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The analyses, opinions and findings of these papers represent the views of the authors, they are not necessarily those of the Banco de Portugal or the Eurosystem

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### An integrated financial amplifier: The role of defaulted loans and occasionally binding constraints in output fluctuations

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#### Abstract

We present a DSGE model for a small euro area economy comprising a banking sector empowered with regulatory capital requirements, defaulted loans and occasionally binding endogenous credit restrictions. Under non-financial shocks no important amplifications arise due to balancing forces: while banks' equity acts as a shock absorber, the observance of regulatory capital requirements acts as a shock amplifier. Under moderately-sized "bad" financial-based shocks defaulted loans increase and banks' value drop. As a result, credit becomes supply constraint for some time, severely amplifying and protracting output downfalls. Endogenous inertia implies a slow recovery in banks' capital and thus an enduring fragility of the banking system. Defaulted loans and credit restrictions are strongly intertwined, since the former severely impact banks' value, hence leveraging the amplification size.

 ${\rm JEL:}\ {\rm E62},\ {\rm F41},\ {\rm H62}$ 

Keywords: DSGE models, euro area, small-open economy, financial accelerator, banks, credit supply restrictions, defaulted loans.

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#### 1. Introduction

We build a Dynamic Stochastic General Equilibrium (DSGE) model for a small euro area economy comprising a banking sector empowered with regulatory capital requirements, defaulted loans and occasionally binding credit restrictions. Defaulted loans are defined herein as bank loans from bankrupted corporations that linger in the balance sheet until they are either recovered or written-off. We construct a benchmark accelerator version inspired in Bernanke et al. (1999)—henceforth BGG—and a non-accelerator version of the model, and compare the macroeconomic dynamics across different models. For nonfinancial shocks the banking model essentially delivers similar dynamics as compared with its BGG-accelerator version; credit restrictions remain slack all the time and defaulted loans are nearly unchanged. Under "bad" financial-based shocks the model is able to substantially increase the size and persistence of output downfalls. Banks' value drops and credit restrictions become binding for some time period. Defaulted loans increase substantially if the shock strongly impacts corporate default, and endogenous inertia implies a slow recovery in banks' capital and thus an enduring fragility of the banking system. Defaulted loans and credit restrictions are strongly intertwined, since a large increase in the former negatively impacts banks' value and thus leverages credit restrictions and output downfalls.

The 2007–2009 international crisis created shockwaves in the economics profession that are far from settled. In the euro area, interactions between financial and non-financial institutions during the 2010s sovereign debt crisis called for in-depth examinations by both the empirical and theoretical literature. Failures in financial markets were pushed again to the forefront by many economists, just like Irving Fischer or J. M. Keynes did when they examined the causes behind the Great Depression. The challenges are very simple: what happened, why, and what can we do to avoid a repetition?

A key criticism of DSGE models was their inability to identify the cumulative vulnerabilities before the worst recession of the postwar period (Christiano *et al.* 2018), let alone to signal meaningful policy warnings. The depth of the recession was outside the predictive density of standard pre-crisis models (Del Negro and Schorfheide 2013) or was only explained through a "cocktail of extremely unlikely shocks" (Lindé *et al.* 2016). Bernanke *et al.* (1999) and Kiyotaki and Moore (1997) are two of the most influential studies on financial frictions as amplification and propagation devices. The former exhibits a financial accelerator whereby the external finance premium—a price effect—enhances the swings in borrowing and thus on output. The later draws directly from the effects on quantities due to collateral constraints.<sup>1</sup>

<sup>1.</sup> Iacoviello (2005) provides an important contribution. See also Iacoviello (2015), who emphasize the importance of "redistribution shocks" (transfers of wealth from savers to borrowers) in the collateral constraint framework.

Theoretical and empirical evaluations of such devices provide some support for the insufficiency of such mechanisms. Some empirical studies favor the BGG approach, but suggest that it provides no clear and compelling improvement over the standard New-Keynesian benchmark (Brzoza-Brzezina and Kolasa 2013; Brzoza-Brzezina *et al.* 2013), or that additional non-linear features are required to properly account business cycle fluctuations (Lindé *et al.* 2016).<sup>2</sup> Models based on collateral constraints, in turn, have been cataloged as quantitatively insignificant (Kocherlakota 2000; Cordoba and Ripoll 2004).

The large Non-Performing Loans (NPL) stock in banks' balance sheets has been a persistent legacy of a number of euro area economies in the aftermath of the financial crisis. Both institutional work (Aiyar et al. 2015; Constâncio 2017) and academic studies (Gerali et al. 2010; Pariès et al. 2011; Benes and Kumhof 2015) have emphasized the link between credit defaults and credit supply restrictions in amplifying output fluctuations.<sup>3</sup> The DSGE modeling literature often deals with credit default flows by assuming that they are fully covered by state contingent interest rate (Bernanke et al. 1999), immediately recognized as impairment losses and written-off (Benes and Kumhof 2015; Clerc et al. 2015), or somehow embodied in an exogenous shock to the value of bank capital (Gerali et al. 2010; Pariès et al. 2011). In practice there is a significant delay between the occurrence of a credit default and its recognition as a loss, something which accrues from the increase in the NPL stock registered over time coupled with inadequate impairment recognition. As a result, defaulted loans often linger on banks' balance sheets for a number of periods, as bankers attempt to recover at least part of the claim though renegotiation, sell it to a third party, or simply delay the recognition of the loss to avoid eroding shareholder equity. However, the question remains on whether a large stock of defaulted loans on banks' balance sheet is able to affect or even magnify business cycle fluctuations. To our knowledge, this issue has never been addressed in the context of a DSGE model.

The inability of financial frictions-based models to properly take into account rare or extreme events and to provide a convincing improvement over simpler and more standard alternatives suggests that some work must be targeted to enhance the size, persistence and asymmetry of business cycle fluctuations (Kocherlakota 2000). We contribute towards the literature by developing an"integrated financial system," where the BGG framework is attached to a banking system where credit supply decisions are simultaneously driven by regulatory capital requirements, defaulted loans, and credit supply restrictions that become binding in shocks that severely affect banks' value. The banking system proposed herein intertwines two strands in the literature

<sup>2.</sup> See Brunnermeier and Sannikov (2014) for an alternative theoretical approach. Recent progresses in macroeconomic models can be found in Quadrini (2011), Brunnermeier and Sannikov (2014) or Christiano *et al.* (2018).

<sup>3.</sup> A recent proposal with bank runs can be found in Gertler and Kiyotaki (2015).

with two completely novel features. Capital requirements follow the approach in Benes and Kumhof (2015), and are coupled to a moral hazard-inspired credit constraint mechanism in the spirit of Gertler and Karadi (2011), Gertler *et al.* (2012), and Gertler and Karadi (2013). However, contrary to these studies which assume an always binding incentive compatibility constraint, we propose and develop herein an occasionally binding mechanism which is slack in the steady state but endogenously affects credit supply decisions when banks' capital is severely affected. Simultaneously, we bring forth into the model a theory of optimal impairment loss recognition, which gives raise to an endogenous defaulted loans stock that bankers manage over time.<sup>4</sup>

In our model, credit is simultaneously driven by price and quantity effects, mixing the properties of both aforementioned literature strands, and the wholesale interest rate premium can be broken down into the contribution of each effect. Credit is mostly demand/price driven in a defaulted loansbanking enhanced BGG world. Loan contracts set an unconditional, nonstate contingent lending rate, creating a non-diversifiable aggregate risk environment which spawns ex-post gains/losses in the lending activity. In addition, wholesale banks face an idiosyncratic shock over their loan portfolio and are subject to regulatory requirements. As a result, they endogenously set capital buffers which allows them to cushion both aggregate and idiosyncratic adverse shocks that negatively affect the value of capital. As compared with the BGG-accelerator model, the banking model has an additional cushion mechanism—banks' equity which acts as a shock absorber—and an additional friction mechanism—the observance of regulatory capital requirements which is reflected into higher wholesale spreads. These forces tend to balance each other, and thus no important amplifications arise.

Defaulted loans emerge endogenously within the model since repossessed assets following default are illiquid and cannot be immediately converted into income. The intuition is that defaulted loans are often associated with auditing expenses, judicial proceedings aimed at recovering some of the claim's value, and costly proof providence to investors respecting their correct valuation. We assume that defaulted loans have an opportunity cost (pay no interest) and an holding cost (expected penalty/reputation cost).<sup>5</sup> Over time, an exogenous fraction of defaulted loans is automatically transformed from illiquid into liquid status at no cost, but bankers can increase the pace of this transformation by requesting a liquidation service from households. The trade-off then consists in balancing the liquidation service cost—aka the associated impairment loss with the expected cost of carrying-over that defaulted loans unit to the

<sup>4.</sup> Gourinchas *et al.* (2016) include NPL in a DSGE model. In contrast with out framework, however, they follow an *ad hoc* approach in which credit losses increase with higher firm and household debt levels and lower output.

<sup>5.</sup> For details on the defaulted loans life-cycle and the due diligence required by the Single Supervisory Mechanism, see ECB (2017).

next period—composed of the opportunity cost and the corresponding capital requirements penalty/reputation costs. It follows that defaulted loans crowds out new bank lending and debt roll-overs, limiting banks' ability to raise funds through external finance.<sup>6</sup>

Credit becomes (endogenously) supply/quantity driven when a "bad" shock depleting banks' value hits the economy. The banker has the option to divert a fraction of funds from the bank, though this only becomes attractive when the bank's value collapses well below the steady-state level. In this case, the initially slack incentive compatibility constraint becomes binding for some time period as depositors restrain the amount of funds placed at the bank to avoid an expansion in loans and consequently the diversion of funds. The model is therefore an asymmetric amplifier by construction. Under "good shocks" credit restrictions remain slack and play no role whatsoever, whereas under "bad shocks" the banks' supply-constrained balance sheet limits the amount of resources made available to the entrepreneurial sector, hence hindering investment and capital accumulation.

In our model, defaulted loans and credit restrictions are strongly intertwined. The increase in the defaulted loans stock on the aftermath of a "bad" financial shock negatively impacts banks' value, thus leveraging credit restrictions and boosting output downfalls. We run a comparative statics exercise which assesses the role of some key parameters in determining the strength of credit restrictions, and conclude that the defaulted loans recovery rate and their expected cost play a key role in the magnitude of credit restrictions and hence on asymmetric output developments. The intuition is that these parameters severely affect the response of banks' value to defaulted loans, thereby enhancing the severity of restrictions to credit and the amplification size.

Our work has obvious policy implications. Firstly, the distribution of shocks matter to explain output fluctuations and in particular output downfalls; the mean is not sufficient. Secondly, a narrow set of negative small-sized financialbased shocks can trigger a deep and protracted recession, which may contribute decisively to enhance the predictive density of DSGE model in crisis periods. However, the opposite is not true: positive financial shocks may not trigger a sizable expansion. Third, our model predicts that defaulted loans mostly accumulate on banks balance sheet on the aftermath of financial shocks, which is in line with facts recorded in a number of euro area economies in the aftermath of the financial crisis. Fourth, the model provides a completely novel framework to analyze policy-oriented measures aimed at increasing the robustness of the financial and banking system, especially during crisis periods.

<sup>6.</sup> Our theory implies that defaulted loans can be written as a generalized AR(1) process with a time-varying mean and autoregressive coefficient.

The article is organized as follows. Sections 2 and 3 present the non-financial and financial blocks of the model, respectively. Section 4 details the model's calibration. Section 5 addresses the size, amplification, and asymmetry macroeffects of the model. Section 6 carries out a comparative statics exercise and discusses some results. Finally, Section 7 concludes.

#### 2. A DSGE model for a small-open euro area economy

The domestic economy is composed of nine types of agents: households, intermediate goods producers (manufacturers), final goods producers (distributors), retailers, capital goods producers, entrepreneurs, banks, the government, and foreign agents (the remaining euro area). This section presents a canonical model for all agents except entrepreneurs and banks, which are analyzed separately in the next section.

Households follow an infinitely-lived structure, renting labor services to manufacturers at a price, paying lump-sum taxes to the government, and earning interest on deposits. They are composed of three member types: workers, entrepreneurs and bankers. There is full consumption insurance within the family. When exiting activity, the last two member types transfer accumulated earnings back to the household. In each period and for each activity the number entries and exits are the same.

Capital goods producers fabricate capital goods and sell them to entrepreneurs. Manufacturers combine capital with labor services to produce intermediate goods, which distributors use as inputs. We consider Calvostaggered price adjustment and sluggish adjustment in hours worked. Distributors combine intermediate goods with imported goods to produce the final good, facing Calvo-type price staggering. Perfectly competitive retailers acquire the final good from distributors and reallocate it to different costumers.

The government keeps the budget balanced at all times, financing public consumption with lump-sum taxes, levied on households. The foreign economy corresponds to the rest of the monetary union. The domestic economy interacts with the foreign economy via goods and financial markets. In the goods market, domestic distributors buy imported goods from abroad to be used in the production stage. Likewise for foreign distributors, who buy export goods from domestic retailers for the same purpose. In the financial market, banks can finance balance sheet operations by trading assets with the foreign economy. Monetary policy is exogenous and unresponsive to domestic developments, a consequence of the small-open economy framework. Hence, developments in euro-area interest rates are orthogonal to domestic developments, as in Adolfson et al. (2007). The nominal exchange rate vis-à-vis the rest of the euro area is irrevocably set to unity.

The exposition in this section omits most technical details, which can be found in or readily adapted from Júlio and Maria (2018). The novel part of the model, dealt with in Section 3, is more detailed.

#### 2.1. Households

Households are composed of workers, entrepreneurs and bankers, and there is perfect consumption insurance within the family. For simplicity, we assume that the percentage of entrepreneurs and bankers is infinitesimally small to avoid keeping track of their mass. A representative household derives utility from consumption  $C_t(h)$ , real money holdings  $dep_t(h) = DEP_t(h)/P_t$ , and disutility from working  $U_t(h)$ . The term  $U_t$  stands for hours worked as a fraction of total time endowment,  $DEP_t(h)$  for nominal deposits, and  $P_t$  for the price paid to retailers for the consumption good, taken as *numéraire*. Expected lifetime utility is

$$E_t \sum_{s=0}^{\infty} (\beta)^s UTIL_{t+s}(h)$$
(1)

where  $E_t$  is the expectation operator and  $0 \le \beta \le 1$  stands for the discount factor. Flow utility is separable in all arguments

$$\text{UTIL}_{t}(h) = (1 - \nu) \log(C_{t}(h) - Hab_{t}) - \frac{\eta_{L}}{1 + \sigma_{L}} (U_{t}(h))^{1 + \sigma_{L}} + \eta_{D} \log(dep_{t}(h))$$

where  $\eta_L, \eta_D > 0$  are utility weights and  $\sigma_L$  is the inverse Frisch elasticity of labor supply. The element  $Hab_t = \nu C_{t-1}$  stands for external habits, where  $\nu$  is a scale parameter. Since money holdings do not affect the intratemporal consumption-leisure choice, money is superneutral in the steady state. Deposits pay a gross nominal interest rate of  $i_t^D$  if held between period t and t + 1.

The household supplies labor services  $U_t(h)$  to manufacturers (through workers), receiving a wage rate  $V_t$ , and pays a lumpsum tax  $LT_t$  to the government. She also recovers  $RBR_t$  for services provided in bankruptcy monitoring and  $DL_t^{imp}$  for services in reducing the defaulted loans portfolio of banks—activities performed at no personal effort—and receives dividends  $D_t^x$ ,  $x \in \{\mathcal{M}, \mathcal{D}, \mathcal{KP}, \mathcal{E}, \mathcal{BK}\}$ . These can originate from manufacturers  $(\mathcal{M})$ , distributors  $(\mathcal{D})$ , capital goods producers  $(\mathcal{KP})$ , entrepreneurs  $(\mathcal{E})$ , banks  $(\mathcal{BK})$ . Over time, an entrepreneur in period t stays an entrepreneur in the next period with probability  $\iota^{\mathcal{E}}$ , and a banker with probability  $\iota^{\mathcal{BK}}$ , independent of history. The remaining fractions become workers and transfer accumulated earnings to their respective household, and are replaced by a similar measure of entrepreneurs and bankers. The household provides these elements with small amount of startup funds. We let  $D_t^{\mathcal{E}}$  and  $D_t^{\mathcal{BK}}$  denote transferred earnings net of startup funds.

Households are not allowed to hold foreign financial assets but may hold private securities, as in Gertler and Karadi (2013). We introduce limits to arbitrage by assuming a holding cost equaling

$$\Phi_t^{L^{\mathcal{H}}(h)} = \frac{\kappa_L}{2} \frac{P_t}{L_t^{\mathcal{H}}(h)} \left(\frac{L_t^{\mathcal{H}}(h)}{P_t} - \frac{\bar{L}^{\mathcal{H}}}{P_t}\right)^2, \ L_t^{\mathcal{H}}(h) \ge \bar{L}$$

for each unit of private securities  $L_t^{\mathcal{H}}(h) \geq \overline{L}^{\mathcal{H}}$  held between periods t and t+1, where  $\kappa_L$  is a parameter. This structure implies incomplete arbitrage by capturing households' limited participation in asset markets.<sup>7</sup> Households lend to corporations indirectly *via* retail branches, who support unexpected gains/losses from the loan activity. Households therefore receive the same return on loans  $i_t^W$  that is charged by wholesale banks to retail branches.<sup>8</sup>

The nominal budget constraint embodying that expenditures cannot exceed revenues is

$$P_{t}C_{t}(h) + DEP_{t}(h) + (1 + \Phi_{t}^{L^{\mathcal{H}}(h)})L_{t}^{\mathcal{H}}(h) \leq i_{t-1}^{D}DEP_{t-1}(h) + i_{t}^{W}L_{t}^{\mathcal{H}}(h) + V_{t}U_{t}(h) - LT_{t} + RBR_{t} + DL_{t}^{imp} + DIV_{t}$$
(2)

where

$$DIV_t = \sum_{\substack{x \in \{\mathcal{M}, \mathcal{D}, \\ \mathcal{KP}, \mathcal{E}, \mathcal{BK}\}}} \int_0^1 D_t^x(i) \mathrm{d}i$$

The optimization problem consists in maximizing expected lifetime utility (1) with respect to  $\{C_{t+s}(h), DEP_{t+s}(h), L_{t+s}^{\mathcal{H}}(h)\}_{s=0}^{\infty}$ , subject to (2). We treat wage setting and labor supply decisions separately below. First-order conditions are trivial and we omit them for brevity. For later reference, let us define the stochastic discount factor between t and t+1 for real payoffs as  $\Lambda_{t,t+1}^{R} = \beta \lambda_{t+1}/\lambda_{t}$ , and for nominal payoffs as  $\Lambda_{t,t+k}^{N} = \Lambda_{t,t+k}^{R} \pi_{t+1}^{-1}$ , where  $\lambda_{t}$  stands for the Lagrange multiplier.

<sup>7.</sup> This framework helps the computational algorithm in finding a solution under the nonlinear structure of occasionally biding credit restrictions.

<sup>8.</sup> A lower return implies that retails branches would finance all loans through households, whereas a higher return implies the opposite. Either case is not an equilibrium. Incomplete arbitrage by households or more competitive financing conditions faced by retail branches drive the interest rate on private securities held by households towards the wholesale rate. The exposition of retail branches is postponed to Section 3.

#### 2.2. Wage setting

Suppose that manufacturer j combines specialized labor supply by households into a homogeneous labor service as follows

$$U_t(j) = \left(\int_0^1 U_t(h,j)^{\frac{\sigma^{\mathcal{U}}-1}{\sigma^{\mathcal{U}}}} \mathrm{d}h\right)^{\frac{\sigma^{\mathcal{U}}}{\sigma^{\mathcal{U}}-1}}$$

where  $\sigma^{\mathcal{U}} \geq 0$  is the elasticity of substitution between labor varieties. Letting  $V_t(h)$  denote the wage charged by household h, each manufacturer solves

$$\min_{U_t(h,j)} \int_0^1 V_t(h) U_t(h,j) \mathrm{d}h \qquad \text{s.t.} \qquad U_t(j) = \left(\int_0^1 U_t(h,j) \frac{\sigma^{\mathcal{U}} - 1}{\sigma^{\mathcal{U}}} \mathrm{d}h\right)^{\frac{\sigma^{\mathcal{U}}}{\sigma^{\mathcal{U}}} - 1}$$

yielding the usual demand for labor variety  $h, U_t(h)$ 

$$U_t(h) = \left(\frac{V_t(h)}{V_t}\right)^{-\sigma^{\mathcal{U}}} U_t \tag{3}$$

where  $U_t$  is aggregate labor demand. We consider Calvo-type frictions and assume that households are unable to reoptimize the wage in each period with probability  $\iota^{\mathcal{U}}$ . The optimal wage  $V_t^*(h)$  follows from the solution to

$$\max_{V_t^*(h)} \mathcal{E}_t \sum_{s=0}^{\infty} (\beta \iota^{\mathcal{U}})^s \left\{ -\frac{\eta_L}{1+\sigma_L} \left[ \left( \frac{V_t^*(h)}{V_{t+s}} \right)^{-\sigma^{\mathcal{U}}} U_{t+s} \right]^{1+\sigma_L} + \frac{\lambda_{t+s}(h)}{P_{t+s}} V_t^*(h) \left[ \left( \frac{V_t^*(h)}{V_{t+s}} \right)^{-\sigma^{\mathcal{U}}} U_{t+s} \right] \right\}$$

We ignored the irrelevant terms from the household's objective function. The element  $\lambda_t$  is the Lagrange multiplier on households' budget constraint. The wage rate is

$$V_t = \left(\iota^{\mathcal{U}} V_{t-1}^{1-\sigma^{\mathcal{U}}} + (1-\iota^{\mathcal{U}})(V_t^*)^{1-\sigma^{\mathcal{U}}}\right)^{\frac{1}{1-\sigma^{\mathcal{U}}}}$$

where

$$V_t^* = \frac{\sigma^{\mathcal{U}}}{\sigma^{\mathcal{U}} - 1} W_t \frac{CV_t^{\mathcal{U}, n}}{CV_t^{\mathcal{U}, d}}$$

The element  $W_t = P_t(\eta_L/\lambda_t)U_t^{\sigma_L}$  is the value of the marginal disutility of labor, while  $CV_t^{\mathcal{U},n}$  and  $CV_t^{\mathcal{U},d}$  are Calvo parameters

$$CV_t^{\mathcal{U},n} = 1 + \beta \iota^{\mathcal{U}} (\pi_{t+1}^V)^{(1+\sigma^L)\sigma^{\mathcal{U}}} \left(\frac{U_{t+1}}{U_t}\right)^{1+\sigma^L} CV_{t+1}^{\mathcal{U},n}$$
$$CV_t^{\mathcal{U},d} = 1 + \Lambda_{t,t+1}^R \iota^{\mathcal{U}} \frac{(\pi_t^V)^{\sigma^{\mathcal{U}}}}{\pi_{t+1}} \frac{U_{t+1}}{U_t} CV_{t+1}^{\mathcal{U},d}$$

with  $\pi_{t+1}^V = V_{t+1}/V_t$  denoting wage inflation.

#### 2.3. The non-financial block

Capital goods producers fabricate and sell productive capital to entrepreneurs, who will own it during the production cycle. Manufacturers combine capital with labor services to produce intermediate goods. These are thereafter sold to distributors to be combined with imported goods, yielding the final good. Capital goods producers are perfectly competitive in both input and output markets, whilst manufacturers and distributors operate in a monopolistically competition environment in the output market. Perfectly competitive retailers acquire and bundle together the different varieties of the final good from distributors and reallocate it to different costumers.

In what follows we use the convention that  $K_t$  represents the stock of capital that is actually used by manufacturers in period t. This quantity is decided one period in advance, *i.e.*, the manufacturers' demand for  $K_t$  is decided at t-1. The quantity  $\bar{K}_t$  represents the total physical capital stock of the economy at t, fabricated by capital goods producers and owned by entrepreneurs during the production cycle. This may differ from the capital stock that is actually used in production since entrepreneurs adjust capital utilization,  $u_t$ . Hence,  $K_t = u_t \bar{K}_t$ .

2.3.1. Capital goods producers. There exists a continuum of capital goods producers indexed by  $i \in [0, 1]$ . In each period, capital goods producers combine the undepreciated installed productive capital stock  $(1 - \delta^{\mathcal{K}})\bar{K}_t(i)$ , bought from entrepreneurs, with investment goods  $I_t^{\mathcal{K}}(i)$ , bought from retailers, to produce new installed productive capital  $\bar{K}_{t+1}(i)$ , according to the following law of motion

$$\bar{K}_{t+1}(i) = (1 - \delta^{\mathcal{K}})\bar{K}_t(i) + I_t^{\mathcal{K}}(i)$$

where  $\delta^{\mathcal{K}}$  is the depreciation rate and  $\bar{K}_t(i)$  represents the capital stock at t. We impose a sluggish pattern for investment by assuming quadratic adjustment costs with the form

$$\Gamma_t^{\mathcal{IK}}(i) = \frac{\varphi_{\mathcal{IK}}}{2} I_t^{\mathcal{K}} \left( \frac{I_t^{\mathcal{K}}(i)}{I_{t-1}^{\mathcal{K}}(i)} - 1 \right)^2$$

where  $I_t^{\mathcal{K}}$  denotes period t overall investment on productive capital and  $\varphi_{\mathcal{IK}}$  is a scaling factor. Capital goods producers select the intertemporal profile  $\{I_{t+s}^{\mathcal{K}}(i)\}_{s=0}^{\infty}$  that maximize the present discounted value of the dividends stream

$$\mathbf{E}_{t} \sum_{s=0}^{\infty} \Lambda_{t,t+s}^{N} \left[ P_{t+s}^{\mathcal{K}} I_{t+s}^{\mathcal{K}}(i) - P_{t+s}^{\mathcal{I}} \left( I_{t+s}^{\mathcal{K}}(i) + \Gamma_{t+s}^{\mathcal{I}\mathcal{K}}(i) \right) \right]$$

2.3.2. Manufacturers. There is a continuum of manufacturing firms  $j \in [0, 1]$ . Each firm produces a specific variety of the intermediate good, which is bought by a continuum of distributor firms  $f \in [0, 1]$ . Let  $Z_t(j, f)$  stand for the time tquantity of variety j produced by manufacturer j and purchased by distributor f. Distributors buy intermediate goods from many manufacturers, bundling them together in a homogeneous intermediate good,  $Z_t(f)$ , to be used in the final goods production. The bundling technology is given by the CES aggregator

$$Z_t(f) = \left(\int_0^1 Z_t(j, f)^{\frac{\sigma^Z - 1}{\sigma^Z}} \mathrm{d}j\right)^{\frac{\sigma^Z}{\sigma^Z - 1}}$$

where  $\sigma^{\mathcal{Z}} \geq 0$  is the elasticity of substitution between varieties of the intermediate good. The resulting demand for intermediate goods faced by firm j is

$$Z_t(j) = \left(\frac{P_t^{\mathcal{Z}}(j)}{P_t^{\mathcal{Z}}}\right)^{-\sigma^{\mathcal{Z}}} Z_t \tag{4}$$

where  $Z_t$  is the aggregate demand for the intermediate good. Each manufacturing firm j combines labor services  $U_t^{\mathcal{Z}}(j)$  with capital  $K_t(j)$  according to the following production function

$$Z_t(j) = \left( \left(1 - \alpha_{\mathcal{U}}\right)^{\frac{1}{\xi_{\mathcal{Z}}}} \left(K_t(j)\right)^{\frac{\xi_{\mathcal{Z}} - 1}{\xi_{\mathcal{Z}}}} + \left(\alpha_{\mathcal{U}}\right)^{\frac{1}{\xi_{\mathcal{Z}}}} \left(A_t T_t U_t^{\mathcal{Z}}(j)\right)^{\frac{\xi_{\mathcal{Z}} - 1}{\xi_{\mathcal{Z}}}} \right)^{\frac{\xi_{\mathcal{Z}}}{\xi_{\mathcal{Z}} - 1}}$$
(5)

where  $\xi_{\mathcal{Z}} \geq 0$  is the elasticity of substitution between capital an labor,  $0 \leq \alpha_{\mathcal{U}} \leq 1$  is a distribution parameter, and  $A_t$  is a stationary labor-augmenting technology shifter. We impose a sluggish adjustment of hours worked through a quadratic adjustment cost function

$$\Gamma_t^U(j) = \frac{\varphi_U}{2} U_t^{\mathcal{Z}} \left( \frac{U_t^{\mathcal{Z}}(j)}{U_{t-1}^{\mathcal{Z}}(j)} - 1 \right)^2 \tag{6}$$

where  $\varphi_U$  is a sector specific scaling factor determining the magnitude of adjustment costs. Manufacturer j sets labor demand  $U_t^{\mathcal{Z}}(j)$  and capital demand  $K_{t+1}(j)$  in each period in order to maximize the present discounted value of the dividends stream

$$\operatorname{E}_{t}\sum_{s=0}^{\infty}\Lambda_{t,t+s}^{N}\left[P_{t+s}^{\mathcal{Z}}(j)Z_{t+s}(j)-R_{t+s}^{\mathcal{K}}K_{t+s}(j)-V_{t+s}\left(U_{t+s}^{\mathcal{Z}}(j)+\Gamma_{t+s}^{U}(j)\right)-P_{t+s}^{\mathcal{Z}}\varpi^{\mathcal{Z}}\right]$$

where  $R_t^{\mathcal{K}}$  is the nominal rental rate and  $P_t^{\mathcal{Z}} \varpi^{\mathcal{Z}}$  is a quasi-fixed cost, subject to the constraints imposed by variety j demand in (4), production technology in (5), and adjustment costs in (6).

Firm is unable to reoptimize the price in each period with probability  $\iota^{\mathcal{Z}}$ , facing a Calvo-type problem. The manufacturer's price-setting problem is

$$\mathcal{L}(\cdot) = \mathbb{E}_t \sum_{s=0}^{\infty} \Lambda_{t,t+s}^N (\iota^{\mathcal{Z}})^s \left\{ \left( P_t^{\mathcal{Z}*}(j) - P_{t+s} \lambda_{t+s}^{\mathcal{Z}} \right) \left( \frac{P_t^{\mathcal{Z}*}(j)}{P_{t+s}^{\mathcal{Z}}} \right)^{-\sigma^2} Z_{t+s} \right\}$$

where  $P_{t+s}\lambda_{t+s}^{\mathcal{Z}}$  reflects the (nominal) marginal value of producing an additional unit of the intermediate good that results from the duality theorem. The intermediate goods price is

$$P_t^{\mathcal{Z}} = \left(\iota^{\mathcal{Z}} \left(P_{t-1}^{\mathcal{Z}}\right)^{1-\sigma^{\mathcal{Z}}} + (1-\iota^{\mathcal{Z}})(P_t^{\mathcal{Z}*})^{1-\sigma^{\mathcal{Z}}}\right)^{\frac{1}{1-\sigma^{\mathcal{Z}}}}$$

where

$$P_t^{\mathcal{Z}*} = \frac{\sigma^{\mathcal{Z}}}{\sigma^{\mathcal{Z}} - 1} \lambda_t^{\mathcal{Z}} \frac{CV_t^{\mathcal{Z},n}}{CV_t^{\mathcal{Z},d}}$$

The element  $\lambda_t^{\mathcal{Z}}$  denotes the real marginal cost, whereas the Calvo elements are

$$CV_{t}^{\mathcal{Z},n} = 1 + \Lambda_{t,t+1}^{R} \iota^{\mathcal{Z}} (\pi_{t+1}^{\mathcal{Z}})^{\sigma^{\mathcal{Z}}} \frac{\lambda_{t+1}^{\mathcal{Z}}}{\lambda_{t}^{\mathcal{Z}}} \frac{Z_{t+1}}{Z_{t}} CV_{t+1}^{\mathcal{Z},n}$$
$$CV_{t}^{\mathcal{Z},d} = 1 + \Lambda_{t,t+1}^{R} \iota^{\mathcal{Z}} \frac{(\pi_{t+1}^{\mathcal{Z}})^{\sigma^{\mathcal{Z}}}}{\pi_{t+1}} \frac{Z_{t+1}}{Z_{t}} CV_{t+1}^{\mathcal{Z},d}$$

with  $\pi_{t+1}^{\mathcal{Z}} = P_{t+1}^{\mathcal{Z}} / P_t^{\mathcal{Z}}$  denoting intermediate goods inflation.

2.3.3. Distributors and retailers. There is a continuum of distributors  $f \in [0, 1]$ , each producing a specific variety of the good, which are bundled together by retailers to form the final good,  $Y_t$ . Let  $Y_t(f)$  stand for the time t quantity of variety f from the final good, purchased by a continuum  $r \in [0, 1]$  of retailers. The sole function of retailers is to bundle together the different varieties fproduced by distributors to form an homogeneous final good  $Y_t$  that can be reallocated to different costumers—households, capital goods producers, government, and foreign distributors. The bundling technology is given by the CES aggregator

$$Y_t(r) = \left(\int_0^1 Y_t(f, r)^{\frac{\sigma^{\mathcal{V}} - 1}{\sigma^{\mathcal{V}}}} \mathrm{d}f\right)^{\frac{\sigma^{\mathcal{V}}}{\sigma^{\mathcal{V}} - 1}}$$

The demand for variety f is

$$Y_t(f) = \left(\frac{P_t^{\mathcal{Y}}(f)}{P_t^{\mathcal{Y}}}\right)^{-\sigma^{\mathcal{Y}}} Y_t \tag{7}$$

where  $\sigma^{\mathcal{Y}} \geq 0$  is the elasticity of substitution between varieties of the final good. Each distributor f combines domestic manufactured goods  $Z_t^{\mathcal{Y}}(f)$  with imported goods  $M_t(f)$  to obtain the final good  $Y_t(f)$ , according to the technology

$$Y_t(f) = \left( \left( \alpha_{\mathcal{Z}} \right)^{\frac{1}{\xi_{\mathcal{Y}}}} \left( Z_t^{\mathcal{Y}}(f) \right)^{\frac{\xi_{\mathcal{Y}} - 1}{\xi_{\mathcal{Y}}}} + \left( 1 - \alpha_{\mathcal{Z}} \right)^{\frac{1}{\xi_{\mathcal{Y}}}} \left[ M_t(f) \left( 1 - \Gamma_t^{IM}(f) \right) \right]^{\frac{\xi_{\mathcal{Y}} - 1}{\xi_{\mathcal{Y}}}} \right)^{\frac{\xi_{\mathcal{Y}} - 1}{\xi_{\mathcal{Y}} - 1}}$$
(8)

where  $\xi_{\mathcal{Y}} \geq 0$  is the elasticity of substitution between domestic manufactured goods and imported good and  $0 \leq \alpha_{\mathcal{Z}} \leq 1$  is the home bias parameter. We impose the following quadratic adjustment cost function on changes in the import content

$$\Gamma_t^{IM}(f) = \frac{\varphi_{IM}}{2} \frac{\left(\mathcal{A}_t^{IM}(f) - 1\right)^2}{1 + \left(\mathcal{A}_t^{IM}(f) - 1\right)^2}, \ \mathcal{A}_t^{IM}(f) = \frac{M_t(f)/Y_t(f)}{M_{t-1}/Y_{t-1}} \tag{9}$$

Each distributor f selects  $\{Z_{t+s}^{\mathcal{Y}}(f), M_{t+s}(f)\}_{s=0}^{\infty}$  to maximize the discounted value of the dividend stream

$$E_t \sum_{s=0}^{\infty} \Lambda_{t,t+s}^N \Big[ P_{t+s}^{\mathcal{Y}}(f) Y_{t+s}^{\mathcal{Y}}(f) - P_{t+s}^{\mathcal{Z}} Z_{t+s}^{\mathcal{Y}}(f) - P_{t+s}^* M_{t+s}(f) - P_{t+s}^{\mathcal{Y}} \left( \Gamma_{t+s}^{P\mathcal{Y}}(f) + T_{t+s} \varpi^{\mathcal{Y}} \right) \Big]$$

where  $P_t^*$  is the foreign price level (the nominal exchange rate is assumed to be irrevocably set to unity), subject to variety f demand (7), technology (8), and adjustment costs (9). With Calvo-type frictions, the firm is unable to reoptimize its price in each period with probability  $\iota^{\mathcal{Y}}$ . The distributor's price-setting problem becomes

$$\mathcal{L}(\cdot) = \mathbb{E}_t \sum_{s=0}^{\infty} \Lambda_{t,t+s}^N (\iota^{\mathcal{V}})^s \left\{ \left( P_t^{\mathcal{V}*}(j) - P_{t+s} \lambda_{t+s}^{\mathcal{Y}} \right) \left( \frac{P_t^{\mathcal{V}*}(j)}{P_{t+s}^{\mathcal{V}}} \right)^{-\sigma^{\mathcal{V}}} Y_{t+s} \right\}$$

where  $P_{t+s}\lambda_{t+s}^{\mathcal{Y}}$  reflects the (nominal) marginal value of producing an additional unit of the final that results from the duality theorem. The final goods price is

$$P_t^{\mathcal{Y}} = \left(\iota^{\mathcal{Y}}(P_{t-1}^{\mathcal{Y}})^{1-\sigma^{\mathcal{Y}}} + (1-\iota^{\mathcal{Y}})(P_t^{\mathcal{Y}*})^{1-\sigma^{\mathcal{Y}}}\right)^{\frac{1}{1-\sigma^{\mathcal{Y}}}}$$

where

$$P^{\mathcal{Y}*} = \frac{\sigma^{\mathcal{Y}}}{\sigma^{\mathcal{Y}} - 1} \lambda_t^{\mathcal{Y}} \frac{CV_t^{\mathcal{Y},n}}{CV_t^{\mathcal{Y},d}}$$

The element  $\lambda_t^{\mathcal{Y}}$  represents the real marginal cost, whereas the Calvo elements are

$$CV_t^{\mathcal{Y},n} = \Lambda_{t,t+1}^R \iota^{\mathcal{Y}} (\pi_{t+1}^{\mathcal{Y}})^{\sigma^{\mathcal{Y}}} \frac{\lambda_{t+1}^{\mathcal{Y}}}{\lambda_t^{\mathcal{Y}}} \frac{Y_{t+1}}{Y_t} CV_{t+1}^{\mathcal{Y},n}$$
$$CV_t^{\mathcal{Y},d} = 1 + \Lambda_{t,t+1}^R \iota^{\mathcal{Y}} \frac{(\pi_{t+1}^{\mathcal{Y}})^{\sigma^{\mathcal{Y}}}}{\pi_{t+1}} \frac{Y_{t+1}}{Y_t} CV_{t+1}^{\mathcal{Y},d}$$

where  $\pi_{t+1}^{\mathcal{Y}} = P_{t+1}^{\mathcal{Y}}/P_t^{\mathcal{Y}}$  denotes final goods inflation. Retailers are perfectly competitive in input and output markets, charging to final costumers the same price paid to distributors, *i.e.*  $P_t = P_t^{\mathcal{Y}}$ . The price  $P_t$  is the numéraire of the economy, and therefore the relative price of final goods is one, *i.e.*  $P_t^{\mathcal{Y}}/P_t = 1$ .

#### 2.4. Fiscal authorities

The government keeps the budget balanced at all times, financing nominal public consumption  $P_tG_t$  with lump-sum taxes  $LT_t$  levied on households. The government budget constraint is simply  $P_tG_t = LT_t$ .

#### 2.5. Rest of the world

Distributors' in the home country import differentiated goods  $M_t$  at the (exogenous) price  $P_t^*$ , to be used in the production process. In a monetary union the real exchange rate is  $\varepsilon_t = P_t^*/P_t$ , implying  $\varepsilon_t/\varepsilon_{t-1} = \pi_t^*/\pi_t$ , where  $\pi_t^* = P_t^*/P_{t-1}^*$  is the imported goods inflation rate.

For tractability, trade and financial flows are restricted to euro area countries. We follow Adolfson *et al.* (2007) and assume that in the rest of the world there exists a continuum of distributors  $m \in [0, 1]$ , who demand  $Y_t^{\mathcal{X}}(m)$ units of the final good from domestic retailers. This good is thereafter combined with foreign intermediate goods  $Z_t^*(m)$  according to the following production function

$$Y_t^*(m) = \left( \left( \alpha_Y^* \right)^{\frac{1}{\xi^*}} \left( Y_t^{\mathcal{X}}(m) \right)^{\frac{\xi^* - 1}{\xi^*}} + \left( 1 - \alpha_Y^* \right)^{\frac{1}{\xi^*}} \left( Z_t^*(m) \right)^{\frac{\xi^* - 1}{\xi^*}} \right)^{\frac{\xi^*}{\xi^* - 1}}$$
(10)

where  $\xi^*$  is the elasticity of substitution between intermediate goods and domestic exports, and  $\alpha_Y^*$  is the home bias parameter. Each foreign distributor selects the quantities  $\{Y_t^{\mathcal{X}}(m), Z_t^*(m)\}_{s=0}^{\infty}$  to maximize the present discounted value of the dividends stream, subject to the production function in (10). The solution yields the demand for export goods  $Y_t^{\mathcal{X}} = \alpha_Y^*(\varepsilon_t)^{\xi^*} Y_t^*$ , where  $\varepsilon_t$  is the real exchange rate.

Banks are allowed to borrow from abroad whenever internal funds do not suffice to meet credit requirements, paying a country-specific risk premium  $\Psi_t$ on the net foreign asset position

$$\Psi_t = 1 - \varphi_{BF} \left[ \frac{B_t^*}{4 \cdot P_t \cdot GDP_t} - \left( B_{GDP}^* \right)^{target} \right]$$
(11)

over the foreign interest rate  $i_t^*$ , with  $\varphi_{BF}$  representing a scale parameter and  $(B_{GDP}^*)^{target}$  the target foreign assets-to-GDP ratio.<sup>9</sup>

#### 3. The financial sector: entrepreneurs and banks

The financial transmission mechanism is modeled along the lines in Bernanke  $et \ al.$  (1999), Christiano  $et \ al.$  (2010), and Kumhof  $et \ al.$  (2010). Financial frictions affect the return on capital and therefore capital demand. Before each production cycle, capital goods producers buy the undepreciated productive capital stock from entrepreneurs, combining it with investment goods bought

<sup>9.</sup> GDP in equation (11) is adjusted by a factor of 4, since the model is quarterly and the net foreign assets ratio is annualized.

from retailers to produce new installed productive capital. This capital is then sold to entrepreneurs, which will own it during the next production cycle. Entrepreneurs do not have access to sufficient internal resources to finance desired capital purchases, but can borrow the difference from banks at a cost. They face an idiosyncratic shock that changes the value of the firm after decisions have been made. When hit by a severe shock, the value of assets collapses and the entrepreneur must declare bankruptcy, handing over the value of the firm to the bank.

The banking system builds on Benes and Kumhof (2015), and is composed of retail branches and wholesale banks. Retail branches operate in a perfectly competitive environment, celebrating loan contracts with entrepreneurs. These contracts set an unconditional, non-state contingent lending rate. Since entrepreneurs are risky, so are the individual loans of retail banks, who therefore charge a spread over the wholesale lending rate—the cost of obtaining funds from the wholesale bank or households—to cover for the losses incurred in the mass of entrepreneurs that declare bankruptcy. Since a given retail branch lends to many entrepreneurs, by the law of large numbers the aggregate loan portfolio is risk-free, and hence *ex-ante* profits are zero. Retail branches are however exposed to non-diversifiable aggregate risk given the non-state contingent lending rate, and thus *ex-post* profits—to be transferred to wholesale banks—may differ from zero.<sup>10</sup>

Wholesale banks finance their activities, i.e. loans to retail branches, through equity, deposits, and foreign funds. We assume that repossessed assets are illiquid and accumulated as defaulted loans on the balance sheet. Over time, an exogenous fraction of defaulted loans is automatically transformed from illiquid into liquid at no cost, but banks can increase the pace of this transformation by requesting a liquidation service—henceforth interpreted as impairment losses—from households.<sup>11</sup> Wholesale banks face an idiosyncratic shock affecting the return on their loan portfolio which, coupled with potential losses from retail branches, may trigger balance sheet effects and/or credit supply restrictions. They are subject to regulatory capital requirements and non-compliance results in adjustment costs and reputational losses. Banks therefore endogenously set capital buffers, which allow them to cushion adverse shocks that negatively affect the value of capital. For simplicity, we rule out bank failure.

Credit supply restrictions arise endogenously from a modified moral hazard/costly enforcement problem inspired in Gertler and Karadi (2011), Gertler *et al.* (2012), and Gertler and Karadi (2013). The banker has the option

<sup>10.</sup> We implicitly assume that retail branches transfer ex-post loan losses to wholesale banks, though some funds lent to entrepreneurs originate from households. This is a simplifying assumption with no important role.

<sup>11.</sup> These amounts can be interpreted as impairment losses since they are a cost to the bank and deduct to the existing defaulted loans stock.

to divert a fraction of funds, though this only becomes attractive when the bank's value collapses well below the steady-state level. Households recognize this fact and restrain the amount of deposits placed at the bank until the banker's incentives to divert funds are aligned with depositors' interests. In this way, wholesale banks become supply constrained with respect to the resources they can make available to the entrepreneurial sector.

In what follows there are a different number of interest rates to take into account. Wholesale banks remunerate deposits from households at the rate  $i_t^D$ , and lend to retail branches at  $i_t^W$ . The premium between wholesale rates and the deposits rate reflects both balance sheet risk and moral hazard/costly enforcement problems. Let  $\tilde{i}_t^W$  denote the corresponding shadow wholesale lending rate that wholesale banks would charge when moral hazard/costly enforcement problems are fully absent. Since wholesale banks face a positive probability of having capital falling short regulatory requirements, the shadow wholesale rate will be at a premium over the deposits rate, *i.e.*  $\tilde{i}_t^W - i_t^D >$ 0. We term this premium capital requirements spread. Moral hazard/costly enforcement problems in turn place a premium on the shadow wholesale rate, *i.e.*  $i_t^W - \tilde{i}_t^W \ge 0$ , since credit restrictions trigger a wedge between the maximum rate retail branches are willing to pay and the minimum rate wholesale banks demand to cope with balance sheet risk. We term this premium credit restrictions spread. Finally, the retail rate  $i_t^R$  charged by branches is at another premium over the wholesale rate, *i.e.*  $i_t^R - i_t^W > 0$ . This is to compensate for the fact that some entrepreneurs will declare insolvency and be unable to meet their debt obligations. We term this difference external finance premium. In short, under non-binding credit restrictions we have  $i_t^D < \tilde{i}_t^W = i_t^W < i_t^R$ , whereas under binding credit restrictions  $i_t^D < \tilde{i}_t^W < i_t^R$ .

#### 3.1. Retail branches and the entrepreneurial sector

There is a continuum of infinitely lived entrepreneurial firms  $l \in [0, 1]$ . At the end of each period, entrepreneurs buy the new capital stock from capital goods producers and rent it, partially or entirely, to manufacturers, for usage in the production process.

The entrepreneurial firm l selects the capital utilization rate,  $u_t(l)$  in each period to maximize the net return per unit of capital,  $[R_t^{\mathcal{K}}u_t(l) - P_ta(u_t(l))]$ , where  $R_t^{\mathcal{K}}$  is the nominal rental rate of capital charged to intermediate goods producers, taken as given. The cost of capital utilization  $a(u_t(l))$  takes the following functional form

$$a(u_t(l)) = \frac{1}{2}\varphi_a\sigma_a(u_t(l))^2 + \varphi_a(1-\sigma_a)u_t(l) + \varphi_a\left(\frac{\sigma_a}{2}-1\right)$$

where  $\varphi_a > 0$  is calibrated to ensure a unitary capital utilization in the steady state and  $\sigma_a > 0$  is a parameter that controls the curvature. Capital effectively

rented to manufacturers and used in production is  $K_t = u_t \bar{K}_t$ , and the resource cost associated with variable capital utilization is  $P_t^{\mathcal{Z}} rcu_t = P_t a(u_t) \bar{K}_t$ .

Entrepreneurs do not have access to sufficient internal funds,  $N_t(l)$ , to finance desired capital purchases, but can cover the funding gap by borrowing  $L_t(l)$  from retail branches at the gross nominal interest rate  $i_t^R$ . They face the following balance sheet constraint

$$P_t^{\mathcal{K}}\bar{K}_{t+1}(l) = L_t(l) + N_t(l)$$

After acquiring the capital stock from capital goods producers (but before selecting the utilization rate), entrepreneurs experience an idiosyncratic shock  $\omega_{t+1}^{\mathcal{K},l}$ ,

$$\ln \omega_{t+1}^{\mathcal{K},l} \sim \mathcal{N}\left(-\frac{1}{2}\left(\sigma_{t+1}^{\mathcal{K}}\right)^2, \left(\sigma_{t+1}^{\mathcal{K}}\right)^2\right)$$

distributed independently over time and across entrepreneurs, affecting the value of capital. Specifically, there exists an endogenous threshold level for the idiosyncratic shock,  $\bar{\omega}_{t+1}^{\mathcal{K},l}$ , separating two distinct outcomes: if  $\omega_{t+1}^{\mathcal{K},l} \geq \bar{\omega}_{t+1}^{\mathcal{K},l}$  the entrepreneur is able to pay off her debts and is therefore solvent, whereas if  $\omega_{t+1}^{\mathcal{K},l} < \bar{\omega}_{t+1}^{\mathcal{K},l}$  the entrepreneur cannot meet her debt obligations and is forced to declare bankruptcy.

Entrepreneurs celebrate a standard debt contract with retail branches, specifying a nominal loan amount  $L_t(l)$  and a non-state contingent gross nominal retail interest rate,  $i_t^R(l)$ , to be paid if  $\omega_{t+1}^{\mathcal{K},l} \geq \bar{\omega}_{t+1}^{\mathcal{K},l}$ . The value for the threshold  $\bar{\omega}_{t+1}^{\mathcal{K},l}$  satisfies the following condition

$$\bar{\omega}_{t+1}^{\mathcal{K},l} \operatorname{Ret}_{t}^{\mathcal{K}} P_{t}^{\mathcal{K}} \bar{K}_{t+1}(l) = i_{t}^{R}(l) L_{t}(l) \tag{12}$$

where  $Ret_t^{\mathcal{K}}$  is the entrepreneurs' *ex-ante* return on capital

$$Ret_{t}^{\mathcal{K}} = \mathbf{E}_{t} \frac{\left[R_{t+1}^{\mathcal{K}} u_{t+1} - P_{t+1} a(u_{t+1})\right] + (1-\delta) P_{t+1}^{\mathcal{K}}}{P_{t}^{\mathcal{K}}}$$

reflecting the expected income from the rental activity in addition to changes in the market value of capital. Equation (12) states that the threshold  $\bar{\omega}_{t+1}^{\mathcal{K},l}$  is such that the gross return on capital is barely enough for the entrepreneur to pay off her debt.

Retail branches must pay savers a unitary repossession cost  $\mu_{t+1}^{\mathcal{K}}$  over the firm value to repossess the capital value of defaulting firms. Let  $\mathfrak{F}_t^{\mathcal{K}}(x) = \Pr[\omega_{t+1}^{\mathcal{K},l} < x]$  denote the cumulative distribution function and  $\mathfrak{f}_t^{\mathcal{K}}(x)$  the

corresponding probability density function of  $\omega_{t+1}^{\mathcal{K},l}$ . Since retail branches are perfectly competitive, their participation constraint corresponds to zero-expected profits

$$\underbrace{[1 - \mathfrak{F}^{\mathcal{K}}(\bar{\omega}_{t+1}^{\mathcal{K},l})]}_{\text{probability}} \underbrace{i_{t}^{R}(l)L_{t}(l)}_{\text{in case of no default}} + \underbrace{(1 - \mu_{t+1}^{\mathcal{K}})}_{\text{rate}}_{\text{rate}} \\ \underbrace{\int_{0}^{\bar{\omega}_{t+1}^{\mathcal{K},l}} \omega_{t+1}^{\mathcal{E},l} \operatorname{Ret}_{t}^{\mathcal{K}} P_{t}^{\mathcal{K}} \bar{K}_{t+1}(l) \mathfrak{f}^{\mathcal{K}}(\omega_{t+1}^{\mathcal{K},l}) \mathrm{d}\omega_{t+1}^{\mathcal{K},l}}_{t+1}}_{\operatorname{Average value of capital}_{\text{in case of default } \omega_{t+1}^{\mathcal{K},l} < \bar{\omega}_{t+1}^{\mathcal{K},l}} = \underbrace{i_{t}^{W} L_{t}(l)}_{\operatorname{Cost of funds}}$$

$$(13)$$

Equation (13) states that the terms of the loan contract are such that expected revenues cover the cost of assessing funds,  $i_t^W$ . This rate—to be determined later—reflects the wholesale banks' financing costs, balance sheet risk, and moral hazard/costly enforcement problems. In equilibrium  $i_t^W$  matches the cost of assessing household funds; otherwise retail branches would opt for the cheapest financing source. The loan portfolio  $L_t$  is composed of wholesale funds  $L_t^{\mathcal{BK}}$  and of household funds  $L_t^{\mathcal{H}}$ . Since capital acquisitions are risky, so are the loans of retail branches, who therefore charge a spread over the wholesale rate  $i_t^W$  to cover for bankruptcy losses and repossession costs, *i.e.*  $i_t^R - i_t^W > 0$ . The existence of identical a priori expectations on the idiosyncratic shock implies that the external finance premium is identical for all entrepreneurs. Even though individual loans are risky, the aggregate portfolio of retail branches is risk free, since each branch is assumed to lend to many entrepreneurs, thus recovering through the credit spread what is lost to bankrupt entrepreneurs.

Entrepreneurs maximize the expected value of terminal wealth. In the current setup, this corresponds to maximize the expected value of assets over the non-default region in every period

$$\int_{\bar{\omega}_{t+1}^{\mathcal{K},l}}^{\infty} \left(\omega_{t+1}^{\mathcal{K},l} - \bar{\omega}_{t+1}^{\mathcal{K},l}\right) \operatorname{Ret}_{t}^{\mathcal{K}} P_{t}^{\mathcal{K}} \bar{K}_{t+1}(l) \mathfrak{f}^{\mathcal{K}}(\omega_{t+1}^{\mathcal{K},l}) \mathrm{d}\omega_{t+1}^{\mathcal{K},l}$$

subject to (13). Following Bernanke  $et \ al.$  (1999), this can be restated as the maximization of

$$\left[1 - \Gamma^{\mathcal{K}}\right] Ret_t^{\mathcal{K}} P_t^{\mathcal{K}} \bar{K}_{t+1}(l)$$

subject to

$$\left[\Gamma^{\mathcal{K}} - \mu_{t+1}^{\mathcal{K}} G^{\mathcal{K}}\right] \operatorname{Ret}_{t}^{\mathcal{K}} P_{t}^{\mathcal{K}} \bar{K}_{t+1}(l) = i_{t}^{W} \left[P_{t}^{\mathcal{K}} \bar{K}_{t+1}(l) - N_{t}(l)\right]$$

where

$$\Gamma_{t+1}^{\mathcal{K}} \equiv \int_{0}^{\bar{\omega}_{t+1}^{\mathcal{K},l}} \omega_{t+1}^{\mathcal{K},l} \mathfrak{f}^{\mathcal{K}}(\omega_{t+1}^{\mathcal{K},l}) \mathrm{d}\omega_{t+1}^{\mathcal{K},l} + \bar{\omega}_{t+1}^{\mathcal{K},l} \int_{\bar{\omega}_{t+1}^{\mathcal{K},l}}^{\infty} \mathfrak{f}^{\mathcal{K}}(\omega_{t+1}^{\mathcal{K},l}) \mathrm{d}\omega_{t+1}^{\mathcal{K},l}$$

and

$$G_{t+1}^{\mathcal{K}} \equiv \int_{0}^{\bar{\omega}_{t+1}^{\mathcal{K},l}} \omega_{t+1}^{\mathcal{K},l} \mathbf{f}^{\mathcal{K}}(\omega_{t+1}^{\mathcal{K},l}) \mathrm{d}\omega_{t+1}^{\mathcal{K},l}$$

The solution, identical for all l, yields

$$\left(1-\Gamma_{t+1}^{\mathcal{K}}\right)\frac{Ret_t^{\mathcal{K}}}{i_t^W} + \left(\frac{\left(\Gamma_{t+1}^{\mathcal{K}}\right)'}{\left(\Gamma_{t+1}^{\mathcal{K}}\right)' - \mu_{t+1}^{\mathcal{K}}\left(G_{t+1}^{\mathcal{K}}\right)'}\right)\left[\left(\Gamma_{t+1}^{\mathcal{K}} - \mu_{t+1}^{\mathcal{K}}G_{t+1}^{\mathcal{K}}\right)\frac{Ret_t^{\mathcal{K}}}{i_t^W} - 1\right] = 0$$

where  $(\Gamma_{t+1}^{\mathcal{K}})' = \partial \Gamma^{\mathcal{K}} / \partial \bar{\omega}_{t+1}^{\mathcal{K}}$  and  $(G_{t+1}^{\mathcal{K}})' = \partial G^{\mathcal{K}} / \partial \bar{\omega}_{t+1}^{\mathcal{K}}$ . This condition, jointly with the retail branches participation constraint

$$\left[\Gamma_{t+1}^{\mathcal{K}} - \mu_{t+1}^{\mathcal{K}} G_{t+1}^{\mathcal{K}}\right] \frac{Ret_t^{\mathcal{K}}}{i_t^{\mathcal{W}}} \frac{P_t^{\mathcal{K}} \bar{K}_{t+1}}{N_t} = \frac{P_t^{\mathcal{K}} \bar{K}_{t+1}}{N_t} - 1$$

defines the demand for loans from the entrepreneurial sector and the associated threshold  $\bar{\omega}_{t+1}^{\mathcal{K}}$  separating bankruptcy from solvency. As net worth is taken as given, capital purchases directly determine the balance sheet composition and therefore leverage and the threshold  $\bar{\omega}_{t+1}^{\mathcal{K}}$ . In turn, the degree of leverage determines the relative risk of the firm and thus the probability of default. If capital and leverage increase, so does the risks faced by financial intermediaries and therefore the cost of external financing.

Let  $P_t \Lambda_t^{\mathcal{K}}$  represent *ex-post* period t loan losses from retail branches on all contracts celebrated with all entrepreneurs in the previous period

$$P_t \Lambda_t^{\mathcal{K}} = i_{t-1}^{W} \left( P_{t-1}^{\mathcal{K}} \bar{K}_t - N_{t-1} \right) - \left( \Gamma_t^{\mathcal{K}} - \mu_t^{\mathcal{K}} G_t^{\mathcal{K}} \right) Ret_{t-1}^{\mathcal{K}} P_{t-1}^{\mathcal{K}} \bar{K}_t$$

This amount—to be transferred to wholesale banks—corresponds to a gain for entrepreneurs, resulting from unexpected events (*i.e.* unforeseen aggregate shocks) that, due to the non-state contingent nature of the interest rate, could not be taken into account in the loan contract. Obviously,  $\Lambda_t^{\mathcal{K}}$  can be negative, a case in which entrepreneurs' loan losses correspond to branches gains. The first element represents interest expenses, whereas the second element are realized revenues.

A fraction  $1 - \iota^{\mathcal{E}}$  of entrepreneurs goes out of business in every period, transferring the residual value of the firm to the household. Net worth  $N_t$  evolves over time according to

An integrated financial amplifier

$$N_t = \iota^{\mathcal{E}} \left[ i_{t-1}^W N_{t-1} + P_{t-1}^{\mathcal{K}} \bar{K}_t \left( Ret_{t-1}^{\mathcal{K}} (1 - \mu_t^{\mathcal{K}} G_t^{\mathcal{K}}) - i_{t-1}^{W} \right) + P_t \Lambda_t^{\mathcal{K}} \right] + WT^{\mathcal{E}}$$

where  $WT^{\mathcal{E}}$  are initial wealth transfers from households to new businessmen.<sup>12</sup> In contrast with Bernanke *et al.* (1999), we assume that repossession costs

$$REP_t = \mu_t^{\mathcal{K}} Ret_{t-1}^{\mathcal{K}} P_{t-1}^{\mathcal{K}} \bar{K}_t G_t^{\mathcal{K}}$$

are not immediately transferred to households but rather accumulated into a vehicle according to

$$REP_t^{acc} = REP_{t-1}^{acc} + REP_t - RBR_t$$

and the accumulated values are distributed to households following the law of motion

$$RBR_t = REP^{\rm ss} + \theta_{rbr}(REP_t^{acc} - REP^{acc,ss})$$

where ss represents steady-state levels and  $\theta_{rbr}$  is a sensibility parameter to steady-state deviations. This setup captures the idea that repossessing and liquidating assets from bankrupted companies takes time, and avoids large swigs in households income driven by changes in the value of repossessed assets.

#### 3.2. Wholesale banks

There is a continuum of infinitely lived wholesale banks  $k \in [0, 1]$ . Each bank k issues deposits  $DEP_t(k)$  to households, combining them with capital or equity  $E_t(k)$  and foreign funds  $B_t^*(k)$ , to lend  $L_t^{\mathcal{BK}}(k)$  to entrepreneurs.<sup>13</sup> Lending is processed through retail branches. Current period defaulted loans correspond to the potential value of repossessed assets net of repossession costs

$$DL_t^{\text{new}}(k) = (1 - \mu_t^{\mathcal{K}}) Ret_{t-1}^{\mathcal{K}} P_{t-1}^{\mathcal{K}} \bar{K}_t G_t^{\mathcal{K}}$$

<sup>12.</sup> Since the solution does not depend on  $\iota^{\mathcal{E}}$ , we only require that the pair  $(\iota_t^{\mathcal{E}}, WT^{\mathcal{E}})$  does not allow net worth to grow indefinitely over time, a situation in which entrepreneurs would no longer need external funding.

<sup>13.</sup> We use the terms banks' capital and equity interchangeably throughout the article.

This amount is assumed to be illiquid and thus not immediately available. In each period, banks can transform an exogenous fraction  $\tau^{\mathcal{BK}}$  of the defaulted loans stock from illiquid to liquid at no cost, but they can increase the pace of this transformation by a fraction  $v_t(k)$  at a cost by requesting a liquidation service from households. Note that a decline in  $\tau^{\mathcal{BK}}$  corresponds to higher defaulted loans-driven financial frictions. For simplicity, defaulted loans that become liquid in the period do not need to be financed with deposits. Letting  $\tilde{DL}_t(k) = [DL_{t-1}(k) + DL_t^{\text{new}}(k)]$  denote start-of-period defaulted loans, the law of motion representing the stock that is carried-out to the next period reads<sup>14</sup>

$$DL_t(k) = (1 - \upsilon_t(k))(1 - \tau^{\mathcal{B}\mathcal{K}})\tilde{DL}_t(k)$$
(14)

whereas liquidation services requested from households, aka impairment losses, to be paid at t + 1, are

$$DL_{t+1}^{\rm imp}(k) = v_t(k)(1 - \tau^{\mathcal{BK}})\tilde{DL}_t(k)$$

Defaulted loans earn to interest, and banks face a positive probability  $\varphi_{dl}$  of facing an inspection by the regulator, case in which they must pay a penalty/reputation cost of  $\log(1/v_t(k))DL_t(k)$  at t + 1. Expected defaulted loans costs are<sup>15</sup>

$$\mathbf{E}_t \Gamma_{t+1}^{DL}(k) = \varphi_{dl} [\log(1/\upsilon_t(k))] DL_t(k)$$

.....

Notice that the recovered amount  $\tau^{\mathcal{BK}} \tilde{DL}_t(k)$  is implicitly taken into account: it is subtracted to bank's assets, who will no longer require deposits or foreign funds to finance that amount. Equity is positively affected due to the impact on the opportunity and reputation/holding costs. Impairment losses are processed alike, except that they also generate a payment to households that is deducted to bank's equity: a cost paid today to avoid larger expected costs in the future.

To wrap up, repossessed assets do not become liquid immediately; they instead accumulate in banks' balance sheet as defaulted loans. These have an

<sup>14.</sup> Our theory of defaulted loans determination implies that they can be written as a generalized AR(1) process,  $DL_t = C_t^{dl} + \rho_t^{dl} DL_{t-1} + \varepsilon_t^{dl}$ , with a time-varying constant  $C_t^{dl} = E_t(1 - v_t)(1 - \tau^{\mathcal{B}\mathcal{K}})DL_t^{\text{new}}$  and autoregressive coefficient  $\rho_t^{dl} = (1 - v_t)(1 - \tau^{\mathcal{B}\mathcal{K}})$ , and where innovations  $\varepsilon_t^{dl}$  are interpreted as surprises to the value of repossessed assets.

<sup>15.</sup> We adopt this specification for convenience since it allows a relatively straightforward calibration while yielding an interior solution,  $v_t(k) \in (0, 1)$ . Note that a higher inspection probability or defaulted loans stock, or a lower transformation rate  $v_t(k)$ , imply a larger cost—to be deducted to the market clearing condition.

opportunity cost (they must be financed with deposits or foreign funds) and an holding cost (the expected penalty) associated. An optimal strategy may thus consist in writing some amount off from the balance sheet at a cost in order to avoid some of these costs, implying  $v_t(k) > 0$ . A more reality-oriented interpretation postulates that a fraction  $v_t(k)$  of defaulted loans will never be received by banks, thus being written off from the balance sheet and deducted to the income statement.<sup>16</sup>

Bank k's balance sheet is

$$L_t^{\mathcal{BK}}(k) + DL_t(k) = DEP_t(k) + B_t^*(k) + E_t(k)$$
(15)

Each bank is exposed to an idiosyncratic shock  $\omega_{t+1}^{\mathcal{BK},k}$ , that changes the return on total loans from

$$i_t^W L_t^{\mathcal{BK}}(k) + DL_t(k)$$

 $\mathrm{to}$ 

$$\omega_{t+1}^{\mathcal{BK},k} \left[ i_t^W L_t^{\mathcal{BK}}(k) + DL_t(k) \right]$$

creating a risky environment. This shock may reflect differing loan recovery rates and differing success at raising non-interest income and minimizing non-interest expenses, and is therefore interpreted as a loan return shock (Benes and Kumhof 2015). The random variable  $\omega_{t+1}^{\mathcal{BK},k}$  follows a log-normal distribution with a mean of unity

$$\ln \omega_{t+1}^{\mathcal{BK},k} \sim \mathcal{N}\left(-\frac{1}{2} \left(\sigma_t^{\mathcal{BK}}\right)^2, \left(\sigma_t^{\mathcal{BK}}\right)^2\right)$$

distributed independently over time and across banks. Let  $\mathfrak{F}^{\mathcal{BK}}(x) = \Pr[\omega_{t+1}^{\mathcal{BK},k} < x]$  denote the cumulative distribution function and  $\mathfrak{f}^{\mathcal{BK}}(x)$  the corresponding probability density function of  $\omega_{t+1}^{\mathcal{BK},k}$ . If a given bank do not comply with regulatory requirements  $\bar{\gamma}_{t}^{\mathcal{BK}}$ , it faces a value/reputation loss  $\bar{\chi}_{t+1}^{\mathcal{BK}}$  for each unit of total assets. Let  $\bar{\omega}_{t+1}^{\mathcal{BK},k}$  denote the threshold loan return shock below which bank k is unable to comply with regulatory requirements

$$\bar{\omega}_{t+1}^{\mathcal{BK},k} \left( i_t^{\mathcal{W}} L_t^{\mathcal{BK}}(k) + DL_t(k) \right) - i_t^D DEP_t(k) - i_t^* \Psi_t B_t^*(k) - DL_{t+1}^{imp}(k) - \Gamma_{t+1}^{DL}(k) - \Xi_{t+1}/\iota^{\mathcal{BK}} \\ = \bar{\gamma}_t^{\mathcal{BK}} \bar{\omega}_{t+1}^{\mathcal{BK},k} \left( i_t^{\mathcal{W}} L_t^{\mathcal{BK}}(k) + DL_t(k) \right)$$
(16)

<sup>16.</sup> We present the defaulted loans satellite of the model according to the former interpretation because it is easier to model impairment losses as a payment to households, a necessary condition to close the model.

The left-hand side is the bank's expected equity at the beginning of period t+1, given a loan return shock of  $\bar{\omega}_{t+1}^{\mathcal{BK},k}$ . It equals the gross return on loans net of gross deposit interest payments, foreign reimbursements, and defaulted loans impairment losses and reputation costs. The element  $\Xi_{t+1}^{\mathcal{BK}}$  is a common equity shock, used in the next section to simulate the dynamics associated with an exogenous drop in banks' capital. The right-hand side is the minimum capital requirement and the element  $\bar{\gamma}_t^{\mathcal{BK}}$  represents the capital ratio requirement. Notice that expected loan gains/losses from retail banks are zero,  $\mathbf{E}_t [P_{t+1} \Lambda_{t+1}^{\mathcal{K}}(k)] = 0.$ 

The expected bank value at t + 1 given that the banker remains in the job is

$$\mathbf{E}_{t} \left[ EV_{t+1}(k) \right] = \mathbf{E}_{t} \left[ i_{t}^{\mathcal{W}} L_{t}^{\mathcal{B}\mathcal{K}}(k) + DL_{t}(k) - i_{t}^{D} DEP_{t}(k) - i_{t}^{*} \Psi_{t} B_{t}^{*}(k) - DL_{t+1}^{imp}(k) - \Gamma_{t+1}^{DL}(k) - \bar{\chi}_{t+1}^{\mathcal{B}\mathcal{K}} \left[ L_{t}^{\mathcal{B}\mathcal{K}}(k) + DL_{t}(k) \right] \mathfrak{F}^{\mathcal{B}\mathcal{K}} (\bar{\omega}_{t+1}^{\mathcal{B}\mathcal{K},k}) - \Xi_{t+1}/\iota^{\mathcal{B}\mathcal{K}} \right]$$

$$(17)$$

Equation (17) is composed of the expected gross return on loans, net of gross deposit interest payments, foreign reimbursements, defaulted loans impairment and reputation costs, expected costs for non-compliance with regulatory requirements, and the equity shock. We consider that the banker will always retain earnings until exiting the industry.<sup>17</sup> The banker's objective is therefore to maximize expected terminal wealth

$$V_t(k) = \mathcal{E}_t \sum_{s=1}^{\infty} (1 - \iota^{\mathcal{B}\mathcal{K}}) (\iota^{\mathcal{B}\mathcal{K}})^{s-1} \Lambda_{t,t+s}^N E V_{t+s}(k)$$
(18)

Iterating forward, Equation (18) can be restated as

$$V_t(k) = \mathcal{E}_t(1 - \iota^{\mathcal{B}\mathcal{K}})\Lambda_{t,t+1}^N E V_{t+1}(k) + \iota^{\mathcal{B}\mathcal{K}}\Lambda_{t,t+1}^N V_{t+1}(k)$$
(19)

The moral hazard/costly enforcement problem has the following structure. At the beginning of each period the banker has the option to divert a fraction  $\theta$  of assets. The decision to divert assets at t + 1 is made at the end of period t, before both individual and aggregate uncertainty at t + 1 are revealed. The intuition is that it takes some time to reallocate assets. If the banker decides to divert funds, the bank defaults on deposits and is shut down. Depositors will be willing to supply funds to bank k if and only if the following incentive compatibility constraint is satisfied

<sup>17.</sup> This is in fact the weakly dominating strategy: under frictionless capital markets the timing of payouts is irrelevant whereas with frictions the banker will prefer to retain earnings to expand the asset base.

$$V_t(k) \ge \theta \left[ L_t^{\mathcal{BK}}(k) + \Delta DL_t(k) \right]$$
(20)

where  $\theta \Delta$  is the fraction of defaulted loans that can be diverted, with  $0 \leq \Delta \leq 1$ . A larger defaulted loans stock implies a greater incentive to divert funds; however defaulted loans are illiquid and hence more difficult to divert than corporate loans, which may justify a value below one for  $\Delta$ . The left-hand side is the bank's value for the banker whereas the right-hand side is the gain from diverting assets. We assume for the time being that the incentive compatibility constraint is always binding. The banker will select the vector

$$\{L_{t+s}^{\mathcal{BK}}(k), DEP_{t+s}(k), B_{t+s}^{*}(k), DL_{t+s}(k)\}$$

in each period to maximize expected terminal wealth in (19), subject to (14), (15), (16), (17), and (20). Let  $\lambda_{t+s}^{BK}$  denote the Lagrange multiplier associated with the incentive compatibility constraint. We impose symmetry and drop the indexer k, and break down the overall spread  $i_t^W - i_t^D$  into the contribution of two elements, risky bank lending,  $\tilde{i}_t^W - i_t^D$ , and moral hazard,  $i_t^W - \tilde{i}_t^W$ . The spread triggered by risky bank lending, *i.e.* that prevail in the absence of moral hazard issues thus implying  $\lambda_t^{BK} = 0$ , is

$$\tilde{i}_{t}^{W} - i_{t}^{D} = \mathbb{E}_{t} \bar{\chi}_{t+1}^{\mathcal{B}\mathcal{K}} \left[ \mathfrak{F}^{\mathcal{B}\mathcal{K}}(\bar{\omega}_{t+1}^{\mathcal{B}\mathcal{K}}) + \mathfrak{f}^{\mathcal{B}\mathcal{K}}(\bar{\omega}_{t+1}^{\mathcal{B}\mathcal{K}}) \zeta_{t}^{\mathcal{B}\mathcal{K}} \left[ i_{t}^{D} - \bar{\omega}_{t+1}^{\mathcal{B}\mathcal{K}} (1 - \bar{\gamma}_{t}) i_{t}^{W} \right] \right]$$
(21)

where  $\zeta_t^{BK}$  is an auxiliary variable

$$\zeta_t^{BK} = \frac{L_t^{\mathcal{BK}} + DL_t}{(1 - \bar{\gamma}_t^{\mathcal{BK}}) \left[ i_t^W L_t^{\mathcal{BK}} + DL_t \right]}$$

This condition states that the return on loans will be at a premium over the cost of funds, given by the interest rate on deposits, to cover for the expected costs triggered by the possible non-compliance of regulatory requirements in case of an adverse shock. The spread is a function of the probability of such event and of the change in that probability triggered by an expansion in the asset base and hence leverage. The first-order condition with respect to loans can then be written as

$$E_t \tilde{\Lambda}_{t,t+1}^{\mathcal{BK}} (i_t^W - \tilde{i}_t^W) = \frac{\lambda_t^{BK}}{1 + \lambda_t^{BK}} \theta$$
(22)

where

$$\tilde{\Lambda}_{t,t+1}^{\mathcal{B}\mathcal{K}} = \Lambda_{t,t+1}^{N} \left[ 1 - \iota^{\mathcal{B}\mathcal{K}} + \iota^{\mathcal{B}\mathcal{K}} \frac{\partial V_{t+1}}{\partial \mathbf{E}_t E V_{t+1}} \right]$$

is the change in the augmented stochastic discount factor as the expected bank value changes. This term augments the households discount factor  $\Lambda_{t,t+1}^N$  by the marginal value of net worth averaged across exiting and continuing states. With probability  $1 - \iota^{\mathcal{BK}}$  the banker exits and transfers retained earnings back to the household, implying a marginal value of net worth of unity. With probability  $\iota^{\mathcal{BK}}$  she continues as a banker, and uses the additional net worth to expand the asset base and thus increase terminal wealth. The first-order condition above states that the return on loans will be at a premium over the shadow rate whenever moral hazard issues are present, and this premium depends on the tightness of the incentive constraint and the fraction of funds that can be diverted.

The first-order condition on foreign bonds collapses to  $i_t^* \Psi_t - i_t^D = 0$ , whereas the first-order condition with respect to defaulted loans yields

$$\mathbf{E}_{t}\tilde{\Lambda}_{t,t+1}^{\mathcal{BK}}\left[-(1-i_{t}^{D})+\frac{\partial\Gamma_{t+1}^{DL}}{\partial DL_{t}}+\frac{\partial CpR_{t}}{\partial DL_{t}}\right] = \mathbf{E}_{t}\tilde{\Lambda}_{t,t+1}^{\mathcal{BK}}-\frac{\lambda_{t}^{BK}}{1+\lambda_{t}^{BK}}\theta$$
(23)

where

$$\begin{split} CpR_t = &\bar{\chi}_{t+1}^{\mathcal{BK}} \left[ L_t^{\mathcal{BK}} + DL_t \right] \mathfrak{F}^{\mathcal{BK}}(\bar{\omega}_{t+1}^{\mathcal{BK}}) \\ &\frac{\partial CpR_t}{\partial DL_t} = \bar{\chi}_{t+1}^{\mathcal{BK}} \left[ \mathfrak{F}^{\mathcal{BK}}(\bar{\omega}_{t+1}^{\mathcal{BK}}) + \mathfrak{f}^{\mathcal{BK}}(\bar{\omega}_{t+1}^{\mathcal{BK}}) \iota_t^{\mathcal{BK}} \left[ i_t^D - 1 + \frac{\partial \Gamma_{t+1}^{DL}}{\partial DL} - \bar{\omega}_{t+1}^{\mathcal{BK}}(1 - \bar{\gamma}_t) \right] \right] \\ &\frac{\partial \Gamma_{t+1}^{DL_t}}{\partial DL} = \varphi_{dl} \left( \frac{1 - \upsilon_t}{\upsilon_t} - \log \upsilon_t \right) \end{split}$$

The right-hand side in Equation (23) is the liquidation service cost net of the incentives to divert funds of transforming one additional unit of defaulted loans from illiquid to liquid, *i.e.* the cost of recognizing that unit as impairment loss. The left-hand side is the expected cost of carrying-over that defaulted loans unit to the next period, and is composed of the opportunity cost and the defaulted loans and capital requirements penalty/reputation costs. Note that larger impairment losses push down the gain from diverting assets, and thus the incentive compatibility condition becomes "less binding."

We can write terminal wealth as

$$V_t = \mathcal{E}_t \Lambda_{t,t+1}^N \left[ 1 - \iota^{\mathcal{B}\mathcal{K}} + \iota^{\mathcal{B}\mathcal{K}} \theta \varphi_{t+1}^{\mathcal{B}\mathcal{K}} \right] EV_{t+1} = \mathcal{E}_t \Lambda_{t,t+1}^{\mathcal{B}\mathcal{K}} EV_{t+1}$$

where

$$\Lambda_{t,t+1}^{\mathcal{BK}} = \Lambda_{t,t+1}^{N} \left[ 1 - \iota^{\mathcal{BK}} + \iota^{\mathcal{BK}} \theta \varphi_{t+1}^{\mathcal{BK}} \right]$$

is the augmented stochastic discount factor and  $\varphi_{t+1}^{\mathcal{BK}} = V_{t+1}/\theta E V_{t+1}$ is independent of bank specific factors along the equilibrium path. The interpretation of  $\Lambda_{t,t+1}^{\mathcal{BK}}$  is similar to that of  $\tilde{\Lambda}_{t,t+1}^{\mathcal{BK}}$ , except that it refers to the average value of net worth instead of the marginal value of net worth. It follows that

$$\frac{\partial V_t}{\partial E V_t} = \mathcal{E}_t \Lambda_{t,t+1}^{\mathcal{B}\mathcal{K}} \left[ (i_t^W - \tilde{i}_t^W) \varphi_t^{\mathcal{B}\mathcal{K}} + i_t^D \left[ 1 + \bar{\chi}_{t+1}^{\mathcal{B}\mathcal{K}} f^{\mathcal{B}\mathcal{K}} (\bar{\omega}_{t+1}^{\mathcal{B}\mathcal{K}}) \iota_t^{\mathcal{B}\mathcal{K}} \right] \right]$$

which is similar to that in Gertler and Karadi (2013) except that it adds a capital requirements factor and the interest rate premium is computed against the shadow interest rate that arises from risky bank lending. With binding credit restrictions, as the bank expands the asset base, it adds to its terminal wealth the premium  $i_t^W - \tilde{i}_t^W$ , corrected for the capital requirements factor that takes into account how the change in risk interacts with this premium. This is discounted by the usual factor augmented by the value of net worth averaged across exiting and continuing states.

To express the incentive compatibility constraint as occasionally binding, we re-write it as

$$\mathbf{E}_t \Lambda_{t,t+1}^{\mathcal{BK}} (i_t^W - \tilde{i}_t^W) L_t^{\mathcal{BK}} \ge \theta \left[ L_t^{\mathcal{BK}} + \Delta D L_t \right] - V_t^{AUX}$$
(24)

where

$$V_t^{AUX} = \mathbb{E}_t \Lambda_{t,t+1}^{\mathcal{BK}} \left[ (\tilde{i}_t^W - i_t^D) L_t^{\mathcal{BK}} + (1 - i_t^D) DL_t + i_t^D E_t - (i_t^* \Psi_t - i_t^D) B_t^* \right]$$
$$-\Gamma_{t+1}^{DL} - DL_{t+1}^{imp} - \mathbb{E}_t \bar{\chi}_{t+1}^{\mathcal{BK}} \left[ L_t^{\mathcal{BK}} + DL_t \right] \mathfrak{F}^{\mathcal{BK}} (\bar{\omega}_{t+1}^{\mathcal{BK}}) - \Xi_{t+1} / \iota^{\mathcal{BK}} \right]$$

Notice that the first-order (Kuhn-Tucker) conditions imply  $i_t^W - \tilde{i}_t^W \ge 0$  and  $(i_t^W - \tilde{i}_t^W)\lambda_t^{BK} = 0$ . Whenever  $\lambda_t^{BK} = 0$ , the incentive compatibility constraint does not bind and we have  $i_t^W = \tilde{i}_t^W$ , *i.e.* there are no credit restrictions whatsoever. Whenever  $\lambda_t^{BK} > 0$ , the wholesale rate is at a premium over the shadow counterpart, and the incentive compatibility constraint binds. Inequation (24) can therefore be re-stated as<sup>18</sup>

<sup>18.</sup> In practice, the non-linear feature of the model may impose convergence issues for sufficiently sizable shocks. In such cases, one can postulate that bankers

$$\mathbf{E}_{t}\Lambda_{t,t+1}^{\mathcal{B}\mathcal{K}}(i_{t}^{W}-\tilde{i}_{t}^{W}) = \max\left\{0,\theta\left[1+\Delta\frac{DL_{t}}{L_{t}^{\mathcal{B}\mathcal{K}}}\right]-\frac{V_{t}^{AUX}}{L_{t}^{\mathcal{B}\mathcal{K}}}\right\}$$

The spread  $i_t^W - \tilde{i}_t^W$  is zero when the constraint does not bind, but is positive whenever the incentive compatibility constraint binds. Intuitively, households restrict the amount they deposit at the bank up to the point where the banker's incentives to divert funds are fully canceled out. This creates a wedge between the rate wholesale banks are willing to supply funds and the rate that creditors are willing to pay for funds. The occasionally binding nature of credit restrictions is able to generate powerful asymmetric responses to financial or banking shocks—those whose nature is endowed with important effects on the banking system. Under "good shocks" credit restrictions remain slack and play no role whatsoever, whereas under "bad shocks" they may become binding for some period of time and greatly affect the model dynamics, amplifying and increasing business cycle persistence.

A fraction  $1 - \iota^{\mathcal{BK}}$  of bankers goes out of business in every period, transferring the residual value to the household. Aggregate equity therefore evolves according to

$$E_{t} = \iota^{\mathcal{B}\mathcal{K}} \left[ (\tilde{i}_{t-1}^{W} - i_{t-1}^{D}) L_{t-1}^{\mathcal{B}\mathcal{K}} + (1 - i_{t}^{D}) DL_{t-1} + i_{t-1}^{D} E_{t-1} - \Gamma_{t}^{DL} - DL_{t}^{imp} - (i_{t-1}^{*} \Psi_{t-1} - i_{t-1}^{D}) B_{t-1}^{*} - \bar{\chi}_{t}^{\mathcal{B}\mathcal{K}} \left[ L_{t-1}^{\mathcal{B}\mathcal{K}} + DL_{t-1} \right] \mathfrak{F}^{\mathcal{B}\mathcal{K}} (\bar{\omega}_{t}^{\mathcal{B}\mathcal{K}}) \right] - \Xi_{t} + W T_{t}^{\mathcal{B}\mathcal{K}}$$

where  $WT_t^{\mathcal{BK}}$  are startup funds provided by the household to new bankers. Reputation losses are

$$P_t^{\mathcal{Z}} pen_t = \bar{\chi}_t^{\mathcal{BK}} \left[ L_{t-1}^{\mathcal{BK}} + DL_{t-1} \right] \mathfrak{F}^{\mathcal{BK}}(\bar{\omega}_t^{\mathcal{BK}})$$

Figure 1 plots the loans market partial equilibrium under binding and nonbinding credit constraints. When the constraint does not bind, the supply of funds is mostly driven by the expected costs associated with a possible violation of capital requirements (the CAR curve). *Ceteris paribus*, an increase in loans implies higher banks' leverage and hence a large wholesale rate premium to cope with expected penalties. We say that in such equilibrium credit is mostly demand-driven or price-determined. When the constraint binds,

are more effectively monitored and thus face lower diversion gains as the spread (hence incentives to divert) increases. In particular, we one can assume  $\theta = \overline{\theta} \left(1 - \theta_a \left(\exp(1 + (i_t^W - \tilde{i}_t^W)/\text{E}_t \pi_{t+1}) - \exp(1)\right)\right)$ , where  $\overline{\theta}$  and  $\theta_a$  are parameters. This functional form smooths the spread dynamics and allows non-linear convergence without affecting the nature of the mechanism.

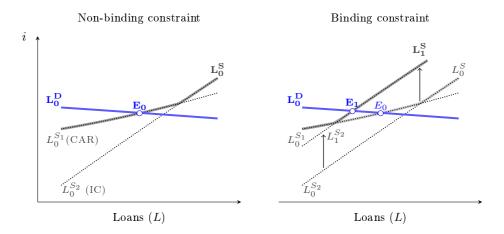


FIGURE 1: Loans market equilibrium under occasionally binding credit restrictions.

Notes: Equilibrium  $E_0$  clears the loan market by accounting for both entrepreneurs' demand  $L_0^D$  (downward sloping blue line) and banks' supply  $L_0^S$  (upward sloping gray line), at period 0, when the constraint is not binding (e.g. in the steady state). Observe that supply is determined by two components: the first evaluates the conditions under which the regulatory requirements (CAR) are taken into account  $(L_0^{S_1}$ , see equation (21)). The second evaluates the incentive compatibility (IC) constraint  $(L_0^{S_2}$ , see equation (22)). Equilibrium  $E_0$  is such that  $L_0^D = L_0^S = L_0^{S_1}$ . The right panel depicts a partial equilibrium movement assuming a parallel shift in the incentive compatibility constraint (from  $L_0^{S_2}$  to  $L_1^{S_2}$ ), which shifts upward or rotates the supply curve  $L_1^S$  over a certain loans interval. The new equilibrium  $E_1$  is such that  $L_0^D = L_1^S = L_1^{S_2}$ . The relevant component of the supply curve changed from  $L_0^{S_1}$  to  $L_1^{S_2}$ , implying higher interest rates and lower lending in comparison with  $E_0$ .

the supply of funds becomes driven by moral hazard issues. To elude the incentives to divert funds, depositors restrict the amounts placed at the bank, and the spread increase arises as a response to a quantity effect. We say that such equilibrium is mostly supply-driven or quantity-determined, due to the comparative importance played by the IC constraint for the equilibrium determination  $vis-\dot{a}-vis$  the CAR curve.

Figure 2 illustrates another key feature of the model in partial equilibrium, the interaction between defaulted loans and credit restrictions. An exogenous increase in the defaulted loans stock impacts reputation costs negatively, and concomitantly the bank's value. The higher leverage position pressures the probability of non-compliance with capital requirements upwards, shifting the CAR curve. In addition, credit restrictions become "more binding" and may play a role in the final equilibrium, from an initial standpoint where they were redundant. An important corollary is that defaulted loans leverage the effects of credit restrictions: any shock to the model which leads to an endogenous increase in defaulted loans will amplify the effects of credit restrictions.

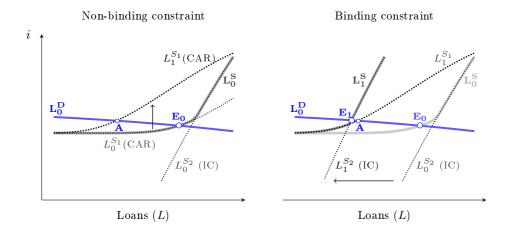


FIGURE 2: The effect of an increase in defaulted loans under occasionally binding credit restrictions.

Notes: Point  $E_0$  represents the initial equilibrium, in which loans' demand  $L_0^D$  equals loans' supply  $L_0^S$ , and the constraint does not bind. The left panel depicts a partial equilibrium situation in which higher defaulted loans shifts and rotates the supply of loans given the initial IC curve (from  $L_0^{S_1}$  to  $L_1^{S_1}$ ). In this case the equilibrium would move from  $E_0$  to point A. The right panel closes the partial equilibrium analysis by depicting the impact of higher defaulted loans on the IC constraint (which shifts from  $L_0^{S_2}$  to  $L_1^{S_2}$ ). The new equilibrium  $E_1$  is such that the constraint becomes binding.

#### 3.3. Market clearing conditions and GDP definition

We close the model through a set of market clearing conditions. Labor market clearing implies  $U_t = U_t^{\mathcal{Z}} + \Gamma_t^{\mathcal{U}} + \Gamma_t^{V}$ . In the intermediate goods market, we have

$$Z_t - \Gamma_t^{PZ} - rcu_t - pen_t - \Gamma_t^{DL} - \Xi_t - \varpi^Z = Z_t^{\mathcal{Y}}$$

In the final goods market

$$Y_t - \Gamma_t^{P\mathcal{Y}} - \Gamma_t^{\mathcal{I}\mathcal{K}} - \varpi^{\mathcal{Y}} = C_t + I_t + G_t + X_t$$

Finally, GDP is

$$GDP_t = C_t + G_t + I_t + X_t - \varepsilon_t M_t$$

where  $\varepsilon_t = P_t^*/P_t$  is the real exchange rate.

	Parameter	Value
Households		
Inverse Frisch elasticity	$\sigma_L$	0.276
Habit persistence	u	0.80
Weight in utility, labor supply	$\eta_L$	1.00
Weight in utility, deposits Discount factor	$\eta_D_{eta}$	$\begin{array}{c} 0.0025 \\ 0.996 \end{array}$
Discount factor	ρ	0.990
Wage and price markups		
Wage markup	$(\sigma^{\mathcal{U}}/\sigma^{\mathcal{U}}-1)-1$	0.32
Intermediate goods price markup	$(\sigma^{\mathcal{Z}}/\sigma^{\mathcal{Z}}-1)-1$	0.21
Final goods price markup	$(\sigma^{\mathcal{Z}}/\sigma^{\mathcal{Z}}-1)-1$ $(\sigma^{\mathcal{F}}/\sigma^{\mathcal{F}}-1)-1$	0.09
EoS and technology		
EoS, intermediate goods	εz	0.99
EoS, final goods	$\varepsilon_{\mathcal{V}}$	1.50
EoS, exports	ε	1.50
Quasi-labor income share	$\alpha^{\mathcal{U}}$	0.60
Home bias in domestic distributors	$\alpha^{\mathcal{Z}}$	0.66
Export market share	$lpha^*$	0.03
Rigidities		
Labor	$arphi_U$	5.0
Investment, productive capital	$\varphi_{IK}$	5.0
Utilization rate	$\sigma_a$	0.3
Import content	arphi IM	2.0
Calvo parameters		
Wage	$\iota^{\mathcal{U}}$	0.75
Intermediate goods	$\iota^{\mathcal{Z}}$	0.75
Final goods	$\mu^{\mathcal{Y}}$	0.50
Indexing factor	idx	0.00
Miscellaneous		
Depreciation rate, productive capital	$\delta^{\mathcal{K}}$	0.025
ECB interest rate target	$i^*$	1.008
ECB Inflation target	$\pi^*$	1.005
Target NFA-to-GDP ratio	$(B^*_{GDP})^{ ext{target}}$	-0.30
NFA risk premium cost	$\varphi_{BF}$	0.0001

TABLE 1. Main parameters (non-financial).

Sources: *Banco de Portugal* data, National accounts data, several studies on the Portuguese and euro area economies, and authors' own calculations.

Notes: EoS—Elasticity of Substitution; NFA—Net Foreign Assets; ECB—European Central Bank. The model is quarterly and parameters are not annualized.

#### 4. Calibration

We calibrate the model to match long-run data or studies for Portugal and euro area economies. Some parameters are exogenously set by taking into consideration common options in the literature, available historical data, or empirical evidence, whilst others are endogenously determined to match great ratios or other measures. Tables 1 and 2 present the model's calibrated parameters, whereas Table 3 exhibits the implied key steady-state relationships.

	Parameter	Value
Entrepreneurs		
Repossession costs	$\mu^{\mathcal{K}}$	0.40
Idiosyncratic shock volatility	$\sigma^{\mathcal{K}}$	0.25
Probability of transition to worker	$1 - \iota^{\mathcal{E}}$	0.04
Startup funds (net worth ratio)	$WT^{\mathcal{E}}/N$	0.00
Sensibility of repossession costs to SS deviations	$\theta_{rbr}$	0.05
Banks		
Idiosyncratic shock volatility	$\sigma^{BK}$	0.02
Probability of transition to worker	$1 - \iota^{BK}$	0.05
Startup funds (Equity ratio)	$WT^{\mathcal{BK}}/E$	0.05
Reputation loss if non-compliance with regulatory requirements	$\overline{\chi}^{\prime \mathcal{BK}}$	0.003
Capital ratio requirement	$\frac{\lambda}{\gamma}\mathcal{BK}$	0.14
Defaulted loans		
Reputation cost	$\varphi_{dl}$	0.03
Recovery fraction	$ au_{\pi}^{arphi_{dl}}$	0.30
Credit restrictions		
Fraction of corporate loans that can be diverted	$\overline{ heta}$	0.16
Smoothness parameter	$\theta_a$	2.0
Relative weight, diversion of defaulted loans	$\bar{\Delta}$	1.0

#### TABLE 2. Main parameters (financial).

Sources: *Banco de Portugal* data, National accounts data, several studies on the Portuguese and euro area economies, and authors' own calculations.

Notes: The model is quarterly and parameters are not annualized. SS stands for steady state.

We set the interest rate target at 3.2 percent per year, matching the precrisis average for the 3-month Euribor. Steady-state inflation is set at 2 percent per year, in line with the ECB's price stability target.

The inverse Frish elasticity  $\sigma_L$  is set to 0.276, and the parameter k indexing habit persistence to 0.8. The discount factor is 0.996, resulting in a net foreign asset position of around -36 percent of GDP for a target ratio of -30 percent and an adjustment cost parameter of  $1 \times 10^{-4}$ . Utility weights are  $\eta_L = 1$  and  $\eta_D = 0.0025$ , which yields a deposits-to-GDP ratio close to 40 percent.

Steady-state price markups are set at 6/19 for wage setting, 4/19 for the intermediate goods sector, and 1/11 for the final goods sector. The elasticity of substitution between capital and labor is set to 0.99, whereas for domestic and foreign goods distributors the elasticity of substitution between inputs is 1.5. The depreciation rate of capital is calibrated at 10 percent per year. The labor quasi-share and the home bias parameters are endogenously calibrated to take into account the actual labor income share and the import share, whereas the export market share is adjusted according to the exports-to-GDP ratio.

The investment and labor adjustment costs are parameterized to ensure plausible dynamics. Likewise for the parameter assessing the cost of underor over-utilization of capital. The import content adjustment costs ensures plausible real exchange rate fluctuations. Calvo parameters imply an average

	Model	Data	Period
Expenditure (GDP ratio)			
Private consumption	0.62	0.65	1995 - 2016
Private investment	0.19	0.18	1995 - 2016
Public consumption & investment	0.23	0.23	1995 - 2016
Exports	0.35	0.32	1995 - 2016
Imports	0.39	0.39	1995 - 2016
Shares (output ratio)			
Import share	0.28	0.30	1995 - 2008
Labor income share	0.60	0.67	1995 - 2016
External account (GDP ratio, in %)			
Net foreign assets (annualized)	-36.7	-83.5	1995 - 2016
Current and capital accounts	-0.7	-5.3	1995 - 2016
Trade balance	-4.4	-5.4	1995 - 2016
Financial sector, ratios			
Deposits-to-GDP ratio	0.42	0.46	1995 - 2016
Financial sector, Entrepreneurs			
Leverage ratio	1.2	1.2	1999 - 2008
Probability of default (in %)	3.6	3.6	1999 - 2008
Retail-wholesale interest rate spread (in p.p.)	1.6	1.7	1999-2008
Financial sector, Banks			
Probability of not fulfilling capital requirements (in %)	4.0	n.a.	
Capital-to-loans ratio (in %)	17.0	n.a.	
Endogenous capital buffer (in %)	3.0	n.a.	
Wholesale-deposits interest rate spread (in p.p.)	0.5	0.6	1999-2008
Financial sector, defaulted loans			
Defaulted loans-to-credit ratio (in %)	1.17	1.15	1999 - 2011
New defaulted loans-to-credit ratio (in %)	0.56	n.a.	
Defaulted loans recovered (in %)	0.52	n.a.	
Impairment-to-credit ratio (in %)	0.04	n.a.	
Immediate impairment losses (credit ratio, in %)	0.37	n.a.	

TABLE 3. Key steady-state relationships.

Sources: *Banco de Portugal* data, National accounts data, and authors' own calculations. Notes: Immediate impairment losses are endogenously calibrated according to the retailwholesale interest rate spread. We adjust the impairment-to-credit ratio to yield an overall loss-given default around 45 percent.

contract duration and intermediate goods average price duration of 1 year, and a final goods average price duration of half a year. We assume no indexing.

On the entrepreneurial side, we calibrate the monitoring cost parameter, the idiosyncratic shock volatility, and transferred earnings to households net of startup funds to match a target leverage (net worth-to-debt ratio) of 1.2, a yearly default probability of 3.6 percent, and a yearly retail lending rate spread of 1.6 percentage points. In practice, we set startup funds to zero and

let the fraction of entrepreneurs going out of business adjust. The resulting repossession cost parameter  $\mu^{\mathcal{K}}$  is 40 percent.<sup>19</sup>

For the banking sector, we set the capital requirement to 14 percent and let banks build an endogenous capital buffer of 3 percentage points, yielding a steady-state capital-to-loans ratio of 17 percent. The probability of noncomplying with capital requirements is set at 4 percent, and the spread between the wholesale interest rate-matched by the 6-month Euribor-and the deposits rate is 0.5 percentage points. The fraction of bankers going out of business is 5 percent, and startup funds amount to 5 percent of banks capital.<sup>20</sup> It follows that a banker stays in the job on average around 5 years. This recalibration endogenously determines the value/reputation losses per unit of asset and the idiosyncratic shock volatility. The reputation cost for each unit of defaulted loans is 0.03, and 30 percent of these loans are recovered in each period. This calibration results in a defaulted loans-to-credit ratio of approximately 1.2 percent. New defaulted loans in each period amount to 0.56 percent of total credit, and in the steady state this matches the amount that is withdrawn from the balance sheet—0.52 percent is recovered and 0.04 percent is recognized as impairment loss and written off. Immediate losses amount to 0.37 percent of total credit. This calibration results in a loss given default slightly above 45 percent.<sup>21</sup>

The parameter  $\overline{\theta}$  is endogenously calibrated so that agency problems do not arise in the steady state, but are triggered in the presence of shocks with large negative impacts on banks' terminal wealth. We achieve this by imposing a slack *sl* in the incentive compatibility constraint in (24)

$$\overline{\theta} \bigg[ 1 + \Delta \frac{DL_t}{L_t^{\mathcal{BK}}} \bigg] - \frac{V_t^{A\,UX}}{L_t^{\mathcal{BK}}} = -sl$$

calibrated at an annualized rate of 0.40 percentage points. Bankers are able to divert 16 percent of total assets. We assume the same diversion rate for both loans and defaulted loans, and perform a comparative statics exercise on  $\Delta$  latter. The smoothing parameter  $\theta_a$  is set to 2, which allows us to carry out all simulations in the article without running into convergence issues.

<sup>19.</sup> The calibration is within values found in the literature, being comprised for instance above the values found in Bernanke *et al.* (1999) or Christiano *et al.* (2014), but below the value in Christiano *et al.* (2011).

<sup>20.</sup> It follows that in the steady state implicit dividends are close to zero.

<sup>21.</sup> Loss given default is understood herein as total losses in each period, both immediate and delayed impairment losses, over the amount at risk given default, which include immediate losses and defaulted loans.

#### 5. Business cycle size, persistence, and asymmetry

In this section we summarize the results of several quantitative experiments and illustrate the workings whereby different model frictions may amplify business cycle fluctuations and endogenously increase persistence. We set up four standard non-financial shocks—on technology  $(A_t)$ , domestic demand  $(G_t)$ , external demand  $(Y_t^*)$ , and monetary policy  $(i_t^*)$ . We then address the effects of a purely financial risk shock  $(\sigma_t^K)$  affecting entrepreneurial returns, and a balance sheet shock impacting expected bank returns  $(\Xi_{t+1})$ . Shocks follow a standard autoregressive process of order 1,

$$x_t = (1 - \rho)x_{ss} + \rho x_{t-1} + \varepsilon_t$$

where  $x_t$  denotes the process at t,  $x_{ss}$  is the steady-state value and  $\varepsilon_t$  is an *i.i.d.* innovation. The autoregressive parameter  $\rho$  is calibrated at 0.8 for all shocks, yielding an half-life of around 4 quarters. We perform also a comparative statics exercise in some shocks, and assume an alternative scenario where the shock process remains constant for two years and only thereafter becomes driven by the aforestated law of motion.

Our interest lies in evaluating the dynamics and amplification effects of the banking model with defaulted loans (henceforth "banking model & DL") and of the banking model with both defaulted loans and credit restrictions ("banking model & DL & CR"). For comparison purposes, we run three benchmark models: the banking model without defaulted loans or credit restrictions (viz "banking model"), the BGG-accelerator model ("accelerator model"), and the plain vanilla model deprived of any financial friction ("no FF model"). To compare the dynamics across different models, we first calibrate and run the "banking model & DL & CR" presented in the previous section. Then we successively deactivate parts of the model while fixing the values for all common parameters. The "banking model" poses a solution issue, since suppressing defaulted loans pushes up equity towards an outcome where capital requirements become redundant and are never violated. We go around this issue by resetting startup funds to new bankers such that the probability of not complying with capital requirements remains unchanged at the yearly rate of 4 percent in the steady state, while keeping all other parameters fixed.<sup>22</sup> We then run the BGG-accelerator model by assuming that households lend directly to retail banks, who charge a state contingent interest rate. Intertemporal smoothing is achieved by assuming that households can borrow from abroad. The absence of bank loans and deposits renders banks completely

35

<sup>22.</sup> We carried out a sensitivity analysis and concluded that the amount of startup funds (or equivalently the steady-state probability of violating capital requirements) lets the qualitative analysis broadly unchanged.

useless. Finally, we run the no financial frictions model, in which capital goods producers sell capital directly to manufacturers, rendering the entrepreneurial sector useless.

As previously noted, the model is asymmetric by construction, but not for every shock. Non-financial shocks endowed with minor impacts in the banking system and in bank returns are not able to generate restrictions to credit and the dynamic response will be symmetric. In such cases, the incentive compatibility constraint remains slack all the time, even for sizable shocks. On the opposite direction, financial and banking shocks endowed with major impacts in the banking system and in bank returns generate asymmetric dynamic responses. The incentive compatibility constraint remains slack all the time under "good shocks," but becomes binding for some period of time under moderately-sized "bad shocks." As a result, small innovations may be able to trigger important model dynamics and substantial output downfalls. In what follows, we calibrate the innovation  $\varepsilon_t$  in all shocks such that GDP declines approximately 0.5 percent vis-à-vis the steady state in the "banking model" at the trough.

## 5.1. Technology shock

The friction mechanisms embodied in the accelerator and banking models weaken business cycle fluctuations under supply side shocks (see Figure B.1 in the appendix). Such result is not novel in the literature and examples can be found were financial frictions dampen the effects of technology shocks (*e.g.* Christensen and Dib 2008). Intuitively, firms partially outweigh the effects of the technology shock on the production function by increasing factor demands, pushing the price of capital and hence net worth upwards. Financial frictions and spreads concomitantly decrease following the downward pressure on leverage, thus hampering the GDP downfall.

On the banking side, the decline corporate default rates is reflected into a lower probability of not complying with regulatory capital requirements, though these effects are sufficiently small to have any sizable impact for the simulated shock (the wholesale spread decline is negligible). It follows that the banking and accelerator models do present indistinguishable impacts.<sup>23</sup> The decline in corporate default also triggers a fall in the amount of defaulted loans. This results in less costs for banks, who respond by charging lower spreads, and the output downfall is marginally dampened. Under technology shocks credit restrictions remain non-binding since the bank valuation (*i.e.* terminal wealth) is barely affected.

<sup>23.</sup> The similarities between banking and accelerator models following fiscal, demand and supply side shocks have been emphasized in Andrle et al. (2015).

## 5.2. Domestic and external demand shocks

The amplification mechanisms comprehended in the accelerator and banking models play minor roles under domestic and external demand shocks (see Figures B.2 and B.3 in the appendix). Common to both simulations is the fact that the price of capital barely moves and thus no substantial frictions arise. The net worth decline and the concomitant leverage uprise are small, the impacts on the default probability are unimportant, and bank returns are hardly affected. Along the same lines, the amount of defaulted loans is nearly unchanged, playing no role in model dynamics.<sup>24</sup> Credit restrictions remain slack at all times as both shocks have minor impacts on banks' value.<sup>25</sup>

## 5.3. Sovereign risk shock

A sovereign risk shock (Figure 3) impacts the real economy by raising the cost of credit and thus leverage.<sup>26</sup> Specifically, the cost of foreign funds increases and is reflected on a nearly one-to-one basis on the wholesale rate (the change in the capital requirements spread is negligible). The default rate increases as do spreads paid by entrepreneurs (*i.e.* the external finance premium), adding a second round of effects that boosts the initial dynamics. In this simulation the banking model *per se* is endowed with a cushion effect, due to the non-state contingent nature of the interest rate. To put differently, the losses incurred by retail branches are split between banks and entrepreneurs, and banks' capital acts as a shock absorber. As a result, investment, capital, loans, and output face a lesser decline *vis-à-vis* the accelerator model, where the retail interest rate is state-contingent.

Defaulted loans and credit restrictions are amplification mechanisms in this case.<sup>27</sup> Defaulted loans amplify the real effects by leveraging the balance sheet impact of corporate defaults, a channel created through the opportunity costs and holding or reputation costs. This translates in a higher expected risk of non-complying with capital requirements and thus larger wholesale spreads.

<sup>24.</sup> Gourinchas et al. (2016), for instance, emphasizes a zero-correlation between NPL and government consumption.

<sup>25.</sup> Notwithstanding, a major difference between models encompassing financial frictions and the "no FF model" is investment, which does not decline in the latter case, a result that can be found in the literature. The size of the effect is insufficient to trigger important differences in GDP or business cycle fluctuations across models.

<sup>26.</sup> This shock resembles a monetary policy shock with no Taylor rule, *i.e.* in which the interest rate follows an exogenous autoregressive process and does not respond to euro area aggregates. Recall that our model is for a small-open euro area small open economy and hence it does not feature a Taylor Rule. Results cannot therefore be directly compared with most literature.

<sup>27.</sup> For instance, a positive correlation between a sovereign risk shock and NPL can be found in Gourinchas  $et \ al. \ (2016)$ .

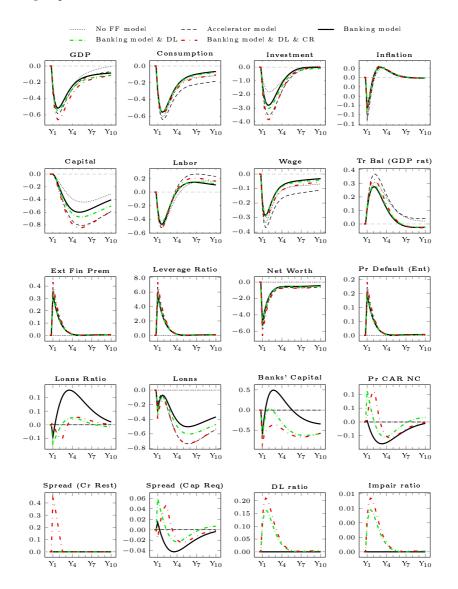


FIGURE 3: Sovereign risk shock.

Notes: The figure represents a 1.2 percentage points increase in the annualized sovereign risk premium, which generates a 0.5 percent GDP decline at the trough in the "banking model". All variables are in percentage deviations from steady-state values except ratios, probabilities, and spreads/premiums, which are in percentage points deviations. NC stands for non-compliance, DL ratio for the ratio of defaulted to total loans, and impair ratio for the impairment-to-loans ratio. Notation  $Y_x$  refers to the first quarter of year x. The loans ratio is defined as post-return loans over post-return equity and the external finance premium as the retail rate minus the wholesale rate,  $i_t^R - i_t^W$ .

Credit restrictions further amplify the cycle by forcing bankers to be more parsimonious in credit decisions following the decline in banks' value. The impact is leveraged by defaulted loans, since these create a cost which is reflected into lower banks' capital, a determinant of banks' value. Less (and more expensive) credit place an additional constraint on entrepreneurs' external finance, who further halt investment decisions. This is reflected into lower capital and hence GDP.

Figure B.4 in the appendix details the dynamics of a more persistent sovereign risk shock. The qualitative effects are essentially similar, except that credit restrictions play a much major role due to their asymmetrical nature. Notice also that the capital requirements spread declines on impact in the "Banking & DL & CR" model, naturally outweighed by the credit restrictions counterpart in the total spread determination. The main idea here is that credit restrictions hinder banks' external finance, which is reflected into a better loans ratio and thus a lower probability of violating regulatory capital requirements.

## 5.4. Risk shock

Figure 4 presents the effects of a risk shock. Both the accelerator and banking models are endowed with similar effects.<sup>28</sup> The latter has an additional soothing mechanism—banks' equity acts as a shock absorber—and an additional friction mechanism—the observance of regulatory requirements is reflected into higher wholesale spreads. The risk shock impacts directly the corporate default probability, leading to more expensive credit through a higher retail spread. However, in the banking model banks bear a fraction of the loss, due to the non-state contingent nature of the retail interest rate. This is the soothing or cushion effect. Additionally, banks respond to the fall in capital by increasing the wholesale spread, as a means to face the increased risk of violating regulatory requirements. This is the friction or amplification mechanism.

In the case of a risk shock, real impacts are substantially amplified by defaulted loans and credit restrictions, since bank returns are severely affected. Greater corporate default leads to a substantial accumulation of defaulted loans. The wholesale spread is therefore further pushed upwards as banks strive to cope with regulatory requirements and defaulted loans opportunity costs and holding/reputation costs. More expensive credit pushes corporate loans down, resulting in fewer investments and less capital accumulation. Additionally, the powerful impact on bank returns and thus on their value forces bankers to hold back on credit, unfolding a large (credit restrictions-driven) wholesale spread hike. Entrepreneurs are forced to withhold investment decisions and hinder capital accumulation as external finance collapses. Persistence is also substantially incremented as banks' capital has its own inertia and recovers

<sup>28.</sup> A result broadly in line with Andrle et al. (2015).

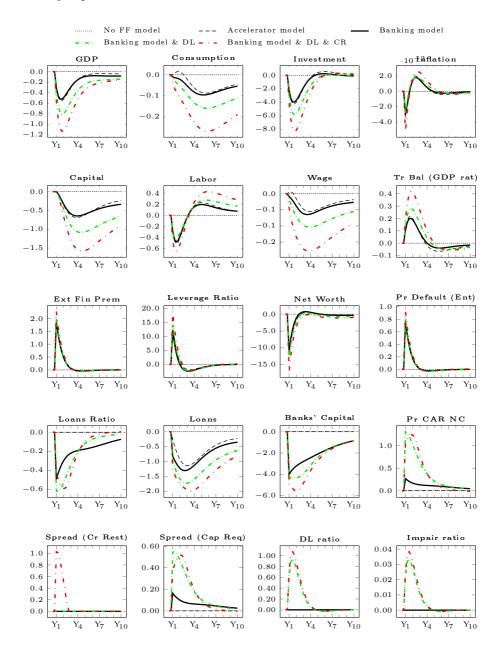


FIGURE 4: Risk shock.

Notes: The figure represents a 14 percent increase in risk, which generates a 0.5 percent GDP decline at the trough in the "banking model". See Figure 3 for additional notes.

slowly. To put differently, it takes several years for banks to recover the revenues lost to the risk shock. As a result, corporate loans also recover gradually as banks strive to keep the loans ratio and thus the probability of not complying with the regulatory requirements under control.

Figure B.5 in the appendix shows the effects a more persistent risk shock. While in the short run the amplification effects of the "banking model & DL & CR" are essentially similar to those on Figure 4, over the medium term credit restrictions trigger a more protracted decline in output vis-à-vis the "banking model." Even though credit restrictions are in general short-lived, they compromise short-term returns that are not fully compensated for in the medium or long run. Specifically, bankers would like to expand corporate loans at a faster pace on the aftermath of credit restrictions, but are tied by regulatory capital requirements which put an halt on the banks' leverage position. This boost the inertia underlying the recovery in banks' capital and thus generates protracted real impacts.

## 5.5. Banks' capital shock

A banks' capital shock (Figure 5) raises the probability of non-compliance with regulatory capital requirements. Wholesale spreads increase and corporate loans are decline as a result. The inertia in the recovery of banks' capital, resulting from the inability of expanding leverage due to regulatory requirements and in this case also from the shock itself, triggers a protracted credit and output response.

The decline in credit and the concomitant spread increase raises corporate default and thus defaulted loans. The effect triggers an additional cost for banks, who respond through a larger wholesale spread in the "banking model & DL." Greater financing costs feed back into the entrepreneurs' cost of external finance, triggering a new round of effects as net worth drops and leverage and the default probability increase. In the "banking model & DL & CR," the decline in banks' value triggers restrictions to credit, which are reflected into even lower corporate loans and larger wholesale spreads. This mechanism plays an important role in macroeconomic dynamics, leading to a deterioration in entrepreneurial returns and hence net worth, hindering capital accumulation, and pushing GDP further downwards. Both defaulted loans and credit restrictions do not substantially enhance persistence in this case; since entrepreneurs are not directly affected, banks are able to recover faster the short run losses. To put differently, notice that for the risk shock in Figure 4 the ratio of defaulted loans-to-total loans increased around 1 percentage point and the capital ratio spread nearly 0.6 percentage points, figures which compare with 0.3 and 0.7 percentage points respectively for the banks' capital shock. Since the entrepreneurial demand for funds is not directly affected in this latter case, banks are able to charge larger spreads and thus recover faster from the nefarious effects of defaulted loans and credit restrictions. However, they do not recover faster from the shock itself, as the "banking model" makes clear.

The shock affects banks' capital directly and its effects outweigh the returns from larger spreads.

Figure B.6 in the appendix plots the dynamics of a more persistent shock, highlighting the short run effects triggered by credit restrictions and the sharper medium term recovery. In particular, the figure illustrates the asymmetrical nature of credit restrictions: the amplification effect *vis-à-vis* the "banking model & DL" is larger when compared with that in Figure 5.

## 6. Discussion

Our article proposes a model with endogenous defaulted loans determination and credit restrictions, both of which able to substantially amplify business cycle downturns. Shocks positively impacting corporate default trigger an increase in defaulted loans, which in turn lead to lower banks' returns and larger wholesale rates. More expensive external finance feeds back into the entrepreneurial sector for a second round of effects: net worth drops further, leverage and default increase. Shocks negatively impacting banks' returns and hence their value trigger restrictions to credit. Corporate loans drop and become more expensive, severely impacting entrepreneurial net worth and defaults for a second round of effects that feed back into the banking system.

The two mechanisms are strongly intertwined: defaulted loans amplify the effects of credit restrictions by pushing banks' value downwards. To explore this feature further, Figure 6 plots the effects of ceteris paribus changes in three parameters related with defaulted loans on restrictions to credit and on the downturn, for the risk shock introduced in the previous section.<sup>29</sup> A larger defaulted loans holding cost ( $\varphi_{dl}$ ) implies a more severe deterioration of banks' value given the defaulted loans increase that follows a risk shock. It becomes comparatively more attractive for the bank manager to divert funds, an outcome which is prevented with more severe credit restrictions. The GDP downfall becomes larger as a result. A lower liquidity transformation rate  $\tau^{\mathcal{BK}}$  has a similar impact, since the change in the defaulted loans stock that follows the risk shock—and hence the decline in the banks' value—becomes larger. An increase in the fraction of defaulted loans that can be diverted ( $\Delta$ ) results directly in more severe restrictions to credit.

Restrictions to credit also interact with capital requirements and the banker's transition probability (Figure 7). For instance, a larger value/reputation loss from violating capital requirements ( $\chi^{\mathcal{BK}}$ ) implies a stronger decline in the banks' value following the shock, to which bankers respond with more severe credit restrictions. A higher probability of transiting

<sup>29.</sup> We select the risk shock for this exercise since it has been recognized as one of the most important business cycle drivers (e.g. Christiano et al. 2014).

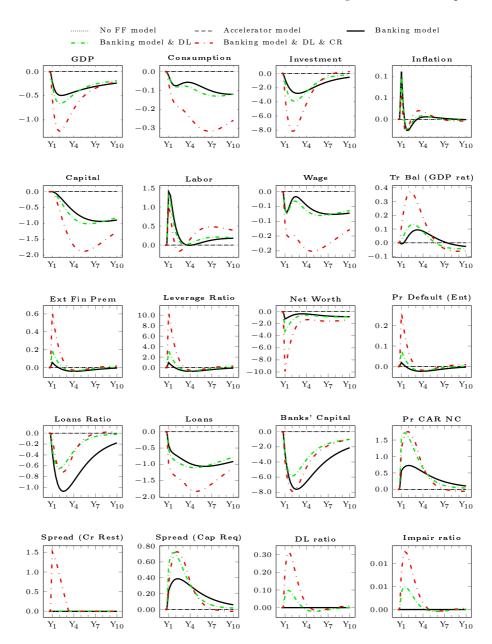


FIGURE 5: Banks' capital shock.

Notes: The figure represents a 2.6 percent decline in banks' capital, which generates a 0.5 percent GDP decline at the trough in the "banking model". See Figure 3 for additional notes.

to worker implies a shorter bankers' time span. The banks' value or terminal

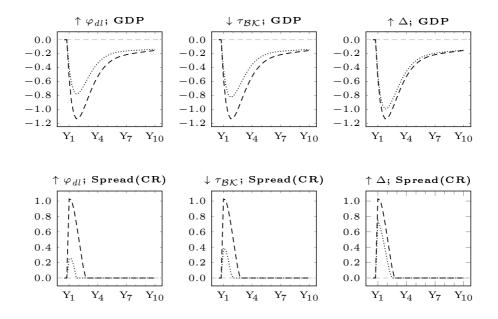


FIGURE 6: The effect of defaulted loans-related parameters on restrictions to credit and the downturn.

Notes: The figure depicts GDP and the spread driven by credit restrictions from a risk shock under different parameterizations related with defaulted loans. The first figure compares  $\varphi_{dl} = 0.02$  versus  $\varphi_{dl} = 0.03$ . The second figure compares  $\tau_{\mathcal{BK}} = 0.35$  versus  $\tau_{\mathcal{BK}} = 0.30$ . The third figure compares  $\Delta = 0.5$  versus  $\Delta = 1$ .

wealth becomes more influenced by short run events, and thus by the shock itself. The stronger decline is reflected into more restrictive credit.

## 7. Concluding remarks

We present an integrated financial amplifier dynamic stochastic general equilibrium model which introduces defaulted loans and occasionally binding credit restrictions into an otherwise standard banking model. We borrow from the literature the endogenous capital requirements and the moral hazardinspired credit constrain mechanisms. We then propose and develop an occasionally binding version of the latter mechanism, which is slack in the steady state but endogenously affects credit supply decisions when banks' capital is severely affected. As a result, credit is mostly demand/price driven but endogenously becomes supply/quantity driven when a "bad" shock depleting banks' value hits the economy. Simultaneously, we bring forth into the model a theory of optimal impairment loss recognition, which gives raise to an

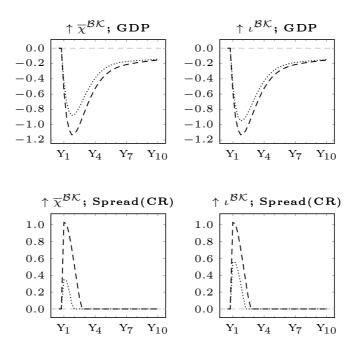


FIGURE 7: The role of capital requirement costs and the transition probability on restrictions to credit and the downturn.

Notes: The figure depicts GDP and the spread driven by credit restrictions from a risk shock under different parameterizations related with the capital requirements cost and the transition probability. The first figure compares  $\overline{\chi}^{\mathcal{BK}} = 0.02$  versus  $\overline{\chi}^{\mathcal{BK}} = 0.03$ . The second figure compares  $\iota = 0.03$  with  $\iota = 0.05$ .

endogenous defaulted loans stock that bankers manage over time. Defaulted loans interact with regulatory capital requirements and credit restrictions.

The model is asymmetric by construction, and our simulations show that is possesses powerful amplification effects over recessions, strengthening and protracting output downfalls under moderately sized "bad" financial shocks. A key prerequisite for amplification is that the shock is able to negatively affect banks' value, boosting the bankers' incentives to divert funds. The mechanism is not able to amplify the effects of non-financial shocks that barely affect the value of banks.

Our work has obvious policy implications and addresses some fragilities often pinpointed in dynamic stochastic general equilibrium models. First, we unify the two literature strands—the financial accelerator and the collateral constrain—into one model. Second, we introduce non-linear behaviors over the business cycle. Third, we provide an environment where a narrow set of small-sized financial-based shocks can trigger a deep and protracted recession, something which may contribute decisively to enhance the predictive density

of dynamic stochastic general equilibrium model in crisis periods. Fourth, we propose and develop a simple theory in which defaulted loans increase sharply after a financial shock, but fluctuate little after non-financial shocks. Such outcome is in line with the important accumulation of defaulted loans in banks' balance sheets registered by number of euro area economies in the aftermath of the financial crisis. Fifth, the model provides a completely novel framework to analyze policy-oriented measures aimed at increasing the robustness of the financial and banking system, especially during crisis periods.

## Appendix A: Solution to Banks' problem

The bank's problem consists in maximizing expected terminal wealth in (19), subject to (14), (15), (16), (17), and (20). The first-order conditions with respect to corporate loans and credit to households can be found in our technical guide Júlio and Maria (2018). Letting

$$CpR_t(k) = \bar{\chi}_{t+1}^{\mathcal{BK}} \left[ L_t^{\mathcal{BK}}(k) + DL_t(k) \right] \mathfrak{F}^{\mathcal{BK}}(\bar{\omega}_{t+1}^{\mathcal{BK},k})$$

the first-order condition with respect to DL reads

$$\mathbf{E}_{t}\tilde{\Lambda}_{t,t+1}^{\mathcal{BK}}\left[\left(1-i_{t}^{D}\right)+1-\frac{\partial\Gamma_{t+1}^{DL}(k)}{\partial DL_{t}(k)}-\frac{\partial CpR_{t}(k)}{\partial DL_{t}(k)}\right]=\frac{\lambda_{t}^{BK}(k)}{1+\lambda_{t}^{BK}(k)}\theta$$

where

$$\begin{aligned} \frac{\partial \Gamma_{t+1}^{DL}(k)}{\partial DL_t(k)} = & \varphi_{dl} \left[ \frac{1 - \upsilon(k)}{\upsilon(k)} - \log \upsilon(k) \right] \\ \frac{\partial CpR_t(k)}{\partial DL_t(k)} = & E_t \bar{\chi}_{t+1}^{\mathcal{BK}} \left[ \mathfrak{F}^{\mathcal{BK}}(\bar{\omega}_{t+1}^{\mathcal{BK},k}) + \mathfrak{f}^{\mathcal{BK}}(\bar{\omega}_{t+1}^{\mathcal{BK},k}) \iota_t^{\mathcal{BK}}(k) \left[ i_t^D - 1 + \frac{\partial \Gamma_{t+1}^{DL}(k)}{\partial DL_t(k)} - \bar{\omega}_{t+1}^{\mathcal{BK},k}(1 - \bar{\gamma}_t) \right] \right] \end{aligned}$$

The change in the augmented stochastic discount factor is

$$\tilde{\Lambda}_{t,t+1}^{\mathcal{BK}} = \Lambda_{t,t+1}^{N} \left[ 1 - \iota^{\mathcal{BK}} + \iota^{\mathcal{BK}} \frac{\partial V_{t+1}}{\partial \mathbf{E}_t E V_{t+1}} \right]$$

It remains to find  $\partial V_t / \partial EEV_t$ . Terminal wealth can be re-stated as  $V_t = E_t \Lambda_{t,t+1}^{\mathcal{BK}} EV_{t+1}$ . Hence

$$\begin{split} \frac{\partial V_t}{\partial \mathbf{E}_t E V_t} = & \mathbf{E}_t \Lambda_{t,t+1}^{\mathcal{BK}} \bigg[ \left( i_t^W - i_t^D - \frac{\partial C p R_t(k)}{\partial L_t(k)} \right) \frac{\partial L_t(k)}{\partial E V_t(k)} \\ & + \left( 1 - i_t^D + 1 - \frac{\partial \Gamma_{t+1}^{DL}(k)}{\partial D L_t(k)} - \frac{\partial C p R_t(k)}{\partial D L_t(k)} \right) \frac{\partial D L_t(k)}{\partial E V_t(k)} \\ & - \left( i_t^* \Psi_t - i_t^D - \frac{\partial C p R_t(k)}{\partial B_t^*(k)} \right) \frac{\partial B^* t(k)}{\partial E V_t(k)} - i_t^D - \frac{\partial C p R_t(k)}{\partial E V_t(k)} \bigg] \end{split}$$

Using the first-order conditions, this simplifies to

$$\frac{\partial V_t}{\partial \mathbf{E}_t E V_t} = \mathbf{E}_t \Lambda_{t,t+1}^{\mathcal{BK}} \left[ \left( i_t^W - \tilde{i}_t^W \right) \left[ \frac{\partial L_t(k)}{\partial E V_t(k)} + \frac{\partial D L_t(k)}{\partial E V_t(k)} \right] + i_t^D + \frac{\partial C p R_t(k)}{\partial E V_t(k)} \right]$$

The incentive compatibility constraint reads

$$\varphi_t^{\mathcal{BK}} EV_t(k) = \left[ L_t^{\mathcal{BK}}(k) + DL_t(k) \right]$$

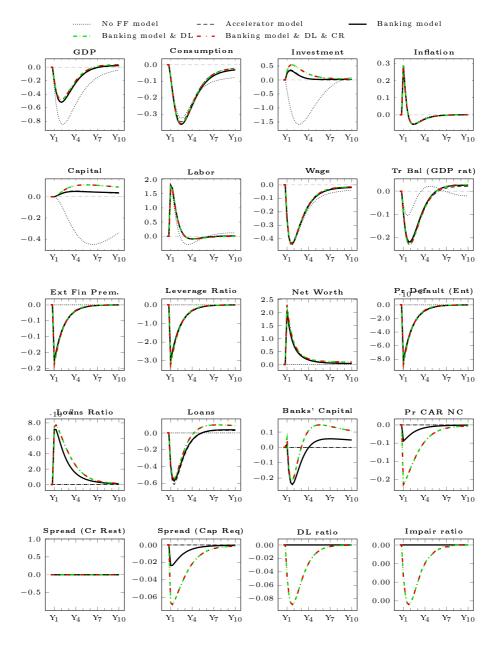
Differentiating yields

$$\varphi_t^{\mathcal{BK}} = \frac{\partial L_t^{\mathcal{BK}}(k)}{\partial E V_t(k)} + \frac{\partial D L_t(k)}{\partial E V_t(k)}$$

implying

$$\frac{\partial V_t}{\partial \mathbf{E}_t E V_t} = \mathbf{E}_t \Lambda_{t,t+1}^{\mathcal{B}\mathcal{K}} \left[ \left( i_t^W - \tilde{i}_t^W \right) \varphi_t^{\mathcal{B}\mathcal{K}} + i_t^D \left[ 1 + \mathbf{E}_t \chi_{t+1}^{\mathcal{B}\mathcal{K}} \left[ \mathbf{j}^{\mathcal{B}\mathcal{K}} (\bar{\omega}_{t+1}^{\mathcal{B}\mathcal{K}}) \iota_t^{\mathcal{B}\mathcal{K}} \right] \right] \right]$$

Finally, notice that this derivation is only valid when the incentive compatibility is biding. If the IC is not bidding, then  $\lambda_t^{\mathcal{BK}} = 0$  and the element  $\tilde{\Lambda}_{t,t+1}^{\mathcal{BK}}$  is not required to characterize the solution.



## Appendix B: Additional figures

FIGURE B.1: Technology shock.

Notes: The figure represents a 1 percent negative technology shock, which generates a 0.5 percent GDP decline at the trough in the "banking model." See Figure 3 for additional notes.

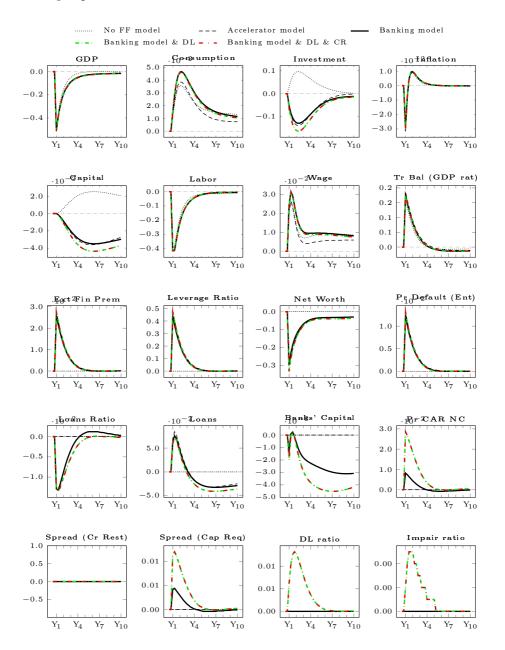


FIGURE B.2: Government consumption shock.

Notes: The figure represents a negative government consumption shock of 0.6 percent of GDP, which generates a 0.5 percent GDP decline at the trough in the "banking model." See Figure 3 for additional notes.

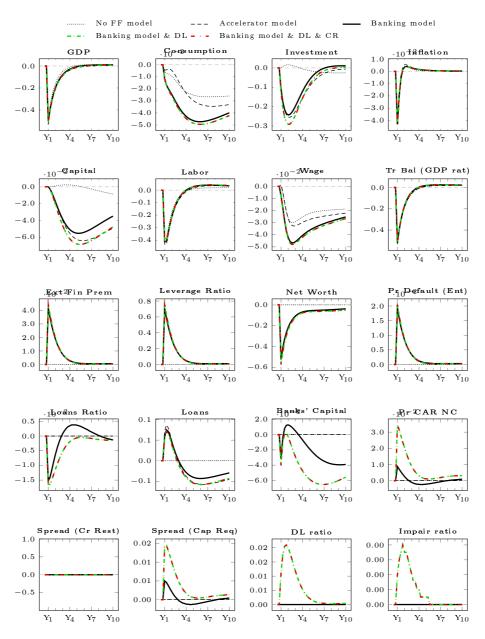


FIGURE B.3: External demand shock.

Notes: The figure represents a negative external demand shock of 2 percent of GDP, which generates a 0.5 percent GDP decline at the trough in the "banking model." See Figure 3 for additional notes.

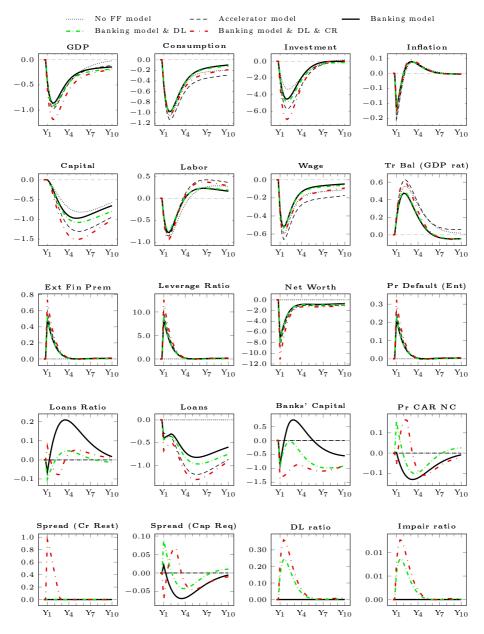


FIGURE B.4: Persistent sovereign risk shock.

Notes: The figure represents a 1.2 percentage point increase in the annualized sovereign risk premium, lasting 2 years and then reverting to the steady state at a rate of approximately 50 percent per year. See Figure 3 for additional notes.

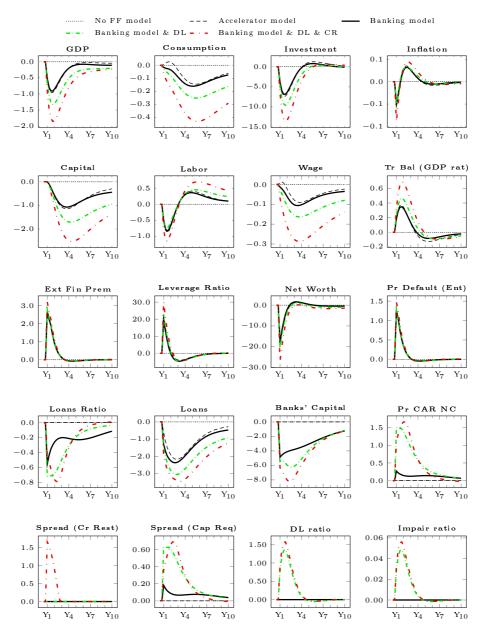


FIGURE B.5: Persistent risk shock.

Notes: The figure represents a 14 percent increase in risk, lasting 2 years and then reverting to the steady state at a rate of approximately 50 percent per year. See Figure 3 for additional notes.

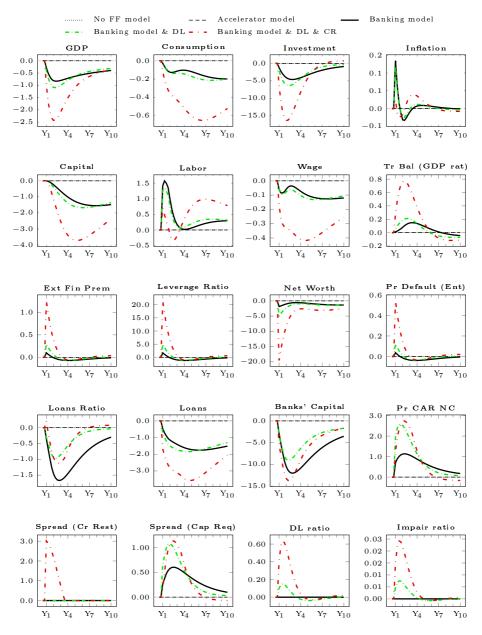


FIGURE B.6: Banks' capital shock.

Notes: The figure represents a 2.6 percent decline in banks' capital, lasting 2 years and then reverting to the steady state at a rate of approximately 50 percent per year. See Figure 3 for additional notes.

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