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The analyses, opinions and fi ndings of these papers represent the views of the authors, they are not necessarily those of the Banco de Portugal or the Eurosystem

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A reexamination of inflation persistence dynamics in OECD countries: A new approach.

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Abstract
This paper introduces a simple and easy to implement procedure to test for changes in persistence. The time-varying parameter that characterizes persistence changes under the alternative hypothesis is approximated by a parsimonious cosine function. The new test procedure is the minimum of a t-statistic, computed from a test regression that considers a set of reasonable values for a frequency term that is used to evaluate the time varying properties of persistence. The asymptotic distributions of the new tests are derived and critical values are provided. An indepth Monte Carlo analysis shows that the new procedure has important power gains when compared to the local GLS de-trended Dickey-Fuller (DF^{GLS}) type tests introduced by Elliott *et al.* (1996) under various data generating processes with persistence changes. Moreover, an empirical application to OECD countries' inflation series shows that for most countries analysed persistence was high in the first half of the sample and subsequently decreased. These results are compatible with modern macroeconomic theories that point to changes in inflation behavior in the early 1980s and also with recent empirical evidence against the $I(1)$-$I(0)$ dichotomy.

JEL: C12 (Hypothesis Testing), C22 (Time-Series Models)
Keywords: Nonstationarity, unit roots, inflation, CPI, change in persistence.

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1. Introduction

Since the seminal work of Nelson and Plosser (1982), a large number of procedures have been developed to infer whether the empirical evidence of nonstationarity of economic and financial variables is due to the lack of flexibility of the linear AR model specifications typically considered. Given that structural breaks and nonlinear dynamics are data features that may occur in practice, the assumption that parameters are constant over the whole sample may be restrictive.

The presence of structural breaks in the data is a consequence of, among other things, the occurrence of exogenous shocks, such as crises or policy decisions, which may have permanent effects on the variables' dynamics. For instance, Perron (1989) showed that the Great Crash of 1929 caused a dramatic decrease in the mean of most aggregate variables of the US economy and further suggested the 1973 oil price shock as a possible cause of changes in the slope of the trend function responsible for the subsequent slowdown of the output's growth rate. Garcia and Perron (1996) reported that the average value of real interest rates suffered two breaks as a consequence of important structural events: the rise in oil prices in 1973, and the high budget deficits in 1981 and 1982. Regarding inflation dynamics, changes in monetary policy, which reflect the evolving preferences for price stability over time, seem to have affected parameter constancy in linear models. Chang *et al.* (2013) and Chen and Hsu (2016), *inter alia*, presented results pointing to the occurrence of structural breaks in the deterministic component of inflation. Levin and Piger (2003) and Beechey and Österholm (2012) proposed methodologies which allow for the assessment of whether shifts in monetary policy caused changes in the autoregressive parameter. Moreover, Leybourne *et al.* (1998) proposed unit root tests that consider the possibility of a gradually rather than abrupt break in the deterministic structure using LSTAR models; and Enders and Granger (1998) and Kapetanios *et al.* (2003) developed unit root tests that consider nonlinear models under the alternative hypothesis of stationarity, allowing for adjustments to deviations from the deterministic component to be asymmetric.

A widely used approach to shed light on the nature of inflation persistence is to empirically assess the order of integration of the inflation rate series (see, for instance, Evans and Wachtel 1993, Culver and Papell 1997, and Crowder and Wohar 1999). However, as mentioned above, the finite sample power performance of traditional unit root tests is far from being satisfactory when the coefficients of linear AR models change. This has led to the development of tests that allow for breaks in the deterministic kernel of the process. Recent unit root tests allow the breaks to be endogenously estimated along with the other parameters of the model, however it is not easy to deal with more than two breaks, since it is complex to derive the asymptotic distributions and to obtain critical values for different combinations of breaks (see Perron, 2005, for an interesting survey). However, many procedures consider that structural changes occur instantaneously, which may not be consistent with the fact that changes in economic aggregates are influenced by changes in the behavior of a very large number of agents that may not react simultaneously to a given shock.

Enders and Lee (2012a) and Rodrigues and Taylor (2012) proposed tests that do not require assumptions about the number of breaks and their exact forms. To this end, Fourier terms have been used to approximate structural changes of unknown functional forms in the deterministic component and, thus, reducing the specification problem to the selection of the appropriate frequency components of the Fourier approximation. Since structural breaks shift the spectral density function towards the zero frequency, low-frequency terms must be employed. A small number of low-frequency terms can capture a great variety of breaks, sometimes even a single frequency is sufficient.

A lot of research has been devoted to the analysis of the interplay between structural changes in the deterministic components and unit roots. However, to the best of our knowledge, work on unit root tests that allow for changes in the autoregressive parameter under the alternative hypothesis is scant (see e.g. Caner and Hansen 2001). The extant literature on testing for changes in the persistence of a time series has in general given priority to processes that switch from stationarity to nonstationarity and vice-versa. Most of the available statistics are related to the residual-based test for stationarity proposed by Kim (2000) (see, for instance, Busetti and Taylor 2004 and Harvey *et al.* 2006). These tests are based on a ratio that uses two partial sum processes of the residuals from regressions of the time series of interest on its deterministic component before and after a given break date. Since the break date is typically unknown, statistics based on the value of the ratio for all possible break dates, such as the maximum Chow-type test, were considered. Harvey *et al.* (2006) showed that existing tests were unable to adequately distinguish between a change in persistence and a constant $I(1)$ process. They proposed modified versions of the ratio based statistics of Kim (2000) that have the same critical values regardless of the order of integration. Thus, the null hypothesis is that of constant $I(0)$ or $I(1)$ persistence and the alternative is that of a change in persistence from $I(0)$ to $I(1)$, from $I(1)$ to $I(0)$ or of unknown direction.

In this paper, the focus is to investigate if changes in the autoregressive parameter may have been responsible for the occurrence of periods in which inflation displayed higher persistence. To this end, we propose an easy to implement approach. The procedure is compared, using Monte Carlo simulations, to the local GLS de-trended unit root tests of Elliott *et al.* (1996) for several types of breaks in persistence. In addition to processes in which the breaks in the autoregressive parameter are approximated by a cosine function, we also investigate the finite sample performance of the tests when abrupt changes at different timings occur.

The simulation results show that the new statistics reject the false null hypothesis significantly more often than the DF^{GLS} tests in the presence of changes in persistence. The relative superiority of our tests becomes even more evident when the sample size increases.

Finally, the results obtained applying the proposed test to G7 countries inflation data are in line with the findings of the simulation analysis. The unit root hypothesis is rejected for considerably more countries than when the DF^{GLS} or the test by Rodrigues and Taylor (2012) are employed.

The remainder of the paper is organized as follows. Section 2 introduces the test procedures and derives their asymptotic distributions under the null and local alternative hypotheses. Section 3 investigates the finite sample properties of the statistics through Monte Carlo simulations. In specific, the impacts of conditional heteroskedasticity, breaks in the innovation variance and serially correlated errors are examined. Section 4 presents an in-depth analysis of inflation data. Section 5 concludes, and finally, a Technical Appendix provides detailed proofs of all results presented throughout the paper.

2. Motivation and proposed statistic

In this work we consider a model for persistence changes in line with Harvey *et al.* (2006), i.e.,

$$y_t = \mathbf{x}_t'\beta + u_t \tag{1}$$

$$u_t = \rho_t u_{t-1} + \varepsilon_t, \tag{2}$$

where $\varepsilon_t \sim iid(0, \sigma^2)$, $\mathbf{x}_t$ is a deterministic kernel which is either a constant or a constant and time trend (i.e. $\mathbf{x}_t := 1$ or $\mathbf{x}_t := [1, t]'$), β is the corresponding vector of parameters that captures the deterministic structure and (2) describes the stochastic behaviour of y_t. Most unit root test procedures available assume that ρ_t is constant under both the null, $H_0 : \rho_t = \rho = 1$, and the alternative hypothesis, $H_a : -1 < \rho_t = \rho < 1$, of stationarity.

2.1. The Test Procedure

In this work we use a simple cosine function with a single frequency in order to mimic the pattern of unknown shifts in the autoregressive parameter ρ_t. In specific, to implement the test procedure, a two-step approach as in Elliott *et al.* (1996) is employed. In the first step the time series of interest, y_t, is locally GLS de-trended based on $\tilde{\rho}_t := 1 + \frac{\tilde{c}}{T}\cos(k,t)$ with

$$\cos(k,t) := \frac{1 + \cos(2\pi kt/T)}{2} = \cos^2(\pi kt/T), \tag{3}$$

where $\tilde{c}$ is fixed and non-positive, and k is fixed[1]. The demeaned/detrended variable is computed as,

$$\hat{u}_{\tilde{c},t} = y_t - \mathbf{x}_t^{'}\hat{\beta}_{\tilde{\mathbf{c}}} \tag{4}$$

where $\mathbf{x}_t = 1$ (constant case) or $\mathbf{x}_t = [1,t]^{'}$ (linear trend case), $\hat{\beta}_{\tilde{c}} = \left(\sum_{t=1}^{T}\mathbf{x}_{\tilde{c},t}\mathbf{x}_{\tilde{c},t}{'}\right)^{-1}\sum_{t=1}^{T}\mathbf{x}_{\tilde{c},t}y_{\tilde{c},t}$, and $y_{\tilde{c},1} = y_1$, $\mathbf{x}_{\tilde{c},1} = \mathbf{x}_1$ and $y_{\tilde{c},t} := y_t - \widetilde{\rho}_t y_{t-1}$, $\mathbf{x}_{\tilde{c},t} := \mathbf{x}_t - \widetilde{\rho}_t\mathbf{x}_{t-1}$, for $t > 1$ (see also Elliott *et al.* (1996)). As the $\cos(k,t)$ function takes values between 0 and 1, this de-trending method can be seen as a local GLS de-trending approach with time-varying weights.

In the second step, the presence of a unit root in $\hat{u}_{\tilde{c},t}$ is investigated considering, for k known and fixed, the t-statistic on φ computed from the test regression

$$\Delta\hat{u}_{\tilde{c},t} = \varphi\cos(k,t)\hat{u}_{\tilde{c},t-1} + \varepsilon_t, \tag{5}$$

where under the null hypothesis of a unit root, $H_0 : \varphi = 0$, and under the alternative hypothesis, $H_a : \varphi < 0$.

Remark 1. Elliott *et al.* (1996) showed that there is no uniformly most powerful unit root test and proposed choosing the noncentrality parameter $\tilde{c}$ as the value at which the test is tangent to the power envelope at 50%. For the proposed test, the noncentrality parameter $\tilde{c}$ will assume different values depending on the deterministic component and also on the frequency parameter k considered (see Remark 2.8). □

Remark 2. The tests proposed by Enders and Lee (2012a) consider breaks in intercept, which affect the conditional and unconditional means of the process. However, breaks in persistence also alter the unconditional variance. Hence, both de-trending and testing steps are influenced by the autoregressive parameter and the assumption of constancy, which when invalid, seems to favor the null hypothesis of a unit root. It is therefore important to propose tests that allow for changes in persistence under the alternative hypothesis. Since the number of breaks and its functional form are typically unknown in practice, trigonometric functions have been considered to approximate parameter changes. For instance, Fourier series, which are linear combinations of sine and cosine functions, are widely used in this context (see e.g. Gallant 1981). □

Remark 3. In our framework, we use a single factor since the increase in flexibility of the functions employed (i.e. more frequency terms) to describe parameter changes has been associated with a deterioration in power performance. For instance, the use of multiple Fourier frequencies to obtain

1. Note that when $k = 0$, $\tilde{\rho}_t$ corresponds to the typical near unit root representation used by Elliott *et al.* (1996).

more precise approximations leads to significant power losses due to over-fitting of the data (see Enders and Lee 2012b).

REMARK 4. The $\cos(k, t)$ function in (3) is crucial to properly approximate, but not identify, structural break dates in persistence. Its shape is entirely determined by the frequency parameter k. Most empirical work using Fourier terms has only considered integer values for k (see e.g. Enders and Lee 2012a and Rodrigues and Taylor 2012), which implies that the starting and ending values of $\cos(k, t)$ are equal. For instance, considering $k = 1$, the resulting function may be useful in cases with two breaks, where an increase in the autoregressive parameter somewhere in the middle of the sample is followed by a decrease of similar magnitude later. However, when there is an increase in persistence at an unknown point in time and the parameter does not return to its initial value, a fractional frequency needs to be considered. For example, choosing $k = 0.5$ may be useful if after an increase in persistence the autoregressive parameter remains close to 1 and the time series exhibits near-unit root behavior for the rest of the sample. □

REMARK 5. Figure 1 illustrates the shape of $\cos(k, t)$ for $k = (0.5, 1, 1.5, 2, 2.5, 3)$. This figure shows that fractional frequency values are useful for the approximation of periods of higher persistence at the end of the sample. For integer values of k, the autoregressive parameter is higher in the middle of the sample and is far from unity at the end of the sample.

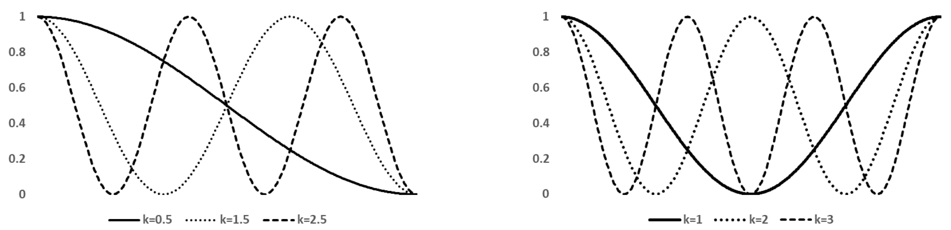

FIGURE 1: The cosine function for non-integer and integer values of k

□

In this work, we consider both integer and fractional values for the frequency parameter k. It is not expected that many breaks in persistence occur given the relatively small number of observations usually available in empirical work. Thus, the proposed test considers that under the alternative hypothesis there are a maximum of three periods of persistence change (this assumption can however be relaxed if necessary).

To test the null hypothesis, $H_0 : \varphi = 0$, in equation (5) when k is unknown (which is the empirically relevant case) the following test statistic is considered,

$$\mathcal{T}_{\hat{k}}^{GLS} := \min_{k\in K} \hat{t}_k^{GLS} = \min_{k\in K} \frac{\sum_{t=2}^{T} \Delta\hat{u}_{\tilde{c},t}\cos(k,t)\hat{u}_{\tilde{c},t-1}}{\left[\hat{\sigma}_k^2 \sum_{t=2}^{T}\cos^2(k,t)\hat{u}_{\tilde{c},t-1}^2\right]^{1/2}}, \tag{6}$$

where $K = \{0.5, 1, 1.5, 2, 2.5, 3\}$ and $\hat{\sigma}_k^2$ is the least-squares estimate of σ^2 obtained from (5) under a fixed k.

Although, under the alternative hypothesis a time series is not weakly stationary, it follows an intrinsically mean-reverting process with some exceptional periods during which the autoregressive parameter was close to unity.

2.2. Short-run dependence

In practice, test regression (5) may not be sufficient to properly describe the dynamics of economic and financial time series, resulting in autocorrelated error terms. Thus, to overcome this problem an augmented version of test regression (5) can be considered, i.e.,

$$\Delta\hat{u}_{\tilde{c},t} = \varphi\cos(k,t)\hat{u}_{\tilde{c},t-1} + \sum_{j=1}^{p} \delta_j \Delta\hat{u}_{\tilde{c},t-j} + \varepsilon_t, \tag{7}$$

where p is the order of augmentation (lag length) selected using some model selection criteria (such as, e.g., AIC, MAIC or BIC). In the augmented Dickey-Fuller (ADF) context, Chang and Park (2002) showed that the asymptotic distribution of the DF test will not change when the true DGP is an ARMA process of unknown order if the test regression is augmented with a sufficient number of lagged differences of $\hat{u}_{\tilde{c},t}$ to ensure that the residuals are approximately uncorrelated. In (7), p denotes the lag truncation order chosen to account (parametrically) for any weak dependence in $\{\varepsilon_t\}$. More generally, when ε_t is a linear process satisfying standard summability and moment conditions, p needs to be such that $1/p + p^3/T \to \infty$ as $T \to \infty$; see Said and Dickey (1984) and Chang and Park (2002). As shown in (7), the difference between the proposed test and that of the DF^{GLS} is due to the presence of some terms that are deterministic when k is known. So, the results of Chang and Park (2002) remain valid in this context.

2.3. Unconditional and Conditional heteroskedasticity

Another important issue is to examine how the proposed tests perform when the conditional variance of the error process ε_t is not constant over time, in order to avoid spurious evidence of changes in the persistence of y_t. In the case of the ADF test, although conditional heteroskedasticity does not affect the asymptotic distribution of the test statistic (Phillips 1987), the presence of ARCH effects does cause size distortions in finite samples (see e.g. Kim and

Schmidt 1993 and Haldrup 1994). Hence, it is important to understand how this feature of the data may impact the proposed test.

Moreover, it is also important to assess the impact of unconditional heteroscedasticity on the proposed test's performance. Hamori and Tokihisa (1997) and Kim *et al.* (2002) showed that a permanent variance shift causes size distortions in the DF tests. Note that if simultaneous increases in persistence and in the innovation variance are observed (two reinforcing effects that cause an increase in σ_y^2) the proposed test may exhibit size distortions. This is because it may be hard to distinguish whether the increase in the unconditional variance of y_t is caused by a true change in persistence or by an exogenous shift in the innovation variance. Moreover, a large increase in the unconditional variance may cause the process to be confounded more often with a unit root process, for which the variance grows with t.

In the presence of (conditional or unconditional) heteroskedasticity, heteroskedasticity-consistent standard errors, as proposed by Eicker-White (EW), are typically employed in response to this problem (see e.g. Demetrescu 2008 and Phillips 1987). The proposed test statistic with EW robust standard errors, considering fixed k and no short-run dependence in ε_t is based on,

$$\hat{t}_{k,EW}^{GLS} := \frac{\sum_{t=2}^{T} \Delta\hat{u}_{\tilde{c},t}\cos(k,t)\hat{u}_{\tilde{c},t-1}}{\left(\sum_{t=2}^{T}\cos^2(k,\mathrm{t})\hat{u}_{\tilde{c},t-1}^2\hat{\varepsilon}_t^2\right)^{1/2}}. \tag{8}$$

Based on this statistics, the following limit results can be stated.

Proposition 1. *Under the null hypothesis, $H_0 : \varphi = 0$, and considering Assumptions 1 and 2 in Demetrescu (2008), as $T \to \infty$,*

$$\hat{t}_{k,EW}^{GLS} - \hat{t}_{k}^{GLS} \xrightarrow{p} 0$$

for a given fixed k, where " $\xrightarrow{p}$ " stands for convergence in probability.

Proposition 2. *Under the alternative hypothesis, $H_a : \varphi = \frac{c}{T}$, as $T \to \infty$,*

$$\hat{t}_{k,EW}^{GLS} - \hat{t}_{k}^{GLS} \xrightarrow{p} 0,$$

for a given fixed k and any fixed non-positive c and $\tilde{c}$.

Since conditional heteroskedasticity has no impact on the de-trending approach used, Propositions 1 and 2 are valid regardless of the deterministic kernel (constant or linear time trend) considered.

An alternative to the EW approach used in (8) which is also widely employed in the literature to deal with, among other things, (unconditional and conditional) heteroskedasticity of unknown form is the Wild bootstrap (see, for instance, Killian and Gonçalves, 2004, Cavaliere and Taylor 2008, Pavlidis *et al.*

2010 and Maki 2015). It consists of using the residuals $\hat{\varepsilon}_t$ computed from (6) and generating a new unit root process as

$$\hat{u}_t^b = \hat{u}_{t-1}^b + v_t^b$$

where $v_t^b := e_t\hat{\varepsilon}_t$ and e_t is such that any heteroskedasticity in $\hat{\varepsilon}_t$ is preserved in the newly created residuals v_t^b. We use $e_t \sim$i.i.d. $N(0,1)$, but the Rademacher distribution is also frequently used. Next, B bootstrap series $\hat{u}_t^b$ are generated and in each iteration the bootstrap t-statistic $\hat{t}_k^{b,GLS}$ is computed based on the auxiliary regression

$$\Delta\hat{u}_t^b = \varphi^b \cos(k,t)\hat{u}_{t-1}^b + \eta_t \tag{9}$$

where η_t is an error term. The bootstrap p-value is computed as

$$P_b\left(\hat{t}_k^{GLS}\right) := \frac{1}{B}\sum_{n=1}^{B} I\left(\hat{t}_k^{b,GLS} > \hat{t}_k^{GLS}\right), \tag{10}$$

where B is the number of bootstrap iterations and $I(.)$ is the indicator function (see e.g. **?**). In the case of short-run dependence in the innovations of the process an augmented framework as in (8) is used.

2.4. Asymptotic Distribution

In this section the asymptotic distributions of the proposed tests are derived under the null hypothesis of a unit root and under the alternative hypothesis of local breaks in persistence. Moreover, the test statistics employed in the construction of the asymptotic local power envelope and their asymptotic distributions are also presented.

THEOREM 1. *Under the null hypothesis of a unit root, $H_0 : \varphi = 0$ ($c = 0$), the limit distribution of the proposed test statistic, as $T \to \infty$, when local GLS demeaning is used is*

$$\mathcal{T}_{\hat{k}}^{GLS_\mu} := \min_{k\in K} \hat{t}_k^{GLS_\mu} \Rightarrow \min_{k\in K} \frac{\cos(k,1)W(1)^2 + \frac{1}{2}(2\pi k)^2\int_0^1 \cos(2\pi kr)\left[W(r)\right]^2 dr - 1}{2\left(\int_0^1 \cos^2(k,r)\left[W(r)\right]^2 dr\right)^{1/2}} \tag{11}$$

and for local GLS detrending,

$$\mathcal{T}_{\hat{k}}^{GLS_\tau} := \min_{k\in K} \hat{t}_k^{GLS_\tau} \Rightarrow \min_{k\in K} \frac{\cos(k,1)\left[W^\tau(1)\right]^2 + \frac{1}{2}(2\pi k)^2\int_0^1 \cos(2\pi kr)\left[W^\tau(r)\right]^2 dr - 1}{2\left(\int_0^1 \cos^2(k,r)\left[W^\tau(r)\right]^2 dr\right)^{1/2}} \tag{12}$$

where k is fixed, $W(r)$ is a standard Brownian motion, $\cos(k,r)$ is as defined in (3) with $r := t/T$ and

$$W^{\tau}(r) = \sigma W(r) - \sigma r \frac{(1-\tilde{c}\cos(k,r))W(1) + \tilde{c}^2 \int_0^1 r\cos^2(k,r)W(r)dr}{\int_0^1 \left[1 - 2\tilde{c}r\cos^2(k,r) + r^2\tilde{c}^2\cos(k,r)\right]dr} +$$

$$\sigma r \frac{\tilde{c}k\pi \int_0^1 r\sin(2\pi kr)W(r)dr}{\int_0^1 \left[1 - 2\tilde{c}r\cos^2(k,r) + r^2\tilde{c}^2\cos(k,r)\right]dr}. \quad (13)$$

Remark 6. As in the traditional unit root testing context, local GLS demeaning has no effect on the proposed test's asymptotic distribution (see the Appendix for details) and, therefore, the asymptotic distribution of $\mathcal{T}_{\hat{k}}^{GLS_\mu}$ given in (11) is equivalent to that of a test statistic computed from a test regression with no deterministics. □

Remark 7. The results in (11) and (12) show that the asymptotic distributions of the proposed tests depends only on the frequency parameter k. When $k = 0$, $\cos(0,r) = 1$, $\forall r \in (0,1)$ and the asymptotic distributions in (11) and (12) correspond to the asymptotic distributions of the local GLS demeaned/detrended DF unit root tests of Elliott *et al.* (1996). □

Remark 8. Table 1 presents the values for $\tilde{c}$ for which the power of the test is tangent to the power envelope at 50% (as recommended by Elliott *et al.* 1996).

k	$\mathbf{x}_t = 1$	$\mathbf{x}_t = [1,t]'$
0.0	-7.0	-13.5
0.5	-15.6	-25.4
1.0	-11.8	-25.8
1.5	-12.7	-26.1
2.0	-10.7	-22.2
2.5	-11.2	-23.3
3.0	-10.2	-20.2

Table 1. Local GLS detrending parameter $\tilde{c}$

Note: Values computed based on 100,000 replications for T=1000.

In order to construct the asymptotic power envelope, an asymptotically equivalent test to the infeasible most powerful invariant LR statistic proposed by Elliott *et al.* (1996) will be used. For a given $\tilde{c}$ and k, it has the form:

$$P_{\tilde{c}} := \frac{\sum_{t=1}^{T} \hat{\varepsilon}_{\tilde{c},t}^2 - \left[1 + \frac{\tilde{c}}{T}\cos(k,t)\right] \sum_{t=1}^{T} \hat{\varepsilon}_{0,t}^2}{\hat{\sigma}^2} \tag{14}$$

where $\hat{\varepsilon}_{0,t}$ and $\hat{\varepsilon}_{\tilde{c},t}$ are the residuals of the model defined by (1) and (2) when, respectively, $\tilde{c} = 0$ and $\tilde{c} < 0$ for $\tilde{\rho}_t := 1 + \frac{\tilde{c}}{T}\cos(\mathrm{k}, \mathrm{t})$.

Theorem 2. *Under $H_0 : \varphi = 0 (c = 0)$ and i.i.d. innovations, the asymptotic distribution of the test statistic presented in (14) is*

$$P_{\tilde{c}}^{\mu} \Rightarrow \tilde{c}^2 \int_0^1 \cos^2(r,k) J_c(r)^2 - \tilde{c}\cos(T,k) J_c(1)^2$$

and

$$P_{\tilde{c}}^{\tau} \Rightarrow \tilde{c}^2 \int_0^1 \cos^2(k,r) \left[J_c^{\tau}(r)\right]^2 + (1 - \tilde{c}\cos(k,T)) \left[J_c^{\tau}(1)\right]^2,$$

where $P_{\tilde{c}}^{\mu}$ and $P_{\tilde{c}}^{\tau}$ correspond to demeaned and de-trended test statistics, respectively, $J_c(r)$ is a standard Ornstein-Uhlenbeck [OU] process, $J_c^{\tau}(r)$ is a local GLS de-trended OU process and $\hat{\sigma}^2 := T^{-1} \sum_{t=1}^{T} \varepsilon_{\tilde{c},t}^2$.

Theorem 3. *Under the local alternative hypothesis $H_a : \varphi = \frac{c}{T} < 0$ the limit distribution of the proposed statistic under local GLS demeaning is*

$$\mathcal{T}_{\hat{k}}^{GLS_{\mu}} = \min_{k \in K} \hat{t}_k^{GLS} \Rightarrow \min_{k \in K} \frac{\cos(k,1) J_c^2(1) + \frac{1}{2}(2\pi k)^2 \int_0^1 \cos(2\pi k r) J_c^2(r) dr - 1}{2\left(\int_0^1 \cos^2(k,r) J_c^2(r) dr\right)^{1/2}} \tag{15}$$

and under local GLS de-trending

$$\mathcal{T}_{\hat{k}}^{GLS_{\tau}} = \min_{k \in K} \hat{t}_k^{GLS_{\tau}} \Rightarrow \min_{k \in K} \frac{\cos(k,1) \left[J_c^{\tau}(1)\right]^2 + \frac{1}{2}(2\pi k)^2 \int_0^1 \cos(2\pi k r) \left[J_c^{\tau}(r)\right]^2 dr - 1}{2\left(\int_0^1 \cos^2(k,r) \left[J_c^{\tau}(r)\right]^2 dr\right)^{1/2}} \tag{16}$$

where $J_c(r)$ is a standard OU process, $k \in K$, $r \in (0,1)$, and $J_c^{\tau}(r)$ is a local GLS detrended OU process.

3. Monte Carlo Analysis

This section investigates the finite sample properties of the tests previously introduced under the null and alternative hypotheses. All simulations are performed in Gauss 10. For the Wild bootstrap procedure, the number of Monte Carlo and bootstrap replications is 1000, whereas for all the other simulations 10,000 Monte Carlo replications were used.

Table 2 presents the critical values for $\mathcal{T}_{\hat{k}}^{GLS_\mu}$ and $\mathcal{T}_{\hat{k}}^{GLS_\tau}$, considering $T \in \{150, 250, 500\}$ and $k \in K = \{0.5, 1, 1.5, 2, 2.5, 3\}$.

	$\mathcal{T}_{\hat{k}}^{GLS_\mu}$			$\mathcal{T}_{\hat{k}}^{GLS_\tau}$		
T	**1%**	**5%**	**10%**	**1%**	**5%**	**10%**
150	-3.266	-2.695	-2.403	-4.092	-3.589	-3.336
250	-3.192	-2.629	-2.346	-4.008	-3.517	-3.268
500	-3.152	-2.592	-2.303	-3.958	-3.467	-3.215
1000	-3.133	-2.574	-2.285	-3.935	-3.438	-3.189

Table 2. Critical values

Notes: For the constant case, critical values were computed from test regressions applied to demeaned data. Reported critical values are based on 100,000 simulations.

In what follows, the finite sample performance of the proposed tests will be compared to that of DF^{GLS_ς}, with $\varsigma = \mu, \tau$, applied to demeaned or de-trended data, respectively. We investigate how the tests perform under iid innovations, in the presence of autocorrelation and, under conditional and unconditional heteroskedasticity.

3.1. IID innovations

To investigate the finite sample properties of the tests we consider two data generation processes (DGPs): i) the first DGP, (henceforth DGP1), is,

$$y_t = \rho_t y_{t-1} + \varepsilon_t \tag{17}$$

with $\rho_t = (1 + \varphi\cos(k,t))$, $\cos(k,t) := (1 + \cos(2\pi k\mathrm{t}/\mathrm{T}))/2$, $\varphi \in \{-0.1, -0.2, 0\}$; and ii) as second DGP (DGP2) we use,

$$\begin{cases} y_t = \rho_1 y_{t-1} + \varepsilon_t & for \quad t = 1, ..., \lfloor \tau_1 T \rfloor \\ y_t = \rho_2 y_{t-1} + \varepsilon_t & for \quad t = \lfloor \tau_1 T \rfloor + 1, ..., \lfloor \tau_2 T \rfloor \\ y_t = \rho_3 y_{t-1} + \varepsilon_t & for \quad t = \lfloor \tau_2 T \rfloor + 1, ..., T, \end{cases} \tag{18}$$

where $\tau_1 \leq \tau_2$, $\tau_1 \in \{0.3, 0.4, 0.6, 0.8\}$, $\tau_2 \in \{0.3, 0.6, 0.7, 0.8\}$, $\rho_1 \in \{0.8,\ 0.9\}$, $\rho_2 \in \{0.99,\ 1\}$ and $\rho_3 \in \{0.8, 0.9, 0.99,\ 1\}$ to investigate the finite sample power and $\rho_1 = \rho_2 = \rho_3 = 1$ ($\varphi = 0$) to examine the finite sample size of the tests. For both cases, $\varepsilon_t \sim N(0,1)$ and $y_1 = \varepsilon_1 \sim N(0,1)$ is used.

Thus, DGP1 implies that the transition from regimes with $\varphi < 0$ to regimes with $\varphi = 0$ is smooth, since the autoregressive parameter of the process is obtained by multiplying φ by time-varying weights defined by the $\cos(k, t)$ function in (3). This function is approximately 1 for the first values of t, which implies that $\tilde{\rho}_t$ is smaller at the beginning of the sample. Table 3 presents the empirical size and power of the proposed test for this DGP when applied to demeaned data. The empirical size is close to the nominal 5% significance level for all cases considered. Regarding the power properties, $\mathcal{T}_{\hat{k}}^{GLS_\mu}$ displays significant power gains relative to DF^{GLS_μ} when there are breaks in persistence. It provides power gains for almost all of the simulation parameters used (the only exception is $k = 0.5$). However, as expected, the proposed test has more difficulties in rejecting the null hypothesis when $\varphi = -0.1$, since the autoregressive parameter is already large before the increase in persistence. But, even in this case, there are significant power gains that increase with the sample size (note that the empirical power of the $\mathcal{T}_{\hat{k}}^{GLS_\mu}$ test is close to 100% for $T = 500$).

Table 4 presents the results for the linear trend case. When $\varphi = -0.1$, the proposed test only presents power gains for all values of k when $T = 500$. Nonetheless, there are some relevant positive differences relative to DF^{GLS_τ} for $k > 0.5$ even when $T = 250$. For $\varphi = -0.2$, there are relevant power gains even in the smaller samples considered ($T = 150$) with $k > 0.5$.

DGP2 is used to investigate the performance of the tests when breaks in persistence occur instantaneously. To save space, we allow for a maximum of two abrupt changes which result in a single period of higher persistence. When $\rho_2 = \rho_3$, the break divides the process into two regimes, with the autoregressive parameter being larger in the last sub-period. If $\rho_1 = \rho_3$, the sub-period of higher persistence occurs in the middle of the time series and the autoregressive parameter returns to the value assumed at the beginning of the process. Table 5 presents the results for the demeaned test statistics. As expected, the power of the two tests considered is lower when $\rho_2 = 1$ for a significant percentage of the sample. For instance, if there is a break in persistence, at $\tau_1 = \tau_2 = 0.6$ the time series behaves as a random walk over the last 40% of the sample. When the sample size is moderate ($T = 250$) and a break occurs, the proposed test displays significant power gains even when $\rho_1 = 0.9$ and $\rho_2 = \rho_3 = 1$.

If $\rho_2 = \rho_3$, the proposed test only provides power gains relative to the DF^{GLS_μ} test when $T \geq 250$. The differences between the rejection rates of these two tests are maximized when the process exhibits higher persistence for a relevant portion of the sample ($\tau_1 = \tau_2 = 0.6$). The same holds when $\rho_1 = \rho_3$, where power gains are observed even in small samples ($T = 150$) when $\tau_1 = 0.3$ and $\tau_2 = 0.7$.

Table 6 presents the empirical power for the detrended statistics with abrupt breaks in persistence. For $\rho_2 = \rho_3$, the advantages of using the proposed test are clear only when the sample is large. On the other hand, for $\rho_1 = \rho_3$, there are positive differences relative to the DF^{GLS_τ} for all cases.

Note that, for the trigonometric function considered, the persistence parameter is always lower at the beginning of the sample. The symmetric cases can be investigated using the time series in reverse chronological order. However, this transformation alters the asymptotic distributions of the proposed test and those of the DF^{GLS} tests. In this work, we only derive the asymptotic distribution of the tests in reverse chronological order for the demeaned case (see Section 5, equation (24)). Since the critical values are very close to those obtained using the normal chronological order, reversing the chronological order when the process starts with a period of higher persistence should lead to results similar to those obtained in this Section

Overall, simulation results suggest that the proposed test performs substantially better than the DF^{GLS} test when the periods of higher persistence occur somewhere in the middle of the sample. The Wild bootstrap was also considered here and the results show that the empirical size and power using this technique are, as expected, very close to those obtained using finite sample critical values (see Table 2).

DGP: $y_t = y_{t-1} + \varphi((1+\cos(2\pi kt/T))/2)y_{t-1} + \varepsilon_t$, with $\varepsilon_t \sim N(0,1)$

	DF^{GLS_μ}			$\mathcal{T}_{\hat{k}}^{GLS_\mu}$			$DF^{GLS_\mu^*}$			$\mathcal{T}_{\hat{k}}^{GLS_\mu^*}$		
	$T=150$											
k/φ	0	−0.1	−0.2	0	−0.1	−0.2	0	−0.1	−0.2	0	−0.1	−0.2
0	0.052	0.925	0.999	0.051	0.707	0.962	0.053	0.918	0.996	0.048	0.679	0.952
0.5	0.052	0.317	0.523	0.051	0.286	0.654	0.053	0.330	0.547	0.048	0.268	0.640
1.0	0.052	0.356	0.601	0.051	0.375	0.769	0.053	0.349	0.619	0.048	0.366	0.748
1.5	0.052	0.336	0.574	0.051	0.354	0.734	0.053	0.336	0.574	0.048	0.337	0.725
2.0	0.052	0.377	0.658	0.051	0.428	0.811	0.053	0.377	0.659	0.048	0.419	0.803
2.5	0.052	0.361	0.635	0.051	0.420	0.796	0.053	0.365	0.624	0.048	0.400	0.762
3.0	0.052	0.405	0.725	0.051	0.461	0.843	0.053	0.423	0.726	0.048	0.465	0.820
	$T=250$											
0	0.052	0.996	1.000	0.052	0.942	0.996	0.040	0.935	0.998	0.040	0.935	0.998
0.5	0.052	0.484	0.674	0.052	0.557	0.942	0.040	0.477	0.663	0.040	0.532	0.930
1.0	0.052	0.555	0.765	0.052	0.689	0.967	0.040	0.549	0.762	0.040	0.685	0.961
1.5	0.052	0.524	0.753	0.052	0.657	0.958	0.040	0.526	0.754	0.040	0.628	0.958
2.0	0.052	0.601	0.854	0.052	0.740	0.974	0.040	0.578	0.851	0.040	0.720	0.971
2.5	0.052	0.588	0.839	0.052	0.717	0.967	0.040	0.577	0.832	0.040	0.703	0.955
3.0	0.052	0.666	0.918	0.052	0.774	0.976	0.040	0.661	0.919	0.040	0.752	0.970
	$T=500$											
0	0.050	1.000	1.000	0.052	0.999	1.000	0.041	1.000	1.000	0.041	1.000	1.000
0.5	0.050	0.683	0.818	0.052	0.961	0.999	0.041	0.684	0.818	0.041	0.950	0.998
1.0	0.050	0.774	0.911	0.052	0.982	1.000	0.041	0.758	0.909	0.041	0.978	1.000
1.5	0.050	0.761	0.913	0.052	0.977	1.000	0.041	0.778	0.923	0.041	0.966	1.000
2.0	0.050	0.859	0.976	0.052	0.988	1.000	0.041	0.852	0.968	0.041	0.984	0.999
2.5	0.050	0.848	0.968	0.052	0.983	1.000	0.041	0.844	0.972	0.041	0.972	1.000
3.0	0.050	0.922	0.995	0.052	0.989	1.000	0.041	0.915	0.992	0.041	0.985	0.999

TABLE 3. Empirical size and power with *iid* errors - constant case

DGP: $y_t = y_{t-1} + \varphi((1+\cos(2\pi kt/T))/2)y_{t-1} + \varepsilon_t$, with $\varepsilon_t \sim N(0,1)$

	DF^{GLS_τ}			$\mathcal{T}_{\hat{k}}^{GLS_\tau}$			$DF^{GLS_\tau^*}$			$\mathcal{T}_{\hat{k}}^{GLS_\tau^*}$		
						$T=150$						
k/φ	0	-0.1	-0.2	0	-0.1	-0.2	0	-0.1	-0.2	0	-0.1	-0.2
0	0.050	0.594	0.991	0.048	0.364	0.859	0.053	0.584	0.988	0.046	0.340	0.839
0.5	0.050	0.184	0.400	0.048	0.153	0.373	0.053	0.195	0.415	0.046	0.146	0.351
1.0	0.050	0.119	0.280	0.048	0.134	0.396	0.053	0.120	0.274	0.046	0.110	0.348
1.5	0.050	0.126	0.298	0.048	0.130	0.376	0.053	0.132	0.297	0.046	0.125	0.338
2.0	0.050	0.134	0.314	0.048	0.147	0.439	0.053	0.133	0.307	0.046	0.142	0.406
2.5	0.050	0.130	0.307	0.048	0.147	0.434	0.053	0.129	0.324	0.046	0.130	0.415
3.0	0.050	0.146	0.348	0.048	0.171	0.481	0.053	0.149	0.343	0.046	0.155	0.446
						$T=250$						
0	0.048	0.960	1.000	0.051	0.725	0.994	0.048	0.969	1.000	0.056	0.706	0.994
0.5	0.048	0.333	0.604	0.051	0.290	0.759	0.048	0.356	0.629	0.056	0.309	0.737
1.0	0.048	0.230	0.478	0.051	0.298	0.819	0.048	0.223	0.456	0.056	0.300	0.787
1.5	0.048	0.246	0.504	0.051	0.280	0.797	0.048	0.259	0.517	0.056	0.264	0.789
2.0	0.048	0.253	0.537	0.051	0.336	0.850	0.048	0.264	0.544	0.056	0.325	0.824
2.5	0.048	0.244	0.542	0.051	0.325	0.833	0.048	0.243	0.527	0.056	0.309	0.818
3.0	0.048	0.282	0.598	0.051	0.369	0.872	0.048	0.275	0.602	0.056	0.361	0.844
						$T=500$						
0	0.050	1.000	1.000	0.048	0.995	1.000	0.049	1.000	1.000	0.046	0.987	1.000
0.5	0.050	0.607	0.806	0.048	0.764	0.999	0.049	0.612	0.808	0.046	0.749	0.997
1.0	0.050	0.486	0.728	0.048	0.827	0.999	0.049	0.490	0.727	0.046	0.807	1.000
1.5	0.050	0.509	0.758	0.048	0.806	0.999	0.049	0.517	0.767	0.046	0.782	0.998
2.0	0.050	0.535	0.811	0.048	0.859	1.000	0.049	0.540	0.793	0.046	0.851	1.000
2.5	0.050	0.540	0.828	0.048	0.847	1.000	0.049	0.550	0.817	0.046	0.837	1.000
3.0	0.050	0.595	0.884	0.048	0.879	1.000	0.049	0.597	0.882	0.046	0.871	0.999

TABLE 4. Empirical size and power with *iid* errors -linear trend case

DGP:

$$y_t = \rho_1 y_{t-1} + \varepsilon_t \;\; for \;\; t = 1, ..., \lfloor \bar{\tau}_1 T \rfloor,$$
$$y_t = \rho_2 y_{t-1} + \varepsilon_t \;\; for \;\; t = \lfloor \bar{\tau}_1 T \rfloor + 1, ..., \lfloor \bar{\tau}_2 T \rfloor,$$
$$y_t = \rho_3 y_{t-1} + \varepsilon_t \;\; for \;\; t = \lfloor \bar{\tau}_2 T \rfloor + 1, ..., T$$
$$\varepsilon_t \sim N(0,1)$$

	DF^{GLS_μ}		$\mathcal{T}_{\hat{k}}^{GLS_\mu}$		DF^{GLS_μ}		$\mathcal{T}_{\hat{k}}^{GLS_\mu}$		DF^{GLS_μ}		$\mathcal{T}_{\hat{k}}^{GLS_\mu}$	
	CV	WB	CV	WB	CV	WB	CV	WB	CV	WB	CV	WB
$T = 150$												
$(\bar{\tau}_1, \bar{\tau}_2)$												
(ρ_1, ρ_2, ρ_3)	$(0.3, 0.3)$				$(0.6, 0.6)$				$(0.8, 0.8)$			
$(0.8, 0.99, 0.99)$	0.233	0.252	0.287	0.287	0.515	0.541	0.724	0.714	0.783	0.792	0.908	0.889
$(0.8, 1, 1)$	0.148	0.165	0.205	0.211	0.408	0.418	0.661	0.655	0.718	0.723	0.897	0.876
$(0.9, 0.99, 0.99)$	0.194	0.207	0.188	0.191	0.398	0.419	0.373	0.384	0.629	0.650	0.553	0.526
$(0.9, 1, 1)$	0.124	0.135	0.133	0.133	0.314	0.308	0.315	0.317	0.561	0.576	0.525	0.502
(ρ_1, ρ_2, ρ_3)	$(0.3, 0.6)$				$(0.4, 0.7)$				$(0.3, 0.7)$			
$(0.8, 0.99, 0.8)$	0.752	0.768	0.880	0.866	0.743	0.746	0.920	0.896	0.607	0.623	0.865	0.838
$(0.8, 1, 0.8)$	0.667	0.676	0.876	0.863	0.658	0.675	0.924	0.905	0.501	0.509	0.873	0.850
$(0.9, 0.99, 0.9)$	0.569	0.578	0.538	0.520	0.568	0.573	0.574	0.544	0.454	0.457	0.500	0.473
$(0.9, 1, 0.9)$	0.485	0.487	0.534	0.517	0.491	0.498	0.581	0.561	0.362	0.360	0.513	0.490
$T = 250$												
(ρ_1, ρ_2, ρ_3)	$(0.3, 0.3)$				$(0.6, 0.6)$				$(0.8, 0.8)$			
$(0.8, 0.99, 0.99)$	0.342	0.345	0.475	0.452	0.640	0.628	0.940	0.933	0.863	0.873	0.992	0.992
$(0.8, 1, 1)$	0.170	0.157	0.286	0.286	0.449	0.448	0.878	0.856	0.763	0.777	0.989	0.989
$(0.9, 0.99, 0.99)$	0.305	0.316	0.332	0.327	0.569	0.557	0.686	0.644	0.798	0.809	0.857	0.842
$(0.9, 1, 1)$	0.149	0.125	0.189	0.180	0.392	0.402	0.573	0.531	0.687	0.698	0.834	0.815
(ρ_1, ρ_2, ρ_3)	$(0.3, 0.6)$				$(0.4, 0.7)$				$(0.3, 0.7)$			
$(0.8, 0.99, 0.8)$	0.856	0.853	0.982	0.981	0.857	0.855	0.990	0.990	0.745	0.732	0.981	0.977
$(0.8, 1, 0.8)$	0.733	0.729	0.981	0.980	0.737	0.730	0.991	0.987	0.570	0.577	0.983	0.979
$(0.9, 0.99, 0.9)$	0.773	0.769	0.831	0.813	0.775	0.777	0.868	0.852	0.659	0.655	0.804	0.793
$(0.9, 1, 0.9)$	0.635	0.638	0.822	0.810	0.633	0.631	0.879	0.864	0.481	0.475	0.820	0.804

Table 5 continued on next page

DGP:

$$y_t = \rho_1 y_{t-1} + \varepsilon_t \;\; for \;\; t = 1, ..., \lfloor \bar{\tau}_1 T \rfloor,$$
$$y_t = \rho_2 y_{t-1} + \varepsilon_t \;\; for \;\; t = \lfloor \bar{\tau}_1 T \rfloor + 1, ..., \lfloor \bar{\tau}_2 T \rfloor,$$
$$y_t = \rho_3 y_{t-1} + \varepsilon_t \;\; for \;\; t = \lfloor \bar{\tau}_2 T \rfloor + 1, ..., T$$
$$\varepsilon_t \sim N(0,1)$$

	DF^{GLS_μ}		$\mathcal{T}_{\hat{k}}^{GLS_\mu}$		DF^{GLS_μ}		$\mathcal{T}_{\hat{k}}^{GLS_\mu}$		DF^{GLS_μ}		$\mathcal{T}_{\hat{k}}^{GLS_\mu}$	
	CV	WB	CV	WB	CV	WB	CV	WB	CV	WB	CV	WB
	$T = 500$											
(ρ_1, ρ_2, ρ_3)	$(0.3, 0.3)$				$(0.6, 0.6)$				$(0.8, 0.8)$			
$(0.8, 0.99, 0.99)$	0.578	0.569	0.752	0.753	0.824	0.816	0.998	0.998	0.948	0.934	1.000	1.000
$(0.8, 1, 1)$	0.177	0.161	0.369	0.355	0.476	0.472	0.959	0.957	0.801	0.800	1.000	1.000
$(0.9, 0.99, 0.99)$	0.550	0.549	0.644	0.657	0.793	0.789	0.978	0.973	0.928	0.921	0.997	0.997
$(0.9, 1, 1)$	0.165	0.147	0.291	0.269	0.440	0.440	0.881	0.877	0.761	0.756	0.995	0.993
(ρ_1, ρ_2, ρ_3)	$(0.3, 0.6)$				$(0.4, 0.7)$				$(0.3, 0.7)$			
$(0.8, 0.99, 0.8)$	0.961	0.965	1.000	1.000	0.962	0.955	1.000	1.000	0.910	0.909	1.000	1.000
$(0.8, 1, 0.8)$	0.784	0.770	1.000	1.000	0.792	0.785	1.000	1.000	0.615	0.615	1.000	1.000
$(0.9, 0.99, 0.9)$	0.941	0.945	0.995	0.994	0.943	0.935	0.997	0.996	0.878	0.866	0.992	0.990
$(0.9, 1, 0.9)$	0.727	0.723	0.992	0.988	0.733	0.730	0.998	0.997	0.560	0.554	0.993	0.993

TABLE 5. Empirical power when the breaks in persistence are abrupt - constant case

DGP:

$$y_t = \rho_1 y_{t-1} + \varepsilon_t \ \ for \ \ t = 1, ..., \lfloor \bar{\tau}_1 T \rfloor,$$
$$y_t = \rho_2 y_{t-1} + \varepsilon_t \ \ for \ \ t = \lfloor \bar{\tau}_1 T \rfloor + 1, ..., \lfloor \bar{\tau}_2 T \rfloor,$$
$$y_t = \rho_3 y_{t-1} + \varepsilon_t \ \ for \ \ t = \lfloor \bar{\tau}_2 T \rfloor + 1, ..., T$$
$$\varepsilon_t \sim N(0, 1)$$

	DF^{GLS_μ}		$\mathcal{T}_{\hat{k}}^{GLS_\mu}$		DF^{GLS_μ}		$\mathcal{T}_{\hat{k}}^{GLS_\mu}$		DF^{GLS_μ}		$\mathcal{T}_{\hat{k}}^{GLS_\mu}$	
	CV	WB	CV	WB	CV	WB	CV	WB	CV	WB	CV	WB
	$T = 150$											
	$(\bar{\tau}_1, \bar{\tau}_2)$											
(ρ_1, ρ_2, ρ_3)	$(0.3, 0.3)$				$(0.6, 0.6)$				$(0.8, 0.8)$			
$(0.8, 0.99, 0.99)$	0.185	0.206	0.211	0.199	0.442	0.460	0.440	0.429	0.712	0.728	0.661	0.658
$(0.8, 1, 1)$	0.143	0.156	0.168	0.154	0.360	0.369	0.386	0.372	0.646	0.660	0.637	0.620
$(0.9, 0.99, 0.99)$	0.129	0.147	0.124	0.100	0.244	0.253	0.186	0.176	0.375	0.381	0.259	0.234
$(0.9, 1, 1)$	0.105	0.119	0.103	0.091	0.198	0.203	0.157	0.155	0.337	0.346	0.241	0.215
(ρ_1, ρ_2, ρ_3)	$(0.3, 0.6)$				$(0.4, 0.7)$				$(0.3, 0.7)$			
$(0.8, 0.99, 0.8)$	0.449	0.450	0.615	0.584	0.461	0.475	0.657	0.604	0.304	0.320	0.531	0.494
$(0.8, 1, 0.8)$	0.376	0.380	0.610	0.575	0.393	0.390	0.651	0.598	0.241	0.248	0.530	0.468
$(0.9, 0.99, 0.9)$	0.228	0.222	0.232	0.208	0.240	0.230	0.230	0.196	0.166	0.175	0.181	0.153
$(0.9, 1, 0.9)$	0.187	0.184	0.231	0.208	0.198	0.185	0.214	0.181	0.125	0.131	0.172	0.135
	$T = 250$											
(ρ_1, ρ_2, ρ_3)	$(0.3, 0.3)$				$(0.6, 0.6)$				$(0.8, 0.8)$			
$(0.8, 0.99, 0.99)$	0.256	0.236	0.383	0.372	0.596	0.614	0.810	0.791	0.846	0.851	0.956	0.952
$(0.8, 1, 1)$	0.174	0.164	0.274	0.257	0.445	0.457	0.713	0.696	0.749	0.760	0.946	0.942
$(0.9, 0.99, 0.99)$	0.195	0.171	0.208	0.215	0.436	0.449	0.381	0.391	0.673	0.694	0.545	0.535
$(0.9, 1, 1)$	0.134	0.132	0.151	0.148	0.313	0.340	0.300	0.321	0.576	0.597	0.506	0.500
(ρ_1, ρ_2, ρ_3)	$(0.3, 0.6)$				$(0.4, 0.7)$				$(0.3, 0.7)$			
$(0.8, 0.99, 0.8)$	0.617	0.611	0.929	0.906	0.632	0.621	0.964	0.951	0.456	0.437	0.902	0.868
$(0.8, 1, 0.8)$	0.483	0.463	0.920	0.900	0.499	0.496	0.964	0.943	0.317	0.297	0.897	0.857
$(0.9, 0.99, 0.9)$	0.433	0.416	0.487	0.480	0.452	0.451	0.516	0.495	0.315	0.287	0.414	0.410
$(0.9, 1, 0.9)$	0.326	0.311	0.480	0.463	0.345	0.326	0.506	0.492	0.211	0.203	0.410	0.393

Table 6 continued on next page

DGP:

$$y_t = \rho_1 y_{t-1} + \varepsilon_t \;\; for \;\; t = 1, ..., \lfloor \bar{\tau}_1 T \rfloor,$$
$$y_t = \rho_2 y_{t-1} + \varepsilon_t \;\; for \;\; t = \lfloor \bar{\tau}_1 T \rfloor + 1, ..., \lfloor \bar{\tau}_2 T \rfloor,$$
$$y_t = \rho_3 y_{t-1} + \varepsilon_t \;\; for \;\; t = \lfloor \bar{\tau}_2 T \rfloor + 1, ..., T$$
$$\varepsilon_t \sim N(0,1)$$

	DF^{GLS_μ}		$\mathcal{T}_{\hat{k}}^{GLS_\mu}$		DF^{GLS_μ}		$\mathcal{T}_{\hat{k}}^{GLS_\mu}$		DF^{GLS_μ}		$\mathcal{T}_{\hat{k}}^{GLS_\mu}$	
	CV	WB	CV	WB	CV	WB	CV	WB	CV	WB	CV	WB
							$T = 500$					
(ρ_1, ρ_2, ρ_3)	$(0.3, 0.3)$				$(0.6, 0.6)$				$(0.8, 0.8)$			
$(0.8, 0.99, 0.99)$	0.412	0.396	0.680	0.678	0.799	0.791	0.991	0.989	0.951	0.938	1.000	1.000
$(0.8, 1, 1)$	0.200	0.192	0.413	0.404	0.509	0.517	0.920	0.912	0.812	0.815	1.000	1.000
$(0.9, 0.99, 0.99)$	0.360	0.339	0.474	0.466	0.723	0.720	0.863	0.855	0.910	0.904	0.968	0.958
$(0.9, 1, 1)$	0.169	0.164	0.267	0.256	0.434	0.440	0.697	0.675	0.749	0.750	0.951	0.940
(ρ_1, ρ_2, ρ_3)	$(0.3, 0.6)$				$(0.4, 0.7)$				$(0.3, 0.7)$			
$(0.8, 0.99, 0.8)$	0.819	0.813	0.999	0.999	0.841	0.833	1.000	1.000	0.699	0.685	0.999	0.999
$(0.8, 1, 0.8)$	0.563	0.580	0.998	0.997	0.582	0.572	1.000	1.000	0.386	0.381	0.997	0.992
$(0.9, 0.99, 0.9)$	0.740	0.732	0.941	0.937	0.757	0.745	0.963	0.950	0.607	0.608	0.913	0.904
$(0.9, 1, 0.9)$	0.479	0.489	0.924	0.917	0.497	0.494	0.966	0.945	0.313	0.309	0.901	0.878

TABLE 6. Empirical power when the breaks in persistence are abrupt - linear trend case

3.2. Serially correlated errors

The finite sample properties of the tests in the presence of autocorrelation are examined considering that the error process follows an ARMA, i..,

$$\varepsilon_t = \delta\varepsilon_{t-1} + \theta e_{t-1} + e_t \tag{19}$$

with $e_t \sim N(0,1)$, $y_1 = \varepsilon_1 \sim N(0,1)$, $\delta \in \{0, 0.3, 0.6\}$ and $\theta \in \{-0.8, -0.4, 0, 0.4, 0.8\}$. The lags of the augmented test regression will be chosen using the MAIC information criteria proposed by Ng and Perron (2001), as it is one of the most popular lag selection criteria used in the literature.

Table 7 summarizes the finite sample results for the demeaned tests in the presence of autocorrelation in ε_t. Although we also performed simulations for $T = 150$ and 500, only the results for $T = 250$ are reported as the conclusions are qualitatively the same for the other sample sizes. The $\mathcal{T}_{\hat{k}}^{GLS_\mu}$ test shows good finite sample size performance. Its empirical size only exceeds the nominal 5% significance level when the MA term is negative and even for those cases the values are close to those (or slightly lower) of the DF^{GLS_μ}.

In the evaluation of the power performance, we considered $\varphi = -0.1$ and that the breaks in persistence are smooth and approximated by cosine functions. The sign of the MA term also affects the power properties of the proposed test. When the MA term is negative, the advantage of using it are less clear. However, if the MA term is nonnegative and $k > 0.5$, there are relevant power gains even for $T = 150$. For larger samples, the power differences relative to DF^{GLS_μ} are more pronounced and occur for all values of k. The $\mathcal{T}_{\hat{k}}^{GLS_\mu}$ test faces more difficulties in rejecting the null hypothesis when $\theta = -0.8$ and $\delta = 0$.

Simulation results for the detrended case are presented in Table 8. As in the demeaned case, the empirical size of the proposed test exceeds the nominal 5% level less than the DF^{GLS_τ} when the MA term is negative. In this case, the two tests are more severely undersized when the MA term is positive. This problem is attenuated as the sample size increases.

The results show that the power of the two tests considered is low when a linear trend is used and the autoregressive parameter is large before the increase in persistence (when the cosine functions is zero, the autoregressive parameter equals 0.9). For $T = 500$, the results are less disappointing and the superiority of the proposed test becomes evident, especially when the MA term is nonnegative.

DGP:		$y_t = y_{t-1} + \varphi((1+\cos(2\pi kt/T))/2)y_{t-1} + \varepsilon_t$ $\varepsilon_t = \delta\varepsilon_{t-1} + \theta e_{t-1} + e_t,\ e_t \sim N(0,1)$					
$T = 250$		DF^{GLS_μ}			$\mathcal{T}_{\hat{k}}^{GLS_\mu}$		
(φ, k)	θ / δ	0	0.3	0.6	0	0.3	0.6
	-0.8	0.078	0.091	0.104	0.063	0.074	0.091
	-0.4	0.052	0.053	0.029	0.055	0.056	0.034
$(0, 0)$	0	0.044	0.044	0.043	0.045	0.046	0.044
	0.4	0.043	0.040	0.039	0.046	0.043	0.044
	0.8	0.039	0.039	0.040	0.045	0.044	0.046
	-0.8	0.235	0.317	0.426	0.164	0.266	0.433
	-0.4	0.361	0.395	0.289	0.378	0.452	0.282
$(-0.1, 0.5)$	0	0.392	0.375	0.351	0.439	0.426	0.411
	0.4	0.356	0.335	0.312	0.403	0.379	0.343
	0.8	0.289	0.283	0.262	0.331	0.322	0.301
	-0.8	0.290	0.389	0.512	0.226	0.350	0.550
	-0.4	0.434	0.480	0.366	0.518	0.604	0.496
$(-0.1, 1.0)$	0	0.473	0.463	0.445	0.605	0.603	0.606
	0.4	0.437	0.420	0.400	0.588	0.576	0.553
	0.8	0.362	0.359	0.342	0.529	0.524	0.509
	-0.8	0.275	0.364	0.492	0.204	0.330	0.523
	-0.4	0.401	0.441	0.316	0.483	0.570	0.436
$(-0.1, 1.5)$	0	0.436	0.418	0.393	0.562	0.561	0.545
	0.4	0.392	0.373	0.344	0.533	0.517	0.486
	0.8	0.321	0.313	0.293	0.481	0.469	0.449
	-0.8	0.323	0.432	0.576	0.249	0.380	0.592
	-0.4	0.487	0.536	0.405	0.582	0.672	0.584
$(-0.1, 2.0)$	0	0.518	0.509	0.498	0.668	0.677	0.676
	0.4	0.489	0.474	0.452	0.658	0.648	0.634
	0.8	0.423	0.419	0.410	0.608	0.605	0.591
	-0.8	0.308	0.411	0.553	0.243	0.378	0.584
	-0.4	0.466	0.512	0.364	0.557	0.645	0.540
$(-0.1, 2.5)$	0	0.493	0.478	0.462	0.641	0.641	0.637
	0.4	0.455	0.439	0.415	0.623	0.610	0.587
	0.8	0.388	0.384	0.369	0.570	0.567	0.549
	-0.8	0.355	0.476	0.642	0.269	0.415	0.624
	-0.4	0.547	0.608	0.464	0.618	0.704	0.635
$(-0.1, 3)$	0	0.585	0.585	0.589	0.706	0.717	0.714
	0.4	0.560	0.553	0.550	0.696	0.689	0.675
	0.8	0.508	0.512	0.515	0.651	0.648	0.633

TABLE 7. Finite sample sizes and power with ε_t autocorrelated - constant case

DGP:	$y_t = y_{t-1} + \varphi((1+\cos(2\pi kt/T))/2)y_{t-1} + \varepsilon_t$ $\varepsilon_t = \delta\varepsilon_{t-1} + \theta e_{t-1} + e_t,\ e_t \sim N(0,1)$						
$T = 250$		DF^{GLS_τ}			$\mathcal{T}_{\hat{k}}^{GLS_\tau}$		
(φ, k)	θ / δ	0	0.3	0.6	0	0.3	0.6
	−0.8	0.066	0.085	0.107	0.029	0.047	0.093
	−0.4	0.046	0.046	0.013	0.050	0.054	0.022
$(0,0)$	0	0.033	0.032	0.034	0.039	0.040	0.043
	0.4	0.030	0.027	0.026	0.037	0.033	0.033
	0.8	0.022	0.022	0.023	0.035	0.033	0.034
	−0.8	0.169	0.229	0.315	0.050	0.093	0.227
	−0.4	0.218	0.254	0.104	0.170	0.228	0.076
$(-0.1, 0.5)$	0	0.231	0.204	0.186	0.196	0.185	0.176
	0.4	0.188	0.161	0.138	0.162	0.136	0.116
	0.8	0.118	0.109	0.099	0.112	0.107	0.102
	−0.8	0.117	0.162	0.235	0.054	0.106	0.247
	−0.4	0.151	0.172	0.067	0.201	0.254	0.123
$(-0.1, 1.0)$	0	0.155	0.138	0.132	0.227	0.218	0.211
	0.4	0.128	0.112	0.097	0.201	0.179	0.160
	0.8	0.078	0.075	0.069	0.159	0.154	0.145
	−0.8	0.133	0.181	0.245	0.055	0.106	0.240
	−0.4	0.160	0.182	0.071	0.190	0.234	0.101
$(-0.1, 1.5)$	0	0.167	0.146	0.133	0.206	0.195	0.184
	0.4	0.130	0.111	0.092	0.174	0.152	0.138
	0.8	0.079	0.073	0.065	0.137	0.134	0.126
	−0.8	0.148	0.201	0.278	0.068	0.128	0.278
	−0.4	0.175	0.201	0.078	0.242	0.295	0.168
$(-0.1, 2.0)$	0	0.181	0.161	0.151	0.268	0.258	0.248
	0.4	0.144	0.127	0.109	0.241	0.219	0.196
	0.8	0.094	0.088	0.082	0.201	0.192	0.178
	−0.8	0.151	0.203	0.275	0.072	0.137	0.278
	−0.4	0.172	0.194	0.068	0.228	0.287	0.157
$(-0.1, 2.5)$	0	0.17	0.151	0.141	0.252	0.246	0.236
	0.4	0.135	0.117	0.099	0.230	0.211	0.190
	0.8	0.089	0.081	0.070	0.187	0.183	0.171
	−0.8	0.177	0.238	0.327	0.080	0.151	0.313
	−0.4	0.209	0.232	0.088	0.277	0.334	0.207
$(-0.1, 3)$	0	0.202	0.186	0.179	0.305	0.293	0.289
	0.4	0.168	0.146	0.134	0.276	0.257	0.237
	0.8	0.113	0.108	0.105	0.234	0.226	0.213

TABLE 8. Finite sample sizes and power with ε_t autocorrelated - linear trend case

3.3. Heteroskedasticity

3.3.1. Conditional heteroskedasticity . In order to investigate the finite sample distortions caused by the existence of conditional heteroskedasticity, we consider that the innovations, ε_t, follow a GARCH(1,1) process

$$\varepsilon_t = e_t \sqrt{h_t}$$

with

$$h_t = \omega + \zeta \varepsilon_{t-1}^2 + \xi h_{t-1},$$

where $y_1 = \varepsilon_1 \sim N(0,1)$, $h_1 = 1$, $e_t \sim N(0,1)$, $\zeta \in \{0.7, 0.8, 0.9\}$, $\xi \in \{0, 0.05, 0.1, 0.2\}$ and $\omega = 1 - \zeta - \xi$, implying an unconditional variance of unity.

Tables 9 and 10 report the empirical size of the $\mathcal{T}_{\hat{k}}^{GLS_\mu}$ and $\mathcal{T}_{\hat{k}}^{GLS_\tau}$ tests. Results show that these tests suffer relevant size distortions in the presence of conditional heteroskedasticity, especially when ζ is high. Its empirical size is slightly higher than that of the DF^{GLS_μ} and considerably exceeds the nominal 5% level for all parameter values considered.

The results using the EW heteroskedasticity-consistent standard errors, presented in Tables 11 and 12 (to save space, only results for $T = 250$ are reported), show that the empirical sizes are close to the nominal 5% significance level for both demeaned and de-trended cases. Moreover, the results obtained employing the Wild bootstrap approach (see Tables 13 and 14), show that the proposed test is relatively well behaved for all cases considered..

Regarding the empirical power, relevant power gains are observed relatively to the DF^{GLS} when EW heteroskedasticity-consistent standard errors are considered. When the Wild bootstrap technique is employed, the superiority relative to DF^{GLS} is less pronounced for all parameter configurations considered.

However, the conclusions are different for the demeaned and de-trended cases when T=500. For this sample size, our tests exhibit considerable power gains relative to the DF^{GLS} tests, mainly when the constant is the only deterministic component used. For some combinations of parameters, the percentages of rejections are over 20% higher than those of the DF^{GLS} test.

When these two techniques (EW and Wild bootstrap), used to control the empirical size, are compared we see that Wild bootstrap provides higher percentages of rejections of the false null hypothesis for almost all cases investigated.

DGP: $y_t = y_{t-1} + \varepsilon_t$
$\varepsilon_t = e_t\sqrt{h_t},\ h_t = \omega + \zeta\varepsilon_{t-1}^2 + \xi h_{t-1}$
$\omega = 1 - \zeta - \xi,\ e_t \sim N(0,1)$

	DF^{GLS_μ}			$\mathcal{T}_{\hat{k}}^{GLS_\mu}$		
			$T = 150$			
ξ/ζ	0.7	0.8	0.9	0.7	0.8	0.9
0	0.063	0.065	0.071	0.083	0.093	0.106
0.05	0.065	0.071	0.079	0.086	0.099	0.114
0.1	0.068	0.075	-	0.089	0.102	-
0.2	0.077	-	-	0.102	-	-
			$T = 250$			
0	0.066	0.073	0.079	0.085	0.094	0.109
0.05	0.068	0.074	0.079	0.088	0.099	0.114
0.1	0.069	0.076	-	0.092	0.104	-
0.2	0.076	-	-	0.102	-	-
			$T = 500$			
0	0.061	0.067	0.071	0.071	0.085	0.096
0.05	0.064	0.068	0.074	0.078	0.091	0.104
0.1	0.066	0.071	-	0.086	0.098	-
0.2	0.069	-	-	0.096	-	-

TABLE 9. Empirical size assuming *iid* errors in the presence of GARCH effects - constant case

DGP: $y_t = y_{t-1} + \varepsilon_t$
$\varepsilon_t = e_t\sqrt{h_t},\ h_t = \omega + \zeta\varepsilon_{t-1}^2 + \xi h_{t-1}$
$\omega = 1 - \zeta - \xi,\ e_t \sim N(0,1)$

			$T = 150$			
	DF^{GLS_τ}			$\mathcal{T}_{\hat{k}}^{GLS_\tau}$		
ξ/ζ	0.7	0.8	0.9	0.7	0.8	0.9
0	0.079	0.091	0.102	0.127	0.146	0.168
0.05	0.084	0.097	0.108	0.130	0.153	0.183
0.1	0.087	0.102	-	0.138	0.164	-
0.2	0.098	-	-	0.157	-	-
			$T = 250$			
0	0.075	0.084	0.099	0.120	0.142	0.168
0.05	0.076	0.089	0.106	0.126	0.149	0.178
0.1	0.081	0.097	-	0.133	0.161	-
0.2	0.093	-	-	0.157	-	-
			$T = 500$			
0	0.070	0.078	0.090	0.103	0.126	0.158
0.05	0.072	0.084	0.100	0.110	0.138	0.173
0.1	0.076	0.091	-	0.120	0.153	-
0.2	0.089	-	-	0.147	-	-

TABLE 10. Empirical size assuming *iid* errors in the presence of GARCH effects - linear trend case

DGP:	$y_t = y_{t-1} + \varphi((1+\cos(2\pi kt/T))/2)y_{t-1} + \varepsilon_t,$ $\varepsilon_t = e_t\sqrt{h_t},\ h_t = \omega + \zeta\varepsilon_{t-1}^2 + \xi h_{t-1}$ $\omega = 1-\zeta-\xi,\ e_t \sim N(0,1)$						
$T = 250$		DF^{GLS_μ}			$\mathcal{T}_{\hat{k}}^{GLS_\mu}$		
	ξ/ζ	0.7	0.8	0.9	0.7	0.8	0.9
$\varphi = 0$	0	0.051	0.049	0.049	0.058	0.054	0.052
	0.05	0.050	0.050	0.048	0.056	0.055	0.050
	0.1	0.050	0.048	-	0.057	0.052	-
	0.2	0.052	-	-	0.053	-	-
				$\varphi = -0.1$			
$k = 0.5$	0	0.396	0.370	0.336	0.434	0.399	0.348
	0.05	0.386	0.352	0.299	0.422	0.379	0.306
	0.1	0.375	0.335	-	0.407	0.352	-
	0.2	0.340	-	-	0.360	-	-
$k = 1$	0	0.454	0.422	0.382	0.571	0.531	0.483
	0.05	0.438	0.406	0.351	0.557	0.513	0.432
	0.1	0.426	0.385	-	0.543	0.486	-
	0.2	0.390	-	-	0.496	-	-
$k = 1.5$	0	0.429	0.396	0.359	0.520	0.482	0.432
	0.05	0.415	0.380	0.325	0.505	0.46	0.391
	0.1	0.400	0.360	-	0.488	0.437	-
	0.2	0.366	-	-	0.449	-	-
$k = 2$	0	0.494	0.459	0.4138	0.613	0.581	0.532
	0.05	0.482	0.440	0.3748	0.603	0.56	0.484
	0.1	0.467	0.418	-	0.589	0.536	-
	0.2	0.428	-	-	0.545	-	-
$k = 2.5$	0	0.475	0.435	0.3921	0.603	0.566	0.514
	0.05	0.459	0.417	0.3579	0.589	0.546	0.462
	0.1	0.444	0.395	-	0.576	0.518	-
	0.2	0.404	-	-	0.531	-	-
$k = 3$	0	0.533	0.491	0.441	0.660	0.625	0.569
	0.05	0.518	0.469	0.398	0.647	0.603	0.517
	0.1	0.498	0.443	-	0.631	0.577	-
	0.2	0.450	-	-	0.587	-	-

Table 11. Empirical size and power in the presence of GARCH effects using White standard errors - constant case

DGP:	$y_t = y_{t-1} + \varphi((1+\cos(2\pi kt/T))/2)y_{t-1} + \varepsilon_t,$ $\varepsilon_t = e_t\sqrt{h_t},\ h_t = \omega + \zeta\varepsilon_{t-1}^2 + \xi h_{t-1}$ $\omega = 1-\zeta-\xi,\ e_t \sim N(0,1)$						
$T = 250$		DF^{GLS_τ}			$\mathcal{T}_{\hat{k}}^{GLS_\tau}$		
	ξ/ζ	0.7	0.8	0.9	0.7	0.8	0.9
$\varphi = 0$	0	0.041	0.037	0.035	0.049	0.044	0.040
	0.05	0.039	0.035	0.033	0.049	0.043	0.040
	0.1	0.038	0.033	-	0.047	0.042	-
	0.2	0.027	-	-	0.044	-	-
$\varphi = -0.1$							
$k = 0.5$	0	0.229	0.200	0.170	0.225	0.201	0.175
	0.05	0.219	0.186	0.150	0.216	0.191	0.156
	0.1	0.207	0.174	-	0.207	0.180	-
	0.2	0.169	-	-	0.184	-	-
$k = 1$	0	0.168	0.147	0.126	0.227	0.210	0.188
	0.05	0.159	0.139	0.114	0.219	0.201	0.174
	0.1	0.153	0.127	-	0.213	0.193	-
	0.2	0.125	-	-	0.196	-	-
$k = 1.5$	0	0.169	0.148	0.126	0.215	0.196	0.176
	0.05	0.161	0.138	0.114	0.208	0.187	0.162
	0.1	0.151	0.131	-	0.200	0.181	-
	0.2	0.124	-	-	0.185	-	-
$k = 2$	0	0.189	0.165	0.139	0.283	0.262	0.244
	0.05	0.180	0.155	0.128	0.276	0.252	0.227
	0.1	0.168	0.145	-	0.265	0.242	-
	0.2	0.138	-	-	0.247	-	-
$k = 2.5$	0	0.186	0.164	0.142	0.282	0.261	0.242
	0.05	0.177	0.153	0.127	0.274	0.252	0.226
	0.1	0.167	0.142	-	0.264	0.242	-
	0.2	0.137	-	-	0.245	-	-
$k = 3$	0	0.203	0.182	0.156	0.332	0.310	0.289
	0.05	0.193	0.169	0.139	0.324	0.299	0.268
	0.1	0.184	0.157	-	0.315	0.287	-
	0.2	0.148	-	-	0.288	-	-

TABLE 12. Empirical size and power in the presence of GARCH effects using White standard errors - linear trend case

DGP:	$y_t = y_{t-1} + \varepsilon_t$ $\varepsilon_t = e_t\sqrt{h_t}, h_t = \omega + \zeta\varepsilon_{t-1}^2 + \xi h_{t-1}$ $\omega = 1 - \zeta - \xi,\ e_t \sim N(0,1)$						
$T = 250$		DF^{GLS_μ}			$\mathcal{T}_{\hat{k}}^{GLS_\mu}$		
	ξ/ζ	0.7	0.8	0.9	0.7	0.8	0.9
$\varphi = 0$	0	0.046	0.055	0.056	0.060	0.064	0.067
	0.05	0.051	0.055	0.055	0.057	0.063	0.064
	0.1	0.052	0.051	-	0.058	0.068	-
	0.2	0.053	-	-	0.058	-	-
				$\varphi = -0.1$			
$k = 0.5$	0	0.442	0.436	0.413	0.468	0.459	0.428
	0.05	0.442	0.427	0.405	0.473	0.453	0.400
	0.1	0.446	0.426	-	0.465	0.44	
	0.2	0.445	-	-	0.458		
$k = 1$	0	0.531	0.518	0.491	0.556	0.528	0.487
	0.05	0.525	0.506	0.469	0.541	0.515	0.448
	0.1	0.516	0.499	-	0.533	0.492	-
	0.2	0.497	-	-	0.513	-	-
$k = 1.5$	0	0.523	0.518	0.488	0.551	0.526	0.494
	0.05	0.523	0.514	0.469	0.544	0.515	0.447
	0.1	0.526	0.514	-	0.525	0.493	-
	0.2	0.524	-	-	0.485	-	-
$k = 2$	0	0.556	0.55	0.534	0.637	0.612	0.554
	0.05	0.553	0.543	0.511	0.627	0.592	0.496
	0.1	0.544	0.532	-	0.619	0.551	-
	0.2	0.538	-	-	0.573	-	-
$k = 2.5$	0	0.544	0.542	0.520	0.621	0.599	0.555
	0.05	0.546	0.535	0.495	0.618	0.587	0.516
	0.1	0.546	0.524	-	0.597	0.569	-
	0.2	0.519	-	-	0.574	-	-
$k = 3$	0	0.613	0.585	0.560	0.684	0.643	0.575
	0.05	0.595	0.582	0.527	0.663	0.621	0.519
	0.1	0.596	0.562	-	0.650	0.598	-
	0.2	0.571	-	-	0.613	-	-

TABLE 13. Empirical size and power in the presence of GARCH effects when the Wild Bootstrap technique is employed - constant case

DGP:	$y_t = y_{t-1} + \varepsilon_t$ $\varepsilon_t = e_t\sqrt{h_t}, h_t = \omega + \zeta\varepsilon_{t-1}^2 + \xi h_{t-1}$ $\omega = 1 - \zeta - \xi,\ e_t \sim N(0,1)$						
$T = 250$		DF^{GLS_τ}			$\mathcal{T}_{\hat{k}}^{GLS_\tau}$		
ξ/ζ		0.7	0.8	0.9	0.7	0.8	0.9
$\varphi = 0$	0	0.049	0.051	0.057	0.064	0.069	0.074
	0.05	0.049	0.055	0.061	0.068	0.068	0.076
	0.1	0.058	0.056	-	0.068	0.075	-
	0.2	0.058	-	-	0.070	-	-
			$\varphi = -0.1$				
$k = 0.5$	0	0.321	0.323	0.311	0.297	0.290	0.275
	0.05	0.323	0.318	0.306	0.296	0.283	0.26
	0.1	0.322	0.307	-	0.295	0.282	-
	0.2	0.308	-	-	0.281	-	-
$k = 1$	0	0.217	0.214	0.219	0.251	0.234	0.224
	0.05	0.218	0.22	0.216	0.245	0.227	0.208
	0.1	0.219	0.218	-	0.237	0.22	-
	0.2	0.226	-	-	0.219	-	-
$k = 1.5$	0	0.238	0.240	0.241	0.246	0.24	0.232
	0.05	0.249	0.243	0.238	0.252	0.242	0.218
	0.1	0.246	0.242	-	0.247	0.237	-
	0.2	0.243	-	-	0.239	-	-
$k = 2$	0	0.250	0.249	0.251	0.265	0.259	0.246
	0.05	0.249	0.249	0.249	0.274	0.258	0.254
	0.1	0.257	0.257	-	0.277	0.259	-
	0.2	0.262	-	-	0.260	-	-
$k = 2.5$	0	0.250	0.247	0.251	0.286	0.267	0.246
	0.05	0.246	0.249	0.245	0.279	0.257	0.228
	0.1	0.250	0.257	-	0.267	0.243	-
	0.2	0.255	-	-	0.259	-	-
$k = 3$	0	0.281	0.284	0.279	0.319	0.297	0.285
	0.05	0.285	0.289	0.285	0.313	0.295	0.260
	0.1	0.287	0.291	-	0.306	0.283	-
	0.2	0.287	-	-	0.293	-	-

TABLE 14. Empirical size and power in the presence of GARCH effects when the Wild Bootstrap technique is employed - linear trend case

3.3.2. Unconditional heteroskedasticity . To analyze the impact of unconditional heteroscedasticity on the performance of the tests, we next consider the simulation design of DGP1 with the following specification for the innovation process:

$$\begin{cases} \varepsilon_{1,t} \sim N(0,\sigma_1^2) & for \quad t = 1,...,\lfloor \bar{\tau}_1 T \rfloor \\ \varepsilon_{2,t} \sim N(0,\sigma_2^2) & for \quad t = \lfloor \bar{\tau}_1 T \rfloor + 1,...,\lfloor \bar{\tau}_2 T \rfloor \\ \varepsilon_{3,t} \sim N(0,\sigma_3^2) & for \quad t = \lfloor \bar{\tau}_2 T \rfloor + 1,...,T, \end{cases} \tag{20}$$

where $y_1 = \varepsilon_{1,t} \sim N(0,\sigma_1^2)$.

Structural breaks in the unconditional volatility was investigated in several works. For instance, McConnell and Perez-Quiros (2000) identified a structural decline in U.S. output volatility in the early 1980's. Sensier and van Dijk (2003) found similar evidence for a large number of macroeconomic time series. Moreover, they claimed that nominal variables such as inflation also witnessed temporary increases in volatility during the 1970s and therefore, may have suffered more than one structural break.

Thus, in order to infer how changes in the innovation variance affect the finite sample properties of the proposed test, we carried out some simulations. To save space, only DGP1 with $\varphi = -0.1$ is examined. Moreover, only two cases were considered. In the first case a single break is allowed that either increases or decreases the innovation variance; and in the other case investigated, two breaks in volatility are allowed, the first causing an increase in the innovation variance and the second a reduction of this parameter to the value it assumed before the occurrence of any break. The values considered in the simulations for the standard deviations allow for large changes in the unconditional volatility.

The size results for the constant case are reported in Table 15. Results show that the proposed test reveals in some cases finite sample size distortions. However, the rejection rates are, overall, not too far from those of the DF^{GLS_μ} test. Over-rejections are more severe when the innovation variance faces a large increase (e.g. $\sigma_2^2 = 4$), even if negative variation of the same magnitude occurs later. Moreover, for breaks closer to the end of the sample empirical size distortions are the largest. For decreases in volatility, there are also small over-rejections of the null, especially when these occur at the beginning of the sample ($\bar{\tau}_1 = 0.3$).

Table 16 presents the size results for the linear trend case. Although the nominal size of the proposed test shows the same patterns reported for the constant case, the rejection rates are larger.

The Wild bootstrap procedure was employed to control the empirical size of the proposed test and the results show that this technique achieves the goal. Table 17 presents the results using the Wild bootstrap for the demeaned case with $T = 250$. The empirical sizes of the proposed test are close to the nominal 5% significance level for all cases investigated. Regarding the power properties, comparing the results in Table 17 with those reported in Table 3, where the innovation variance is constant, we observe that the power loss is greater for an increase in volatility, for instance, for $(\sigma_1, \sigma_2, \sigma_3) = (1, 2, 2)$, with $k = 0.5$ or $\bar{\tau}_1 = 0.7$. When a decrease in the unconditional variance is observed, the power loss is greater if $k > 0.5$ and $\bar{\tau}_1 = 0.3$. Finally, the occurrence of two breaks in volatility causes more severe power losses when $k = 1$ and the magnitude of the changes is larger ($\sigma_2 = 2$).

The superior power performance of the proposed test relatively to DF^{GLS_μ} also depends on k, $\bar{\tau}_1$ and σ. For instance, when the innovation variance increases, the percentage of rejections of the proposed test is greater for $k > 0.5$ and the difference relative to the DF^{GLS_μ} test reaches its maximum for $\bar{\tau}_1 = 0.3$. For a decrease in volatility, the proposed test shows better power properties for all k and the superiority relative to DF^{GLS_μ} is more prominent for $\bar{\tau}_1 = 0.7$. On the other hand, if two breaks in volatility occur, the positive difference in the percentage of rejection relative to the DF^{GLS_μ} test is greater when $k = 1$ and the change in the innovation variance is smaller ($\sigma_2 = 1.5$).

Table 18 presents the results for the linear trend case with $T = 250$. Power losses (when compared to Table 4) caused by the occurrence of breaks in volatility are similar to those reported for the constant case. Regarding the comparison with the DF^{GLS_τ} test, we see that power gains are small or nonexistent here. For instance, if $k > 0.5$ the percentage of rejections is always higher for the DF^{GLS_τ} test and for the combinations of parameter where there are some power gains, the differences relative to the DF^{GLS_τ} test are small. Thus, simulations show that, for the linear trend case, it is extremely difficult to reject the null of a unit root in small and moderate samples when $\varphi = -0.1$ if, besides the breaks in persistence, the time series also faced breaks in the innovation variance. In this case, the advantages of using the proposed test only becomes clear for larger samples ($T = 500$).

DGP: $y_t = y_{t-1} + \varepsilon_{i,t},\ i = 1,2,3$
$\varepsilon_{1,t} \sim N(0, \sigma_1^2)\ if\ t \leq \bar{\tau}_1 T,$
$\varepsilon_{2,t} \sim N(0, \sigma_2^2)\ if\ \bar{\tau}_1 T < t \leq \bar{\tau}_2 T,$
$\varepsilon_{3,t} \sim N(0, \sigma_3^2)\ if\ t \leq \bar{\tau}_2 T.$
$\boldsymbol{\sigma} = (\sigma_1, \sigma_2, \sigma_3)$ and $\bar{\boldsymbol{\tau}} = (\bar{\tau}_1,\ \bar{\tau}_2)$

	DF^{GLS_μ}	$\mathcal{T}_{\hat{k}}^{GLS_\mu}$	DF^{GLS_μ}	$\mathcal{T}_{\hat{k}}^{GLS_\mu}$	DF^{GLS_μ}	$\mathcal{T}_{\hat{k}}^{GLS_\mu}$
$T = 150$						
$\boldsymbol{\sigma}/\bar{\boldsymbol{\tau}}$	$(0.3, 0.3)$		$(0.5, 0.5)$		$(0.7, 0.7)$	
$(1, 2, 2)$	0.068	0.077	0.070	0.082	0.069	0.083
$(2, 1, 1)$	0.059	0.071	0.062	0.073	0.057	0.052
$\boldsymbol{\sigma}/\bar{\boldsymbol{\tau}}$	$(0.3, 0.6)$		$(0.4, 0.7)$		$(0.3, 0.7)$	
$(1, 1.5, 1)$	0.058	0.059	0.058	0.062	0.060	0.066
$(1, 2, 1)$	0.068	0.078	0.065	0.079	0.071	0.084
$T = 250$						
$\boldsymbol{\sigma}/\bar{\boldsymbol{\tau}}$	$(0.3, 0.3)$		$(0.5, 0.5)$		$(0.7, 0.7)$	
$(1, 2, 2)$	0.069	0.077	0.069	0.080	0.072	0.088
$(2, 1, 1)$	0.063	0.070	0.061	0.069	0.056	0.058
$\boldsymbol{\sigma}/\bar{\boldsymbol{\tau}}$	$(0.3, 0.6)$		$(0.4, 0.7)$		$(0.3, 0.7)$	
$(1, 1.5, 1)$	0.056	0.059	0.059	0.058	0.060	0.063
$(1, 2, 1)$	0.069	0.072	0.069	0.078	0.069	0.080
$T = 500$						
$\boldsymbol{\sigma}/\bar{\boldsymbol{\tau}}$	$(0.3, 0.3)$		$(0.5, 0.5)$		$(0.7, 0.7)$	
$(1, 2, 2)$	0.065	0.075	0.071	0.083	0.068	0.089
$(2, 1, 1)$	0.058	0.066	0.057	0.068	0.057	0.060
$\boldsymbol{\sigma}/\bar{\boldsymbol{\tau}}$	$(0.3, 0.6)$		$(0.4, 0.7)$		$(0.3, 0.7)$	
$(1, 1.5, 1)$	0.051	0.058	0.058	0.061	0.054	0.059
$(1, 2, 1)$	0.061	0.072	0.070	0.078	0.063	0.073

TABLE 15. Empirical size assuming constant volatility when one or two changes in the variance of ε_t occur - constant case

DGP: $y_t = y_{t-1} + \varepsilon_{i,t},\ i = 1,2,3$
$\varepsilon_{1,t} \sim N(0, \sigma_1^2)\ if\ t \leq \bar{\tau}_1 T,$
$\varepsilon_{2,t} \sim N(0, \sigma_2^2)\ if\ \bar{\tau}_1 T < t \leq \bar{\tau}_2 T,$
$\varepsilon_{3,t} \sim N(0, \sigma_3^2)\ if\ t \leq \bar{\tau}_2 T.$
$\boldsymbol{\sigma} = (\sigma_1, \sigma_2, \sigma_3)$ and $\bar{\boldsymbol{\tau}} = (\bar{\tau}_1,\ \bar{\tau}_2)$

	DF^{GLS_μ}	$\mathcal{T}_{\hat{k}}^{GLS_\mu}$	DF^{GLS_μ}	$\mathcal{T}_{\hat{k}}^{GLS_\mu}$	DF^{GLS_μ}	$\mathcal{T}_{\hat{k}}^{GLS_\mu}$
$T = 150$						
$\boldsymbol{\sigma}/\bar{\boldsymbol{\tau}}$	$(0.3, 0.3)$		$(0.5, 0.5)$		$(0.7, 0.7)$	
$(1, 2, 2)$	0.074	0.074	0.077	0.088	0.081	0.107
$(2, 1, 1)$	0.074	0.083	0.076	0.079	0.070	0.066
$\boldsymbol{\sigma}/\bar{\boldsymbol{\tau}}$	$(0.3, 0.6)$		$(0.4, 0.7)$		$(0.3, 0.7)$	
$(1, 1.5, 1)$	0.061	0.063	0.059	0.065	0.058	0.065
$(1, 2, 1)$	0.074	0.096	0.070	0.102	0.069	0.095
$T = 250$						
$\boldsymbol{\sigma}/\bar{\boldsymbol{\tau}}$	$(0.3, 0.3)$		$(0.5, 0.5)$		$(0.7, 0.7)$	
$(1, 2, 2)$	0.063	0.073	0.065	0.089	0.068	0.108
$(2, 1, 1)$	0.059	0.077	0.058	0.075	0.052	0.063
$\boldsymbol{\sigma}/\bar{\boldsymbol{\tau}}$	$(0.3, 0.6)$		$(0.4, 0.7)$		$(0.3, 0.7)$	
$(1, 1.5, 1)$	0.058	0.063	0.054	0.062	0.058	0.062
$(1, 2, 1)$	0.069	0.091	0.065	0.101	0.071	0.093
$T = 500$						
$\boldsymbol{\sigma}/\bar{\boldsymbol{\tau}}$	$(0.3, 0.3)$		$(0.5, 0.5)$		$(0.7, 0.7)$	
$(1, 2, 2)$	0.065	0.078	0.066	0.088	0.067	0.105
$(2, 1, 1)$	0.059	0.075	0.056	0.070	0.054	0.062
$\boldsymbol{\sigma}/\bar{\boldsymbol{\tau}}$	$(0.3, 0.6)$		$(0.4, 0.7)$		$(0.3, 0.7)$	
$(1, 1.5, 1)$	0.058	0.062	0.062	0.069	0.059	0.064
$(1, 2, 1)$	0.071	0.089	0.077	0.103	0.074	0.093

TABLE 16. Empirical size assuming constant volatility when one or two changes in the variance of ε_t occur - linear trend case

DGP: $y_t = y_{t-1} + \varphi((1+\cos(2\pi kt/T))/2)y_{t-1} + \varepsilon_{it}, i = 1,2,3$
$\varepsilon_{1,t} \sim N(0,\sigma_1^2)\ if\ t \leq \bar{\tau}_1 T,$
$\varepsilon_{2,t} \sim N(0,\sigma_2^2)\ if\ \bar{\tau}_1 T < t \leq \bar{\tau}_2 T,$
$\varepsilon_{3,t} \sim N(0,\sigma_3^2)\ if\ t \leq \bar{\tau}_2 T.$
$\bar{\boldsymbol{\tau}} = (\bar{\tau}_1,\ \bar{\tau}_2)$

	DF^{GLS_μ}			$\mathcal{T}_{\hat{k}}^{GLS_\mu}$		
$T = 250$	$\sigma_1 = 1,\ \sigma_2 = 2$ and $\sigma_3 = 2$					
$(\varphi, k)/\bar{\boldsymbol{\tau}}$	$(0.3, 0.3)$	$(0.5, 0.5)$	$(0.7, 0.7)$	$(0.3, 0.3)$	$(0.5, 0.5)$	$(0.7, 0.7)$
$(0, 0)$	0.051	0.054	0.055	0.058	0.046	0.044
$(-0.1, 0.5)$	0.372	0.312	0.313	0.323	0.276	0.276
$(-0.1, 1.0)$	0.428	0.575	0.650	0.590	0.654	0.676
$(-0.1, 1.5)$	0.519	0.552	0.463	0.581	0.571	0.423
$(-0.1, 2.0)$	0.604	0.538	0.534	0.689	0.649	0.640
$(-0.1, 2.5)$	0.569	0.510	0.570	0.652	0.597	0.577
$(-0.1, 3.0)$	0.607	0.641	0.589	0.736	0.716	0.654
	$\sigma_1 = 2,\ \sigma_2 = 2$ and $\sigma_3 = 1$					
$(0, 0)$	0.057	0.053	0.041	0.052	0.037	0.049
$(-0.1, 0.5)$	0.624	0.683	0.651	0.683	0.699	0.671
$(-0.1, 1.0)$	0.582	0.452	0.463	0.606	0.552	0.576
$(-0.1, 1.5)$	0.490	0.491	0.583	0.564	0.549	0.685
$(-0.1, 2.0)$	0.486	0.573	0.562	0.591	0.685	0.688
$(-0.1, 2.5)$	0.532	0.613	0.599	0.601	0.670	0.712
$(-0.1, 3.0)$	0.597	0.606	0.670	0.638	0.658	0.733
	$\sigma_1 = 1,\ \sigma_2 = 1.5$ and $\sigma_3 = 1$					
$(\varphi, k)/\bar{\tau}$	$(0.3, 0.6)$	$(0.4, 0.7)$	$(0.3, 0.7)$	$(0.3, 0.6)$	$(0.4, 0.7)$	$(0.3, 0.7)$
$(0, 0)$	0.046	0.052	0.056	0.042	0.052	0.050
$(-0.1, 0.5)$	0.505	0.460	0.486	0.520	0.455	0.482
$(-0.1, 1.0)$	0.422	0.458	0.424	0.569	0.588	0.548
$(-0.1, 1.5)$	0.528	0.610	0.566	0.663	0.685	0.671
$(-0.1, 2.0)$	0.667	0.616	0.629	0.754	0.692	0.696
$(-0.1, 2.5)$	0.574	0.550	0.580	0.695	0.679	0.700
$(-0.1, 3.0)$	0.640	0.653	0.667	0.729	0.747	0.749
	$\sigma_1 = 1,\ \sigma_2 = 2$ and $\sigma_3 = 1$					
$(0, 0)$	0.052	0.056	0.052	0.059	0.058	0.057
$(-0.1, 0.5)$	0.505	0.434	0.461	0.509	0.387	0.436
$(-0.1, 1.0)$	0.352	0.401	0.339	0.442	0.482	0.444
$(-0.1, 1.5)$	0.520	0.647	0.581	0.625	0.684	0.667
$(-0.1, 2.0)$	0.707	0.592	0.604	0.727	0.643	0.648
$(-0.1, 2.5)$	0.558	0.514	0.573	0.653	0.618	0.650
$(-0.1, 3.0)$	0.602	0.617	0.649	0.677	0.710	0.740

TABLE 17. Empirical size and power employing the Wild Bootstrap technique when changes in the variance of ε_t occur - constant case

DGP:	$y_t = y_{t-1} + \varphi((1+\cos(2\pi kt/T))/2)y_{t-1} + \varepsilon_{it}, i = 1,2,3$
	$\varepsilon_{1,t} \sim N(0,\sigma_1^2)\ if\ t \leq \bar{\tau}_1 T,$
	$\varepsilon_{2,t} \sim N(0,\sigma_2^2)\ if\ \bar{\tau}_1 T < t \leq \bar{\tau}_2 T,$
	$\varepsilon_{3,t} \sim N(0,\sigma_3^2)\ if\ t \leq \bar{\tau}_2 T.$
	$\bar{\boldsymbol{\tau}} = (\bar{\tau}_1,\ \bar{\tau}_2)$

	DF^{GLS_μ}			$\mathcal{T}_{\hat{k}}^{GLS_\mu}$		
$T = 250$	$\sigma_1 = 1,\ \sigma_2 = 2$ and $\sigma_3 = 2$					
$(\varphi, k)/\bar{\boldsymbol{\tau}}$	$(0.3, 0.3)$	$(0.5, 0.5)$	$(0.7, 0.7)$	$(0.3, 0.3)$	$(0.5, 0.5)$	$(0.7, 0.7)$
$(0, 0)$	0.045	0.054	0.051	0.050	0.050	0.060
$(-0.1, 0.5)$	0.259	0.230	0.200	0.188	0.178	0.176
$(-0.1, 1.0)$	0.175	0.262	0.332	0.234	0.285	0.341
$(-0.1, 1.5)$	0.260	0.317	0.244	0.254	0.287	0.222
$(-0.1, 2.0)$	0.294	0.258	0.236	0.279	0.263	0.256
$(-0.1, 2.5)$	0.276	0.241	0.322	0.274	0.242	0.265
$(-0.1, 3.0)$	0.266	0.315	0.273	0.311	0.319	0.269
	$\sigma_1 = 2,\ \sigma_2 = 2$ and $\sigma_3 = 1$					
$(0, 0)$	0.044	0.044	0.051	0.057	0.049	0.046
$(-0.1, 0.5)$	0.470	0.520	0.493	0.391	0.386	0.352
$(-0.1, 1.0)$	0.264	0.202	0.171	0.302	0.228	0.203
$(-0.1, 1.5)$	0.245	0.199	0.259	0.234	0.192	0.290
$(-0.1, 2.0)$	0.213	0.250	0.255	0.218	0.262	0.264
$(-0.1, 2.5)$	0.223	0.269	0.226	0.235	0.279	0.244
$(-0.1, 3.0)$	0.245	0.230	0.288	0.279	0.262	0.304
	$\sigma_1 = 1,\ \sigma_2 = 1.5$ and $\sigma_3 = 1$					
$(\varphi, k)/\bar{\tau}$	$(0.3, 0.6)$	$(0.4, 0.7)$	$(0.3, 0.7)$	$(0.3, 0.6)$	$(0.4, 0.7)$	$(0.3, 0.7)$
$(0, 0)$	0.051	0.049	0.049	0.053	0.051	0.050
$(-0.1, 0.5)$	0.365	0.317	0.326	0.290	0.254	0.262
$(-0.1, 1.0)$	0.163	0.180	0.156	0.184	0.208	0.181
$(-0.1, 1.5)$	0.231	0.270	0.249	0.249	0.284	0.278
$(-0.1, 2.0)$	0.312	0.268	0.266	0.320	0.289	0.282
$(-0.1, 2.5)$	0.238	0.218	0.235	0.300	0.247	0.262
$(-0.1, 3.0)$	0.261	0.263	0.283	0.327	0.313	0.353
	$\sigma_1 = 1,\ \sigma_2 = 2$ and $\sigma_3 = 1$					
$(0, 0)$	0.053	0.053	0.054	0.043	0.050	0.049
$(-0.1, 0.5)$	0.361	0.302	0.317	0.267	0.208	0.245
$(-0.1, 1.0)$	0.126	0.146	0.124	0.126	0.128	0.124
$(-0.1, 1.5)$	0.220	0.278	0.254	0.226	0.278	0.252
$(-0.1, 2.0)$	0.321	0.267	0.253	0.302	0.279	0.255
$(-0.1, 2.5)$	0.228	0.210	0.226	0.276	0.194	0.237
$(-0.1, 3.0)$	0.240	0.253	0.278	0.276	0.270	0.316

TABLE 18. Empirical size and power employing the Wild Bootstrap technique when changes in the variance of ε_t occur - linear trend case

4. Empirical application

In this section, we investigate inflation rate persistence, which is defined here as the speed at which inflation converges to equilibrium after a shock in the disturbance term. Thus, the parameter φ in the test regression

$$\Delta\hat{\pi}_t = \varphi\cos(k,t)\hat{\pi}_{t-1} + \sum_{j=1}^{p}\delta_j\Delta\hat{\pi}_{t-j} + \varepsilon_t, \tag{21}$$

is a reasonable indicator of persistence dynamics and its statistical significance is tested using the procedure introduced in Section 2, i.e.,

$$\mathcal{T}_{\hat{k}}^{GLS_\mu} = \min_{k\in K}\hat{t}_k^{GLS_\mu},\ K = \{0.5, 1, 1.5, 2, 2.5, 3\}\,. \tag{22}$$

Although a large literature, mostly based on U.S. data, has investigated the nature of inflation persistence over the last decades, findings are not consensual. One possible reason is the use of different measures of persistence, by different authors.

An important branch of this literature considered the order of integration as a measure of inflation persistence. Thus, traditional unit root tests have been widely used in this context in order to determine whether inflation rate series are $I(0)$ or $I(1)$, (see, for instance, Evans and Wachtel 1993, Culver and Papell 1997, and Crowder and Wohar 1999). In the $I(0)$ case, shocks have only a transitory impact, while for $I(1)$ they are highly persistent. Applications of these tests to inflation data has led to mixed evidence about the order of integration, which is not surprising given their poor power performance in finite samples when the coefficient of the underlying linear model changes.

More recently, in response to the difficulty of conventional unit root tests to reject the unit root hypothesis, new methodologies have been proposed to infer about the order of integration of inflation series. One possible way is to employ stationarity or unit root tests that take the existence of structural breaks and non-linearities, which may be present in the variables' dynamics, into account. Chang *et al.* (2013) employed the stationarity tests proposed by Becker *et al.* (2006), which use Fourier functions to approximate structural breaks in the deterministic terms. Their results show that inflation series exhibit mean reversion behavior for all 22 OECD countries investigated. Chen and Hsu (2016) present similar conclusions using the unit root tests developed by Leybourne *et al.* (1998) which employ a logistic function aiming at capturing the occurrence of smooth breaks in the deterministic component. However, it should be noted that they do not discard the questionable hypothesis that inflation rates have a deterministic trend (linear or nonlinear). Moreover, these authors also found evidence, applying TAR and ESTAR-type nonlinear unit root tests, supporting the existence of nonlinear dynamics in inflation behavior.

A possible cause of change in persistence is the interdependence between monetary policy decisions and the persistence of inflation. Alogoskoufis and Smith (1991) found evidence that inflation persistence is a positive function of the degree of monetary and exchange-rate accommodation. Higher accommodation following a price shock has been associated not only with high inflation values but also with higher persistence of inflation. Therefore, specifications that allow for the order of integration to exogenously change over time must also be considered in this context. This statement is supported by more recent works. For instance, Harvey *et al.* (2006) found evidence, using tests for a change in persistence, that CPI inflation in the U.S. suffered a change in persistence from $I(1)$ to $I(0)$. Halunga *et al.* (2009) applied the same tests to the UK and US inflation data and reported similar results. They attempt to circumvent the single change in persistence limitation of these tests by partitioning the sample when a break is found. The findings achieved using this approach point to the existence of a first change from $I(0)$ to $I(1)$ in the early 1970s and a subsequent reversion to $I(0)$ in the early 1980s, suggesting that the nonstationary dynamics of inflation lasted only about ten years.

Lastly, some works use the order of fractional integration as a measure of inflation persistence, but considered that the order of integration is time-varying. For instance, Kumar and Okimoto (2007) and Martins and Rodrigues (2014) opted for this approach and detected a change from more persistent to less persistent behavior over the last decades in the inflation rate series.

An alternative branch of literature uses AR model-based measures, such as, the largest autoregressive root (LAAR) and the sum of the autoregressive coefficients (SARC). Taylor (2000) estimated both LAAR and SARC and found a large decline in persistence in the early 80's. Levin and Piger (2003) argue that if structural breaks in the intercept of the AR equation are considered, the estimated SARC is lower and far from that of a random walk process. Thus, they conclude that high inflation persistence is not an inherent characteristic of industrial economies. Beechey and Österholm (2012) used an ARMA model of inflation with a time-varying autoregressive parameter for the US and suggested that the preference for inflation stability since the 1980s lead to a decrease in inflation persistence (see also Pivetta and Reis 2007).

Almost all of the recently proposed methodologies provide strong evidence against the statement that inflation dynamics is described by a pure $I(1)$ process. This work intends to contribute to this literature, where the focus has been mostly to attenuate the effects of structural breaks in the deterministic structure, by proposing an easy to implement approach for which the considered specification under the alternative hypothesis is sufficiently flexible to allow for the dynamics of inflation to change between integrated and mean reversion over the sample. Thus, the proposed test does not simply classify inflation as either an $I(0)$ or $I(1)$ process, but is closely related to the literature that investigate the occurrence of changes in the order of integration (Kim 2000 and Harvey *et al.* 2006). However, as our test retains the whole sample in a unified estimation, we expect it to provide relevant power gains when changes in inflation persistence are gradual.

Therefore, the main objective of this section is to gather empirical evidence of structural breaks in inflation persistence by applying the proposed test procedures to inflation data from several industrial economies. Our approach allows for a maximum of three breaks, which seems adequate given the sample sizes available in practice (this can however be relaxed if needed).

Our sample consists of quarterly CPI data for the G7 countries obtained from OECD Statistics for the period from 1955Q1 to 2018Q2. The quarterly CPI is then used to compute the year-on-year and the quarterly growth rate of the CPI data.

According to Hassler and Demetrescu (2005), the power performance of the ADF test crucially depends on whether inflation is assumed to be equal to the year-on-year growth or to the quarterly growth of CPI. For instance, when the quarterly growth is stationary, the year-on-year growth of the CPI introduces a noninvertible moving average component in the resulting time series, which is responsible for a loss of power of the ADF tests. However, since the year-on-year growth of CPI is relevant for monetary policy decisions (inflation targeting regimes typically observe its evolution) we will consider the two definitions.

Figures 2 and 3 display, respectively, the year-on-year and the quarterly growth rates of the CPI time series for the G7 countries. Simple visual inspection suggests that the 1970s was the period with the highest inflation rates for almost all countries. Over these years, which have as a milestone the collapse of Bretton Woods, the world economy faced a period of turbulence in which the option for a highly accommodative monetary policy seems to have triggered an unusual increasing inflation persistence only attenuated in the early 80's with the shift to a more restrictive monetary policy. This option, characterized by the introduction of inflation targeting as a framework for monetary policy, contributed to a long period of low and stable inflation in developed countries. Since the 2008 global financial crisis, price stability is once more a concern but this time because of the risk that inflation could remain too low for too long. In order to avoid deflation and to bring inflation back to a desirable level, central banks implemented an expansionary monetary policy, which may have had an impact on inflation persistence. Thus, for the sample period considered in this work, two breaks in persistence may have occurred associated with periods of turbulence and relevant changes in monetary policy.

Since breaks in the autoregressive parameter have impact on the deterministic component of the process, we also consider the unit root test proposed by Rodrigues and Taylor (2012) to infer if allowing for exogenous structural breaks in the intercept is sufficient to gather evidence against the null hypothesis. Moreover, Wild bootstrap p-values of the proposed test are also computed in order to prevent that the results are influenced by breaks in the unconditional volatility (or even by the presence of GARCH effects). The values considered in the simulation studies for the variance parameters are more extreme than, for instance, the changes in the standard deviation reported by Sensier and van Dijk (2003) for consumer prices time series.

Table 19 reports the results for the year-on-year growth rate of CPI. Considering the proposed test, the null hypothesis of a unit root is rejected for three countries (Canada, France and Germany) at the 5% level of significance, and for the UK at the 10% level of significance. The chosen frequencies ($\hat{k} = 1.5$ and 2) suggest that there were two periods of higher persistence. When $k = 1.5$, the trigonometric function employed to approximate the breaks reaches its minimum values (zero) for $t/T \cong 0.33(3)$ and for $t/T \cong 1$. Therefore, for Canada, France and Germany there is statistically evidence that persistence is higher before the middle and at the end of the sample. For $k = 2$, the minimum occurs for $t/T \cong 0.25$ and for $t/T \cong 0.75$ and persistence is, if the unit root test rejects the null hypothesis, far below unity at the end of the sample.

As the timing of the breaks influences the power performance of the proposed unit root test, we also considered the time series in reverse chronological order. It is easy to prove that in this case the proposed test has the following asymptotic distribution (see, for instance, **?**):

$$\mathcal{T}_{\hat{k},r}^{GLS_\mu} := \min_{k\in K} \hat{t}_k^{GLS_\mu} \Rightarrow$$
$$\min_{k\in K} \frac{-\cos(k,0)W(1)^2 + \frac{1}{2}(2\pi k)^2 \int_0^1 \cos(2\pi kr)W(r)^2 dr - 1}{2\left(\int_0^1 \cos^2(k,r)W(r)^2 dr\right)^{1/2}} . \tag{23}$$

The values of $\tilde{c}$ for a given $k \in K = \{0.5, 1, 1.5, 2, 2.5, 3\}$ and the critical values of the $\mathcal{T}_{\hat{k},r}^{GLS_\mu}$ test are very close to those obtained using the normal chronological order (which are not presented here for the sake of space). This approach also allows, when fractional values of k are considered, to approximate the cases in which the process starts with a period of higher persistence.

With the exception of Japan, the proposed test with this transformation rejects at the 5% level of significance for all countries. Comparing with previous evidence using the normal chronological order, the unit root hypothesis is rejected for two additional countries at the 5% level of significance. The estimated $\hat{k}$ parameter equals one for all countries, suggesting that attenuating the effect of a period of higher persistence somewhere around the middle of the sample is sufficient to gather strong evidence against the unit root hypothesis. These results may be related with the findings using the ESTAR model previously reported. The shape of the exponential transition function (as function of the lagged dependent variable) is similar to the shape considered by the cosine function with $k = 1$. The main difference between the two approaches is the assumption made about the causes that lead to breaks in inflation persistence. The ESTAR model employed in this context assumes that the switch from the stationary to the highly persistent regime is endogenous, a function of the transition variable. So, for this model to be globally stationary, there must be a mean-reverting mechanism towards equilibrium, which implies the existence of a unique natural rate of inflation. This hypothesis is not compatible with the assumption that there have been exogenous changes in monetary policy which had permanent effects on the inflation dynamics.

Thus, regarding the apparent power gains when the reverse chronological order is used, the results in Table 19 seem to confirm that the proposed test has more difficulties in detecting breaks in persistence at the beginning of the sample (as suggested by the simulation studies) since a simple reversion of the chronological order leads to three additional rejections of the unit root hypothesis (for Italy, Japan and the US). Table 5 shows that when the process is highly persistent in the interval $0.4 < t/T < 0.7$, the rate of rejection of the null hypothesis is larger than when the same occurs in the interval $0.3 < t/T < 0.6$. With this in mind, assume that the 1970s was the the only period of highly persistent inflation. The first observations of this decade correspond to $t/T \cong 0.23$ and the last ones to $t/T \cong 0.39$. Thus, reversed chronological order suggests that the break in persistence occurred at $t/T \cong 0.61$, which is associated with better power performance. Therefore, these results appear to reveal that there was an important structural change in persistence before the first half of the sample when the normal chronological order is considered. Nonetheless, regardless of the chronological order, the proposed test always rejects more often the unit root than the other two tests employed in this empirical application.

However, as we computed the test statistic in normal, $\mathcal{T}_{\hat{k}}^{GLS_\mu}$, and in reverse chronological order, $\mathcal{T}_{\hat{k},r}^{GLS_\mu}$, it is expected that the empirical size is above the nominal 5% level. Therefore, we also considered the critical values of the minimum between these two statistics (see note to Table 19), and we observe that the unit root hypothesis is rejected at the 1% significance level for Canada, France, Germany and the US, at the 5% significance level for Italy and the UK and at the 10% significance level for Japan. For France and Germany, using the time series in normal chronological order leads to stronger rejection of the null hypothesis. For these cases, two breaks in persistence seem to have occurred. One before the middle of the sample and the other at the end of the sample, suggesting that the 2008/2009 financial crisis originated a statistically significant change in inflation persistence.

Table 20 reports the results for the quarterly growth CPI. The test proposed in this paper rejects the null hypothesis of nonstationarity for three countries (Canada, France and the UK) at the 5% significance level and for the US at the 10% significance level. The estimated frequency k is the same as for the year-on-year inflation series for six of the seven countries. Using the data in reverse chronological order leads to similar conclusions as those drawn from Table 19. When the minimum between $\mathcal{T}_{\hat{k}}^{GLS_\mu}$ and $\mathcal{T}_{\hat{k},r}^{GLS_\mu}$ is considered, there is statistical evidence against the unit root hypothesis for all countries at the 5% significance level.

It is important to note that the proposed cosine term can only provide a rough approximation of the exogenous breaks in persistence and, for the chosen values of k, it implies that the persistence parameter only assumes two values: one close to unity when the function reaches its minimum value and a lower persistence value when the function reaches its maximum value. Furthermore, considering $\hat{k} = 3$ as the maximum value for the frequency parameter implies that there were a maximum of three periods of shorter duration with higher persistence in inflation. Given the sample length available in practice, this number may even be large in some cases. Lastly, simulations show that the power gains achieved by the new test are maximized when k is smaller. Even with these limitations, we found relevant evidence against the unit root hypothesis of the inflation rate series for several industrial countries.

Summing up, the proposed test provides statistical evidence of breaks in persistence for several countries considering both year-on-year and quarterly growth of CPI. When time series in reverse chronological order are considered, the null hypothesis is rejected more times and $\hat{k} = 1$ for most countries.

Comparing the results obtained using normal and reverse chronological orders seems to suggest that the difficulty in rejecting the $I(1)$ hypothesis is largely due to the Great Inflation of the 1970s (beyond the limitations of available unit root tests such as, for instance, low power in small samples). Regarding the last ten years, the plausible change in persistence after the crisis seems not to have such a large influence, since the proposed unit root test continues to provide strong rejections of the unit root hypothesis even when a frequency parameter which is not able to approximate a break in persistence occurring in this period is chosen.

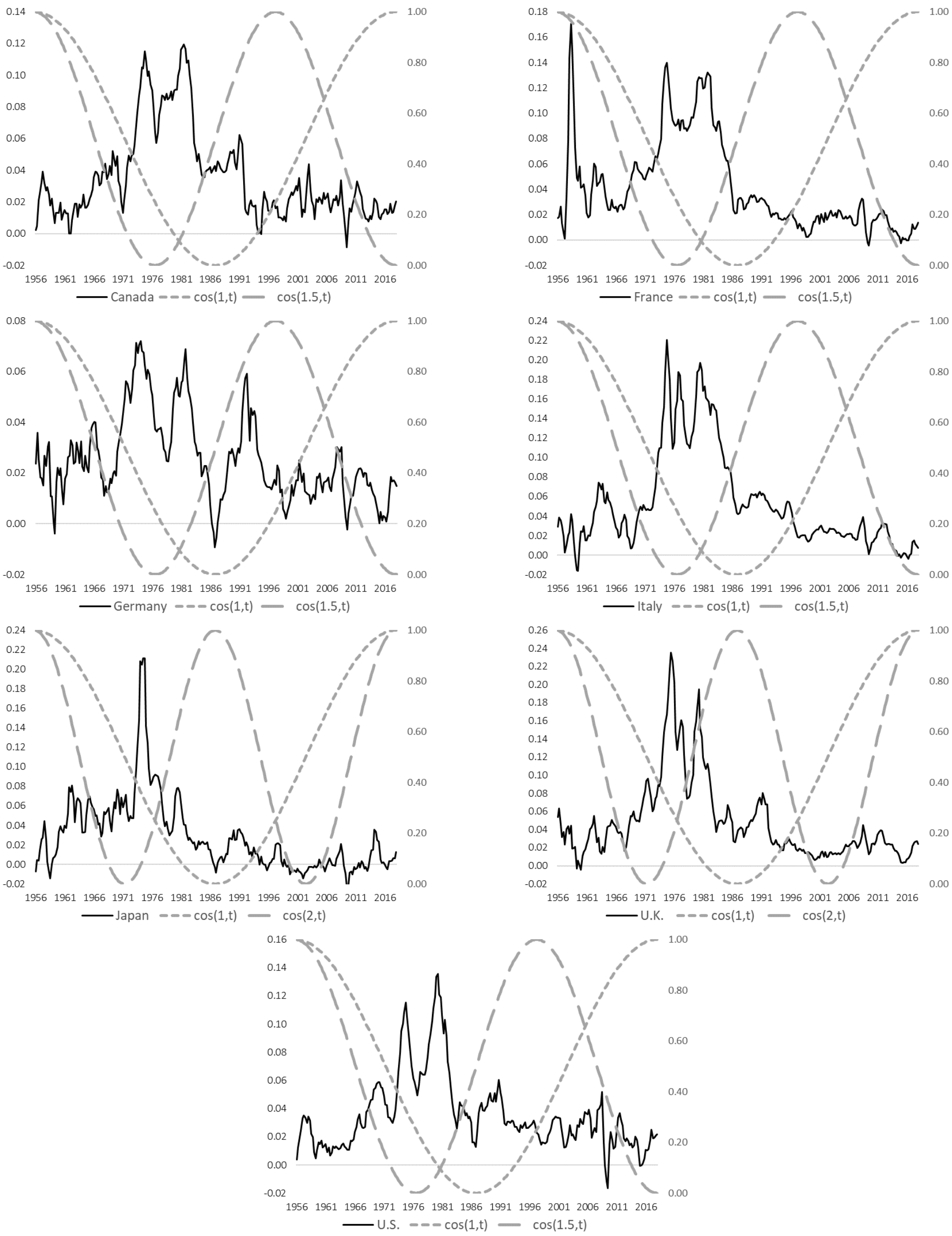

Figure 2: Year-on-year growth of the CPI (percentage change relative to the same quarter of the previous year) for the G7 countries.

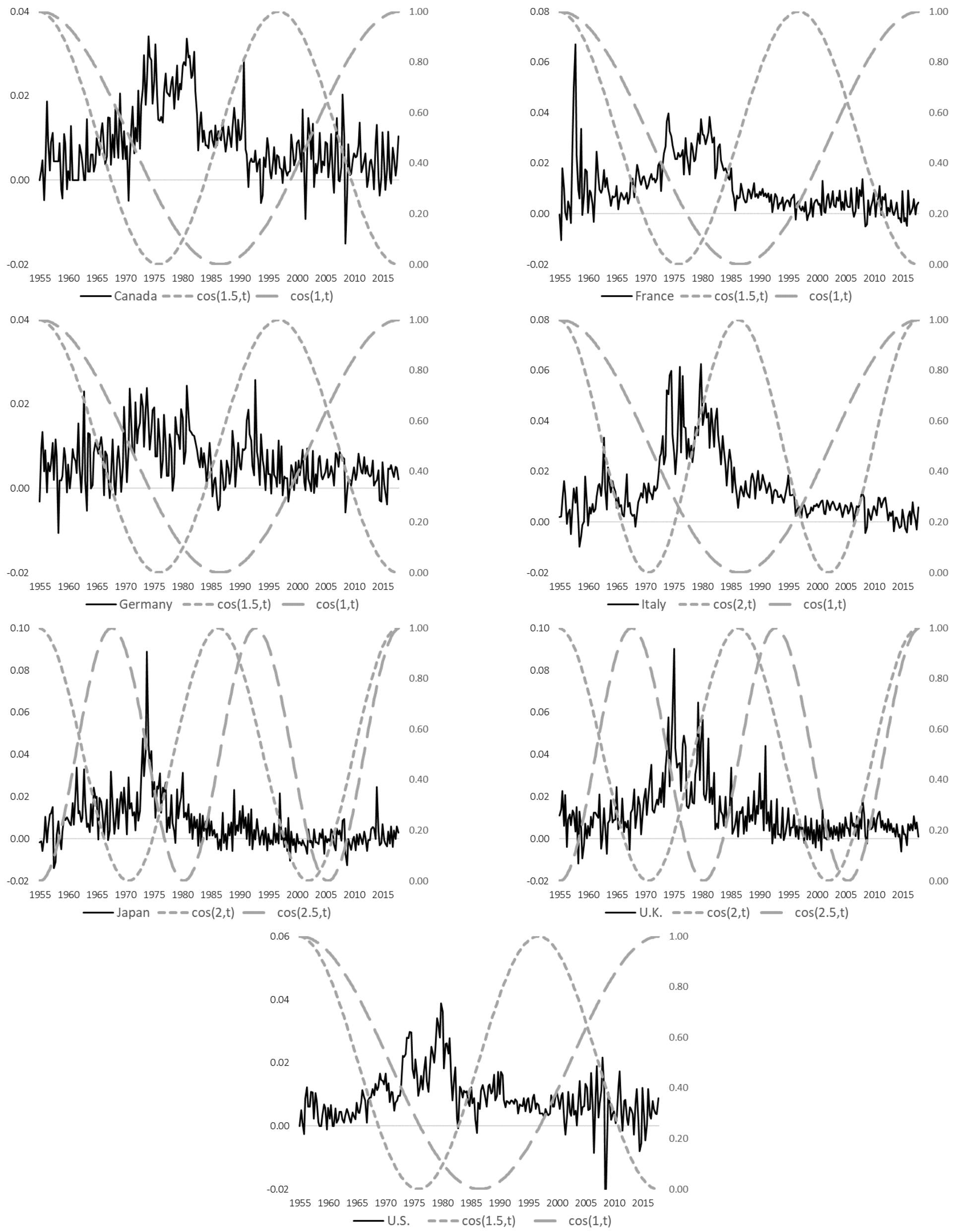

FIGURE 3: Quarterly growth of the CPI (percentage change relative to the previous quarter) for the G7 countries.

Country	DF^{GLS_μ}	$\hat{k}$	$\mathcal{T}_{\hat{k}}^{GLS_\mu}$	WB p-value	k	$t_\alpha^{ERS_f^\mu}$
Canada	−1.040	1.5	−2.900 [b]	0.028	1	−2.276
France	−1.348	1.5	−3.494 [a]	0.016	1	−2.724
Germany	−2.481 [b]	1.5	−3.692 [a]	0.002	1	−3.040 [c]
Italy	−1.719 [c]	1.5	−2.280	0.161	1	−3.109 [c]
Japan	−1.320	2	−2.002	0.230	1	−2.378 [c]
United Kingdom	−1.392	2	−2.446 [c]	0.111	1	−1.951
United States	−1.065	1.5	−2.031	0.235	1	−2.343
reverse chronological order						
Canada	−1.348	1	−3.748 [a]	0.000	1	−2.432
France	−1.109	1	−3.108 [b]	0.045	1	−2.381
Germany	−2.051 [b]	1	−3.583 [a]	0.007	1	−3.104 [c]
Italy	−1.432	1	−3.174 [b]	0.033	1	−3.114 [c]
Japan	−1.647	1	−2.586 [c]	0.091	1	−2.805
United Kingdom	−1.278	1	−2.894 [b]	0.054	1	−2.068
United States	−1.471	1	−4.096 [a]	0.002	1	−2.533

Table 19. Results for the year-on-year growth of the CPI

Country	DF^{GLS^μ}	$\hat{k}$	$\mathcal{T}_{\hat{k}}^{GLS_\mu}$	WB p-value	k	$t_\alpha^{ERS_f^\mu}$
Canada	−1.499	1.5	−3.021 [b]	0.016	1	−2.790
France	−1.303	1.5	−3.313 [a]	0.034	1	−2.713
Germany	−0.901	1.5	−1.706	0.325	1	−1.905
Italy	−1.375	2	−2.243	0.170	1	−2.487
Japan	−1.355	2	−1.967	0.219	1	−2.561
United Kingdom	−1.383	2	−3.178 [b]	0.029	1	−2.108
United States	−1.417	1.5	−2.528 [c]	0.235	1	−2.460
reverse chronological order						
Canada	−1.745 [b]	1	−2.697 [b]	0.044	1	−1.887
France	−1.005	1	−3.013 [b]	0.046	1	−2.046
Germany	−1.635	1	−3.125 [b]	0.017	1	−3.156 [b]
Italy	−1.591	1	−3.205 [a]	0.037	1	−2.447
Japan	−1.654	2.5	−3.016 [b]	0.036	1	−3.055 [c]
United Kingdom	−0.988	2.5	−2.516 [c]	0.124	1	−2.079
United States	−1.798 [b]	1	−3.411 [a]	0.007	1	−2.329

Table 20. Results for the quarterly growth of the CPI

Notes for Tables 19 and 20: (1) a, b and c denote significance at the 1%, 5% and 10% respectively; (2) $t_\alpha^{ERS_f^\mu}$ is the test proposed by Rodrigues and Taylor (2012) which assumes that the frequency k is known ($k = 1, 2, ..., 5$); (3) the lags of the unit root tests were chosen using the MAIC information criterion; (4) the values of the critical values of $\mathcal{T}_{\hat{k}}^{GLS_\mu}$ with reverse chronological order equals, when T=250, -3.198, -2.634 and -2.352 for 1%, 5% and 10% respectively; (5) the critical values for the minimum between $\mathcal{T}_{\hat{k}}^{GLS_\mu}$ with normal and reverse chronological order are, when T=250, -3.382, -2.839 and -2.568 for 1%, 5% and 10%.

5. Conclusions

In this paper, we propose a simple approach to detect potential persistence changes and allow for the possibility of the occurrence of up to three breaks in persistence (note that more breaks can be allowed for if deemed necessary). The unknown shape and timing of the breaks are approximated using a cosine term. The test procedure is based on a one-sided t-statistic where this statistic is minimized over a set of values chosen *a priori* for the frequency parameter k. We considered fractional and integer values for k, since using integer values only may be restrictive as it implies that the autoregressive parameter has the same value at the beginning and at the end of the sample.

We find via Monte Carlo simulations that our proposed test has interesting power performance when compared to local GLS detrended unit root tests when breaks in persistence are present. The advantages of the $\mathcal{T}_{\hat{k}}^{GLS}$ are more pronounced when applied to demeaned data and moderate sample sizes ($T = 250$).

In addition to the DGPs suggested by the specification of the alternative hypothesis, which implies smooth breaks in persistence, we also investigated the power properties of the proposed test when abrupt breaks in the autoregressive parameter occur and the results remain favorable. The power gains relative to the DF^{GLS} test are even greater if increases in persistence induce temporary nonstationary behavior. Moreover, we also performed simulations to investigate the effects of conditional heteroskedasticity and of changes in the innovation variance (unconditional heteroscedasticity). Although our proposed test shows some size distortions when the homoskedasticity assumption does not hold, this problem is attenuated using the Wild bootstrap which produces empirical sizes close to the nominal 5% level.

Applications of the proposed test to the G7 countries' inflation data provided relevant statistical evidence of breaks in persistence. When year-on-year growth of CPI in reverse chronological order is considered, the null hypothesis of a unit root is rejected for all countries. Comparing these results with those obtained considering the normal chronological order suggests that the evidences of nonstationarity of the inflation series previously reported in the literature is possibly due to the occurrence of a period of higher persistence in the first half of the sample.

Summing up, this paper alerts for the consequences of ignoring the occurrence of breaks in persistence. Most of the work on unit root testing that employed Fourier series to approximate smooth structural breaks has focused on changes in the constant parameter. However, changes in the behaviour of economic and financial variables caused by, for instance, exogenous events, shifts in monetary policy or improvements in the available technology may have altered not only the equilibrium value but also the speed of reversion to equilibrium after a shock. As previously mentioned, changes in the persistence parameter affect the conditional mean, the unconditional mean and the unconditional variance of the process. Thus, it is possible that some of the evidence in the literature regarding the occurrence of structural breaks in volatility (e.g, Sensier and van Dijk 2003, McConnell and Perez-Quiros 2000) may have been influenced by the occurrence of changes in persistence.

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Appendix

Proof of Theorem 2.1

a) No deterministics

From a test regression as in (5) with no deterministic terms (that is when $\hat{u}_{\tilde{c},t} = y_t$), it follows that the OLS t-statistics to test the significance of φ, $H_0 : \varphi = 0$, is

$$\hat{t}_k^{GLS} := \frac{\sum_{t=2}^{T} \Delta\hat{u}_{\tilde{c},t} \cos(k,t) \hat{u}_{\tilde{c},t-1}}{\left[\hat{\sigma}_k^2 \sum_{t=2}^{T} \cos^2(k,\mathrm{t}) \hat{u}_{\tilde{c},t-1}^2\right]^{1/2}}. \quad (24)$$

Considering the identity $\hat{u}_{\tilde{c},t} = \Delta\hat{u}_{\tilde{c},t} + \hat{u}_{\tilde{c},t-1}$, squaring both sides and multiplying by $\cos(k,t)$ leads to
$\cos(k,t)\hat{u}_{\tilde{c},t}^2 = \cos(k,t) \left[(\Delta\hat{u}_{\tilde{c},t})^2 + 2\Delta\hat{u}_{\tilde{c},t}\hat{u}_{\tilde{c},t-1} + \hat{u}_{\tilde{c},t-1}^2\right]$.
Summing over t and rearranging, gives

$$\begin{aligned} &\sum_{t=2}^{T} \Delta\hat{u}_{\tilde{c},t} \cos(k,t) \hat{u}_{\tilde{c},t-1} = \\ &\frac{1}{2} \left[\sum_{t=2}^{T} \cos(k,t)\hat{u}_{\tilde{c},t}^2 - \sum_{t=2}^{T} \cos(k,t)\hat{u}_{\tilde{c},t-1}^2 - \sum_{t=2}^{T} \cos(k,t)(\Delta\hat{u}_{\tilde{c},t})^2\right]. \end{aligned} \quad (25)$$

Since under the null $\Delta\hat{u}_{\tilde{c},t} = \hat{\varepsilon}_t$, it follows that,

$$\begin{aligned} &\sum_{t=2}^{T} \Delta\hat{u}_{\tilde{c},t} \cos(k,t) \hat{u}_{\tilde{c},t-1} = \\ &\frac{1}{2} \left[\cos(k,T)\hat{u}_{\tilde{c},T}^2 - \cos(k,2)\hat{u}_{\tilde{c},1}^2 - \sum_{t=3}^{T} \Delta\cos(k,t)\hat{u}_{\tilde{c},t-1}^2 - \sum_{t=2}^{T} \cos(k,t)\hat{\varepsilon}_t^2\right]. \end{aligned} \quad (26)$$

In what follows the following limit results will prove useful. In specific, as $T \to \infty$,

$$\cos(k,T)\frac{1}{T}\hat{u}_{\tilde{c},T}^2 \Rightarrow \sigma^2 \cos(k,1) W(1)^2; \quad (27)$$

$$\cos(k,2)\frac{1}{T}\hat{u}_{\tilde{c},1}^2 \Rightarrow \sigma^2 \cos(k,0) W(0)^2 = 0; \quad (28)$$

$$\frac{1}{T}\sum_{t=3}^{T} \Delta\cos(k,t)\hat{u}_{\tilde{c},t-1}^2 \Rightarrow \frac{\sigma^2}{2}(2\pi k)^2 \int_0^1 \cos(2\pi k r) W(r)^2 dr; \quad (29)$$

$$\frac{1}{T}\sum_{t=2}^{T} \cos(k,t)(\Delta\hat{u}_{\tilde{c},t-1})^2 \to \sigma^2 \int_0^1 \cos(k,r) dr = \frac{\sigma^2}{2}. \quad (30)$$

Note that the result in (29) is obtained given that $\Delta\cos(k,t) = -\frac{1}{2}(2\pi k/T)\sin(2\pi kt/T) + o(1)$ (see Enders and Lee, 2012), and Lemma A.1

in Bierens (1997)). Recall that $\cos(k,t) := \frac{1}{2}(1+\cos(2\pi kt/T))$. Hence, for (26) we establish, as $T \to \infty$, that,

$$\begin{aligned}&\frac{1}{T}\sum_{t=2}^{T}\Delta\hat{u}_{\tilde{c},t}\cos(k,t)\hat{u}_{\tilde{c},t-1} \Rightarrow \\ &\frac{\sigma^2}{2}\left\{\cos(k,1)W(1)^2+\frac{1}{2}(2\pi k)^2\int_0^1\cos(2\pi kr)W(r)^2dr-1\right\}.\end{aligned} \tag{31}$$

Finally, for the denominator of (24) it follows from the CMT that,

$$\frac{1}{T^2}\sum_{t=2}^{T}\cos^2(k,t)\hat{u}^2_{\tilde{c},t-1} \Rightarrow \sigma^2\int_0^1\cos^2(k,r)W(r)^2dr. \tag{32}$$

Taking the results in (31) and (32) it follows, under joint convergence, that the statistic in (24) converges to,

$$\hat{t}_k^{GLS} \Rightarrow \frac{\cos(k,1)W(1)^2+\frac{1}{2}(2\pi k)^2\int_0^1\cos(2\pi kr)W(r)^2dr-1}{2\left(\int_0^1\cos^2(k,r)W(r)^2dr\right)^{1/2}}, \tag{33}$$

where k is a fixed value.

b) Local GLS demeaning

To analyse the effects of local GLS demeaning on the limit distribution of the statistics, we consider estimation of the parameter vector $\boldsymbol{\beta}$ in (1) using $\mathbf{x}_t = 1$. Hence, for local GLS demeaning as in Elliott *et al.* (1996), we consider $y_{\tilde{c},1} := y_1$, $y_{\tilde{c},t} := y_t - \tilde{\rho}_t y_{t-1}$, $\mathbf{x}_{\tilde{c},1} := \mathbf{x}_1$, $\mathbf{x}_{\tilde{c},t} := \mathbf{x}_t - \tilde{\rho}_t\mathbf{x}_{t-1}$, and compute the OLS estimates as,

$$\hat{\boldsymbol{\beta}}_{\tilde{c}} = \left[\sum_{t=1}^{T}\mathbf{x}_{\tilde{c},t}\mathbf{x}_{\tilde{c},t}{}'\right]^{-1}\left[\sum_{t=1}^{T}\mathbf{x}_{\tilde{c},t}y_{\tilde{c},t}\right].$$

Consequently, the local GLS demeaned data is,

$$\hat{u}_{\tilde{c},t} = y_t - \mathbf{x}_t^{'}\hat{\boldsymbol{\beta}}_{\tilde{c}} = u_t - \mathbf{x}_t^{'}(\hat{\boldsymbol{\beta}}_{\tilde{c}}-\boldsymbol{\beta}) \tag{34}$$

or equivalently,

$$\hat{u}_{\tilde{c},t} = u_t - \mathbf{x}_t\left[\sum_{t=1}^{T}\mathbf{x}_{\tilde{c},t}\mathbf{x}_{\tilde{c},t}{}'\right]^{-1}\left[\sum_{t=1}^{T}\mathbf{x}_{\tilde{c},t}u_{\tilde{c},t}\right]. \tag{35}$$

Now, given that,

$$\sum_{t=1}^{T}\mathbf{x}_{\tilde{c},t}\mathbf{x}_{\tilde{c},t}{}' = 1+\left(\frac{\tilde{c}}{T}\right)^2\sum_{t=2}^{T}\cos^2(\mathrm{k,t}) \tag{36}$$

$$= 1+o(1). \tag{37}$$

and

$$\begin{aligned}\sum_{t=1}^{T}\mathbf{x}_{\tilde{c},t}u_{\tilde{c},t} &= u_1-\left(\frac{\tilde{c}}{T}\right)\sum_{t=2}^{T}\cos(k,t)\left[\Delta u_t-\frac{\tilde{c}}{T}\cos_(k,t)u_{t-1}\right]\\ &= u_1-\left(\frac{\tilde{c}}{T}\right)\sum_{t=2}^{T}\cos(k,t)\Delta u_t+\left(\frac{\tilde{c}}{T}\right)^2\sum_{t=2}^{T}\cos^2(k,t)u_{t-1}\quad(38)\end{aligned}$$

It follows that,

$$\frac{1}{\sqrt{T}}\left[\sum_{t=1}^{T}\mathbf{x}_{\tilde{c},t}\mathbf{x}_{\tilde{c},t}{}'\right]^{-1}\sum_{t=1}^{T}\mathbf{x}_{\tilde{c},t}u_{\tilde{c},t}\to 0.$$

Note that since $\frac{1}{T}\sum_{t=2}^{T}\cos^2(k,t)\ \to\ \int_0^1\cos^2(k,r)dr$, it follows that $\left(\frac{\tilde{c}}{T}\right)^2\sum_{t=2}^{T}\cos^2(k,t)=o(1)$.

c) Local GLS detrending

Regarding local GLS detrending the procedure is similar to that adopted with only a constant term. Thus, we have to prove the convergence of the expressions $D_T\sum_{t=1}^{T}\mathbf{x}_{\tilde{c},t}\mathbf{x}_{\tilde{c},t}{}'D_T$ and $D_T\sum_{t=1}^{T}\mathbf{x}_{\tilde{c},t}u_{\tilde{c},t}$ where the scaling is given by the diagonal matrix $D_T:=\text{diag}\left(1,T^{-1/2}\right)$.

It can be shown that

$$\begin{aligned}D_T\sum_{t=1}^{T}\mathbf{x}_{\tilde{c},t}\mathbf{x}_{\tilde{c},t}{}'D_T &= D_T\mathbf{x}_1\mathbf{x}_1{}'D_T+D_T\sum_{t=2}^{T}\mathbf{x}_{\tilde{c},t}\mathbf{x}_{\tilde{c},t}{}'D_T\\ &= \begin{bmatrix}1 & T^{-1/2}\\ T^{-1/2} & T^{-1}\end{bmatrix}+D_T\begin{bmatrix}\Xi_1 & \Xi_2\\ \Xi_2 & \Xi_3\end{bmatrix}D_T\end{aligned}$$

$$\Rightarrow\ \begin{bmatrix}1 & 0\\ 0 & \int_0^1\left[1-2\tilde{c}r\cos^2(k,r)+r^2\tilde{c}^2\cos^2(k,r)\right]dr\end{bmatrix},\tag{39}$$

where, $\mathbf{x}_1=(1,1)'$ and $\mathbf{x}_{\tilde{c},t}=\left(-\tilde{c}\cos(k,t)T^{-1},1-(t-1)\tilde{c}\cos(k,t)T^{-1}\right)'$ for $t>1$, with

$$\begin{aligned}\Xi_1 &:= \left(\frac{\tilde{c}}{T}\right)^2\sum_{t=2}^{T}\cos^2(k,t),\\ \Xi_2 &:= \left(1-\frac{(t-1)\tilde{c}\cos(k,t)}{T}\right)\frac{\tilde{c}}{T}\cos(k,t),\\ \Xi_3 &:= \left(1-\frac{(t-1)\tilde{c}\cos(k,t)}{T}\right)^2.\end{aligned}$$

Moreover, note that,

$$\frac{1}{T}\sum_{t=2}^{T}\cos(k,t) \quad \Rightarrow \quad \int_0^1 \cos(k,r)dr; \tag{40}$$

$$\frac{1}{T^2}\sum_{t=2}^{T}t\cos(k,t) \quad \Rightarrow \quad \int_0^1 r\cos(k,r)dr; \tag{41}$$

$$\frac{1}{T^3}\sum_{t=2}^{T}t^2\cos^2(k,t) \quad \Rightarrow \quad \int_0^1 r^2\cos^2(k,r)dr; \tag{42}$$

$$T^{-5/2}\sum_{t=3}^{T}t\cos(k,t)u_{t-1} \quad \Rightarrow \quad \int_0^1 r\cos(k,r)W(r), \qquad 0 \leq r \leq 1. \tag{43}$$

Finally,

$$D_T\sum_{t=1}^{T}\mathbf{x}_{\tilde{c},t}u_{\tilde{c},t} = \begin{bmatrix} \Xi_4 \\ \Xi_5 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 0 \\ \sigma W(1)(1-\tilde{c}\cos(k,T)) - \sigma\pi k\tilde{c}\int_0^1 r\sin(2\pi kt/T)W(r)dr \\ +\sigma\tilde{c}^2\int_0^1 r\cos(k,r)W(r)dr \end{bmatrix}$$

where Ξ_4 is defined as in (41) and since $u_{\tilde{c},t} = \Delta u_{\tilde{c},t} - \tilde{c}\cos(k,t)T^{-1}$,

$$\Xi_5 = u_1 + u_T - u_1 - \frac{\tilde{c}}{T}\sum_{t=2}^{T}\cos(k,t)u_{t-1} -$$

$$\frac{\tilde{c}}{T}\sum_{t=2}^{T}(t-1)\cos(k,t)\Delta u_t + \left(\frac{\tilde{c}}{T}\right)^2\sum_{t=2}^{T}(t-1)\cos(k,t)^2 u_{t-1}$$

$$= u_T - \frac{\tilde{c}}{T}\sum_{t=2}^{T}\cos(k,t)u_{t-1}$$

$$+ \left(\frac{\tilde{c}}{T}\right)^2\left[\sum_{t=2}^{T}t\cos(k,t)^2u_{t-1} - \left(\frac{\tilde{c}}{T}\right)^2\sum_{t=2}^{T}t\cos(k,t)^2u_{t-1}\right]$$

$$- \frac{\tilde{c}}{T}\left[T\cos(k,T)u_T - 2\cos(k,\mathit{2})u_1 - \sum_{t=3}^{T}t\Delta\cos(k,t)u_{t-1}\right]$$

$$- \frac{\tilde{c}}{T}\left[-\sum_{t=3}^{T}\cos(k,t-\mathit{1})u_{t-1} - \frac{\tilde{c}}{T}\sum_{t=2}^{T}\cos(k,t)\Delta u_t\right]$$

$$= (1-\tilde{c}\cos(k,T))u_T + 2\tilde{c}\cos(k,\mathit{2})u_1T^{-1} - \frac{\pi k}{T}\frac{\tilde{c}}{T}\sum_{t=3}^{T}t\sin(2\pi \mathrm{kt/T})u_{t-1}$$

$$+ \frac{\tilde{c}}{T}\sum_{t=3}^{T}\cos(k,t-\mathit{1})u_{t-1} - \frac{\tilde{c}}{T}\sum_{t=2}^{T}\cos(k,t)u_{t-1} + \left(\frac{\tilde{c}}{T}\right)^2\sum_{t=2}^{T}t\cos(k,t)^2u_{t-1}$$

$$- \left(\frac{\tilde{c}}{T}\right)^2\sum_{t=2}^{T}\cos(k,t)^2u_{t-1} + \frac{\tilde{c}}{T}\sum_{t=2}^{T}\cos(k,t)\Delta u_t.$$

It follows from the FCLT and CMT that

$$T^{-1/2}\hat{u}_{[Tr]} = T^{-1/2}u_{[Tr]} - T^{-1/2}\mathbf{x}_{[Tr]}'\left[D_T\sum_{t=1}^{T}\mathbf{x}_{\tilde{c},t}\mathbf{x}_{\tilde{c},t}{}'D_T\right]^{-1}\left[D_T\sum_{t=1}^{T}\mathbf{x}_{\tilde{c},t}u_{\tilde{c},t}\right]$$

$$\Rightarrow \sigma W(r) - \sigma r\left[\frac{(1-\tilde{c}\cos(k,r))W(1) + \tilde{c}^2\int_0^1 r\cos^2(k,r)W(r)dr - \tilde{c}k\pi\int_0^1 r\sin(2\pi \mathrm{kr})W(r)dr}{\int_0^1\left[1-2\tilde{c}r\cos^2(k,r)+r^2\tilde{c}^2\cos(k,r)\right]dr}\right]. \quad (44)$$

■

Proof of Proposition 1

An extension of the FCLT near integrated process, $\rho_t = 1 - \frac{c}{T}$, states that,

$$\frac{1}{\sqrt{T}} u_{[Tr]} \Rightarrow \sigma J_c(r), \qquad 0 \leq r \leq 1,$$

where J_c is a standard OU process (see Phillips, 1987). The $P_{\tilde{c}}$ test statistic is given by

$$P_{\tilde{c}} = \frac{\sum_{t=1}^{T} \hat{\varepsilon}_{\tilde{c},t}^2 - \left[1 + \frac{\tilde{c}}{T}\cos(k,t)\right] \sum_{t=1}^{T} \hat{\varepsilon}_{0,t}^2}{\hat{\sigma}^2},$$

where $\hat{\varepsilon}_{0,t}$ is the residual term under $H_0 : \tilde{\rho}_t = 0$ and $\hat{\varepsilon}_{\tilde{c},t}$ is the residual term under $H_1 : \tilde{\rho}_t := 1 + \frac{\tilde{c}}{T}$ for a given $\tilde{c}$. The null hypothesis is rejected for small values of this statistic. Note that, in the case of demeaning,

$$\hat{\varepsilon}_{\tilde{c},t} = y_t - \left(1 + \frac{\tilde{c}}{T}\cos(\mathrm{k,t})\right) y_{t-1} - \beta_1 \left(1 - \left(1 + \frac{\tilde{c}}{T}\cos(\mathrm{k,t})\right)\right)$$
$$\to \Delta u_t - \frac{\tilde{c}}{T}\cos(\mathrm{k,t}) u_{t-1},$$

$$\hat{\varepsilon}_{\tilde{c},t}^2 \to \Delta u_t^2 - \frac{2\tilde{c}}{T}\Delta u_t \cos(\mathrm{k,t}) u_{t-1} + \left(\frac{\tilde{c}}{T}\right)^2 \cos^2(k,t) u_{t-1}^2,$$

$$\hat{\varepsilon}_{0,t}^2 = (\Delta u_t)^2,$$

Putting these results together we have that,

$$\sum_{t=1}^{T} \hat{\varepsilon}_{\tilde{c},t}^2 - \left[1 + \frac{\tilde{c}}{T}\cos(\mathrm{k,t})\right] \sum_{t=1}^{T} \hat{\varepsilon}_{0,t}^2 = -\frac{2\tilde{c}}{T} \sum_{t=2}^{T} \Delta u_t \cos(k,t) u_{t-1}$$
$$-\frac{\tilde{c}}{T} \sum_{t=2}^{T} \cos(k,t) \hat{\varepsilon}_{0,t}^2 + \left(\frac{\tilde{c}}{T}\right)^2 \sum_{t=2}^{T} \cos^2(k,t) u_{t-1}^2,$$

and given that

$$-\frac{2\tilde{c}}{T} \sum_{t=2}^{T} \Delta u_t \cos(\mathrm{k,t}) u_{t-1} \Rightarrow \tilde{c}\left[\sigma^2 \int_0^1 \cos(\mathrm{k,r}) dr - \sigma^2 \cos(\mathrm{k,T}) J_c^2(1)\right],$$

$$\left(\frac{\tilde{c}}{T}\right)^2 \sum_{t=2}^{T} \cos^2(\mathrm{k,t}) u_{t-1}^2 \Rightarrow \tilde{c}^2 \sigma^2 \int_0^1 \cos^2(\mathrm{k,r}) J_c^2(r),$$

$$\frac{\tilde{c}}{T} \sum_{t=2}^{T} \cos_j(k,t) \hat{\varepsilon}_{0,t}^2 \Rightarrow \tilde{c}\sigma^2 \int_0^1 \cos(k,r) dr,$$

the asymptotic distribution of $P_{\tilde{c}}$ is

$$P_{\tilde{c}} \Rightarrow \tilde{c}^2 \int_0^1 \cos^2(\mathrm{k,r}) J_c^2(r) - \tilde{c}\cos(\mathrm{k,T}) J_c^2(1).$$

Finally, when detrending is considered

$$\hat{\varepsilon}_{\tilde{c},t} = y_t - \rho_t y_{t-1} - \beta_1(1-\rho_t) - \beta_2(t - \rho_t\,(t-1)).$$

Thus, using the FCLT result presented previously it follows that,

$$P_{\tilde{c}} \Rightarrow \tilde{c}^2 \int_0^1 \cos^2(\mathrm{k,r})\,[J_c^{\tau}(r)]^2 + (1 - \tilde{c}\cos(\mathrm{k,T})\,[J_c^{\tau}(1)]^2\,,$$

where J_c^{τ} is the local GLS detrended OU process. ■

Proof of Proposition 2

The proposed test statistic with White standard errors is defined as,

$$\hat{t}_{k,W}^{GLS} := \frac{\sum_{t=2}^{T} \Delta\hat{u}_t \cos(k,t)\hat{u}_{t-1}}{\left(\sum_{t=2}^{T}\cos^2(k,\mathrm{t})\hat{u}_{t-1}^2\hat{\varepsilon}_t^2\right)^{1/2}}.$$

As the numerator is the same as in equation (6), we only need to examine the denominator. Hence, considering

$$\frac{1}{T^2}\sum_{t=2}^{T}\cos^2(\mathrm{k,t})\hat{u}_{t-1}^2\hat{\varepsilon}_t^2 = \frac{1}{T^2}\sum_{t=2}^{T}\cos^2(\mathrm{k,t})\hat{u}_{t-1}^2\sigma^2 + \frac{1}{T^2}\sum_{t=2}^{T}\cos^2(\mathrm{k,t})\hat{u}_{t-1}^2\left(\hat{\varepsilon}_t^2 - \sigma^2\right) \tag{45}$$

and noting that,

$$\sigma^2\frac{1}{T^2}\sum_{t=2}^{T}\cos^2(\mathrm{k,t})u_{t-1}^2 \Rightarrow \sigma^2\int_0^1 \cos^2(\mathrm{k,r})W(r)^2,$$

we only need to prove that the second term in (45) is $o_p(1)$. Thus, from the result in Demetrescu (2008),

$$\frac{1}{T^2}\sum_{t=2}^{T}\hat{u}_{t-1}^2\left(\hat{\varepsilon}_t^2 - \sigma^2\right) \xrightarrow{p} 0,$$

it follows that,

$$\frac{1}{T^2}\sum_{t=2}^{T}\cos^2(\mathrm{k,t})\hat{u}_{t-1}^2\left(\hat{\varepsilon}_t^2 - \sigma^2\right) \leq \frac{1}{T^2}\sum_{t=2}^{T}\hat{u}_{t-1}^2\left(\hat{\varepsilon}_t^2 - \sigma^2\right),$$

and therefore $\frac{1}{T^2}\sum_{t=2}^{T}\cos^2(\mathrm{k,t})\hat{u}_{t-1}^2\left(\hat{\varepsilon}_t^2-\sigma^2\right)$ is also $o_p(1)$ since $\cos^2(k,t)\leq 1$ for fixed $k>0$. ■

Proof of Proposition 3

We need to show that,

$$\frac{1}{T^2}\sum_{t=2}^{T}\hat{u}_{t-1}^2\left(\hat{\varepsilon}_t^2-\sigma^2\right)\xrightarrow{p}0$$

is still true when the process is near integrated. That is, when $u_t=(1-c\cos(k{,}t)/T)u_{t-1}+\varepsilon_t$. Since, $y_0=0$, we have

$$u_t=\sum_{i=0}^{t-1}\left(1-\frac{c}{T}\cos(k,\mathrm{i})\right)^i\varepsilon_{t-i}$$

$$\left(1-\frac{c}{T}\cos(k{,}i)\right)^i=1-\frac{c}{T}\,i\cos(k{,}i)+O(T^{-1}),$$

and

$$u_t=\sum_{i=0}^{t-1}\varepsilon_{t-i}-\frac{c}{T}\sum_{i=0}^{t-1}i\cos(k,\mathrm{i})\,\varepsilon_{t-i}+O(T^{-0.5}).$$

Since $i/T=O(1)$, the result can be derived in the same way as in the proof of Proposition 2. ■

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