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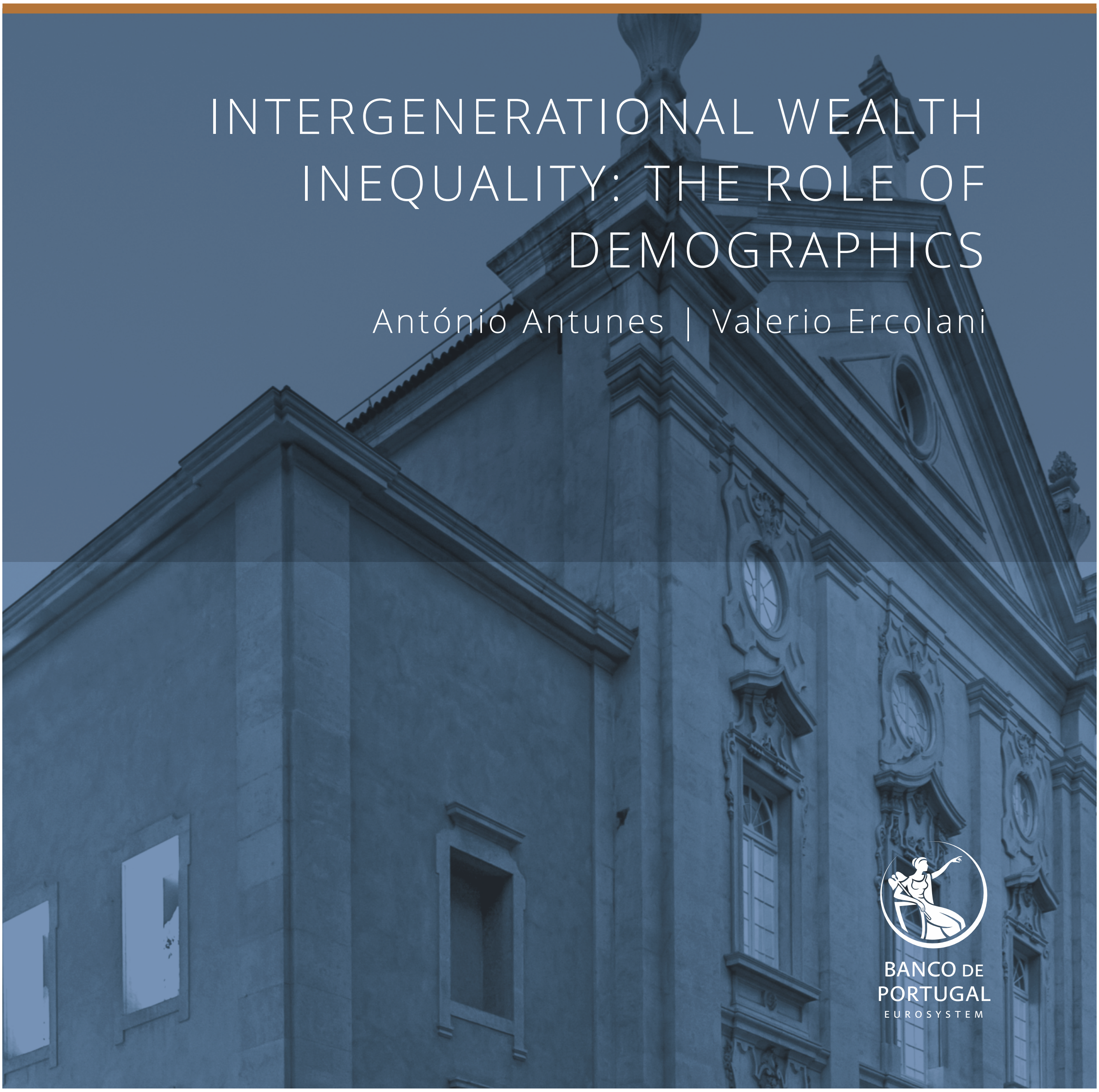

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Intergenerational wealth inequality: the role of demographics

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Abstract
During the last three decades in the US, the older part of the population has become significantly richer, in contrast with the younger part, which has not. We show that demographics account for a significant part of this intergenerational wealth gap rise. In particular, we develop a general equilibrium model with an OLG structure which is able to mimic the wealth distribution of the household sector in the late 1980s, conditional on its age structure. Inputting the observed rise of life expectancy and the fall in population growth rate into the model generates an increase in wealth inequality across age groups which is between one third and one half of that actually observed. Furthermore, the demographic factors help explain the change of the wealth concentration conditional on the age structure; for example, they account for more than one third of the rise of the share of the elderly within the top 5% wealthiest households. Finally, consistent with a stronger life-cycle motive and an increase of the capital-labor ratio, the model produces an interest rate fall of 1 percentage point.

JEL: E21, D15, J1

Keywords: Intergenerational wealth inequality, wealth concentration, real interest rate, demographics.

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1. Introduction

Studying wealth inequality has been central in the recent academic and policy debates (Piketty 2015, among others). We focus on one particular dimension of wealth inequality: that occurring across age groups. Panel A of Figure 1 shows that, in the US, the gap between the average wealth of the older part of the population (above 55 years old) and that of the younger part (20–54) has significantly increased over the last three decades: in 2016 it was more than twice the one observed in 1989.[1] A finer decomposition shows that the pre-retirement group (that is, between 55 and 65 years old) and the retirees have significantly increased their average wealth while the youngest group has decreased it (Panel B). Furthermore, the age structure of the wealthy has changed over time, as the 55+ group are now roughly three fourths of the top 5% wealthiest households, up from only half in 1989 (Panel C). Part of these facts have also been highlighted by business magazines and think tanks; see, among others, Hove (2018) and Taylor *et al.* (2011).

Investigating wealth inequality across age groups and how this projects into the future is important for several reasons. An increasing intergenerational wealth gap coupled with the well-known rise of the old-age-dependency ratio could put at risk the sustainability of the social security system; the younger share of the population, which becomes poorer and poorer, can find it more difficult to finance, through a pay-as-you-go system, the mass of pensions of the retirees, whose number is steadily increasing. Further, it is known that the transmission of monetary and fiscal policies depends on how wealth is distributed across households (see Kaplan and Violante 2014; Kaplan *et al.* 2018, among others). Hence, wealth dispersion or inequality along the age dimension can drive as well the effectiveness of these policies. Understanding the sources of this intergenerational wealth gap is a necessary preliminary step for any analysis or policy dealing with wealth-distributional related issues.

Many causes could have concurred to explain the evolution of this intergenerational wealth gap, including heterogeneous choices in asset allocation, housing prices or the effects of the great recession; see, among others, Glover *et al.* (2019), Taylor *et al.* (2011), and Boshara *et al.* (2015). As for the great recession, we note that, over the period 2007–2010, it significantly influenced the wealth dynamics of both the younger and the older groups, but the sign of its effects on intergenerational wealth inequality is not totally clear from visual inspection of Panels A and B in Figure 1. In fact, the diverging wealth dynamics across age groups had manifested itself already well before the onset of the financial crisis; it could be that long-run factors are at the base of these dynamics. Demographic forces seem a natural candidate, and indeed a direct implication of the life-cycle theories is that part of the wealth distribution observed in the society is explained by the age structure. We hence ask if the demographic changes observed over roughly the last

1. The intergenerational wealth gap rises for both total wealth and financial wealth.

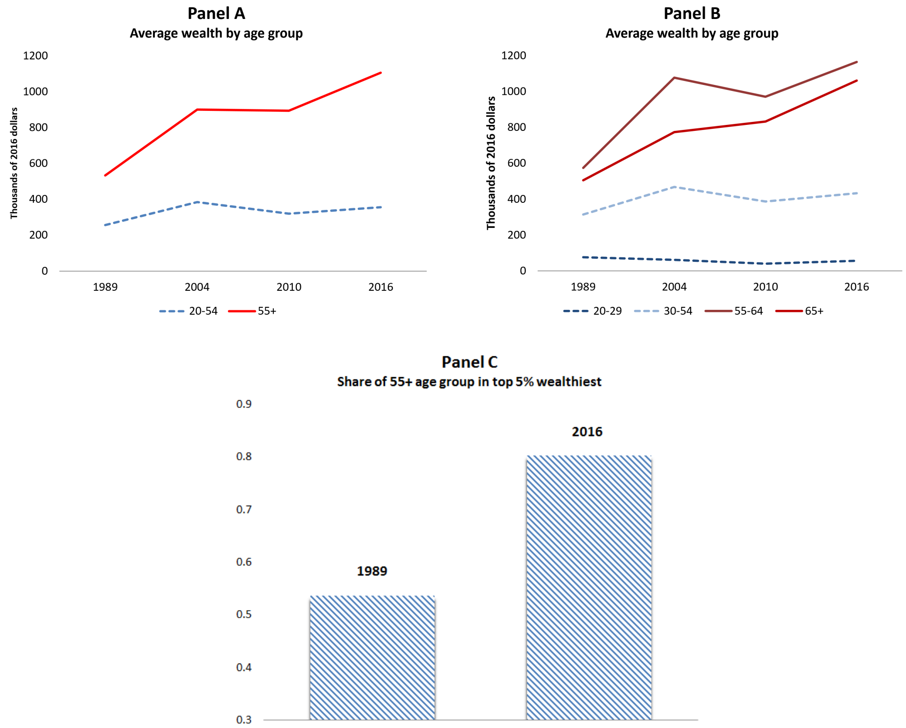

Figure 1: Evolution of the average wealth across age groups and of the age structure of the top 5% wealthiest. Panels A and B show the evolution of the average wealth across age groups. Panel C shows the evolution of the share of the older part of the population (above 55) within the top 5% wealthiest. Data on household net wealth, which is measured in 2016 dollars, are taken from the Survey of Consumer Finances.

three decades, namely increasing life expectancy and a falling population growth rate, have contributed to feed this intergenerational wealth gap and, consequently, to modify the age structure of the household wealth distribution.

We perform our analysis within a general equilibrium, incomplete-markets model with heterogenous households in terms of wealth and productivity which follows the long-standing tradition of Bewley (1986) and Aiyagari (1994). We then superimpose an OLG stochastic structure characterized by five age groups: the "children" or young dependents (0-19 years old), the young workers (20-29), the mature workers (30-54) (who generate children and rear the bulk of them), the pre-retirement workers (55-64) and the retirees (65 and above).[2] Agents in the latter group are at risk of death, in which case they leave bequests. The age structure

2. Each age group corresponds to a specific generation in the sense that it is composed of individuals approximately with the same age. The conflation of the terms "age group" and "generation", while admissible, was avoided whenever possible.

represents a good compromise between complexity and the necessary parsimony to keep the model tractable computationally.

We calibrate the model so as to replicate the US distribution of households' wealth conditional on the age structure observed in 1989. This economy is characterized by a life expectancy of 75 years and a population yearly growth rate of 1%. Assuming that the young dependents do not possess any wealth, our benchmark calibration matches the fact that the young and mature workers are significantly wealth-poorer than retirees and pre-retirement workers, with the latter being the wealthiest. Given the parsimonious parametric structure of the model, the joint distribution of wealth and age is in our view an encouraging sign of the relevance of the particular modeling choices we made to tackle our research question.

Inputting the observed rise of life expectancy (from 75 to 80 years) and the fall in population growth rate (from 1 to 0.7%) into our model produces a new stationary equilibrium with the following characteristics. First, the age structure tilts towards the elderly. The share of retirees (workers) increases (decreases) in such a way that the old-age-dependency ratio (the ratio between retirees and the working-age population) rises from 0.18 to 0.27, which is consistent with the data. Second, the demographic changes generates a rise in the wealth inequality across age groups which is between one third and half of that actually observed during roughly the last three decades. Third, the increase of the share of the old part of the population (55 years old and above) is more prominent among the wealthier classes of the population than among the poor; for example, the demographic forces explain more than one third of the observed increase of the share of the 55+ group among the 5% wealthiest households. Finally, the real interest rate falls by 1 percentage point. Notice that, consistently with the observed evidence over the period under scrutiny, throughout all simulations we keep constant the payroll tax rates paid by workers and firms that finance the pension system.

One can argue that the larger share of the elderly in the economy mechanically drives the increase in wealth inequality across age groups. To investigate that issue, we decompose the increase of wealth inequality across age groups into two factors: change in wealth, which is generated by the reactions of the households to the demographic changes; and change in population, which is an exogenous force in the context of our model. The former explains most of the rise in wealth dispersion, leaving a minor role to population. This is true both in the data and in our simulations.

Various channels fuel the above-described results. Three of them—life-cycle motives, the change in the disposable income of retirees, and price effects—play a crucial role. Living longer pushes the saving of each age group up. However, the pre-retirement group saves more than the younger groups because the former are closer to retirement and, being the wealthiest in the economy, have a higher propensity to save. Furthermore, the disposable income of retirees falls significantly because (i) asset income is lower due to the fall in the interest rate, and (ii) the pension income is lower due to a non trivial fall in the replacement rate. Pre-retirement workers, above everyone else, save more in order to guarantee a smooth profile of consumption when they retire.

The demographic process generates lower (higher) interest (wage) rates, which induce heterogeneous effects across age groups. While the lower interest rate produces a less steep consumption path over the life cycle, it also generates a negative income effect for both the pre-retirement group and the retirees, who rely more on income from saving. Both groups react to that by further cutting consumption; the pre-retirement group also saves and works more. On the other hand, the age group of the youngest workers, which is also the poorest and hence relying more on labor income, experiments a positive income effect through higher wages; it consumes more while working less. In addition to that, a higher wage could act as if it relaxed the borrowing constraint of the poorest, who are also the youngest, allowing their level of precautionary saving to decrease. Prices effects thus boost the saving rate of the pre-retirement group while decreasing the saving rate of the young.

Finally, we would highlight another effect. Because of the fall of the population growth rate caused by the lower birth rate, there is an alleviation the burden of rearing children, especially for the mature workers, who can hence consume more and save less.

Notice, wealthy workers—who are also the oldest workers in our context—save more while poor workers—who are also the youngest—save less or borrow more. This is compatible with the "saving glut of the rich" narrative highlighted by Mian *et al.* (2019), that is, over the last four decades in the US, national saving was fueled by the richest portions of the population.

It is worth noting that the saving motive of the elderly seems almost unaffected by all the highlighted mechanisms; this is because retirees have a rather limited scope of actions to smooth consumption, as they essentially deplete over time their previously accumulated wealth after setting their optimum level of bequests.

All the described effects contribute to increasing the wealth of pre-retirement workers, but also of retirees, who indeed benefit from the marked building up of assets during the pre-retirement phase. Instead, the wealth of the two younger groups decreases. These actions magnify intergenerational wealth inequality and make the presence of the elderly more prominent in the richest portions of the population.

As for the interest rate, its change is consistent with the behavior of the variables both at the aggregate and the individual level. While a stronger life-cycle motive generates an appetite for assets among households, thus depressing the interest rate, the decrease of the share of workers makes physical capital less productive, reinforcing the fall in the interest rate.

Our work is connected to several papers that try to understand both qualitatively and quantitatively the causes of wealth inequality. De Nardi (2004), De Nardi and Yang (2016), and De Nardi and Fella (2017) study wealth inequality at one point in time. They find that mechanisms like bequests or intergenerational correlation in ability help match observed wealth inequality.

Other papers, like Hubmer *et al.* (2019), Kaymak and Poschke (2016), Gabaix *et al.* (2016) and Heathcote *et al.* (2010), study wealth inequality over time. An important finding is that wage dispersion and decreasing tax progressivity explain a relevant part of the observed increase of wealth inequality over the last 50 years. While these papers mainly target unconditional wealth inequality, we target intergenerational wealth dispersion and ask if demographics can be a driver thereof. Vandenbroucke (2016) is to our knowledge the first scholar to use a simple OLG model to investigate the effects of demographics on unconditional wealth inequality. He finds that the sign of the effect of the whole demographic process on wealth inequality is ambiguous. Other than having a different target from ours, his model abstracts from relevant economic characteristics—such as idiosyncratic income uncertainty, progressive taxation, and social security—which happen to be crucial for answering our quantitative questions. Di Nola and Ferrari (2018) study how a worsening of the economic outlook affects wealth inequality through influencing both birth rates and intergenerational transfers. Our objective is different in that we want to quantify by how much slow moving forces, like demographics, explain the observed changes in both the wealth intergenerational gap and the age structure of the wealthy.

There is a lively stream of the literature that studies the effects of demographics on the interest rate. Among others, Carvalho *et al.* (2016) and Papetti (2019), using OLG models, and Ferrero *et al.* (2019), using a purely empirical approach, show that demographic forces have contributed to depress the real interest rate in the previous decades.[3] Our findings support their results through the lens of a model with an endogenous wealth distribution. Specifically, Carvalho *et al.* (2016) find a fall in the rate of roughly 1.5 percentage points while Papetti (2019) finds a value around 1.3 percentage points, both close to our own results. Very recently, Auclert *et al.* (2020) use an OLG framework applied to several countries, including the US, to show that population ageing significantly contributes to explain the evolution of wealth-to-income ratios, real interest rates and global imbalances.

The paper is structured as follows. Section 2 presents the model. Section 3 presents the results for benchmark economy calibrated to the 1989 US economy. Section 4 reports the main results about the effects of demographics on the economy and uncovers the channels at the base of these results. Section 5 concludes.

3. Carvalho *et al.* (2016) study the effects of the demographics on the real interest rate using a two-agent (young and old) model applied to an area of selected developed countries (including the US), between 1990 and 2014. Papetti (2019) shares the same objective but uses a large-scale OLG model applied to the euro area between 1990 and 2030.

2. Model

To tackle the questions we are interested in we use a heterogeneous-agent, incomplete-market model with an overlapping generations structure that features demographic growth. This implies modeling two of the most important characteristics of demographics: the fertility rate and the mortality rate, or equivalently the birth rate and life expectancy. The generational burden stemming from fertility and mortality, that is, the allocation of resources to children and retirees, is important because it directly affects consumption and saving of all households. Furthermore, the model features a bequest motive which helps to discipline the intergenerational wealth profile. To keep the model manageable, we leave out features such as migration and intra-household decisions related to fertility, which is taken as exogenous, and labor supply in the extensive margin.

2.1. Formal description

We use a standard Aiyagari-Bewley-Huggett model with elastic labor supply and a demographic structure superimposed. Time is discrete and each period is one year.

Utility. Each agent is endowed with one unit of time, to be split between labor n and leisure $1-n$. Agents benefit from consumption c and dislike labor according to utility function

$$u(c,n) = \frac{c^{1-\gamma}-1}{1-\gamma} + \psi\frac{(1-n)^{1-\eta}}{1-\eta}\,.$$

The vector of parameters (γ, η, ψ) will be defined so as to match the equilibrium behavior of model agents to empirically observed outcomes.

Production. Each agent rents labor at rate w per efficient unit to a representative firm. The agent's labor idiosyncratic productivity has two components. One takes values in a finite set with n_z elements, that is, $z \in \mathcal{Z} = \{z_1, \dots, z_{n_z}\}$, and follows an exogenous Markov process described by the n_z-by-n_z transition matrix Π^z. The other component is age specific and will be described below. The representative firm has production function $Y = AK^{\alpha}N^{1-\alpha}$ and chooses efficient labor, N, and capital, K, taking factor prices as given, according to:

$$r^K = \alpha A\left(\frac{N}{K}\right)^{1-\alpha}, \text{ where } r = r^K - \delta \tag{1}$$

$$(1+\tau_f)w = (1-\alpha)A\left(\frac{K}{N}\right)^{\alpha}, \tag{2}$$

where A is total factor productivity (TFP) and τ_f is the rate of the social security contribution by the firm; see more details below in the paragraph about social policies.

Demographics. Each agent faces a lifetime expectancy at birth of l and goes through n_g age groups. Age groups 2 through n_g-1 correspond to the time during which the agent works. The first age group is composed of the young, non-working agents, or while the last age group corresponds to the retirement period. The processes of aging and dying are stochastic and governed by the n_g-by-(n_g+1) matrix

$$\Pi^{\mathsf{pop}} = \begin{bmatrix} 1-\pi_{1,2}-\pi_{1,n_g+1} & \pi_{1,2} & 0 & 0 & \cdots & 0 & \pi_{1,n_g+1} \\ 0 & 1-\pi_{2,3}-\pi_{2,n_g+1} & \pi_{2,3} & 0 & \cdots & 0 & \pi_{2,n_g+1} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & 1-\pi_{n_g,n_g+1} & \pi_{n_g,n_g+1} \end{bmatrix}$$

where $\pi_{g,g+1}$ is the probability of aging in age group g and it is understood that the last column contains the probabilities of dying per period of each age group. We assume, consistently with the data, that the mortality of age groups other than the last's is negligible, so that $\pi_{g,n_g+1} = 0$ for all $g < n_g$.

Population grows at rate p per period, the crude birth rate is b and the crude death rate is d, so that $p = b - d$. Let λ_g be the population of age group g as a share of total population. Using the standard notation of denoting next period's quantities by an apostrophe, population dynamics is described by

$$(1+p)\lambda_1' = \lambda_1(1-\pi_{1,2}) + b \tag{3a}$$

$$(1+p)\lambda_g' = \lambda_g(1-\pi_{g,g+1}) + \lambda_{g-1}\pi_{g-1,g} \quad \text{for } 1 < g < n_g \tag{3b}$$

$$(1+p)\lambda_{n_g}' = \lambda_{n_g} + \lambda_{n_g-1}\pi_{n_g-1,n_g} - d\,. \tag{3c}$$

Demographic projections often assume a certain evolution for life expectancy at birth and the fertility rate. We define life expectancy at birth as the mean length of life of a person assumed to have, from birth through death, the same age-specific mortality rates as currently observed. The fertility rate is defined as the average number of children born to a woman over her lifetime assuming she keeps current age-specific fertility rates through her lifetime, and survives from birth through the end of her reproductive life. Life expectancy at birth and the fertility rate of the agent are given by

$$l = \sum_{g=1}^{n_g} \frac{1}{\pi_{g,g+1}} \tag{4}$$

$$f = \sum_{g\in\mathcal{R}} \frac{b_g}{\pi_{g,g+1}} \tag{5}$$

where $\mathcal{R}$ is the set of age groups with reproductive capacity and b_g is the birth rate of age group g, defined as the number of children born per household in that age group during one period.

Households. A household is initiated when an agent ages from the first to the second group, and is dissolved when the agent dies. Households have to cater for the consumption of children. For simplicity we assume that the number of children within a household can be fractional, which corresponds to assuming that all identical households collectively support their children's consumption. Each household is therefore composed of one adult agent and a fractional number of children. We further assume that children consume the same amount as the adult in their household. Upon death of the household's head, its children are randomly assigned to another household.

The model household structure is a simplification of its real counterpart: there is no matching between agents; there is no intra-household decision-making; the number of children is homogeneous across households with the same wealth, age and ability. Explicitly modeling these margins would considerably complicate the model. However, the model retains the two fundamental demographic features we want to capture: one is the overlapping generations structure; and the other is the extra burden of rearing children that befalls on specific age groups.

Specifics of demography. In order to account for the main phases of an agent's life, we divide it in five age groups. The first encompasses individuals between zero and 20 years old, which we call "children" or young dependents. The common characteristic among them is that they do not earn income, possess no wealth, and depend on their parents. The second age group includes individuals between 20 and 30 years old. The salient feature of these workers is that, while working, they are assumed to not have started reproducing. The third age group spans individuals between 30 and 55 years old. They constitute the bulk of the working force and are the ones to whom children are born, as in, for example, De Nardi and Yang (2016), who assume that children are born to agents at age 35. As a consequence, agents in this age group have the burden of rearing most of the individuals of the first group. The fourth group includes pre-retirement workers. Finally, the fifth group is constituted by retired workers. These individuals do not work and have non-zero death probability.

Age also determines part of an agent's productivity, which is a common feature in the life-cycle productivity literature and a stand-in for labor experience (see, for example, Huggett 1996; Guvenen 2007). Total idiosyncratic productivity is the product of a persistent idiosyncratic component, z, which is described in a previous paragraph, and an age-specific productivity level, $h \in \{h_g\}_{g \in \mathcal{G}}$, where $\mathcal{G} = \{1, \ldots, n_g\}$. For convenience, we assume that $h_1 = h_{n_g} = 0$.

Inheritance. The assets of agents that die in the current period are transmitted to agents alive in the next period. In each period, each individual inherits with probability p_b and gets zero bequest otherwise; consequently, a household inherits with probability $p_b w_g$, where w_g is the size of the household including its dependents, a quantity we also call below "burden". The larger the household, the higher the probability of inheritance. In case the agent inherits, the bequest amount is taken from the distribution of wealth of agents who died during the transition to the current period. No negative bequests are allowed and there is an insurance scheme run by the Government to compensate creditors of individuals who die in debt. An agent can age and die without ever receiving inheritance. Elderly agents formulate savings and consumption decisions based on a bequest motive, as in

De Nardi and Yang (2016), subsumed in function

$$\varphi(a) = b_1 \frac{(b_2 + a)^{1-\gamma}}{1-\gamma}$$

where a is the bequest amount. Parameter b_1 determines the strength of the bequest motive, while parameter b_2 sets the degree to which bequest is considered a luxury good. The curvature parameter is set at the same value as for consumption utility.

Credit. Agents can borrow and lend riskless securities supplied by the Government, with a maturity of one period and which are perfect substitutes of capital. Total supply B of these bonds is kept constant throughout. An agent entering the period with a units of the security receives $(1+r)a$ units of the consumption good. Agents can borrow up to $\underline{a}$ so that $a' \geq -\underline{a}$, where a' is the amount of securities acquired in the current period and maturing in the next. For simplicity, we assume that the space of bond holdings has an upper limit sufficiently large so as not to constraint the agents' decisions in equilibrium, that is $a \in \mathbb{A}$, where $\mathbb{A} = [-\underline{a}, \overline{a}]$ and $\overline{a} \gg 0$. This hypothesis will have to be confirmed numerically.

Social policies. In most advanced economies, pensions are financed through private or public schemes autonomously from the general Government budget. We follow Jeske and Kitao (2009) and assume that retirees benefit from a pension financed by social security contributions from workers and firms. In the US these contributions are flat-rate taxes on labor income paid by workers and firms, which we denote by τ_w and τ_f, respectively. Actual social security programs in advanced economies are typically history dependent. In order to avoid an additional high-dimensional state variable, we adopt the simplification that the pension received by a beneficiary is a function of their productivity at retirement and of an endogenously determined replacement rate. This assumption allows us to retain the main characteristic of pension schemes: transfers do not depend on retirees' decisions. Social policies are then specified by the function

$$T(z, n_g) = m_r z h_{n_g - 1} \overline{n}(z, n_g - 1) w \tag{6}$$

where m_r is the replacement rate and $\overline{n}(z, n_g - 1)$ is the average labor effort of current pre-retirement workers with the same level z as that of the pensioner at retirement. Consistently, we assume that the agent's ability level z is frozen upon retirement. As we restrict social programs to pensions, $T(z, g) = 0$ if $g < n_g$.

Taxation. There is a tax system which finances general government spending. We abstract from estate taxation and consider taxation of income summarized by function $\tau(y)$, taken from Gouveia and Strauss (1994),

$$\tau(y) = \tau_1 \left[1 - (\tau_2 y^{\tau_3} + 1)^{-\frac{1}{\tau_3}}\right]$$

for gross income $y \geq 0$. Guner *et al.* (2014) show that this function accounts well for the structure of the US tax system; they also provide estimates for the parameters.

The Government purchases a certain amount G of output so that the debt level B is constant and consistent with the level of taxes collected.

Generational burden. Define w_g as the burden of agents of age group g, that is, the ratio between all agents on that age group and all its workers. Agents of the first age group (0–19 years old) whose parents are alive depend on agents of the third group (30–54 years old) and older. For simplicity, we will assume that agents of the first age group whose parents died—the young orphans—are assigned randomly to all other households. To compute the burden of each age group, with a slight abuse of notation let us define λ_{g,g_p} as the fraction of agents in age group g whose parents are in age group $g_p \in \{\mathcal{G}, n_g + 1\}$, where $g_p = n_g + 1$ denotes deceased parents. The burden of age group $g \in \{2, \ldots, n_g\}$ is given by:

$$w_g = (1 + o)\frac{\lambda_{1,g} + \lambda_g}{\lambda_g}\,, \tag{7}$$

where $1 + o$ accounts for the extra burden of young orphans and it is understood that $\lambda_{1,2} = 0$. One also has, by definition, $w_1 = 0$. Imposing the equality $\sum_g w_g \lambda_g = 1$ implies that $1 + o = \left(1 - \lambda_{1,n_g+1}\right)^{-1}$.

Optimization problem of the agent. Let $V(a, z, g)$ be the lifetime utility of an agent with asset holdings a and labor productivity z in age group g. We assume that the observed ability of the agent does not change once it enters retirement, so as to make its social security transfers dependent on its labor income prior to retirement.

Defining $x = (z, g)$, the agent's optimization problem for age groups 2 to n_g is given by

$$\begin{aligned} V(a, x) = \max_{c,a',n} u(c, n) + \beta\,\mathbf{E}\left[(1 - \pi_{g,n_g+1})((1 - p_b w_g)V(a', x')\right. \\ \left. + p_b w_g V(a' + \hat{e}'_n, x')) + \pi_{g,n_g+1}\varphi(a')|x\right] \end{aligned} \tag{8}$$

subject to

$$\begin{aligned} y &= ra + zh_g nw + T(x) \\ g(y) + a &\geq w_g c + (1 + p)a' + \tau_w z h_g nw \\ a' &\geq -\underline{a} \\ \hat{e}'_n &= \max\{0, \hat{e}'\} \\ \hat{e}' &\sim \lambda'(\cdot|n_g) \end{aligned}$$

where $g(y) = (1 - \tau(y))y$ is income net of taxation and $\tau(y)$ is the average tax rate paid out of income y. $\hat{e}'$ is the estate left to the agent in case it inherits; its value is taken from next period's asset distribution of all agents currently at risk of death, $\lambda'(\cdot|n_g)$. If $\hat{e}'$ is negative, the agent can renege on it. The final estate delivered to the agent is $\hat{e}'_n$.

The expectation conditional on x is governed by the transition matrices Π^z and Π^{pop}. Note that $h_{n_g} = 0$ forces labor income of retirees to be zero; since labor effort is disliked, they do not work.

2.2. Equilibrium

The steady-state equilibrium of this economy is defined as follows. Let $\mathcal{B}(X)$ be the Borel σ-algebra of $X = \mathbb{A} \times \mathcal{Z} \times \mathcal{G}$. Given a transition matrix Π^z for idiosyncratic productivity, a transition matrix Π^{pop} for population across age groups, a set of government policies (τ_w, τ_f, B, G), and a credit constraint $\underline{a}$, we define a *recursive competitive equilibrium* as a belief system H, a pair (r, w) of prices, a replacement rate m_r, a probability of inheriting p_b, a measure defined over the set of possible states $\lambda : \mathcal{B}(X) \to [0, 1]$, a joint measure of agents and parents $\{\lambda_{g,g_p}\}_{g \in \mathcal{G}, g_p \in \{\mathcal{G}, n_g+1\}}$ with the associated burdens $\{w_g\}_{g \in \mathcal{G}}$, and individual policy functions $c = f_c(a, x)$, $a' = f_a(a, x)$ and $n = f_n(a, x)$ such that:

a) The individual policy functions solve problem (8);
b) The representative firm maximizes profits taking prices and social security contributions as given, that is, (1) and (2) hold;
c) The bond market clears,

$$\int f_a(a, x) d\lambda = K + B\,;$$

d) The labor market clears,

$$\int z h_g f_n(a, x) d\lambda = N\,;$$

e) In each period the number of bequests equals the number of recipients:

$$\sum_g \pi_{g,n_g+1} \lambda_g = p_b \sum_g (1 - \pi_{n_g,n_g+1}) w_g \lambda_g\,;$$

f) The general Government constraint holds,

$$rB + G + \pi_{n_g,n_g+1} \int_{f_a(a,z,n_g)<0} f_a(a, z, n_g) d\lambda(\cdot, \cdot, n_g) = \int \tau(y) y d\lambda\,;$$

g) The pension scheme is solvent,

$$(\tau_w + \tau_f) w \int z h_g f_n(a, x) d\lambda = \int T(x) d\lambda$$

where transfers policy $T(x)$ is given by (6);
h) The belief system H is consistent with the aggregate law of motion implied by the individual policy functions;
i) The measure λ is constant over time.

Some details on the computational method for calculating the equilibrium are in Section A of the Appendix.

	share of age group					65+ dep. ratio
	0-19	20-29	30-54	55-64	65+	
data 1989	0.29	0.17	0.33	0.09	0.13	0.19
model	0.33	0.15	0.30	0.11	0.10	0.18

Table 1. Age structure in the model and in the data. Shares of the different age groups are reported. The ratio between the number of retirees (age 65 and above) and the working-age population is also shown. Data values are taken directly from United Nations (2017).

3. Benchmark calibration

The model was calibrated using a mix of off-the-shelf parameter values and procedures aimed at specific targets of the real US economy in the late 1980s. We started our analysis in 1989 because it represents the first date for which the Survey of Consumer Finances has sufficient detail for the purposes of our analysis. Furthermore, going further back in time would amount to comparing economies that could have very different structures, and not only with respect to the demography; for example, Figure 4 in Guner *et al.* (2014) shows that, in the US, the degree of tax progressivity hardly changed between 1989 and 2000, while it was very different in 1980.

The first set of parameters describes the demography during that period. Given the age structure described before, composed of five age groups, matrix Π^{pop} is given by:

$$\Pi^{\text{pop}} = \begin{bmatrix} 0.95 & 0.05 & 0 & 0 & 0 & 0 \\ 0 & 0.9 & 0.1 & 0 & 0 & 0 \\ 0 & 0 & 0.96 & 0.04 & 0 & 0 \\ 0 & 0 & 0 & 0.9 & 0.1 & 0 \\ 0 & 0 & 0 & 0 & 0.9 & 0.1 \end{bmatrix}.$$

Notice that Π^{pop} and equation (4) imply a life expectancy of 75 years. The set of equations (3), together with the assumption of a population growth rate p of 1 percent a year, the fact that only retirees die in our model ($d = \pi_{n_g,n_g+1}\lambda_{n_g}$), and the normalization $\sum_{g=1}^{n_g} \lambda_g = 1$ imply that the crude death rate and the crude birth rate are $d = 0.01$ and $b = 0.02$, respectively. The shares in the total population of the different age groups match quite well the actual shares observed in 1989; see Table 1. The old-age dependency ratio, that is, the ratio between retirees and the working-age population, is 0.18 in our model, very similar to the observed number. Overall, this represents a good match despite our model having some simplifying assumptions regarding age-specific fertility rates, absence of migration, and absence of infertile households. Relaxing these assumptions would allow us to closely match the aggregate US values, as the demographic equations in the model are accurate for large numbers.

A second set of parameters was chosen using a standard approach. In the case of the curvatures of the utility function, γ and η, we used conventional off-the-shelf values. The intercept of the leisure component of utility, ψ, was set so that households spend on average $1/3$ of their time working. Time discount β was picked so that the equilibrium real return on capital is 4 percent. The intercept of the production function was normalized to one, and the share of capital in production was set to $1/3$. The depreciation rate of capital, δ, was taken to be that value which yields an equilibrium capital-to-output ratio of 3.

As for the fiscal system parameters, we first set the firm and worker social security contribution out of labor income, τ_f and τ_w, to their statutory values of the US economy during the late 1980s. We obtain a replacement rate of pensions relative to the last wage income of retirees of 41 percent, which compares with the actual value of 43.5% reported by Social Security Administration (2014).[4] The parameters of the Gouveia and Strauss (1994) tax function were chosen so as to reproduce the income tax schedule reported by Guner *et al.* (2014) during the same period. The ratio of the government purchases to output is set to 0.17 so that the steady-state value of public debt is 60% of GDP, close the levels of the early 1990s. Part of this information is included in Table 2.

The last set of parameters were chosen so as to match as well as possible a set of moments of the wealth distribution. The counterpart of wealth in the US economy we use is net wealth, defined as the difference between assets and liabilities of a household. To discipline the overall behavior of idiosyncratic shocks while keeping the number of parameters manageable, we assume that the log of the idiosyncratic labor productivity of agents follows a 5-state Rouwenhorst-discretized AR(1) process with persistence parameter $\rho = 0.98$ and unconditional variance $\sigma^2 = 0.72$, as in Krueger and Perri (2005), augmented by a low-probability, high-productivity, persistent state independent of the other states.

There are thus eight free parameters remaining: three from the extra state of the idiosyncratic labor productivity (probability of occurrence, persistence, and level); two from the age-specific profile $\{h_g\}_{g=1,\ldots,n_g}$, as there are three levels for each working age group but one can be arbitrarily set as a normalization; two from the bequest function; and the borrowing limit $\underline{a}$. The eight parameters were jointly chosen so as to minimize the difference between a set of model-based moments and their empirical counterparts. The moments are (i) the average net wealth of each age group, and (ii) the fraction of total wealth held by households delimited by quantiles 20, 40, 60, 80, 90, 95 and 99 of the wealth distribution.[5]

There are more targets than parameters so the final calibration is a compromise between the different targets. Some of the parameters have a very large impact on specific targets. For example, the age-specific productivity levels mostly affect the age profile of wealth for working age groups; and the bequest function parameters

4. The replacement rate varies with individual labor income. To keep the model manageable we use a single replacement rate as in Jeske and Kitao (2009). The reported number refers to the replacement rate for "medium" earnings as estimated by Social Security Administration (2014) in 1990.

5. We exclude the group of young dependents, which in the US data is shown to possess an extremely tiny fraction of total wealth.

Parameter	Value	Target/source/comment
β	0.9812	Match real interest rate 4 percent
γ	2	Assumed
η	2	Assumed
ψ	8.4	Average labor supply 0.33 of time
α	0.33	Assumed
A	1	Normalization
δ	0.0711	K/Y of 3
b_1	4000	Moments of wealth distribution
b_2	30	Moments of wealth distribution
$\underline{a}$	-0.1	Moments of wealth distribution
τ_f	0.062	US social security
τ_w	0.062	US social security
τ_1	0.5	Match tax schedule (Guner et al 2014)
τ_2	1.1	Match tax schedule (Guner et al 2014)
τ_3	0.964	Match tax schedule (Guner et al 2014)
G/Y	0.17	B/Y of 0.6

Table 2. Summary of benchmark calibration.

affect the drop in wealth of the retirees relative to pre-retirement agents and the concentration of wealth in the top percentiles; the borrowing limit significantly affects the fraction of wealth held by the first quintile of wealth. Part of the final outcome of this process is summarized in Table 2 and in the following expressions:

$$\Pi^z = \begin{bmatrix} 0.9601 & 0.0388 & 0.0006 & 0.0000 & 0.0000 & 0.0005 \\ 0.0097 & 0.9604 & 0.0291 & 0.0003 & 0.0000 & 0.0005 \\ 0.0001 & 0.0194 & 0.9605 & 0.0194 & 0.0001 & 0.0005 \\ 0.0000 & 0.0003 & 0.0291 & 0.9604 & 0.0097 & 0.0005 \\ 0.0000 & 0.0000 & 0.0006 & 0.0388 & 0.9601 & 0.0005 \\ 0.0200 & 0.0200 & 0.0200 & 0.0200 & 0.0200 & 0.9000 \end{bmatrix}$$
$$z = \begin{bmatrix} 0.1834 & 0.4283 & 1 & 2.3348 & 5.4514 & 30 \end{bmatrix}$$
$$z_g = \begin{bmatrix} 0 & 0.3 & 0.8 & 1.8 & 0 \end{bmatrix}.$$

Further, Figure 2 depicts the profile of the average wealth across age groups, both for the model and the US economy using data from the Survey of Consumer Finances. Unsurprisingly, the model age profile of wealth is very similar to the data given the free parameters in age-specific labor productivity and, in the case of retirees, the bequest motive parameters.

Table 3 compares model and data in terms of non-targeted wealth statistics across age groups: the Gini coefficient and the dispersion index. Define first the mass of households in age group $g \in \{2, \ldots, n_g\}$ renormalized to 1 as $\overline{\lambda}_g = \left(\sum_{j=2}^{n_g} \lambda_j\right)^{-1} \lambda_g$. The Gini coefficient across age groups is obtained by sorting age groups by average wealth $\overline{a}_{\tilde{g}}$, where $\tilde{g} \in \{2, \ldots, n_g\}$ is a permutation of indices

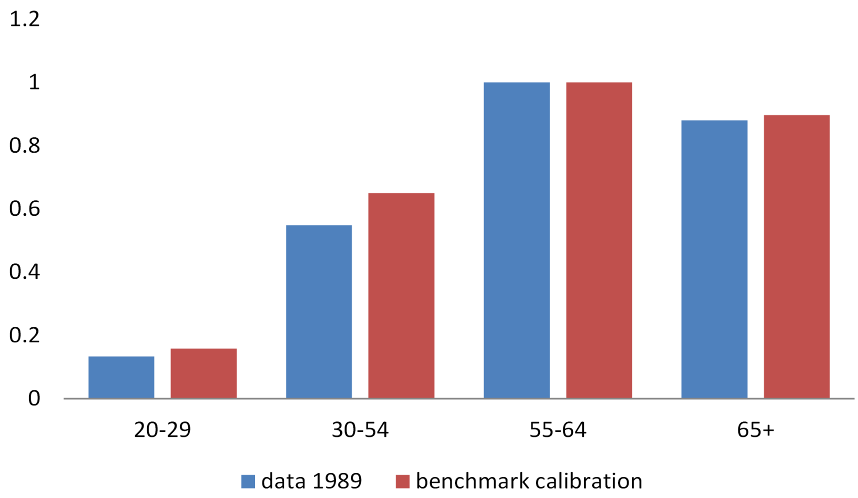

Figure 2: Average wealth across age groups in the data and in the model. Data on household net wealth is measured in 2016 dollars and taken from the Survey of Consumer Finances. The wealth of each age group is normalized to that of the richest group in the economy, which is the pre-retirement group (55-64) both in the data and in the benchmark calibration.

	wealth distribution across age groups		
	Gini	dispersion index	55+ in top 5%
data 1989	0.235	0.64	0.53
model	0.243	0.67	0.48

Table 3. Conditional wealth distribution by age group, in the model and in the data. The Gini coefficient and the dispersion index for the wealth distribution conditional on age groups are calculated as explained in the text. The share of the elderly (age 55 and above) among the top 5% wealthiest is also shown.

such that $\overline{a}_{\tilde{g}} \leq \overline{a}_{\tilde{g}+1}$, and then applying the Gini formula for discrete data,

$$1 - S_{n_g}^{-1} \sum_{\tilde{g}=2}^{n_g} \overline{\lambda}_{\tilde{g}} (S_{\tilde{g}-1} + S_{\tilde{g}}) \tag{10}$$

where $S_{\tilde{g}} = \sum_{j=2}^{\tilde{g}} \overline{\lambda}_{\tilde{j}} \overline{a}_j$ with $S_1 = 0$. The dispersion measure is the population-weighted standard deviation of log average wealth by age group

$$\sqrt{\sum_{g=2}^{n_g} \overline{\lambda}_g \left(\ln(\overline{a}_g) - \sum_{h=2}^{n_g} \overline{\lambda}_h \ln(\overline{a}_h) \right)^2}. \tag{11}$$

Notice that we exclude age group 0–19 from these computations.

The model-implied dispersion measures of wealth across age groups are similar to the ones obtained from data. This is foreshadowed by the results in Figure 2

	Gini	quintiles					top (%)		
		1st	2nd	3rd	4th	5th	90-95	95-99	99-100
data 1989	79	-0.2	1.2	5.2	13.0	80.7	12.9	24.3	29.9
model	81	-0.7	0.3	4.0	13.3	83.0	14.7	28.9	24.3

Table 4. Wealth held by population groups sorted by wealth, in the model and in the data. Gini coefficient of the unconditional wealth distribution also shown. Data are from the Survey of Consumer Finances. All values in percentage.

	age groups			
	20-29	30-54	55-64	65+
burden	1.05	1.76	1.53	1.37
saving intensity	0.09	0.08	0.23	-0.25
labor intensity	1.19	0.97	0.78	0.00

Table 5. Selected characteristics at the household level by age group. Saving intensity is the average saving for each age group normalized to per capita output. Labor intensity is the average labor effort for each age group normalized to the average labor effort of the economy. The burden of each age group is defined by equation (7).

but these values give a sense of the numerical similarity between the model and the data. Another independent statistic is the share of agents above 55 years old in the top 5% wealthiest households. This value is roughly 1/2, both in the model and in the data.

Table 4 presents statistics closely related to targets of the calibration related to the unconditional wealth distribution. The moments of the unconditional wealth distribution are hard to match and we do see some discrepancies in specific moments. However, the model does a good job at mimicking the overall wealth inequality summarized by the Gini coefficient and the fact that the wealth distribution is strongly skewed to the right. The match is overall reasonable in view of the relatively small number of free parameters compared to the targets of interest.

Table 5 reports other characteristics at the household level. The burden of the mature workers' age group (30–54 years old) is the largest due to the assumption that it is the only group that generates children. The corresponding number of children per household for this age group is 1.52, which is close to the observed value of 1.40 extracted from the Survey of Consumer Finances. The table conveys additional interesting information. As expected, the younger (and generally poorer) age groups are characterized by a stronger labor effort than the pre-retirement agents, which are the most wealthy and thrifty. These facts are consistent with the typical shape of the consumption and labor policy functions within the class of the incomplete-markets heterogeneous-agents models. The retirees dissave, although at a much lower rate than if bequest motives were absent.

4. The effects of the demographic change

In this section we describe the effects of the demographic changes through our model. In particular, starting from our benchmark calibration we change only the parameters that control the demographic structure of the economy and let the new economy adjust to that. In brief, we produce two further steady-state simulations; the first features longer life expectancy; the second encompasses the whole demographic process, which features both lengthier life and lower population growth rate.

We report steady-state simulations since we are interested in comparing the economy in two relatively distant moments, more than studying the transition between these two moments. And while the transition is computationally feasible, it would still pose some numerical challenges in terms of convergence.

Further, we also note that there is a lot of similarity between the demographic features of the final steady state and those currently observed for the US economy. This is true not only in terms of life expectancy and population growth rate—as would be expected given that these two values are targeted in the calibration of the final steady state—but also in terms of the relative size of each age group and the old-age-dependency ratio; see section 4.1. This very close mapping between the model demography in the final steady state and the current US demography suggests that comparing steady states is reasonable. Moreover, agents entering the labor force in 1989 will already take into account in their saving and consumption decisions the old-age-dependency ratio, life expectancy, and the generational burden that will determine their pension outcomes thirty or forty years later; hence, the final steady state is, so to speak, front-loaded in the transition. Cooley and Henriksen (2018) make a similar argument for comparing steady states.

Notice that, consistently with the observed evidence over the period under scrutiny, we keep the payroll tax rates paid by workers and firms constant throughout all simulations.

The first part of the section describes the changes of the age structure; the second reports the main results in terms of wealth inequality and concentration, and prices; the third investigates the channels or mechanisms undergirding these results.

4.1. Demographics at work

During the last three decades, the US economy, together with the rest of the developed countries, has continued its ageing process. In particular, since the end of the 1980s, life expectancy at birth grew by almost 5 years in the US, while the population growth rate fell from roughly 1% to 0.7%.

Departing from the benchmark stationary distribution calibrated in Section 3, we produce two other simulations: the "ageing" economy, which features a life expectancy of 80 years with all the other parameters set at their benchmark values; the "ageing + population growth fall" economy, which features both the increased

	age group					other indicators			
	0-19	20-29	30-54	55-64	65+	life exp.	pop. growth	birth rate	65+ dep. ratio
benchmark calibration	0.33	0.15	0.30	0.11	0.10	75	0.010	0.02	0.18
ageing	0.33	0.15	0.29	0.10	0.13	80	0.011	0.02	0.25
ageing + pop. growth fall	0.30	0.14	0.30	0.11	0.15	80	0.007	0.017	0.27

Table 6. Shares of the population across age groups and other demographic indicators. The values are reported for different scenarios.

	age group				
	0-19	20-29	30-54	55-64	65+
benchmark calibration	0	1.05	1.76	1.53	1.37
ageing	0	1.04	1.77	1.53	1.33
ageing + pop. growth fall	0	1.04	1.67	1.46	1.29

Table 7. The household burden under different scenarios. The burden is defined by equation (7).

life expectancy and the above-mentioned lower population growth rate of 0.7%. In the second exercise the birth rate has to be consistent with the general population growth rate. Table 6 displays this information together with the change in the structure of the population.

As expected the increase in life expectancy increases the relative size of the 65+ age group (the retirees), at the expense of other age groups' sizes. The demographic process, as a whole, significantly shrinks the prevalence of children in the population. As a consequence, the old-age-dependency ratio goes from 0.18 in the benchmark economy to 0.27 in the "ageing + population growth fall" economy, which is close to the observed pattern. Indeed, the old-age-dependency ratio was around 0.19 by the end of the 1980s and currently is around 0.25. Further, the shares of the population are similar both in the data and in the simulations.[6]

Table 7 shows how the burden for each age group varies across the three economies. In the "ageing" economy such burden does not differ substantially from that of the benchmark economy. In the "ageing + population growth fall" economy, the burden changes considerably because the growth rate of the population, and hence the birth rate, adjusts downward. In particular, the burden decreases for each group, especially for the mature workers (30–54 years old). As argued below, the change of the burden will play a role in our results.

6. For example, notice that, in 2016, the shares of the 0-19, 20-29, 30-54 and 65+ age groups were 0.26, 0.14, 0.32, 0.13 and 0.15, respectively (United Nations 2017) which are quite close to those of the "ageing + population growth fall" economy reported in Table 6.

4.2. Main results

This section reports the main results of the paper in terms of the effects of the demographic process on intergenerational wealth inequality, on wealth concentration conditional on the age structure and finally, on prices and other aggregate variables.

Wealth inequality and the top rich. Figure 3 presents the distribution of the average wealth across age groups in the three simulations above described. The striking result is that, because of the demographics changes, wealth increases for the already wealthiest age groups (the pre-retired and the retirees) while decreasing for the younger (and poorer) ones.

The changes described above inevitably make wealth inequality across age groups larger. Indeed, Table 8 shows two measures of intergenerational wealth inequality both in the data and in the simulations; in particular, we report the Gini coefficient and the dispersion index already described in Section 3. Interestingly, our model, through the demographic forces, explains a significant part of the observed increase in wealth inequality: roughly half of the increase in the Gini coefficient (0.035 of 0.068) and roughly one third of the increase in the dispersion index (0.08 of 0.27).

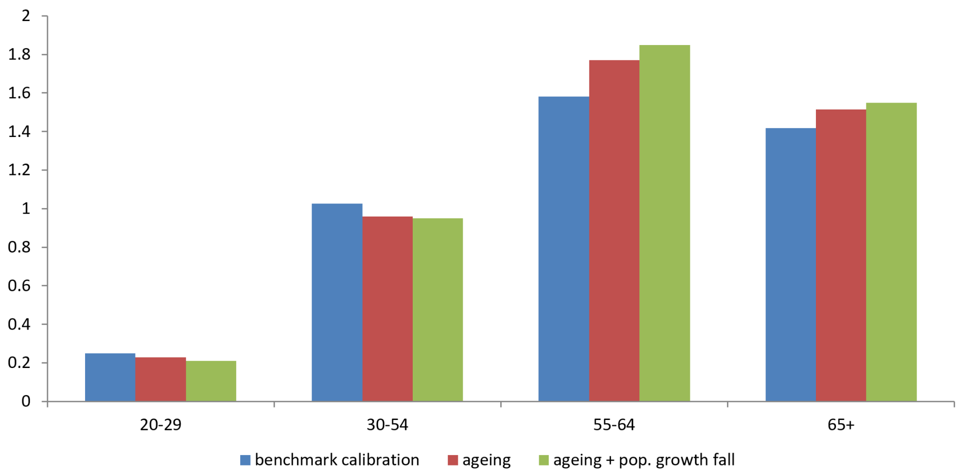

Figure 3: Average wealth across age groups under different scenarios. The wealth for each age group is then normalized by the average wealth of the benchmark calibration.

The changes in the dispersion measures depend both on the change of the population structure and on the change of the wealth distribution. If the richest age group becomes wealthier but, at the same time, its share of the total population shrinks, then the effect on wealth inequality across age groups will be ambiguous: while the wealth factor contributes to exacerbate wealth inequality, the fall of the share of that age group works in the opposite direction. In order to isolate the change in intergenerational wealth inequality generated by the households' reactions

data			model		
	Gini	disp. index		Gini	disp. index
1989	0.235	0.64	benchmark calibration	0.243	0.67
2016	0.303	0.91	ageing + pop. growth fall	0.278	0.75
change	0.068	0.270	change	0.035	0.080
of which:			of which:		
wealth effect	0.096	0.325	wealth effect	0.051	0.095
pop. effect	-0.028	-0.056	pop. effect	-0.016	-0.022

Table 8. Dispersion measures across age groups, both in the data and under different model scenarios. The calculation of the Gini coefficient and dispersion index, and the decomposition of the changes in wealth and population factors are explained in the text.

we follow Vandenbroucke (2016) and decompose our measures of dispersion along two dimensions: wealth and age structure. Taking the set of average wealth by age group, $\overline{a} = \{\overline{a}_g\}_{g\in\{2,\ldots,n_g\}}$ and the respective mass of households, $\overline{\lambda} = \{\overline{\lambda}_g\}_{g\in\{2,\ldots,n_g\}}$, one can compute the Gini coefficient and the dispersion index using expressions (10) and (11). More generally, the change in any statistic $D(\overline{a},\overline{\lambda})$ can be decomposed in wealth and population effects according to:

$$D(\overline{a}',\overline{\lambda}') - D(\overline{a},\overline{\lambda}) = \underbrace{\left(D(\overline{a}',\overline{\lambda}') - D(\overline{a},\overline{\lambda}') + D(\overline{a}',\overline{\lambda}) - D(\overline{a},\overline{\lambda})\right)/2}_{\text{wealth effect}} + \underbrace{\left(D(\overline{a}',\overline{\lambda}') - D(\overline{a}',\overline{\lambda}) + D(\overline{a},\overline{\lambda}') - D(\overline{a},\overline{\lambda})\right)/2}_{\text{population effect}} \qquad (12)$$

where the apostrophe denotes the final allocation. As Table 8 shows, the rise of intergenerational wealth inequality is attributable to actual changes of wealth by age group, and not to the change in the population structure which, in this specific case, has worked in the opposite direction, that is, has reduced intergenerational inequality or dispersion.

Another interesting point is the change of the age structure of the wealthy which occurred in the last three decades. Figure 4 shows that the 55+ age group represented about half of the top 5% wealthiest in 1989; this share rose up to roughly 80% in 2016. The demographic forces, through our model, explain more than one third of this increase.

This is a general effect that can be observed for other age groups, both in the data and with the model. Table 9 gives more details on how the share of the 55+ age group, for different wealth quantiles, has been affected by the demographic changes. Each number in the table represents the elasticity of the share of the elderly in that given quantile to the share of the 55+ age group in the whole population.[7] These elasticities, on average, are larger for the wealthiest portions of

7. Specifically, as for the data, these values represent the ratio between the percent change of the share of the 55+ age group within that given quantile, between 2016 and 1989, and the percentage

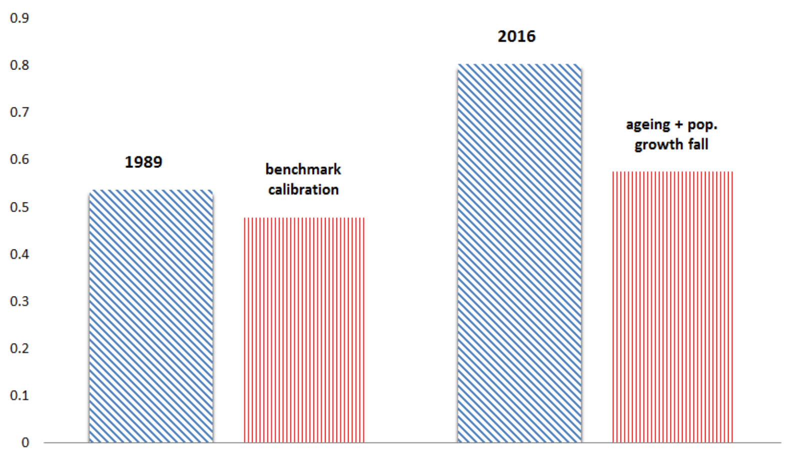

Figure 4: Share of the 55+ age group among the wealthy in the data and in the model. Wealthy are defined to be those households belonging to the top 5% quantile of net wealth. As for the actual data, household net wealth, which is measured in 2016 dollars, is taken from the Survey of Consumer Finances.

	quantiles of the wealth distribution (%)			
	bottom 40	bottom 60	top 20	top 5
data	0.17	0.43	0.70	1.57
model	0.44	0.36	1.28	0.84

Table 9. Elasticities of the share of the 55+ age group in a given quantile to the share of that group in the whole population. Calculation details are given in footnote 7.

the population, both in the data and in the model. That is, the demographic forces, through our model, make the share of the elderly grow more among the wealthy than among the poor. For example, in our simulations that elasticity among the top 20% wealthiest households is roughly 1.3, which compares to around 0.35 for those who possess less than 4% of the total wealth, which correspondes to the 60% poorest.

Aggregate shifts. Table 10 shows the effects of the demographic process on prices and other aggregate variables. The final economy is 13% wealthier with respect to the benchmark economy and is also more capital intensive. The real

change of the share of the 55+ age group in the whole population over the same period. As for the model, the calculations follow the same logic and apply to the shares of the 55+ age group in the "ageing + population growth fall" economy and in the benchmark economy.

	benchmark	ageing	ageing + pop. growth fall
r	0.04	0.035	0.030
w	1.00	1.02	1.05
K/N	5.20	5.56	5.96
K/Y	3.00	3.14	3.29
total assets	1.00	1.05	1.13
m_r (repl. rate)	0.41	0.28	0.25

Table 10. Aggregate results under different scenarios. The wage rate, w, and the total assets, $\int f_a(a,x)d\lambda$, are normalized to their respective value in the benchmark economy.

interest rate falls by 1 percentage point due to the demographic process, and, symmetrically, the wage rate increase by roughly 5%. As pointed out in the Introduction, the fall in the interest rate is similar to that obtained by Carvalho *et al.* (2016) and Papetti (2019), ranging between 1.3 and 1.5 percentage points, though they analyze a different set of countries.

Given that social security contributions are kept constant and the old-age-dependency ratio increases, the replacement rate (m_r) falls endogenously to 25%. Comparing the fall of our replacement rate with the one that actually occurred is not, for different reasons, an easy task. First, as explained in Section 3, the replacement rate varies with the individual labor income while the model features a single replacement rate. Second, the replacement rate depends on the evolution of wages which, in turn, are decisively influenced by many things outside the model, as, for example, market competition and labor tax policy. In our experiments the sole driver of change is the demographic process, which induces changes in policy functions and prices. In any case, according to Social Security Administration (2014) the average fall across the replacement rates of different income classes between 1990 and 2020, with the last few years being forecasts, has been around 10 percentage points, which is in the same ballpark of our own results.

Public debt is kept constant in terms of output at the level of the benchmark economy. This implies a marginal change in the level Government spending of 0.2 percentage points of output.

4.3. Inspecting the mechanisms

This section investigates the channels at the base of a larger intergenerational wealth inequality and of the fall in the real interest rate.

Higher life expectancy. Table 11 shows the behavior of various variables in three economies: (a) the benchmark economy, (b) the "ageing" economy at fixed prices, and (c) the "ageing" economy in general equilibrium.[8]

8. The fixed prices economy, which is an off-equilibrium simulation, is generated by keeping both the interest and the wage rate at their values in the benchmark economy. In practice, we iterate on policy functions conditional on the same factor prices and higher life expectancy.

age	wealth			saving			consumption			labor effort			dispos. income		
group	(a)	(b)	(c)	(a)	(b)	(c)	(a)	(b)	(c)	(a)	(b)	(c)	(a)	(b)	(c)
20–29	0.25	0.27	0.23	0.09	0.10	0.08	0.37	0.37	0.38	1.19	1.19	1.17	0.48	0.48	0.47
30–54	1.03	1.14	0.96	0.08	0.09	0.08	0.51	0.51	0.51	0.97	0.98	0.98	0.98	0.99	0.97
55–64	1.58	1.97	1.77	0.23	0.39	0.41	0.85	0.82	0.81	0.78	0.83	0.85	1.52	1.65	1.65
65+	1.42	1.76	1.51	-0.25	-0.24	-0.25	0.81	0.76	0.73	0	0	0	0.86	0.77	0.71

Table 11. Averages of selected variables, by age group, under different scenarios. (a) represents the benchmark economy, (b) the ageing economy at fixed prices, and (c) the ageing economy in general equilibrium. Wealth is normalized by average wealth of the benchmark economy; saving, consumption and disposable income are normalized by per capita output of the benchmark economy; labor effort is normalized by the average labor effort of the benchmark economy.

Moving from columns labeled (a) to columns labeled (b) we can evaluate the effect, by age group, of living longer on a selected number of variables, while holding prices constant. All the working-age classes save more because they know that, on average, they will have to consume for an additional 5-year period later in life. Furthermore, as typical in life-cycle models, retirees have a much smaller pension income than their previous labor income, irrespective of the rise in life expectancy. Finally, as presented in Table 10, the demographic process pushes the replacement rate further down, giving households an additional motive for saving. The last two facts, as visible in the penultimate column of Table 11, contribute to the marked fall of the disposable income of retirees.

Who saves the most? The pre-retirement group, and for two main reasons. They are the richest group in the economy and have, on average, the highest propensity to save. They also are closer to the retirement phase than other working-age groups. To save more, the pre-retirement group supplies more labor and consumes less. The youngest group can save more thanks to a slightly lower burden (see Table 7), while mature workers supply a bit more work.

Retirees adjust downward their level of consumption and on average dissave a bit less. Higher saving levels during the working phase allow retirees to avoid larger cuts in consumption.

Moving from columns (b) to columns (c) we see the effect of adjusting prices. The combination of lower (higher) interest (wage) rate causes heterogeneous effects across age groups. On average, a lower interest rate generates a negative income effect, especially for the wealthy (the pre-retirement group and the retirees), because it makes their wealth less remunerative; both groups react to that by further cutting consumption, the pre-retirement group also saving and working more. Furthermore, a lower interest rate generates a flatter consumption path over the life cycle, which makes households, especially the young, substitute future consumption for current consumption. On the other hand, the wage change influences more those households who significantly rely on labor income, which are typically the poorest and also, in our context, the youngest. Higher wages produce a positive income effect for the youngest workers; they consume more while working less. In addition to that, such effect acts as if it relaxed the borrowing constraint, especially for

age	wealth		saving		consumption		labor effort		dispos. income	
group	(c)	(d)	(c)	(d)	(c)	(d)	(c)	(d)	(c)	(d)
20-29	0.23	0.21	0.08	0.05	0.38	0.39	1.17	1.14	0.47	0.46
30-54	0.96	0.95	0.08	0.05	0.51	0.54	0.98	0.95	0.97	0.95
55-64	1.77	1.85	0.41	0.44	0.81	0.84	0.85	0.85	1.65	1.67
65+	1.51	1.55	-0.25	-0.28	0.73	0.73	0.00	0.00	0.71	0.66

Table 12. Averages of selected variables, by age group, under different scenarios. (b) represents the ageing economy, and (c) the ageing + population growth economy. Wealth is normalized by average wealth of the benchmark economy; saving, consumption and disposable income are normalized by per capita output of the benchmark economy; labor effort is normalized by the average labor effort of the benchmark economy.

those age groups closer to it, which implies that the level of precautionary saving of these agents may decrease. Prices effects thus boost the saving rate of the pre-retirement group while causing a drop of the youngest groups's. The mature workers' age group is a "mixture" of the two previous groups: the price effects seem to cancel out and they eventually save slightly less while maintaining the same level of consumption.

Let us clarify the role of price effects by taking the youngest group as an example. It is obvious that *ceteris paribus* higher wages positively influence the present value of the wealth of this type of workers. However, the youngest's reactions to the price changes (both less saving and less work) happen to have a negative effect on their wealth that is larger than the positive effect generated by the wage increase.

Lower population growth rate. Moving from columns (c) to columns (d) in Table 12 allows us to identify the effect of the lower population growth on selected model variables. As a general observation, the effects described above become stronger.

On one hand, the old-age-dependency ratio continues to rise, pushing down the replacement rate and the disposable income of retirees: this encourages saving for all agents, especially for those who are close to retirement. On the other hand, the fall (increase) in the interest rate (wage) is larger, reinforcing the "redistribution income effect" described above. Finally, the fall of the birth rate lowers the burden of each age group, especially that of the mature workers, who generate children. The lower burden represents somehow a positive income effect that, for example, may allow consuming more.

Eventually, the qualitative results presented before show up again: life-cycle motives and the interest rate fall mostly affect the older age groups, while the wage rise mostly influences the younger age groups.

All in all, the demographic process contributes to deplete the wealth of the younger age groups (young and mature workers) while increasing that of pre-retirees and retirees, the latter benefitting from the marked build up of assets during the pre-retirement phase.

As for the interest rate, its behaviour across the different simulations is consistent with the just described changes. The stronger life-cycle motive generates an appetite for assets among households, thus depressing the interest rate, while the decrease of the share of workers contributes to reduces the capital-labor ratio, making physical capital less productive and reinforcing the fall in the interest rate.

5. Conclusions

The current work has focused on the rise of the intergenerational wealth inequality during the last three decades. The older part of the population has become significantly richer, in contrast with the younger part, which has not. We challenge the demographic factor as a possible driver for such an inequality.

We build an OLG model with incomplete financial markets that is able to match quite well the wealth distribution of the household sector conditional on its age structure during the late 1980s.

The demographic changes occurred in the last thirty years, that is, higher life expectancy and lower population growth, explain, through our model, a significant part of the rise in the intergenerational wealth inequality. Further, our simulations account well for the fact that, during the period under scrutiny, the elderly became more prevalent among the wealthy.

The key issue is that the effects induced by such demographic process—including, but not restricted to, stronger life-cycle motives and changes in prices—influence the various age groups in very different ways. It so happens that the already wealthiest age groups, composed of the older part of society, accumulate wealth, while the already poorest, the younger, deplete it.

Further, consistent with a higher appetite for assets by households and a higher capital-labor ratio, the real interest rate is estimated to fall by roughly one percentage point due to demographics alone.

The current results call for several extensions. For example, the life expectancy increase makes the young workers' age group wealth-poorer, though with a higher wage; it would be interesting to investigate whether such age group is more likely to be liquidity-constrained with today's demographics than thirty years ago. Or, put differently, how the interaction between market incompleteness and demographic changes impacts on the welfare of the various age groups. As a further step, United Nations (2017) forecast an even higher (lower) life expectancy (population growth rate) for all the developed countries during the next decades; it would be interesting to have an estimate on the evolution of wealth inequality and concentration conditional on such enduring demographic process.

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Appendix: Some elements of the computational method

The numerical procedure to solve for the steady state iterates on the individual first-order conditions (8) and aggregate constraints presented in the definition of equilibrium in subsection 2.2. The individual policy functions are then used to compute the density measure across asset holdings, labor productivity and age group.

We first set up a grid $\tilde{\mathbb{A}}$ of asset holdings with overall negative and positive limits $\underline{\underline{a}}$ and $\overline{a}$, and make sure that the upper limit is not binding in any of the calibrations. We set these bounds to -1 and 100. The grid is exponential, therefore oversampling low values, and each point is given by $a_i = \exp\left(\frac{i-1}{n_a-1}\log\left(\overline{a}-\underline{\underline{a}}+1\right)\right)+\underline{\underline{a}}-1$, $i=1,\ldots,n_a$, where $n_a = 50$ is the number of points. This grid is augmented with the value $a = 0$ for computational reasons. The stochastic process for the idiosyncratic productivity is modeled using Rouwenhorst's method (Kopecky and Suen 2010) plus a special high-productivity, low-probability state, in a total of 6 points; see details on Section 3, also for the calibration of the borrowing limit.

The discrete measure $\tilde{\lambda}$ defined on the grid can be computed iteratively using the policy functions.

The method to solve the numerical problem hinges on iterating in the first-order conditions of the individual problem (8), and then updating the discrete measure $\tilde{\lambda}$ using the individual policy functions.

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