + e^{\pi_t} c_g(\ell_{t-1}^g) M_t + e^{\pi_t} c_b(\ell_{t-1}) (\sigma (D_t + L_t) + F_t).$$

Below we will assume that $T_t = \Omega_t + x_t$ is constant over time and only the composition of transactions is determined in equilibrium. For example if banks and the government are more levered, then consumption must be lower.

The market for payment instruments clears if deposits and credit lines offered by banks equal payment instruments demanded by households plus credit lines arranged by banks and the government. The model does not introduce a functional difference between credit lines and deposits. To emphasize their similarity, we focus on equilibria with $i_t^L = \hat{\delta}_t - (i_t^D - \pi_{t+1})$ and treat the two types interchangeably. Market clearing in the market for payment instruments requires that $D_t^h + L_t^h + L_t^b + L_t^g = D_t + L_t$. If the three goods- market liquidity constraints (for households, banks and the government) all bind, the total real quantity of payment instruments satisfies

$$P_t T_t = \bar{v} (D_t + L_t). \quad (10)$$

In order to for society to handle the transactions T_t , banks must supply a positive amount of payment instruments in real terms. Given a finite real value of amount of collateral, banks thus

incur leverage costs. As a result, payment instruments are costly to endusers and endusers' liquidity constraints bind: (10) holds with equality and works like a quantity equation that relates the price level to the nominal supply of payment instruments.

While enduser liquidity constraints always bind, banks' liquidity constraints may or may not bind, depending on how many real reserves are available relative to transactions T_t as well as other collateral. We say that reserves are *scarce* at date t if the threshold shock is smaller than the upper bound of the shock distribution, $\lambda_{t+1} < \bar{\lambda}$, so that the bank liquidity constraint binds with positive probability at date $t + 1$. In contrast, reserves are *abundant* if $\lambda_{t+1} \geq \bar{\lambda}$ so banks are sure that the constraint will not bind. In principle, reserves could be scarce at date t even though some banks are constrained at date t itself.

The appendix derives a system of equations that characterize equilibrium. It describes the dynamics of the endogenous prices – the interest rates i_t and i_t^L , the price of bank trees Q_t^j , the nominal price level P_t – and three variables that describe bank balance sheets – aggregate payment instruments $D_t + L_t$ as well as the liquidity ratio λ_t and the leverage ratio ℓ_t , both of which are equal across banks in equilibrium. The appendix further derives an approximation to the system that works as long as rates of return are small numbers, which can be guaranteed by assumptions on exogenous policy parameters. The following sections then study the steady state of that system for different exogenous parameters.

Neutrality of nominal government liabilities

Government policy determines the total amount of nominal liabilities $M_t + B_t^g$ as well as their composition. If the path of both reserves and government debt are increased by the same factor, then the path of the nominal price level P_t is also increased by that factor. Mechanically, reserves and government debt appear in the equations characterizing equilibrium only in the form of real reserves $m_t = M_t/P_t$ and real borrowing B_t/P_t . This is because our model assumes flexible prices and no nominal rigidities in the private sector.

The neutrality property here differs from that in cash-in-advance models in which reserves (or currency) directly provide liquidity services to households. In the latter models, an increase in the money supply alone increases the price level in the same proportion – the outstanding amount of nominal government debt is not important. This is not true in our model because banks can use government debt as an additional nominal collateral to produce nominal payment instruments.⁴

3.4 Timeline

To interpret the model and connect to the motivating facts in Section 2, suppose time is divided into days. At the beginning of the day, endusers have prearranged payment instruments with banks: they have money in a deposit account or a credit line. Those funds are more liquid than securities – enduser would have to first sell their securities in exchange for payment instruments. Enduser pay for their transactions T in the morning before selling any securities. They pay by making payment instructions to their bank, either arranging a transfer out their deposit

⁴The special role of government thus comes from the endogeneity of deposits and their exclusive use as a medium of exchange. It does not come from “non-Ricardian” fiscal policy. In our model, government surplus is not exogenous but instead adjusts so as to satisfy the government budget constraint.

account or asking to draw on their credit line.

Table 1: Bank balance sheet at beginning of day t

Assets	Liabilities
Reserves M_t	Equity
Overnight lending B_t	Overnight borrowing F_t
Trees $\int_{\Theta^b} Q_t^j \theta_{t-1}^j$	Deposits D_t

off balance sheet: loan commitments L_t

Banks enter the period with a balance sheet as in Table 1. The asset side consists of reserves, overnight lending, and trees. The liability side has equity, overnight borrowing, and deposits. In addition, the bank has extended loan commitments (credit lines) that are formally off-balance sheet. To clarify the role of these loan commitments, it is helpful to compare two extreme versions of a bank. If $L_t = 0$, we have a textbook bank-balance sheet where payment instruments are all in the form of on-balance sheet debt. However, we can also imagine a bank with zero deposits, $D_t = 0$. If the bank has made loan commitments L_t , it has assembled a portfolio of collateral that is funded with equity. Another off-balance sheet item that we include in our definition of payment instruments are money-market mutual funds sponsored by the bank. Shares in these funds are typically held in trust for the client. These items make our concept of leverage quite different from an accounting leverage ratio.

Consider the following timeline for the opening of markets. In the morning, after all shocks are realized and payment instructions as well as required interbank transfers are known, the first market to open is the Federal Funds market – banks decide whether to lend or borrow reserves, or keep holding them. Next, enduser payment instructions are executed. At this point, an intraday credit facility opens – this could be a private netting system such as CHIPS that lends at zero interest for repayment in the evening. Banks obtain intraday credit proportional to unborrowed plus borrowed reserves $M + F$. At mid-day, banks make payments to other banks: some of these involve transfers of reserves, whereas others are credited to other banks' intraday credit positions. Finally, in the evening the intraday credit positions are settled and banks adjust securities portfolios as well as capital structure.

Table 2: Bank balance sheet at mid-day on day t : bank with $\tilde{\lambda}_t > 0$

Assets	Liabilities
Reserves $\max \left\{ M_t - \tilde{\lambda}_t \bar{v} (D_t + L_t), 0 \right\}$	Equity
Overnight lending B_t	Overnight borrowing F_t
Trees $\int_{\Theta^b} Q_t^j \theta_{t-1}^j$	Deposits $(D_t + L_t) \left(1 - \tilde{\lambda}_t \bar{v} \right)$
Intraday loans to endusers L_t	New borrowing $F_{t+1} = \max \left\{ \tilde{\lambda}_t \frac{\bar{v}(D_t + L_t)}{1 + \gamma} - M_t, 0 \right\}$
	Intraday borrowing $\min \{ \tilde{\lambda}_t \bar{v} (D_t + L_t) - M_t, \gamma M_t \} + \gamma F_{t+1}$

Table 2 shows the balance sheet of a bank at mid-day after all payments have been executed. We consider a bank that has received a positive shock $\tilde{\lambda}_t$ and therefore experiences an outflow of funds. There are several changes relative to the beginning of the day. First, credit lines have been drawn: they now show up on the asset side as intraday loans to endusers. On the liability side, drawn credit lines add to deposits – lending creates inside money. However, a share $\tilde{\lambda}_t \bar{v}$ of all deposits then leaves to other banks. How the bank handles the outflow depends on the size of the shock. While the bank is indifferent between reserves and intraday credit within some range, we illustrate the mechanics assuming a “pecking order”: banks first exhaust reserves, then turn to intraday credit and finally tap the overnight market.

For a small outflow $\tilde{\lambda}_t \bar{v} (D_t + L_t) < M_t$, the deposit outflow simply shortens the balance sheet – fewer deposits are matched by fewer reserves. For an intermediate outflow, $M_t < \tilde{\lambda}_t \bar{v} (D_t + L_t) < M_t(1 + \gamma)$, unborrowed reserves are sufficient as collateral for intraday credit to fund the outflow. There is no new overnight borrowing, but the drop in deposits outflow is offset partly by the reduction of reserves to zero and partly by intraday borrowing. Finally, for a large outflow $\tilde{\lambda}_t \bar{v} (D_t + L_t) > M_t(1 + \gamma)$, the bank turns in addition to the overnight market, in addition to the above position changes there is also an increase in new overnight borrowing.

A bank that receives an inflow of funds ($\tilde{\lambda}_t < 0$) shows an increase of deposits on the liability side of the balance sheet. On the asset side, three positions may increase to offset this increase: reserves, new borrowing B_{t+1} and a positive position relative to the intraday credit system. Lender banks are indifferent between these options. All that is determinate in equilibrium is that the Federal funds market clears and that the intraday credit system nets out to zero. Because the shock $\tilde{\lambda}_t$ is iid with mean zero, we can always choose lender positions with these properties.⁵

4 Steady state equilibrium

We now consider steady state equilibria with constant output $\Omega + x$ and constant rates of return. We thus restrict attention to policies that imply constant growth rates for the nominal quantities M_t and B_t^g so that B_t^g/M_t is also constant. With constant rates of return, the marginal rate of substitution of wealth across dates is constant, we have $\hat{\delta}_t = \delta$. Moreover, the key ratios chosen by banks, leverage ℓ and the liquidity ratio λ , are constant over time. With fixed output, payment instruments and the price level also grow at same rate as nominal government liabilities. The nominal price level and the real quantity of reserves are endogenous and depend on how many payment instruments banks produce for a given level of reserves.

Our analysis below relies on comparative statics of steady states. The time period in the model should be thought of as very short, such as a day. Moreover, the model has limited scope for transition dynamics: Appendix A.4 provides conditions such that the transition from one steady state to the next takes only one period.⁶ In particular, changes in the date 0 price level

⁵If we had written the model with an explicit Fed Funds market with its own interest rate, then the overnight rate and the Fed Funds rate would be equated in equilibrium.

⁶The reason that there is any scope for transition dynamics comes from the presence of interbank credit in bank balance sheets when reserves are scarce. We show that open market policy can offset this effect and guide the economy to the new steady state after one period. The size of the extra policy depends on outstanding

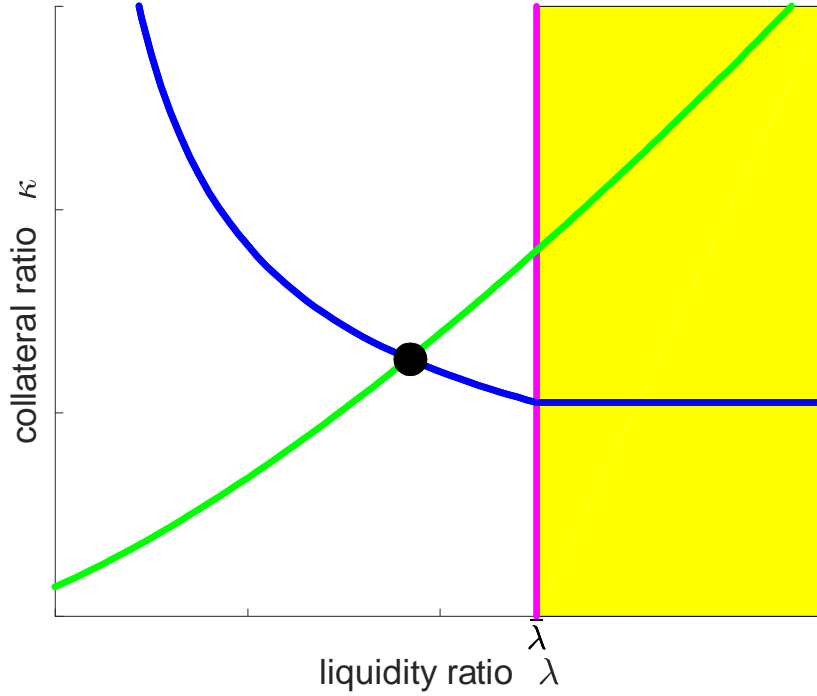


Figure 3: The green line is the capital structure curve. The blue line is the liquidity management curve. In the yellow shaded region, reserves are abundant, $\lambda \geq \bar{\lambda}$.

across steady states can be interpreted as inflation or deflation in response to an unanticipated permanent shock. We are therefore comfortable using the model for thinking about the behavior of asset prices, payments and credit over a sequence of years, such as the recent boom bust cycle.

4.1 Graphical analysis

The predictions of the model can be characterized by reducing the system of equations characterizing equilibrium to only two equations in two key variables: the bank collateral ratio, which we defined as the inverse $\kappa = 1/\ell$ of the leverage ratio, and the liquidity ratio λ . The first equation – the capital structure curve – describes the amount of collateral κ that banks need to handle transactions T for given liquidity λ . It depends on policy because the government can change the mix of collateral available to banks.

The second equation – the liquidity management curve – describes liquidity management: banks' optimal choice of reserves λ that equates the opportunity costs of holding reserves to its collateral and liquidity benefits given the collateral ratio κ . The curve depends on policy because the government can change the opportunity costs of holding reserves. Properties of equilibrium can be studied by plotting the two curves in the (λ, κ) -plane, as in Figure 3. Equilibrium is described by their intersection and its welfare properties can also be studied graphically since the welfare costs of leverage can be written as functions of λ and κ .

interbank credit; it is zero when reserves are abundant.

A figure in (λ, κ) -plane helps think about not only the liquidity and collateral ratios, but also a number of other variables that are related in simple ways to those ratios in equilibrium. In particular, location of the equilibrium along the vertical axis provides insight on real asset values, whereas location along the horizontal axis contains information about bank balance sheets and the price level. We now explain these relationships. Derivations of all equations are collected in the appendix.

The scarcity of reserves and the overnight credit market

The (λ, κ) -plane splits into two halves that correspond to scarce and abundant reserves, respectively. The vertical pink line in Figure 3 marks the critical liquidity ratio $\bar{\lambda}$ such that reserves are abundant if $\lambda_t \geq \bar{\lambda}$ (the yellow region shown to the right) and reserves are scarce if $\lambda_t < \bar{\lambda}$. If reserves are abundant, then banks do not borrow overnight. With scarce reserves, bank borrowing follows from integrating over all banks that receive a liquidity shock beyond λ_t :

$$\frac{F_t}{D_t + L_t} = \frac{\bar{v}}{1 + \gamma} \int_{\lambda_t}^{\bar{\lambda}} (\tilde{\lambda}_t - \lambda_t) dG(\tilde{\lambda}_t) := \frac{\bar{v}}{1 + \gamma} f(\lambda_t).$$

The function f is decreasing: if the banking system has more real reserves to handle a given amount of transactions, then less overnight credit is needed. Overnight borrowing by banks can therefore be read on the horizontal axis going left. Moreover, if more transactions can be netted within the day (higher γ), then banks again run out of reserves less often and borrow less overnight.

Asset market participation and intermediary asset pricing

Except for the restriction that reserves must be held by banks, we have not made assumptions on asset market segmentation. Instead, participation patterns of intermediaries and households are determined endogenously in equilibrium. Throughout, the basic principle at work is familiar from other models with short sale constraints: assets are held – and hence priced – by the investor who likes them the most, either because that investor derives nonpecuniary (liquidity or collateral) benefits from them or later also because that investor has more optimistic beliefs.

Consider participation in overnight credit and tree markets. Since the production of payment instruments entails leverage costs, banks obtain a positive collateral benefit from assets they are eligible to hold. As a result, households do not invest in any asset markets that banks can invest in: banks bid up the price of any asset accessible to banks until its return is below the discount rate and the asset is unattractive to households. For overnight credit and bank trees, banks are thus the only marginal investors.

The real overnight interest rate satisfies the bank Euler equation

$$\delta - (i - \pi) = \text{mb}(\kappa). \quad (11)$$

where $\text{mb}(\kappa)$ is the marginal benefit of a unit of collateral $c'_b(1/\kappa)/\kappa^2$. Since the collateral ratio $\kappa = 1/\ell$ is inversely related to leverage, the condition resembles other “intermediary asset pricing” equations in the literature, as it draws a connection between returns and leverage. Importantly, our concept of leverage is special and reflects the provision of payment instruments as opposed to book or market leverage in an accounting sense.

The real interest rate $i - \pi$ is positively related to the collateral ratio κ . In Figure 3, the real rate can be read on the vertical axis going up. Intuitively, less collateralized banks are willing to pay more for collateral and bid up prices, thus lowering returns. The same logic applies to bank trees, which are priced by a similar Euler equation. In steady state, cash flows on bank trees, denoted x^b are discounted at a low rate that accounts for the collateral benefits: their steady state value is $x^b / (\delta - \rho \text{mb}(\kappa))$. In the figure, it can be read on the vertical axis going down.

Payment instruments and the nominal price level

Location along the horizontal axis contains information about payment instruments and the price level. The “money multiplier” $(D + L)/M$ is inversely proportional to λ and can be read on the horizontal axis moving left. The nominal price level is determined by nominal government liabilities. At date zero we have

$$P_0 = \frac{1}{\lambda} \frac{1 + \gamma}{1 + B_0^g/M_0} \frac{M_0 + B_0^g}{T}. \quad (12)$$

For given nominal liabilities $M_0 + B_0^g$, we can read off the price level on the horizontal axis moving left as long as the split of government debt into reserves and bonds B_0^g/M_0 does not change over time. For comparative statics that change that mix, for example when we look at the result of open-market operations, the price level no longer changes one for one with λ , but the figure is still helpful to see the effect on the price level, as explained below.

The capital-structure curve

How much liquidity λ is required in order for banks to manage transactions T with given bank collateral κ ? Steady state liquidity and collateral are linked by

$$\kappa = \frac{\lambda \left(1 + \frac{B_0^g}{M_0}\right) \frac{\bar{v}}{1+\gamma} + \rho \bar{v} \frac{x^b}{(\delta - \rho \text{mb}(\kappa))T} + \frac{\bar{v}}{1+\gamma} f(\lambda)}{\sigma + \frac{\bar{v}}{1+\gamma} f(\lambda)}. \quad (13)$$

We refer to pairs (λ, κ) that satisfy this relationship it as the *capital-structure curve*. Since B_0^g/M_0 is exogenous, the curve also tells us how much government leverage is required to handle transactions with given bank leverage.

The capital-structure curve slopes up in the (λ, κ) -plane, as shown as green line in Figure 3. The basic intuition is that the real quantity of payment instruments $\sigma T / \bar{v}$ is pinned down by the volume of transactions. Since reserves contribute to collateral, we have that the collateral ratio increases with λ . As a stark example, consider a “narrow bank” which holds no trees or overnight credit – its only collateral is reserves or short term government debt. The collateral ratio is then simply the ratio of reserves to payment instruments: $\kappa = (1 + B_0^g/M_0) \lambda \bar{v} / \sigma (1 + \gamma)$. For a narrow bank, the capital-structure curve is thus a straight upward-sloping line through the origin. For a bank which holds mostly trees as collateral, the capital-structure curve is almost horizontal.

In general, two more subtle effects further contribute to an upward slope. First, banks with higher liquidity ratios run out of reserves less often, which results in lower outstanding interbank credit and hence a higher collateral ratio. Indeed, since every dollar of interbank

credit is both an asset and a liability to the banking sector, a reduction in interbank credit also increases overall collateral. Second, more collateralized banks compete less for trees, so tree prices and hence collateral values fall. As the liquidity ratio increases, the collateral ratio has to rise to compensate this effect.

The capital-structure curve shifts up if banks obtain access to more collateral other than reserves. One example is an increase in the ratio of government bonds to reserves B_0^g/M_0 : if more government bonds are available to back payment instruments, then banks require fewer real reserves to achieve any given collateral ratio. An increase in the cash flow from bank trees x^b has the same effect: if more private sector assets are available to banks, then fewer reserves are required. Both shifts also entail an increase in interbank credit for given leverage ratio: this increase – which contributes to leverage – is feasible because of the increase in collateral.

The capital-structure curve is steeper if banks have more collateral that is fixed in nominal terms. Consider in particular the region with abundant reserves, where reserves and government bonds are the only nominal assets. If banks have more bonds relative to reserves, then any given change in the liquidity ratio λ implies a larger change in the collateral ratio κ , that is, we have a steeper capital-structure curve. Intuitively, an increase in λ corresponds to a decrease in the nominal price level (12) and thus an increase in the value of money. The more nominal collateral banks, the more an increase in the value of money increases the value of collateral.

The location of the capital-structure curve also depends on velocity as well as interbank netting arrangements. For a given liquidity ratio λ , an economy with higher velocity has higher collateral ratios κ – this is because fewer payment instruments are needed to handle a given amount of transactions. Similarly, for a given liquidity ratio λ , an economy with more intraday netting (higher γ) also has higher collateral ratios. This because that economy requires less overnight credit. This effect is limited to the scarce reserves region – if reserves are abundant then netting is irrelevant for collateral.

The liquidity-management curve

Optimal liquidity management equates the opportunity cost of reserves to their collateral and liquidity benefits:

$$\delta - (i^R - \pi) = \text{mb}(\kappa) + (1 - G(\lambda)) (\text{mc}(\kappa) - \text{mb}(\kappa)), \quad (14)$$

where $\text{mc}(\kappa)$ is the marginal cost of leverage $c_b(\ell) + c'_b(\ell)\ell$ which is increasing in leverage ℓ and thus decreasing in collateral $\kappa = 1/\ell$. For a given real return on reserves, this equation represents pairs (λ, κ) such that banks optimally choose their liquidity ratio given collateral κ . We refer to it as the *liquidity-management curve*.

The left hand side of (14) is the opportunity cost to banks of holding reserves, relative to shrinking the balance sheet and paying back shareholders who demand the return on equity δ . In equilibrium, it must equal the benefit of reserves as collateral as well as for liquidity purposes. The collateral benefit is $\text{mb}(\kappa)$ which is decreasing in the collateral ratio κ . The second term on the right hand side is the liquidity benefit: with probability $1 - G(\lambda)$, the bank runs out of reserves and obtains the benefit $\text{mc}(\kappa) - \text{mb}(\kappa)$.

The liquidity benefit of reserves is decreasing in the real quantity of reserves – in fact, once reserves are abundant, the liquidity benefit shrinks to zero. As long as reserves are scarce, the liquidity benefit of reserves $\text{mc}(\kappa) - \text{mb}(\kappa)$ is positive and decreasing in the collateral ratio. In

equilibrium, every dollar of interbank credit entails leverage cost for the borrowing bank but adds collateral benefits for the lending bank. Since the former effect is larger, having to borrow overnight implies a penalty for running out of reserves that decreases with collateral.

Putting together these effects, the liquidity-management curve (14) behaves much like a money demand function for banks. It consists of two pieces, as shown in Figure 3. As long as reserves are scarce, it describes a downward sloping curve in the (λ, κ) plane: a higher liquidity ratio lowers the liquidity benefit, and is consistent with the same opportunity cost only if the marginal benefit of collateral is higher – which means that the collateral ratio must be lower. Once reserves are abundant, the liquidity-management curve is flat: the economy has reached a collateral ratio such that the opportunity cost of reserves is just compensated by the collateral benefit alone.

The location of the liquidity-management curve depends on the real return on reserves $i^R - \pi$. In particular, a higher return on reserves shifts the curve up, that is, banks choose to hold more collateral for a given liquidity ratio. Intuitively, lower opportunity costs of holding reserves make it cheaper for banks to handle transactions. As a consequence, banks can afford to hold more collateral.

The location of the liquidity-management curve also depends on interbank netting arrangements, at least as long as reserves are scarce. Greater “netting efficiency” (higher γ) shifts the curve to the left: if intraday credit is more effective at making payments, a lower liquidity ratio is enough to handle them. A leftward shift implies that the curve remains unchanged in the abundant reserves region where netting is not important. At the same time, the mix of collateral available in the economy – for example how many trees are available relative to government liabilities – is not relevant for liquidity management and does not affect the location of the curve.

4.2 Equilibrium & changes in policy

Equilibrium real reserves and leverage are determined by the intersection of the capital structure and liquidity management curves. Whether reserves are scarce or abundant in equilibrium is determined by the interaction of policy and the availability of collateral. In particular, the government can set the real rate on reserves so as to select its preferred region. For example, increasing the real rate on reserves shifts the liquidity-management curve down towards an equilibrium with abundant reserves.

At the same time, holding fixed policy, an equilibrium with abundant reserves obtains when collateral is relatively scarce: as discussed above, a decrease in the cash flow of trees accessible to banks x^b or a decline in the relative amount of government bonds B_t^g/M_t shifts the capital-structure curve up, and reserves become relatively more abundant.

We now consider comparative statics of steady states. The time period in the model should be thought of as very short, such as a day. We are therefore comfortable using the model for thinking about the behavior of asset prices, payments and credit over a sequence of years, such as the recent boom bust cycle. In particular, we are interested in the effect of shocks to agents’ belief about future asset payoffs (such as those on claims to housing) as well as monetary policy responses and study those effects as a sequence of comparative statics.

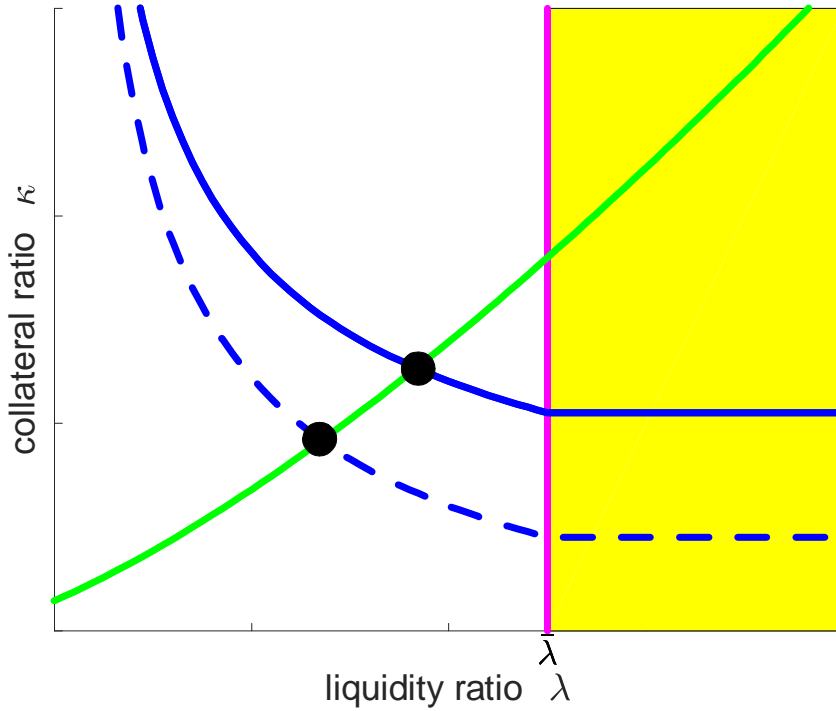


Figure 4: Faster growth rate of nominal liabilities when reserves are scarce.

Monetary/fiscal policy & interest on reserves

Since the steady state rate of inflation equals the growth rate of reserves, a government that commits to a growth rate of nominal liabilities effectively controls the real rate on reserves. This specification helps think about two policy regimes. One is traditional: the government sets debt and reserves while interest on reserves is zero. The other – more recently popular – regime is one where the central bank has issued so many reserves that they are abundant, and the spread between i and i^R is zero. The central bank then makes policy by moving around the interest rate on reserves which can be positive or negative.

Consider how changes in monetary policy affect equilibrium in the two regions. First, suppose we start from an initial equilibrium with $i^R = 0$ and scarce reserves – the typical situation in many countries before the recent financial crisis. Suppose now the government decides on faster growth of nominal liabilities. Mechanically, the capital-structure curve remains unchanged, whereas the liquidity-management curve shifts down. Figure 4 shows the shifted curve as a dashed line.

Banks optimally respond to higher inflation and hence higher opportunity costs of holding reserves by holding less collateral for any given liquidity ratio. They provide more nominal payment instruments relative to reserves (the “money multiplier” increases), which is inflationary in the short run. Since reserves are more scarce, overnight credit also increases. At the same time, banks compete more for collateral and bid up asset prices: the price of bank trees increases and the real overnight interest rate falls.

In comparison, consider raising the reserve rate when reserves are already abundant, as the Fed did in December 2015. Assume further that the government continues to commit to the

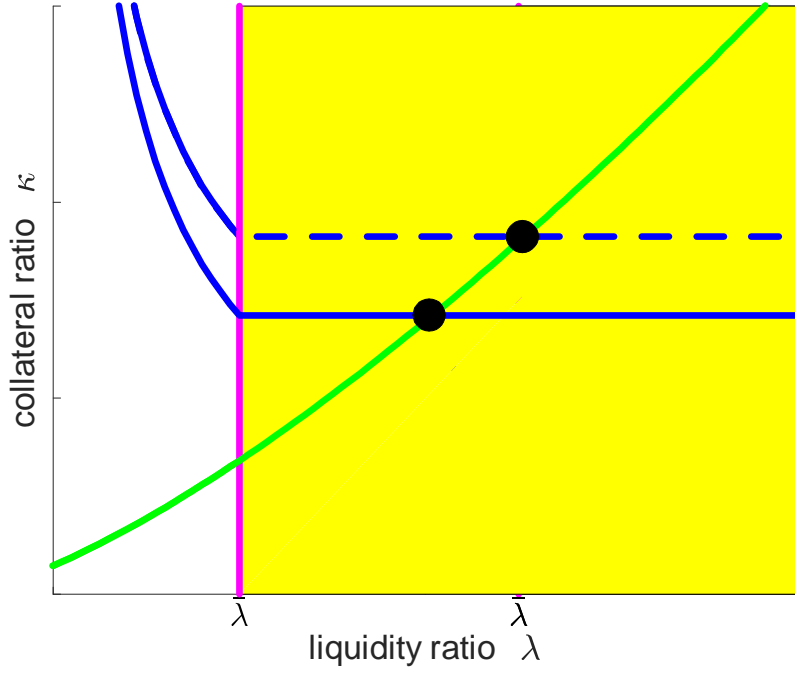


Figure 5: Higher interest on reserves i_R when reserves are abundant

same constant growth rate π of nominal liabilities. The mechanical effect is the opposite of the above: as the real rate on reserves increases, the liquidity-management curve shifts up, as in Figure 5. As the opportunity cost of holding reserves falls, banks find it profitable to handle the same payments with more collateral. They produce fewer payment instruments relative to reserves, the money multiplier shrinks and there is deflationary pressure in the short run. Since reserves are already abundant and now become even more attractive, the overnight credit market remains inactive. Banks also value additional collateral less, so that asset prices decline.

The two examples show that changes to the opportunity costs of holding reserves have intuitive effects. In particular, expansionary policy (such as faster money growth) lowers interest rates and is inflationary in the short run, whereas contractionary policy (such as a higher nominal rate on reserves) does the opposite. We emphasize that the effects are *permanent* – they result from comparative statics across steady states. This is in contrast to many models with sticky prices or segmented markets, where “liquidity effects” on the real interest rate are temporary phenomena. Permanent effects arise in our model because the opportunity costs of holding reserves lead banks to change the way they produce payment instruments, with effects on the cost of leverage and the value of collateral.

Open market operations & the collateral mix

By engaging in open-market policy – for example, a swap of reserves for government bonds – the government can alter the tradeoff between bank leverage and government leverage required to handle transactions. In other words, it shifts the capital-structure curve. This is different from changes to the real return on reserves that alters the tradeoffs in liquidity management. To illustrate, consider a comparative static that increases reserves and offsets this change by an equal change in bonds: we move from an initial equilibrium with (B_0^g, M_0) to a new equilibrium with $(\tilde{B}_0^g, \tilde{M}_0)$, where $\tilde{M}_0 - M_0 = B_0^g - \tilde{B}_0^g > 0$. As before, reserves and bonds then grow at

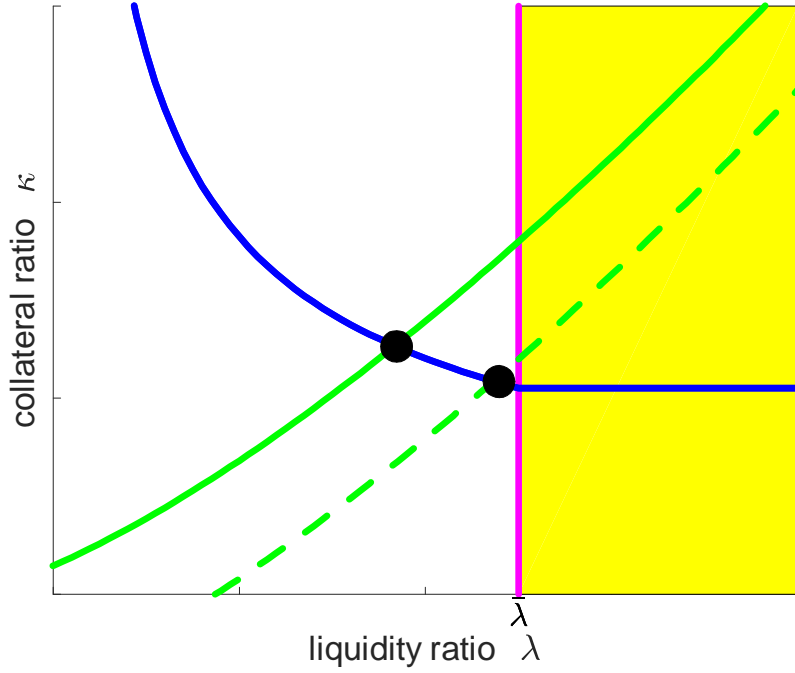


Figure 6: Open market operations when reserves are scarce

the same rate π throughout.

The expansionary open-market operation can be summarized by a decrease in the ratio B_t/M_t which shifts the capital-structure curve to the right. If reserves are abundant in the initial equilibrium, then the policy has no effects: it does not change nominal liabilities so the price level remains unchanged, as do leverage and interest rates. The horizontal shift in the curve thus reflects the change in nominal reserves $\tilde{M}_0 - M_0$.

In contrast, when reserves are scarce as in Figure 6, then a purchase of bonds with reserves is inflationary in the short run and lowers the real interest rate. Mechanically, since the liquidity-management curve slopes down, the change in real reserves is smaller than $\tilde{M}_0 - M_0$, indicating an increase in the price level. Intuitively, when more liquid reserves are available, competition drives banks to produce more payment instruments, pushing the price level up. As a result, the real value of nominal collateral falls – banks have less collateral and bid up the prices of collateral including short bonds, a permanent “liquidity effect” on the real interest rate.

4.3 The role of the nominal interest rate

Some central banks conduct monetary policy by following a nominal interest rate rule. In practice, the rule is typically implemented by open-market policy. For example, during the scarce reserves regime in place in the US until 2008, the New York Fed’s trading desk bought and sold bonds of various maturities in exchange for reserves in order to move the overnight interest rate (the Federal Funds rate) close to the Fed’s target. More recently, as reserves have become abundant, the Fed Funds rate and the interest rate on reserves have been essentially the same, and the policy lever is the interest rate on reserves. It is then tempting to simply transfer

existing analysis of interest-rate rules to the abundant reserves environment even though the policy implementation is different.

In many monetary models, the details of how the central bank implements the interest-rate rule are indeed irrelevant – the nominal interest rate alone summarizes the stance of monetary policy. In particular, many models use households’ optimal choice between currency and short-term bonds to derive optimal real balances as a function of the nominal interest rates and consumption. At the same time, intertemporal asset pricing equations – and possibly price setting equations – imply a path for inflation. The path for the money supply can then be inferred ex post so as to generate the implied path for real balances, but is often omitted from the analysis altogether. In particular, it does not matter whether policy is implemented with open-market purchases or interest on reserves.

In our model, policy cannot be summarized by the nominal interest rate alone. As discussed above, policy matters in two ways. Policy can change the nominal rate on reserves. Moreover, policy can change the collateral mix between reserves and government bonds, which matters as long as reserves are scarce. Both policy actions affect the nominal interest rate. First, with scarce reserves, the same nominal interest rate can be achieved with many combinations of interest on reserves and open-market purchases that have different implications for real interest rates, inflation, and real reserves. Second, with abundant reserves when open-market purchases are irrelevant, interest on reserves is the key policy tool.

Interest rate policy with scarce reserves

Consider first the case of scarce reserves. We start from an initial equilibrium in that region; it is generated by initial parameters i^R, π and B_0/M_0 and implies some initial overnight rate. Holding fixed i^R , we now choose a new target overnight rate that is above the reserve rate and ask how π and B_0/M_0 can change to implement it. To proceed graphically, we combine the first-order condition for overnight lending (11) and the liquidity-management curve (14) to trace out all equilibrium pairs (λ, κ) that are consistent with the spread $i - i^R$:

$$i - i^R = (1 - G(\lambda)) (\text{mc}(\kappa) - \text{mb}(\kappa)). \quad (15)$$

The spread is the opportunity cost of holding reserves rather than lending overnight. It must be equal to the liquidity benefit of reserves on the right hand side since the collateral benefits of the two assets are the same.

The key properties of the curve in (λ, κ) plane described by equation (15) are illustrated in Figure 7. First, the red curve is downward sloping: if banks have less collateral, then overnight credit is more costly; in order to maintain the same opportunity cost of reserves $i - i^R$, there must be more liquidity λ to lower the probability $1 - G(\lambda)$ with which banks run out of reserves. Second, the curve never enters the abundant reserves region. As reserves become more abundant, collateral must fall to maintain a positive spread. Third, the new curve lies above the liquidity-management curve at the initial equilibrium. In order for the new overnight rate to be lower requires a lower liquidity benefit, which requires more collateral for given liquidity λ . Finally, the curve is independent of π and B_0/M_0 , the two parameters describing policy.

The equilibrium pair (λ, κ) not only satisfies (15), but must also lie on the capital structure and liquidity management curves. How can the government change policy to move to the new

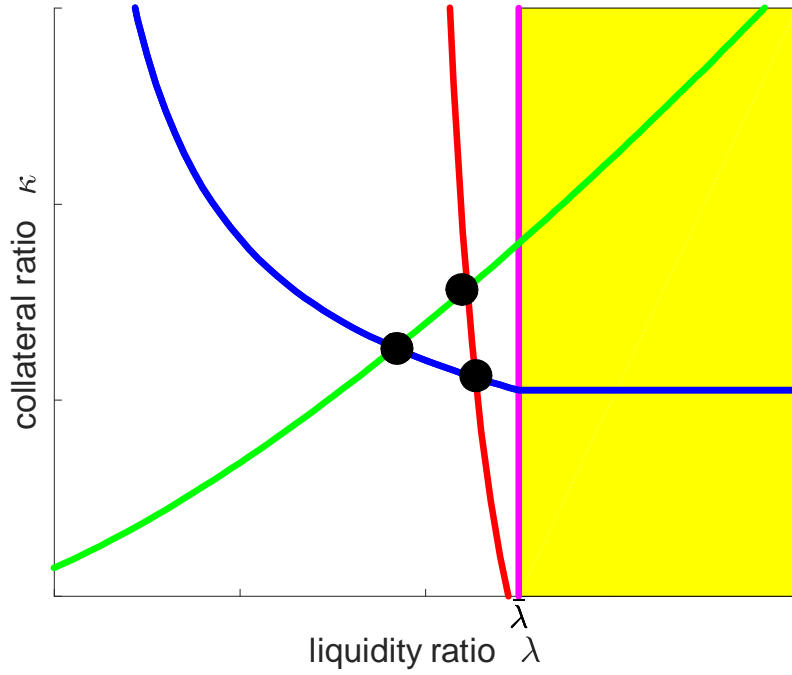


Figure 7: The red curve is equation (15)

lower overnight rate? There are two stark options. The first option is for the government to announce a lower growth rate of nominal liabilities π , shifting the liquidity-management curve up until all three curves intersect. This policy leaves the collateral mix unchanged but only lowers the opportunity cost of reserves. The second option is for the government to engage in expansionary open-market operations, shifting the capital-structure curve to the right until all three curves intersect. This policy leaves the opportunity costs on reserves unchanged but changes the collateral mix.

The two extreme policies produce the same change in the overnight nominal rate. At the same time, they have very different implications for the real interest rate and inflation, as well as for real reserves and overnight credit. In the first option, lower money growth reduces inflation which contributes to the decline in the nominal interest rate. The effect is less than one-for-one however – the equilibrium real interest rate actually increases. The reason is that lower opportunity costs of reserves increase bank collateral and thus lower the collateral benefit of overnight lending. In the other option, open-market purchases leave inflation unchanged. However, fewer bonds implies less collateral.

In addition to the two extreme policies just sketched, many other policies are also consistent with the new target nominal overnight rate. Indeed, we can combine open-market purchases with announcement of future growth of liabilities: we then shift both curves at once, rather than one at a time as for the extreme policies. The only requirement on the shifts is that the new equilibrium ends up on the curve described by equation (15). In particular, the same nominal interest rate is compatible with an entire range of bank liquidity ratios in equilibrium.

A key difference between our model and other models of scarce outside money is that the *only* medium of exchange for endusers is payment instruments produced by banks. Outside

money – here reserves – is only one input into the production of payment instruments. In particular banks also use government bonds as collateral to back payment instruments. As a result, the spread between the overnight rate and the reserve rate measures the scarcity of reserves for banks; it does not measure the scarcity of payment instruments in the economy as a whole. In particular, there is not a unique amount of reserves implied by a given volume of transactions and a spread.

Interest rate policy with abundant reserves

Consider policy in the abundant reserve regime. Can the government describe policy only by the single interest rate $i = i^R$? Equilibrium is described by (11) and (13), which determine λ and κ for a given real return on reserves. As a result, a nominal reserve rate alone cannot pin down bank liquidity and collateral. Similarly, a feedback rule that relates the rate on reserve to inflation, for example $i = g(\pi)$, does not uniquely determine λ , κ , inflation and the real interest rate. This is true even if we directly select a rule for the real rate as a function of π , thus eliminating possible multiplicity coming from the shape of g that has been discussed in the literature.

Our model differs from other monetary models in what happens once outside money becomes abundant. Consider first models with bonds and currency only. At the zero lower bound in such models, bonds and outside money become perfect substitutes to endusers, so the medium of exchange (currency) loses its liquidity benefit. In the current model, endusers hold neither bonds nor outside money – both are held only by banks. Equating i and i^R makes bonds and outside money perfect substitutes for banks, but does not remove the liquidity benefit of the medium of exchange, namely payment instruments produced by banks.

There are also models in which reserves, bonds and currency coexist. In such models, $i = i^R > 0$ makes bonds and reserves perfect substitutes. At the same time, currency remains a scarce medium of exchange that is relevant for some transactions. The reserve rate represents the spread between reserves and currency; it measures the scarcity of currency and relates the demand for real balances to real variables such as consumption. The tradeoff between currency and reserves is what enables those models to work with interest rate rules in the usual way even when reserves are abundant.

4.4 Optimal policy

The optimal payment system minimizes the loss of resources due to leverage. Our technological assumptions say that a given volume of transactions requires a fixed amount of payment instruments supplied by banks, as well as some outside money supplied by the government that can in turn serve as collateral for banks. Total consumption lost every period in steady state can be expressed as a function of leverage and liquidity:

$$c_g \left(\left(1 + \frac{B_0}{M_0} \right) \frac{\lambda}{1 + \gamma} \frac{T}{\Omega} \right) \left(1 + \frac{B_0}{M_0} \right) \frac{\lambda}{1 + \gamma} T + c_b (\kappa^{-1}) T \left(\frac{\sigma}{\bar{v}} + \frac{f(\lambda)}{1 + \gamma} \right). \quad (16)$$

Provided that the leverage cost of the government slopes up fast enough, the indifference curves are upward sloping and convex, as shown in Figure 8.

We consider the best steady state equilibrium that the government can select by choice

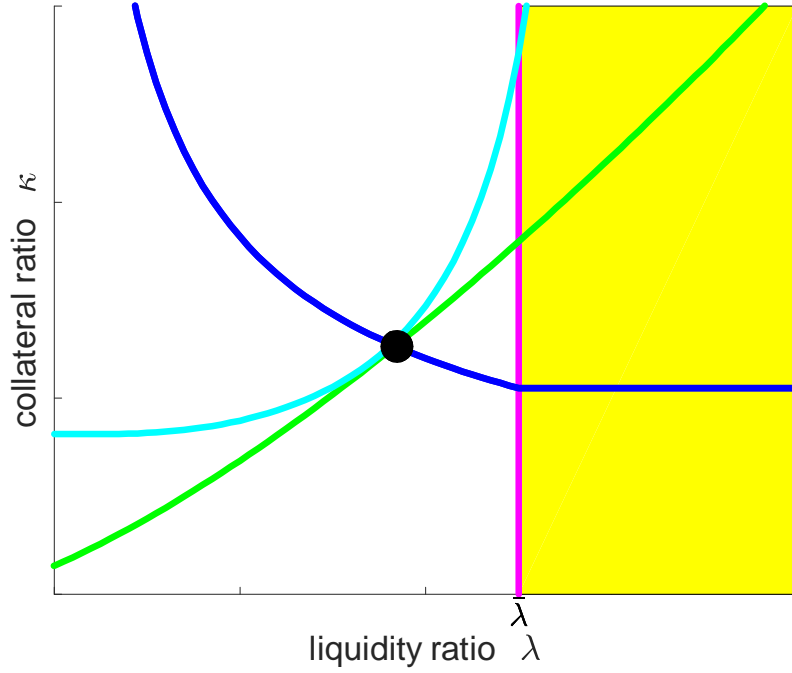


Figure 8: The cyan line is an indifference curve implied by equation (16)

of its two policy instruments, the collateral mix represented by B_t/M_t and the real return on reserves $i_R - \pi$, which determines the opportunity costs of holding reserves. The optimal policy problem is to choose those instruments together with λ and κ to minimize losses (16) subject to the capital-structure curve (13) and the liquidity-management curve (14).

If the government can freely choose the ratio of bonds to reserves, it is optimal to set $B_t/M_t = 0$. Indeed, while bonds and reserves provide the same collateral services, reserves also provide liquidity services, which lowers the need for interbank borrowing and hence costly bank leverage. Since the model focuses on the provision of payment services, there is no benefit of government bonds per se, nothing is lost by just issuing reserves. More generally, the way fiscal policy is conducted independently of monetary policy may imply that there is a constraint on B_t/M_t . We can then view the welfare costs as a function in λ and κ with B_t/M_t a fixed parameter.⁷

The return on reserves directly affects neither the welfare cost nor the capital-structure curve. We can therefore find the optimal solution in two steps. We first find a point (λ, κ) on the capital-structure curve (for given B_t/M_t) that minimizes (16). The optimal real return on reserves is then whatever return shifts the liquidity-management curve so that the equilibrium occurs precisely at that optimal point. If the indifference curves are convex and the capital-structure curve is curved less – which is a reasonable assumption if the effects of interbank credit are relatively small – then we obtain an interior solution as shown in Figure 8.

Should reserves be abundant? The figure suggests that this is not necessarily the case.

⁷ Alternatively, we could capture fiscal policy by a given real amount of bonds, say b . The welfare cost can then be written with total debt equal to $m + b$ as opposed to $m(1 + B_t/M_t)$. The basic tradeoff remains the same.

Indeed, if the government leverage cost curve slopes upward very steeply, then it may be optimal to run a system with scarce reserves, in which real government leverage is much lower than the debt required to run the payment system. It is better to have banks rely on other collateral in order to back payment instruments. However, if the government can borrow cheaply at will, so that its leverage cost is close to zero, then it makes sense to move towards narrow banking where reserves make up the lion's share of bank portfolios.

5 Securities markets and the payment system

In this section we consider the interplay between securities markets and the payment system. We maintain throughout our focus on steady states. We first introduce uncertainty premia, the key source of fluctuations in asset prices. This allows to discuss the effect of uncertainty shocks on the supply of payment instruments as well as unconventional monetary policy – the government buys trees that carry uncertainty premia. These questions can be studied even if the only link between tree (that is, securities) markets and the payment system is that banks invest in trees.

We then extend the model to introduce two additional links: banks lend overnight to institutional investors and institutional investors use payment instruments to trade assets. To clarify the effect of each link in isolation, we introduce two types of asset management firms: *carry traders* buy trees on margin, whereas *active traders* face liquidity constraints for some asset purchases. Both types of firms otherwise work like banks: they are competitive firms owned by households that have access to a subset of trees and maximize shareholder value.

5.1 Introducing uncertainty and collateral quality

We capture a change in uncertainty as a change in beliefs about asset payoffs: we assume that households behave *as if* tree dividends x are permanently lower by s percent from the next date on. Actual tree payoffs remain constant throughout. One way to think about these beliefs is that households are simply pessimistic. Our preferred interpretation is ambiguity aversion: households contemplate a range of models for payoffs, and evaluate consumption plans using the worst case model. In either case, the key effect of pessimistic valuation is to generate premia on assets: an observer (such as an econometrician measuring the equity premium) will observe low prices relative to payoffs and hence high average returns.

To define equilibrium, we must take a stand not only on beliefs about exogenous variables, but also about endogenous variables such as the nominal price level and asset prices. We follow Ilut and Schneider (2015) who also capture the presence of uncertainty with low subjective mean beliefs about exogenous variables: beliefs about endogenous variables follow from agents' knowledge of the structure of the economy. In particular, agents know the policy rule of the government and that banks maximize shareholder value given the households' discount factor. Households' worst case beliefs thus also affect bank decisions; shareholder value (3) is replaced by its worst case expectation.

Appendix A.5 characterizes steady state equilibrium with uncertainty. Mechanically, agents

live in a steady state with all variables constant, yet act as if the economy is on a transition path to a worst case steady state with lower tree payoffs in the future. In general, dynamics are characterized by first finding the transition path to derive the law of motion for endogenous variables, and then combining that law of motion with the true dynamics of the exogenous variables. We show that in the present model, the transition path converges to the worst case steady state after one period, and that the bank ratios in the actual and the worst case steady state coincide. These properties allow simple graphical analysis, as in the case without uncertainty.

Uncertainty premia on assets

Asset prices reflect uncertainty in two ways. First, agents act *as if* payoffs will be lower and hence value trees less. At the same time, however, they discount future payoffs at a lower rate. This is because they fear higher inflation and hence a higher future cost of liquidity. Indeed, while the government commits to a growth rate g of nominal liabilities, agents act as if output will fall. Real returns on nominal assets thus reflect *worst case* inflation $\tilde{\pi}$ which is higher than actual inflation. In particular, since the nominal rate on reserves is fixed, the cost of liquidity next period is perceived to be high.

It is helpful to introduce notation to compare the worst case loss of tree payoffs to the worst case drop in output. A drop in payoff of s percent implies that agents expect output to be permanently lower by $s_y = \tau s$ percent, where $\tau = x/(\Omega + x)$ is the share of tree payoffs in output. We assume that τs is a small decimal number, whereas s itself need not be. We thus allow for large losses on trees, but keep the expected loss of output bounded. This approach is designed to focus on large disruptions to the financial sector that are not accompanied by large drops in real activity.

We summarize the equilibrium effect of agents' concern with high liquidity cost by the effective discount rate

$$\hat{\delta} = \delta - (\tilde{\gamma} - \gamma) = \delta + g - \tilde{\pi} = \delta + s_y,$$

where $\tilde{\gamma}$ is the worst case log marginal utility of wealth next period. Worst case liquidity costs $\bar{v}\tilde{\gamma}$ are thus higher than current liquidity costs $\bar{v}\gamma$ if worst case inflation $\tilde{\pi}$ is higher than the growth rate of nominal liabilities g . The latter is driven by the fear of lower tree payoffs, and the effect is smaller if tree payoffs are a smaller share of output.

The presence of uncertainty generates premia on assets to compensate investors. The steady state equilibrium price of a tree held by households is

$$\frac{Q_j}{P} = u \frac{x}{\delta}; \quad u = \frac{1-s}{1-s_y} \tag{17}$$

The factor u reflects compensation for uncertainty. If $s = 0$, then $s_y = 0$ and $u = 1$, and the price is the present value x/δ . The same result obtains if $\tau = 1$ and hence $s_y = s$: if all output comes from trees, then the cash flow and discount rate effects on asset prices exactly offset. In the interesting case where tree payoffs are some nondegenerate share of output, u is strictly between zero and uncertainty lowers prices.

To see how uncertainty generates premia on assets, consider an econometrician who observes

tree prices as well as payoffs. The return on the tree measured in steady state is

$$\frac{Q_j/P + x}{Q_j/P} = 1 + \delta/u.$$

As payoff uncertainty s increases, u declines and the return on the tree increases to compensate investors. If there was also a second “safe” tree held by households that earns exactly the discount rate, the econometrician would measure an equity premium on the uncertain tree. In terms of comparative statics, an increase in uncertainty captured by an increase in s leads to higher premia and lower prices.

Uncertainty and collateral quality

It is natural to assume that trees that are more uncertain also represent worse collateral. In this section, we make the weight that trees receive in the aggregation of collateral explicitly a decreasing function $\rho(s)$ of payoff uncertainty s . A change in uncertainty thus has two effects on banks tree portfolios. There is a direct effect on prices. In addition, the fact that the trees are uncertain makes them worse collateral per dollar of funds invested in them.

The bank first-order condition for trees accessible by banks is now

$$\hat{\delta} = r_j + \rho(s) \text{ mb}(\kappa).$$

In the presence of uncertainty, banks still hold all assessable trees in equilibrium because of their collateral benefits. Returns on assets held by banks are affected by two opposing forces: compensation for uncertainty born by shareholders tends to increase returns, while the collateral benefit tends to lower returns. The presence of uncertainty thus also puts an additional wedge between the return on trees held by banks and the return on safer overnight credit.

In terms of our graphical analysis, the only change is to the value of trees in the capital-structure curve. In particular, the second term in the numerator on the right hand side of the collateral ratio (13) now becomes

$$\frac{ux^b}{(\delta - \rho(s) \text{ mb}(\kappa))T}$$

An increase in payoff uncertainty s lowers the value of bank trees and thus the value of banks’ collateral. For the collateral ratio κ to remain the same, the value of reserves must increase, which is deflationary in the short run. It follows that the capital-structure curve shifts to the right, as in Figure 9. The liquidity-management curve is not affected by changes in s .

5.2 An increase in uncertainty and policy responses

The value and collateralizability of trees affects the scarcity of reserves even if policy (described by $i_R - \pi$ and B_t/M_t) does not change. Indeed, starting from an equilibrium with scarce reserves, an increase in uncertainty lowers the price level. This is because the drop in collateral values makes it more expensive for banks to create payment instruments. As a result, the supply of payment instruments declines and generates deflation. In the new equilibrium, bank collateral ratios are lower, bank liquidity ratios are higher and the ratio of interbank credit to payment instruments is lower.

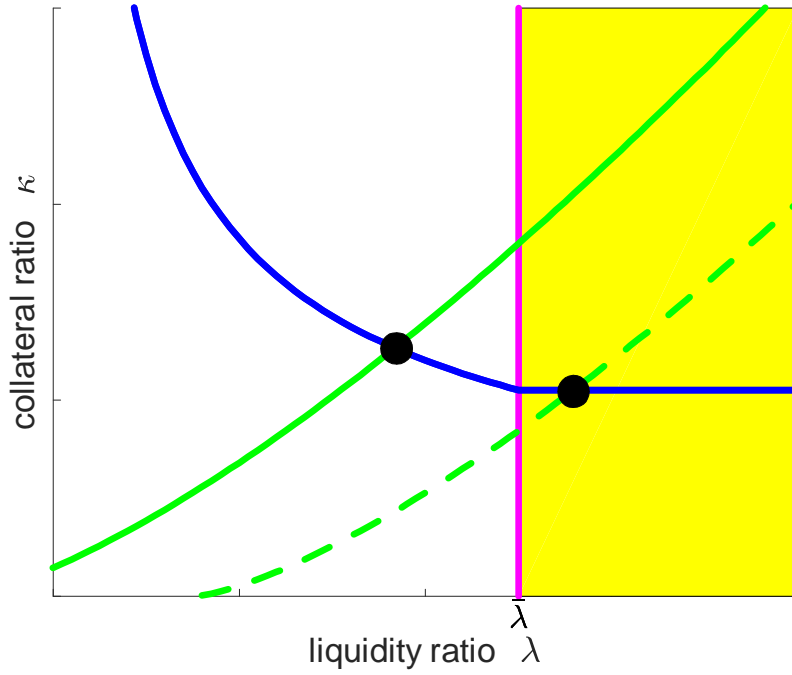


Figure 9: An increase in uncertainty shifts the solid green capital structure curve to the right, resulting in the dotted line.

If the increase in uncertainty is large enough, the capital-structure curve shifts so far to the right as to push the economy into the abundant reserves region, as shown in Figure 9. At this point, further increases in uncertainty no longer change the overnight interest rate. The real quantity of reserves is now so high relative to the real quantity of transactions that all liquidity shocks can be handled with reserves and intraday credit alone. The interbank overnight market shuts down entirely. Further increases in uncertainty do still lower the nominal quantity of payment instruments and the price level.

An increase in uncertainty is an attractive candidate for a shock that could have occurred at the beginning of the recent financial crisis. It is consistent with an increase in asset premia, a drop in uncertain asset prices, a decline in the overnight interest rate all the way to the reserve rate as well as a decline in bank collateral and an effective shutdown of interbank Federal Funds lending. However, we did not see a large deflation – after an initial small drop in late 2008 the price level remained quite stable over time.

An expansion of reserves

According to our model, a candidate for the absence of deflation is monetary policy. Suppose the Treasury issues a lot of new debt that is then purchased by the central bank in exchange for reserves. Suppose further that this is perceived as a one time change, with a stable path of nominal liabilities thereafter. In terms of the model, this policy corresponds to an increase in the outstanding nominal quantity of reserves.

In the abundant reserve regime, where reserves and other debt are perfect substitutes, an increase in reserves is neutral. The real interest rate and leverage do not change, and the economy remains in the abundant reserves regime. The only variable that changes is the price

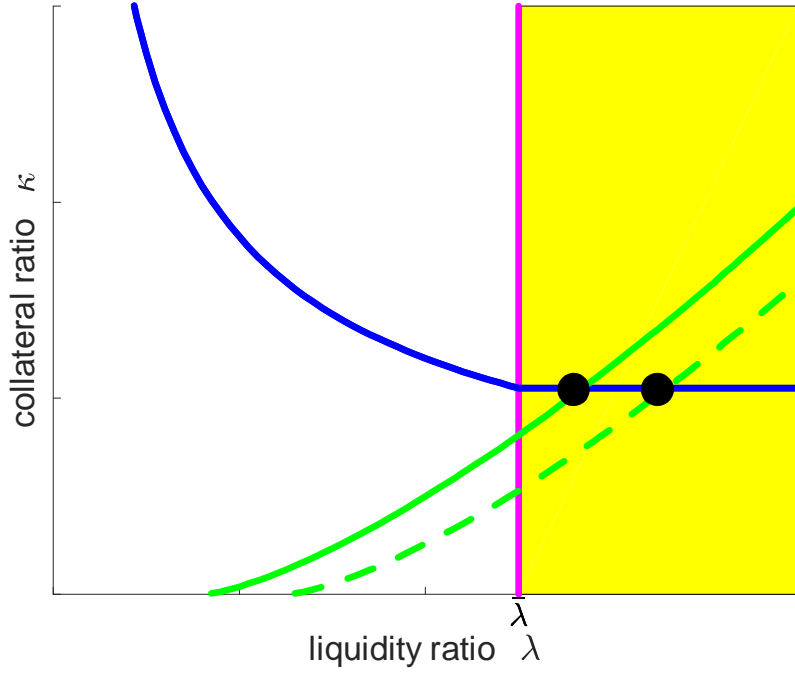


Figure 10: Central bank purchase of trees when reserves are abundant.

level which rises in proportion to the increase in reserves. We conclude that an increase in uncertainty coupled with a large injection of reserves can move the economy into a period of abundant reserves with low asset prices, collateral and real rates, and the move is not accompanied by drop in the price level.

Unconventional monetary policy

An alternative response by central banks to a decline in asset values – a collateral shortage – has been to purchase low quality collateral, such as risky mortgage backed securities. We now consider what happens when the government purchases risky trees instead. We set up an experiment analogously to the open-market purchase above. We start from an initial equilibrium with abundant reserves M_0 , price level P_0 and collateral ratio κ .

We assume that the government injects reserves to purchase all trees from banks' balance sheet, that is, new reserves are chosen such that, at the new equilibrium with reserves \tilde{M}_0 , price level \tilde{P}_0 and collateral ratio $\tilde{\kappa}$, we have

$$\tilde{M}_0 - M_0 = \frac{\tilde{P}_0 u x^b}{\delta + \rho(s) \text{mb}(\tilde{\kappa})}.$$

The effect of the purchase are just like in Figure 10. As trees are removed from bank balance sheets, the capital-structure curve moves to the right.

After the additional injection, reserves continue to be abundant, so the policy has no effect on collateral (so $\tilde{\kappa} = \kappa$) and real asset values. However, the policy does help stabilize the price level – it counteracts the deflationary effect of higher uncertainty. Indeed, collateral in the new equilibrium is \tilde{M}_0/\tilde{P}_0 which equals the real value of old reserves M_0/\tilde{P}_0 plus the full real value of trees. In contrast, collateral in the initial equilibrium was given by the real value of reserves

M_0/P_0 as well as the value of trees multiplied by the collateral quality weight $\rho(s) < 1$. Since the collateral ratio is the same in the two equilibria, it must be that $\tilde{P}_0 > P_0$.

Unconventional policy thus works by replacing low quality real collateral on bank-balance sheets with high quality nominal collateral. Since reserves continue to be abundant and the real return on reserves stays the same, this does not actually lead to an increase in real collateral. However, backed by the new reserves, banks provide more nominal payment instruments, which is inflationary in the short run. Compared with an outright increase in reserves, the inflationary effect of tree purchases is smaller since at the same trees are removed from the collateral pool.

Tree purchases by the central bank have two additional, more subtle, effects. First, it makes the capital-structure curve steeper. The slope in turn matters for the inflation response to changes in the interest rate on reserves: indeed, the steeper the capital-structure curve, the less does an increase in the return on reserves push the price level down. Second, removing trees from bank balance sheets reduces banks' exposure to further shocks to asset quality. In particular, suppose that after all trees have been bought by the government, the uncertainty shock is reversed and asset prices increase. The payment system would not react to this shock as trees no longer serve as collateral to produce payment instruments. The economy would remain in an abundant reserves environment even though the asset-market turbulence that has sent it there in the first place has actually subsided.

5.3 Carry traders

So far, the effect of asset values on bank balance sheet is direct: it requires bank investment in trees. In this section, we introduce institutional investors who borrow short term from banks in order to invest in trees. We call these investors “carry traders” – they do not actively trade trees but roll over their debt. This creates an additional link between securities markets and banks that operates even if banks only engage in short term lending. Monetary policy can affect carry traders' funding cost.

Carry traders are competitive firms that issue equity, borrow overnight and invest in the subset of Θ^* , which is distinct from the subset accessible to banks. Like banks, carry traders face leverage costs, captured by an increase concave function c^* that could be different from the cost function c_b assumed for banks.⁸ The leverage ratio of carry trader i is defined as overnight credit $F_{i,t}^*$ divided by the market value of the tree portfolio

$$\ell_{i,t}^* = \frac{F_{i,t+1}^*}{\int_{\Theta^*} Q_t^j \theta_{i,j,t}^* dj}.$$

Carry traders' marginal collateral benefit is $\text{mb}^*(\kappa)$ and their marginal cost of leverage is $\text{mc}^*(\kappa)$, respectively.

We assume further that carry traders are more optimistic about the payoff of trees in Θ^* than households: they perceive uncertainty s whereas households perceive $s^* > s$. The

⁸We do not consider welfare effects of leverage for carry traders – instead we focus on the positive implications of margin trading. We thus assume for simplicity that leverage costs of carry traders are paid lump sum to households so that they have no impact on welfare.

idea here is that the firm employs specialized employees who households trust to make asset-management decisions. As a result, the spread relevant for investment in carry trader trees *indirectly through investment by carry traders* carries the uncertainty premium s that makes these trees as desirable as other trees held directly by households.

Optimal investment and borrowing

Carry traders' first-order condition for overnight credit resembles that of banks in (A.6), except that overnight borrowing does not provide liquidity benefits: the return on equity must be smaller than the real overnight rate plus the marginal cost of leverage. We focus on steady states only and drop time subscripts. Since we already know that the real rate is lower than δ in equilibrium, it is always optimal for carry traders to borrow and we directly write the condition as an equality:

$$\delta = i - \pi + mc^*(\kappa_i^*). \quad (18)$$

It follows that all carry traders choose the same collateral ratio and we drop subscripts i from now on. Moreover, carry trader collateral is higher in equilibrium when interest rates are high.

Like banks, carry traders hold all trees accessible to them. This is due not only to the collateral benefit conveyed by trees, but also to carry traders' relative optimism. The first-order condition for tree j is

$$\delta = r^j - s + mb^*(\kappa^*).$$

When interest rates or uncertainty is low, carry traders apply a lower effective discount rate to trees, which results in higher tree prices.

The amount of carry trader borrowing in steady state equilibrium is

$$F^* = \ell^* \frac{x^*}{\delta + s - mb^*(\kappa^*)}. \quad (19)$$

Here x^* represents total dividends on carry trader trees. Lower interest rates increase both leverage and the value of collateral, and therefore increase borrowing. Moreover, an increase in uncertainty (that is, an increase in s) lowers collateral values and borrowing.

Equilibrium with carry traders

Our graphical analysis of equilibrium remains qualitatively similar when carry traders are added to the model. The only change is that carry trader borrowing now enters on the asset side of the banking sector. We can thus add in the numerator of the collateral ratio (13) a term $B^*(\kappa)$ that expresses carry trader borrowing as a function of bank collateral. We obtain the function B^* by substituting for κ^* in (19) from (18) and then substituting for the interest rate from the bank first-order condition from (A.10). The function B^* is decreasing: if banks have more collateral, the interest is higher and carry traders borrow less.

We can now revisit the effect of changes in beliefs and monetary policy in an economy with carry traders. Suppose first that, starting from an equilibrium with scarce reserves, there is an increase in uncertainty. The new effect is that, as carry traders value trees less, they demand fewer loans from banks. This lowers bank collateral and shifts the capital-structure curve to the right. The liquidity-management curve does not change. In the new equilibrium, bank collateral is even lower and the interest rate is lower, as is the price level. The deflationary effect therefore amplifies the increase in uncertainty about bank trees considered earlier. The

additional prediction is that we should see a decline in funding of institutional investors with short-term credit from payment intermediaries, such as a decline in repo extended by money-market mutual funds to broker-dealers.

It is also interesting to reconsider the effect of monetary policy. Suppose policy engineers a change in the mix of bank assets or their value that lowers the real overnight interest rate. Carry traders borrow more and bid up the prices of the trees they invest in. As one segment of the tree market thus increases in value, the aggregate value of trees also rises: there is a tree market boom. Importantly, this is not a real interest rate effect: the discount rate of households, which is used to value trees held by households, is unchanged. The effect comes solely from the effect of monetary policy on the overnight rate and hence on carry traders' funding costs.

5.4 Active traders

Carry traders provide collateral to banks and thus affect the supply of payment instruments. We now introduce another group of institutional investors who demand payment instruments. "Active traders" periodically rebalance their portfolios as their view of tree payoffs changes, and they require cash to pay for new trees. The demand of active traders for payment instruments counteracts the supply side effects due to changes in the collateral values or the borrowing by carry traders. At the same time, monetary policy also affects their funding costs.

Active traders are competitive firms that issue equity and invest in payment instruments as well as a subset of trees $\hat{\Theta}$. There are many active traders and each is optimistic about one particular "favorite" tree: the trader perceives uncertainty $\hat{s} < s$ about this tree. In contrast, households and other active traders perceive uncertainty s . Active traders also perceive uncertainty s about all other trees. Every period, the identity of the favorite tree within the subset $\hat{\Theta}$ changes to some other tree in the subset.

To generate a need for payment instruments, we assume that active traders must pay for new tree purchases with prearranged payment instruments or intraday credit. Active trader i faces the liquidity constraint

$$\int_{\hat{\Theta}} Q_t^j \hat{\theta}_{i,j,t} dj = I_{i,t} + (\hat{D}_{i,t} + \hat{L}_{i,t}),$$

where I_i is the intraday credit position, \hat{D}_i are deposits that the fund keeps at its bank together with credit lines $\hat{L}_{i,t}$.

Like a bank, active trader i faces a limit on intraday credit

$$I_{i,t} \leq \hat{\gamma}(\hat{D}_{i,t} + \hat{L}_{i,t}),$$

where $\hat{\gamma}$ is a parameter that governs netting in tree transactions. It is generally different from the parameter γ that governs netting among banks, since it captures netting by a clearing and settlement system for the securities that active traders invest in.

Active traders choose payment instruments, trees and their shareholder payout. We focus on equilibria in which every active trader always holds only his favorite tree – we can assume

that the perceived uncertainty on other trees is high enough. Since payment instruments are costly – the real rate on deposits is below the discount rate – active traders arrange as few payment instruments as necessary in order to purchase the entire outstanding amount of their new favorite tree in case the identity of their favorite tree changes. It follows that the intraday credit limit binds in equilibrium, a form of “cash-in-the-market pricing”.

Optimal investment and deposits

Much like households, active traders equate their marginal liquidity benefit to the marginal cost of payment instrument, given by the rate on credit lines i_t^L or equivalently the opportunity cost of deposits $\hat{\delta}_t - (i_t^D - \pi_{t+1})$. The liquidity benefit in turn is due to traders’ ability to invest in their favorite tree, which carries a return that compensates them for cost of liquidity. As in the previous section, tree prices also reflect uncertainty about output and hence inflation. It is convenient to denote the worst loss of output by s_y – it depends on the weighted average of losses on the different trees in the economy.

The steady state price of trees held by active traders can then be written as

$$\frac{Q_j}{P} = \hat{u} \frac{\hat{x}}{\delta + \frac{i^L}{1+\hat{\gamma}}}; \quad \hat{u} = \frac{1 - \hat{s}}{1 - s_y}$$

Here the first factor is again compensation for uncertainty – it takes the same form as in (17) and is less than one if the expected loss in payoff from active trader trees is larger than the expected drop in output. The second term shows that prices reflect traders’ need for inside money: prices are higher if the enduser cost of liquidity i^L is lower and when netting is more efficient (higher $\hat{\gamma}$).

Equilibrium payment instruments arranged by active traders are proportional to the market value of active traders’ favorite trees:

$$\hat{D} + \hat{L} = \frac{\hat{u} \hat{x}}{(1 + \hat{\gamma}) \delta + i^L},$$

The demand for inside money by active traders is interest elastic, in contrast to the inelastic demand from households. This is a stark way to capture the idea that financial institutions responds more strongly to changes in liquidity costs.

Since the household and active trader sector differ in their demand for inside money, the share of inside money used in the goods versus the asset market changes over time. We define active traders’ share of insider money as

$$\alpha = \frac{\hat{D} + \hat{L}}{D + L} = \frac{\hat{v} / (1 + \hat{\gamma})}{\Omega + x + \hat{v} / (1 + \hat{\gamma})}.$$

If enduser liquidity becomes cheaper, the value of active traders’ trees increases and their share of the total supply of inside money goes up. In equilibrium, the share is an increasing function $\tilde{\alpha}(\lambda, \kappa)$ of the two bank ratios λ and κ , since both lower the cost of enduser liquidity.

Equilibrium with active traders

We focus on local changes to equilibria with abundant reserves. Our graphical analysis of equilibrium bank ratios remains qualitatively similar when active traders are added to the

model. The price level is determined by

$$P_0 = \frac{1 - \tilde{\alpha}(\lambda, \kappa)}{\lambda} \frac{1}{1 + B_0^g/M_0} \frac{1 + \gamma}{\bar{v}} \frac{M_0 + B_0^g}{T}.$$

The key difference to (12) is the presence of active traders' share, which works like velocity. If the cost of liquidity is lower, then active traders absorb more inside money. As less money is used in the goods market, the price level declines.

We can now revisit the effect of changes in beliefs and monetary policy in an economy with active traders. Suppose first that there is an increase in uncertainty. As active traders value trees less, they demand fewer payment instruments. This increases the collateral ratio of banks and shifts the capital-structure curve to the left. The liquidity-management curve does not change. In the new equilibrium, bank collateral and the interest rate are higher, as is the price level. In other words, active traders are a force that generate the opposite response to a change in uncertainty from banks and carry traders. Since in the typical economy all traders are present to some extent, we can conclude that their relative strength is important. An additional prediction is that we should see a decline in payment instruments – either deposits or credit lines – provided to institutional investors.

We can also reconsider the effect of monetary policy. Suppose once more that policy lowers the real overnight interest rate. The opportunity cost of holding deposits falls and active traders demand more payment instruments. At the same time, they bid up the prices of the trees they invest in. Again a segment of the tree market increases in value and the aggregate value of trees also rises: there is a tree market boom. Again the effect is not due a change in the discount rate, but instead a change in the funding cost: here it affects active traders' strategy which requires payment instruments in order to wait for trading opportunities.

6 Related literature

In this section we discuss how our results relate to existing work in monetary economics.

Balance sheet effects and government liabilities

In our model, welfare costs derive from "balance sheet effects" and policy matters by changing the asset mix in the economy. This theme is familiar from other work on unconventional monetary policy. Several papers study setups where banks are important to channel funds to certain borrowers. By purchasing the bonds of these borrowers, policy can effectively substitute public credit when weak balance sheets constrain private credit (e.g. Cúrdia and Woodford 2010, Christiano and Ikeda 2011, Gertler and Karadi 2011, Gertler, Kiyotaki and Queralto 2012).⁹ Our model differs from this literature in how banks add value – their special ability is not lending, but the handling of payment instructions.

Since the price level depends on the supply of payment instruments, shocks to bank assets have deflationary effects in our model. If all payment instruments are taken to be deposits, we obtain a collapse of the money multiplier along the lines of Friedman and Schwartz (1963).

⁹Buera and Nicolini (2014) also consider the effect on monetary policy on balance sheets in a model of entrepreneurs who face collateral and cash-in-advance constraints.

Brunnermeier and Sannikov (2016) also consider the link between asset values and the supply of inside money by banks. In their model, banks' special ability is to build diversified portfolios and deposits are a perfect substitute to outside money as a store of value. In contrast, in our model inside money is a medium of exchange for endusers, and outside money works like an intermediate good for producing inside money, rather than a substitute.

Asset pricing and money

In our model, collateral benefits generate market segmentation. We thus arrive endogenously at "intermediary asset pricing" equations that are reminiscent of those in He and Krishnamurthy (2013) or Bocola (2016). Unlike our banks, banks in those models are investors with limited net worth who are assumed to hold some assets because they have special investment abilities.

The interaction of liquidity and collateral benefits in our model also generates permanent liquidity effects. In contrast, the literature on monetary policy with partially segmented asset markets (for example, Lucas 1990, Alvarez, Atkeson and Kehoe 2002) obtains temporary liquidity effects; collateral benefits play no role there.

Asset values in our model depend on the cost of payment instruments to institutional investors. A permanent effect of monetary policy on asset values also obtains in Lagos and Zhang (2014) where the inflation tax discourages trade between heterogeneous investors; this alters which investor prices assets in equilibrium. The effect we derive is different because the cost of liquidity to endusers is not captured by the inflation tax; instead the cost of payment instruments depends on banks' cost of leverage. In particular, our mechanism is also operative when reserves are abundant.

Bank liquidity management and monetary policy

With scarce reserves, bank liquidity management matters for asset valuation, policy impact and welfare in our model. The liquidity management problem arises because banks cannot perfectly insure against liquidity shocks due to customer payment instructions, as in Bhattacharya and Gale (1987).¹⁰ Recent work has discussed the interaction of monetary policy and liquidity management with scarce versus abundant reserves (for example, Whitesell 2006, Keister, Monnet and McAndrews 2008)¹¹. While these papers consider more detail that is useful to understand the cross section of banks, our stylized model tries to capture the main tradeoff and its interaction with other features of the economy.

Several papers have incorporated bank liquidity management into DSGE models. Cúrdia and Woodford (2011) study optimal monetary policy in a New Keynesian model.¹² In their

¹⁰A related literature asks whether government supplied liquidity is useful when firms cannot perfectly insure shocks to investment opportunities (for example, Woodford 1990, Holmstrom and Tirole 1998). An alternative approach to bank liquidity, following Diamond and Dybvig (1983), considers optimal contracts offered by banks to endusers. This approach typically abstracts from interbank transactions; the focus is instead on optimal dependence of contracts on enduser liquidity needs, given information problems as well as the scope for multiple equilibria that include bank runs.

¹¹Similar tradeoffs have been developed in the literature on the dynamics of the Federal Funds market (e.g. Ho and Saunders 1985, Hamilton 1996, Afonso and Lagos 2015.)

¹²Other notions of bank liquidity have also been explored in DSGE settings. Gertler and Kiyotaki (2010) consider a model in which bank borrowing not only depends on bank net worth but also is fragile and subject to runs. Del Negro, Eggertsson, Ferrero and Kiyotaki (2013) study a model in which assets become illiquid in the sense of being harder to sell, as in Kiyotaki and Moore (2008).

setup, reserve policy can be stated in terms of a rule for the overnight interest rate and the reserves rate; there is no need to formulate policy in terms of the quantity of reserves. Our setup is different because of market segmentation: the nominal interest rate is not directly connected to a household marginal rate of substitution, but rather to bank leverage. Rules for interest rates are then not enough to characterize the behavior of inflation – the supply of nominal government liabilities is also relevant.

Bianchi and Bigio (2014) study a quantitative model in which banks have a special ability to lend and face an inelastic demand for debt as well as idiosyncratic liquidity shocks. Monetary policy changes the tradeoff between reserves and interbank credit and hence the willingness of banks to make loans. In contrast, the demand for payment instruments in our model comes from their role as a medium of exchange for goods and securities; monetary policy affects the cost of payment instruments to endusers, not only to banks.

In Drechsler, Savov and Schnabl (2016), banks are investors with relatively low risk aversion who issue debt subject to aggregate liquidity shocks. Monetary policy changes the cost of self-insurance via reserves and thereby affects banks' willingness to take leveraged positions in risky assets as well as the risk premium on those assets. In our model, monetary policy affects not only the funding cost of banks, but also that of banks' institutional clients; the two channels have opposite effects on uncertainty premia.

The role of interest on reserves as a policy tool has recently received renewed attention. A number of papers ask when the price level remains determinate (Sargent and Wallace 1985, Hornstein 2010, Ennis 2014). Woodford (2012) and Ireland (2014) consider macroeconomic effects of interest on reserves in a New Keynesian framework. Kashyap and Stein (2012) consider a model with a financial sector; they emphasize the presence of quantity and price tools for macroeconomic and financial stability, respectively.

Multiple media of exchange and liquidity premia

While our model allows for both deposits and credit lines in the enduser layer, we assume that those instruments are perfect substitutes. An interesting related literature asks which instruments are used in which transactions. In particular, Telyukova and Wright (2008) consider a model in which both credit and money are used and explain apparently puzzling cost differences with convenience yields. Lucas and Nicolini (2015) distinguish currency and interest bearing accounts and show that a model that makes this distinction can better explain the relationship between interest rates and payment instruments. Nosal and Rocheteau (2011) survey models of payment systems.

In our model, asset values reflect collateral benefits to banks and hence indirectly benefits to the payments system. Moreover, government policy can matter by changing the scarcity of collateral that effectively backs payment instruments. Similar themes appear in "new monetarist" models with multiple media of exchange.¹³ In models based on Lagos and Wright (2005), assets that are useful in decentralized exchange earn lower returns. Several papers have recently studied collateralized IOUs as media of exchange, following Kiyotaki and Moore (2005).

For example, in Williamson (2012, 2014) some payments are made with claims on bank portfolios that contain money, government bonds or private assets; banks moreover provide

¹³See Lagos, Rocheteau and Wright (2014) for a recent survey.

insurance to individuals against liquidity shocks. Rocheteau, Wright and Xiao (2015) consider payment via money or government bonds (or, equivalently in their setup, IOUs secured by bonds). These models give rise to regimes of scarcity or abundance for each medium of exchange. The real effects of scarcity can be different for, say, bonds and money because money is used to purchase a different set of goods.

While we also study how policy affects the scarcity of different assets like bonds and reserves, the mechanisms we emphasize as well as our welfare conclusions differ in important ways.¹⁴ Indeed, in our model only one medium of exchange helps in enduser transactions – payment instruments supplied by banks. Since any bank commitment is costly, payment instruments are never abundant and collateral to back them is always scarce – only the degree of scarcity changes and affects welfare. In contrast, reserves can be scarce or abundant depending on their role in bank liquidity management.

In our two layer setup, whether scarcity affects asset prices or welfare thus depends crucially on features of the banking system. For example, scarcity of bonds has different effects from scarcity of reserves because reserves change banks’ liquidity management problem and the leverage costs of tapping the overnight market. In addition, the price level in our model is related to the supply of nominal payment instruments by banks and hence the nominal collateral that banks hold. For example, the quantity of nominal collateral available to banks shapes the price level response to policy.

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¹⁴In addition, the new monetarist literature considers explicit models of decentralized exchange, rather than reduced form liquidity constraints as we do. At the same time, Andolfatto and Williamson (2015) consider a cash-in-advance model with bonds and money and show that several key effects from the more complex papers can be seen already there.

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A Appendix

In this appendix we provide all the equations to accompany the analysis in the text. Section A.1 derives banks' first-order conditions. Section A.2 derives a system of equations characterizing equilibrium, Section A.3 considers its steady state and Section A.4 provides conditions such that transition to a steady state happens in one period. Section A.5 introduces uncertainty and Section A.6 adds active traders.

A.1 Bank optimization

This section studies the optimal choice of banks that maximize (3) subject to (7), (4) and (9).

Paying for leverage costs, marginal leverage costs and marginal collateral benefit

Consider first banks' choice of credit line to pay for leverage costs. Credit lines granted by other banks do not contribute to collateral and hence do not show up in the leverage ratio (8). As long as the interest rate on credit lines is positive, the constraint then binds in equilibrium: banks arrange for a line that is just large enough to cover the leverage costs that will accrue next period.

Using the budget constraint and the binding liquidity constraint, the last two terms in the bank budget equation (7) describe the cost of leverage chosen in the previous period and can be written as

$$-e^{\pi_t}(1 + i_{t-1}^L/\bar{v})c_b(\ell_{t-1})(\sigma(D_t + L_t) + F_t). \quad (\text{A.1})$$

Since banks have to arrange a credit line, their effective cost of leverage also includes the cost of the line.

To derive bank first-order conditions, it is helpful to define the marginal cost of leverage as the derivative of the discounted effective leverage cost (A.1) with respect to total commitments (the numerator in (8)) and the marginal benefit of collateral as the (discounted) derivative with respect to the denominator:

$$\begin{aligned} \text{mc}(\kappa_t) &= e^{-\hat{\delta}_t}(c_b(\ell_t) + c'_b(\ell_t)\ell_t)\{1 + i_t^L/\bar{v}\}, \\ \text{mb}(\kappa_t) &= e^{-\hat{\delta}_t}c'_b(\ell_t)\ell_t^2\{1 + i_t^L/\bar{v}\}. \end{aligned} \quad (\text{A.2})$$

where $\kappa_t = 1/\ell_t$ is the collateral ratio.

An extra unit invested in assets at date t that contributes to collateral earns not only a pecuniary return, but also the collateral benefit $\text{mb}(\kappa_t)$. Similarly, committing to a unit payable entails the extra leverage cost $\text{mc}(\kappa_t)$. Concavity implies that both $\text{mb}(\kappa)$ and $\text{mc}(\kappa)$ are decreasing in collateral since $2c'_b(\ell) + c''_b(\ell)\ell > 0$. Since the cost function c slopes up sufficiently fast that banks always choose $\ell < 1$, we also have that for a given level of leverage, the marginal cost of leverage is higher than the marginal collateral benefit: $\text{mc}(\kappa) > \text{mb}(\kappa)$.

Bank first-order conditions for assets

The typical bank's first-order conditions describe the key trade-offs of portfolio and capital structure choice. Since shareholders are risk neutral, the expected marginal benefits or costs of all assets and liabilities are compared to the required return on equity $\hat{\delta}$. If the portfolio of the

bank is chosen optimally, the marginal benefit from any asset cannot be larger than $\hat{\delta}$: if not, then the bank would choose to invest more. Moreover, the marginal benefit is equal to $\hat{\delta}$ if the bank optimally holds a positive position in the asset. If the marginal benefit is below $\hat{\delta}$ then the bank does not invest: it is better to pay dividends instead.

Consider the first-order condition for overnight lending which not only earns the real interest rate, but also the collateral benefit. The return on equity must be higher than the marginal benefit:

$$\hat{\delta}_t \geq i_t - \pi_{t+1} + \text{mb}(\kappa_t), \quad (\text{A.3})$$

with equality if the bank lends overnight. In the latter case, low real interest rates imply that overnight credit is costly, so banks optimally choose lower collateral. Put differently, highly levered banks obtain a high benefit from overnight lending as collateral and thus require a lower return on credit.

The first-order condition for trees is similar. The real rate of return on tree j held by banks is $r_{t+1}^j = \log(Q_{t+1}^j + P_{t+1}x_{t+1})/Q_t^j - \pi_{t+1}$. Since trees also deliver collateral benefits, we must have

$$\hat{\delta}_t \geq r_{t+1}^j + \rho \text{mb}(\kappa_t),$$

with equality for trees held by the bank. Since the collateral benefit lowers the return on trees held by the bank, it raises the price of the trees relative to payoff. Indeed, holding fixed the payoff $Q_{t+1}^j + P_{t+1}x_{t+1}$, the price of a tree held by the bank is $Q_t^j = (Q_{t+1}^j + P_{t+1}x_{t+1}) / (\hat{\delta}_t - \rho \text{mb}(\kappa_t))$. In particular, if banks holding tree j are more levered, then those trees are more valuable collateral, their cash flows are discounted at a lower rate and their price is higher.

Reserves differ from overnight lending and trees in that they not only provide returns $i_t^R - \pi_{t+1}$ and collateral benefits, but also liquidity benefits – they can be used for payments. The liquidity benefit depends on the Lagrange multiplier on the liquidity constraint (4). Writing μ_t for that Lagrange multiplier divided by the price level, the first-order condition for reserves is

$$\hat{\delta}_t \geq i_t^R - \pi_{t+1} + \text{mb}(\kappa_t) + (1 + \gamma) e^{-\hat{\delta}_t - \pi_{t+1}} E[\mu_{t+1}]. \quad (\text{A.4})$$

The liquidity benefit (the second term) is higher the higher the expected discounted Lagrange multiplier $E[\mu_{t+1}]$ and the more payments can be made per dollar of reserves (higher γ).

Bank first-order conditions for liabilities & credit lines

Consider now banks' choice to finance themselves with deposits or overnight credit. If the capital structure of the bank is chosen optimally, the marginal cost of any liability type cannot be lower than $\hat{\delta}_t$, the cost of issuing equity: if not, banks would choose to borrow more. Moreover, the marginal cost is equal to $\hat{\delta}_t$ if the bank holds a positive position in that particular liability type. If the marginal cost is above $\hat{\delta}_t$ then the bank does not issue the liability: it is better to issue equity instead.

The first-order condition for deposits says that the equity return must be smaller than the sum of the real deposit rate plus the marginal leverage and liquidity costs of deposits

$$\hat{\delta}_t \leq i_t^D - \pi_{t+1} + \sigma \text{mc}(\kappa_t) + e^{-\hat{\delta}_t - \pi_{t+1}} (1 + \gamma) E[\mu_{t+1} \tilde{\lambda}_{t+1}], \quad (\text{A.5})$$

with equality if the bank issues deposits. Leverage costs increase with overall leverage through $\text{mc}(\kappa_t)$ and are also scaled by the parameter σ which makes deposits cheaper than other borrowing. Liquidity costs arise in those states next period when positive liquidity shocks $\tilde{\lambda}_{t+1} > 0$ coincide with a binding intraday credit limit $\mu_{t+1} > 0$.

The first-order condition for overnight borrowing says that the equity return must be smaller than the sum of the real overnight rate plus the marginal leverage cost, *less* the liquidity benefit provided by overnight credit:

$$\hat{\delta}_t \leq i_t - \pi_{t+1} + \text{mc}(\kappa_t) - \mu_t(1 + \gamma). \quad (\text{A.6})$$

For banks that borrow overnight, the condition holds with equality. This can happen because for those banks the intraday credit limit binds and $\mu_t > 0$. In contrast, banks with sufficient reserves have $\mu_t = 0$ and do not borrow.

Finally, consider banks' choice to extend credit lines, a form of implicit liability. The fee earned for extending the line must at least compensate the bank for the leverage cost occurred as well as the expected liquidity costs

$$i_t^L \geq \sigma \text{mc}(\kappa_t) + \bar{v} e^{-\hat{\delta}_t - \pi_{t+1}} E \left[\mu_{t+1} \tilde{\lambda}_{t+1} \right]. \quad (\text{A.7})$$

As discussed above, we focus on equilibria with $i_t^D - \pi_{t+1} = \hat{\delta}_t - i_t^L$ so that deposits and credit lines are equivalent from the perspective of households. Comparing (A.5) and (A.7), the two payment instruments are then also equivalent from the perspective of banks.

A.2 Characterizing equilibrium

In this section, we derive a difference equation that summarizes the equilibrium dynamics. Policy is described by a growth rate for reserves g and a path for the ratio of bonds to money B_t/M_t . The endogenous variables are the (i) asset prices, that is, interest rates i_t and i_t^L , tree prices Q_t^j , (ii) the nominal price level P_t and the quantity of payment instruments $D_t + L_t$, (iii) bank ratios that are equated across banks in equilibrium, that is, the liquidity ratio λ_t and the collateral ratio κ_t (iv) constrained banks' marginal benefit of liquidity $\tilde{\mu}_t$ and finally the household marginal utility of wealth $e^{-\gamma_t}$ which determines the effective discount rate $\hat{\delta}_t$. The initial condition is the liquidity ratio λ_0 which implies also the predetermined initial nominal price level.

We note that $\tilde{\mu}$ represent the liquidity benefit that obtains per real dollar for any bank for which the liquidity constraint binds. In contrast, the symbol μ used above is the Lagrange multiplier in the first-order condition of an individual bank (A.4). Of course, in the cross section of banks, there are banks for whom the constraint does not bind and for those banks we have $\mu = 0$.

Asset pricing

Consider participation in overnight credit and tree markets. In an equilibrium with positive consumption, banks must supply payment instruments. Since payment instruments entail leverage costs, banks obtain a positive collateral benefit from assets they are eligible to hold.

As a result, households do not invest in any asset markets that banks can invest in: at least one bank will bid up the price of any asset accessible to banks until its return is below the discount rate and the asset is unattractive to households.

The collateral ratio is the same for all active banks in equilibrium¹⁵. The equilibrium rate of return on a tree j accessible to banks is then related to the aggregate collateral ratio by the bank “Euler equation”

$$\hat{\delta}_t = r_{t+1}^j + \rho \text{ mb}(\kappa_t). \quad (\text{A.8})$$

The flip side of lower returns is higher prices. Indeed, let x_t^b denote the sum of dividends on all trees accessible to banks at date t and let $v^b = \int_{\Theta^b} Q_t^j dj / P_t$ denote the real value of those trees. In equilibrium, the value of these bank trees reflects not only the present value the future payoff on the tree, but also the collateral value:

$$v_t^b = e^{-\hat{\delta}_t} (v_{t+1}^b + x_{t+1}^b + \rho \text{ mb}(\kappa_t) v_t^b). \quad (\text{A.9})$$

Using our conventions, pricing effectively works as if payoffs are discounted at the lower rate $\hat{\delta}_t - \rho \text{ mb}(\kappa_t)$.

Liquidity benefits and reserve scarcity

Bank liquidity management and their activity in the overnight credit market depends crucially on the scarcity of reserves. Indeed, with abundant reserves, the first-order condition for reserves (A.4) implies that the real rate on reserves is equal to $\hat{\delta}_t - \text{mb}(\kappa_t)$. From the first-order condition for overnight lending (A.3), this is the same overnight interest rate that obtains if banks lend in the overnight market – if reserves are abundant, they are a perfect substitute to overnight paper and must earn the same interest rate.¹⁶

The overnight interest rate is connected to collateral holdings through a bank Euler equation analogous to (A.8),

$$\hat{\delta}_t - (i_t - \pi_{t+1}) = \text{mb}(\kappa_t). \quad (\text{A.10})$$

As long as there is some government debt B_t , this equation holds in any equilibrium, whether reserves are abundant or not. At the same time, the inequality (A.3) implies that households never participate in the overnight market – as with banks trees, banks bid up the price of overnight paper to the point where the asset is unattractive to households. In particular, a regime of abundant reserves has the property that reserves and overnight credit are perfect substitutes from the perspective of banks, but neither asset is ever held by households.

Consider a bank that must borrow at the current date, that is, it receives a liquidity shock $\tilde{\lambda}_t$ beyond the liquidity ratio $\lambda_t = M_t(1 + \gamma) / \bar{v}(D_t + L_t)$ from (5). The bank’s liquidity benefit from overnight credit follows from (A.3) and (A.6):

$$\tilde{\mu}_t(1 + \gamma) = \text{mc}(\kappa_t) - \text{mb}(\kappa_t) > 0. \quad (\text{A.11})$$

¹⁵Indeed, suppose bank 1 has higher leverage than bank 2, and hence a higher collateral benefit $\kappa(\ell)$. In equilibrium, bank 2 does not invest in illiquid assets (trees or overnight credit); due to the lower collateral benefit it is willing to pay strictly less than bank 1 for those assets. To be active, bank 1 must hold some assets, so suppose it holds only reserves. However, the liquidity benefit it earns from reserves cannot be higher than that of the more leveraged bank 2, a contradiction.

¹⁶Of course, reserves still flow across banks. In particular an amount of reserves $\bar{\phi}\bar{v}(D + L) / (1 + \gamma)$ still serves to buffer liquidity shocks. However reserves beyond this amount are equivalent to overnight paper that cannot be used to handle payments instructions.

Tapping the overnight market entails both a leverage cost, and an opportunity cost in terms of foregone collateral value. A banking system with less collateral thus faces a larger penalty of running out of reserves.

The spread between overnight and reserve rate reflects the expected liquidity benefit of holding reserves. Substituting into the first-order condition for reserves, we obtain

$$i_t - i_t^R = (1 - G(\lambda_{t+1})) e^{-\hat{\delta}_t - \pi_{t+1}} \tilde{\mu}_{t+1} (1 + \gamma). \quad (\text{A.12})$$

A liquidity benefit obtains only when the withdrawal shock exceeds λ_{t+1} , that is, with probability $1 - G(\lambda_{t+1})$. In this case, a bank holding an extra dollar of reserves saves the excess cost of overnight lending relative to equity.

In equilibria with scarce reserves $i_t > i_t^R$, the spread (A.12) shows how banks choose the reserve-deposit ratio as a function of interest rates and leverage. The interest rate i plays a dual role here. On the one hand, it affects the opportunity cost of reserves. Indeed, holding fixed the liquidity benefit, the equation works much like a money demand equation: if the overnight rate is higher, then it is more costly to hold reserves and banks choose smaller λ_t , or fewer reserves per dollar of deposits and credit lines. On the other hand, the interest rate affects the liquidity benefit itself: holding fixed the collateral ratio κ_t and the spread, a higher interest rate increases the liquidity benefit of reserves and leads banks to choose more reserves to avoid the higher penalty of running out.

Liquidity and leverage

Bank leverage depends on how much of each source of collateral is available, and how much banks must borrow apart from issuing deposits. In particular, interbank credit contributes both to collateral (for lender banks) and raises costs (for borrower banks). Given collateral κ and a liquidity ratio $\lambda \leq \bar{\lambda}$, the equation for interbank borrowing (6) delivers the ratio of outstanding interbank credit to transactions

$$\frac{F_t}{D_t + L_t} = \frac{\bar{v}}{1 + \gamma} \int_{\lambda_t}^{\bar{\lambda}} (\tilde{\lambda} - \lambda_t) dG(\tilde{\lambda}) =: \frac{\bar{v}}{1 + \gamma} f(\lambda_t). \quad (\text{A.13})$$

The function f is decreasing in λ_t : if interest rates are such that banks hold a lot of reserves, then λ_t is high and banks rarely run out of reserves, so outstanding interbank credit is low. In this sense, reserves and overnight are substitutes in liquidity management.

Collecting promises due to payment instruments, real reserves, trees and interbank credit, as well as collateral in the form of reserves, government debt, trees, and interbank credit, equilibrium collateral satisfies

$$\kappa_t = \frac{M_{t+1} + B_{t+1}^g + P_t \rho v_t^B + (D_t + L_t) \frac{\bar{v}}{1 + \gamma} f(\lambda_t)}{\sigma(D_{t+1} + L_{t+1}) + (D_t + L_t) \frac{\bar{v}}{1 + \gamma} f(\lambda_t)}. \quad (\text{A.14})$$

Holding fixed the value of collateral from outside the banking system – reserves, government debt and trees – scarcity of reserves requires more leverage to support a given quantity of transactions. Indeed, we have assumed a cost function c_b such that the economy operates in the range $\kappa^{-1} = \ell \leq 1$; the presence of an interbank market adds an equal amount of debt and collateral and hence increases leverage.

The cost of payment instruments

From banks' first-order condition, the equilibrium rate on credit lines satisfies

$$i_t^L = \sigma \text{mc}(\kappa_t) + \bar{v} e^{-\hat{\delta}_t - \pi_{t+1}} \tilde{\mu}_{t+1} (1 + \gamma) \int_{\lambda_{t+1}}^{\bar{\lambda}} \tilde{\lambda} dG(\tilde{\lambda}). \quad (\text{A.15})$$

The two terms represent a leverage and a liquidity component. When reserves are abundant, we have $\lambda_t > \bar{\lambda}$ so the liquidity component is zero and the cost of a credit line simply reflects banks' cost of leverage.

When reserves are scarce, banks incur additional costs when they run out of reserves. Those costs are larger if velocity is higher and there is less netting among banks. They further depend on the marginal benefit of liquidity as well as on the liquidity shock the bank receives in the next period. The costs banks incur when providing payment instruments also lower the deposit rate $i_t^D - \pi_{t+1}$ by the same amount.¹⁷

Consider now the valuation of payment instruments by the household. If the liquidity constraint binds, the household first order condition for deposits is

$$e^{-\gamma_t} = \beta \left(e^{i_t^D - \pi_{t+1}} e^{-\gamma_{t+1}} + e^{-\pi_{t+1}} \bar{v} (1 - e^{-\gamma_{t+1}}) \right)$$

Investing a unit of wealth in deposits delivers not only a real pecuniary return – the first term on the right hand side – but also a liquidity benefit – the second term. The liquidity benefit is positive since the liquidity constraint is binding so that $e^{-\gamma_{t+1}} < 1$.

Using the definition of $\hat{\delta}_t = \delta - (\gamma_{t+1} - \gamma_t)$, we can write endusers' liquidity benefit as approximately

$$\bar{v} \gamma_{t+1} = \hat{\delta}_t - (i_t^D - \pi_{t+1}) = i_{t+1}^L \quad (\text{A.16})$$

where we have again used that rates of return are small. The liquidity benefit is thus equated to the household's liquidity cost, represented by the opportunity cost of deposits or the fee on a credit line.

Equilibrium

An equilibrium is characterized by the seven equations (10) and (A.9)-(A.12) and (A.14)-(A.16) together with equation (5) that defines the liquidity ratio as well as the two equations (A.2) that define marginal leverage cost and collateral benefit and the definition of $\hat{\delta}_t$. If policy is described by a path for the money supply, then no additional equations are needed. The twelve equations determine twelve endogenous variables $i, i_L, v_B, P, D + L, \lambda, \kappa, \text{mc}(\kappa), \text{mb}(\kappa), \tilde{\mu}, \hat{\delta}$ and γ . The difference equation characterizing equilibrium has only two state variables: aggregate nominal reserves M_t and aggregate nominal payment instruments $D_t + L_t$.

Alternatively, we can consider equilibria that obtain when policy is described by an interest rate rule. Since output is exogenous, we focus on rules that depend on inflation, $i_t = \tilde{i}(\pi_t)$ for some function \tilde{i} . The interest rate rule then provides an additional equation, and M_t is an additional endogenous variable. Since the interest rate rule conditions on inflation at the

¹⁷The term “deposit rate” should be interpreted broadly here: it is the only cost endusers pay for payments services in our model, since we do not explicitly model other costs such as account and transaction fees.

previous date, the past price P_{-1} also becomes part of the initial conditions of the difference equation.

In order to compute equilibrium consumption and welfare at date 0, we need to know additional predetermined variables: initial leverage ratios ℓ_{-1} and ℓ_{-1}^g for banks and the government, respectively as well as banks' outstanding overnight borrowing F_0 and lending B_0 . Those variables are needed to compute the real resources $c_G(\ell_{-1}^G) M_0/P_{-1}$ and $c_b(\ell_{-1})(D_0 + L_0 + F_0)/P_{-1}$ purchased by the government and banks to pay leverage costs at date 0. At the same time, they do not affect any endogenous variables other than consumption which does not enter the difference equation; as a result there is no need to treat them explicitly as state variables.

Consider equilibria in which reserves are abundant at all times, so $\lambda_t \geq \bar{\lambda}$ for all t . The three liquidity management equations (A.11)-(A.13) are then redundant, the variables λ and μ can be removed from the system and the overnight interest rate achieves its lower bound $i = i_R$. The remaining six equations then determine $i_L, v_B, P, \kappa, mc(\kappa)$ and $mb(\kappa)$.

Approximating equilibria with small rates of return

We have simplified formulas above by assuming that rates of return are small decimals so their products can be ignored. To guarantee that small rates obtain in equilibrium, we scale the leverage cost function and bound the real rate on reserves. Let \bar{r}_R denote a lower bound on the real rate on reserves, a small decimal number. In what follows, we guarantee this bound by appropriate assumptions on monetary policy.

The highest possible leverage ratio that can obtain in any equilibrium then satisfies

$$\hat{\delta} - \bar{r}_R = c'_b(\bar{\ell}) \bar{\ell}^2 (1 + \sigma(c_b(\bar{\ell}) + c'_b(\bar{\ell}) \bar{\ell})).$$

This leverage ratio corresponds to an equilibrium where the reserve rate hits the bound and reserves are abundant. If reserves were scarce, the benefit of reserves would include liquidity components and leverage would have to be lower.

We now choose the cost function c such that $\bar{\ell} < 1$ and $c'(\bar{\ell}) \bar{\ell}$ is a small decimal number, much like $\hat{\delta} - \bar{r}_R$. It follows that the marginal cost of leverage $mc(\kappa)$ and the marginal benefit of collateral $mb(\kappa)$ are also small decimal numbers in any possible equilibrium. Moreover, the effect of the interest rate on credit lines on $mc(\kappa)$ and $mb(\kappa)$ is second order. We thus use the additional approximation

$$\begin{aligned} mc(\kappa) &= c_b(\ell) + c'_b(\ell) \ell, \\ mb(\kappa) &= c'_b(\ell) \ell^2, \end{aligned} \tag{A.17}$$

where both mc and mb are decreasing function of the collateral ratio $\kappa = 1/\ell$. We thus abstract from effects of the interest rate i_L on banks' cost of leverage. Those effects are small and not economically interesting; omitting them altogether makes for cleaner formulas below.

The fact that equilibrium mc and mb are small has several convenient implications. First, we can omit the factor $e^{-\hat{\delta}-\pi}$ on the right hand sides of (A.12) and (A.15). The timing of the liquidity benefit has only a second-order effect. Second, combining (A.17) and (A.10), we obtain a one-to-one relationship between the real overnight interest rate and the collateral ratio: banks increase collateral if interest rates are higher. We use both properties when characterizing equilibria further below.

Collecting equations

We restate here the equations characterizing equilibrium, making use of the approximations justified above. It is convenient to organize them in three blocks. The first block describes *bank behavior*: it says how banks respond to market prices in equilibrium by setting their liquidity and collateral ratios. It is given by

$$\begin{aligned}
i_t - i_R &= (1 - G(\lambda_{t+1})) \frac{\tilde{\mu}_{t+1}}{1 + \gamma} \\
\hat{\delta}_t - (i_t - \pi_{t+1}) &= \text{mb}(\kappa_t) \\
i_t^L &= \sigma \text{mc}(\kappa_t) + \bar{v}(1 + \gamma) \tilde{\mu}_{t+1} \int_{\lambda_{t+1}}^{\bar{\lambda}} \tilde{\lambda} dG(\tilde{\lambda}) \\
v_t^b &= \exp(-\hat{\delta}_t + \rho \text{mb}(\kappa_t)) (v_{t+1}^b + x_{t+1}^b). \\
\tilde{\mu}_{t+1}(1 + \gamma) &= \text{mc}(\kappa_{t+1}) - \text{mb}(\kappa_{t+1})
\end{aligned} \tag{A.18}$$

Since the bank behavior block describes responses to current conditions, it is entirely forward-looking, that is, it does not involve any state variables.

The second block consists of the evolution of household marginal utility of wealth.

$$\begin{aligned}
\bar{v}\gamma_{t+1} &= i_t^L \\
\hat{\delta}_t &= \delta - (\gamma_{t+1} - \gamma_t)
\end{aligned} \tag{A.19}$$

Household discounting uses the rate δ unless there is a difference in the cost of liquidity between different dates. In particular, when liquidity is more costly next period then future payoffs are discounted at a lower rate – future consumption is effectively more expensive.

Finally, consider the quantity block that relates bank ratios and the money supply to the price level. Here it is convenient to eliminate inside money ($D_t + L_t$) and the price level so the only state variable is the liquidity ratio λ_t . Denoting the growth rate of reserves by g , we have

$$\begin{aligned}
\kappa_t &= \frac{e^{g_{t+1}} \lambda_t (1 + B_{t+1}^g / M_{t+1}) \frac{\bar{v}}{1 + \gamma} + \rho \bar{v} \frac{v_t^B}{\Omega_t + x_t} + \frac{\bar{v}}{1 + \gamma} f(\lambda_t)}{\sigma e^{g_{t+1}} \frac{\lambda_t}{\lambda_{t+1}} + \frac{\bar{v}}{1 + \gamma} f(\lambda_t)} \\
e^{\pi_{t+1}} \frac{\Omega + x_{t+1}}{\Omega + x_t} &= e^g \frac{\lambda_t}{\lambda_{t+1}}
\end{aligned} \tag{A.20}$$

Since we have eliminated $D_t + L_t$ and can write mb and mc just as functions of κ_t , we are left with 9 equations in the 9 variables $i_t, v_t^B, i_t^L, \lambda_t, \kappa_t, \tilde{\mu}_t, \pi_t, \hat{\delta}_t, \gamma_t$ and $\tilde{\mu}_t$.

A.3 Steady state

This section derives the equations characterizing steady state. We assume the same exogenous growth rate g for the nominal quantities M_t and B_t^g so the ratio B_t^g / M_t is constant over time.

With constant rates of return, the marginal rate of substitution of wealth across dates equals the discount rate, that is $\hat{\delta}_t = \delta$. Moreover, the key ratios chosen by banks, collateral κ and liquidity λ , are constant over time. With a constant money multiplier and output, the price level grows at the rate $\pi = g$.

Omitting time subscripts in (A.18) we obtain the steady state relationships between asset prices and bank ratios:

$$\begin{aligned}
i - i_R &= (1 - G(\lambda)) \frac{\tilde{\mu}}{1 + \gamma} \\
\delta - (i - \pi) &= \text{mb}(\kappa) \\
i^L &= \sigma \text{mc}(\kappa) + \bar{v}(1 + \gamma) \tilde{\mu} \int_{\lambda}^{\bar{\lambda}} \tilde{\lambda} dG(\tilde{\lambda}) \\
v^b &= \frac{x^b}{\delta - \text{mb}(\kappa)} \\
\tilde{\mu}(1 + \gamma) &= \text{mc}(\kappa) - \text{mb}(\kappa)
\end{aligned} \tag{A.21}$$

To derive the liquidity-management curve (14), we combine the first two equations:

$$\delta - (i^R - \pi) = \text{mb}(\kappa) + (1 - G(\lambda)) (\text{mc}(\kappa) - \text{mb}(\kappa)). \tag{A.22}$$

Banks' opportunity cost of holding reserves (the difference between the return on equity and the real return on reserves) is equated to the sum of the collateral and liquidity benefits the bank earns on reserves. Equation (15) – used in the text to discuss interest rate policy – is the first equation in (A.21).

Consider now the capital-structure curve relationship (13). Dropping time subscripts and substituting for the value of trees from (A.21), the first equation in (A.20) becomes

$$\kappa = \frac{e^g \lambda \left(1 + \frac{B_t^g}{M_t}\right) \frac{\bar{v}}{1 + \gamma} + \rho \frac{x^b}{(\delta - \rho \text{mb}(\kappa))T} + \frac{\bar{v}}{1 + \gamma} f(\lambda)}{\sigma e^g + \frac{\bar{v}}{1 + \gamma} f(\lambda)}. \tag{A.23}$$

Since the growth rate of nominal liabilities g is a small decimal number, we omit it in the formula in the text. We thus ignore small effects on collateral ratios that derive from the difference in timing between when collateral is measured and when deposits provide services.

Given the solution for the money multiplier λ , the price level evolves in steady state as

$$P_t = \frac{1}{\lambda} \frac{1}{1 + B_t^g/M_t} \frac{1 + \gamma}{\bar{v}} \frac{M_t + B_t^g}{Y}$$

Across steady states, changes in the availability collateral alter the money multiplier and hence the steady state price level. The next section shows that the transition across steady states takes one period so that we can think of the change in price across steady states as a one time inflation response.

A.4 Transition dynamics

Consider solutions to the difference equation (A.18)-(A.20) for $t \geq 0$ given initial condition λ_0 . We focus on equilibria in which the government commits to a constant growth rate g of reserves and to a ratio of bonds of reserves B/M that is fixed from date $t = 2$ onwards. We show that for any initial conditions close enough to the steady state there exists a bond-money ratio B_1/M_1 chosen at date 0 such that the equilibrium settles down at steady state from date $t = 1$ onwards. The additional open market operation is required only when reserves are scarce and is small if outstanding interbank lending is small.

Suppose we have a sequence of variables that enters steady state at $t = 1$. In particular, the variables v_1^b , $\tilde{\mu}_1$, λ_1 and γ_1 are at their steady state values. It follows from the first equation in (A.19) and the first and third equation in (A.18) that κ , i and i^L must achieve their steady state values already at date 0. The steady state interest rate i and collateral ratio κ thus satisfy the pair of equations

$$\begin{aligned}\delta - (i - g) &= mb(\kappa) \\ \hat{\delta}_0 - (i - \pi_1) &= mb(\kappa)\end{aligned}\tag{A.24}$$

where we have used the fact that steady state inflation must be equal to the growth rate of nominal liabilities g .

It follows that the date 0 effective discount rate can be written as

$$\hat{\delta}_0 = \delta + g - \pi_1 = \delta - \log(\lambda_0/\lambda)$$

where we have used the second equation in (A.20) to substitute for π_1 . If the date 0 liquidity ratio is above that in steady state ($\lambda_0 > \lambda$), then the money multiplier will rise from date 0 to date 1 and inflation will be high. As a result, liquidity will be more costly to hold in the future and future payoffs are discounted at a lower rate.

We can further solve for the price of trees

$$v_0^b = e^{-\hat{\delta}_0 - \rho mb(\kappa)} \frac{x}{1 - e^{-\delta - \rho mb(\kappa)}} = \frac{x}{\delta - mb(\kappa)} \frac{\lambda_0}{\lambda},$$

If the date 0 liquidity ratio is higher than that in steady state, lower discounting increase the value v^B . We obtain γ_0 from the second equation in (A.19). If inflation is close enough to the steady state inflation rate g , then we have $\gamma_0 > 0$.

We now have a sequence of variables that satisfies all equations except possibly the first equation in (A.20). The variables entering that equation are all pinned down: λ_0 is predetermined, κ_0 and λ_1 are equal to steady state κ and λ and v_0^B is as above. We must therefore have

$$\kappa_0 = \frac{e^g \lambda_0 (1 + B_1^g/M_1) \frac{\bar{v}}{1+\gamma} + \rho \bar{v} \frac{x^b(1-s)}{\delta - mb(\kappa)} \frac{1}{\Omega+x} \frac{\lambda_0}{\lambda_1} + \frac{\bar{v}}{1+\gamma} f(\lambda_0)}{\sigma e^g \frac{\lambda_0}{\lambda_1} + \frac{\bar{v}}{1+\gamma} f(\lambda_0)}$$

If reserves are abundant then $f(\lambda_0) = 0$, the predetermined liquidity ratio λ_0 cancels and the equation is equivalent to the steady state relationship (A.23). Since κ_1 and λ_1 are their steady

state values, the equation holds provided that B_1/M_1 is also at its steady state value. Moreover, if reserves are scarce and the initial liquidity ratio λ_0 is close enough to the steady state value λ_1 , then we can choose B_1/M_1 to satisfy the equation. In general this requires B_1/M_1 different from the steady state value – the government intervention offsets the transition dynamics that would arise from the fact that interbank lending is a predetermined form of collateral. As long as interbank lending is a small share of the balance sheet, the intervention is also small.

We have thus found an equilibrium that transits to steady state in one step. The only endogenous variables that are different from steady state at date 0 are the effective discount rate $\hat{\delta}_0$ and the log marginal utility of wealth γ_0 . Moreover, the inflation rate π_1 from date 0 to date 1 produces a change in the price level from the old to the new steady state and is thus different from the inflation rate that obtains from date 2 on.

A.5 Steady state with uncertainty

We find a steady state with uncertainty in three steps, following Ilut and Schneider (2015). We first determine the "worst case" steady state, that is, the steady state to which the economy would converge were the worst case scenario contemplated by agents to actually occur. Here this worst case scenario is that the payoff of the trees is multiplied by $1 - s$. Worst case steady state prices and bank ratios follow from the equations in the previous subsection, with x^b replaced by $x^b(1 - s)$. We indicate worst case steady state values by stars. In particular, the worst case steady state ratios λ^* and κ^* are determined by the intersection of the liquidity management and capital structure curves (A.22)-(A.23), with x^b replaced by $x^b(1 - s)$.

The second step characterizes agents' perceptions of the equilibrium dynamics. Agents observe every period the true tree payoff x^b which is larger than the worst case steady state payoff. As a result, agents always act *as if* they are on a transition path away from an initial transitory shock x^b that is higher than the (perceived) steady state value $x^b(1 - s)$. The actual steady state value of the endogenous state variable λ is such that agents' behavior leaves λ constant over time given their worst case beliefs. Formally, we are looking for a solution to the system (A.18)-(A.20) together with an initial condition for the endogenous state variable λ such that (i) the solution converges to the worst case steady state, and (ii) the endogenous state variable λ is constant in the first period of the transition path.

The third step then characterizes the actual dynamics by combining the law of motion for the endogenous variables implied by the second step with the actual dynamics of the exogenous variables. Compared to the model studied in Ilut and Schneider (2015), the second step is simple in the sense that the transition path moves to the worst case steady state after one period. As a result, the actual steady state liquidity ratio λ is equal to the worst case ratio λ^* . Moreover, the third step is simple in that the actual evolution of output – the ambiguous exogenous variable – enters only one equation, namely the last equation in (A.20) that relates inflation to output growth.

Transition to the worst case steady state

Suppose that the solution in fact settles at the worst case steady state after one period. In particular, from date 1 on, the liquidity ratio is λ^* and the liquidity multipliers are $\tilde{\mu}^*$ and γ^* remain constant at the worst case steady state values. Given those values, the actual steady

state values of i, κ and i^L must be equal to their worst case steady state counterparts. Indeed, those variables follow from

$$\begin{aligned} i - i_R &= (1 - G(\lambda^*)) \frac{\tilde{\mu}^*}{1 + \gamma}, \\ i^L &= \sigma mc(\kappa) + \bar{v}(1 + \gamma) \tilde{\mu}^* \int_{\lambda^*}^{\bar{\lambda}} \tilde{\lambda} dG(\tilde{\lambda}), \\ \gamma^* &= i^L. \end{aligned} \tag{A.25}$$

Let $\tilde{\pi}$ denote the worst case inflation rate expected by agents in the first period as the system transits to the worst case steady state. From (A.24) we have that the steady state effective discount rate is $\hat{\delta} = \delta + g - \tilde{\pi}$. We can therefore write the price of a tree in the actual steady state and in the worst case steady state as

$$\begin{aligned} v^b &= e^{-\hat{\delta} - \rho mb(\kappa)} (v^{b*} + x^b(1 - s)) \\ v^{b*} &= e^{-\delta - \rho mb(\kappa)} (v^{b*} + x^b(1 - s)) \end{aligned}$$

The only difference here is the discount rate applied to payoffs between the current and next period.

Combining these equations and the relationship between the discount rate, money growth and worst case inflation, the steady state tree payoff is

$$\begin{aligned} v^b &= e^{-\hat{\delta} - \rho mb(\kappa)} \frac{x^b(1 - s)}{1 - e^{-\delta - \rho mb(\kappa)}} \\ &= \frac{x^b}{\delta - mb(\kappa)} \frac{\Omega + x}{\Omega + (1 - s)x} \frac{\lambda}{\lambda^*}, \end{aligned} \tag{A.26}$$

where in the second line we have used the second equation in (A.20) and the fact that $\delta - mb(\kappa)$ is a small decimal number.

To verify that the actual steady state liquidity ratio is $\lambda = \lambda^*$, we now substitute the actual steady state value v^b into the first equation of (A.20) for date $t = 0$. Given the worst case steady state λ^* , the actual steady state ratios λ and κ are related by

$$\kappa = \frac{e^g \lambda (1 + B^g/M) \frac{\bar{v}}{1 + \gamma} + \rho \bar{v} \frac{x^b(1 - s)}{\delta - mb(\kappa)} \frac{1}{\Omega + x(1 - s)} \frac{\lambda}{\lambda^*} + \frac{\bar{v}}{1 + \gamma} f(\lambda)}{\sigma e^g \frac{\lambda}{\lambda^*} + \frac{\bar{v}}{1 + \gamma} f(\lambda)} \tag{A.27}$$

If $\lambda = \lambda^*$, then this equation reduces to (A.23), but with tree payoffs x and x^b replaced by worst case payoffs $x(1 - s)$ and $x^b(1 - s)$, respectively. Since $\kappa = \kappa^*$, it is therefore satisfied at $\lambda = \lambda^*$.

We have now derived a solution to (A.18)-(A.20) with the desired properties, thus verifying our conjecture above. Starting from $\lambda = \lambda^*$, a transitory shock that increases all tree payoffs by a factor of $1/(1 - s)$ induces banks to maintain the liquidity ratio constant at λ^* . Intuitively, agents expect a higher cost of liquidity, output is expected to fall and inflation is temporarily higher than the growth rate of nominal government liabilities. As a result, future tree payoffs

are discounted at a lower rate. This effect implies that the ratio of tree value to output is the same as in the worst case steady state.

Graphical analysis and steady state asset prices

The graphical analysis of optimal ratios works the same as in the absence of uncertainty. The CS curve is described by (A.27) with $\lambda^* = \lambda$. The liquidity management curve is still described by (A.22). Given numbers for $\lambda = \lambda^*$ and $\kappa = \kappa^*$, the short interest rate and the rate on credit lines follows from (A.25). Finally, the inflation rate in the actual steady state is $\pi = g$ from the second equation in (A.20). This rate is lower than the worst case rate of inflation $\tilde{\pi}$ contemplated by agents. This is because agents fear a drop in output that will translate into higher inflation for given growth of nominal liabilities.

The price of trees follows from (A.26). Using $\lambda = \lambda^*$ and the definition of $\tau = x / (\Omega + x)$, we have

$$v^b = \frac{x^b}{\delta - mb(\kappa)} \frac{1 - s}{1 - \tau s} = u(s) \frac{x^b}{\delta - mb(\kappa)}$$

We also note that while the realized real interest rate is $i - g = \delta + mb(\kappa)$ as before, the ex ante real interest rate $i - \pi_1 = \hat{\delta} + mb(\kappa)$ reflects the lower discount rate induced by output and hence inflation uncertainty.

Welfare

Consider welfare in the presence of uncertainty. We compute the expected loss due to leverage cost under agents' worst case perception of the future. We thus take into account the fact that agents act *as if* output will be lower starting next period, which also implies lower leverage costs. Let $\Delta = (\Omega + xe^{-s}) / (\Omega + x)$ denote the drop in output between dates 0 and 1 under worst case expectations. It is also helpful to introduce notation for the total flow cost of leverage

$$K(T) = c_g \left(\left(1 + \frac{B}{M} \right) \frac{\lambda}{1 + \gamma} \frac{T}{\Omega} \right) \left(1 + \frac{B}{M} \right) \frac{\lambda}{1 + \gamma} T + c_b(\kappa^{-1}) T \left(\frac{\sigma}{\bar{v}} + \frac{f(\lambda)}{1 + \gamma} \right)$$

The steady state loss can then be written as

$$K(T) + \frac{\beta}{1 - \beta} K(\Delta T)$$

Since the output loss simply reduces transactions proportionally, the tradeoff between government leverage and private sector leverage (or equivalently λ and κ^{-1}) is qualitatively the same as in the absence of uncertainty. As a result, the graphical analysis of welfare remains the same as in the text.

A.6 Equilibrium with active traders

Active traders maximize shareholder value, with nominal dividends given by

$$(\hat{Q}_t + \hat{x}_t)\theta_{t-1} - \hat{Q}_t\theta_t - i_{t-1}^L \hat{L}_t - \hat{D}_{t+1} + \hat{D}_t (1 + i_{t-1}^D)$$

where θ_t is the number of trees bought at date t and \hat{L}_t is the credit line arranged at $t - 1$ in order to pay for trees at t and \hat{D}_t are deposits made at $t - 1$.

The first order conditions for active traders can be written as

$$\begin{aligned} i_t^L &= \hat{\delta}_t - (i_t^D - \pi_{t+1}) = \hat{\mu}_{t+1} (1 + \hat{\gamma}) \\ \hat{v}_t (1 + \hat{\mu}_t) &= e^{-\hat{\delta}_t} (\hat{v}_{t+1} + \hat{x}_{t+1} (1 - \hat{s})) \end{aligned} \quad (\text{A.28})$$

where $\hat{\mu}_t$ is the multiplier on active traders' liquidity constraint multiplied by the price level. The interest rate on credit lines – endusers' cost of liquidity – is equated to active traders' liquidity benefit. The expected payoffs of active trader trees in the second equation incorporate the worst case payoff on trees.

To capture the quantity of deposits absorbed by active traders, we define their share $\alpha_t = (\hat{D}_t + \hat{L}_t) / (D_t + L_t)$. The equilibrium share satisfies

$$\alpha_t = \frac{\hat{v}_t}{\hat{v}_t + (1 + \hat{\gamma}) \bar{v} (\Omega + x_t)} \quad (\text{A.29})$$

Active traders absorb more inside money if the netting system is less efficient (low $\hat{\gamma}$). It is also helpful to denote aggregate goods market transactions by $Y_t = \Omega + x$.

Since the money held by active traders does not directly affect the price level, the leverage and inflation equations contain only the share of inside money that circulates in the goods market:

$$\begin{aligned} \kappa_t &= \frac{e^{g_{t+1}} \lambda_t (1 + B_{t+1}^g / M_{t+1}) + \rho \bar{v} \frac{v_t^B (1 - \alpha_t)}{Y_t} + \frac{\bar{v}}{1 + \gamma} f(\lambda_t)}{\sigma e^{g_{t+1}} \frac{\lambda_t}{\lambda_{t+1}} + \frac{\bar{v}}{1 + \gamma} f(\lambda_t)} \\ e^{\pi_{t+1}} \frac{Y_{t+1}}{Y_t} &= e^g \frac{\lambda_{t+1}}{\lambda_t} \frac{1 - \alpha_{t+1}}{1 - \alpha_t} \end{aligned} \quad (\text{A.30})$$

The system with active traders is thus given by (A.18), (A.19) and (A.28). Endogenous state variables are now both the banks' liquidity ratio λ and the active trader money share α .

Worst case steady state

To characterize steady state equilibrium, the first step is again to derive the worst case steady state, that is, the steady state if $\hat{x}_t = x^* = \hat{x} (1 - \hat{s})$ and $Y_t = Y^*$, where the worst case output reflects low payoffs on all trees including bank trees and active trader trees. The bank equations (A.21) and therefore the bank ratios (λ, κ) still lie on the liquidity management curve described by (A.22). The steady state liquidity benefit, and money share as well as the price of trees held by active traders satisfy

$$\begin{aligned} i^{L*} &= \hat{\mu}^* (1 + \hat{\gamma}) \\ \hat{v}^* &= \frac{\hat{x} (1 - \hat{s})}{\delta + \hat{\mu}^*} \\ \alpha^* &= \frac{\hat{v}^*}{\hat{v}^* + (1 + \hat{\gamma}) \bar{v} (\Omega + x)} \end{aligned}$$

To describe the capital structure curve, we first write the cost of enduser liquidity and the active trader money share as functions of the bank ratios λ and κ :

$$\begin{aligned}\tilde{i}^L(\lambda^*, \kappa^*) &:= \sigma \text{mc}(\kappa) + \bar{v}(1 + \gamma)(\text{mc}(\kappa^*) - \text{mb}(\kappa^*)) \int_{\lambda^*}^{\bar{\lambda}} \tilde{\lambda} dG(\tilde{\lambda}) \\ \tilde{\alpha}(\lambda^*, \kappa^*) &:= \frac{\hat{x}(1 - s)}{\hat{x}(1 - s) + \bar{v}(\Omega + x + \hat{x}(1 - \hat{s}))((1 + \hat{\gamma})\delta + \tilde{i}_L(\lambda^*, \kappa^*))}\end{aligned}$$

The function \tilde{i}_L is decreasing in both arguments: higher liquidity or collateral ratios lower endusers' cost of liquidity. As a result, the active trader money share is increasing in both λ^* and κ^* . In the region of (λ, κ) -plane where reserves are abundant both functions are independent of λ .

We find the worst case bank ratios from the intersection of the liquidity management curve and the new capital structure curve described by

$$\kappa^* = \frac{e^g \lambda (1 + B^g/M) \frac{\bar{v}}{1+\gamma} + \rho \bar{v} \frac{x^b(1-s)}{\delta - \text{mb}(\kappa)} \frac{1}{Y^*} (1 - \tilde{\alpha}(\lambda^*, \kappa^*)) + \frac{\bar{v}}{1+\gamma} f(\lambda^*)}{\sigma e^g + \frac{\bar{v}}{1+\gamma} f(\lambda^*)} \quad (\text{A.31})$$

The difference to (A.23) is the presence of the factor $1 - \alpha$ in the numerator. In the region where reserves are abundant, $\tilde{\alpha}$ is independent of $\tilde{\alpha}$ and the curve is upward sloping in the (λ, κ) -plane without further assumptions. With scarce reserves, the presence of active traders reduced the slope of the curve. To guarantee an upward slope, we need the share of nominal assets sufficiently large or the payoff of active trader small enough relative to output.

Actual steady state

The second step in the characterization of equilibrium is to find the transition path from the actual to the worst case steady state. We conjecture again that this transition takes only one period, so that $\lambda = \lambda^*$ and $\alpha = \alpha^*$. Equations (A.25) hold as before and deliver actual steady state i , κ and i^L . To derive the value of bank trees, we apply the same argument as in Section A.5 but use the new equation for inflation – the second equation in (A.30) – to obtain

$$v^b = \frac{x^b(1-s)}{\delta - \text{mb}(\kappa)} \frac{Y}{Y^*} \frac{\lambda}{\lambda^*} \frac{1 - \alpha^*}{1 - \alpha}.$$

Substituting into the first equation in (A.30), the actual steady state ratios λ and α satisfy

$$\kappa = \frac{e^g \lambda (1 + B^g/M) + \rho \bar{v} \frac{x^b(1-s)}{\delta - \text{mb}(\kappa)} \frac{1 - \alpha^*}{Y^*} \frac{\lambda}{\lambda^*} + \frac{\bar{v}}{1+\gamma} f(\lambda)}{\sigma e^g \frac{\lambda}{\lambda^*} + \frac{\bar{v}}{1+\gamma} f(\lambda)}$$

The equation is satisfied if $\lambda = \lambda^*$ since we have $\alpha^* = \tilde{\alpha}(\lambda^*, \kappa^*)$ and the equation is otherwise identical to (A.31).

It remains to check the second equation in (A.28) as well as (A.29). The actual steady state value of active trader trees follows from (A.29) as

$$\hat{v} = \frac{\alpha^*}{1 - \alpha^*} Y$$

Given this value, we can find the actual steady state multiplier $\hat{\mu}$ to satisfy (A.28). We have shown that if the initial conditions are given by the worst case steady value α^* and λ^* and there is a transitory shock that increases tree payoffs, agents respond by choosing again worst case steady state α and λ .

We summarize the actual steady state for the economy with active traders as follows. Actual inflation is given by the growth rate of nominal government liabilities $\pi = g$. Since $\kappa = \kappa^*$ and $\lambda = \lambda^*$, the actual steady state bank ratios are determined from the intersection of the liquidity management and capital structure curves (A.22) and (A.31). The value of active trader trees and the evolution of the nominal price level are given by

$$\hat{v} = \hat{v}^* \frac{Y}{Y^*} = (1 - \hat{s}) \frac{Y}{Y^*} \frac{\hat{x}}{\delta + \hat{\mu}^*},$$

$$P_t = \frac{1 - \tilde{\alpha}(\lambda, \kappa)}{\lambda} \frac{1}{1 + B_t^g/M_t} \frac{1 + \gamma}{\bar{v}} \frac{M_t + B_t^g}{Y}.$$