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DECEMBER 2022
The analyses, opinions and findings of these papers represent the views of the authors, they are not necessarily those of the Banco de Portugal or the Eurosystem.

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Lisboa, 2022 • www.bportugal.pt

Working Papers | Lisboa 2022 • Banco de Portugal Rua do Comércio 148 | 1100-150 Lisboa • www.bportugal.pt •
Edition Banco de Portugal • ISBN (online) 978-989-678-845-2 • ISSN (online) 2182-0422

The Labor Share and the Monetary Transmission

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Abstract
We show that the effectiveness of monetary policy changes with the labor income share. We do this in the context of a continuous time cash-in-advance model with heterogeneous agents and market segmentation. It turns out that the current price level depends on future interest rates through an integral equation. The solution of this integral equation reveals that, after an increase in interest rates, a larger income share implies larger reductions in money, prices and inflation. Monetary policy is more powerful in countries with a higher labor income share.

JEL: E3, E4, E5, C6

Keywords: monetary policy, labor income share, market segmentation, integral equation.

Acknowledgements: The views in this paper are those of the authors and do not necessarily reflect the views of the Banco de Portugal. We thank Zachary Bethune and Chao Gu for comments and discussion. This work was funded by Fundação para a Ciência e a Tecnologia (UID/ECO/00124/2019, UIDB/00124/2020 and Social Sciences DataLab, PINFRA/22209/2016), POR Lisboa and POR Norte (Social Sciences DataLab, PINFRA/22209/2016).

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1. Introduction

Many are the determinants of the monetary transmission mechanism. We find that the labor income share affects significantly the monetary transmission mechanism. As the labor income share has changed overtime and is different across countries it can explain the differences in the monetary transmission mechanism overtime and across countries.

The monetary transmission mechanism refers to the process through which monetary policy decisions such as changes in interest rate affect the economy. There are many studies that investigate the monetary transmission mechanism. For instance, Berben *et al.* (2004) find heterogeneity in the responses of 12 countries of the euro area to a monetary policy shock and conclude that economic, financial and structural differences across countries explain this diversity. Dedola e Lippi (2005) present evidence of cross-industry heterogeneity in the transmission of monetary policy for 21 manufacturing industries in 5 OECD countries (France, Germany, Italy, the UK and the US). They find that these patterns are related to industry output durability, investment intensity and the interest payment burden of firms. Moreover, regions inside the same country can have different monetary transmission mechanisms. For instance, Kouparitsas (2001) estimates that 2 out of the 8 U.S. Bureau of Economic Analysis regions have different responses to common shocks.

Adão *et al.* (2004) consider three types of restrictions on agent decisions in a standard dynamic general equilibrium model. Agents may be restricted in the setting of prices, wages, or in the choice of portfolio composition. Each of these restrictions corresponds to a different transmission mechanism. The severity of the restrictions determines the strength of the transmission mechanism. For instance, when monopolistic competitive firms are restricted in the way they set prices, an unanticipated monetary injection can raise production. This increase in output is larger the smaller is the fraction of firms that can adjust freely their prices. Similarly, changes in output will be larger if the fraction of agents that can freely adjust their portfolios is smaller. Thus, if countries have different degrees of price stickiness, or different degrees of financial participation, then they will have different monetary transmission mechanisms.

Here, we focus on the liquidity of the payments to the production factores and endogenous market segmentation. To study the relation between the labor income share and monetary transmission mechanism, we use the model in Silva (2012). This model combines the transactions demand for money models in Baumol (1952) and Tobin (1956) with the general equilibrium market segmentation models in Grossman e Weiss (1983), Rotemberg (1984), and Alvarez *et al.* (2009). The non-neutrality of money is an implication of the fact that at any given moment only a fraction of the agents are in the asset market. And the strength of the monetary policy is a function of the size of the fraction of agents in the market. We obtain, every thing else the same, that at any moment the fraction of agents in the market is smaller, the larger is the labor income share.

The payments to labor have a higher liquidity as they are made at a higher frequency than the payments to capital. Wages and salaries tend to be paid often, weekly or monthly, while dividends are typically paid once a year. As a result, economies with a higher labor income share have a higher proportion of income that is automatically available for spending. This impacts on the monetary transmission mechanism, as other things being equal, it increases the size of the period the agents are not in the asset market. In other words, it decreases the number of agents that are at a given time in the asset market, i.e. increases market segmentation. In this context, an unanticipated increase in the nominal interest rate implies larger decreases in the price level, inflation, and money, if the labor income share is larger.[1]

Attaining these results requires solving a second-order nonlinear integral equation, which we transform into a second-order ordinary differential equation. There are few examples in economics where the equilibrium is the solution of an integral equation. A first-order linear integral equation appears in Edmond (2008). As in Edmond, our integral equation appears as an equilibrium condition when we aggregate the agents in the economy. On the other hand, in addition to the aggregation across agents, the integral appears in our case because each agent needs to take into account the future path of interest rates. We have not found any higher-order integral equation in the literature. Another difference is that we do not rely on numerical methods to solve the integral equation. We reduce the integral equation into a differential equation and solve the corresponding differential equation.

The rest of the paper is organized as follows. Section 2 describes the labor income share distribution across countries and its relation with money. Section 3 describes the model. Section 4 characterizes the steady-state equilibrium. Section 5 characterizes the equilibrium after a monetary policy shock. Sections 6 and 7 present the responses of macroeconomic variables to a permanent and temporary shocks to the nominal interest rate. Section 8 concludes.

2. The distribution of labor shares across countries

National income is the sum of all income available to the residents of a given country in a given year. The labor income share is the ratio between labor income and aggregate income, hence it corresponds to the part of national income allocated to labor compensation. Different measures of labor share have been used in the literature (for instance, Bridgman (2018), Guerriero (2019), Karabarbounis e Neiman (2014)). Usually, the measures differ in the way they impute a wage component to self-employment income. Guerriero (2019) compares six measures of labor share for 151 countries for the period 1970–2015. The results show that the

1. Adão e Silva (2020) describe a related result. They argue that the historical downward trend in the nominal interest rate increased market segmentation, which ceteris paribus strengthened monetary policy.

six measures are highly correlated and there is a substantial heterogeneity across countries for all measures.

The Penn World Table includes the computation of the labor income share for 138 countries over time. Figure 1 shows the labor shares and the logarithm of income per capita for all counties in 2019. There is no clear relation between the variables. The labor share varies between 0.81 and 0.16, with an mean of 0.51 and a standard deviation of 0.12. There is a dispersion of labor shares even between developed countries. Canada and Switzerland have a labor share of 0.65 and 0.68 respectively, while Ireland, Italy and Norway have labor shares of 0.32, 0.52, and 0.53, respectively.

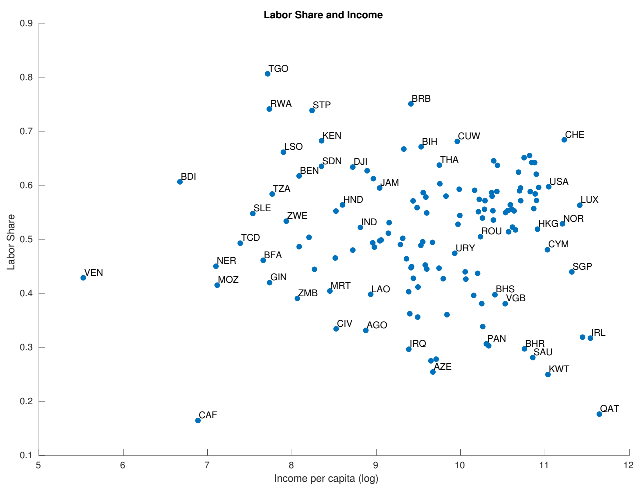


Figure 1: Labor shares and income per capita for 138 countries. Values for 2019. Source: Penn World Table.

Figure 2 shows the evolution of the mean labor share for 34 OECD countries between 1970 and 2019. It shows as well the evolution of the labor share of the United States for reference. For the majority of the countries, the labor share decreased during the period, which is reflected in a decreasing average for the OECD. The standard deviation shows that there is a significant heterogeneity in the labor shares of the OECD countries. This is in accordance with the literature, which documents the existence of considerable heterogeneity across economies (Guerriero (2019)) and a general decline in the labor share in the world, in particular since the mid-1980s (for instance, Karabarbounis e Neiman 2014 and Elsby *et al.* 2013).[2]

2. The dataset used adjusts for the labor income or proprietors, which addresses a possible underestimation of the labor share because of the attribution of proprietor's income to capital instead of to labor, as stated by Gollin (2002). The decreasing trend of the labor share could become steeper when accounting for sole proprietorship because of the decrease in the proportion

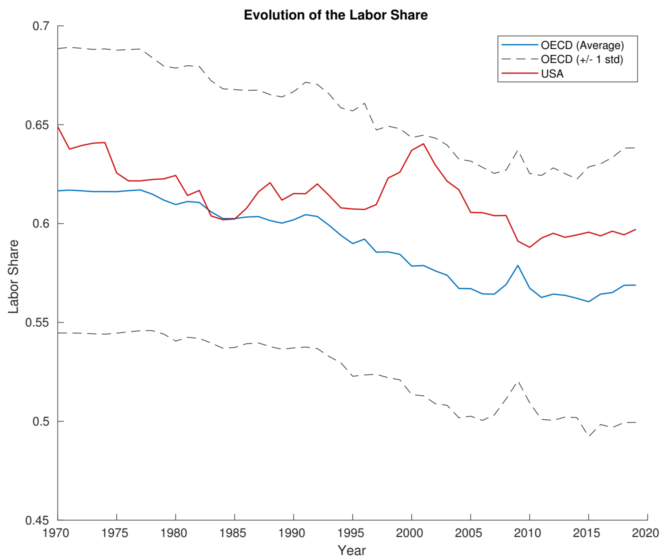


Figure 2: Evolution of the labor shares in the OECD. Source: Penn World Table.

There is literature on the determinants of the labor share. For instance, Heckscher and Ohlin (Ohlin 1933) argued that countries specialize in the production of goods that are intensive in the factor where they have a relatively higher endowment. Accordingly, a country that is relatively abundant in labor will have a larger labor share. Other models provide alternative ways in which international trade may affect labor shares. The higher competition due to international trade may decrease markups of prices and profits. This would result in higher real wages and an increase in the labor share. On the other hand, Rodrik (1998) argues that reduced barriers to trade benefit capital owners and highly skilled workers, as those factors of production can more easily cross borders.

There are other determinants of the labor share. For instance, according to Karabarbounis e Neiman (2014), the trend in capital-augmenting technological progress can explain the declining trend in labor share. Koh *et al.* (2020), Chiavari e Goraya (2020) and Zhang (2019) put the rise in intangible capital at the core of this process, albeit through different mechanisms: changes in the accounting procedure related to intellectual property, frictions in the allocation of intangible capital, and the decrease in investment costs for intangible capital coupled with an elasticity of substitution above one in the production function, respectively. Kehrig e Vincent (2021) find that changes in the composition of capital are responsible

of small and medium sized business. This, however, is contested by Gutiérrez e Piton (2020), who argues that the harmonized series remain stable or increase in all major advanced economies except the United States and Canada.

for the labor share decline: an aggregate reallocation of value added towards the lower end of the labor share distribution is responsible for the pattern. Autor *et al.* (2017), De Loecker e Eeckhout (2018) and De Loecker *et al.* (2021) relate industry concentration (in the products market) with a decrease in the aggregate labor share. Azar *et al.* (2017) find that concentration also increases labor market power, which represents another explanatory mechanism for the decrease in the labor share. Deb *et al.* (2020) compare monopsony (in the labor market) and monopoly (in the product market) as explanations for real wage stagnation and find that the dominant force is monopoly. More broadly, Bergholt *et al.* (2022) consider four possible mechanisms - wage markups, price markups, automation and investment specific technology - which include almost all of those previously mentioned, to find which contributed the most for the observed changes. They find that most of the pre-crisis decline in the labor share can be attributed to automation, but the rising market power of firms became the main source since the Great Recession.

We do not provide an alternative explanation for the labor share or its evolution. We simply study whether the labor share is an important determinant of the monetary policy transmission mechanism. The link between monetary policy and the labor share over the business cycles has been studied by Cantore *et al.* (2020), who showed that monetary policy tightenings increase the labor share and decrease real wages, raising issues about the compatibility between New Keynesian models and the data. We look at how permanent changes in the labor share impact monetary policy.

To study the impact of the labor share on the transmission mechanism of monetary policy, we consider that the fraction of personal income that is received as wage and salary disbursements is automatically deposited into household bank accounts. These deposits, either demand or time deposits, are included in the monetary aggregates, and can be easily accessible for consumption purposes. On the other hand, in general, the payments to the other production factors are made with lower frequency, and are less accessible to be used for consumption. In the model specified in the next section, we assume that households have two types of accounts, a banking account and a brokerage account. The payments to labor are deposited in the banking account and the payments to the other production factors are deposited in the brokerage account.

The interpretation is that the bank accounts in the model represent retail banking, whereas the brokerage accounts represent those institutions where households hold securities like stocks, bonds and mutual funds. They correspond to the array of actual brokerage accounts, mutual funds, pension funds, and life insurance reserves. We follow Alvarez *et al.* (2009) in this distinction between bank and brokerage accounts. According to Alvarez *et al.*, U.S. households pay a substantial cost in terms of foregone interest to hold assets in retail banks relative to short-term Treasury securities. This cost is in the order of two percentage points. At the same time, the distinction between demand deposits in M1 and the components of M2 is unimportant as there is no significant difference in the opportunity cost of those different types of deposits.

3. The model

There is a continuum of infinitely-lived households with measure one. There is an asset market and a goods market. There are two assets: money and bonds. Bonds are traded in the asset market and goods are traded in the goods market. Only money can be used to buy goods. The government sets the nominal interest rate and performs open market operations.

Each household has a brokerage account and a bank account. The brokerage account is used to manage the activities in the asset market and the bank account is used to manage the activities in the goods market. The financial frictions show up when households transfer resources between the asset market and the goods market. The size of the financial frictions is given by the transfer cost, Γ, that the agents need to pay whenever they transfer resources between the brokerage account and the bank account. The transfer cost is paid with the resources in the brokerage account and it is independent of the volume transferred.

Time is a continuous variable, $t \geq 0$. It is convenient that time is continuous as it avoids integer constraints on the timing of the transfer decision. At $t = 0$, each household has M_0 money holdings in the bank account and B_0 in bonds in the brokerage account. Households are indexed by their initial holdings of money and bonds, $s = (M_0, B_0)$. There is a distribution F of s.

Each household s has three members, a worker, a capital owner and a shopper, as in Lucas (1990). The capitalist and the worker together produce $Y(t)$ goods.[3] The goods are sold for money at the price $P(t)$. A fraction of revenues, $\alpha P(t)Y(t)$, is paid to the worker and is deposited in the bank account. The remaining fraction, $(1-\alpha)P(t)Y(t)$, is paid to capital owners and is deposited in the brokerage account. The parameter α, $0 \leq \alpha \leq 1$, is the labor income share.[4]

As money deposited in the brokerage account cannot be used to buy goods and does not receive interest, it is immediately used by the shopper to buy bonds. Periodically the shopper transfers cash from the brokerage account to the checking account. If such a transfer is made, the shopper has to sell the required quantity of bonds. The shopper uses the available cash in the bank account to buy goods in the goods market. The consumption of goods is shared by the three members of the household. Thus, household s decides consumption, $c(t,s)$, the timing of transfers, $T_j(s)$, $j = 1, 2, \ldots$, money and bond holdings, $M(t,s)$ and $B(t,s)$, and the transfers amounts between the two accounts, $z(t,s)$.

Define a holding period as an interval (T_j, T_{j+1}), $j = 1, 2, \ldots$ Let $r(t)$ denote the nominal interest rate at time t. During a holding period, bond holdings in the

3. As in Silva (2012) we could have considered instead that $Y(t)$ is produced according to a Cobb-Douglas production function $AK(t)^{(1-\alpha)}L(t)^{\alpha}$, where $K(t)$ and $L(t)$ are the stock of capital and labor hours, respectively. The results would not change but the analysis would be more cumbersome.

4. Typically in the literature α denotes the capital share but here it denotes the labor share, because it is shorter notation than the alternative, $(1-\alpha)$.

brokerage account follow the equation

$$\dot{B}(t,s) = r(t)B(t,s) + (1-\alpha)P(t)Y, \ t \geq 0, \ t \neq T_1(s), \ T_2(s), \ \ldots, \qquad (1)$$

where $\dot{x}$ is the derivative of x with respect to time. If there are no transfers, bond holdings in the brokerage account increase, as the household simply accumulates the interest and the fraction $(1-\alpha)$ of the income. Let $B^-(T_j(s), s)$ represent bond holdings just before a transfer at $t = T_j(s)$ and $B^+(T_j(s), s)$ represent bond holdings just after the transfer, $B^-(T_j(s), s) = \lim_{t \to T_j, t < T_j} B(t,s)$, and $B^+(T_j(s), s) = \lim_{t \to T_j, t > T_j} B(t,s)$. At $t = T_1(s), T_2(s), \ \ldots,$ the constraint on the brokerage account is

$$z(T_j(s), s) + P(T_j(s))\Gamma = B^-(T_j(s), s) - B^+(T_j(s), s), \qquad (2)$$

When there is a positive transfer to the bank account, $z(T_j(s), s) > 0$, and $B^-(T_j(s), s) > B^+(T_j(s), s)$, i.e. bond holdings decrease after the transfer.

During a holding period, money holdings in the bank account follow

$$\dot{M}(t,s) = \alpha P(t)Y(t) - P(t)c(t,s), \ t \geq 0, \ t \neq T_1(s), \ T_2(s), \ \ldots \qquad (3)$$

Thus, if there are no transfers, money holdings decrease with goods purchases and increase with the income transfers from sales.

Analogously to the definitions for bond holdings, let $M^+(T_j(s), s)$ and $M^-(T_j(s), s)$ denote money holdings just after a transfer and money holdings just before a transfer. We have $z(T_j(s), s) \equiv M^+(T_j(s), s) - M^-(T_j(s), s)$. If the transfer, $z(T_j(s), s)$, is positive, then $M^+(T_j(s), s) > M^-(T_j(s), s)$. For the transfer dates, the money holdings obey the equation,

$$\dot{M}(T_j(s), s)^+ = \alpha P(t)Y(t) - P(t)c^+(T_j(s), s), \ t = T_1(s), \ T_2(s), \ \ldots \qquad (4)$$

where $\dot{M}(t,s)^+$ is the corresponding right time derivative for cash and $c^+(T_j(s), s)$ is consumption just after the transfer.

Household s makes transfers $z(T_j(s), s)$ so that the resulting money holdings $M^+(T_j(s),\, s)$ covers purchases during holding period (T_j, T_{j+1}) and allows money holdings at the end of the period to be $M^-(T_{j+1}(s), s), s)$. Equation (3) implies

$$M^+(T_j(s), s) = \int_{T_j(s)}^{T_{j+1}(s)} [P(t)c(t,s) - \alpha P(t)Y(t)]dt + M^-(T_{j+1}(s), s) \qquad (5)$$

for $j = 1, 2, \ldots$ Household $s = (M_0, B_0)$ starts with M_0 money holdings and these balances for consumption purchases until the first transfer, at $T_1(s)$. For the first holding period $[0, T_1(s))$, the condition equivalent to (5) is

$$M_0 = \int_0^{T_1(s)} [P(t)c(t,s) - \alpha Y]dt + M^-(T_1(s), s). \qquad (6)$$

It can be the case that the household chooses to make the first transfer at $t = 0$. For example, if $a = 0$ and $M_0 = 0$. In this case, $T_1(s) = 0$, and $M^-(T_1(s), s) = 0$.

Let $Q(t)$ denote the price at time zero of a bond that pays one dollar at time t. Given the nominal interest rate $r(t)$, then $Q(t) = e^{-\int_0^t r(s)ds} \equiv e^{-R(t)}$, where $R(t) \equiv \int_0^t r(s)ds$. Using (1), we can write $B^-(T_j(s), s)$ as a function of the interest payments accrued during $[T_{j-1}, T_j)$. Substituting recursively in (2) and using the no-Ponzi condition $\lim_{j\to+\infty} Q(T_j) \times B^+(T_j(s), s) = 0$, we obtain the intertemporal constraint on the brokerage account,

$$\sum_{j=1}^{\infty} Q(T_j(s)) \left[z(T_j(s), s) + P(T_j(s))\Gamma \right] \leq W_0(s), \tag{7}$$

where $z(T_j(s), s) = M^+(T_j(s), s) - M^-(T_j(s), s)$ and $W_0(s) \equiv B_0(s) + \int_0^\infty Q(t)(1-\alpha)P(t)\, Y(t)dt$. Constraint (7) states that the present value of gross cash transfers (transfers plus the transfer costs) is equal to the initial bonds plus the present value of deposits in the brokerage account.

The problem of household s that starts with assets $s = (M_0, B_0)$, is to choose consumption, $c(t, s)$, cash, $M(t, s)$, bonds, $B(t, s)$ and transfers, $z(T_j(s), s)$, that maximize the intertemporal utility

$$\sum_{j=0}^{\infty} \int_{T_j(s)}^{T_{j+1}(s)} e^{-\rho t} u(c(t, s))dt \tag{8}$$

subject to (1)–(6), where $u(c(t, s))$ is the instantaneously utility function, with $M(t, s) \geq 0$, $c(t, s) \geq 0$, and $\rho > 0$.

We abstract from government consumption or taxes to concentrate on the effects of monetary policy. The government executes monetary policy through open market operations in the asset market. The government supplies aggregate cash $M(t)$. An increase in the supply of cash generates revenue $\dot{M}(t)/P(t)$. Let B_0^G denote the supply of government bonds at $t = 0$. The government intertemporal budget constraint is $B_0^G = \int_0^\infty Q(t)\dot{M}(t)dt$. This equation says that the initial public debt must be equal to the present value of future money injections.

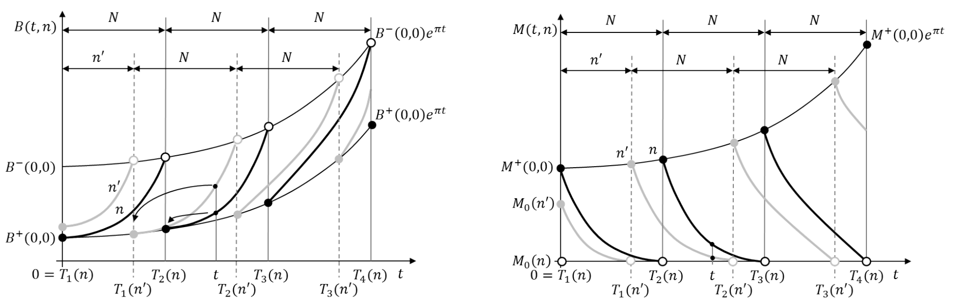
The market clearing condition for cash is given by $\int M(t, s)dF(s) = M(t)$, where F is the distribution of s. Similarly, the market clearing condition for bonds is given by $B_0^G = \int B_0(s)dF(s)$. The market clearing condition for goods takes into account that goods are used to pay the transfer cost. Let $A(t, \delta) \equiv \{s : T_j(s) \in [t, t+\delta]\}$ denote the set of households that make a transfer during the period $[t, t+\delta]$. The volume of goods necessary to pay the transfer costs incurred during the period $[t, t+\delta]$ is given by $\int_{A(t,\delta)} \frac{1}{\delta}\Gamma dF(s)$. The limit of this expression gives the transfer cost at time t. Thus, the time t market clearing condition for goods is $\int c(t, s)dF(s) + \lim_{\delta\to 0} \int_{A(t,\delta)} \frac{1}{\delta}\Gamma dF(s) = Y(t)$.

The equilibrium of this economy is defined as a vector of prices $\{P(t), Q(t)\}$, and allocations $\{M(t, s)$, $B(t, s)$, $c(t, s)\}$ such that $\{M(t, s), B(t, s), c(t, s)\}$ solve household s maximization problem given $\{P(t), Q(t)\}$, for all s in the support of $F(s)$; the government budget constraint is satisfied; and the market clearing conditions for cash, bonds, and goods hold.

4. The steady state equilibrium

Here we focus on the steady state equilibrium, an equilibrium in which the nominal interest rate is constant at r and the inflation rate is constant at π. The steady state equilibrium can be interpreted as the equilibrium of an economy that has not been exposed to shocks for a long time. Later, we study the effects of a monetary shock when the economy is in the steady state.

When inflation and interest rate are constant, all households choose holding periods with the same size, N, and have the same profile of consumption during the holding periods. At the beginning of a holding period, all households start with the same amount of cash, which they spend until the end of the holding period, at which time they make a new transfer.[5] To simplify the exposition, we assume the economy is in the steady state and reindex households by their positions in a holding period. Let $n \in [0, N)$ denote the household that makes transfers at $T_1(n) = n$, $T_2(n) = n + N$ and so on. Figure 3 describes the evolution of the brokerage accounts and banking accounts. A household with $M_0(n)$ makes transfers at $t = n, n + N, \ldots$, and so on. The initial money holdings $M_0(n)$ increase with n. Households that make their first transfer later must have more initial money holdings at date $t = 0$. Analogously, the initial value in the brokerage account $B_0(n)$ decreases with n. A household with a smaller n has a higher stock of bonds and makes the first trade of bonds for money earlier than a household with a larger n.



(a) Bonds (b) Money

Figure 3: Bond and money holdings for agents n and n'. Agent n starts with $M_0(n) = 0$. Agent n' starts with $M_0(n') > 0$.

Additionally, to simplify the analysis, we assume $Y(t) = Y$, $\Gamma = \gamma Y$, $\gamma > 0$, and $u(c) = \log(c)$. Under these assumptions, we now characterize the consumption pattern of each household. As in Adão e Silva (2020), a first order condition of

5. Households engage in (S, s) policies. They start with a large amount of cash balances and make a new transfer only when cash balances reach a minimum level. This pattern is repeated over time.

household n's problem is

$$P(t)c(t,n) = \frac{e^{-\rho t}}{\lambda(n)\,Q(T_j(n))}, \tag{9}$$

$t \in (T_j(n), T_{j+1}(n))$, $j \geq 1$, where $\lambda(n)$ is the Lagrange multiplier of household n intertemporal budget constraint (7). Let c_0 denote the consumption at the beginning of a holding period. In the steady state, the price, $P(t)$, increases at a constant rate π, the bond price, $Q(t)$, decreases at constant rate, r, and $r = \rho + \pi$, where ρ is the discount factor in the intertemporal utility function. Equation (9) implies that consumption during a holding period is given by $c(t,n) = c_0 e^{-r(t-T_j(n))}$, for j such that $t \in [T_j(n), T_{j+1}(n))$. Thus, consumption decreases at rate r, from an upper bound c_0 to a lower bound $c_0 e^{-rN}$ within holding periods.

By integrating the expression of $c(t,n)$ across households we obtain aggregate consumption,

$$C(t) = c_0 \frac{1-e^{-rN}}{rN}. \tag{10}$$

Even though consumption of households changes during the holding period, aggregate consumption is constant over time. As aggregate consumption is constant, at any time the same number of households must be starting a new holding period. Otherwise, aggregate consumption would vary over time. As a result, the distribution of households must be uniform along $[0, N)$, with density $1/N$.[6]

The value of c_0 is obtained from the market clearing condition for goods. The market clearing condition for goods implies $\frac{1}{N}\int_0^N c(t,n)dn + \frac{\Gamma}{N} = \frac{1}{N}\int_0^N Y dn$. After substituting the expression for $c(t,n)$ into the integral, and using $r = \rho + \pi$, we obtain the consumption-income ratio at the beginning of a holding period, $\hat{c}_0 \equiv c_0/Y$, $\hat{c}_0 = \left(1 - \frac{\gamma}{N}\right)\left(\frac{1-e^{-rN}}{rN}\right)^{-1}$.

The first order conditions of the problem of the households for $T_j(n)$, $j = 2, 3, \ldots$, provide an equation that characterizes the optimal interval between transfers N. Proposition 1 formally describes this equation.[7]

Proposition 1 *The optimal interval between transfers, N, is the solution to the equation*

$$\rho\gamma + \hat{c}_0(r,N)rN\left[\frac{1-e^{-\rho N}}{\rho N}\right] + \alpha r N\left[\frac{e^{rN}-1}{rN} - \frac{e^{(r-\rho)N}-1}{(r-\rho)N}\right] = \hat{c}_0(r,N)rN, \tag{11}$$

where $\hat{c}_0(r,N) = \left(1 - \frac{\gamma}{N}\right)\left(\frac{1-e^{-rN}}{rN}\right)^{-1}$.

6. In terms of welfare, however, the staggered behavior of individual households consumption creates distortions. Higher nominal interest rates increase the concentration of spending at the beginning of holding periods. This variation of spending decreases welfare. For further analysis of the welfare cost of inflation in related economies, see Silva (2012) and Adão e Silva (2019).

7. For the details see Adão e Silva (2020).

We write $\hat{c}_0(r, N)$ to emphasize that the consumption-income ratio just after the transfer is a function of the interest rate and the size of the interval between transfers.[8]

Equation (11) says that, at the optimal value of N, the marginal gain of delaying the transfer, on the left hand side, must be equal to the marginal cost of delaying the transfer, on the right hand side. The marginal gain has three terms. The first term is the gain of deferring the transfer cost by an instant. The second term is the gain from postponing the decrease in the brokerage account. The third term is the net effect on the money receipts of a delay in the transfer. The right hand side is the loss in utility caused by the delay in the transfer.

It follows from (11) that the interval between transfers, N, is a function of the nominal interest rate, the transfer cost, and the labor share. The next result, Proposition 2 establishes how the optimal interval between transfers changes with the nominal interest rate, the transfer cost and the labor share. The results are intuitive. The size of the holding period is larger, the lower is the nominal interest rate (which measures the opportunity cost of money) and the higher are the transfers costs. Finally, the households make transfers less often if the fraction of payments that goes directly to the checking account is higher. [9]

Proposition 2 *N is such that (i) $\frac{\partial N}{\partial r} < 0$, (ii) $\frac{\partial N}{\partial \gamma} > 0$ and (iii) $\frac{\partial N}{\partial \alpha} > 0$.*

Aggregate money holdings are equal to $M(t) = \frac{1}{N}\int_0^N M(t,n)dn$, where $M(t,n) = \int_t^{T_j(n)} P(\tau)c(\tau,n)d\tau$. The aggregate money-income ratio, $m = \frac{M(t)}{P(t)Y}$, is constant at the steady state as aggregate money holdings grow at the same rate as inflation. As shown in Adão e Silva (2020), the aggregate money-income ratio can be rewritten as

$$m(r;\rho,\gamma,\alpha) = \frac{\hat{c}_0 e^{-rN}}{\rho}\left[\frac{e^{rN}-1}{rN} - \frac{e^{(r-\rho)N}-1}{(r-\rho)N}\right] - \frac{\alpha}{r-\rho}\left[\frac{e^{(r-\rho)N}-1}{(r-\rho)N} - 1\right]. \quad (12)$$

We write $m(r;\rho,\gamma,\alpha)$ to emphasize that the aggregate money-income ratio is a function of the nominal interest rate, r, the real interest rate, ρ, financial costs, γ, and the labor share, α, since N and $\hat{c}_0$ are also functions of these variables.

It turns out that the money-income ratio decreases with α. The intuition for this result is not immediate. On the one hand, a higher value for α implies that agents receive a higer proportion of their income in a liquid form. On the other hand, when α is large agents do not need to hold high levels of money balances, as they receive more frequently a large proportion of their income as money. The net effect over the money-income ratio is negative, as the second effect is stronger than the first.

8. The formulas were arranged to facilitate the identification of the terms $\frac{e^x-1}{x} \approx 1 + \frac{x}{2}$ and $\frac{1-e^{-x}}{x} \approx 1 - \frac{x}{2}$.

9. For the details see Adão e Silva (2020).

If agents received all their income at the frequency of their consumption, i.e. $\alpha = 1$, then the demand for money would be null. In this case the flow of consumption of each agent would be equal to his or her income flow, which would be paid exclusively with liquid assets, and the first best would be achieved. For values of $\alpha < 1$, the consumption flow will be larger than the income flow received under the form of liquid assets. The difference between the two flows determines the demand for money. The demand for money will be an increasing function of the difference between the consumption flow and the income flow paid with liquid assets. Thus, the smaller the α the larger will be the demand for money.

This result links the trend in the decrease in the labor share to the trend in the decrease in the velocity of money. The model predicts, as a consequence of a lower labor share in the last few decades, an increase in the real value of the money assets and also in the ratio of money to GDP, and thus a decrease in the velocity of money, which is exactly what one finds in the data. This is not the only reason behind the decrease in the velocity of money. As in all Baumol-Tobin cash inventory models, a long-term decrease in nominal interest rates implies higher money holdings and therefore lower velocity of money too. Thus, by considering a more general Baumol-Tobin model, with $\alpha > 0$, we find an additional explanation for the long-term decrease in the velocity of money.

Just as in Adão e Silva (2020), there is neutrality of money in the steady-state, with inflation equal to the nominal interest rate minus the discount rate, $(\pi = r - \rho)$, which means the closed-form solution for prices is given by $P(t) = P_0\, e^{(r-\rho)t}$ with P_0 being the price at $t = 0$.

We follow the literature in the calibration of the parameters. We set the real interest rate, ρ, equal to 3 percent per year, following Lucas (2000), and set the labor share of income α initially to 0.6, following Alvarez *et al.* (2009) and Khan e Thomas (2015). We consider the proxies for M and r, to be M1 and the commercial paper rate, respectively. The parameter γ is set so that the theoretical aggregate money-income ratio, $m(r; \rho, \gamma, \alpha)$, is equal to the average historical money-income ratio when r is equal to the geometric mean of the historical nominal interest rate. This is the procedure followed by Lucas (2000). Similarly, Alvarez *et al.* (2009) parameterize the holding period such that the theoretical demand for money approximates the average velocity in the data.

Figure 4 shows the money-income ratio $m(r)$ predicted by the model along with U.S. data. As a validation of the model, the model implies a money-income ratio that follows the observed pattern of the data. The dataset is similar to the one used in Lucas (2000), Lagos e Wright (2005) and others.[10] The results for $m(r)$ imply

10. The nominal interest rate is given by the commercial paper rate and the monetary aggregate is given by M1. Data are annual from 1900 to 1997 (the last year in which the commercial paper data are available from the same source). M1 and commercial paper rate were also used by Lucas (2000), Lagos e Wright (2005), Craig e Rocheteau (2008), among others. We use a similar dataset to facilitate the comparison of results. As it is known, money demand changed substantially at

an interest-elasticity of $-1/2$, which agrees with the findings in Guerron-Quintana (2009), Alvarez e Lippi (2009) and others.

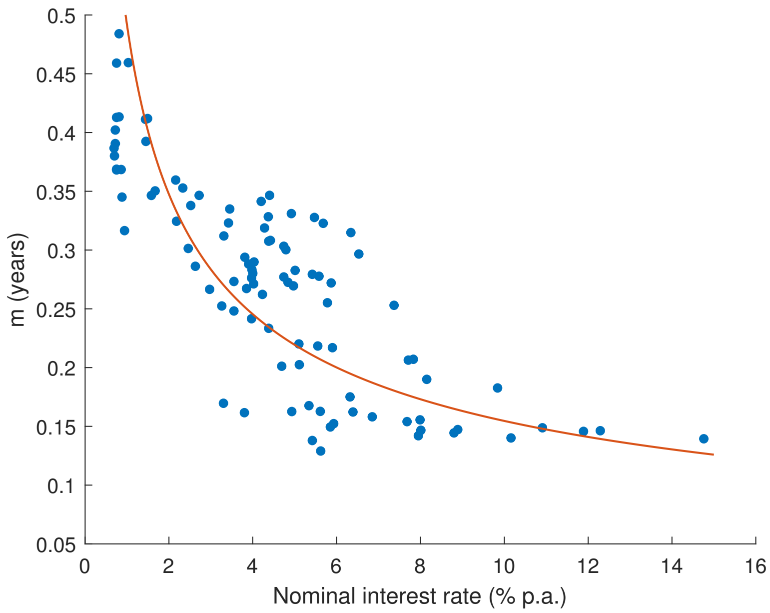


Figure 4: Money as predicted by the model and US data.

The model implies a large interval between transfers, $N = 440$ days, as is common in market segmentation models (Edmond e Weill 2016). Notice that financial transfers are transfers from high-yielding assets to currency, not ATM withdrawals, which change the allocations of checking deposits and currency, but do not change the quantity of money. Although large, the transfer intervals agree with the frequency with which U.S. households trade in high-yield assets (Vissing-Jorgensen 2002).[11]

5. Monetary policy shocks

We now study the effect of a monetary policy shock. The monetary policy shock happens at $t = 0$, with the economy in the steady state. In the experiment the N, which is obtained from the steady state r and α, is fixed. The monetary policy is summarized by the nominal interest rate path $r(t)$, $t \geq 0$. Since a change in

the beginning of 1990s. Teles e Zhou (2005) and Lucas e Nicolini (2015) propose new monetary aggregates to account for the changes in the demand for money.

11. The large N is in accordance with other models of market segmentation. Alvarez *et al.* (2009) have calibrations with N equal to 24 and 36 months; Khan e Thomas (2015) have the average N varying from 15 to 30 months.

$r(t)$ affects the aggregate demand for cash, the central bank has to adjust the money supply accordingly when setting the interest rate path. The central bank supplies $M(t)$ to satisfy the market clearing condition for cash. The interest rate path determines bond prices $Q(t) = e^{-R(t)}$, where $R(t) = \int_0^t r(s)ds$. We focus our attention on small shocks, for which the change in N with the interest rate is small.

For $t \geq N$, all agents adjust their portfolio with full knowledge of the nominal interest rate ahead. For $t \geq N$, monetary neutrality holds and the real interest rate is equal to the steady state real interest rate. On the other hand, for any $t < N$, there will be households that have made a transfer since the shock, at $t = 0$, and other households that have not made a transfer yet. Households that have not made the transfer yet, must do transactions using what is left out of $M_0(n)$. Households that have already made the transfer are households with smaller values of $n \in [0, N)$, as they make the first transfer at $T_1 = n$. During the period $[0, N)$ the real interest rate will be different from the steady state real interest rate.

In this context, the problem of household n is to maximize intertemporal utility, (8), subject to the intertemporal budget constraint, (7), and the cash in advance constraint for the period $[0, N)$, (6). Adão e Silva (2020) derive the household n's consumption that solves this problem and compute the aggregate consumption for all households,

$$C(t) = \frac{1}{N}\int_0^t \frac{e^{-\rho t}}{\lambda(n)Q(T_1(n))P(t)}dn + \frac{1}{N}\int_t^N \frac{e^{-\rho t}}{\mu(n)P(t)}dn, \ 0 \leq t < N, \quad (13)$$

where $\lambda(n) = 1/(P_0 c_0(n))$ is the Lagrange multiplier associated with the intertemporal budget constraint, (7), and $\mu(n)$ is the Lagrange multiplier associated with the cash in advance constraint of the first holding period, (6); the value of $\mu(n)$ depends on $M_0(n)$. The first term of equation (13) determines most of the aggregate consumption when t is close to N, and the second term explains most of the aggregate consumption when t is close to zero. The interpretation is that, when most households have not yet made the transfer, i.e. for t close to zero, the value of $\mu(n)$ is important to determine consumption and ultimately to determine prices.[12]

The equilibrium price $P(t)$ is obtained from the market clearing condition for goods. Proposition 3 provides the expression for the price level.

Proposition 3 *The evolution of prices in this setting obeys the integral equation*

$$\begin{cases} P(t) = f(t) + \alpha Y \frac{\rho}{N\Upsilon} e^{-\rho t} \displaystyle\int_t^N \frac{\int_0^n P(\tau)d\tau}{1 - e^{-\rho t}}dn, & t \in [0, N], \\ P(t) = P(N)\, e^{R(t-N)-\rho(t-N)}, & t > N, \end{cases} \quad (14)$$

12. We assume as a boundary condition that $\lambda(n)$ is the same across agents.

with

$$f(t) \equiv \frac{P_0 c_0}{N\Upsilon} \int_0^t e^{-\rho t} e^{R(n)} dn + \frac{P_0 c_0}{N\Upsilon} \frac{e^{-\rho t} - e^{-\rho N}}{\rho} - \frac{\rho P_0 \alpha Y e^{-\rho t}}{N\Upsilon} \int_t^N \frac{n}{1 - e^{-\rho n}} dn. \quad (15)$$

As shown in Appendix A, the agents exhaust their money stocks at the time of their first transfer. Therefore, the inequality $M_0 \geq \int_0^{T_1(n)} (\frac{e^{-\rho\tau}}{\mu(n)} - \alpha P(\tau) Y) d\tau$ holds with equality. This allows us to find $\mu(n)$, which we can then replace on equation (13), together with the market-clearing condition for goods, to obtain the equation for $P(t)$ when $t < N$. For $t \geq N$ we consider the solution that guarantees that the real interest rate $\pi(t) - r(t)$ is equal to the discount rate ρ, as all the agents have now adjusted their stocks under the new market conditions and money neutrality must hold.

In order to study the effect of changes in the nominal interest rates on prices and money stock, it is necessary to solve the integral equation (14) for $P(t)$ when $t \in [0, N]$. That is, an equation for which is the unknown is the function $P(t)$, and for which the integral of the unknown function appears. This is a second-order Volterra integral equation of the second kind.[13]

Intuitively, the term with the integral appears because agents receive $P(t)\alpha Y$ at every time directly to their bank accounts. They take into account the changes in the price level to calculate how much in real terms they will receive from the transfers. When $\alpha = 0$, this integral term disappears and the equation is simplified.[14]

The strategy to obtain the equilibrium price level is to transform equation (14) into an ordinary differential equation. Proposition 4 establishes that is possible to obtain a differential equation equivalent to equation (14).

Proposition 4 *For $t \in (0, N]$ the integral equation in* (14) *for $P(t)$ is equivalent to the differential equation*

$$\ddot{P}(t) + p(t)\dot{P}(t) + q(t)P(t) = g(t) \quad (16)$$

with

$$p(t) \equiv \rho \frac{2e^{\rho t} - 1}{e^{\rho t} - 1}, \quad (17)$$

$$q(t) \equiv \frac{\rho}{e^{\rho t} - 1} \left(\rho e^{\rho t} + \frac{\alpha Y}{N\Upsilon} \right), \quad (18)$$

$$g(t) \equiv \frac{1}{N\,\Upsilon} \left(\ddot{f}(t) + \rho \frac{2e^{\rho t} - 1}{e^{\rho t} - 1} \dot{f}(t) + \rho \frac{\rho e^{\rho t}}{e^{\rho t} - 1} f(t) \right). \quad (19)$$

13. Volterra integral equations appear also in the study of the risk of insolvency in actuarial science. See for instance Wang *et al.* (2021).

14. Adão e Silva (2020) consider the case $\alpha = 0$, but allow for different groups of agents to represent heterogeneity across firms.

The proof of Proposition 4 is in Appendix B.

In this economy monetary policy is non-neutral, as an unexpected change in the nominal interest rate will affect the real interest rate. An unexpected change of policy is going to be absorved directly only by the fraction of agents that are in the market at the time of the change in policy. Individual money circulation speeds are greater for those agents who are closer to going to the bank, because their ratio between consumer spending and money is higher. Consider for instance a decrease in the nominal interest rate. The agents that are at the bank when the interest rate goes down are going to choose to hold a larger quantity of money has the opportunity cost of it decreased. Thus, those agents, who have just come from the bank (and absorbed the injection) have a lower circulation velocity than they would if there was no policy shock. Therefore, an unexpected decrease in the nominal interest rate leads to an injection of money, which leads to a decrease in the economy's aggregate velocity of circulation because it increases relatively more the money in the hands of those who have just come from the bank. As money velocity decreased, the price responds less than proportionally to the increase in currency.

This difference between the behavior of prices and money is greater the longer the interval between visits to the bank. The fraction of agents in the market at any moment in time is smaller the larger the interval between visits to the bank. The smaller the fraction of agents that is absorving the unexpected change in the monetary policy, the larger will be the real macro effects of that change. As a higher labor share is associated with less agents in the market, in those economies monetary policy is more powerful.

6. Permanent shock

We proceed to describe the equilibrium path when there is an unexpected permanent increase in the nominal interest. The economy is in a steady state, with zero inflation, $r(t) = \rho$, when a surprise shock hits the economy. The effect on the price level is given by the system of equations (14). The solution of this system depends on the function $r(t)$ defined for $t > 0$ which is fully determined by the monetary authority.

The function $r(t)$ for the permanent shock is given by $r(t) = r_2 = r_1 + r_e$ where r_1 is the nominal interest rate before the shock, and r_e is a positive constant. The values of the initial money holdings $M_0(n)$ correspond to a steady state as described in section 4. The price at $t = N$ can be obtained directly from equation (14) and the time derivative of the price level at $t = N$ is given by $\dot{P}(t)|_{t=N} = r_e P(N)$.[15]

15. For these boundary conditions, this equation has an analytical solution. However, the results obtained using a numerical approximation (Euler-Chromer, Runge-Kutta) are virtually indistinguishable from those obtained through the analytical solution and much less computationally

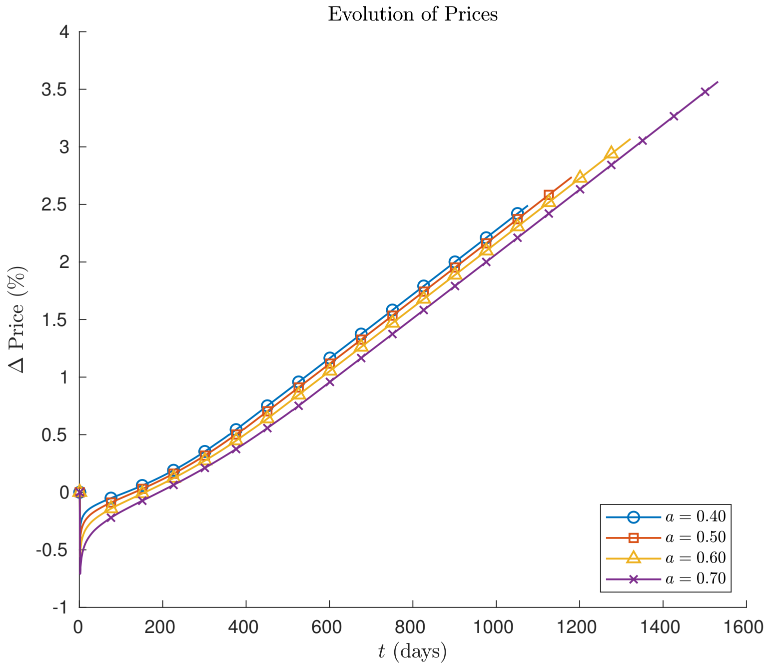


Figure 5: Time path of prices for different values for the labor share α as a response to a permanent shock $r(t) = r_2$, until prices conform to the steady-state solution (for higher values of α prices take longer to conform to the steady-state solution).

In what follows, we calibrate the holding period N for each country, which is a function of the labor share α, the transfer cost γ, and the nominal interest rate r. We take the γ to be the same for all countries (equal to the value for the US), $r_1 = \rho = 3\%$ per year, and $r_2 = r_1 + r_e$, which corresponds to 4% per year. Figure 5 shows the time path of the price for different values of the labor share. According to proposition 2, the transition period to a new steady state is longer the larger the labor share is. The short term behavior of prices is also a function of the labor share.

The evolution of prices in figure 5 shows the fundamental driving forces at play. When faced with an increase in interest at $t = 0$, all agents suffer a substitution effect that drives them to save (the fact that the interest rate is higher than the intertemporal discount rate makes savings more appealing) and an income effect that drives them to consume. The value of α determines the size of the income effect. The smaller the α, the larger the income effect.[16] When $\alpha = 0$ the income effect is at its maximum, and in this case it cancels the substitution effect at $t = 0$.

demanding. Therefore, results obtained using a Runge-Kutta numerical approximation method will be used throughout.

16. The agents in economies with smaller αs have higher levels of savings under the form of bonds and money.

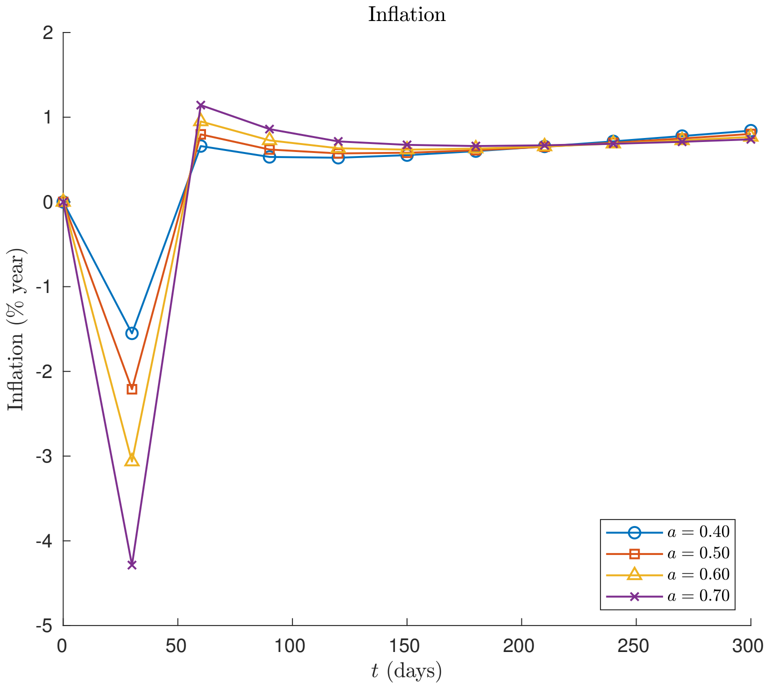


Figure 6: Monthly inflation values for different values for the labor share α as a response to a permanent shock $r(t) = r_2$.

Hence, no decrease in price takes place in this case. For higher values of α, the price level jumps down. The absolute value of the jump is increasing in α.

The discontinuities in the price level lead to instantaneous infinite values of inflation. However, in practice this is not measured. Actual measures of inflation compare price levels monthly or annually, not on a daily basis. Thus, a negative jump in the price level followed by an immediate sustained increase is interpreted as negative inflation only if the price level remains below its initial value the first time it is measured. To account for this, the values for monthly inflation are presented in figure 6 for several values of α. In the period immediately following a monetary shock, a significant drop in inflation is perceived, in this case from 0, at the steady state, to negative values. This is a short-term adjustment, as inflation ends up conforming to monetary neutrality in the long run, with N being the transition period. In the new steady state, inflation is 1% per year. These findings are consistent with Uribe's (2022) empirical results for the long-run effect of a permanent shock.

As only a fraction of the agents is in the market at any moment, they are going to react differently, which makes the real interest rate move more slowly than if there was no market segmentation. As the real interest rate is equal to the difference between the nominal interest rate and the rate of inflation, the real

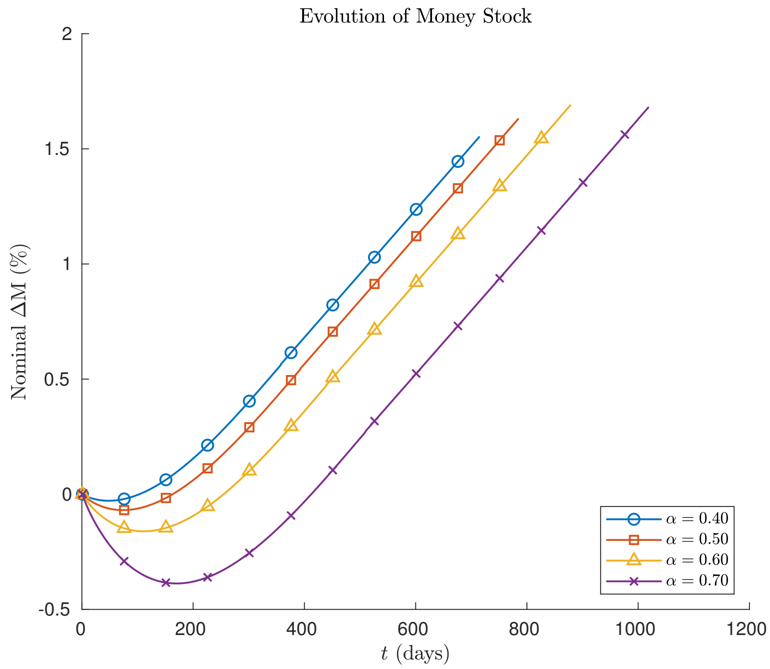


Figure 7: Nominal money stock for different values for the labor share α as a response to a permanent shock $r(t) = r_2$, until the money stock conforms to the steady-state solution. For higher values of α prices take longer to conform to the steady-state solution.

interest rate increases together with the nominal interest rate just after a positive shock.[17]

Economies with larger labor shares have larger reductions in inflation and higher increases in the real interest rate, i.e. the monetary shock has a stronger impact. As discussed above, this result is somewhat surprising: an increase in the labor share - which may be seen as a reduction in the financial friction - ends up enhancing the effects of monetary policy. The fact that economies with higher labor shares have lower steady-state stocks means that the income effect of any interest rate change is lower, and because this income effect is countering the substitution effect that is driving the real effects of monetary policy, the overall effect is a higher sensitivity of prices to any given change in the interest rate.

These results have policy implications. They suggest that countries suffering from below-target inflation and near zero nominal rates can achieve reflation by setting a permanent higher nominal interest rate. However, if this change in the interest rate is not credible or unanticipated, the transition to a new steady state, with higher inflation and higher interest rates, will be longer and the costs higher,

17. The slow response of prices and the increase in the real interest rate after the increase in the nominal interest rate is found in many empirical studies. Among others, Cochrane (1994), Christiano *et al.* (1999), Kahn *et al.* (2002), Bernanke *et al.* (2005), and Uhlig (2005).

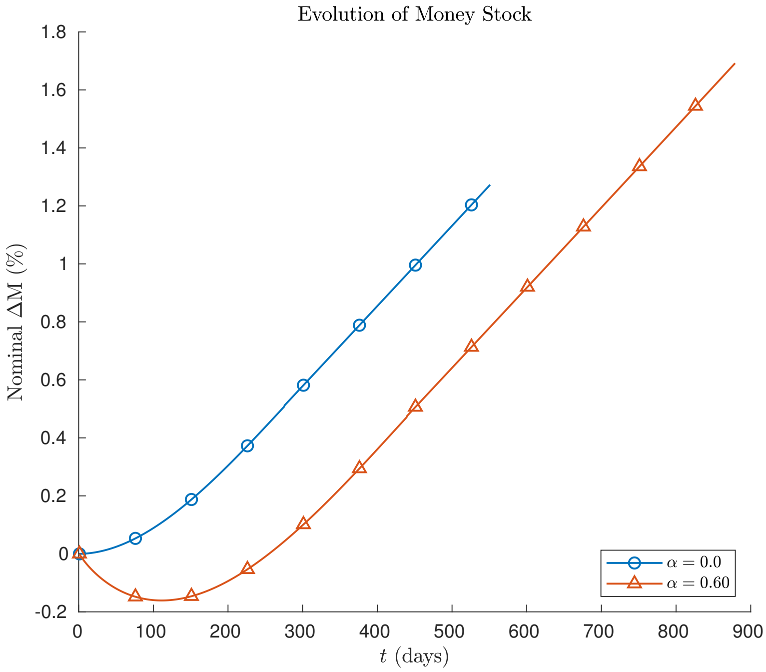


Figure 8: Nominal money stock for two values for the labor share α as a response to a permanent shock $r(t) = r_2$, until the money stock conforms to the steady-state solution.

the larger the labor share is. Moreover, the costs are very sensitive to the value of the labor share. For instance, according to figure 6, just after the shock, the monthly inflation is about -2.2 percent when $\alpha = 0.5$, but if $\alpha = 0.7$, then deflation almost doubles (4.3%).

Once equilibrium price level $P(t)$ is obtained, we can solve the problem of agent n and obtain $M(n,t)$. Then, the aggregation of $M(n,t)$ over all the agents yields the total nominal money stock M in the time domain $0 < t \leq N$:

$$M(t) = \frac{1}{N}\left(\int_0^t \int_t^{n+N} \frac{e^{R(n)-\rho\tau}}{\lambda} d\tau dn + \int_t^N \int_t^n \frac{e^{-\rho\tau}}{\mu(n)} d\tau dn\right) - \frac{\alpha Y}{N}\left(\int_0^t \int_t^{n+N} P(\tau) d\tau dn + \int_t^N \int_t^N P(\tau) d\tau dn\right). \quad (20)$$

Figure 7 shows the values of the nominal money stock for different values of the labor share, α. The short run reduction in the nominal money stock is larger, for larger values of α. The liquidity efffect, i.e. the short term reduction in the nominal money stock due to an unexpected increase in the nominal interest rate, increases with the labor share.[18] These results stress the importance of the labor share: nominal payments sent directly to agents are key to materialize the real effects of monetary policy.

The nominal money stock decreases in the short run, whereas the real money stock has an immediate discontinuity as a result of the sudden decrease in prices. After an adjustment period, the nominal money stock ends up increasing at the long-term inflation rate ($r_e = r_2 - \rho$) and the real money stock converges to a constant value, just as expected.

Notice that in economies with a higher labor share, agents have better access to their income, and therefore their money and bond stocks are lower. Consequently, a higher labor share reduces the income effect associated with changes in the interest rate. This income effect dampens the substitution effect that drives the real effects of monetary policy in this framework. Therefore, in economies with a higher labor share, the effects of monetary policy are larger.

To highlight this relation, figure 8 presents a comparison between the money stock evolution for the extreme case $\alpha = 0$ and the conventional case $\alpha = 0.6$. With $\alpha = 0$ there is no reduction in the money stock at all.

7. Temporary shock

We now study the effects of the monetary policy shock shown in figure 9. This shock follows the estimates made by Uhlig (2005) of the monetary policy shocks imposed by the Federal Reserve. Adão e Silva (2020) modeled these shocks as $r(t) = r_1 + (r_e + Bt)\, e^{-\eta t}$.[19]

Figure 10 shows the time path of the price for different values of the labor share. When comparing these results with those presented in the previous section, one difference is immediately apparent: the discontinuity created by the shock is much smaller. This is to be expected: a temporary shock would have a smaller impact on agent's decisions, as it is relatively short-lived, and therefore leads to smaller changes in allocations.

As above, we show monthly inflation in Figure 11 for several values of α. Given the shorter duration of the shock, the measurements of monthly inflation are taken every fifteen days. We observe that, as a consequence of the smaller discontinuities in price, the absolute value of inflation attains much lower values in the first month than in the case of the permanent shock.

When the shock is temporary, the policy implications discussed in the previous section are maintained. The impact of the shock increases with the labor share. A decrease in the labor share from $\alpha = 0.7$ to $\alpha = 0.6$, which is in the range of the

18. In fact, a positive labor share is required to obtain a liquidity effect

19. Temporary shocks are modeled as dampened oscilator. They are obtained as the solution of a differential equation in the form $m\ddot{r} + c\dot{r} + kr = 0$, where $\eta \equiv c/2m$.

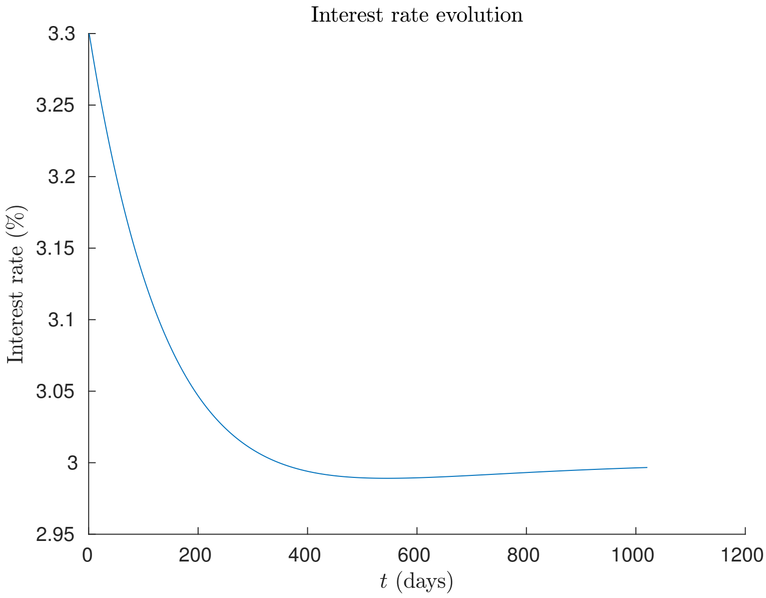


Figure 9: Time path of interest rates, given by $r(t) = r_1 + (r_e + Bt)\, e^{-\eta t}$.

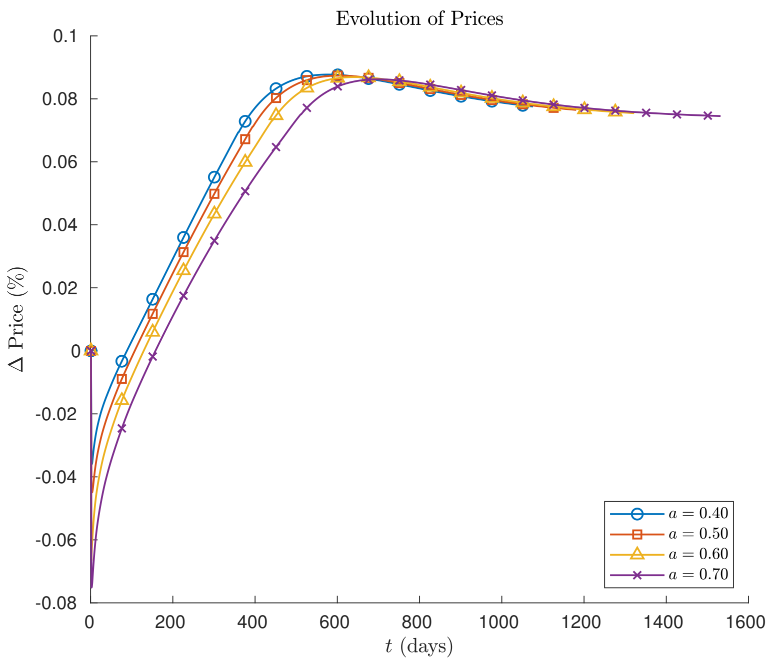


Figure 10: Time path of prices for different values for the labor share α as a response to a temporary shock $r(t) = r_1 + (r_e + Bt)\, e^{-\eta t}$, until prices conform to the steady-state solution (for higher values of α prices take longer to conform to the steady-state solution).

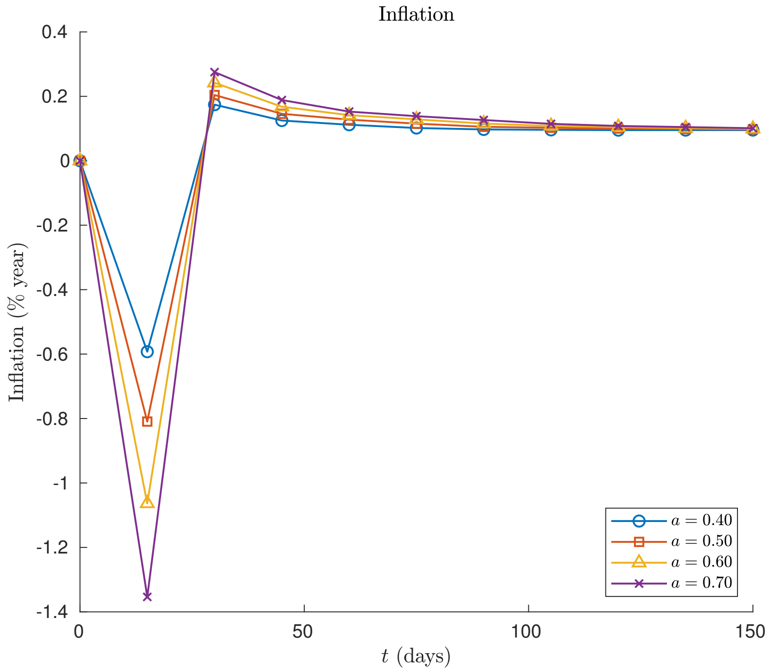


Figure 11: Monthly inflation values for different values for the labor share α as a response to a temporary shock $r(t) = r_1 + (r_e + Bt)\, e^{-\eta t}$.

decrease in the labor share in the last decades, significantly decreases the effect of the shock on prices on the first month.

Figure 12 shows the values of the nominal money stock for different values of the labor share, α. The short-run reduction in the nominal money stock is larger for larger values of α, but as $r(t)$ returns to its steady-state value, so does the money stock. The quantity of money does not change as much as the price or monthly inflation, but money takes much longer to return to the steady-state equilibrium. For all values of α considered, monthly inflation and prices converge to their steady-state values after about two months, but that is not the case with money stocks. Instead, for most values of α, the effects of the shock are still quite significant for a long period when it comes to the money stock. Market segmentation introduces substantial inertia in the money stock path. Even after inflation returned to its steady-state, the composition of the agents portfolio is still different from the steady-state.

In economies with different labor shares to attain the same real results, monetary policy shocks must be larger when the labor share is small, and smaller when the labor share is large. Morever, the shock interacts nonlinearly with the labor share. For the same shock, the absolute change in the money stock increases more than proportionally with the labor share. For the same shock, as can be seen in Figure 12, an increase in the labor share from $\alpha = 0.6$ to $\alpha = 0.7$ more than doubles the maximum decrease in the money stock.

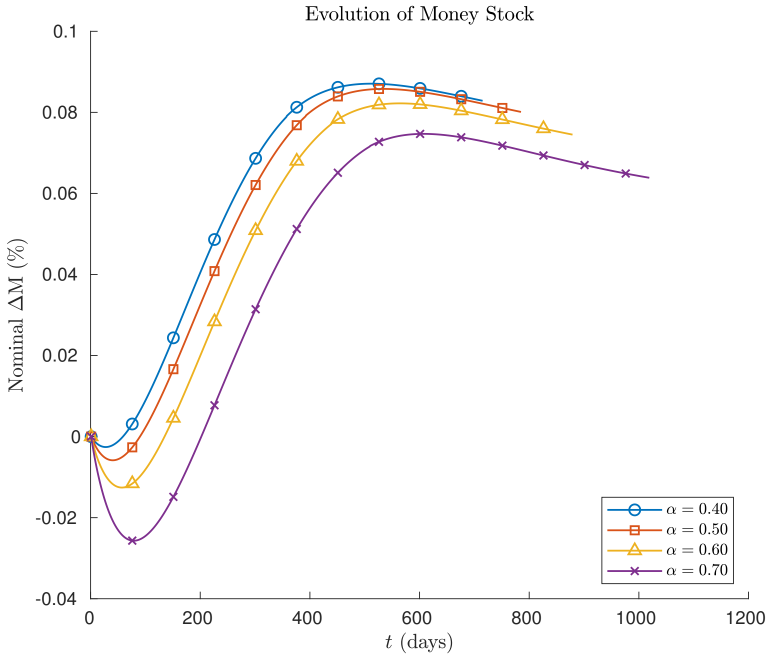


Figure 12: Nominal money for different values for the labor share α as a response to a temporary shock $r(t) = r_1 + (r_e + Bt)\,e^{-\eta t}$, until the money stock conforms to the steady-state solution. For higher values of α, prices take longer to conform to the steady-state.

8. Conclusions

We uncover a new determinant of the monetary transmission mechanism, the labor income share. Countries with a higher labor income share have a higher proportion of income that is automatically available for spending. This liquid fraction of income affects the impact of an unexpected increase in the nominal interest. The immediate decrease in inflation is larger, the larger the labor share is. Thus, a larger labor income share makes the monetary policy more powerful. Moreover, the short-term reduction in the nominal money stock is larger if the labor share is larger, which demonstrates the crucial importance of the size of the labor share for the monetary policy transmission.

We make an additional contribution to the literature on the monetary transmission mechanism by presenting an economy where a decrease in financial frictions ends up enhancing the real effects of monetary policy. Economies with higher labor share can be thought of as economies with lower financial frictions, as agents have an easier access to their income. The result that a shock has a larger effect when there are lower frictions is new. Usually, as in Adão *et al.* (2004), a shock has stronger effects when the economy has more frictions.

Calculating the effects of a monetary policy shock under the framework presented requires solving a second-order nonlinear integral equation, which we transformed into a second-order ordinary differential equation. There are scant references to solutions of integral equations in the economic literature. To the best of our knowledge, none of them regarding a second-order or higher integral equation. We contribute to the literature by outlining the analytical procedure to transform a second-order integral equation into an ODE numerically solvable with familiar methods.

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Appendix A: Proof that agents exhaust their money holdings just before their first transfer

Proof: The first order condition for consumption of the problem (1)–(5) may be used to obtain the following relation for consumption during periods $[T_j(n), T_{j+1}(n))$,

$$c(t,n) = \frac{e^{-\rho t}}{\lambda_j(n)P(t)}. \tag{A.1}$$

If the path of interest rates and prices is foreseen before any given transfer then, assuming $r(t) > 0$ for all t, it will never be optimal to finish that holding period with a positive amount of money. This implies the following relation for any transfer under these conditions $(j > 0)$,

$$B^{+}(T_j(n),n) - B^{-}(T_j(n),n) = \int_{T_j(n)}^{T_{j+1}(n)} [P(\tau)c(\tau,s) - \alpha P(\tau)Y(\tau)]d\tau + \Gamma P(T_j(n)). \tag{A.2}$$

The optimal control problem (for $j > 0$) may now be presented only in terms of the control sequence $\lambda_j(n)$ and state sequence $B^{-}(T_j(n),n)$. As we are considering the problem for agent n, these terms will be termed λ_j and B_j for clarity. For the same reason, all other references to n are implicit,

$$\begin{aligned}H_j = &\int_{T_j}^{T_{j+1}} e^{-\rho\tau} \log\Big(\frac{e^{-\rho\tau}}{\lambda_j P(\tau)}\Big)d\tau + \psi_j \times \Big(B_j e^{R(T_{j+1})-R(T_j)} + \\ &\int_{T_j}^{T_{j+1}} (1-\alpha)YP(\tau)e^{R(T_{j+1})-R(\tau)}d\tau - \int_{T_{j+1}}^{T_{j+2}} (\frac{e^{-\rho\tau}}{\lambda_{j+1}} - \alpha P(\tau)Y)d\tau - \Gamma P(T_j)\Big).\end{aligned} \tag{A.3}$$

For H_0, however, the problem of the agent is different. The agent did not know $r(t)$ and $P(t)$ at the time of the transfer, and thus it could be the case that the agent does not exhaust the money to which he was endowed in the prior steady state (by himself prior to the new circumstances). H_0 is not related to the moment T_0 of the transfer prior to shock in the same way as H_j for $j > 0$ are related to T_j. Instead, H_0 relates to $t = 0$, the moment the agent reevaluates decisions based on the new information regarding prices and interests. Therefore, H_0 is given by

$$\begin{aligned}H_0 = &\int_0^{T_1} e^{-\rho\tau}\, ln\Big(\frac{e^{-\rho\tau}}{\lambda_0 P(\tau)}\Big)d\tau + \psi_0 \\ &\Big(B_0\, e^{R(T_1)-R(T_0)} + \int_{T_0}^{T_1} (1-\alpha)YP(\tau)e^{R(T_1)-R(\tau)}d\tau - \int_{T_1}^{T_2} (\frac{e^{-\rho\tau}}{\lambda_1} - \alpha P(\tau)Y)d\tau - \Gamma P(T_0)\Big) \\ &+ \psi_0\Big(M_0 - \int_0^{T_1} (\frac{e^{-\rho\tau}}{\lambda_0} - \alpha P(\tau)Y)d\tau\Big) + \mu_0\Big(M_0 - \int_0^{T_1} (\frac{e^{-\rho\tau}}{\lambda_0} - \alpha P(\tau)Y)d\tau\Big),\end{aligned} \tag{A.4}$$

where μ_0 is related to the inequality $M_0 \geq \int_0^{T_1} (\frac{e^{-\rho\tau}}{\lambda_0} - \alpha P(\tau)Y)d\tau$, and M_0 is the stock of money in agent n's account at the moment of the shock.

To find M_0, one considers the steady-state solution for the problem. This yields

$$M_0 = \left((1-\alpha)Y - \frac{\Gamma}{N} \right) P(0) \, \frac{(e^{(r-\rho)T_1} - 1)}{(r-\rho)}. \tag{A.5}$$

Regardless of M_0, the corner solution is immediate and it means that the agent will indeed exhaust all money at the end of the first holding period. To show that this is the overall solution, one has to show that the interior solution is the same or violates the inequality presented above.

The first order conditions for B_j lead to

$$\psi_j = \psi_{j+1} \; e^{R(T_{j+1}) - R(T_j)}, \tag{A.6}$$

which in turn leads to the closed-form solution

$$\psi_j = \psi_0 \; e^{-(R(T_j) - R(T_0))}. \tag{A.7}$$

For $j > 0$, the first-order conditions for λ_j lead to $\lambda_j = \psi_{j-1}$. Thus,

$$\lambda_j = \psi_0 \; e^{-(R(T_{j-1}) - R(T_0))}. \tag{A.8}$$

For $j = 0$, first-order conditions for λ_0 yield

$$-\frac{1}{\lambda_0} \int_{T_0}^{T_1} e^{-\rho\tau} d\tau + \frac{\psi_0}{\lambda_0^2} \int_{T_0}^{T_1} e^{-\rho\tau} d\tau + \frac{\mu_0}{\lambda_0^2} \int_{T_0}^{T_1} e^{-\rho\tau} d\tau = 0 \tag{A.9}$$

The interior solution implies $\lambda_0 = \psi_0$.

This implies that, for $j > 0$, B_j obeys the relation

$$\begin{aligned} B_{j+1} = B_j \, e^{R(T_{j+1}) - R(T_j)} &+ \int_{T_j}^{T_{j+1}} (1-\alpha) Y P(\tau) e^{R(T_{j+1}) - R(\tau)} d\tau \\ &- \int_{T_{j+1}}^{T_{j+2}} \left(\frac{e^{-\rho\tau} \;\; e^{R(T_j) - R(T_0)}}{\psi_0} - \alpha P(\tau) Y \right) d\tau - \Gamma P(T_j), \end{aligned} \tag{A.10}$$

with

$$\begin{aligned} B_1 &= B_0 \, e^{R(T_1) - R(T_0)} + \int_{T_0}^{T_1} (1-\alpha) Y P(\tau) e^{R(T_1) - R(\tau)} d\tau \\ &- \int_{T_1}^{T_2} \left(\frac{e^{-\rho\tau}}{\psi_0} - \alpha P(\tau) Y \right) d\tau - \Gamma P(T_1) + M_0 - \int_0^{T_1} \left(\frac{e^{-\rho\tau}}{\psi_0} - \alpha P(\tau) Y \right) d\tau. \end{aligned} \tag{A.11}$$

The closed-form solution for those equations is given by

$$\begin{aligned} B_j = &\Big(B_0 e^{R(T_1) - R(T_0)} + M_0 - \textstyle\int_0^{T_1} \left(\frac{e^{-\rho\tau}}{\psi_0} - \alpha P(\tau) Y \right) d\tau \Big) e^{R(T_j) - R(T_1)} + \\ &\textstyle\sum_{i=1}^{j} e^{R(T_j) - R(T_i)} \\ &\Big(\textstyle\int_{T_{i-1}}^{T_i} (1-\alpha) Y P(\tau) e^{R(T_i) - R(\tau)} d\tau - \int_{T_i}^{T_i+1} \left(\frac{e^{-\rho\tau} \;\; e^{R(T_{i-1}) - R(T_0)}}{\psi_0} - \alpha P(\tau) Y \right) d\tau - \Gamma P(T_i) \Big). \end{aligned} \tag{A.12}$$

It it then possible to consider the transversality condition $\lim_{j\to\infty} B_j\psi_j = 0$. It yields

$$0=\lim_{j\to\infty}\psi_0\, e^{-(R(T_j)-R(T_0))}\Bigg[\Big(B_0e^{R(T_1)-R(T_0)}+M_0-\int_0^{T_1}(\tfrac{e^{-\rho\tau}}{\psi_0}-\alpha P(\tau)Y)d\tau\Big)e^{R(T_j)-R(T_1)}$$
$$+\sum_{i=1}^{j} e^{R(T_j)-R(T_i)}$$
$$\Big(\int_{T_{i-1}}^{T_i}(1-\alpha)YP(\tau)e^{R(T_i)-R(\tau)}d\tau-\int_{T_i}^{T_{i+1}}\Big(\tfrac{e^{-\rho\tau}\ e^{R(T_{i-1})-R(T_0)}}{\psi_0}-\alpha P(\tau)Y\Big)d\tau-\Gamma P(T_i)\Big)\Bigg]. \tag{A.13}$$

For $t>N$ we consider $P(t)\approx P(N)e^{R(t)-R(N)-\rho(t-N)}$. Define $P_0\equiv P(N)\,e^{\rho N-R(N)}$, to simplify the expression. The transversality condition then yields

$$0=\lim_{j\to\infty}\Bigg[B_0\psi_0+M_0\psi_0e^{R(T_0)-R(T_1)}-\frac{1-e^{-\rho T_1}}{\rho}e^{R(T_0)-R(T_1)}$$
$$+\ \alpha\psi_0Ye^{R(T_0)-R(T_1)}\int_0^{T_1}P(\tau)d\tau+\sum_{i=1}^{j}\psi_0e^{R(T_0)}\int_{T_{i-1}}^{T_i}(1-\alpha)YP_0e^{-\rho\tau}d\tau$$
$$+\sum_{i=1}^{j}\int_{T_i}^{T_{i+1}}e^{-\rho\tau}e^{R(T_{i-1}-R(T_i))}d\tau$$
$$+\ \psi_0e^{R(T_0)}\sum_{i=1}^{j}\int_{T_i}^{T_{i+1}}\alpha YP_0e^{R(\tau)-\rho\tau-R(T_i)}d\tau-\psi_0e^{R(T_0)}\sum_{i=1}^{j}\Gamma P_0e^{-\rho T_i}\Bigg]. \tag{A.14}$$

For the permanent shock, with $R(t)=r_2\,t$, this implies

$$\frac{1}{\psi_0}=\rho\Bigg[B_0\,e^{r_2N}+M_0+\alpha Y\int_0^{T_1}P(\tau)d\tau+(1-\alpha)Y\frac{P_0}{\rho}+e^{r_2T_1}P_0\Big(\frac{1}{1-e^{-\rho N}}\Big)\Big(Y\frac{e^{(r_2-\rho)N}-1}{(r_2-\rho)}-\Gamma\Big)\Bigg]. \tag{A.15}$$

Plugging this value in the inequality, $M_0\ge\int_0^{T_1}(\frac{e^{-\rho\tau}}{\lambda_0}-\alpha P(\tau)Y)d\tau$, we obtain

$$M_0+\alpha Y\int_0^{T_1}P(\tau)d\tau\ge(e^{-\rho T_1}-1)B_0\,e^{r_2N}+$$
$$(1-\alpha)YP_0\frac{e^{\rho T_1}-1}{\rho}+e^{(r_2+\rho)T_1}P_0\frac{1-e^{-\rho T_1}}{1-e^{-\rho N}}\Big(Y\frac{e^{(r_2-\rho)N}-1}{(r_2-\rho)}-\Gamma\Big). \tag{A.16}$$

To prove the inequality is unfeasible, we may ignore the term associated with B_0, considering the worst case scenario where $B_0=0$. Using equation (A.5) to replace M_0 we find

$$\Big((1-\alpha)Y-\frac{\Gamma}{N}\Big)P(0)\frac{(e^{(r_1-\rho)T_1}-1)}{(r_1-\rho)}+\alpha\int_0^{T1}P(\tau)d\tau\ge$$
$$(1-\alpha)YP_0\frac{e^{\rho T_1}-1}{\rho}+e^{(r_2+\rho)T_1}P_0\frac{1-e^{-\rho T_1}}{1-e^{-\rho N}}\Big(Y\frac{e^{(r_2-\rho)N}-1}{(r_2-\rho)}-\Gamma\Big). \tag{A.17}$$

This means that either

$$\left((1-\alpha)Y-\frac{\Gamma}{N}\right)P(0)\,\frac{(e^{(r_1-\rho)T_1}-1)}{(r_1-\rho)}\geq(1-\alpha)YP_0\frac{e^{\rho T_1}-1}{\rho} \tag{A.18}$$

or

$$\alpha Y\int_0^{T1}P(\tau)d\tau\geq e^{(r_2+\rho)T_1}P_0\frac{1-e^{-\rho T_1}}{1-e^{-\rho N}}\left(Y\frac{e^{(r_2-\rho)N}-1}{(r_2-\rho)}-\Gamma\right). \tag{A.19}$$

As $P(0)\leq P_0$ and $\Gamma>0$, then, for any $r_1<2\rho$, the first inequality does not hold. Regarding the second inequality, while the right side must be lower than consumption in the interval $(0,T_1)$, the left side of the inequality must be higher for any $r_2>0$. This violates the inequality, which means the interior solution is not feasible.

For the temporary shock, when assuming that, for large values of t, $r(t)\approx 0$, the same result is obtained.

Appendix B: Proof of Proposition 4

Proof: To solve the integral equation it is helpful to recast it as

$$P(t)=f(t)+z(t)\,d(P(t),t) \tag{B.1}$$

with

$$z(t)\equiv(NY)^{-1}\rho\,\alpha\,e^{-\rho t} \tag{B.2}$$

$$d(P(t),t)\equiv\int_t^N\frac{\int_0^n P(\tau)d\tau}{1-e^{-\rho n}}dn. \tag{B.3}$$

A rearrangement of terms in equation (B.1) leads to

$$d(P(t),t)=\frac{P(t)-f(t)}{z(t)}. \tag{B.4}$$

Differentiating equation (B.1) with respect to time yields

$$\dot{P}(t)=\dot{f}(t)+\dot{z}(t)d(P(t),t)+z(t)\dot{d}(P(t),t). \tag{B.5}$$

Replacing equation B.4 in equation B.5 yields

$$\dot{P}(t)=\dot{f}(t)+\frac{\dot{z}(t)}{z(t)}P(t)-\frac{\dot{z}(t)}{z(t)}f(t)+z(t)\dot{d}(P(t),t). \tag{B.6}$$

As $\frac{\dot{z}(t)}{z(t)}=-\rho$ and $\dot{d}(P(t),t)=-\frac{\int_0^t P(\tau)d\tau}{1-e^{-\rho t}}$, equation B.6 leads to

$$\dot{P}(t)=\dot{f}(t)+\rho f(t)-\rho P(t)-z(t)\frac{\int_0^t P(\tau)d\tau}{1-e^{-\rho t}}. \tag{B.7}$$

We may also define $l(t) \equiv \frac{1-e^{-\rho t}}{z(t)}$. Thus, multiplying both sides of (B.7) by $l(t)$ (both $l(t)$ and $(l(t))^{-1}$ are defined and have no real roots for all positive values of t), and differentiating with respect to time we get

$$\ddot{P}(t)l(t) + \dot{P}(t)\dot{l}(t) = \ddot{f}(t)l(t) + \dot{f}(t)\dot{l}(t) + \rho\dot{f}(t)l(t) + \rho f(t)\dot{l}(t) - \rho\dot{P}(t)l(t) - \rho P(t)\dot{l}(t) - P(t). \quad \text{(B.8)}$$

Dividing both sides by $l(t)$ and rearranging terms, equation B.8 yields the ODE presented in Proposition 4.

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