# Banks' Liquidity Management and Systemic Risk[1]

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[1]The analyses, opinions and findings of this paper represent our own views, and are not necessarily those of Banco de Portugal or the Eurosystem.

# Motivation

- Why do banks hold excess liquidity at times of systemic financial crises?
- Existing explanations focus on precautionary savings against:
  - Aggregate real shocks (Ashcraft et al., 2011; Acharya and Merrouche, 2013)
  - Counterparty risk (Heider et al., 2015)
- Key role of self-fulfilling expectations as triggers of systemic financial crises
- Particularly important in the banking system, where liquidity and maturity transformation make banks vulnerable to self-fulfilling depositors' runs
  - Argentina 2001
  - Greece 2015
  - U.S. 2007-2009: money market funds (Gorton and Metrick, 2012) and life insurance funds (Foley-Fisher et al., 2015)

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## Aim

*Study banks' liquidity management in a theory of self-fulfilling bank runs*

# Introduction

## Existing Literature

- "1$^{st}$ generation" models of self-fulfilling bank runs (Cooper and Ross, 1998; Ennis and Keister, 2006):
  - Sunspots
  - Liquidity management, but no excess liquidity unless run-proof
- "2$^{nd}$ generation" models of self-fulfilling bank runs (Rochet and Vives, 2004; Goldstein and Pauzner, 2005):
  - Global games
  - No liquidity management or excess liquidity

## Our Contribution:

- Banks hold liquidity to provide insurance
- The concept of excess liquidity is well defined
- Banks' liquidity management and depositors' expectations jointly and endogenously determined

# Preview of the Environment

- A Diamond-Dybvig economy with uncertainty
  - Idiosyncratic shocks
  - Aggregate real shocks
- Banks provide insurance against both shocks
  - Offer a deposit contract
  - Financed by investing in liquid and partially illiquid risky assets
- With perfect information (PI), banks hold excess liquidity: $L^{PI} > \lambda c^{PI}$
- Strategic complementarities in depositors' withdrawing decisions give rise to multiple equilibria
- Solve multiplicity via a "global game" (Morris and Shin, 1998)
  - Depositors observe a noisy signal $\sigma$ about the aggregate state
  - They run if $\sigma < \sigma^*$
  - The threshold signal $\sigma^*$ is increasing in the terms of banks' deposit contract
  - It is decreasing in banks' liquidity

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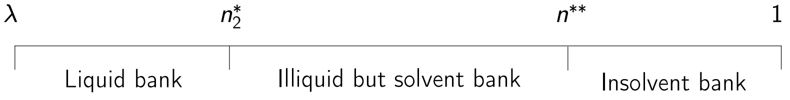
# Preview of the Results

- Banks facing runs decide the pecking order to serve the depositors:
  1. {Liquidation, Liquidity}
  2. {Liquidity, Liquidation}
- Trade off between the costs of:
  - Liquidating the productive asset
  - Employing liquidity and reduce insurance against aggregate real shocks
- We characterize the sufficient conditions for a unique threshold recovery rate $\tilde{r}$ below which the pecking order is {Liquidity, Liquidation}
- As the number of withdrawers increases:



# Preview of the Results

- Assume that $r < \tilde{r}$, so that the pecking order is $\{$Liquidity, Liquidation$\}$
- Ex ante, the banks choose a deposit contract and asset portfolio
- With respect to perfect information (PI), in the banking equilibrium (BE) the banks offer:
  - Lower insurance against idiosyncratic shocks: $c^{BE} < c^{PI}$
  - Higher insurance against aggregate real shocks: $L^{BE} > L^{PI}$
  - Higher excess liquidity: $L^{BE} - \lambda c^{BE} > L^{PI} - \lambda c^{PI}$
- A novel channel: More risk $\Rightarrow$ Excess liquidity

| $\lambda c^{PI}$ | $L^{PI} > \lambda c^{PI}$ | $L^{BE} > L^{PI}$ |
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| Idiosyncratic shocks | + Aggregate real shocks | + Self-fulfilling runs |

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# Plan of the Presentation

- Introduction
- Environment
- Bank Runs
- Banking Equilibrium
- Concluding Remarks

# Environment

- A Diamond and Dybvig (1983) economy
- Three periods $t = 0, 1, 2$
- A continuum of ex-ante identical agents, with endowment $e = 1$ at $t = 0$
- At $t = 1$, they are hit by a private idiosyncratic shock $\theta$, taking value 0 with probability $\lambda$ ("early" consumers) and 1 with probability $1 - \lambda$ ("late" consumers)
- LLN holds
- The shock affects the point in time when they want to consume:

$$U(c_1, c_2, \theta) = (1 - \theta)u(c_1) + \theta u(c_2)$$

with $u(c)$ increasing and concave, with $u(0) = 0$, $RRA > 1$ and satisfying the Inada conditions

# Banks and Technologies

- A large number of banks, in a perfectly-competitive market with free entry
- At $t = 0$, they collect the endowments and issue deposit contracts $\{c, c_L(A)\}$
- The banks invest $L$ in liquidity, yielding 1 in $t = 1$, and $1 - L$ in a productive asset, yielding $r \leq 1$ in $t = 1$ and $A$ in $t = 2$

$$A = \begin{cases} R & \text{with prob p} \\ 0 & \text{with prob 1-p} \end{cases}$$

where $p \sim U[0, 1]$ and $\mathbb{E}[p]R > 1$

▸ Technologies in the banking literature ▸ Perfect information

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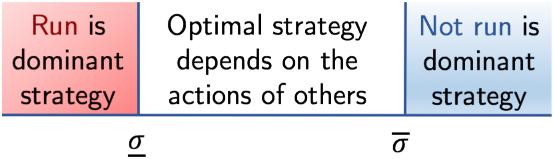
# Bank Runs

- At $t = 1$, the depositors observe a private signal about $p$:

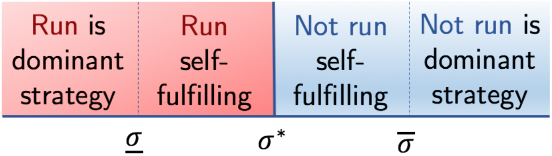
$$\sigma = p + e$$

  where the noise $e \sim U[-\epsilon, +\epsilon]$, with $\epsilon$ positive but small
- The signal allows the depositors to derive:
  - The posterior believes about the true value of $p$
  - The believes of the other depositors
- Assume the existence of two extreme regions of signals



# Threshold Strategies

- They ensure the existence of a threshold signal $\sigma^*$ in the intermediate region (Goldstein and Pauzner, 2005)




- $\sigma^*$ is the threshold signal that ensures that $\mathbb{E}[v(A, n)|\sigma^*] = 0$
- Pecking order $\Rightarrow v(A, n) \Rightarrow \sigma^*$

# Timing

**t=0**:

- The banks collect the deposits, and choose the deposit contract $\{c_1, c_2(A)\}$ and the asset portfolio $\{L, 1 - L\}$

**t=1**:

- The banks choose the pecking order:
  1. {Liquidation, Liquidity}
  2. {Liquidity, Liquidation}
- The depositors observe $\{\theta, \sigma\}$, and late consumers decide whether to run

**t=2**:

- Late consumption with the available resources

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**t=2**:

- Late consumption with the available resources

# Pecking Order 1: {Liquidation, Liquidity}

$$v_1(A,n) = \begin{cases} \sigma u\left(\frac{R(1-L-\frac{nc}{r})+L}{1-n}\right) + (1-\sigma)u\left(\frac{L}{1-n}\right) - u(c) & \text{if } \lambda \leq n < n_1^* \\ \sigma u\left(\frac{r(1-L)+L-nc}{1-n}\right) + (1-\sigma)u\left(\frac{r(1-L)+L-nc}{1-n}\right) - u(c) & \text{if } n_1^* \leq n < n_1^{**} \\ -u\left(\frac{L+r(1-L)}{n}\right) & \text{if } n_1^{**} \leq n < 1 \end{cases}$$

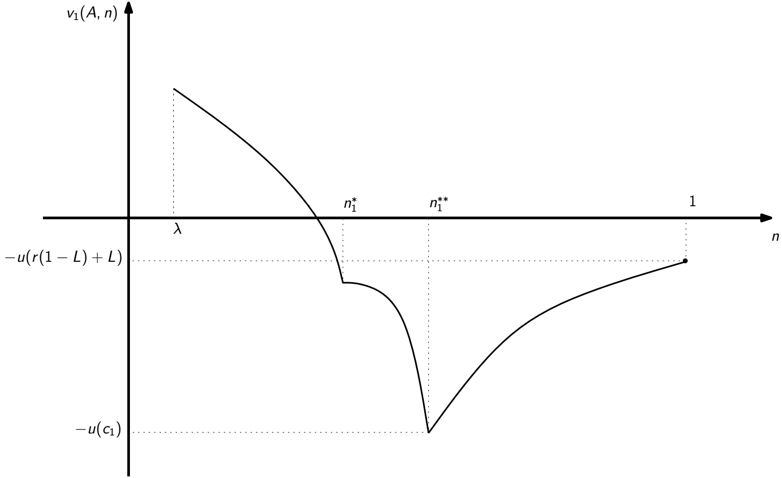
where:

$$n_1^* = \frac{r(1-L)}{c}$$

$$n_1^{**} = \frac{L+r(1-L)}{c}$$

▸ Details

# Pecking Order 1: {Liquidation, Liquidity}



# Pecking Order 1: {Liquidation, Liquidity}

- Given the definition of $\sigma^*$, we get:

$$\sigma_1^* = \frac{\int_\lambda^{n_1^{**}} u(c)dn + \int_{n_1^{**}}^1 u\left(\frac{L+r(1-L)}{n}\right) - \int_\lambda^{n_1^*} u\left(\frac{L}{1-n}\right) dn - \int_{n_1^*}^{n_1^{**}} u\left(\frac{r(1-L)+L-nc}{1-n}\right) dn}{\int_\lambda^{n_1^*} \left[u\left(\frac{R(1-L-\frac{nc}{r})+L}{1-n}\right) - u\left(\frac{L}{1-n}\right)\right] dn}$$

Comparative statics:

$$\frac{\partial \sigma_1^*}{\partial c} > 0$$
$$\frac{\partial \sigma_1^*}{\partial L} < 0$$

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Comparative statics:

$$\frac{\partial \sigma_1^*}{\partial c} > 0$$

$$\frac{\partial \sigma_1^*}{\partial L} < 0$$

# Pecking Order 2: {Liquidity, Liquidation}

$$v_2(A,n) = \begin{cases} \sigma u\left(\frac{R(1-L)+L-nc}{1-n}\right) + (1-\sigma)u\left(\frac{L-nc}{1-n}\right) - u(c) & \text{if } \lambda \leq n < n_2^* \\ \sigma u\left(\frac{R(1-L-D)}{1-n}\right) - u(c) = \sigma u\left(\frac{R\left(1-L-\frac{nc-L}{r}\right)}{1-n}\right) - u(c) & \text{if } n_2^* \leq n < n_2^{**} \\ -u\left(\frac{L+r(1-L)}{n}\right) & \text{if } n_2^{**} \leq n < 1 \end{cases}$$

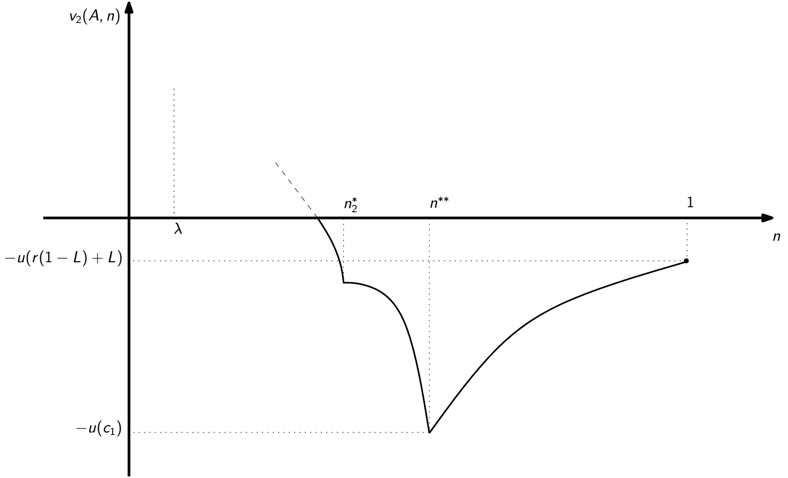
where:

$$n_2^* = \frac{L}{c}$$

$$n_2^{**} = \frac{L+r(1-L)}{c} = n_1^{**} \equiv n^{**}$$

▸ Details

# Pecking Order 2: {Liquidity, Liquidation}



# Pecking Order 2: {Liquidity, Liquidation}

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Comparative statics:

$$\frac{\partial \sigma_2^*}{\partial c} > 0$$
$$\frac{\partial \sigma_2^*}{\partial L} < 0$$

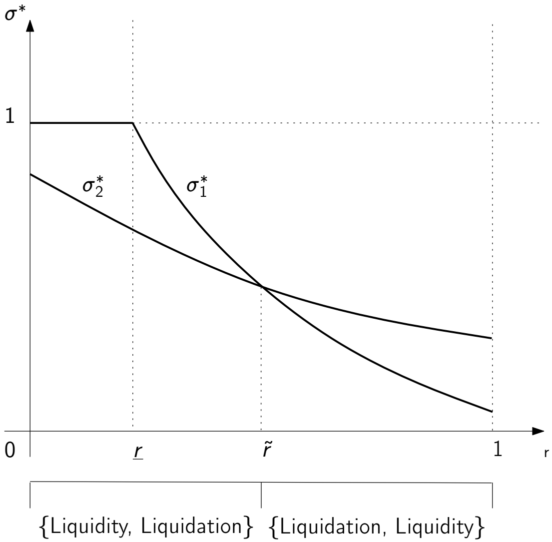
# Optimal Pecking Order

- Banks choose the pecking order that maximizes expected welfare, for given deposit contract and asset portfolio
- Equivalent to minimizing the threshold signal $\sigma^*$

## Proposition

*Assume that the utility function is CRRA, and that the coefficient of relative risk aversion is sufficiently high. Then, there exists a threshold $\tilde{r} \in [0,1]$ such that, for any $r \leq \tilde{r}$, the optimal pecking order is $\{$Liquidity, Liquidation$\}$, and for any $r > \tilde{r}$, the optimal pecking order is $\{$Liquidation, Liquidity$\}$.*

# Optimal Pecking Order: Intuition




▸ More intuition

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# Banking Problem

- Assume that $r \leq \tilde{r}$, so that the pecking order is $\{$Liquidity, Liquidation$\}$
- At date 0, the bank solves:

$$\max_{c,L} \int_0^{\sigma_2^*} u(L+r(1-L))dp + \int_{\sigma_2^*}^1 \left[\lambda u(c) + (1-\lambda)\left[pu\left(\frac{R(1-L)+L-\lambda c}{1-\lambda}\right) + (1-p)u\left(\frac{L-\lambda c}{1-\lambda}\right)\right]\right] dp$$

subject to the liquidity constraint $L \geq \lambda c$

# Banking Equilibrium

## Lemma

*The banking equilibrium features excess liquidity: $L^{BE} > \lambda c^{BE}$. The deposit contract and asset portfolio satisfy the distorted Euler equation:*

$$\int_{\sigma_2^*}^{1} \left[u'(c) - pRu'(c_L(R))\right] dp + \sigma_2^*(1-r)u'(L + r(1-L)) = \left[\frac{\partial \sigma_2^*}{\partial L} + \frac{1}{\lambda}\frac{\partial \sigma_2^*}{\partial c}\right] \Delta U(c, L)$$

## Proposition

*Assume that $u(c)$ is invertible, and $\lambda > u(1)$. Then, the banking equilibrium exhibits more excess liquidity than the equilibrium with perfect information:*

$$c^{BE} < c^{PI}$$

$$L^{BE} > L^{PI}$$

$$L^{BE} - \lambda c^{BE} > L^{PI} - \lambda c^{PI}$$

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# Concluding Remarks

- A novel channel explaining excess liquidity at times of systemic financial crises
- A reaction to self-fulfilling bank runs
- A $2^{nd}$ generation model of self-fulfilling bank runs, with:
  - Idiosyncratic and aggregate real shocks
  - Bank liquidity holding
- When the recovery rate is sufficiently low:
  - Higher liquidity lowers the probability of a self-fulfilling run
  - A rationale for Liquid → Illiquid but solvent → Insolvent
  - A positive probability of a self-fulfilling run cause liquidity hoarding

▸ Work in progress

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# Technologies in the Banking Literature

**Diamond and Dybvig (1983); Goldstein and Pauzner (2005)**

- Perfectly liquid productive asset
- No liquid asset

**Anhert and Elamin (2014)**

- Perfectly liquid productive asset
- Liquid asset at $t = 1$ to transfer resources in case of runs

**Deidda and Panetti (2016)**

- Partially illiquid productive asset (Altman et al., 2004)
- Liquid asset at $t = 0$ and $t = 1$

Back

# Banking Equilibrium with Perfect Information

- The bank solves:

$$\max_{c,L,D} \lambda u(c)+(1-\lambda)\int_0^1 \left[pu\left(\frac{R(1-L-D)+L+rD-\lambda c}{1-\lambda}\right)+(1-p)u\left(\frac{L+rD-\lambda c}{1-\lambda}\right)\right]dp$$

subject to $D \geq 0$ and the liquidity constraint:

$$L + rD \geq \lambda c$$

## Lemma

*In the banking equilibrium with perfect information, $D^{PI} = 0$. Moreover, $L^{PI} > \lambda c^{PI}$, i.e. the bank holds excess liquidity. In equilibrium:*

$$u'(c) = \mathbb{E}[p]Ru'(c_L(R))$$

Back

# Pecking Order 1: {Liquidation, Liquidity}

- The threshold signal $\underline{\sigma}_1$ characterizing the lower dominance region is the one that equalizes:

$$u(c) = \underline{\sigma}_1 u\left(\frac{R\left(1 - L - \frac{\lambda c}{r}\right) + L}{1-\lambda}\right) + (1 - \underline{\sigma}_1)u\left(\frac{L}{1-\lambda}\right)$$

- Rearranging, we obtain::

$$\underline{\sigma}_1 = \frac{u(c) - u\left(\frac{L}{1-\lambda}\right)}{u\left(\frac{R\left(1-L-\frac{\lambda c}{r}\right)+L}{1-\lambda}\right) - u\left(\frac{L}{1-\lambda}\right)}$$

# Pecking Order 1: {Liquidation, Liquidity}

$$\frac{\partial v_1}{\partial n} = \begin{cases} \sigma u'(c_L(R,n))\frac{-\frac{R}{r}c(1-n)+\left[R\left(1-L-\frac{nc}{r}\right)+L\right]}{(1-n)^2} + \frac{(1-\sigma)u'(c_L(0,n))L}{(1-n)^2} & \text{if } \lambda \leq n < n_1^* \\ u'(c_L^L(n))\frac{r(1-L)+L-c}{(1-n)^2} < 0 & \text{if } n_1^* \leq n < n_1^{**} \\ u'(c^B(n))\frac{c^B(n)}{n} > 0 & \text{if } n^{**} \leq n < 1 \end{cases}$$

- In the intermediate interval, $\partial v_1 / \partial n < 0$ becausase $c > L$
- If that was not case, the contract would be run-proof

# Pecking Order 1: {Liquidation, Liquidity}

- What is the sign of the derivative in the first interval?

$$\frac{\partial v_1}{\partial n} = \sigma u'(c_L(R,n)) \frac{R\left(1 - L - \frac{nc}{r}\right) - \frac{R}{r}c(1-n)}{(1-n)^2} + \frac{L}{(1-n)^2}\left[\sigma u'(c_L(R,n)) + (1-\sigma)u'(c_L(0,n))\right]$$
$$= \frac{L}{(1-n)^2}\left[\sigma u'(c_L(R,n)) \frac{R\left(1 - L - \frac{c}{r}\right)}{L} + \sigma u'(c_L(R,n)) + (1-\sigma)u'(c_L(0,n))\right]$$

- This derivative is negative whenever the term in the square brackets is negative, or:

$$L < \frac{\sigma u'(c_L(R,n))R\left(\frac{c}{r} - 1\right)}{\sigma u'(c_L(R,n))(1-R) + (1-\sigma)u'(c_L(0,n))}$$

# Pecking Order 1: {Liquidation, Liquidity}

- The term on the RHS of this last expression is larger than 1, implying that the condition is always satisfied. To see that, rewrite it as:

$$\sigma u'(c_L(R,n))\left[\frac{Rc}{r}-1\right] > (1-\sigma)u'(c_L(0,n))$$

- Notice that:

$$\frac{\sigma}{1-\sigma}\left[\frac{Rc}{r}-1\right] > \frac{u'(c_L(0,n))}{u'(c_L(R,n))} > 1$$

- This implies:

$$\sigma\left[\frac{Rc}{r}-\cancel{1}\right] > 1-\cancel{\sigma} \qquad \Rightarrow \qquad \frac{\sigma Rc}{r} > 1$$

- A sufficient condition for this to be true is that $\sigma R > 1$. By definition, remember that $\sigma = p + e$, so $\sigma R = pR + pe$, which is larger than 1 – provided that $\epsilon$ is sufficiently small – as $\mathbb{E}[p]R > 1$ by assumption.

Back

# Pecking Order 2: {Liquidity, Liquidation}

$$\frac{\partial v_2}{\partial n} = \begin{cases} \sigma u'(c_L(R,n))\frac{c_L(R,n)-c}{1-n} - (1-\sigma)u'(c_L(0,n))\frac{c-c_L(0,n)}{1-n} & \text{if } \lambda \leq n < n^* \\ \sigma u'(c_L^D(R,n))\frac{c_L^D(R,n)-\frac{Rc}{r}}{1-n} < 0 & \text{if } n^* \leq n < n^{**} \\ u'(c^B(n))\frac{c^B(n)}{n} > 0 & \text{if } n^{**} \leq n < 1 \end{cases}$$

- In the intermediate interval, $\partial v_2/\partial n < 0$ as $c_L^D(R,n) < \frac{Rc}{r}$
- If that was not case, we would end up with $n^{**} > 1$, which is impossible

# Pecking Order 2: {Liquidity, Liquidation}

- What is the sign of the derivative in the first interval?
- Remember that $u(c)$ is strictly concave on an open internal X if and only if:

$$u(x) - u(y) < u'(y)(x - y)$$

for all x and y in X. Hence, when $\lambda \leq n < n^*$:

$$\frac{\partial v}{\partial n} < \sigma \frac{u(c_L(R,n)) - u(c)}{1-n} - (1-\sigma)\frac{u(c) - u(c_L(0,n))}{1-n}$$
$$< \frac{\sigma u(c_L(R,n)) + (1-\sigma)u(c_L(0,n)) - u(c)}{1-n} = \frac{v(\sigma,n)}{1-n}$$

- Thus, when $v_2(A,n) \leq 0$, the derivative is negative

# Pecking Order 2: {Liquidity, Liquidation}

- What is the sign of $v_2(A, n_2^*)$?

$$v(A, n_2^*) = \sigma u\left(\frac{R(1-L)}{c-L}c\right) - u(c)$$

- This is negative whenever:

$$\sigma < \frac{u(c)}{u\left(\frac{R(1-L)}{c-L}c\right)} \equiv \bar{\sigma}$$

where $\bar{\sigma} > 1$ whenever $R < \frac{c-L}{1-L}$

- The condition on $R$ is always satisfied in the banking equilibrium
- Hence, $v_2(A, n_2^*) < 0$

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# Optimal Pecking Order: Intuition

- At the threshold $n^{**} = \frac{L+r(1-L)}{c}$, a late consumer who does not run gets:

$$\begin{aligned} \{\text{Liquidation, Liquidity}\}: &\quad c_L^L(n^{**}) = 0 \\ \{\text{Liquidity, Liquidation}\}: &\quad c_L^D(R, n^{**}) = 0 \end{aligned}$$

- Hence, increasing $r$, by the Inada conditions, have:
  - A large positive effect on the utility of not running
  - By concavity of $u(c)$, this effect is decreasing in $r$
- Hence, both $\sigma_1^*$ and $\sigma_2^*$ are decreasing and convex functions of $r$

# Optimal Pecking Order: Intuition

- Liquidation of the productive asset is costly in terms of:
  - Forgone resources at date $t = 1$ due to $r < 1$
  - Forgone late consumption in the good state of the world
- Using liquidity is costly in terms of
  - Forgone late consumption in the bad state of the world
- If $r \to 0$:
  - Late consumers who do not run are worse off under $\{Liquidation, Liquidity\}$
  - Hence, $\sigma_1^* > \sigma_2^*$
- If $r \to 1$:
  - Late consumers who do not run are worse off under $\{Liquidity, Liquidation\}$
  - If sufficiently risk averse, depositors do not care about forgone late consumption in the good state of the world
  - $\sigma_2^* > \sigma_1^*$

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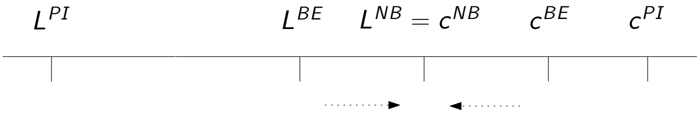
# Policy Implications (Work in Progress)

- Assume that $r \leq \tilde{r} \Rightarrow$ The pecking order is $\{$Liquidity;Liquidation$\}$
- Impose the "Narrow Banking" constraint $L \geq c$
- The banks become completely run-proof
- In equilibrium, $L^{NB} = c^{NB}$
- However, insurance against the idiosyncratic shocks is lower: $c^{NB} < c^{BE}$
- Would $L^{NB} \lesseqgtr L^{BE}$?

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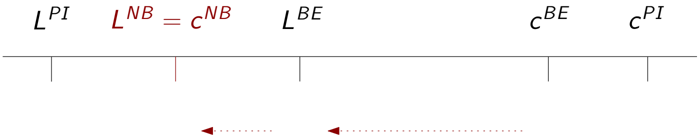


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