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Content

Editor's note[1]

Pedro Duarte Neves

July 2023

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1. This issue of the Banco de Portugal *Economic Studies* contains three studies. The first study provides natural interest estimates in the euro area and discusses its informative content in terms of the conduct of monetary policy. The second study describes the impact of an unconventional monetary policy on the economy. Finally, the third study analyses the determinants of firms' sales.

2. The concept of the natural rate of interest dates back to the nineteenth century and is traditionally attributed to the Swedish economist Knut Wicksell.[2],[3] From several possible definitions for this concept, the editor chooses that of the economist John H. Williams:[4] "*The natural rate is an abstraction: like faith, it is seen by its works. One can only say that if the bank policy succeeds in stabilizing prices, the bank rate has been brought in line with the natural rate, but if it does not, it must not have been*".

The concept of a natural rate of interest corresponds to a real – rather than a nominal – interest rate, generally of a short term nature. In the latest World Economic Outlook, April 2023, the International Monetary Fund (IMF) included a chapter entitled "The Natural Rate of Interest: *Drivers and Implications for Policy*." IMF economists conclude that the natural rate of interest[5] has shown a consistent – and relatively synchronised –

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1. The analyses, opinions and conclusions expressed in this editorial are entirely those of the editor and do not necessarily coincide with those of Banco de Portugal or the Eurosystem.

2. Wicksell's original work (1898) is in Swedish. The English translation is much more recent, from 1936.

3. Although the concept of the natural rate of interest is usually traced back to Wicksell, Claudio Borio mentioned the following: "The concept of the natural rate of interest goes back a long way in the history of economic thought. While it is most closely associated with Wicksell (1898), the seeds of the idea can already be found in John Stuart Mill". See Borio, Claudio (2021), "Navigating by r*: safe or hazardous?", Keynote lecture, SUERF, Bocconi, OeNB workshop on "How to raise r*?", 15 September 2021.

4. Williams, John H., "The Monetary Doctrine of J. M. Keynes", *The Quarterly Journal of Economics* 45, no. 4 (August 1931), 547-587. This quote is taken from a recent speech by the President of the Federal Reserve Bank of New York, John C. Williams, "Measuring the Natural Rate of Interest: Past, Present, and Future", of 19 May 2023.

5. The IMF study uses several alternative methods to estimate the natural rate of interest, which is obviously a consequence of the uncertainty surrounding this estimation (as it is not an observable variable). In this editorial, reference is made to the results obtained with the Platzer and Peruffo (2022) model. See

decline in most advanced economies over the past 50 years. Estimates over the period 2015-19 point to natural interest rates close to 0% for France and Japan, around 0.5% for Germany and the United Kingdom, and around 1% for the United States. Compared with the 1975-79 period, the natural rate of interest fell by around 3 percentage points in the United States, between 1 and 2 percentage points in other advanced economies (France, Germany and the United Kingdom) and remained roughly constant in Japan, China and India.

The Federal Reserve Bank of New York publishes natural interest rate estimates for the United States and advanced economies on a regular basis.[6] In the case of the US, these estimates are currently close to 1%, having been around 0.5% in the period following the Great Financial Crisis; in advanced economies they have been close to 0.5% over the past ten years. These figures are thus very similar to those of the IMF.

Finally, the members of the Federal Open Market Committee present their individual views on future interest rate developments, including those corresponding to future economic stability. For example, on 14 June this year the measures of central location for distribution of individual 'long-term' estimates stood at around 2.5%, corresponding – for an inflation target of 2% – to a natural rate of 0.5%, close to the aforementioned values.

In the World Economic Outlook, the IMF identifies the key factors – macroeconomic and financial – that drive developments in the natural rate of interest. To assess each factor's importance for the change in the natural rate of interest, the authors use a macroeconomic model[7] – calibrated for the eight largest economies in the world – making it possible to estimate the quantitative contribution of each factor. The study concludes that the decline in the natural rate of interest in advanced economies was almost entirely the result of reduced productivity and demographic developments.

3. The opening study of this issue of the Banco de Portugal Economic Studies, by Carvalho, presents estimates of the natural rate of interest in the euro area. The author presents the concept of the natural rate of interest as the real interest rate that stabilises output and inflation.

Platzer, Josef, and Peruffo, Marcel (2022), "Secular Drivers of the Natural Rate of Interest in the United States", IMF Working Paper 2022/030, International Monetary Fund.

6. The Federal Reserve Bank of New York releases estimates based upon two models: Laubach, Thomas and Williams, John C. (2003), "Measuring the Natural Rate of Interest", *Review of Economics and Statistics* 85, no. 4, November, 1063-70 and Holston, Kathryn, Laubach, Thomas, and Williams, John C. (2017), "Measuring the Natural Rate of Interest: International Trends and Determinants", Journal of International Economics 108, Supplemental 1 (May): S39-S75. The release of these estimates has recently been resumed after being suspended during the pandemic.

7. See again Platzer, Josef, and Peruffo, Marcel (2022), "Secular Drivers of the Natural Rate of Interest in the United States", IMF Working Paper 2022/030, International Monetary Fund.

The main results essentially confirm the results found in the literature:

(a) The natural interest rate in the euro area fell sharply over the period, from around 3% in the early 1970s to values currently close to 0.5%;
(b) These values are very close to other existing estimates for the current natural rate of interest in the euro area;
(c) The natural interest rate gap – defined as the difference between the actual real interest rate and the natural interest rate – tends to shrink during economic downturns as monetary policy becomes more accommodative, which was especially evident in the period following the great financial crisis;
(d) Finally, the real interest rate gap has been close to zero following the increase in the ECB's intervention rates initiated in July 2022.

4. How useful could the concept of the natural rate of interest be to conduct economic policy?

In terms of economic policy implications, the IMF study concludes that: "*All else equal, higher natural rates typically decrease fiscal space – that is, higher primary surpluses (smaller deficits) are required to stabilise debt ratios. But they also free up some monetary policy space. Higher natural rates imply higher nominal rates over the long term, providing central banks with more space to react to negative demand shocks without hitting the effective lower bound*." In terms of this second point, the ECB's monetary policy strategy review[8] has concluded accordingly: "*In particular, the pronounced trend decline in the equilibrium real interest rate, if persistent, implies that the effective lower bound on nominal interest rates will more frequently constrain the conduct of monetary policy. The facilitation of cross-country macroeconomic adjustment within the euro area, downward nominal wage rigidities and measurement bias also warrant an inflation buffer*."

The importance of the concept of the natural rate of interest for the conduct of monetary policy has practical implications. For example, estimates of the natural rate of interest play a key role in monetary policy rules, of which the most prominent is the Taylor rule. The Federal Reserve Bank of Cleveland regularly releases prescriptions of seven monetary policy rules for which an estimate of the natural rate of interest is required.[9] Furthermore, and as discussed in the Carvalho study, the natural interest rate gap can signal the monetary policy stance (accommodative, neutral or tightening).

Naturally, some caution is warranted when directly using the natural rate of interest concept in conducting monetary policy. First, different analytical approaches lead to quantitatively different results, therefore it is not always clear whether the actual real interest rate is above the natural rate of interest or not. Moreover, it is also important

8. See "The ECB's monetary policy strategy statement", ECB (2021), the ECB's website.

9. Only one of the seven monetary policy rules does not include the natural rate of interest. This is the first-difference rule, where the assumption of a constant natural rate of interest causes it to disappear from this monetary policy rule.

to consider – as argued, for example, by Claudio Borio of the BIS in several working papers,[10] one of them suggestively entitled "Navigating by r*: safe or hazardous?" – that monetary policy can play a non-neutral role in the behaviour of an economy, even in the medium or long term; i.e. the natural rate of interest may not be independent of monetary policy. This is relevant if monetary policy has an impact on the financial cycle, in particular by contributing to excessive risk taking.

5. The second study, by Adão and Martín, also focuses on monetary policy, this time on quantities rather than prices. The study analyses the consequences of changes in the size of the central bank's balance sheet and not of changes in intervention rates. The authors, using a theoretical model – describing how unconventional monetary policy affects the dynamic equilibrium of the economy – yield significant results: central bank intervention reduces the economy's risk premium, ensures less investment and consumption fluctuation, and greater consumer well-being. In other words, the study explains the benefits arising from unconventional monetary policy measures, such as the acquisition by the central bank of very significant amounts of financial assets.

6. The focus of the study on the risk premium of the economy is also explained by the information on the stage of the business cycle that this concept can provide. At the end of the 1990s, the economists James Stock and Mark Watson – in a study[11] that is considered the most complete synopsis of the stylised facts of business cycles in the US economy – analysed the statistical properties of two indicators: the risk premium as indicated by different maturities (by comparing investments in ten-year and three-month government debt instruments respectively); the risk premium as indicated by different issuers (by comparing six-month investments in private and government bonds). They concluded that the risk premium generally has a countercyclical behaviour and the ability to provide forward-looking guidance on the functioning of the economy.

The sharp increase in private non-financial debt over the past 30 years – graphically hockey stick-shaped[12] – has chronologically coincided with changes in business cycles' statistical properties,[13] in several aspects, as well as in the ability of some variables to continue to provide forward-looking indicators. The current phase of the business cycle, especially in a period of transition from a low interest rate environment over a long period to a quantitatively very fast spike in interest rates, is therefore an opportunity to confirm – or to review – the information that the economy's risk premium can provide

10. See again Borio, Claudio (2021), "Navigating by r*: safe or hazardous?", Keynote lecture, SUERF, Bocconi, OeNB workshop on "How to raise r*?", 15 September 2021.

11. "Business Cycle Fluctuations in US Macroeconomic Time Series", James H. Stock and Mark W. Watson, (1999), Handbook of Macroeconomics, editors John B. Taylor and Michael Woodford, Volume 1A, chapter 1, pages 3-64, North Holland.

12. See Jordà, Òscar, Schularick, Moritz, and Taylor, Alan M. (2017), "Macrofinancial History and the New Business Cycle Facts", NBER Macroeconomics Annual, Vol. 31, 213-263, The University of Chicago Press.

13. ibid.

about future developments in the economy.

7. The final study of this Banco de Portugal *Economic Studies*, by Silva and Moreira, is a review of the literature on the determinants of sales growth of non-financial corporations and assesses their explanatory ability in the Portuguese economy. The study's empirical contribution is based upon the use of a combination of several micro-databases – the Central Balance Sheet Database of the Banco de Portugal, the Central Credit Register, the Integrated System of Securities Statistics and the *Quadros de Pessoal* (linked employee-employer dataset) – making it possible to monitor around 190,000 firms over the period 2006-21.

The authors classify the determinants of sales growth in non-financial corporations into three categories: individual firms' characteristics, decision-making features throughout the firm's life cycle and characteristics of the firm's external environment. The results are broadly in line with those found in the literature. However, there is an exception: the study pinpoints a negative relationship between sales growth and productivity. The interpretation of this outcome requires further in-depth analyses.

Finally, the authors conclude that the model's predictive ability – evaluated over the 2018-21 period – is marginally higher than that of a simple model using the median sales growth rate as a forecast.

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Non-technical summary

July 2023

The euro area natural interest rate — Estimation and importance for monetary policy

Alexandre Carvalho

Since 2022, the European Central Bank has raised monetary policy interest rates with the aim of controlling inflation. For this, it is necessary to assess the appropriate level of interest rates that restrict economic activity and reduce pressure on prices. At the same time, it is also relevant to identify the level at which interest rates may stand when inflation returns to its target as set by the European Central Bank.

The concept of natural interest rate, defined as the real interest rate that neither stimulates nor reduces economic activity or inflation, is particularly useful for answering these questions. The difference between the interest rate observed in the economy and the natural interest rate — the real interest rate gap — allows an assessment on how more restrictive or more accommodative monetary policy is. At the same time, the natural interest rate can be seen as the level to which interest rates should converge over time.

This study presents estimates for the natural interest rate in the euro area based on two modifications to the Holston *et al.* (2017) model. First, the model is adapted for the pandemic period. Second, the model explicitly includes the role of inflation expectations.

Despite the uncertainty associated with the results, the estimates presented in Figure 1 suggest that the natural interest rate in the euro area shows a downward trend, going from levels around 3% in the early 1970s, to values around 0.5% in 2022. Downward movements tend to occur following periods of economic crisis, especially after the global financial crisis of 2007/08. This result is in line with other estimates of the natural interest rate for the euro area and other European economies.

Evaluating the results for the real interest rate gap, it is possible to conclude that monetary policy in the euro area was accommodative after periods of economic crisis, particularly in 2008. Since 2022, this gap has increased significantly, in line with the policy followed by the European Central Bank, which is seeking to restrict economic activity and, therefore, reduce inflation.

The use of the natural interest rate in the conduct of monetary policy presents some challenges. Seeking to follow this rate at all times may not be desirable, as there is high uncertainty surrounding any estimate. However, the use of the natural interest rate in monetary policy communication can play a crucial role in ensuring that interest rate expectations are consistent with a return of inflation to its target. The example of the Federal Reserve shows that communication about the expected level of the policy interest rate in the long-term — based on the natural interest rate — can influence market expectations of longer-term interest rates.

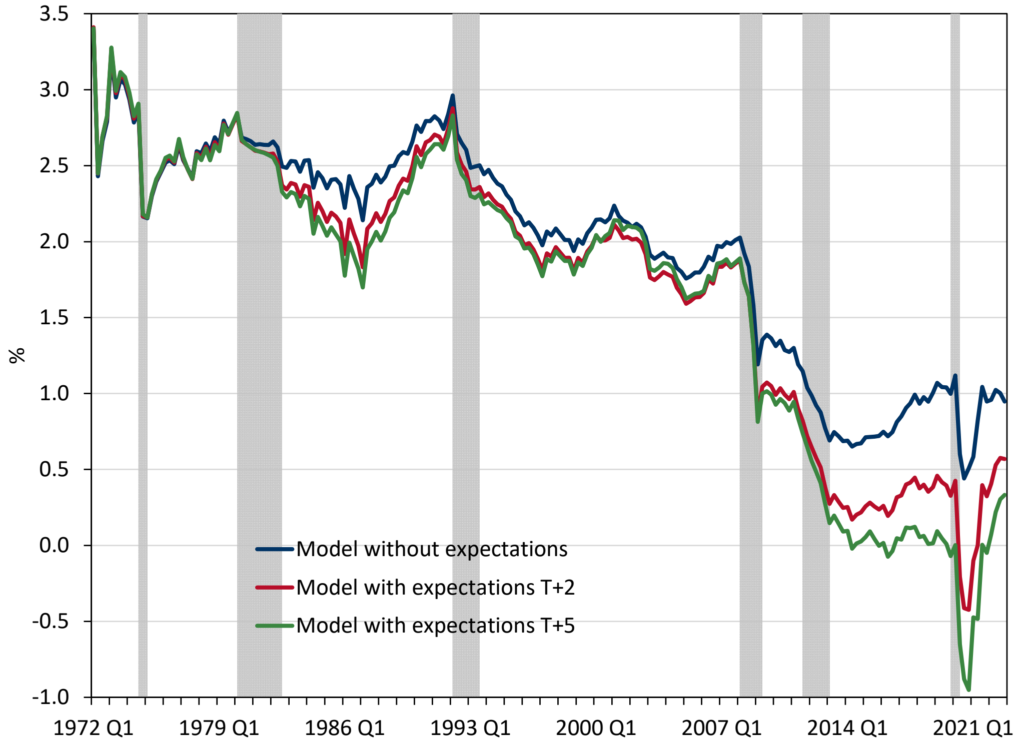

FIGURE 1: Euro area natural interest rate estimates

Notes: Estimates based on modified versions of the Holston *et al.* (2017) model. Gray bars mark the periods of recession in the euro area according to the Euro Area Business Cycle Network — EABCN
Sources: ECB, Eurostat, EABCN, Refinitiv and author calculations.

The euro area natural interest rate – Estimation and importance for monetary policy

Alexandre Carvalho
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Abstract
This study presents estimates of the euro area natural interest rate for the last 50 years. The results are obtained using a more general version of the Holston *et al.* (2017) model adjusted for the pandemic period that incorporates the role of inflation expectations. The estimated natural interest rate shows a downward trend from around 3% in the early 1970s to levels around 0.5% in 2022. The real interest rate gap – defined as the difference between the observed interest rate and the natural interest rate – allows an assesment on the degree of monetary policy accommodation. The estimates obtained indicate an accommodative monetary policy after the start of the global financial crisis, in a progressive way until the recent sharp rise in interest rates. The communication of the level of the natural interest rate by central banks can be relevant to influence expectations of long-term interest rates. **Keywords:** Natural interest rate, semi-structural models, Kalman filter, monetary policy. (JEL: C32, E43, E52)

1. Introduction

Monetary policy since 2022 has been, in most advanced economies, marked by the increase in policy interest rates to ensure the return of inflation to levels consistent with the price stability objectives. Monetary policy uses interest rates to restrict demand and activity and consequently reduce the pressure on prices. However, a correct calibration of monetary policy implies a correct measurement of the degree of accommodation generated by the policy interest rate. At the same time, and after the prolonged period of low interest rates registered in most advanced economies, it is also relevant to assess the level at which interest rates should converge when inflation returns to levels consistent with the objectives of central banks.

The concept of natural interest rate — defined as the real interest rate that neither stimulates nor reduces economic activity or inflation — is particularly useful for the conduct of monetary policy. By comparing this rate with the real interest rate observed

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in the economy, it is possible to assess the degree of monetary policy accommodation. At the same time, the natural interest rate can be seen as the level to which interest rates should converge in the long term.

Since the natural interest rate is an unobservable variable, it is necessary to use estimates to assess its evolution. Among the various methodologies presented in the literature, the model developed by Laubach and Williams (2003) and extended in Holston *et al.* (2017) — henceforth HLW — is among the most used. This semi-structural model estimates the natural interest rate based on the relationship between the real interest rate gap (measured by the difference between the observed real interest rate and the natural interest rate), the output gap and inflation. The model estimates a Phillips curve and an IS curve and uses the Kalman filter to obtain estimates of the natural interest rate.

This study discusses and presents estimates of the euro area natural interest rate for the last 50 years based on two modifications to the Holston *et al.* (2017) model. Firstly, the original HLW model is adapted for the pandemic period, as presented by the authors in Holston *et al.* (2020), including a variable in the measurement of the level of potential output to capture part of the fluctuations recorded in this period. The Covid-19 pandemic and the imposed lock-down measures caused quite sharp fluctuations in the main euro area macroeconomic aggregates that may not be correctly captured by the model.

Secondly, the original HLW model does not include inflation expectations. This study discusses a modification to the original structure of the model to explicitly include the role of inflation expectations. This modification follows closely the work developed by Lopez-Salido *et al.* (2020) that explores the role of inflation expectations in the HLW model for the United States.

This study presents estimates for the natural interest rate based on three specifications: The first includes only the adaptation to the pandemic period, while the second and third combine the adaptation for the pandemic period with forward-looking inflation expectations over two different horizons. Despite the uncertainty associated with the results, estimates indicate a downward trend in the natural interest rate in the euro area since the early 1970s, moving from levels around 3% to levels around 0.5% in 2022.

The study is organised as follows. Section 2 presents a summary of the literature related to the natural interest rate, discussing the concept, the different methodologies used in its estimation, its evolution over time and the main determinants of this evolution. Section 3 begins by presenting the original HLW methodology, followed by a discussion of the two modifications explored in the study. Section 4 presents the results of the three estimated specifications. Section 5 seeks to draw some conclusions about the use of the natural interest rate in the conduct of monetary policy and Section 6 concludes.

2. Literature Review

2.1. *Concept*

The natural interest rate is a central concept in macroeconomics. Originally presented by Wicksell (1898) who discussed the existence of *"a certain rate of interest on loans which is neutral in respect to commodity prices, and tends neither to raise nor lower them. This is necessarily the same as the rate of interest which would be determined by supply and demand if no use were made of money and all lending were effected in the form of real capital goods"*.

Woodford (2003) introduced the concept of the natural interest rate into modern macroeconomics in the Neo-Keynesian framework, characterizing it as a short-term real interest rate that guarantees price stability every period in a model without nominal frictions. Laubach and Williams (2003) present the natural interest rate as the short-term real interest rate consistent with output at its potential level and with stable inflation over the medium term. This interpretation for the natural interest rate can be seen as the result from monetary policy rules, such as the Taylor rule (Taylor (1993)), when the output level is at its potential level and inflation is at its defined target by the central bank.

Regardless of the definition considered, the concept of natural interest rate can be seen as the real interest rate that stabilises the level of production and inflation. If the real interest rate observed in an economy is above (below) the natural interest rate then demand is expected to decrease (increase) and consequently inflation will decrease (increase). As a result, and since the main monetary policy instrument in advanced economies is the interest rate, the natural interest rate can be used as a metric for the degree of monetary policy accommodation: if the real interest rate resulting from monetary policy is above (below) the natural interest rate then monetary policy can be classified as contractionary (expansionary).

This study seeks to contribute to the understanding and estimation of the natural interest rate in the euro area according to the Laubach and Williams (2003) definition. This assessment is particularly relevant in the current context of high inflation in the euro area where monetary policy is using interest rates to control inflation, after a significant period of low interest rates.

2.2. *Estimation*

The natural interest rate is an unobservable variable, so it has to be estimated. There are different methodologies, depending on the type of model and the definition of the inherent natural interest rate. This literature can be divided into two groups. The first uses structural general equilibrium models, such as overlapping generation models — OLG — or dynamic stochastic general equilibrium models — DSGE — to estimate the natural interest rate. These two types of models employ the definition of natural interest rate presented in Woodford (2003). The OLG models estimate the natural interest rate as the real rate of return on capital that balances saving and investment in an economy in the absence of nominal frictions and reallocation shocks. Since this type of model does not allow an evaluation of the evolution of the natural interest rate over the economic cycle, the results can be interpreted as the long-term trend of the natural interest rate. These models are well suited to directly assess the role of demographic trends (such as average life expectancy, birth rates, etc.) in the evolution of the natural interest rate (see Bielecki *et al.* (forthcoming) and Papetti (2021) for results for the euro area). In the DSGE models, the estimation seeks to recover the real interest rate that would prevail in a counterfactual economy with flexible prices and wages, subject to shocks. This methodology makes it possible to assess the evolution of the natural interest rate over the economic cycle (see Del Negro *et al.* (2017) or Barsky *et al.* (2014) for estimates for the United States and Neri and Gerali (2019) for the euro area).

The second group of methodologies for estimating the natural interest rate uses semi-structural models, which impose restrictions on the data based on relationships present in macroeconomic theory and, through econometric techniques, estimate the natural interest rate. In this group, the main reference is the Laubach and Williams (2003) model where a Phillips curve and an intertemporal IS curve are estimated and, using the Kalman filter, obtain an estimate for the natural interest rate in the United States. This model seeks to extract the natural interest rate through low-frequency movements in the data that are consistent with the imposed macroeconomic structure, so the results should be interpreted as the level of the natural interest rate in the medium term where changes in the economic cycle have faded. The Laubach and Williams (2003) model was subject to several adaptations with the aim of improving its structure. Lopez-Salido *et al.* (2020) estimate the model for the United States using measures of forward looking inflation expectations rather than adaptive expectations. Brand and Mazelis (2019) apply the model to the euro area by adding a Taylor rule, a block for the labour market and relaxing the assumption of a unit root in the Phillips curve. Krustev (2019) extends the model in order to capture the impact that the financial cycle has on the determination of the natural interest rate. Pedersen (2015) estimates the natural interest rate for Denmark by modifying the model for a small open economy.

The present study seeks to contribute to this literature by estimating the natural interest rate in the euro area by modifying the Laubach and Williams (2003) model to better capture the pandemic shock in 2020 and considering measures of forward looking inflation expectations, in line with Lopez-Salido *et al.* (2020).

2.3. *Evolution and determinants*

Despite the existence of several interpretations for the natural interest rate and different methodologies for its estimation, the various models consistently point to a reduction in the natural interest rate in the euro area (as well as in most advanced economies[1]) since the 1970s, in particular after the global financial crisis in 2007/08 and the sovereign debt crisis in 2012/13. Since then, the natural interest rate has remained at relatively

1. IMF (2023) presents a revision of natural interest rate estimates for a wide range of advanced and emerging economies.

low values, around 0%[2]. The impact of the pandemic on the natural interest rate is still uncertain, since it was a shock of extreme magnitude that makes it difficult to estimate any unobservable variable, with authors arguing that it may have contributed to its rise or fall (see Jorda *et al.* (2022) and Luzzetti *et al.* (2022)).

The downward trend of the natural interest rate is consistent with the determinants of this variable identified in the literature[3]. The natural interest rate is usually seen as a variable that is determined by real factors, and therefore independent of nominal factors such as monetary policy. Among the main factors, the evolution of productivity and the level of potential output are considered the most relevant. In neoclassical models of economic growth with agents with constant relative risk aversion — CRRA — the equilibrium real interest rate in the savings market is given by the following equation:

$$r^* = \frac{1}{\sigma}g + \theta \tag{1}$$

where r^* is the equilibrium real interest rate, σ represents the intertemporal elasticity of substitution of consumption, g is the potential output growth and θ represents the agents' time preference. As can be seen in equation (1), the natural interest rate is intrinsically linked to the growth of potential output, and consequently to productivity. The intuition of this result is obtained through the intertemporal consumption decision: if the growth of potential output increases, consumers will save less to carry out the same level of consumption in the future, which implies an increase in the interest rate to guarantee the equilibrium between savings and investment. Thus, part of the decline in the natural interest rate recorded in the euro area may be related to the slowdown in potential output and productivity growth, especially after the global financial crisis of 2007/08.

Another factor often presented as a determinant for the evolution of the natural interest rate is the demographic trend. This has a direct impact on the evolution of the workforce in an economy, thereby influencing the growth of potential output, and also influence agents' savings decisions. Thus, the ageing population registered in most advanced economies — characterised by a decrease in birth rates and an increase in average life expectancy — is identified as a factor that led to a decrease in the natural interest rate, since it determines a lower growth of potential output and leads to a greater propensity for agents to save to cope with a longer retirement. Despite the fact that the total number of working-age agents making savings decreases with the ageing of the population (which could lead to an increase in the natural interest rate), the previous effect tends to be identified as dominant.

2. The methodologies discussed so far usually start from some measure of risk-free market interest rates. However, the natural rate of interest should also reflect the real rate of return on capital. Reis (2022) argues that measures of return on capital have shown a relatively constant evolution in contrast to measures of market rates, highlighting the importance of evaluating other ways of estimating the natural interest rate.

3. Brand *et al.* (2018) discuss several factors that influence the natural interest rate in the euro area. Rachel and Smith (2015) provide a global assessment of factors that influence the natural interest rate, as well as a quantification of the contribution of various factors to the evolution of the global natural interest rate.

A final set of factors identified as having an impact on the natural interest rate is related to financial factors. The global financial crisis of 2007/08 and the subsequent sovereign debt crisis in the euro area led to a significant increase in uncertainty and demand for safe assets by economic agents. This leads simultaneously to an increase in savings and a decrease in investment, potentially resulting in a reduction in the natural interest rate. At the same time, changes in banking regulation and supervision adopted also led to an increase in demand for safe assets by banking institutions, which also reinforces this trend.

All these factors contribute to explaining the reduction in the natural interest rate and its permanence at substantially low levels until 2019[4]. The pandemic crisis in 2020 generated a set of shocks of an extreme nature that make it difficult to assess the evolution of the natural interest rate in the recent period. The climate transition, which has been at the centre of discussions of policy makers, may also have an (still uncertain) impact on the natural interest rate (see Mongelli *et al.* (2022)). This study seeks to contribute with estimates for the natural interest rate in the euro area, especially for the period after the pandemic crisis.

3. Methodology

3.1. Laubach and Williams Model

This study adopts a modified version of the model developed in Laubach and Williams (2003) and later extended in Holston *et al.* (2017). Thus, throughout the study, the definition that will be considered for the natural interest rate is a short-term real interest rate consistent with the level of production at its potential level and with stable inflation in the medium term, as the impact of the shocks dissipates.

The structure imposed by the model starts from the equilibrium conditions for the level of inflation and the output gap present in Neo-Keynesian models, as presented in Gali (2008). These conditions can be described by a Phillips curve:

$$\pi_{H,t} = \beta E_t[\pi_{H,t+1}] + \kappa \bar{y}_t \tag{2}$$

And by an IS equation:

$$\bar{y}_t = E_t[\bar{y}_{t+1}] - \frac{1}{\sigma}(i_t - E_t[\pi_{H,t+1}] - r_t^n) \tag{3}$$

where $\pi_{H,t}$ represents the inflation of domestically produced goods and services, $\bar{y}_t$ represents the gap between the observed output level and the level of output which would prevail in a counterfactual economy without nominal frictions, i_t the nominal risk-free interest rate and r_t^n the natural equilibrium interest rate in a counterfactual

4. Other factors are also pointed out in the literature for the reduction of the natural interest rate, such as the increase in income inequality, the fall in the relative price of capital goods and the increase in demand for safe asset (*savings glut*) of some emerging economies, but the estimated impact of these factors tends to be smaller than those mentioned above (see Rachel and Smith (2015)).

economy without nominal frictions. β, κ and σ represent the discount factor, consumer preferences and technology factor. Based on these equilibrium conditions, HLW use the following equations as measurement equations for estimating the natural interest rate:

$$\pi_t = b_\pi \pi_{t-1} + (1 - b_\pi)\pi_{t-2,4} + b_y \tilde{y}_{t-1} + \varepsilon_{\pi,t} \tag{4}$$

$$\tilde{y}_t = a_{y,1}\tilde{y}_{t-1} + a_{y,2}\tilde{y}_{t-2} + \frac{a_r}{2}\sum_{j=1}^{2}(r_{t-j} - r^*_{t-j}) + \varepsilon_{\tilde{y},t} \tag{5}$$

where π_t represents inflation in the period, $\pi_{t-2,4}$ represents the average inflation in the three previous periods, $\tilde{y}$ represents the output gap, that is, the difference between the observed level of output y_t and the estimated measure of potential output y^*_t. r_t represents the short-term real interest rate *ex-post* and r^*_t the natural interest rate. Comparing equations (2) and (3) with equations (4) and (5) one can see the distinction between the interpretations for the natural interest rate in Neo-Keynesian models and the one used by HLW. In equations (2) and (3) the natural interest rate captures all shocks[5] that affect inflation and the level of output. Once the real interest rate gap is closed (in the equation (3): $i_t - E_t[\pi_{H,t+1}] - r^n_t$) in each period, inflation and the level of output stabilise. In equations (4) and (5) there can be shocks that affect inflation and the level of output but that do not necessarily affect the natural interest rate (captured in the equations above by $\varepsilon_{\pi,t}$ and $\varepsilon_{\tilde{y},t}$). Thus, the natural interest rate will only be affected by more persistent shocks in the relationship between the real rate gap and the output gap.

Based on these equations, the HLW model uses the Kalman filter to estimate the natural interest rate and the level and growth of potential output. The transition equations of the model are:

$$r^*_t = g_t + z_t \tag{6}$$

$$y^*_t = y^*_{t-1} + g_{t-1} + \varepsilon_{y^*,t} \tag{7}$$

$$g_t = g_{t-1} + \varepsilon_{g,t} \tag{8}$$

$$z_t = z_{t-1} + \varepsilon_{z,t} \tag{9}$$

where g is the growth rate of potential output and z captures other determinants that can influence the natural interest rate[6]. The transition equations assume that potential

5. Usually, in structural models, the natural interest rate is seen as the real interest rate that would prevail in an economy without nominal price and wage frictions, in the absence of *mark-up* shocks on goods and on labour.

6. Note that equation (6) is based on equation (1) assuming that the coefficient of elasticity of consumption substitution is equal to 1. Laubach and Williams (2003) estimate this parameter and conclude that it is very close to 1 for the United States. Holston *et al.* (2017) also assume this value for the parameter in the euro area.

output growth and the variable z follow a random walk and potential output in level follows a random walk with *drift*. It is assumed that the variables $\varepsilon_{y^*,t}$, $\varepsilon_{g,t}$ and $\varepsilon_{z,t}$ follow a normal distribution with standard deviations σ_{y^*}, σ_g and σ_z and are not correlated. As observed variables, the model uses a measure of output (GDP), inflation and a measure of the real interest rate. HLW uses inflation excluding food and energy (hereafter, core inflation) and calculates the observed real interest rate as an *ex-post* rate given by the difference between a short-term (three-month) interest rate and the inflation observed on average over the last four quarters.

3.2. Adjustment for the pandemic crisis

The years 2020 and 2021 were marked by the pandemic crisis associated with Covid-19. During this period, social distancing measures were imposed, which led to very sharp fluctuations in the variables used in the model[7]. A shock of this magnitude is not correctly captured by the HLW methodology, since the hypothesis that the errors in the measurement and transition equations follow a normal distribution and are uncorrelated is not respected. Recognizing this limitation, Holston *et al.* (2020) propose a correction to the model to mitigate this impact. This modification involves adding the stringency index developed by the University of Oxford[8] in the model with the objective of capturing the effects of the measures imposed to contain the pandemic. This variable is added to the model when measuring the level of potential output, which is now expressed as follows:

$$y_t^{*'} = \begin{cases} y_t^* + \varphi d_t, & \text{if } t = 2020Q1 \text{ or } 2020Q2 \\ y_t^*, & \text{otherwise} \end{cases} \tag{10}$$

And equation (5) is now rewritten as:

$$(y_t - y_t^{*'}) = a_{y,1}(y_{t-1} - y_{t-1}^{*'}) + a_{y,2}(y_{t-2} - y_{t-2}^{*'}) + \frac{a_r}{2}\sum_{j=1}^{2}(r_{t-j} - r_{t-j}^*) + \varepsilon_{\tilde{y}',t} \tag{11}$$

Where d_t is the level of the stringency indicator for the quarter t calculated as the aggregation of the quarterly averages of daily data from the euro area countries weighted by the weight of the population of each country in 2019. With this modification, the strong changes recorded in GDP throughout 2020 that the original model would classify as marked changes in the level of potential output are partly captured by the variable d_t. Figure 1 shows the estimates for potential output growth generated by the model with and without adjustment for the pandemic period.

7. In the second quarter of 2020, GDP in the euro area declined by 11.5% quarter on quarter and grew by 12.4% in the following quarter.

8. This indicator was developed to measure the intensity of the measures imposed to contain the Covid-19 pandemic. A higher level of this indicator means that the containment measures were more restrictive and, therefore, may have led to a greater disruption of economic activity. For more information on this indicator, consult Hale *et al.* (2021)

As expected, the results of the model without the adjustment for the pandemic period show a sharp drop in potential GDP growth, while in the model with the adjustment the drop is less pronounced. Thus, and since the potential output growth rate is a determinant of the natural interest rate in the model, the adaptation of the model tries to capture potential output growth without it being strongly influenced by the pandemic period. It should be noted that the results of the adjusted model are highly dependent on the periods chosen for the inclusion of the stringency index. As can be seen in the equation (10), in the study it is considered that the adjustment variable takes the value of the stringency index in the first and second quarters of 2020.[9]

9. Since the authors in Holston *et al.* (2020) omit the periods in which the variable is present, the choice becomes somewhat arbitrary. The choice made in this study involved comparing the model's estimates of the output gap level with estimates from other international organizations, trying to find the one that best replicates the level and evolution of these estimates.

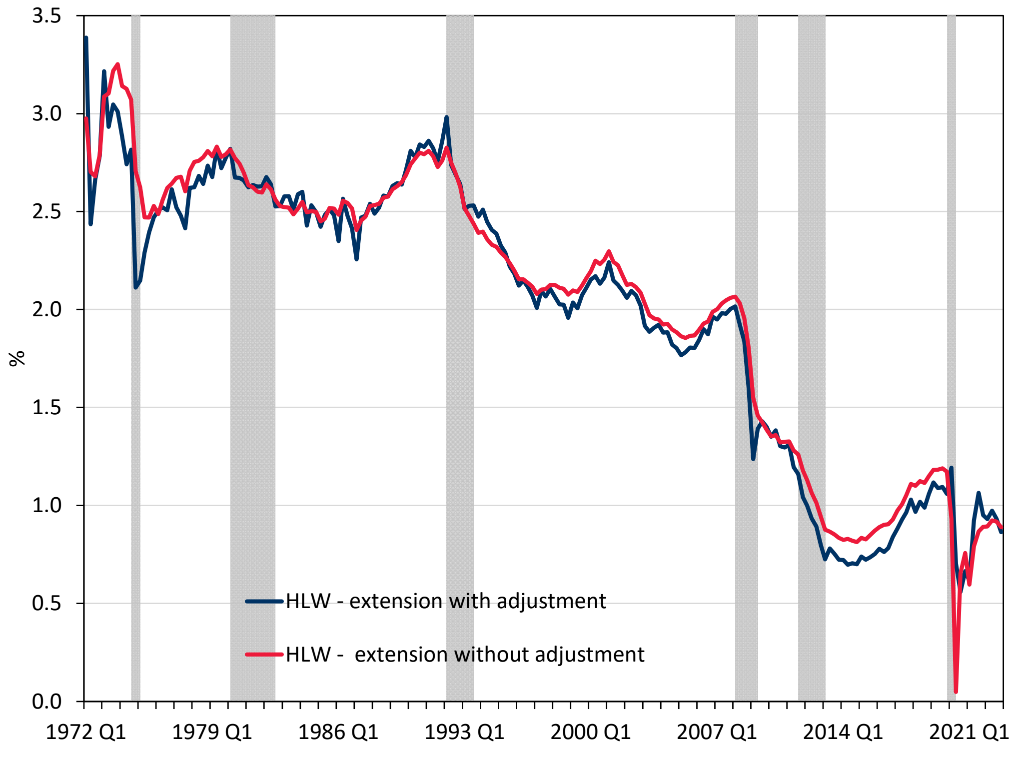

FIGURE 1: Estimates for the euro area potential output growth (g_t)

Notes: Estimates obtained through the HLW model. Extension without adjustment corresponds to the estimated replication of the HLW model until 2022 Q4. Adjusted estimate corresponds to the model discussed in Section 3.2, which includes the stringency index in the euro area in the first and second quarters of 2020. Bars in gray mark the periods of recession according to the dating of the Euro Area Business Cycle Network — EABCN.
Sources: Eurostat, EABCN, Refinitiv and author's calculations.

3.3. Model with inflation expectations

As previously described, the equations that serve as the basis for the specification used by HLW come from two equations from the standard Neo-Keynesian model (equations (2) and (3)). However, making a comparison between these equations and the measurement equations adopted by HLW it is possible to observe a difference in the role of inflation expectations. In equation (2) the level of inflation for the period t is dependent on the inflation expectation formed in that period for the inflation in the period $t+1$. In equation (4), a linear combination between the observed inflation in the period $t-1$ and an average of the observations in $t-2$, $t-3$ and $t-4$ is used. Thus, while in equation (2) inflation expectations are forward looking, in equation (4) expectations are adaptive.

In this study, the model is adapted to take into account forward looking inflation expectations. This adaptation consists of considering a measure of inflation expectations in the equation (4)[10]. Thus, the new Phillips curve is now written as:

$$\pi_t = b_\pi \pi_{t-1} + (1 - b_\pi) E_t[\pi_{t+h}] + b_y \tilde{y}_{t-1} + \varepsilon_{\pi,t} \tag{12}$$

Where h represents the horizon to which inflation expectations correspond. In this way, the evolution of inflation expectations starts to play a role in estimating the natural interest rate.

In addition to the role of inflation expectations, this formulation has an additional advantage. In the original HLW model, when the output gap is zero, inflation remains constant. However, the model does not specify the level around which inflation stabilises. This formulation may not be consistent with economies where central banks have well-defined inflation targets. With the inclusion of forward-looking inflation expectations, when the output gap is zero, inflation, as described by equation (12), converges to the expected level of inflation. Assuming the credibility of the central bank's inflation target, longer-term inflation expectations should approach these inflation targets. So, the equation (12) can be rewritten as:

$$\pi_t = b_\pi \pi_{t-1} + (1 - b_\pi) \pi^* + b_y \tilde{y}_{t-1} + \varepsilon_{\pi,t} \tag{13}$$

where π^* is the inflation target. This equation can also be rewritten as a function of the gap between observed inflation and π^*:

$$\tilde{\pi}_t = b_\pi \tilde{\pi}_{t-1} + b_y \tilde{y}_{t-1} + \varepsilon_{\pi,t} \tag{14}$$

Where $\tilde{\pi}_t$ equals $(\pi_t - \pi^*)$. With this formulation, the measurement of the natural interest rate is not only dependent on the evolution of the inflation level, as in the original HLW model, but also becomes dependent on the evolution of deviations of inflation from the central banks' objective.

10. A similar exercise is carried out by Lopez-Salido *et al.* (2020) for estimating the natural interest rate in the United States.

4. Results

4.1. Data

This section presents the results of estimates for the natural interest rate in the euro area based on three modifications of the original HLW model. The first is the model with adaptation for the pandemic period — Model without expectations — discussed in Section 3.2. and the second and third consist of estimates based on the model presented in Section 3.3 with two different measures of inflation expectations — Model with expectations T+2 and Model with expectations T+5 — also using the adjustment for the pandemic period. Quarterly data for GDP, core inflation and three-month nominal interest rate from 1970 to 2022 are used for the estimation. The data is obtained from the ECB's Euro Area-Wide Model database (Fagan *et al.* (2001)) updated with data from Eurostat and Refinitiv. For the euro area stringency index it is used the quarterly average of daily data calculated as the weighted average of the stringency indices of the euro area countries weighted by the population share in 2019. The inflation expectations considered are the expectations for total inflation from the ECB's Survey of Professional Forecasters — SPF — at fixed horizons two and four/five years ahead[11]. A detailed description of the variables used is provided in the appendix[12].

4.2. Estimates

Table 1 presents the estimated parameters of the three specifications mentioned in the previous section. The parameters a_y and b_π are close to one and statistically significant, which indicated that both the output gap and the inflation exhibit high persistence. The parameter φ associated with the control variable for the pandemic period is also significant and with a negative sign. This result is expected since this variable reflects part of the sharp drop in GDP in the first two quarters of 2020.

The parameters a_r and b_y are particularly relevant for estimating the natural interest rate as they measure the sensitivity of the output gap to the real interest rate gap and of inflation to the output gap, respectively. The estimates show the expected sign. If the real rate gap is positive — that is, if the observed real interest rate is above the natural interest rate — the output gap becomes more negative, and a more negative

11. The use of expectations for total inflation and not for core inflation has to do with sample availability, as core inflation expectations in the SPF have only started to be released since 2016. At the same time, it should be noted that expectations for total and core inflation at the horizons considered in the study follow a similar behaviour. It would also be possible to use inflation expectations one year ahead of the SPF, without significant change in the results. However, this series shows greater volatility than expectations at longer horizons, which translates in higher volatility of the natural interest rate. In addition, the interpretation of inflation expectations as a central bank's inflation target described in section 3.3. is more obvious with longer term expectations.

12. The choice of core inflation over total inflation is also made by HLW. The objective is to use an inflation measure that responds more directly to the output gap and the real interest rate gap so that the natural interest rate estimate is more robust. Total inflation, by including the components of energy and food, is more subject to external shocks that may not be well captured by the structure of the model.

Parameter	Model without expectations	Expectations T+2	Expectations T+5
λ_g	0.059	0.065	0.060
λ_z	0.005	0.012	0.015
$\sum a_y$	0.94	0.94	0.94
a_r	-0.05^{*}	-0.05^{*}	-0.05^{*}
	(0.03)	(0.03)	(0.03)
b_y	0.06^{*}	0.06^{**}	0.06^{**}
	(0.03)	(0.03)	(0.03)
b_π	0.81^{***}	0.80^{***}	0.80^{***}
	(0.05)	(0.05)	(0.05)
φ	-0.19^{***}	-0.19^{***}	-0.19^{***}
	(0.00)	(0.00)	(0.00)
$\sigma_{\tilde{y}}$	0.42	0.43	0.43
σ_π	0.84	0.84	0.84
σ_{y^*}	0.37	0.36	0.37
σ_g	0.09	0.09	0.09
σ_z	0.05	0.11	0.14
σ_{r^*}	0.10	0.14	0.16
Monte Carlo simulations Standard error (mean)			
r^*	1.2	1.8	2.3
g	0.3	0.3	0.3
y^*	1.5	1.5	1.5
Monte Carlo simulations Standard error (2022 Q4)			
r^*	1.6	2.6	3.3
g	0.4	0.5	0.4
y^*	2.0	2.1	2.1

TABLE 1. Parameter estimates

Notes: Estimates based on quarterly data from 1971 Q1 to 2022 Q4. Numbers in parentheses correspond to estimated standard deviations. $***$, $**$ and $*$ next to coefficients significantly different from zero at a confidence level of 99%, 95% and 90%, respectively. Estimation performed using the maximum likelihood method (MLE). Mean and 2022 Q4 standard errors calculated with the Monte Carlo procedure according to Hamilton (1986) which captures the uncertainty of the parameters and the Kalman filter.

output gap translates into less inflation. However, these parameters have a much smaller magnitude when compared to the other parameters. In the first specification of the model these coefficients are only statistically significant at a confidence level of 90%. In other specifications, the coefficient b_y slightly increases its significance. Holston *et al.* (2017) also report a lower significance of the parameters a_r and b_y in the models for the euro area, Canada and the United Kingdom.

The higher uncertainty around a_r and b_y justifies the high uncertainty associated with the estimates of the natural interest rate. The bottom part of the Table 1 shows the standard errors of Monte Carlo simulations for the level of the model's unobservable variables. As in Holston *et al.* (2017), these standard errors are high, which calls for caution when interpreting the natural interest rate estimates obtained. Uncertainty around the natural interest rate is higher in the model specifications with inflation expectations. This fact is associated with the greater variation in the errors of the transition equation of the variable z, as can be seen in the table.

Figure 2 presents the estimates of the natural interest rate in the euro area based on the three specifications considered. As identified in the literature, the natural interest rate in the euro area shows a downward trend since 1970, going from levels of 3% to values around 0.5% until the beginning of 2020. The natural interest rate tends to decreasing after periods of recession, registering particularly significant declines after the global financial crisis of 2007/08 and the sovereign debt crisis in 2013/14. These results are in line with the determinants identified in the literature, where the decline in potential output growth and the aging of the population support the downward trend in the level of natural interest rate, accentuated by the search for risk-free assets and changes in rules of banking supervision after the global financial crisis of 2007/08.

Estimates of the natural interest rate based on the different specifications begin to diverge more significantly from 2008 onwards. This divergence can be explained by two reasons. The first is directly related to data for SPF inflation expectations that only begin in 1999 (two-year ahead expectations) and 2001 (four/five-year ahead expectations), so before these dates the three models are being effectively estimated with the same data.

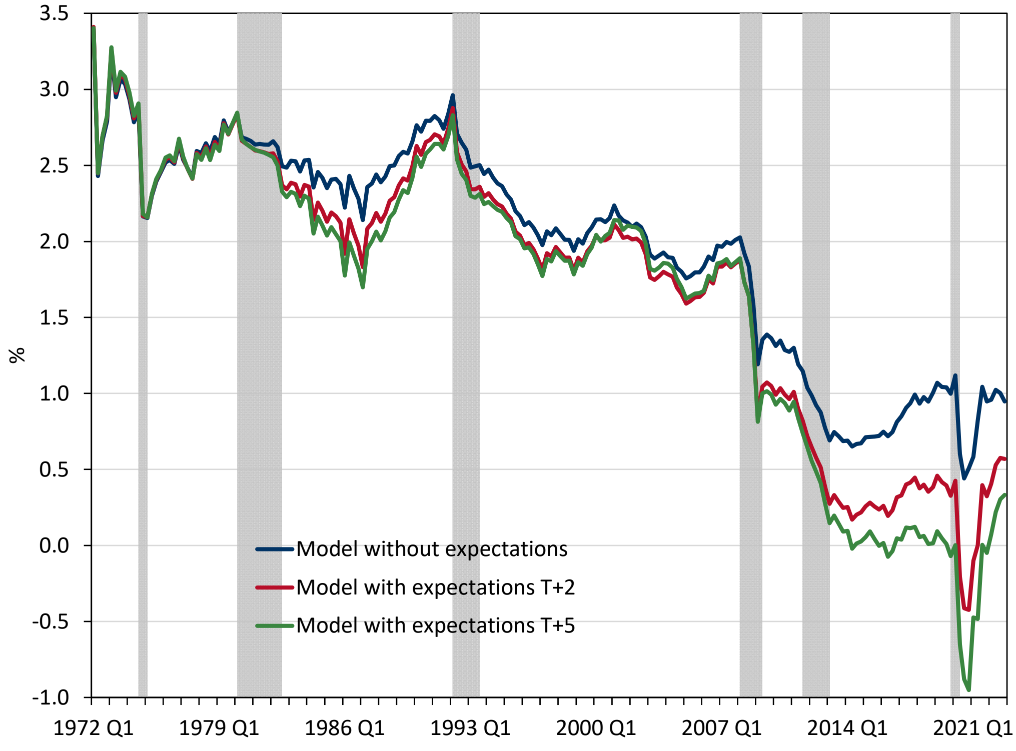

FIGURE 2: Euro area natural interest rate estimates (r_t^*)

Notes: Estimates obtained based on the HLW model. Model without expectations corresponds to the model discussed in Section 3.2 which includes the stringency index in the euro area in the first and second quarter of 2020. Models with expectations T+2 and T+5 correspond to the model discussed in Section 3.3 with inflation expectations two and four/five years ahead of the SPF. Bars in gray mark the periods of recession according to the dating of the Euro Area Business Cycle Network — EABCN.
Sources: ECB, Eurostat, EABCN, Refinitiv and author's calculations.

However, this does not in itself explain the difference from 2008. The second reason is directly related to the evolution of inflation in the euro area. After 2008 headline inflation and core inflation in the euro area declined and remained at relatively low levels until 2019[13]. This decrease leads to a greater difference between the model's inflation measure and the level of inflation expectations considered, which impacts the natural interest rate estimates in the specifications that consider inflation expectations. In fact, estimates based on these specifications show a lower level than those obtained only with the adjustment for the pandemic from 2008 onwards. With the inflation gap having an explicit impact on these specifications and with inflation standing below of the measures of inflation expectations, the model identifies these periods as less accommodative, which would imply a natural interest rate closer to the observed level of the real interest rate[14]. Since the inflation gap is even larger in the model with expectations four/five years ahead, the estimate of the natural interest rate is even lower.

Estimates show that during 2020 the natural interest rate decreased. This evolution is explained by the reduction in potential output growth, since part of the drop in GDP recorded in the euro area during this period continues to be passed on to the natural interest rate, despite the adjustment for the pandemic. In the models with inflation expectations, the drop is even more significant, since in addition to the potential output effect, inflation during this period also dropped significantly, creating another downward pressure on the natural interest rate. Despite this impact, the estimate of the natural interest rate quickly returned to its pre-pandemic level. It is important to note that this model is not suited to fully capture the dynamics of potential output after a shock such as the pandemic. Thus, a more robust assessment of the impact of the pandemic on the natural interest rate will require a model capable of fully capturing its effects.

The most recent estimates at the end of 2022 point to a natural interest rate level in the euro area around 0.5%[15].While the estimate of the model with the adjustment for the pandemic appears to be stable, the estimates based on the models with expectations of inflation have been showing an upward trend. This effect is the opposite of that observed after the 2008 crisis. With the significant rise in inflation observed in the euro area since the beginning of 2022, it is currently above the level of inflation expectations, which the model identifies as a period of greater monetary accommodation, which is converted in the model into an estimate of the natural interest rate above the observed real interest rate.

13. Average core inflation measured by the year-on-year change in the HICP excluding food and energy in the euro area between 2001 and 2007 was 1.8% while between 2008 and 2019 this average was 1.2%.

14. The period after 2008 and up to 2019 was marked by a significant reduction in interest rates from the European Central Bank's policy to fight low inflation. Thus, the *ex-post* real interest rate measure used in the model is negative during this period, on average -0.8%.

15. These results are in line with recent estimates, namely those made by IMF (2023), which put the natural interest rate in Germany and France between 0% and 0.5%.

The assessment of the evolution of the natural interest rate can be complemented by the evolution of the real interest rate gap, measured as the difference between the observed real interest rate and the natural interest rate.

Figure 3 displays the estimates for this gap [16]. The estimated gap was positive from 1980 to 2002 and temporarily between 2006 and 2008. Simultaneously, it is also visible that during periods of recession, the gap tends to decrease. In other words, monetary authorities tend to make monetary policy more accommodative in times of recession. Since 2008, the gap in the euro area has been negative, reflecting the ECB's monetary policy which kept interest rates low in view of the low inflation observed during this

16. The results for the real interest rate gap are not only dependent on the estimate of the natural interest rate used, but also on the calculation method used for the observed real interest rate. In the Graph 3 the measure of the real interest rate is an ex-post rate, which is the one used in the estimation of the model, for consistency.

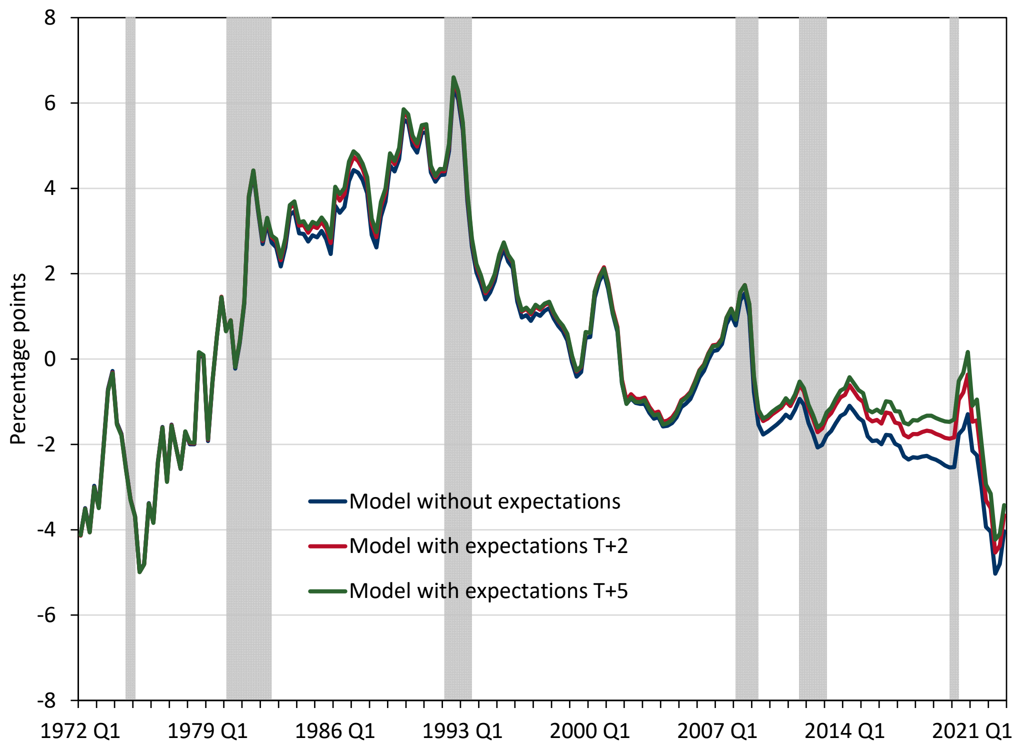

FIGURE 3: Estimates for the euro area ex-post real rate gap $(r_t - r_t^*)$

Notes: Estimates obtained based on the HLW model. Model without expectations corresponds to the model discussed in Section 3.2 which includes the stringency index in the euro area in the first and second quarter of 2020. Models with expectations T+2 and T+5 correspond to the model discussed in Section 3.3 with inflation expectations two and four/five years ahead of the SPF. The real interest rate ex-post is calculated as the difference between the nominal 3-month interbank interest rate in the euro area and average inflation observed in the previous four quarters. Bars in gray mark the periods of recession according to the dating of the Euro Area Business Cycle Network — EABCN.
Sources: ECB, Eurostat, EABCN, Refinitiv and author's calculations.

period. However, it is possible to observe differences between the models, in line with the previous discussion. In the models with inflation expectations, the real rate gap is less negative, taking into account the role of the inflation gap in the model estimation.

In 2020, and taking into account the drop observed in the natural interest rate estimates, the real interest rate gap became less negative, indicating a less accommodative monetary policy. This result was reversed almost instantly, with this gap registering a very significant reduction since 2021. This drop is not just the result of the evolution of nominal interest rates, given that the ECB has in fact raised policy interest rates since July 2022, but it is due to a substantial rise in inflation which leads to a strong decrease in the observed real interest rate. Thus, according to this measure, monetary policy remained accommodative until 2022.

This measure of the real interest rate gap can also be calculated using an *ex-ante* measure of the real interest rate instead of an *ex-post*. Figure 4 shows the real rate gap where real interest rates are calculated as the difference between the three-month nominal interest rate and inflation expectations two years ahead. Comparing the two, it is visible that the gap calculated with real rates *ex-ante* and *ex-post* present very similar behaviours. However, in 2022, the measure with *ex-ante* rates registers a strong rise, despite still remaining at negative levels. This rise reflects not only the less pronounced rise in longer-term inflation expectations when compared to actual inflation, but also the action of the European Central Bank, which has raised interest rates to remove monetary accommodation in order to guarantee the return inflation to its medium-term objective.

It should be noted that measuring the degree of monetary policy accommodation through these measures of the real interest rate gap only measures the accommodation generated by the policy interest rate. This measure does not capture the additional impact that unconventional monetary policy measures have on the monetary policy stance[17]. This distinction should be particularly taken into account in the euro area, where unconventional monetary policy was widely used in the period of low inflation.

5. Considerations for monetary policy

The estimation of the natural interest rate is particularly relevant for monetary policy. Central banks use nominal interest rates as their main instrument to achieve their mandates, with the aim of stimulating or slowing down economic activity and, as a consequence, influence the behaviour of inflation. The natural interest rate makes it possible to infer the levels at which rates should be situated in order to bring about the desired degree of accommodation. However, the practical applicability of the natural interest rate presents some challenges. First, as discussed earlier, the natural interest rate is an unobservable variable with highly uncertain estimates that use observations of economic variables with some time lag. Second, estimates of the natural interest rate

17. Some authors present methodologies to measure the degree of monetary policy accommodation that explicitly capture the impact of unconventional monetary policy measures. See for example Wu and Xia (2016) and Krippner (2013).

show some variability, so if central banks wanted to base their policy decisions on the natural interest rate, they might have to make relatively sharp increases and decreases of interest rates regardless of economic and financial conditions. Thus, the uncertainty surrounding the estimation of the natural interest rate makes its use difficult.

However, the natural interest rate can play an important role in central bank communication to anchor long-term interest rate expectations. By definition, observed interest rates should tend towards levels around the natural interest rate in the long term. This leads to several considerations relevant to monetary policy.

First, it allows an assessment of the ability of the policy interest rates to stimulate the economy. The downward trend observed in the natural interest rate in the euro area and in other advanced economies implies that policy interest rates are at lower levels than those recorded in the past, which implies that the margin available for central banks to lower rates rate will be lower given the existence of an effective lower bound.

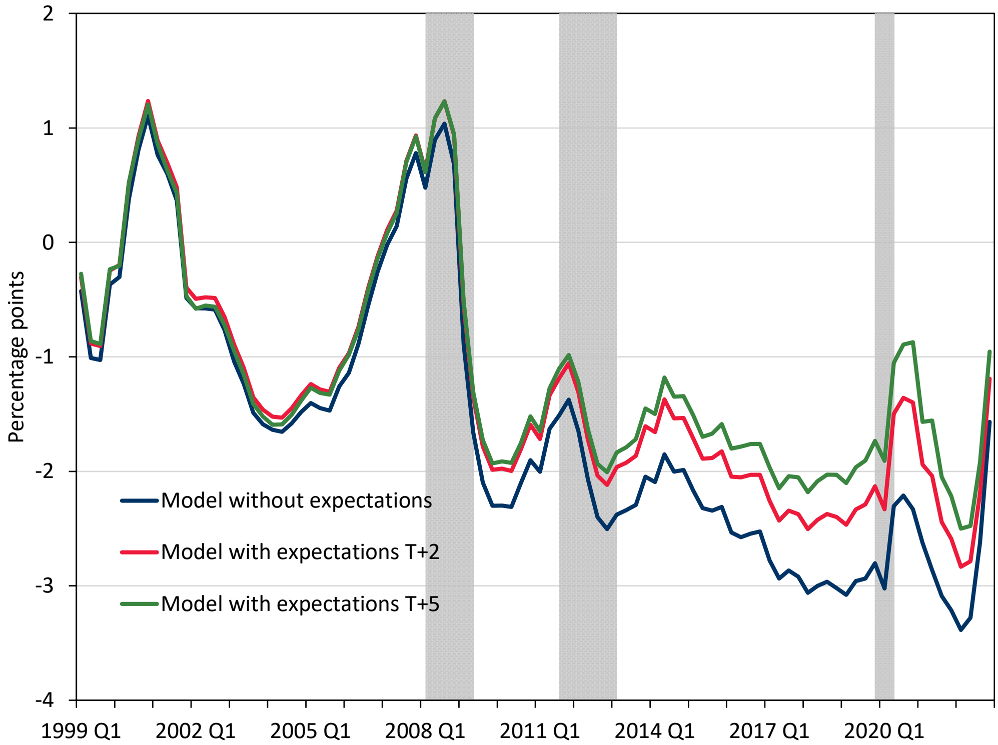

FIGURE 4: Estimates for the euro area ex-ante real rate gap $(r_t - r_t^*)$

Notes: Estimates obtained based on the HLW model. Model without expectations corresponds to the model discussed in Section 3.2 which includes the stringency index in the euro area in the first and second quarter of 2020. Models with expectations T+2 and T+5 correspond to the model discussed in Section 3.3 with inflation expectations two and four/five years ahead of the SPF. The ex-ante real interest rate is calculated as the difference between the 3-month interbank nominal interest rate in the euro area and the total inflation expectation two years ahead of the SPF. Data since 1999, in line with the availability of SPF expectations. Bars in gray mark the periods of recession according to the dating of the Euro Area Business Cycle Network — EABCN.

Sources: ECB, Eurostat, EABCN, Refinitiv and author's calculations.

In fact, the implications of a lower natural interest rate on the performance of central banks is a widely discussed topic[18] and that partly motivated the Federal Reserve's monetary policy strategy review in 2019 and the ECB's monetary policy strategy review in 2020/21[19].

Second, the natural interest rate can anchor the level of interest rates in the long run, which in turn is important to ensure the stabilisation of inflation expectations at a level consistent with the inflation targets of central banks. The long-term relationship between the level of inflation and the interest rate is usually described by the Fisher relation:

$$i_t = r_t + E_t[\pi_{t+1}] \tag{15}$$

where i_t represents a risk-free nominal interest rate, r_t the equilibrium real interest rate (natural) and π_{t+1} the expected inflation rate in the following period. Under the assumption that central banks cannot influence the natural interest rate, there is a direct relationship between the level of inflation and the nominal interest rate in the long run. Thus, and using the natural interest rate estimates presented in Section 4 together with the ECB's medium-term inflation objective of 2%, we obtain that long-term nominal interest rates consistent with this relationship should be situated around 2.5%. This result is particularly important in the current context in which the ECB is raising its policy rates to control the sharp rise in inflation. Any path chosen for the policy interest rate must imply a return to levels consistent with the level of the natural interest rate and the inflation target. If the expectation is created that interest rates will remain at a higher level for a long period of time, it may be the case that inflation expectations adjust upwards to ensure long-term consistency. Thus, a communication that clarifies the level of interest rates in the long term can support the action of central banks to achieve their price stability objectives.

Although the ECB does not explicitly do so, other central banks communicate their expectations for the level of interest rates in the long term. For example, since 2012 the Federal Reserve has released together with the Summary of Economic Projections the perspectives of the members of the Federal Open Market Committee (FOMC) for the level of the policy rate (federal funds rate) in the long run. Although this level of rates is not directly an estimate for the US natural interest rate, it is usually interpreted as the FOMC members' estimate of the natural interest rate plus the Federal Reserve's 2% inflation target. There is evidence (Hillenbrand (2023)) that revisions to the FOMC members' outlook for the long-term policy rate have a significant impact on long-term

18. Andrade *et al.* (2021) discuss the optimal inflation target taking into account the level of the natural interest rate in the area of the euro. The conclusion supports that a lower natural interest rate should lead to a higher inflation target in order to prevent the economy from hitting the effective lower bound of interest rates.

19. In the launch of the ECB's strategy review it is said "Since 2003 the euro area and the world economy have been undergoing profound structural changes. Declining trend growth, on the back of slowing productivity and an ageing population, as well as the legacy of the financial crisis, have driven interest rates down, reducing the scope for the ECB and other central banks to ease monetary policy by conventional instruments in the face of adverse cyclical developments".

market rates. This result may indicate that central banks may benefit from providing some type of guidance on longer-term interest rate levels with a view to ensuring stability in inflation expectations and, thus, facilitating the pursuit of their inflation targets.

6. Conclusion

The estimation of the natural interest rate is relevant for the conduct of monetary policy. The natural interest rate not only allows assessing the degree of monetary policy accommodation, but can also be seen as the level to which interest rates in an economy should converge in the long term. However, as it is an unobservable variable, any inference about its evolution has to be made based on estimates.

This study presents estimates for the natural interest rate in the euro area based on the methodology developed by Holston *et al.* (2017), adapting it for the pandemic period and adding an explicit role for inflation expectations. Despite the uncertainty inherent in this methodology, estimates indicate that the natural interest rate in the euro area has shown a downward trend from the early 1970s until 2019, in line with estimates for other advanced economies. Downward movements tend to follow periods of economic crisis, especially after the global financial crisis in 2007/08. The results indicate that at the end of 2022 the natural interest rate in the euro area was around 0.5%.

Evaluating the results for the real interest rate gap, it is possible to conclude that monetary policy in the euro area tended to become more accommodative after periods of economic crisis. In particular, between 2008 and 2019, the estimate of this gap was always negative, in line with the policy of the European Central Bank. Recently, this measure evaluated with real rates ex-post is at very negative levels, signalling a high degree of accommodation. This accommodation is not directly related to monetary policy decisions, but to the rapid and significant rise in inflation registered in the euro area, which leads to a significant drop in observed real rates. Evaluating the real rate gap with real rates ex-ante it is visible a much more significant increase in the real rate gap during 2022, in line with the policy followed by the European Central Bank which is reducing the degree of monetary policy accommodation to reduce inflation and achieve its objective of price stability.

The use of the natural interest rate in the explicit conduct of monetary policy presents some challenges. A monetary policy that seeks to bring the interest rate to the level of the natural interest rate may not be desirable, since there is high uncertainty about any estimate and may lead to decisions that may not be the most appropriate in the given economic context. However, the use of the natural interest rate in monetary policy communication can play a crucial role in ensuring the consistency of interest rate expectations with inflation targets. Any policy path that does not imply the return of the policy interest rate to levels consistent with the natural interest rate and with the inflation target may lead to losses in the credibility of central banks. The example of the Federal Reserve is evidence that communication about the level of long-term interest rate can influence long-term market interest rate expectations.

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Appendix: Data description

This appendix presents a summary table of the data used to estimate the natural interest rate according to the methodologies discussed in the study.

Variable	Data	Source
GDP	Euro area GDP (1970-2022)	Area Wide Model and Eurostat
Inflation	HICP headline (1970-1987); HICP exc. energy (1988-1995) and HICP exc. energy and food (1996-2022) in the euro area	Area Wide Model and Eurostat
Nominal interest rate	3 month Euribor (1970-2022)	Area Wide Model and Refinitiv
Stringency	Weighted quarterly average of the OSI of euro area countries based on population	Oxford Stringency Index
Inflation expectations		
Model without expectations	-	-
Model with expectations T+2	Average observed inflation in the past four quarters (1970-1998); Headline inflation expectations 2 year ahead (1999-2022)	Area Wide Model and ECB (SPF)
Model with expectations T+5	Average observed inflation in the past four quarters (1970-2000); Headline inflation expectation 4/5 year ahead (2001-2022)	Area Wide Model and ECB (SPF)

TABLE A.1. Data description

As shown in the table, the inflation series considered uses data from three different HICP series. This occur sdue to the lack of data since 1970 for core inflation in the Euro Area-Wide Model. Thus, an index is constructed based on the available aggregates. From 1970 to 1987 the measure considered is total inflation, between 1988 and 1995 the HICP measure excluding energy goods is used and since 1996 the HICP measure excluding food and energy. It is important to note that this aggregation implies that the inflation series used in the model potentially has two structural breaks. This treatment is also used by Holston *et al.* (2017) in estimating the natural interest rate in the euro area.

Non-technical summary

July 2023

Risk reallocation under Central Bank's large-scale asset purchases

Bernardino Adão and Alberto López Martín

Following the economic crisis of 2008, of unprecedented size in the post-war period, central banks resorted to unconventional monetary policy measures. One of these measures was to buy very large amounts of financial assets to improve the financing conditions of economic agents. This study illustrates, using a simple model, that the central bank, by making large-scale purchases of assets, can reduce the risk in the economy, stabilize macroeconomic variables and increase consumer welfare.

The model considers two types of participants in financial markets: consumers and financial intermediaries (banks). Consumers do not have the necessary know-how to buy risky assets, so instead they deposit their financial wealth with banks, which earns a risk-free interest rate. Banks can buy risky assets with an average rate of return higher than that of deposits.

In a recession, the return on risky assets decreases. Therefore, the relative wealth of banks also decreases, increasing the relative wealth of consumers. The increase in the relative wealth of consumers lowers the risk-free interest rate and increases the premium that investors demand to invest in risky assets (the so-called "risk premium").

The model assumes that the central bank issues the risk-free asset and uses the funds thus obtained to buy the risky asset; additionally, it distributes the net dividends of this financial operation, via the Treasury, among economic agents. As part of the dividends is channeled to consumers, who do not participate in the risky asset market, the total risk in the market decreases. Thus, because of the central bank action, the risk premium in the economy decreases. Investment and consumption fluctuations are also smaller, and consumers have a higher level of well-being than they would have if the central bank did not make large-scale asset purchases.

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Risk reallocation under Central Bank's large-scale asset purchases

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July 2023

Abstract
Crises have some common features: increases in risk premia, decrease of real risk-less interest rates, and flight to quality assets, among others. This paper studies the effects of large-scale asset purchases on the market price of risk and the risk-free rate. We observe how, when the central bank buys risky assets using risk-less debt, there is a reduction of risk-taking in the economy, as the risk is transferred to non-market participants. Large-scale asset purchases by the central bank reduce the exposure of intermediaries' balance sheets to capital shocks, leading to a reduction in the risk premium and an increase in the risk-free rate. (JEL: E21, E60, F40)

1. Introduction

In recessions, risk premia increase and risk-free real interest rates decrease. Below we present figures that illustrate these stylized facts. Figure 1 shows the evolution of the TED spread, which is defined as the difference between the interest rate on interbank loans and the rate on 3-month U.S. government debt ("T-Bills"). The TED spread is an indicator of perceived credit risk in the general economy, since T-bills are considered risk-free, while LIBOR reflects the credit risk of lending to commercial banks. An increase in the TED spread is a sign that lenders believe the risk of default on interbank loans is higher. In turn, interbank lenders demand a higher rate of interest, or accept lower returns on safe investments such as T-bills.

Another risk premium measure is the difference between the return on a risky asset, like a long-term Treasury bond, and the return on a low risk asset, like a short-term Treasury bond. Figure 2 shows the evolution of the difference between the yield on the 10-year Treasury bond and the 3-month T-Bill. It shows that this variable increases in recessions periods.

The risk-free real interest rate decreases during recessions. Figure 3 shows the evolution of the risk-free real interest rate measured as the difference between the 3-month T-bill rate and an indicator of expected inflation, the Sticky Price Consumer Price

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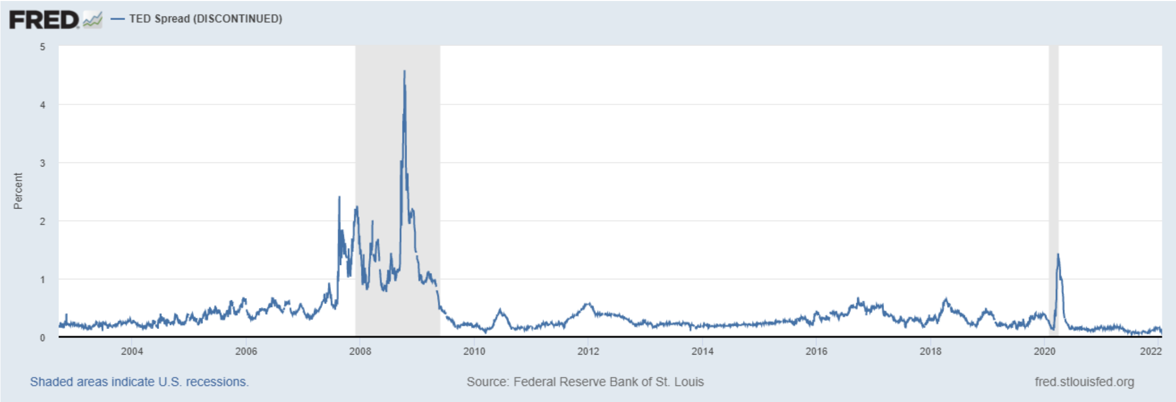

FIGURE 1: TED spread: 3M USD LIBOR – 3M Treasury Bill

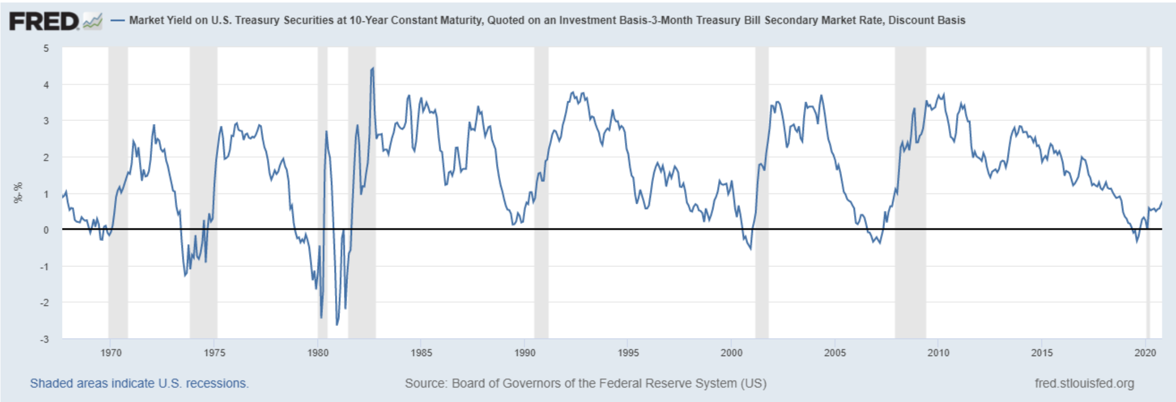

FIGURE 2: 10Y Treasury yield – 3M Treasury Bill

Index. The Sticky Price Consumer Price Index (CPI) is calculated from a subset of goods and services included in the CPI that change price relatively infrequently. These goods and services are thought to incorporate expectations about future inflation to a greater degree than goods and services whose prices change on a more frequent basis. See: Bryan and Meyer (2010).

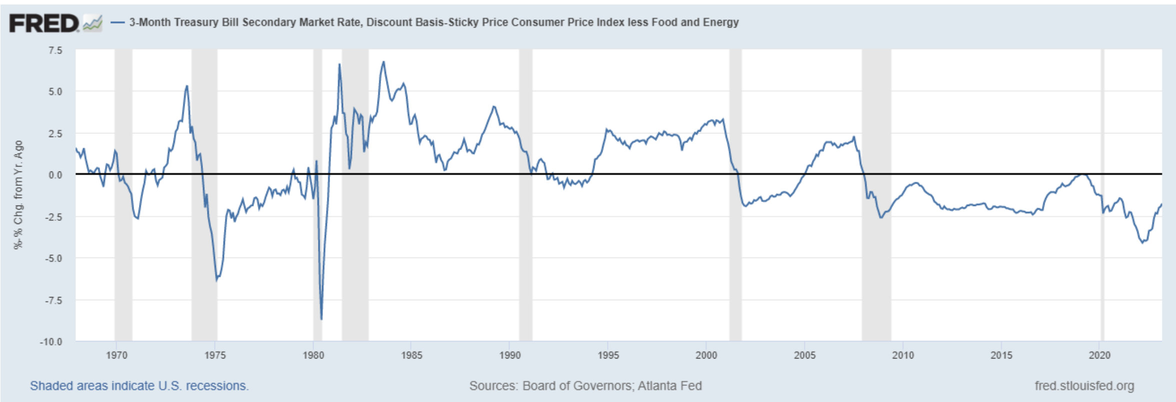

FIGURE 3: 3M T-bill – CPI (less food and energy) inflation

The crisis of 2007-08 required central banks around the globe to expand their monetary policy toolbox in an attempt to ease credit conditions and compress risk

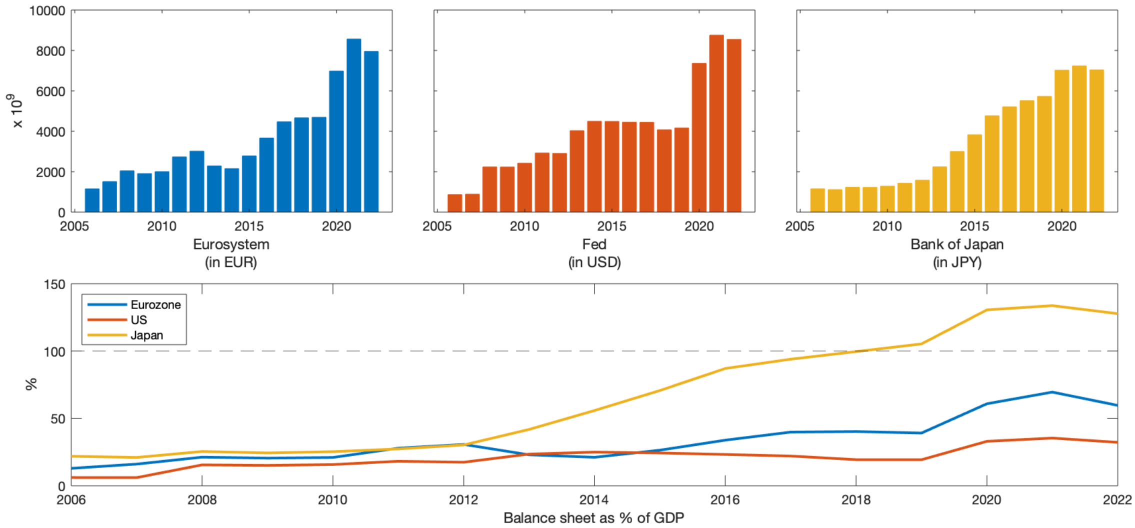

FIGURE 4: Balance sheets of the Eurosystem, the FED and the BoJ

premia. The shock in 2007-08, coupled with the sovereign debt crisis that followed, drove economies to situations in which conventional monetary policy instruments were unable to support a sustained economic recovery. Taylor rules would recommend cutting nominal interest rates well below zero but that was not possible due to the lower bound on the nominal interest rate. Moreover, close to the lower bound the previously well-functioning relationship between changes in official interest rates and market interest rates was no longer reliable.

Quantitative easing, and the purchase of large quantities of financial assets, became the tool of choice of monetary authorities facing the zero lower bound of policy rates. By reducing the supply to the private sector of risk-bearing assets (for example, due to their long maturity), and increasing the supply of less risky assets (for example, bank reserves) central banks expected to lower longer-term market interest rates, and hence channel more lending to consumers and businesses. The purchases of risky assets took many forms: long-term Treasury securities, corporate bonds, and stocks held by financial institutions, among others. Additionally, central banks provided funds to agents that normally do not have access to central bank money, like Commercial Paper issuers, Money Market Mutual Funds, agency bonds and agency Mortgage Backed Securities issuers, among others. All central banks in advanced economies implemented these kind of measures in different proportions to alleviate the effects of the global financial crisis of 2008.

Non-standard monetary policies conducted in response to the 2008 crisis continued to be used in various degrees until the recent pandemic crisis in 2020. Reflecting the net asset purchase programs in place, this led to a substantial increase in the central banks balance sheet. Figure 4 shows the evolution of the balance sheets of the Eurosystem, the FED and the BoJ (in absolute values and as a percentage of GDP), between these two landmarks.

The contribution of this paper is theoretical. We consider a simple model that relies on the model with heterogeneous agents introduced by Brunnermeier and Sannikov (2017), which, in turn follows Basak and Cuoco (1998), to unravel some of the macro effects of unconventional monetary policy measures. In particular we show how unconventional monetary policy affects the risk premium and the risk-less interest rate.

We consider an economy with two heterogeneous rational agents: a market expert, with the required know-how to accumulate a risky asset (capital), and a household, that finances the expert's purchase of risky assets by holding expert-issued risk-less debt. In addition, we introduce a central bank. The central bank redistributes risk in the economy by issuing risk-less bonds to purchase capital, and transferring dividends (via Treasury) to the households.

When solving this model, we obtain analytical expressions for the risk-free interest rate and the evolution of the expert sector's relative wealth. From here, we are able to reason how, after a negative shock to the capital of intermediaries, their ability to continue holding risky assets decreases and, as a consequence, risk premium increases. We show how, when the central bank buys risky assets using risk-less debt, there is a reduction of risk taking in the economy, as this risk is transferred to non-market participants. The asset purchases performed by the central bank change the equilibrium in the economy, leading to a decrease of the market price of risk and an increase in the risk-free interest rate.

The remainder of this paper is organized as follows. Section 2 reviews some of the relevant literature. Section 3 presents the model, describes the equilibrium in the economy and explains how non-conventional monetary policy affects the equilibrium. Section 4 concludes.

2. Literature Review

We start this section by discussing why the size and composition of the central bank's balance sheet might modify the equilibrium. Neil Wallace wrote the pioneer paper on this issue with the title "A Modigliani-Miller Theorem for Open-Market Operations". Wallace (1981) established, in the context of an overlapping generations model, that, under the assumption that markets are complete, neither the size nor the composition of the central bank's balance sheet affects the economy's equilibrium. In his model money serves only as a store of value, as it does not facilitate transactions. Other authors confirmed this result in either different or more general environments. Peled (1985) established that open market operations between money and indexed bonds do not matter in a real sense despite their different risk characteristics. Chamley and Polemarchakis (1984) extended Wallace's result to incomplete markets. Sargent and Smith (1987) extended the neutrality result to a particular economy where money is dominated in return.

The neutrality result is puzzling because open market operations instrument has been the main instrument of monetary policy, and monetary policy is taken to affect the economy. In fact, the targets of monetary policy have changed through time, by either targeting the price of reserves or the monetary aggregates, but the main instrument has always been open market operations, Bindseil (2014). Thus, this theoretical result seemed not to apply to the operations of actual central banks.

Eggertsson and Woodford (2003) explain why in general Wallace (1981) neutrality result does not apply to open market operations with reserves. Reserves (and base money more generally) are assets that have non-pecuniary returns as they help mitigate transaction frictions. Unlike other financial assets, base money provides transactions services, by relaxing constraints that would otherwise restrict the transactions that the holders of the asset can perform.

However, there is a situation where Wallace's neutrality still holds even for open market operations with reserves. When the nominal interest rate is zero, there is no longer any shortage of cash as its opportunity cost is zero. As the non-pecuniary returns of the reserves are zero, then open market operations with reserves should have no effects either. Thus, given that since 2008 until recently the nominal interest rate has been approximately zero, then the quantitative easing strategy used during this period should not have been effective in providing monetary stimulus to the economy.

The neutrality result relies on the existence of frictionless financial markets. In this case the price of an asset equals the present value of its future stochastic returns, where the present value is calculated using a standard stochastic discount factor. For example, in a simple model economy, with complete markets, the stochastic discount factor is unique and is determined by the households' marginal rate of substitution between consumption today and consumption in the different future states of nature. If the trades of assets between the central bank and the private sector do not change the real quantity of resources available for consumption in each state of nature, the households' marginal rate of substitution in the different states of nature should not change either. Thus, the discount factor should not change, and the price of the assets should not change because their returns in each state of nature have not changed.

Suppose, for instance, that the central bank decides, through an open-market operation, to hold a portfolio with more risk, which results in private investors holding a portfolio with less risk. To make things more concrete suppose that after the open-market operation the central bank's portfolio pays a low return in the event of a pandemic, while the portfolio of the private sector pays a similar return in all states of nature. This change in the central bank's portfolio does not make the risk disappear from the economy. The central bank's returns on its portfolio will be lower in the state with a pandemic, and this will imply lower dividends distributed to the Treasury, which in turn means that lower transfers will be disbursed to the private sector in that state. Therefore, the households' income after transfers will remain unchanged in that state and in all the other states of nature. This means the stochastic discount factor will be unchanged too, and thus the open-market operation will fail to change the asset prices.

The irrelevance result of the central bank's balance sheet is easier to prove if there is a representative household, but it remains true even if that is not the case. The result is true even if households are heterogeneous, they may have different risk aversion, different time profiles of income, different types of non-tradeable income risk, etc. The crucial assumption, as Chamley and Polemarchakis (1984), Cúrdia and Woodford (2011), d'Avernas *et al.* (2019) and Silva (2020) show, is that all investors can purchase or sell arbitrary quantities of the same assets at the same prices.

Under this assumption, if the central bank does an open-market operation that will change the households state-contingent income, then households should want to trade in the financial markets to undo the effects of the central bank's trade. Suppose a share x_h of the returns on the central bank's portfolio is distributed, by the Treasury, to household h. If the central bank decides, through an open-market operation, to hold a portfolio with more risk, which results in private investors holding a portfolio with less risk, then household h should choose a trade that cancels exactly fraction x_h of the central bank's trade, as a means to obtain the state-contingent consumption stream he had before the central bank's intervention.

The relevant notion of the households' wealth is the sum of their own portfolio and the present value of the discounted future transfers of the central bank to the households, via the Treasury. Hence, the relevant risk exposure of households includes both the financial risk from their portfolio as well as the risk resulting from the central bank intervention transmitted to households through transfers via the Treasury. If with an open market operation, the central bank increases its holdings of risky assets, investors have an incentive to reduce their own exposure to risk, to keep their total risk exposure constant. Because total risk exposure does not change, asset prices and macroeconomic variables do not change either. Note that this result is easier to prove with complete markets, but still holds under incomplete markets. Thus, the neutrality result holds provided transfers can be replicated (or undone) by a portfolio of tradable financial assets.

In contrast, most of the empirical literature seems to conclude that quantitative easing had effects on the economies.[1] There is a wealth of papers that study the effects of large-scale asset purchases empirically and for different jurisdictions. Krishnamurthy and Vissing-Jorgensen (2011) evaluate the effect of the Federal Reserve's quantitative easing programs on interest rates. According to the authors' findings, the Fed's influence was greater on the risk premia of the assets being purchased. For example, QE1 (2008-09) had a large effect on the reduction of mortgage rates, partly due to the fact that QE1 involved large purchases of agency backed mortgage-backed securities (MBS). In turn, QE2 (2010-11), which involved only Treasury purchases, impacted mainly Treasury and agency bond rates, and less so MBS and corporate rates.

Work has also been done in identifying different channels through which LSAPs transmit their effects to the real economy Koijen *et al.* (2017); Krishnamurthy and Vissing-Jorgensen (2011); Vayanos and Vila (2021). Krishnamurthy *et al.* (2017) focus on the effect of the European Central Bank (ECB) programs, namely the Securities Markets Programme (SMP), the Outright Monetary Transactions (OMT) framework, and the three-year long-term refinancing operations (LTROs). Their interest is on the channels through which the SMP and OMT affected sovereign bond yields of so-called GIIPS countries, as well as stock returns. A number of other authors have also studied the

1. For a skeptical interpretation of the evidence, see Stroebel and Taylor (2012) and Taylor (2009).

effect of LSAP programmes on bank lending to the private sector (Andrade *et al.* (2015); Carpinelli and Crosignani (2017); Fonseca *et al.* (2015); Garcia-Posada and Marchetti (2016)). Empirical studies have also been performed to assess the effect of QE in different asset types (De Santis and Zaghini (2021); Albertazzi *et al.* (2021); Balcilar *et al.* (2020); Farinha and Vidrago (2021); Lewis and Roth (2019); Guo *et al.* (2020)).

With financial imperfections the Wallace's neutrality result does not hold. A simple model modification assumed by many authors, like Chamley and Polemarchakis (1984), Cúrdia and Woodford (2011), d'Avernas *et al.* (2019), and Silva (2020), is that financial markets are segmented. These authors assume that investors are heterogeneous, i.e., not all investors have equal opportunities to invest in all assets on the same terms. It could be that only certain specialists have the expertise required to invest in some assets, or by law some agents do not have access to a particular market: for instance, shadow banks do not have access to central bank reserves. Thus, when the central bank purchases that specific asset, the central bank transfers to investors, via Treasury, have more income risk that is correlated with that specific asset returns. Investors may want to hold less of that risk, but since they do not have that asset in their portfolio and cannot go short on it, that is not possible.

In our model we assume that not all financial market participants can cost-less trade the same set of financial instruments. We assume that some participants lack the expertise required to directly extend credit to firms, so that they must instead deposit funds with competitive intermediaries who are in turn able to offer loan contracts to the firms. By conducting open market operations, the central bank can extend credit to firms and issue risk-free debt. As shocks affect the relative wealth of intermediaries, they affects the investment and the supply of risky assets in the economy. The central bank can modify the risk in the economy by buying the risky asset, supplying the risk-less asset and channeling the risky dividends to non market participants.

3. The model

Our model builds on the model of Brunnermeier and Sannikov (2017) which in turn is based on the paper of Basak and Cuoco (1998). The original model studies the equilibrium dynamics of an economy with financial frictions and the effect of financial disintermediation in times of crises. The model is an infinite horizon economy with production, heterogeneous agents and financial frictions. We extend the original model by introducing a central bank that does non-conventional monetary policy. In addition to the central bank, there are two other type of agents: financial experts and households. There is a continuum (with mass one) of each type of agents. The production technology uses capital to produce final goods and there are investment adjustment costs. There is one risky asset and a risk-free asset. Financial markets are segmented, only the experts and the central bank can trade the risky asset. The central bank and the experts finance their holdings of the risky asset by issuing bonds to households. Non-conventional monetary policy consists in buying capital with the revenue obtained from the issuance of bonds, and transferring the profits (or losses) associated with this operation to households.

The transfers should be interpreted as being done via Treasury after receiving the dividends from the central bank. To simplify the analysis these transfers are assumed to be received entirely by the households, but the qualitative results remain unchanged as long as the households receive some fraction of these transfers. The larger the fraction of the transfers received by the households, the larger will be the quantitative effect of the non-conventional monetary policy on the risk-free interest rate and the risk premium.

The assumption that the households cannot participate in the risky asset market is an extreme assumption that simplifies the analysis. However, in order for non-conventional policy to have effects in the economy it is just enough that households have limited access to the risky asset market, either because they do not have the necessary expertise or lack the relevant information. It is important that the actions of the agents that receive the transfers from the central bank be restricted, so that these agents cannot undo the effect of the central bank non-conventional policy. For instance, if the households were allowed to trade without any restrictions the risky asset then they would choose an efficient portfolio. In that case, whenever there is an open market operation the households would want to change their portfolio, so that the return on the new portfolio together with the central bank transfers would be equal to the return on the original portfolio.

3.1. Financial experts

Experts have a linear constant returns to scale production function of the form

$$Y_t = Ak_t, A > 0. \tag{1}$$

The capital stock evolves according to the law of motion:

$$\frac{dk_t}{k_t} = (\Phi(\iota_t) - \delta)dt + \sigma dZ_t, \tag{2}$$

where ι_t is the investment rate of capital, and the investment function Φ satisfies the usual assumptions: $\Phi(0) = 0, \Phi'(0) = 1, \Phi' > 0$, and $\Phi'' < 0$. The concavity of Φ reflects capital adjustment costs. The investment technology Φ transforms $\iota_t k_t$ units of output into $\Phi(\iota_t)k_t$ units of capital. δ is the depreciation rate of capital. The last term is a growth rate shock which follows a Brownian motion with volatility σ. This term can be interpreted as the risk of holding capital. It is the only shock in the economy.

In order to finance their holdings of the risky asset, experts issue non-contingent bonds which are bought by households. Let θ_t, $\theta_t < 0$, be the expert's short position on bonds, r_t be the interest rate on the bonds, r_t^K be the rate of return on capital, c_t^e be the expert's consumption and n_t^e be the expert's wealth. The law of motion of the expert's wealth is

$$\frac{dn_t^e}{n_t^e} = \left(-\frac{c_t^e}{n_t^e} + \theta_t r_t\right)dt + (1-\theta_t)dr_t^K. \tag{3}$$

The dynamics of r_t^K are described below in (7). The representative expert has logarithmic utility, and discounts at rate $\rho^e \geq 0$ utility flows from future consumption c_t^e:

$$\mathbb{E}_0\left[\int_0^\infty e^{-\rho^e \cdot t}\log(c_t^e)dt\right]. \tag{4}$$

The representative expert's problem amounts to deciding the investment rate ι_t, consumption flow c_t^e, and short position on bonds θ_t, so as to maximize (4) subject to the evolution of the wealth in (3) and the initial value of wealth, n_0^e.

The price of capital, denoted by q_t, follows a stochastic process described by:

$$\frac{dq_t}{q_t} = \mu_t^q dt + \sigma_t^q dZ_t, \tag{5}$$

where the drift and volatility, (μ_t^q, σ_t^q), are determined in equilibrium. The instantaneous rate of return on capital is given by

$$dr_t^K = \frac{A - \iota_t}{q_t} dt + \frac{d(q_t k_t)}{q_t k_t}, \tag{6}$$

where the first term represents the dividend yield and the second term the capital gains. We can simplify this expression to obtain:

$$dr_t^K = \left(\frac{A - \iota_t}{q_t} + \mu_t^q + \Phi(\iota_t) - \delta + \sigma_t^q \sigma \right) dt + (\sigma_t^q + \sigma)\, dZ_t, \tag{7}$$

where $\mu_t^q + \Phi(\iota_t) - \delta + \sigma_t^q \sigma$ denotes the expected capital gain and $\sigma_t^q + \sigma$ the volatility of the return. The volatility term is the total risk, composed by the fundamental risk σ and the price risk σ_t^q. For convenience in later calculations, we denote the drift of this stochastic process by $\mu_t^{r^K}$ and the volatility by $\sigma_t^{r^K}$.

As the measure of experts is one, the aggregate consumption of the expert sector is $C_t^e = \int_0^1 c_{i,t}^e di$.

3.2. *Central bank*

The central bank follows the rule of holding a share of the economy's stock of capital, which is financed by issuing risk-less debt, and distributing the dividends to households. Formally, if K_t^c is the stock of capital own by the central bank and K_t the aggregate capital in the economy then $K_t^c = \varepsilon K_t$, $0 \leq \varepsilon < 1$. To simplify the analysis we assume the central bank has zero net wealth. The central bank invests in capital, K_t^c, by issuing risk-less bonds, B_t^c, so that $q_t K_t^c = B_t^c$. These bonds, B_t^c, pay the same rate, r_t, as those issued by the financial expert. The proceeds obtained from this portfolio are rebated to households as a lump-sum transfer, T_t,

$$T_t = q_t K_t^c r_t^K - B_t^c r_t = \varepsilon q_t K_t (r_t^K - r_t). \tag{8}$$

3.3. *Households*

The financial friction in the model is the restriction that households do not have access to the market for risky securities. Households do not have the necessary expertise to hold risky securities, but may hold bonds.

The representative household can buy risk-less debt, which pays an interest rate, r_t. Additionally, the household receives a transfer, T_t, from the central bank and consumes, c_t^h. The law of motion for the household's wealth is

$$dn_t^h = (n_t^h r_t - c_t^h)dt + dT_t. \tag{9}$$

Households' preferences are also represented by a logarithmic utility function

$$\mathbb{E}_0\left[\int_0^\infty e^{-\rho^h\cdot t}\log(c_t^h)dt\right], \tag{10}$$

with $\rho^h \geq 0$.

The households' problem amounts to choosing their consumption flow c_t^h so as to maximize (10) subject to (9), and an initial value n_0^h.

As we also assume the measure of households is one, the aggregate consumption of the households sector is $C_t^h = \int_0^1 c_{i,t}^h di$.

3.4. *State variable*

Let K_t be the economy's capital. If we denote by K_t^e the experts' aggregate capital, then

$$K_t^e = \int_0^1 k_{i,t}^e di, \tag{11}$$

where $k_{i,t}^e$ denotes the i-th individual expert's capital stock. As the only risk in the model is at the aggregate level, we have that $k_{i,t}^e = k_t^e$. Capital is held by the expert sector and the central bank, so

$$K_t = K_t^e + K_t^c.$$

Let N_t be the economy's aggregate net worth. Then $N_t = q_t K_t$. Let N_t^e and N_t^h be the aggregate wealth levels of the expert and the household sectors, respectively, $N_t = N_t^e + N_t^h$ (remember that we assume the central bank's wealth, N_t^c, is zero). Since there is no idiosyncratic risk $N_t^e = n_t^e$ and $N_t^h = n_t^h$. Finally, we define the (relative) wealth of the expert sector, which we take as the aggregate state variable of the economy,

$$\eta_t \equiv \frac{N_t^e}{N_t} = \frac{N_t^e}{q_t K_t}. \tag{12}$$

As it will be useful in later calculations, we denote the drift of the stochastic process of the state variable by μ_t^η and the volatility by σ_t^η.

Households are assumed to be more patient than financial experts, $\rho^e > \rho^h$, as in Kiyotaki and Moore (1997) to prevent a degenerate stationary distribution of the relative net-worth of experts. As we will see later if $\rho^e = \rho^h$ then in the long run η_t converges to one.

3.5. *Equilibrium*

An equilibrium in this economy is defined as paths for price $\{q_t\}$, expert decisions $\{\iota_t, \theta_t, c_t^e\}$, household decisions $\{c_t^h\}$, central bank decisions $\{\varepsilon, T_t\}$, and net worths $\{n_t^e,\, n_t^h\}$ such that: (i) both agents maximize their objective functions, given relevant restrictions, and (ii) all markets clear.

3.6. *The financial expert's problem*

The relationship between the price and the replacement cost of capital, known in the literature as the Tobin's q formula is given by the expert's problem's first-order condition with respect to investment

$$\Phi'(\iota_t) = \frac{1}{q_t}. \tag{13}$$

In the case of logarithmic utility, the expert's problem can be solved analytically, yielding optimal policies for the expert's consumption and portfolio composition. The first order condition for the choice of consumption

$$c_t^e = \rho^e n_t^e. \tag{14}$$

and the condition for the choice of the optimal portfolio

$$(1-\theta_t) = \frac{\mu_t^{r^K} - r_t}{(\sigma_t^q + \sigma)^2}. \tag{15}$$

Equation (14) says that the consumption flow is a constant share of net-worth. Equation (15) says that the share of the risky asset in the expert's portfolio is the ratio between the expected excess return and the variance of the risky return. From (15) we can obtain the Sharpe ratio. This ratio is the expected return earned in excess of the risk-free rate per unit of volatility or total risk. It turns out that it is equal to the volatility of the expert's net worth:

$$\frac{\mu_t^{r^K} - r_t}{\sigma_t^{r^K}} = \frac{\frac{A-\iota_t}{q_t} + \mu_t^q + \Phi(\iota_t) - \delta + \sigma_t^q \sigma - r_t}{\sigma_t^q + \sigma} = \sigma_t^{n^e}. \tag{16}$$

3.7. *The household's problem*

From the first order condition of the household's problem we derive the consumption choice,

$$c_t^h = \rho^h n_t^h. \tag{17}$$

The consumption flow is a constant share of the net worth.

3.8. *Solving for the equilibrium*

In order to solve the model we need to specify the functions. We start by specifying the investment function. We assume the investment function:

$$\Phi(\iota_t) = \frac{1}{\varphi}\log(\varphi\iota_t + 1), \tag{18}$$

where φ is a capital's adjustment cost parameter.[2] From the solution to the expert's problem (13), we get the optimal investment rate as

$$\iota_t = \frac{q_t - 1}{\varphi}. \tag{19}$$

2. With this functional form, if φ tends to 0, then $\Phi(\iota_t)$ tends to ι_t, and there are no adjustment costs.

Using the capital market clearing condition and the first order conditions of the agents, we have that $C_t = C_t^e + C_t^h = \rho^e N_t^e + \rho^h(q_t K_t - N_t^e)$. Dividing both sides of this equation by N_t we get

$$\frac{C_t}{N_t} = \rho^e \eta_t + \rho^h (1 - \eta_t) \equiv f(\eta_t), \tag{20}$$

where for convenience we define $f(x) \equiv \rho^e x + \rho^h(1-x)$.

The total supply of the consumption good is $(A - \iota_t)K_t$. Market clearing of the consumption good implies

$$f(\eta_t) = \frac{A - \iota_t}{q_t}. \tag{21}$$

From here, we obtain the price of capital,

$$q(\eta_t) = \frac{1 + \varphi A}{1 + \varphi f(\eta_t)}. \tag{22}$$

After substituting this expression back in (19) we get the expression for investment:

$$\iota(\eta_t) = \frac{A - f(\eta_t)}{1 + \varphi f(\eta_t)}. \tag{23}$$

The fraction of the experts' wealth invested in the risky asset is obtained from the clearing condition of the capital market: $N_t^e(1 - \theta_t) = (1 - \varepsilon) q_t K_t$ or, equivalently,

$$(1 - \theta_t) = (1 - \varepsilon) \frac{1}{\eta_t}. \tag{24}$$

The risk-free rate is determined from the equation that characterizes the expert's optimal portfolio (15) and the market clearing condition (24):

$$r_t = -(1 - \varepsilon) \frac{1}{\eta_t} \left(\sigma_t^q + \sigma\right)^2 + \mu_t^{r^K}. \tag{25}$$

The derivation of the drifts and the volatilities of the relevant variables: the state variable, the price of capital, and the rate of return of capital are straightforward but too lengthy. As such they are derived and specified in the Appendix.

3.9. Numerical example

We adopt the values in the literature for the parameters, for example Silva (2020) and Brunnermeier and Sannikov (2017). We set the level of technology $A = \frac{1}{3}$, which corresponds to a capital-output ratio of 3. The depreciation rate δ equals 0.05. We will assume households to be more patient than financial experts, and so $\rho^e = 0.05 > 0.02 = \rho^h$. Finally, the fundamental risk $\sigma = 10\%$ and the capital cost parameter $\varphi = 10$.

Figure 5 depicts the price of capital, the risk-free rate, and the drift and volatility of the wealth share of experts η_t for different values of the parameters. We start by describing the case, represented by the blue line, in which households are as patients as experts, $\rho^h = \rho^e = 0.05$, and there is no intervention of the central bank ($\varepsilon = 0$). The results in this case agree with the results in Brunnermeier and Sannikov (2017). We observe in the top left panel that the price of capital, q, is constant and equal to $\frac{1+\varphi A}{1+\varphi \rho^e}$. Experts' balance sheet concentrates all risk in the economy, as in this case experts are the

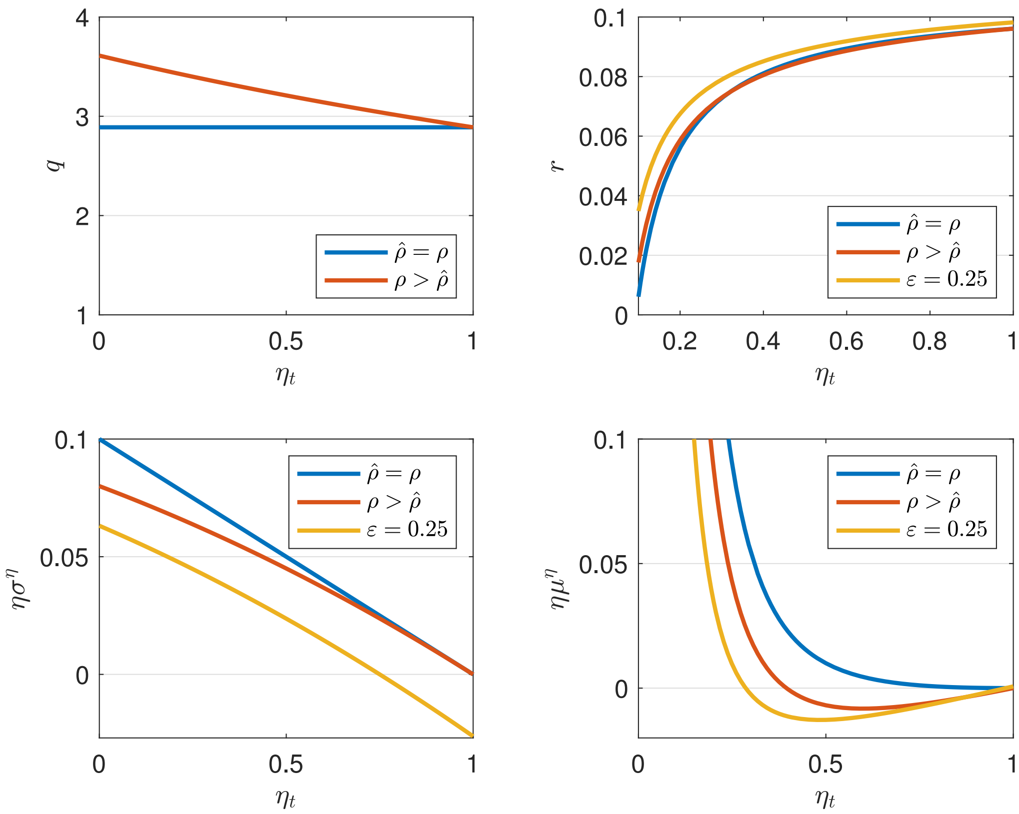

FIGURE 5: Price of capital, risk-free rate, drift and volatility of η_t.

only ones holding capital. In the event of a negative shock on capital, experts' relative wealth decreases. In that case experts demand a higher risk premium, and the risk-free rate decreases, as observed in the top right panel of Figure 5. With a lower risk-free rate, experts pay less to households on their loans, and so experts' relative wealth moves quickly back to higher values, as can be seen in the blue line in the bottom right panel. In the long run the experts relative wealth converges to one.

Let us look now at the case, represented by the red line, when households are more patient than financial experts, $\rho^e > \rho^h$, but still without central bank intervention. In this case, the price of capital is no longer constant, depends on η_t. In fact, in the top left panel of Figure 5, we observe $q'(\eta_t) < 0$. With a negative shock, as experts' relative wealth decreases, the price of capital increases. Differences in the discount rates of the agents lead to an increase in the risk-free rate, as observed when we compare the blue and red lines in the top right panel of Figure 5 and the left panel of Figure 6. As we now have differences in the consumption patterns of agents, if experts' relative wealth increases substantially, their higher consumption rate, when compared to households, will make them lose wealth over time, on average (see bottom right panel in Fig. 5). As a result, and contrary to the case above, the economy is no longer dominated in the long run by the experts' sector. Finally, in the bottom left panel, we observe how, in this case, we have a mitigation of the effect of a negative shock on the capital rate of return. As a result the volatility of the aggregate variable is lower in this case.

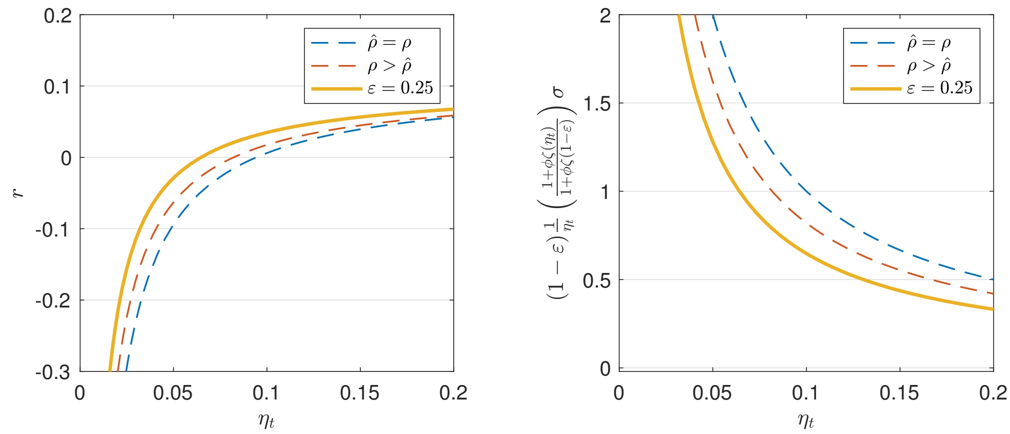

FIGURE 6: Risk-free rate and Sharpe ratio (detail, $0 < \eta_t \leq 0.2$).

At last, we look at the full model, in which the central bank owns part of the capital in the economy. This case is represented by the yellow line. Here we assume the proportion of capital owned by the central bank is $1/4$ of the aggregate capital, i.e. $\varepsilon = 0.25$, which is equivalent to 3/4 of GDP. In this case, the price of capital is unchanged, i.e. is equal to the previous case. As we can observe in the bottom left panel, there is a lower volatility in the relative wealth of experts. As in the previous case, as experts' wealth decrease, their net-worth risk increases. However, as the experts are no longer the only agents holding capital, the increase in risk is lower than in the previous cases. As a result, the risk-free rate is higher and exhibits a more convex profile. The reduction in risk for experts has a direct impact on the risk premium.

In Figure 6, we can observe how a negative shock on capital leads to an increase in the risk premium, measured by the Sharpe ratio (right panel). In the case with central bank intervention the market price of risk increases but by less than in the case without central bank intervention, hence demonstrating the positive effect of the non conventional monetary policy. In the left panel, we observe how this dampening effect on the risk premium translates into a risk-free rate higher than in the cases without non-conventional monetary policy.

3.10. Results for the steady state

In this section we study how different levels of non-conventional policy affect the steady state equilibrium of the economy when households are more patient than financial experts. We start by determining the steady state distribution of the aggregate state variable. The steady state distribution of the state variable is obtained by simulating the discretized version of the law of motion for η for a fine time interval until convergence is achieved. A fine grid of points in the interval $(0, 1)$ was considered for the initial values of the aggregate state variable. All these initial values of the state variable converged to the same steady state distribution.

Figure 7, displays the steady state distribution of the aggregate variable for 3 different levels of non-conventional monetary policy, $\varepsilon = 0.01, 0.25$ and 0.5. It shows that an

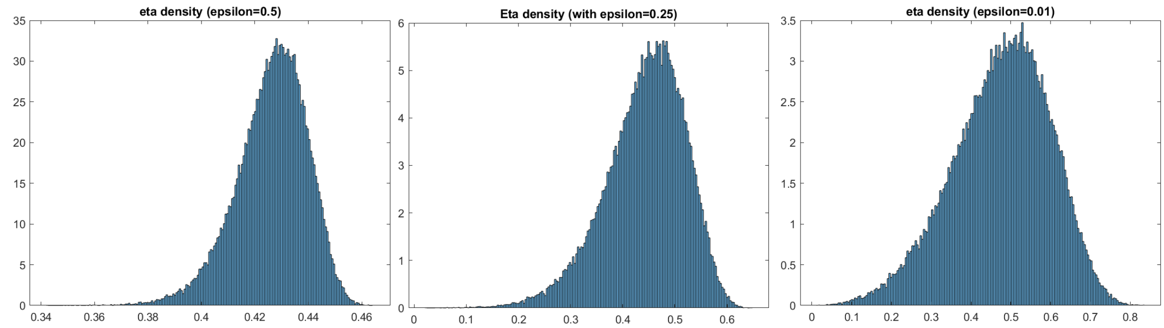

FIGURE 7: Steady state density of η_t

increase in the size of the open market operations decreases the dispersion of the aggregate state variable, which implies lower dispersion of the risk-free interest rate, price of capital, risk premium, investment rate and Sharpe ratio. Additionally, the mean of the relative wealth of the expert decreases with the size of the open market operations.

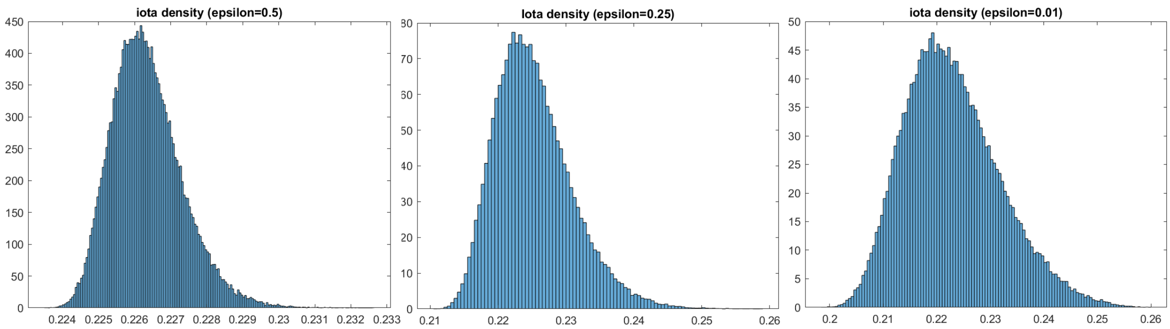

FIGURE 8: Steady state density of ι_t

Figure 8 displays the steady state distribution of the investment rate for the same 3 different levels of non-conventional monetary policy. The investment rate dispersion decreases with the size of the non-conventional policy and the mean investment rate increases with the size of the non-conventional policy. However, the impacts are quantitatively small.

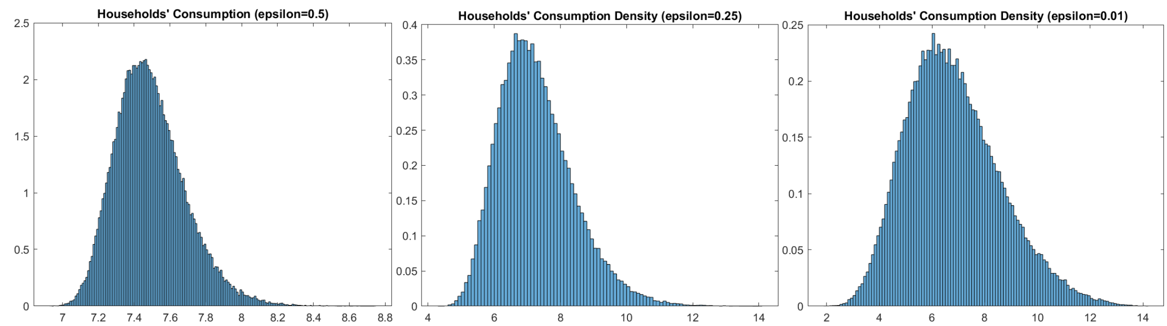

FIGURE 9: Steady state distribution of c_t^h

Finally, we investigate the effects of the non-conventional policy over the consumptions of the two types of households. Figures 9 and 10 show how the steady

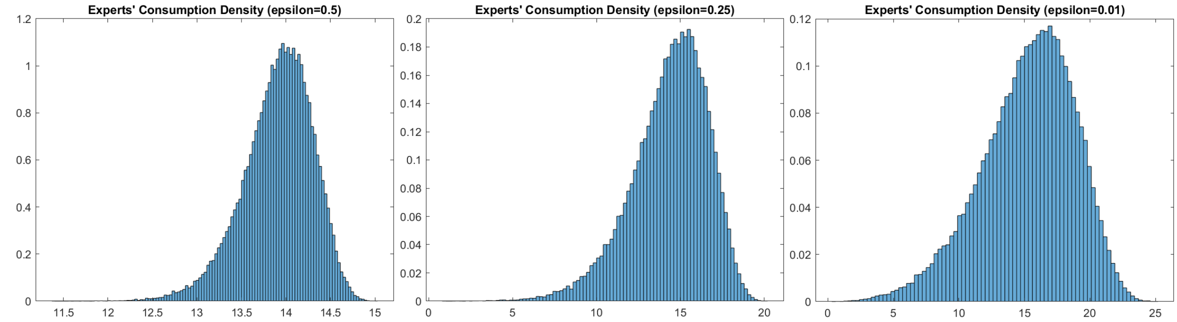

FIGURE 10: Steady state density of c_t^e

state densities of consumptions change with the non-conventional monetary policy. The dispersion of consumptions decreases substantially as the size of the non-conventional policy increases.

Using these stationary densities we compute that the expected utility of the households increases with the size of the open market operations, while the expected utility of experts decreases with the size of the open market operations. The reason for the mean investment rate to increase with the size of the open market operations is because households save more than the experts, and the average relative wealth of households increases.

4. Conclusion

After a negative shock on the capital of intermediaries, their ability to hold risky assets decreases and, as a consequence the market price of risk (or risk premium) increases and the risk-less interest rate decreases. In this paper we show that unconventional monetary policy can mitigate these effects. We unravel the effects of large-scale asset purchases (LSAPs) by central banks in a very simple model where some agents, the households are restricted from participating in risky financial markets. In this context LSAPs redistribute risk in the economy, reducing the exposure of intermediaries' balance sheets to capital shocks, leading to a reduction in the risk premium and an increase in the risk-free rate.

LSAPs stabilize the economy too: the volatilities of consumptions, investment and GDP decrease with the size of the non-conventional monetary policy. As LSAPs allow the households to indirectly access a market they did not have access to, and break the monopoly of the intermediaries in that market, the average net worth of experts decreases while the average net worth of households increases. The expected utility of households increases, expected utility of intermediaries decreases and the average investment rate increases.

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Appendix

We start by deriving the dynamics of the aggregate state variable, the share of experts' wealth, $\eta_t = \frac{N_t^e}{q_t K_t}$, with $0 \leq \eta_t \leq 1$. For that, we make use of Itô's lemma and the definition of stochastic discount factor. The stochastic discount factor for this economy is given by

$$\frac{d\xi_t}{\xi_t} = -r_t dt - \sigma_t^{n^e} dZ_t, \tag{A.1}$$

Given the absence of risks other than the aggregate risk, we have $N_t^e = n_t^e$. Then,

$$\begin{aligned}\frac{dN_t^e}{N_t^e} = \frac{dn_t^e}{n_t^e} &= \left(-\frac{c_t^e}{n_t^e} + \theta_t r_t + (1-\theta_t)\mu_t^{r^K}\right) dt + (1-\theta_t)\left(\sigma_t^q + \sigma\right) dZ_t \\ &= \left(-\frac{c_t^e}{n_t^e} + r_t + (1-\theta_t)\sigma_t^{n^e}\left(\sigma_t^q + \sigma\right)\right) dt + (1-\theta_t)\left(\sigma_t^q + \sigma\right) dZ_t,\end{aligned} \tag{A.2}$$

and using

$$\begin{aligned}\frac{d(q_t k_t)}{q_t k_t} &= \left(\mu_t^q + \Phi(\iota_t) - \delta + \sigma_t^q \sigma\right) dt + \left(\sigma_t^q + \sigma\right) dZ_t \\ &= \left[\left(r_t - \frac{A - \iota_t}{q_t}\right) + \sigma_t^{n^e}\left(\sigma_t^q + \sigma\right)\right] dt + \left(\sigma_t^q + \sigma\right) dZ_t,\end{aligned} \tag{A.3}$$

we obtain

$$\frac{d\eta_t}{\eta_t} = \left(-\frac{c_t^e}{n_t^e} + \frac{A - \iota_t}{q_t} - \theta_t\left(\sigma_t^q + \sigma\right)\left(\sigma_t^{n^e} - \left(\sigma_t^q + \sigma\right)\right)\right) dt - \theta_t\left(\sigma_t^q + \sigma\right) dZ_t. \tag{A.4}$$

Using the fact that $\sigma_t^{n^e} = (1-\theta_t)\left(\sigma_t^q + \sigma\right)$, we get an alternative expression for the law of motion of η_t:

$$\frac{d\eta_t}{\eta_t} = \left(\frac{A - \iota_t}{q_t} - \rho^e + \theta_t^2\left(\sigma_t^q + \sigma\right)^2\right) dt - \theta_t\left(\sigma_t^q + \sigma\right) dZ_t. \tag{A.5}$$

Now we proceed with the determination of the law of motion for the price of capital $q_t = q(\eta_t)$, which we assumed to follow a stochastic process with drift μ_t^q and volatility σ_t^q. By Itô's lemma we get

$$\frac{dq(\eta_t)}{q(\eta_t)} = \frac{q'(\eta_t)\mu^\eta \eta_t + \frac{1}{2}q''(\eta_t)(\sigma^\eta \eta_t)^2}{q(\eta_t)} dt + \frac{q'(\eta_t)}{q(\eta_t)}\eta_t \sigma^\eta dZ_t. \tag{A.6}$$

From the capital market equilibrium condition (24), and (A.5):

$$\sigma^q(\eta_t) = \frac{q'(\eta_t)}{q(\eta_t)}\eta_t \sigma^\eta = \frac{q'(\eta_t)}{q(\eta_t)}(1 - \varepsilon - \eta_t)\left(\sigma_t^q + \sigma\right). \tag{A.7}$$

After solving (A.7) for the volatility of the price we get

$$\sigma^q(\eta_t) = \frac{(1 - \varepsilon - \eta_t)\frac{q'(\eta_t)}{q(\eta_t)}}{1 - (1 - \varepsilon - \eta_t)\frac{q'(\eta_t)}{q(\eta_t)}}\sigma. \tag{A.8}$$

After substituting in (A.8) the expressions for $q(\eta_t)$ in (22) (and its first derivative) we get

$$\sigma^q(\eta_t) = \frac{\varphi(\rho^h - \rho^e)(1 - \varepsilon - \eta_t)}{1 + \varphi f(1 - \varepsilon)}\sigma, \tag{A.9}$$

where $f(1-\varepsilon) = \rho^e(1-\varepsilon) + \rho^h\varepsilon$. For convenience, we use the notation $f(x) \equiv \rho^e x + \rho^h(1-x)$. The drift of the process, $\mu^q = \mu^q(\eta_t)$, can be computed also. It can be established that

$$\mu^q(\eta_t) = \varphi(\rho^h - \rho^e)\left[\frac{\eta_t(1-\eta_t)}{1+\varphi f(\eta_t)}(\rho^h - \rho^e) + \frac{(1+\varphi\rho^h)}{\eta_t}\left(\frac{(1-\varepsilon-\eta_t)\sigma}{1+\varphi f(1-\varepsilon)}\right)^2\right], \qquad \text{(A.10)}$$

where $f(\eta_t) = \rho^e\eta_t + \rho^h(1-\eta_t)$. According to (25) in order to get r_t we need to determine $\mu_t^{r^K}$, which is specified in (7) as $\mu_t^{r^K} = \left(\frac{A-\iota_t}{q_t} + \mu_t^q + \Phi(\iota_t) - \delta + \sigma_t^q\sigma\right)$.

After replacing in (25) the expressions for μ_t^q, σ_t^q and $\mu_t^{r^K}$ we obtain the expression for the risk-free interest rate, which is only dependent on the model's parameters and the state variable,

$$\begin{aligned} r_t = f(\eta_t) + \frac{1}{\varphi}\log\left(\frac{1+\varphi A}{1+\varphi f(\eta_t)}\right) - \delta + \frac{\varphi(\rho^h-\rho^e)\sigma^2}{1+\varphi f(1-\varepsilon)}\left((1-\varepsilon)-\eta_t\right) \\ - (1-\varepsilon)\frac{1}{\eta_t}\left(\frac{1+\varphi f(\eta_t)}{1+\varphi f(1-\varepsilon)}\right)^2\sigma^2 \\ + \varphi(\rho^h-\rho^e)\left[\frac{\eta_t(1-\eta_t)(\rho^h-\rho^e)}{1+\varphi f(\eta_t)} + \frac{(1+\varphi\rho^h)}{\eta_t}\left(\frac{(1-\varepsilon)-\eta_t}{1+\varphi f(1-\varepsilon)}\sigma\right)^2\right]. \end{aligned} \qquad \text{(A.11)}$$

Finally, after substituting in (A.5) the expression for σ_t^q we obtain the law of motion of η_t,

$$\begin{aligned} \frac{d\eta_t}{\eta_t} = \left((\rho^h-\rho^e)(1-\eta_t) + \left(\frac{(1-\varepsilon)-\eta_t}{\eta_t}\frac{1+\varphi f(\eta_t)}{1+\varphi f(1-\varepsilon)}\right)^2\sigma^2\right)dt \\ + \left(\frac{1+\varphi f(\eta_t)}{1+\varphi f(1-\varepsilon)}\right)\frac{(1-\varepsilon)-\eta_t}{\eta_t}\sigma dZ_t. \end{aligned} \qquad \text{(A.12)}$$

The Sharpe ratio of risky investment is

$$\begin{aligned} \frac{\frac{A-\iota_t}{q_t} + \mu_t^q + \Phi(\iota_t) - \delta + \sigma_t^q\sigma - r_t}{\sigma_t^q + \sigma} &= (1-\varepsilon)\frac{1}{\eta_t}\left(\sigma_t^q + \sigma\right) \\ &= (1-\varepsilon)\frac{1}{\eta_t}\left(\frac{1+\varphi f(\eta_t)}{1+\varphi f(1-\varepsilon)}\right)\sigma. \end{aligned} \qquad \text{(A.13)}$$

Non-technical summary

July 2023

On the forecasting power of corporate sales growth determinants

Nuno Silva and Pedro Dias Moreira

This study evaluates the contribution to corporate sales forecasting of the determinants presented in the literature. This is done by estimating a model for the 1-year sales real growth rate and then evaluating the model performance. The study considers 1.2 million observations and 189 thousand unique firms over 14 years (growth rates between 2008 and 2021) and covers several firms' characteristics, detailed financial information (balance sheet and income statement), loan and security level information and human capital characteristics. The study uses a sample of firms corresponding to 77% of sales and 73% of the number of workers in Portugal (average values per year).

The results obtained are broadly in line with those found in the literature. The variables size, age (firm, employees, and managers) and gender (female, both for employees and managers) show a negative relation with corporate sales growth, while the variables access to external funding, profitability, belonging to an economic group, employees' education, lagged investment and financing flows, and the external environment dynamics (industry, local and macroeconomic aggregates) show a positive relation. The relation with the leverage ratio is positive up to a certain level and then negative. The forecasted growth rate depends on the debt composition, being higher in firms where debt from related parties has a greater weight and lower in firms with a higher proportion of financial debt or debt to the government. A noteworthy exception is productivity. While most studies point either to a positive or insignificant relation, we find a significant negative relation: the lower the productivity of firms relative to their peers, the greater the sales growth will be. This result prevails whenever considering different productivity measures.

The period between 2008 and 2017 is used to estimate the model parameters, while the years between 2018 and 2021 are used to evaluate the model performance. As common in forecasting studies, the model performance is compared with a benchmark. The benchmark used in the article is the median sales real growth rate between 2008 and 2017. The average difference, in absolute terms, between the predicted and observed sales real growth rates for the years between 2018 and 2021 is 0.223. This compares with 0.235 in the case of the benchmark, which corresponds to an improvement of 4.9%.

Figure 1 presents a binscatter of the predicted and observed sales real growth rates for each firm, for the years between 2018 and 2021. The binscatter is a type of scatter plot

— specifically applied in situations with a very large number of observations — where the data is aggregated in bins to increase its readability. The observations allocated to each group are determined based on the rank of predicted sales growth rates — x-axis variable — and in a way to guarantee that each point corresponds to the same number of observations (4% of all observations, so there are 25 bins). The blue dots represent the mean and the red dots represent the median values of the variable in the y-axis (observed values) for several groups of observations (thus, there are 25 blue points and 25 red points on the figure). The value in the x-axis corresponds to the mean of the predicted sales growth rate for those bins. A 45 degrees line is added to facilitate the figure interpretation. Points above (below) the 45 degrees line suggest that the model is underestimating (overestimating) the observed sales growth rate.

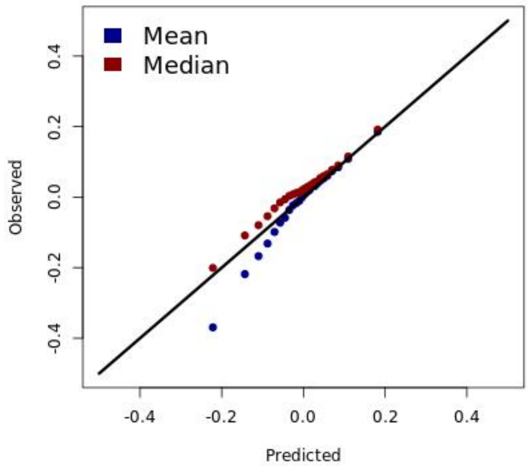

FIGURE 1: Binscatter of predicted and observed sales real growth rates (2018-2021).

Notes: The binscatter gives the mean (blue dots) and the median (red dots) value of the variable in the y-axis variable for groups of observations chosen based on the variable on the x-axis. The value presented in the x-axis corresponds to the average of those observations. All values are in natural units. The observations associated with each point are determined based on the rank of the variable in x-axis so that each of the 25 points represents the same number of observations (4% of all observations).

Except for large negative sales growth predictions, the mean and the median observed sales growth rates are close to the sales growth rate predicted by the model. For those cases where the model predicts a large negative growth rate, the observed growth rate is on average considerably lower than the predicted growth rate, something that does not occur in the case of the median. This result suggests that the model is not able to adequately predict extreme negative growth rates, which indicates that the distribution of observed sales growth rates is more skewed than the distribution of the forecasted sales real growth rate.

The variables that contribute the most to improve sales forecasts are those related with the external environment, in particular the macroeconomic aggregates. Except for investment-related variables, most other firm-level variables cited in the literature have a negligible forecasting power.

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On the forecasting power of corporate sales growth determinants

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Abstract
This article presents a panel regression model to evaluate the forecasting power of the determinants of corporate sales growth cited in the literature. The model is estimated using information on 189 thousand unique firms over 2008-2021. The results point for a negative relation with size, age (firm, employees, and managers), employees and managers' gender (female) and productivity, and a positive relation with access to external funding, profitability, belonging to an economic group, the shareholder being simultaneously a worker, employees' education, lagged investment and financing flows, as well as changes in the external environment (industry, local and macroeconomic). The relation with leverage is concave and depends on debt composition. The effect of autocorrelation depends on the activity sector and it is typically positive for larger firms. Our specification outperforms a model where the growth rate of sales is the same for all firms. The variables that contribute the most to the model performance are those related with the external environment, in particular the growth rate of domestic demand and exports. Except for investment-related variables, most other firm-level variables cited in the literature have a negligible forecasting power. (JEL: C53, D22, G30, L25)

1. Introduction

Forecasting sales is of great importance within all firms. Sales forecasts affect investment decisions, inventory and human resources management and financial planning. Within firms, they can also be used as a criterion to evaluate performance. Sales forecasts are relevant as well for several stakeholders outside the firm, such as prospective shareholders, who rely on these to value the firm, and lenders that often take sales forecasts into account when deciding whether to grant credit and to determine the loan credit spread. Sales forecasts can also be useful for policy makers. Among other things, they can be used to evaluate whether credit is being allocated to the most promising firms, as an input to measure the credit risk in banks' corporate portfolio or to improve macroeconomic forecasts.

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There is a very significant literature, mostly in industrial economics and corporate finance, which explores the determinants of firms' growth. This literature focuses mostly on firms' characteristics like size, age, access to external funding, financial health, profitability, productivity, ownership structure, access to foreign markets and human capital characteristics. There are also some papers that explore firms' dynamics, looking at the effect of autocorrelation patterns, investment, and financing decisions, as well as changes in the external environment. The latter includes looking at fluctuations in industry, local and macroeconomic aggregates. Most studies in the literature measure corporate growth by looking to the growth rate of either the number of employees or real sales.

Though the literature has been able to show that several of these determinants are relevant in explaining corporate growth, in-sample goodness-of-fit measures are typically very low, especially in the case of very small firms. This has led Geroski (2005), among others, to argue that firms' size reflects mostly a sequence of purely stochastic shocks, similar to a random walk, an assumption also made by several structural corporate finance models (e.g. Eisdorfer *et al.* 2019). While there is a vast literature on corporate growth, the number of studies addressing firms' sales forecasting is relatively small, often focused on specific sectors or applied to public companies and frequently consider time horizons below one year.

In this article, we estimate a panel regression model for the 1-year growth rate of firms' sales using a sample of firms corresponding to 77% (73%) of sales (employees) in Portugal (average values per year). We contribute to the literature in two ways. First, we confirm most of the results in the literature. It is the case of size, age (firm, employees, and managers) and gender (female, both for employees and managers), which show a negative relation with corporate sales growth, and access to external funding, profitability, belonging to an economic group, employees' education, lagged investment and financing flows, and external environment dynamics (industry, local and macroeconomic), which show a positive relation. The relation with leverage is concave and depends on debt composition. The effect of autocorrelation depends on the activity sector and it is typically positive for larger firms. A noteworthy exception is productivity. While most studies point either a positive or insignificant relation, we find a significant negative relation for all productivity measures considered. Our results are based on a large database composed by roughly 1.2 million observations and 189 thousand unique firms spread over 14 years (growth rates between 2008-2021) and covers several firms' characteristics, detailed financial information (annual balance sheet and income statement), loan and security level information and human capital characteristics. We are not aware of any study with this level of information for such a large number of private firms.

Second, while most previous studies only explore whether specific factors determine corporate growth or maximise the forecasting performance, we take these two questions jointly. We estimate the model using growth rates between 2008 and 2017 and evaluate its performance using those between 2018 and 2021. The out-of-sample mean absolute difference between observed and forecasted sales growth rates equals 0.223, 0.011 below a model where the growth rate of sales is the same for all firms. This corresponds to a relative mean absolute deviation of 0.951. The variables that contribute the most to improve sales forecasts are those related with the external environment, in particular the macroeconomic aggregates. Except for investment-related variables, most other firm-level determinants cited in the literature have a negligible forecasting power. In contrast with the literature, autocorrelation has no forecasting power, something that is related with the strong negative effect of the pandemic shock in 2020 which is carried over to 2021 forecasts, as a result of the positive autocorrelation coefficient in several sectors.

The remainder of the paper is organised as follows. In section 2 we review the literature on corporate growth determinants, highlighting our contribution. In section 3 our dataset is described along several descriptive statistics. Section 4 presents the econometric model. Section 5 analyses the forecasting performance of the model and section 6 concludes.

2. Related literature and contribution

2.1. Firms' characteristics

Size and age are by far the two most studied corporate growth determinants. Most of the literature on the relation between size and corporate growth is organized around Gibrat's law, which posts that corporate growth rates are independent of size. There are however some arguments that may justify differences in growth rates across sizes. For the transaction and technological theories, firms grow faster until reaching their optimal size. As smaller firms are more likely to be below their optimal size, or minimum efficient level, one should expect them to grow faster.[1] In contrast, the Marris' managerial theory highlights that due to agency problems larger firms focus more on growth than in profitability. Finally, industrial organization theories see sales growth as the result of a competitive process (Shapiro 1989). While smaller firms benefit from greater flexibility, lower wages, political sympathy, and closer managers, they also have higher financing costs and more difficulty to finance their projects, especially during recessions.[2]

Though earlier papers, mostly based on large manufacturing firms found that Gibrat's law was a good approximation of reality, from the 80s onwards the empirical evidence has been more in favour of a slightly negative relation (Hall 1987; Evans 1987a,b, among many others). Some studies show however that this negative relation holds only for samples composed mostly of small firms (Geroski and Gugler 2004) or

[1] These theories differ on the rationale behind the optimal size. For the transaction cost theory, this is determined by the costs and benefits of internalising some function as compared to buying it in the market. For the technological theory, the optimal size is determined by the interplay of the increasing returns to scale associated with production technology and the diseconomies of scale stemming from the spread of managerial input, loss of control and information, less flexibility and less motivating environments.

[2] Some studies highlight that public policies often discriminate negatively larger firms through progressive taxation, higher firing costs and ineligibility to public support measures, ultimately affecting smaller firms' willingness to grow.

that it declines very markedly when controlling for firms' age (Lawless 2014). Rossi-Hansberg and Wright (2007) analyse the impact of size in capital-intensive and labour-intensive sectors and find that the gap between the growth rate of small and large firms is significantly smaller in the latter. Consistent with this Daunfeldt and Elert (2013) find that Gibrat's law is rejected mostly in industries with high minimum efficiency levels.

Firms' age and sales growth are generally conceived to be negatively related. This is the case in Jovanovic (1982) and Ericson and Pakes (1995) models, where firms gradually learn about their productivity level and choose output and investment accordingly. As uncertainty is higher in the first years, these models predict stronger growth for those younger firms that stay in the market. More recently, Coad (2018) argues that age captures several predictable changes in non-observable firms' characteristics like improvements in internal organization and reputation, which tend to be more pronounced in the early ages of the firm leading to faster growth. In addition, older firms are less responsive to market opportunities, less willing to adjust the already established routines, and may have difficulty adjusting their product profile to the evolution of tastes.

The negative relation between age and sales growth has empirical support in the literature (Evans 1987a,b). Several studies point that the effect of age on sales growth is concentrated in the first 5-7 years and that this non-linearity should be addressed in the econometric specification (Huynh and Petrunia 2010; Lawless 2014). In line with most previous studies, we find a negative and non-linear relation between sales growth rates and both size and age, even controlling for employees' age, a variable found to be correlated with firms' age (Ouimet and Zarutskie 2014). On average the gap between the growth rate of small and large firms is higher in sectors of activity with higher capital intensity. Size and age show however a negligible contribution to explain corporate growth and to improve the model forecasting accuracy.

Access to external funding has also been pointed as an important determinant of corporate growth, as firms may not be able to generate the financial means needed to make the investments required to grow. The literature on this regard is vast and analyses the contribution of business angels (Levratto *et al.* 2018), venture capital and private equity funds (Paglia and Harjoto 2014), trade credit (Yazdanfar and Öhman 2015), banks (Rahaman 2011) and public capital markets (Borisov *et al.* 2021). Two important conclusions are worth pointing. First, firms that have access to external sources of funding grow faster because they rely less on internal financing to make their investments (Aghion *et al.* 2007).[3] Second, firms that are heavily dependent on bank credit grow less than others when these institutions face a negative shock (Chava and Purnanandam 2011; Dimelis *et al.* 2017).

Access to external funding is often determined by financial health. The empirical literature on the impact of leverage has reached mixed results. While Huynh and Petrunia (2010) find a positive relationship between sales growth and firm leverage,

[3] The only exception is suppliers' credit. In this case, most studies associate higher levels of accounts payable with difficulties obtaining cheaper and more stable sources of funding (Petersen and Rajan 1997).

Lang *et al.* (1996) point for a negative relation. One possibility for this discrepancy is the existence of different channels that relate leverage and sales growth leading to a non-monotonic relation (Coricelli *et al.* 2012). While high levels of debt may reflect external creditors confidence on the firm projects, too high leverage ratios may lead the firm to reject profitable projects, incur in bankruptcy costs or sell some of its assets at below market prices. Though statistically significant, firms that have bank loans or issue debt securities grow only slightly more than others. Sales growth is found to have a concave relation with leverage. The net effect is nevertheless positive except for very high values and depends considerably on debt composition. Access to external funding, leverage and debt composition together have a relevant contribution to increase the model fit but contribute marginally to improve sales forecasts.

The theoretical literature on firm growth, in particular the growth of the fittest theory, sees profitability and productivity as relevant corporate growth determinants. In a world with imperfect capital markets, one expects more profitable firms to face less financial constraints than firms that must resort to external sources to make their investments. Coad (2009) emphasises that more profitable firms are also more motivated to grow as their business model proved to be successful. In corporate valuation, firms' long term (or 'sustained') growth rate is often determined by the level of retained profits multiplied by the return on equity. Regarding productivity, industrial organization theories emphasize that competitive pressures should lead more productive firms to grow more than others. The empirical evidence on a positive relation between profitability/productivity and sales growth is nevertheless scarce. Coad (2009) concludes that profitability and productivity are able to discriminate mostly whether firms are going to fail and Bottazzi *et al.* (2008) find a barely weak significant relation between sales growth and both profitability and productivity in a large sample of Italian firms.[4] Using rank variables clustered by industry and year, we find that the more profitable and the less productive is the firm relative to its peers, the higher is its sales growth rate. Together these variables contribute modestly to improve the model fit and forecasts.

Some studies explore whether the ownership structure affects corporate growth. Harhoff *et al.* (1998) find that limited liability firms have higher expected growth rates and relate this with the risk-taking incentives resulting from shareholders wealth not being at stake. Following the managerial theory suggestion that managers attach utility to size and growth, Hay and Kamshad (1994) find that owner-controlled small and medium-sized enterprises (SMEs) have lower growth rates. Carney (2005) highlights that family control imposes capital constraints that can inhibit corporate growth and Salvato (2004) argues that family businesses tend to put continuity before growth. Beck *et al.* (2005) find a positive, borderline significant relation between sales growth and foreign ownership. Finally, there is some empirical evidence that firms holding partnership arrangements with other firms grow faster (Variyam and Kraybill 1992).We find that firms with crossholdings with other firms grow considerably more than others.

[4]Coad (2009) points that increasing productivity often requires the firm to downsize its activities. A negative relation between productivity and sales growth may also be the result of less productive firms increasing their size to become more productive.

Additionally, we identify whether the shareholder works in the firm and consider this as a proxy of whether the owner controls the firm. In contrast with the literature, this variable is found to have a positive significant coefficient. These variables have a negligible contribution to increase the model fit and do not contribute to improve sales forecasts.

The relation between access to foreign markets and corporate growth has also been addressed in the literature with most studies pointing for a positive relation (Beck *et al.* 2005). One reason for this is that exporting firms are likely more productive and innovative (Golovko and Valentini 2011) before becoming exporters (Wagner 2007). Conversely, exporting may lead to productivity gains because of higher competition and learning in foreign markets. The effect is however not always significant as it is the case of Liu *et al.* (1999). We find that exporting firms grow faster, but this variable contribution to improve the model fit and forecasts is almost null.

Finally, the role of human capital characteristics on corporate growth has also been explored, mostly by those following a resource-based view of organization growth. Among the most studied attributes are age, education and gender. Studies on the relation between owner-managers' (owners and/or managers) age and corporate growth generally point for a negative relation, something that has been justified with younger CEOs higher propensity to enter into M&A activity, investments and internationalization strategies as a result of their lower risk aversion (Serfling 2014), career concerns (Navaretti *et al.* 2022) and lower likelihood of adverse health events (Bennedsen *et al.* 2020). A negative relation has also been found for employees. In this case, this has been mainly related with older employees' reduced ability and willingness to innovate and adopt new technologies, outdated skills and higher risk aversion (Ouimet and Zarutskie 2014). Education has been found to have a positive relation with corporate growth, both for owner-managers (Wiklund and Shepherd 2003) and employees (Magoutas *et al.* 2012). Some studies specifically associate this pattern with greater innovation and openness to change (Wiersema and Bantel 1992) and the capacity to run internationalisation strategies (Herrmann and Datta 2005), in the case of owner-managers, and higher productivity and capacity to adapt to new technologies, in the case of employees. Last, studies on the relation between owner-managers' gender and corporate growth generally find that male-led firms have higher growth rates, something that liberal feminist theories relate with woman lack of access to important resources (education, professional experience, working hours due to domestic responsibilities) and social feminist theories link with different attitudes towards risk and different goals (Fischer *et al.* 1993 and Gottschalk and Niefert 2013). In the case of employees, there are still very few studies and results are less consensual (see Koch *et al.* 2013). In line with the literature, we find a negative relation with age and gender (female), both for managers and employees. We also find a positive relation with the share of higher education, but only in the case of employees. Taken together, these variables have a modest contribution to increase the model fit and do not contribute to improve the model forecasts.

2.2. *Autocorrelation, lagged investment and lagged financing*

Autocorrelation in firms' growth rates have also been a topic of research, both in industrial economics and sales forecasting literature. While earlier studies mostly based on large manufacturing firms find positive autocorrelation, more recent research highlights that autocorrelation depends on some firms' characteristics, in particular size and industry. Coad (2007a) points that small firms' growth rates are very erratic and, as a result, these firms exhibit negative autocorrelation. In contrast, larger firms make medium-term strategic plans and respond less to the ongoing external environment leading to positive autocorrelation. Fairfield *et al.* (2009) find significant differences in the speed of mean reversion in sales growth rates across industries and show that a model which takes these differences into account has a better performance. We observe significant differences in autocorrelation depending on size and sector of activity. In particular, micro firms and firms in the construction, real estate and agriculture sectors show autocorrelation coefficients below others.[5] Autocorrelation has a relevant contribution to the model fit but no contribution to increase its forecasting power unless we exclude 2021, the year of the recovery from the pandemic shock.

Some studies use lagged investment decisions to predict sales growth. Kesavan *et al.* (2010) use the lagged growth rate in the number of stores to predict next year sales of US retailers whenever contemporaneous store growth forecasts are not available. Coad (2007b) finds a significant positive relation between sales growth rates and lagged growth rates of employment using a reduced-form vector autoregressive model. Geroski *et al.* (1997) and Bottazzi *et al.* (2001) explore the impact of the investment in intangible assets (R&D, innovation, advertising) and find it to be non-significant, despite positive. Geroski and Toker (1996) focus on the role of advertisement and innovation expenditure and conclude that these variables are especially important to preserve market leadership. We find that lagged investment variables, in particular the growth rate of the inventory stock and of the number of employees, have positive and significant coefficients and sizable explanatory and predictive power. Interestingly, for most lagged investment variables, the impact of a decrease is greater than the impact of an increase, something that we have not seen highlighted in the literature.

As corporate growth requires investment, which must be preceded by financing, it is likely that lagged financing flows predict corporate growth. Frank and Sanati (2021) explore the role of profitability and lagged external financing flows in asset growth using aggregate and firm-level information. They show using vector-autoregressive models that only lagged equity issuances predict asset growth. Lagged profitability is statistically not significant despite its sizable impact and debt issuance tends to follow asset growth instead of preceding, something that they relate with creditors collateral requirements. Huang and Ritter (2021) study whether different sources of financing

[5] Coad (2007a) uses quantile regressions to show that positive (negative) shocks are more likely after very strong negative (positive) shocks and that this effect is more relevant for smaller firms. In unshown work, we tried to accommodate this finding by including a quadratic term. Though significant, this was found to contribute negatively to the model out-of-sample performance.

have different objectives and conclude that most equity issuers and an overwhelming majority of net debt issuers would face immediate cash depletion without external financing suggesting that they would have cancelled investments or sold assets if they had not received additional cash. We find a significant positive relation between sales growth rates and lagged external cash flows, and a surprising negative small relation with the cash flow from operations. Only changes in total debt contribute to improve the model fit and forecasting performance.

2.3. *External environment*

It is widely acknowledged that the performance of a firm is influenced by its external environment. The literature has consistently emphasized the significance of three key environmental characteristics in shaping the performance of firms, namely industry performance, local dynamics, and macroeconomic variables.

The effect of industry on corporate performance is addressed in the literature in three different ways. The most popular is to consider industry fixed effects (or industry-time fixed effects). This is the case of Geroski and Gugler (2004), Fairfield *et al.* (2009) and Lawless (2014). These are intended to capture several industry characteristics like life-stage, level of innovation, competition, and market concentration. For forecasting purposes one can either assume that past performance is going to be repeated and use the industry fixed effect or consider it as a control variable and assume the same growth rate across all industries. A common alternative, which we follow, is to use the average (or median) growth rate of the industry at each moment in time. This is the case of Audretsch and Mahmood (1994), Audretsch (1995) and Geroski and Toker (1996). This approach requires however projections of industry growth rates to produce firm-level forecasts. Finally, some studies explicitly consider industry characteristics as firm growth determinants. This is the case of Audretsch (1995), who shows that new firms grow more in sectors where the minimum efficient scale is higher, and Geroski and Toker (1996), who find a positive relation between market concentration and corporate growth. Studies that focus mostly on persistent industry characteristics, either taken explicitly or through fixed effects, usually have little explanatory power (Coad 2009), in contrast to those that account for the industry performance over the business cycle. Industry growth, taken as the difference between the median industry real sales growth rate and the real gross domestic product growth rate, is by far the variable that contribute the most to explain the variation in firms' growth rates. Its contribution to improve the out-of-sample forecasting accuracy is more limited, likely reflecting the relatively low accuracy of the auxiliar industry growth model.

Except for a minority of cases, countries exhibit relevant local heterogeneities emerging from socio-demographic characteristics and dynamics, labour force characteristics, infrastructure supply, investment opportunities and the level of local administration efficiency, availability of business-to-business services and financial support. These differences can be accounted through dummy variables or considering actual growth rates. Dougal *et al.* (2015) study the impact of local firms' investment on the investment of other firms headquartered nearby controlling for industry dynamics as well as firm, time and local fixed effects. They find that the contemporaneous impact of local dynamics is almost half of the one found for the industry. Moreover, while industry impact is mostly contemporaneous, local dynamics have a more gradual passthrough. Jannati (2020) shows that productivity shocks to the largest firms in the US economy affect firms that are geographically close in the following year. They show evidence that these spillovers occur not only through direct relationships (e.g. firms in the same sector or suppliers/clients), but also through knowledge externalities, impact on local public government budgets and changes in collateral values. Parsons *et al.* (2020) show that the sales growth rate of firms within the same city have a degree of comovement of roughly one-fourth of the one found for firms in the same industry. In line with these studies, we find a positive relation with the difference between the median municipality real sales growth rate and the real gross domestic product growth rate. In the case of micro firms, where local dynamics have a stronger impact, the effect is roughly two thirds of the one found for the contemporaneous industry growth rate. This variable has a moderate explanatory power and a detrimental contribution to the model forecasting performance.

The effect of macroeconomic dynamics on corporate growth has been largely measured by using time fixed effects, as seen in the works of Coad (2007a) and Oliveira and Fortunato (2006), among many others. Exceptions to these include Higson *et al.* (2002), Higson *et al.* (2004), Beck *et al.* (2005) and Hölzl and Huber (2009). The first two studies analyse the sensitivity of growth rates to aggregate shocks conditioning on firm size for the US and UK, respectively. They find that firms in the middle range of the growth range are more affected by aggregate shocks than others. Beck *et al.* (2005) use several macroeconomic variables as control variables and find a positive significant relation between firms' 3-years sales growth rates and average GDP growth and inflation. Their coefficients are nevertheless small in size suggesting that these variables are relevant only on the time dimension. Hölzl and Huber (2009) study the evolution and cyclical dependency of the cross-sectional distribution of firm-level job creation rates from 1975 to 2004 for the Austrian private sector and conclude that the smallest firms are largely unaffected by the business cycle. None of these studies differentiates the impact of domestic demand and exports growth rates on corporate sales. We find that macro variables contribute significantly both to explain and to forecast firm-level sales growth. Micro firms are found to depend slightly less on the business cycle but differentiating based on size does not contribute to increase the forecasting power.

3. Data

Our main source of information is the Central Balance Sheet Harmonised Panel (CBHP). This panel is based on *Informação Empresarial Simplificada* (IES), the system through which corporations report mandatory information to the tax administration and statistical authorities. CBHP provides financial information (annual balance sheet and income statement) as well as several firms' characteristics like sector of activity, number of employees, age, legal form, access to foreign markets, whether the firm belongs to any economic group and location for virtually all non-financial firms operating in Portugal. *Central de Responsabilidades de Crédito* (CRC) is used to assess whether the firm has obtained credit from resident financial institutions, the loan amount, and if it has any amount overdue. *Sistema Integrado de Estatísticas de Títulos* (SIET) is used to assess whether the firm has access to capital markets and the amounts issued. *Quadros de Pessoal* (QP) supplies information on the age, qualifications and gender of employees and managers. We merge QP with all our datasets for the period between 2006 and 2013. For the remaining years, in the case of firms born up to 2013, it is assumed that all workers (employees and managers) do not change their characteristics, except for age, which is assumed to increase 0.5 years every year. This value is compatible with an increase in our age-related variables between 2014 and 2020 similar to the one observed between 2006 and 2013. For all firms born after 2013, we use an unmatched QP database (6% of all observations) and consider mean values across clusters determined by year of birth, location (NUTS 2), number of workers, legal form, and sector of activity.[6] Finally, historical information on macroeconomic aggregates is obtained from *Instituto Nacional de Estatística* (INE) and macroeconomic forecasts correspond to those regularly published by *Banco de Portugal* in its December Economic Bulletin.

Our initial dataset consists of all private non-financial firms in activity at the end of the year between 2006 and 2021. This corresponds to more than 825 thousand unique firms and approximately 6.2 million observations. After the computation of sales growth rates, we apply six filters to this dataset. First, we eliminate all observations where the total asset value is different from the sum of total liabilities and equity (0.6% of initial observations). Second, we eliminate all observations with sales and assets below 100 thousand euros at 2021 prices (63.6% of initial observations). Third, we eliminate firms with less than two remunerated employees working full-time and total salary expenditure less than two times the minimum national wage (5.3% of initial observations). Fourth, we eliminate firms with reporting gaps in any year between the first and the last time the previous two conditions are fulfilled (6.5% of initial observations). Fifth, since some variables are computed based on the variation of past accounting figures and we need one year to compute the growth rate of sales, we restrict our analysis to firms that reported IES for at least three consecutive times (4.3% of initial observations). Finally, we eliminate observations with zero sales in the forecasted year (0.1% of initial observations).

The imposed restrictions have two objectives in mind. First, we do not want to capture neither self-employment dynamics nor firms with intermittent activity or whose existence is mostly motivated by tax efficiency reasons. Second, we want to avoid our results to be overaffected by micro-firms that represent 88% of our initial dataset but only 16% of total revenues as this would turn our results less comparable to other works. Our final dataset is an unbalanced panel composed by 189 thousand unique firms and

[6]The corresponding group is determined based on the Gower distance between CBHP and QP observations to mitigate the impact of methodological differences in the two databases.

1.2 million observations (6.4 growth rate observations per firm) covering approximately 77% of total sales, 56% of total assets and 73% of employees in the initial dataset. The lower figure observed for assets results from a high number of firms in the real estate sector with sizable assets and a low level of sales. Micro, small, medium and large firms represent 59%, 34%, 6% and 1% of the observations. In approximately 73% of the observations, firms have credit in CRC or credit lines with maturity above 1-year. In only 0.6% of the observations, firms have some amount due in SIET. A high number of firms (26% of the observations) belongs to some economic group.[7] Based on the highest level of aggregation in the European classification of economic activities methodology (NACE), trade is the most represented sector corresponding to more than one third of all observations, followed by manufacturing and construction. Except for accommodation and food services and consulting, all other sectors represent less than 5% of observations each. Electricity, Mining and Water are the smallest sectors representing less than 0.5% of the observations. The list with all activity sectors is presented in section 5 (Table 4).

Table 1 presents descriptive statistics for most variables used in the econometric model presented in section 4. All variables are winsorized at the 1% and 99% levels, including the sales growth rate, which is computed as the log-change in real sales. Hereafter, we refer to real sales as sales (2021 prices). In line with most other studies, the sales growth rate has heavier tails than the Normal distribution, a pronounced left skew leading to a negative mean and a high concentration around the median, which is slightly below zero.

The mean and the median leverage ratio are 66% and 67%, respectively. A distinctive feature of Portuguese firms is that on average (median) financial debt, which comprises liabilities in the CRC and SIET, represents only 28% (21%) of total debt. Trade credit and liabilities to the government represent 29% (25%) and 10% (5%), respectively. The remaining debt, which represents 33% (26%), corresponds mostly to liabilities to related parties, notably loans from shareholders and group companies. Firms have credit overdue in CRC in 4.1% of all observations, varying between 6.1% in 2012 and 1.7% in 2020. [8] While exports represent only 9% of total sales, they are positive in 33% of the observations. Regarding human capital characteristics, we observe a higher share of men, especially in management positions. Workers in these positions are typically older and more educated, though they have higher education only in a quarter of the observations. Shareholders work for the firm in 59% of the observations.[9]

Investment-related variables show a significant level of concentration around the median, which is either zero or close to it, and fat tails. A pronounced positive skew is

[7] A firm is considered as a member of an economic group if it is owned or own shares in another firm in the period between 2014 and 2018.

[8] We consider a firm has credit overdue whenever credit in arrears in December is at least 100 euros.

[9] We consider as managers all workers that are simultaneously employers, directors, and top executives in QP. This criterion is sequentially relaxed for those firms without managers after applying this rule. For those firms without any employers, directors or top executives we use salary and age. Our median (mean) firm has 1 (1.5) managers. We consider as employees all workers that are not managers. Those workers that are employers are assumed to be shareholders that work for the firm.

Variable	Mean	SD	P5	P25	P50	P75	P95
$\Delta \ln$(Sales)	-0.06	0.39	-0.71	-0.16	-0.01	0.12	0.43
Sales	3 058	32 059	137	268	541	1 376	8 284
Assets	3 732	65 082	129	249	510	1 325	8 595
Number of employees	20.8	137.6	2.0	4.0	7.0	15.0	60.0
Age (firm)	17.5	13.0	3.0	8.0	14.0	24.0	43.0
Total debt / Asset	0.66	0.33	0.14	0.44	0.67	0.85	1.16
Financial debt / Asset	0.19	0.21	0.00	0.00	0.12	0.31	0.61
Trade credit / Asset	0.19	0.19	0.00	0.05	0.14	0.28	0.59
Liab. to the gov. / Asset	0.05	0.07	0.00	0.01	0.03	0.06	0.18
Liab. to related parties / Asset	0.23	0.25	0.00	0.05	0.14	0.32	0.74
Median age (managers)	46.6	9.7	31.0	40.0	46.0	53.5	63.5
Median age (employees)	39.6	7.9	27.0	34.0	39.0	44.5	53.0
Share higher educ. (managers)	0.27	0.43	0.00	0.00	0.00	0.50	1.00
Share higher educ. (employees)	0.12	0.24	0.00	0.00	0.00	0.14	0.68
Share female (managers)	0.27	0.40	0.00	0.00	0.00	0.50	1.00
Share female (employees)	0.41	0.35	0.00	0.08	0.33	0.68	1.00
Capex/Asset	0.07	0.16	0.00	0.00	0.01	0.06	0.34
$\Delta \ln$(Inventory)	0.02	0.63	-0.99	-0.10	0.00	0.15	1.09
$\Delta \ln$(Number of employees)	0.04	0.27	-0.37	-0.03	0.00	0.12	0.56
CFO / Asset	0.06	0.20	-0.26	-0.02	0.06	0.15	0.40
Net capital injection / Asset	0.00	0.09	-0.09	0.00	0.00	0.00	0.09
(ΔTotal Debt)/Asset	0.09	0.35	-0.25	-0.07	0.01	0.13	0.66

TABLE 1. Descriptive statistics.

Notes: All variables are winsorized at the 1% and 99% yearly values. All statistics are computed after the application of the filters. Whenever a firm is liquidated (born), we annualize the last (first) year of sales to account for the number of days in operation. All monetary values are stated at 2021 prices (thousands of euros). All variables are in natural units. All age-related variables are measured in years and correspond to the firm-level median value. Financial debt corresponds to the sum of debt in CRC and debt in SIET. Liabilities to related parties are computed as total liabilities minus financial debt, liabilities to the government and trade credit. We consider as workers all those that are full-time paid. See footnote 9 for a detailed explanation on how managers are identified. Capex is computed as the difference between fixed assets in two consecutive years plus depreciations. CFO refers to the cash flow from operating activities.

observed in the case of capex and the log-change in the number of employees. Regarding financing variables, the cash flow from operating activities (CFO) and the net capital injection distributions are symmetric around their median/mean values. While CFO is positive on average, the net capital injection is on average close to zero with shareholders injecting capital (receiving net dividends) in 39% (50%) of the observations. In contrast, the change in total debt distribution is right-skewed. While the number of positive and negative changes is very similar, debt increases tend to be significantly larger.

4. Econometric model

In this article, we aim at measuring the forecasting power of the most cited corporate growth determinants. For this reason, we split the dataset in two. The first ten years (i.e. growth rates between 2008 and 2017) are used to estimate the model. The remaining four years (i.e. growth rates between 2018 and 2021) are used to evaluate its performance. Our first sales growth rate corresponds to the log-change in sales between 2007 and 2008. The year of 2006 is lost to account for autocorrelation in sales growth and to compute investment and financing-related variables. To ease comparison with other studies in the corporate growth literature we opt to use a linear model. As several other articles in the literature (e.g. Bottazzi *et al.* (2011) and Coad (2007a,b) we estimate the model using least absolute deviation (LAD) instead of ordinary least squares (OLS). Our choice is motivated by better forecasting performance and results presented in other studies in the literature, which have found that LAD performs better than OLS when the data shows fat tails and is not symmetric, two common characteristics in sales growth datasets.

Table 2 presents our econometric specification, where we denote the growth rate of firm i at year $t+1$ as Δ ln(Sales$_{i,t:t+1}$). In addition to the coefficient estimates and the respective significance level, we present the difference between the coefficient of determination with and without each variable and its category based on the Koenker and Machado (1999) measure. The latter is preferred over the traditional R^2 for theoretical consistency reasons. A positive value in each row indicates that the variable/category contributes to better explain the variation in sales.

We organize the determinants in the literature around three types, namely firms' characteristics (size, age, access to external funding, leverage and debt composition, relative fit, ownership structure, access to foreign markets, and human capital characteristics), firm-specific dynamics (autocorrelation, lagged investment and lagged financing), and external environmental dynamics (industry, local, and macroeconomic aggregates growth rates). In the latter case, observed contemporaneous values are used to estimate the model and then replaced by forecasts to compute the model predictions. In line with the literature, we consider heterogenous effects by sector of activity in the case of size and sector of activity and size in the case of autocorrelation. We use the highest level of aggregation in NACE, which comprises 17 sectors in our dataset. Trade, which is the most populated sector, is the omitted category in the econometric output. For simplicity, Table 2 presents only the coefficients on those sectors of activity which represent more than 5% of the observations. We do not include any fixed effects. Despite the high number of variables, except for the leverage ratio for which we include a squared term, all other variables show a generalized variance inflation factor (VIF) below 5 after adjusting for the degrees of freedom using the standard formula and squaring back the term. Most terms present a value close to 1 suggesting a low risk of multicollinearity. The null hypothesis of non-stationary is rejected when applying the IPS unit root panel test.

With a few exceptions, all terms are found to be statistically significant whenever we compute standard errors using the Huber sandwich estimate implemented in the `rq` function in `R`. The model has a coefficient of determination of 0.081 based on Koenker and Machado (1999) measure and 0.121 based on the more traditional R^2. The same model estimated using OLS has an R^2 equal to 0.156. The goodness of fit measures obtained are in line with those presented in the literature for SMEs.[10] Whenever considering the forecasted environmental variables, something that it is not done by the referred goodness of fit measures, we observe a correlation across time of 64% (i.e.

[10] As a robustness check of whether our results are biased by the removal of the initial and final years of the considered firms, we reestimate the model adding back those observations with less than 100 thousand euros of sales and assets (real terms) of those firms that enter the final dataset and still fulfil all other restrictions (11% of the final database). We do not find any material difference in the estimated coefficients.

between the yearly median observed and predicted sales growth rate) and a correlation on the cross-section of 31% (i.e. between firms' median observed and predicted growth rates). Removing in turn all variables related with firms' characteristics, firm-specific dynamics and the external environment lead the Koenker and Machado (1999) measure to decrease by 0.008, 0.02 and 0.045, respectively.

4.1. Firms' characteristics

We measure size and age using the logarithm of sales and age, as it is common in the literature. As expected, we find a negative relation with sales growth in both cases. Taking the trade sector as reference, firms in the 5th percentile of sales and age distributions are found to grow 2.3 and 0.9 percentage points more than firms in the 95th percentile. The gap between the growth rate of small and large firms is typically higher in sectors of activity with higher capital intensity.[11] Though significant, size and age have a negligible contribution to improve the model fit.

Access to external funding is measured through two dummy variables stating whether the firm has access to bank loans and the debt securities market, respectively. A firm is considered to have access to bank loans whenever it has credit in CRC (granted or potential) with original maturity above one year. The firm has access to the debt securities market if it has any amount due in SIET. Both terms are positive and statistically significant, but have almost no explanatory power.

Our results point for a concave relation between leverage, measured by the ratio between total debt and assets, and sales growth. Leverage contribution depends however on debt composition. In particular, the lower the share corresponding to financial debt and liabilities to the government and the higher the share of credit granted by related parties, the higher the sales growth rate. The positive relation with loans granted by related parties suggests that this type of credit has some characteristics in common with equity. Whenever the share of financial debt, liabilities to the government and liabilities to related parties equals the average values in the dataset a positive net effect is obtained for leverage ratios up to 75%. For this firm, the highest net effect is observed for leverage ratios close to 40% and equals 0.5%. The dummy variable indicating whether the firm has overdue loans has a sizable negative coefficient and taken alone is the firm characteristic that contributes the most to improve the model fit. The leverage ratio and debt composition have also a relevant explanatory power.

We measure profitability and productivity using the firm rank within its industry and year (see industry definition in section 4.3). For profitability, we use the ratio of earnings before interest taxes and depreciations (EBITDA) and assets. For productivity, we consider the cost shares approach presented in Foster *et al.* (2016). In line with the theory, we find a sizable positive relation between profitability and sales growth. Firms in the 95th percentile grow 3.2 percentage points more than firms in the 5th percentile. In contrast to theory, we find a negative relation between the productivity rank and sales

[11] We find a correlation of -0.33 (0.27) between the median capital (labour) weight across the 17 sectors of activity and the estimated sector-specific coefficients.

Dependent variable: Δ ln(Sales$_{i,t:t+1}$)			Coefficient	Add. fit by variable	Add. fit by category
Intercept			0.1315***		
Firms' characteristics	Size	ln(Sales$_{i,t}$)	-0.0055***	0.000	0.000
		Manufacturing*ln(Sales$_{i,t}$)	-0.0002***	0.000	
		Construction*ln(Sales$_{i,t}$)	0.0003***		
		Acc. and food*ln(Sales$_{i,t}$)	0.0005***		
		Consulting*ln(Sales$_{i,t}$)	-0.0003***		
	Age	ln(Age$_{i,t}$)	-0.0034***	0.000	0.000
	Access to external funding	AccessBankLoans$_{i,t}$	0.0026***	0.000	0.000
		AccessDebtSecurities$_{i,t}$	0.0036*	0.000	
	Leverage and debt composition	TotalDebt$_{i,t}$/Asset$_{i,t}$	0.0341***	0.001	0.005
		(TotalDebt$_{i,t}$/Asset$_{i,t}$)^2	-0.0366***		
		FinDebt$_{i,t}$/Asset$_{i,t}$	-0.0085***	0.000	
		LiabGovernment$_{i,t}$/Asset$_{i,t}$	-0.1066***	0.000	
		LiabRelatedParties$_{i,t}$/Asset$_{i,t}$	0.0204***	0.000	
		FinancialDebtOverdue$_{i,t}$	-0.1035***	0.002	
	Relative fit	ProfitabilityRank$_{i,t}$	0.0353***	0.001	0.001
		ProductivityRank$_{i,t}$	-0.0198***	0.000	
	Ownership structure	EconomicGroup$_{i,t}$	0.0182***	0.001	0.001
		WorkingEmployer$_{i,t}$	0.0042***	0.000	
	Access to foreign markets	Exporter$_{i,t}$	0.005***	0.000	0.000
	Human capital characteristics	AgeManagers$_{i,t}$	-0.0007***	0.000	0.001
		AgeEmployees$_{i,t}$	-0.0007***	0.000	
		HigherEducationManagers$_{i,t}$	-0.0007	0.000	
		HigherEducationEmployees$_{i,t}$	0.0079***	0.000	
		FemaleManagers$_{i,t}$	-0.004***	0.000	
		FemaleEmployees$_{i,t}$	-0.0075***	0.000	
Autocorrelation, lagged investment and lagged financing	Autocorrelation	Δ ln(Sales)$_{i,t-1:t}$	0.0416***	0.000	0.003
		Small*Δ ln(Sales)$_{i,t-1:t}$	0.0222***	0.000	
		Medium*Δ ln(Sales)$_{i,t-1:t}$	0.0537***		
		Large *Δ ln(Sales)$_{i,t-1:t}$	0.0527***		
		Manufacturing *Δ ln(Sales)$_{i,t-1:t}$	-0.0871***	0.003	
		Construction *Δ ln(Sales)$_{i,t-1:t}$	-0.204***		
		Acc. and food *Δ ln(Sales)$_{i,t-1:t}$	-0.0378***		
		Consulting *Δ ln(Sales)$_{i,t-1:t}$	-0.0811***		
	Lagged investment	Capex$_{i,t}$/Asset$_{i,t-1}$*$1_{(Capex_t>0)}$	0.0587***	0.001	0.009
		Capex$_{i,t}$/Asset$_{i,t-1}$*$1_{(Capex_t<0)}$	0.3691***		
		Δ ln(Inv$_{i,t-1:t}$)*$1_{\Delta Inv_{i,t-1:t}>0}$	0.0326***	0.005	
		Δ ln(Inv$_{i,t-1:t}$)*$1_{\Delta Inv_{i,t-1:t}<0}$	0.0508***		
		Δ ln(NumEmp$_{i,t-1:t}$)*$1_{\Delta NumEmp_{i,t-1:t}>0}$	0.0541***	0.003	
		Δ ln(NumEmp$_{i,t-1:t}$)*$1_{\Delta NumEmp_{i,t-1:t}<0}$	0.1133***		
		AdvertisementRank$_{i,t}$	0.0257***	0.000	
	Lagged financing	CFO$_{i,t}$/Asset$_{i,t}$	-0.0201***	0.000	0.002
		NetCapInj$_{i,t}$/Asset$_{i,t-1}$*$1_{NetCapInj_{i,t}>0}$	0.1039***	0.000	
		NetCapInj$_{i,t}$/Asset$_{i,t-1}$*$1_{NetCapInj_{i,t}<0}$	0.0208***		
		Δ Debt$_{i,t-1:t}$/Asset$_{i,t-1}$*$1_{\Delta Debt_{i,t-1:t}>0}$	0.0319***	0.002	
		Δ Debt$_{i,t-1:t}$/Asset$_{i,t-1}$*$1_{\Delta Debt_{i,t-1:t}<0}$	0.137***		

Dependent variable: Δ ln(Sales$_{i,t:t+1}$)			Coefficient	Add. fit by variable	Add. fit by category
External environment	Industry	g_Ind$_{j,t:t+1}$-g_GDP$_{t:t+1}$	0.965***	0.000	0.020
		Small$_{i,t}$*(g_Ind$_{j,t:t+1}$-g_GDP$_{t:t+1}$)	-0.987***	0.000	
		Medium$_{i,t}$*(g_Ind$_{j,t:t+1}$-g_GDP$_{t:t+1}$)	-0.1057***		
		Large$_{i,t}$*(g_Ind$_{j,t:t+1}$-g_GDP$_{t:t+1}$)	-0.1121***		
	Local	g_Local$_{h,t:t+1}$-g_GDP$_{t:t+1}$	0.6149***	0.000	0.002
		Small$_{i,t}$*(g_Local$_{h,t:t+1}$-g_GDP$_{t:t+1}$)	-0.1698***	0.000	
		Medium$_{i,t}$*(g_Local$_{h,t:t+1}$-g_GDP$_{t:t+1}$)	-0.3453***		
		Large$_{i,t}$*(g_Local$_{h,t:t+1}$-g_GDP$_{t:t+1}$)	-0.635***		
	Macro	ShareDom$_{i,t}$*g_DD$_{t:t+1}$	0.5244***	0.000	0.004
		ShareExp$_{i,t}$*g_Exp$_{t:t+1}$	0.0652***		
		Small$_{i,t}$*ShareDom$_{i,t}$*g_DD$_{t:t+1}$	0.0537***	0.000	
		Medium$_{i,t}$*ShareDom$_{i,t}$*g_DD$_{t:t+1}$	0.0563***		
		Large$_{i,t}$*ShareDom$_{i,t}$*g_DD$_{t:t+1}$	0.0987***		
		Small$_{i,t}$*ShareExp$_{i,t}$*g_Exp$_{t:t+1}$	0.1209***		
		Medium$_{i,t}$*ShareExp$_{i,t}$*g_Exp$_{t:t+1}$	0.1554***		
		Large$_{i,t}$*ShareExp$_{i,t}$*g_Exp$_{t:t+1}$	0.2218***		

TABLE 2. Determinants of firms' growth rates (1-year ahead).

Notes: *** indicates that the variable is significant at 1% level. ** indicates that the variable is significant at 5% level. * indicates that the variable is significant at 10% level. The standard errors are computed using the Huber sandwich estimate implemented in the rq function in R. All monetary variables are deflated using the consumer price index. Trade is the omitted sector when interacting sector of activity with the logarithm of sales. Micro firms in the trade sector is the omitted group when interacting sector of activity with the autoregressive term. For simplicity, we do not present those sectors of activity which represent less than 5% of the observations (12 sectors). AccessBankLoans and AccessDebtSecurities are dummy variables taking 1 if the firm has at least 50 euros in credit (granted or potential) with an original maturity above one year in December of each year and the amount due in SIET is higher than zero, respectively. FinancialDebtOverdue is a dummy variable taking 1 when the firm has credit overdue of at least 100 euros in December of each year. HigherEducationManagers/HigherEducationEmployees and FemaleManagers/FemaleEmployees correspond to the share of managers and employees with higher education and female gender, respectively. ShareDom and ShareExp correspond to the proportion of sales to domestic and foreign markets, respectively. g_DD and g_Exp correspond to the real growth rate of domestic demand and exports. The additional fit associated with ln(Sales$_{i,t}$), g_Ind$_{j,t:t+1}$-g_GDP$_{t:t+1}$ and the combination of ShareDom$_{i,t}$*g_DD$_{t:t+1}$ with ShareExp$_{i,t}$*g_Exp$_{t:t+1}$ is zero by definition due to the presence of heterogenous effects by size. If this was not the case, these variables would have a contribution roughly equal to the value presented in the category.

growth with firms in the 5$^{\text{th}}$ percentile growing 1.8 percentage points more than firms in the 95$^{\text{th}}$ percentile. This finding is robust to the consideration of other productivity measures such as labour and capital productivity and it is consistent with the findings in Banco de Portugal (2019). Profitability and productivity have nevertheless a modest explanatory power.

Belonging to an economic group is significant and has a sizable impact on the sales growth rate with firms that belong to some economic group growing more 1.8 percentage points than others. Firms whose shareholders work for the firm grow 0.4 percentage points more. Assuming that this measure is a good proxy of whether the firm is controlled by its owners, our results go against the literature cited in section

2. Exporting firms are found to grow 0.5 percentage points more than others. These variables have a negligible impact on the model fit.

In line with the literature, a negative relation with age and gender (female) is found both for employees and managers. When the median manager (employee) age equals 31 (27), firms are estimated to grow 2.2 (1.8) percentage points more than when it equals 64 (53). Also, an all-female firm is estimated to grow 0.4 and 0.8 percentage points less than an all-male firm depending on whether one considers management positions or other employees, respectively. Surprisingly, the share of higher education is found to be significant only in the case of employees. In this case, a firm where all employees have higher education is estimated to grow 0.8 percentage points more. Human capital characteristics contribute modestly to improve the model fit, even taken altogether. As a robustness check, we compare our results with the ones estimated considering a dataset up to 2013 (i.e. the last year we are able to merge our dataset with QP without taking additional assumptions) and similar coefficients are obtained.

4.2. Autocorrelation, lagged investment and lagged financing

We consider the effect of lagged sales growth rate with heterogenous effects by size and sector of activity. The omitted group, which corresponds to micro firms in the trade sector, shows a small positive autocorrelation coefficient (approximately 0.04). As expected, autocorrelation increases with firm size with medium and large firms presenting coefficients significantly higher than micro and small firms. Micro firms in roughly half of all sectors of activity exhibit negative autocorrelation. Autocorrelation is especially negative in the construction, real estate and agriculture sectors. Autocorrelation is higher and positive for all corporate sizes in the education, health and transportation and storage sectors. Overall, the obtained autocorrelation coefficients are in line with those typically presented in studies focusing on SMEs and considerably below the ones reported for large public companies. Autocorrelation has a relevant contribution to model fit.

We use several variables to proxy for investment, namely, the ratio of capex and lagged assets, the log-change in inventory, the log-change in the number of employees and the ratio of advertisement expenses and lagged assets, ranked by industry and year. Except for the latter, we distinguish the impact of increases and decreases with decreases having a stronger impact on sales growth. All these variables are statistically significant with the log-changes in inventory and the number of employees contributing to improve the model fit considerably. Firms in the 95^{th} percentile of these variables are estimated to grow 8.5 and 7.3 percentage points more than firms in the 5^{th} percentile, respectively.

We use 3 variables to measure the impact of financing variables, namely, the ratio of operating cash flows and lagged assets, the ratio of net capital injections and lagged assets and the ratio of changes in debt and lagged assets. All variables are found to be significant with the latter two variables exhibiting the expected positive signal and the ratio of operating cash flows and lagged assets showing a surprising negative impact. As for investment-related variables we distinguish the impact of increases and decreases. While positive net capital injections have a greater impact than negative net capital injections, the opposite occurs in the case of changes in total debt. This result should stem from firms typically exhibiting negative capital injections (reflecting dividend payments) and either positive or small negative changes in debt (reflecting new credit or gradual amortizations). A different pattern may signal confidence in the firms' ongoing projects, in the case of a positive capital injection, or financial constraints, in the case of negative changes in debt. In contrast with the literature, where the role of equity issuances is usually more emphasized, changes in debt stand out as the most important of the three variables. In this case, firms in the 95th percentile are found to grow 5.5 percentage points more than firms in the 5th percentile. This compares with 1.1 and -0.9 percentage points in the case of net capital injections and operating cash flows, respectively. Only lagged debt financing has a relevant contribution to the model fit.

4.3. External environment

We consider three categories of environmental variables, namely, the median industry sales growth rate in excess of the real gross domestic product growth rate (g_Ind$_{j,t:t+1}$-g_GDP$_{t:t+1}$), the median municipality sales growth rate in excess of the real gross domestic product growth rate (g_Local$_{h,t:t+1}$-g_GDP$_{t:t+1}$) and the real growth rate in two macroeconomic aggregates, namely domestic demand (g_DD$_{t:t+1}$) and aggregate exports (g_Exp$_{t:t+1}$). We interact these variables with corporate size. In addition, in the case of domestic demand and exports real growth rates, we interact these variables also with the firm-level share of domestic and foreign sales, respectively.

All external environmental variables are contemporaneous to our dependent variable. To avoid obtaining biased coefficients, we estimate the model using the actual values. Auxiliar models are developed to make predictions in the case of the industry and municipality growth rates (Online Appendix A) and official Banco de Portugal forecasts are used in the case of macroeconomic aggregates. The auxiliar industry and municipality models are simpler versions of the firm-level model where we consider only autocorrelation and the impact of the real growth rate in domestic demand and aggregate exports with heterogenous impacts by sector of activity and district, respectively. Macroeconomic forecasts correspond to those regularly published by Banco de Portugal in its December Economic Bulletin. In the case of 2020, the June Economic Bulletin forecasts are used to turn the model performance in this year, which is strongly affected by the surprising pandemic shock, more comparable with other years.[12]

The yearly industry median growth rate of firms' sales (g_Ind$_{j,t:t+1}$) is computed considering 313 industries. For most firms, the industry corresponds to the 5-digits NACE disaggregation. A cluster analysis algorithm is used to merge sector codes with less than 50 firms per year in all years with other sector codes at the 5-digits level.[13]

[12] Between December 2019 and June 2020, the forecasted growth rates of domestic demand and exports for 2020 fell from 2.6% to -8.2% and from 2.6% to -25.3%, respectively.

[13] The scclust algorithm (size-constrained clustering) is used to determine industries. All levels of NACE hierarchical structure are used as determinants to avoid that the algorithm unnecessarily merges very distant sectors. The sales median growth rate by year is used to guarantee a unique solution. Approximately 55% of the clusters correspond to a single 5-digits sector of activity.

Micro firms show the highest sensitivity to the industry with a coefficient equal to 0.96. This compares with approximately 0.86 for small, medium and large firms. The industry excess growth rate is the variable that contributes the most to improve the model fit.

The yearly municipality median sales growth rate (g_Local$_{h,t:t+1}$) is computed for the 308 municipalities composing the Portuguese territory. As expected, the impact of the municipality depends crucially on firms' size. While in the case of micro firms it is found to be around 0.6, in the case of large firms it is broadly zero. The municipality excess growth rate has a moderate contribution to improve the model fit.

Regarding macroeconomic aggregates, we find that corporate growth is more sensitive to domestic demand than to exports even before multiplying by the respective share on total sales. In the case of a large firm that exports half of its sales, sales growth increases roughly twice as much in the case of a 1 percentage point increase in domestic demand comparatively to the same increase in aggregate exports. While smaller firms are more affected by local and industry dynamics, larger firms are the most impacted by country-wide aggregates, in particular exports. The real growth rate of macroeconomic aggregates is among the variables that contribute the most to explain firms' sales growth.

5. Forecasting performance

Table 3 presents some descriptive statistics on the observed and predicted sales growth rates over the testing years. On average, the model overestimates firms' sales growth rates. The median predicted growth rate is nevertheless below the observed. This pattern is not observed in all years. In 2018 and 2019, the mean and the median predicted sales growth rates are relatively close to the corresponding observed values. In 2020, the year of the pandemic shock, on average, the model significantly overestimates sales growth rates, something that does not occur when considering the median. This overestimation occurs despite the forecasted growth rates in domestic demand and exports being lower than the observed and should be related with the unique characteristics of the pandemic shock, which affected some sectors in an unusual way.[14] The model underestimates sales growth rates in 2021, the year of the recovery from the pandemic shock, whenever considering both the mean and the median. This underestimation results in great part from the positive autocorrelation coefficient included both in the firm and in the auxiliary models, which pushes the predictions downwards for those firms, industries and municipalities most affected by the pandemic in the previous year. Observed values are significantly more dispersed than predicted and considerably more skewed to the left.

Table 4 presents the out-of-sample mean absolute deviation (MAD) and relative mean absolute deviation (RMAD) by year, age, size and sector of activity. The RMAD is computed as the ratio between the MAD of the model and the MAD of an alternative model that assumes that the growth rate of sales is the same for all firms. This alternative

[14] The June 2020 Banco de Portugal Economic Bulletin forecasts equal -8.2% for domestic demand and -25.3% for exports. These compare with observed growth rates of -4.0% and -22.6%, respectively.

		Mean	P50	SD	Skewness
2018	Observed	0.00	0.02	0.31	-1.69
	Predicted	0.03	0.03	0.07	0.08
2019	Observed	0.01	0.03	0.30	-1.55
	Predicted	0.02	0.02	0.06	0.03
2020	Observed	-0.17	-0.08	0.45	-1.65
	Predicted	-0.08	-0.07	0.09	-0.20
2021	Observed	0.09	0.09	0.33	-1.15
	Predicted	0.01	0.01	0.07	0.56
Total	Observed	-0.02	0.02	0.37	-1.69
	Predicted	0.00	0.00	0.08	-0.35

TABLE 3. Descriptive statistics on observed and predicted sales growth rates.

Notes: Whenever one considers the December 2019 Economic Bulletin forecasts, the mean predicted sales growth rate in 2020 equals 0.02 instead of -0.08. All values are in natural units.

model is compatible with the hypothesis made in several structural corporate finance models that the logarithm of sales follows a random walk with drift. For the same reasons presented for the main model, we estimate this growth rate using LAD instead of OLS obtaining -2% (OLS leads to -7.3%). This corresponds to the historical median between 2008 and 2017, which is lower than the one observed for the whole period. RMAD is particularly suited to make comparisons among groups of individuals because it controls for the level of dispersion within each group.

The model leads to a MAD of 0.223 (in natural units), a value 0.011 below the alternative model. This corresponds to a RMAD of 0.951, a value close to the one found whenever applying the model to the years between 2008 and 2017 (0.956). The root mean squared error (RMSE) equals 0.353, which compares with 0.369 in the alternative model (relative RMSE equals 0.957).[15] These results suggest that the determinants pointed in the literature have a modest forecasting power. This conclusion is not surprising given the low goodness of fit measures presented in section 4 and in the literature, which has led some authors to argue that firms' growth rate is largely random. The fact that the RMAD obtained is consistently below 1, both in-sample and out-of-sample, suggests however that forecasting firms' growth rates may be worthwhile. Unfortunately, we are not able to formally test whether the obtained RMAD is significant due to the low number of out-of-sample years.

To better evaluate the model performance, we compare the results with those obtained by an autoregressive model with order 1, AR(1) model, a simplified version of our model, where we consider only those variables that contribute to improve the forecasting performance and a naïve model where the predicted sales growth rate equals the previous year growth rate. The first two models are presented in the Online Appendix B. The AR(1) RMAD equals 1.006 whenever considering all years and slightly below 1 when excluding 2021. The simplified model RMAD equals 0.951, a value

[15]The same model and the alternative estimated using OLS lead, respectively, to a MAD of 0.235 and 0.232 (RMAD equals 1.013) and a RMSE of 0.353 and 0.37 (relative RMSE equals 0.956), all above or roughly equal the corresponding values estimated using LAD.

	Mean Absolute Deviation (MAD)	Relative Mean Absolute Deviation (RMAD)
Year		
2018	0.183	0.943
2019	0.182	0.938
2020	0.295	0.975
2021	0.230	0.938
Age		
Age<=5	0.298	0.941
Age>5 & Age<=20	0.224	0.952
Age>20	0.190	0.957
Size		
Micro	0.234	0.959
Small	0.212	0.941
Medium	0.195	0.933
Large	0.165	0.928
Sector of activity		
Agriculture	0.239	0.960
Mining	0.239	0.964
Manufacturing	0.201	0.948
Electricity	0.232	0.999
Water	0.246	0.969
Construction	0.284	0.956
Trade	0.181	0.954
Transportation and storage	0.209	0.940
Accommodation and food services	0.295	0.908
Information and communication	0.256	0.951
Real estate	0.375	0.960
Consulting	0.235	0.964
Administrative	0.318	0.931
Education	0.206	1.024
Health	0.182	1.021
Art and sports	0.392	0.961
Other services	0.193	0.976
Total	0.223	0.951

TABLE 4. MAD and RMAD by corporate year, age, size and sector of activity.
Notes: All values are in natural units.

roughly equal to the main model. The naïve model RMAD equals 1.43. Finally, as a robustness check, we reestimate the model using different firms in the training and test datasets with RMAD increasing by 0.001.

MAD is lower in the years before the pandemic likely reflecting a lower level of uncertainty. RMAD is similar in all years except 2020, where RMAD equals 0.975. Though higher than average, the latter is lower than 1.017, which is the value obtained when considering the forecasts in the December 2019 Economic Bulletin.

Likely reflecting their lower risk profile, the MAD is lower for older and larger firms. Firms in the health and trade sectors show the lowest MAD, while firms in the real estate and arts and sports sectors show the highest. RMAD is lower for younger and larger firms and in the case of firms in the accommodation and food services sector, followed by the administrative sector. The model is not able to beat the alternative model in the case of the education and health sectors.

Figure 1 presents a binscatter of predicted and observed sales growth rates. A binscatter is a type of scatter plot where the data is aggregated into bins to increase its readability. In this case, the mean (blue dots) and the median (red dots) value of the variable in the y-axis (observed sales growth rate) are presented for groups of observations determined based on the variable on the x-axis (predicted sales growth rate). The value presented in the x-axis corresponds to the average value for those observations. The observations associated with each point are determined based on the rank of the variable in x-axis so that each point represents the same number of observations. A 45 degrees line is added to help on the graph interpretation. Points above (below) the 45 degrees line suggest that the model is underestimating (overestimating) the observed sales growth rate. The predicted values are close to the average and median observed values for positive and slightly negative growth rates. For large negative sales growth rates predictions though, the observed growth rate is on average considerably lower than the predicted growth rate, something that does not occur in the case of the median. This result suggests that the distribution of observed sales growth rates is very asymmetric for those cases where the model predicts significant negative growth rates, likely from the model not being able to adequately predict extreme negative growth rates ending up not capturing well the significant skewness in the data.

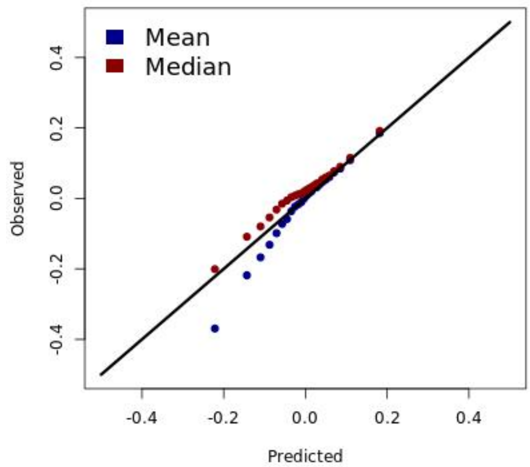

FIGURE 1: Binscatter of predicted and observed sales real growth rates (2018-2021).

Notes: The binscatter gives the mean (blue dots) and the median (red dots) value of the variable in the y-axis variable for groups of observations chosen based on the variable on the x-axis. The value presented in the x-axis corresponds to the average of those observations. All values are in natural units. The observations associated with each point are determined based on the rank of the variable in x-axis so that each of the 25 points represents the same number of observations (4% of all observations).

Table 5 explores the contribution of each variable and category to the RMAD. Each row corresponds to the difference between the RMAD with and without the corresponding variable and category. For simplicity reasons, we present only those variable categories which contribute to decrease RMAD by at least 0.001.

Dependent variable: Δ ln(Sales$_{i,t:t+1}$)			Additional RMAD by variable	Additional RMAD by category
Firms' characteristics	Leverage and debt composition	TotalDebt$_{i,t}$/Asset$_{i,t}$	0.000	-0.002
		(TotalDebt$_{i,t}$/Asset$_{i,t}$)^2		
		FinDebt$_{i,t}$/Asset$_{i,t}$	0.000	
		LiabGovernment$_{i,t}$/Asset$_{i,t}$	0.000	
		LiabRelatedParties$_{i,t}$/Asset$_{i,t}$	0.000	
		FinancialDebtOverdue$_{i,t}$	-0.002	
Lagged investment and lagged financing	Lagged investment	Capex$_{i,t}$/Asset$_{i,t-1}$*$1_{(Capex_t>0)}$	0.000	-0.009
		Capex$_{i,t}$/Asset$_{i,t-1}$*$1_{(Capex_t<0)}$		
		Δ ln(Inv$_{i,t-1:t}$)*$1_{\Delta Inv_{i,t-1:t}>0}$	-0.004	
		Δ ln(Inv$_{i,t-1:t}$)*$1_{\Delta Inv_{i,t-1:t}<0}$		
		Δ ln(NumEmp$_{i,t-1:t}$)*$1_{\Delta NumEmp_{i,t-1:t}>0}$	-0.003	
		Δ ln(NumEmp$_{i,t-1:t}$)*$1_{\Delta NumEmp_{i,t-1:t}<0}$		
		AdvertisementRank$_{i,t}$	-0.001	
	Lagged financing	CFO$_{i,t}$/Asset$_{i,t}$	0.000	-0.003
		NetCapInj$_{i,t}$/Asset$_{i,t-1}$*$1_{NetCapInj_{i,t}>0}$	0.000	
		NetCapInj$_{i,t}$/Asset$_{i,t-1}$*$1_{NetCapInj_{i,t}<0}$		
		Δ Debt$_{i,t-1:t}$/Asset$_{i,t-1}$*$1_{\Delta Debt_{i,t-1:t}>0}$	-0.003	
		Δ Debt$_{i,t-1:t}$/Asset$_{i,t-1}$*$1_{\Delta Debt_{i,t-1:t}<0}$		
External environment	Industry	g_Ind$_{j,t:t+1}$-g_GDP$_{t:t+1}$	0.000	-0.002
		Small$_{i,t}$*(g_Ind$_{j,t:t+1}$-g_GDP$_{t:t+1}$)	0.000	
		Medium$_{i,t}$*(g_Ind$_{j,t:t+1}$-g_GDP$_{t:t+1}$)		
		Large$_{i,t}$*(g_Ind$_{j,t:t+1}$-g_GDP$_{t:t+1}$)		
	Macro	ShareDom$_{i,t}$*g_DD$_{t:t+1}$	0.000	-0.013
		ShareExp$_{i,t}$*g_Exp$_{t:t+1}$		
		Small$_{i,t}$*ShareDom$_{i,t}$*g_DD$_{t:t+1}$	0.000	
		Medium$_{i,t}$*ShareDom$_{i,t}$*g_DD$_{t:t+1}$		
		Large$_{i,t}$*ShareDom$_{i,t}$*g_DD$_{t:t+1}$		
		Small$_{i,t}$*ShareExp$_{i,t}$*g_Exp$_{t:t+1}$		
		Medium$_{i,t}$*ShareExp$_{i,t}$*g_Exp$_{t:t+1}$		
		Large$_{i,t}$*ShareExp$_{i,t}$*g_Exp$_{t:t+1}$		

TABLE 5. Contribution to out-of-sample performance of explanatory variables.

Notes: A negative value implies an improvement to the model accuracy relative to the hypothesis that the growth rate of sales is the same for all firms. The additional RMAD of g_Ind$_{j,t:t+1}$-g_GDP$_{t:t+1}$ and the combination of ShareDom$_{i,t}$*g_DD$_{t:t+1}$ with ShareExp$_{i,t}$*g_Exp$_{t:t+1}$ are zero by definition due to the presence of heterogenous effects by size. If this was not the case, these variables would contribute to improve the model performance by roughly the value presented in the category. This justifies why these variables are included in the simplified model presented in the Online Appendix B. See variables definitions on notes to Table 2.

Most firms' characteristics have almost no impact on RMAD. The only exception is the flag variable indicating whether the firm has debt overdue. Whenever one excludes all firms' characteristics from the model, RMAD increases by 0.003.

Autocorrelation does not improve the RMAD, something that results from the detrimental contribution in 2021. Excluding this year, autocorrelation contributes to decrease RMAD by 0.002. All lagged investment variables have sizable contributions to decrease RMAD. The growth rate in inventory and the number of employees stand out as the most important contributing with -0.004 and -0.003, respectively. A similar contribution is found for the lagged variation in total debt, the only lagged financing variable that contributes to improve sales forecasts. RMAD increases by 0.014 whenever one excludes autocorrelation, lagged investment and lagged financing variables simultaneously.

External environmental variables stand out as the variables that contribute the most. Altogether they contribute to decrease RMAD by 0.038. The largest contribution comes from the macro variables, followed by the industry. Municipality growth rates are slightly detrimental to the model forecasts. Though positive to the model performance, the industry growth rate contribution is relatively modest compared to its contribution to explain observed growth rates, something that reflects the difference between observed and forecasted industry growth rates (see Figure A.1 in the Online Appendix B). The inclusion of heterogeneous impacts by size has a negligible impact in RMAD for all external environmental variables.

6. Conclusion

In this study, we evaluate the out-of-sample forecasting power of the sales growth determinants cited in the literature using a database composed mostly by micro and small firms. This is done using a linear regression model as in most papers in the empirical literature in industrial economics. The growth rates between 2008 and 2017 are used to estimate the model and those between 2018 and 2021 are used to evaluate its performance. Except for productivity, for which a negative relation is found, we confirm most of the results found in the literature based on the training dataset. This is the case of size, age (firm, employees, and managers) and gender (female, both for employees and managers), which show a negative relation with corporate sales growth, and access to external funding, profitability, belonging to an economic group, employees' education, lagged investment and financing flows, and external environment (industry, local and macroeconomic), which show a positive relation. The relation with leverage is concave and depends on debt composition. The effect of autocorrelation depends on the activity sector and it is typically positive for larger firms.

Our model leads to a mean absolute deviation of 0.223, 0.011 less than an alternative model that considers that the growth rate of sales is the same for all firms. This corresponds to a relative mean absolute deviation equal to 0.951. Our results suggest that the sales growth determinants pointed in the literature have a modest forecasting power. This conclusion is not surprising given the low in-sample fit of this type of models, which has led some authors to argue that firms' growth rate is largely random. The variables that contribute the most to improve sales growth forecasts are those related with the external environment, in particular the growth rate of domestic demand and exports. Except for investment-related variables, most other firm-level variables cited in the literature have a negligible forecasting power.

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