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The analyses, opinions and findings of these papers represent the views of the authors, they are not necessarily those of the Banco de Portugal or the Eurosystem

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Inside **PESSOA**—A Detailed Description of the Model*

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Abstract

This article presents a detailed description of *PESSOA*—**P**ortuguese **E**conomy **S**tructural **S**mall **O**pen **A**nalytical model. *PESSOA* is a dynamic general equilibrium model that can be applied to any small economy integrated in a monetary union. The main theoretical reference behind its structure is Kumhof, Muir, Mursula, and Laxton (2010). The model features non-Ricardian characteristics, a multi-sectoral production structure, imperfect market competition, and a number of nominal, real, and financial rigidities. *PESSOA* has been calibrated to match Portuguese and euro area data and used to illustrate a number of key macroeconomic issues, ranging from the effects of structural reforms to alternative fiscal policy options.

JEL classification: E62, F41, H62
Keywords: DSGE models; euro area; small-open economy.

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Be complete in everything, for to be complete in anything is to be right.

Fernando Pessoa

1 Introduction

Macroeconomics is nowadays inseparable from the formal rigor of analytical models, since they allow for the analysis of the fundamental driving forces behind aggregate economic phenomena and for well-grounded diagnosis of economic performances. It is therefore natural that an increasing number of organizations and policy-making institutions are using well-founded macroeconomic models to aid policy recommendations and decision making.

A myriad of macroeconomic models emerged over the past 30 years, both in academia and policy-making institutions, and a vast literature has been devoted to the development of richer and more realistic frameworks to study a different number of issues. Some consensus has emerged recently around New Keynesian general equilibrium models as a reference tool. These models rely on strong theoretical microeconomic foundations—in the spirit of the seminal real business cycle model pioneered by Kydland and Prescott (1977, 1982)—and comprise market imperfections as well as a large number of nominal and real rigidities (Calvo, 1983; Rotemberg, 1982; King, 1991; Rotemberg and Woodford, 1995; Erceg, Henderson, and Levin, 2000; Abel, 1990; Christiano, Eichenbaum, and Evans, 2005) that mimic important evidence on macroeconomic dynamics.

New Keynesian general equilibrium models are able to replicate a satisfactory number of stylized facts, and real rigidities and imperfect market competition create a role for economic stabilization policies. Macro-financial linkages stemming from liquidity channels or from balance sheet channels of borrowers or lenders—an operational constraint that can embed a financial accelerator effect—are also a growing field in macroeconomics. Asymmetric information, as in Bernanke, Gertler, and Gilchrist (1999b), and limited commitment problems, as in Kiyotaki and Moore (1997), are two typical theoretical origins behind the modelling of credit frictions.

General equilibrium models are currently the most appealing tool for structural macroeconomic policy analysis, being widely used in policy-making institutions. Examples include the IMF's Global Integrated Monetary and Fiscal model to assess the impact of fiscal stimulus in the context of the ongoing global crisis (Freedman et al., 2009a,b), the Bank of Sweden's RAMSES model to evaluate policy options and perform forecasting exercises (Adolfson et al., 2007b), the Bank of Finland's AINO model to analyze the macroeconomic impacts of aging and aid in regular projection exercises (Kilponen and Ripatti, 2006), and the ECB's New Area Wide model used for a wide range of purposes within the Eurosystem (Christoffel, Coenen, and Warne, 2008). The relative performance of seven widely used structural models to alternative fiscal policies can be found in Coenen et al. (2010).

DSGE models are not free from caveats, facing a challenging trade-off between parsimony, that allows for analytical and computational tractability, and complexity, required to capture realistic macroeconomic dynamics. Parsimony is often achieved through a number of simplifying assumptions. For instance, replacing rational expectations—a somewhat strong assumption—by bounded rationality has led to a series of models that lack the ability to deal with policy issues, due to their parsimoniousness in many other dimensions. Regardless of how the model matches the data facts, one must be aware, when evaluating policy options or making policy recommendations, that DSGE models are stylized representations of reality, and do not embrace all the complex and relevant economic features.

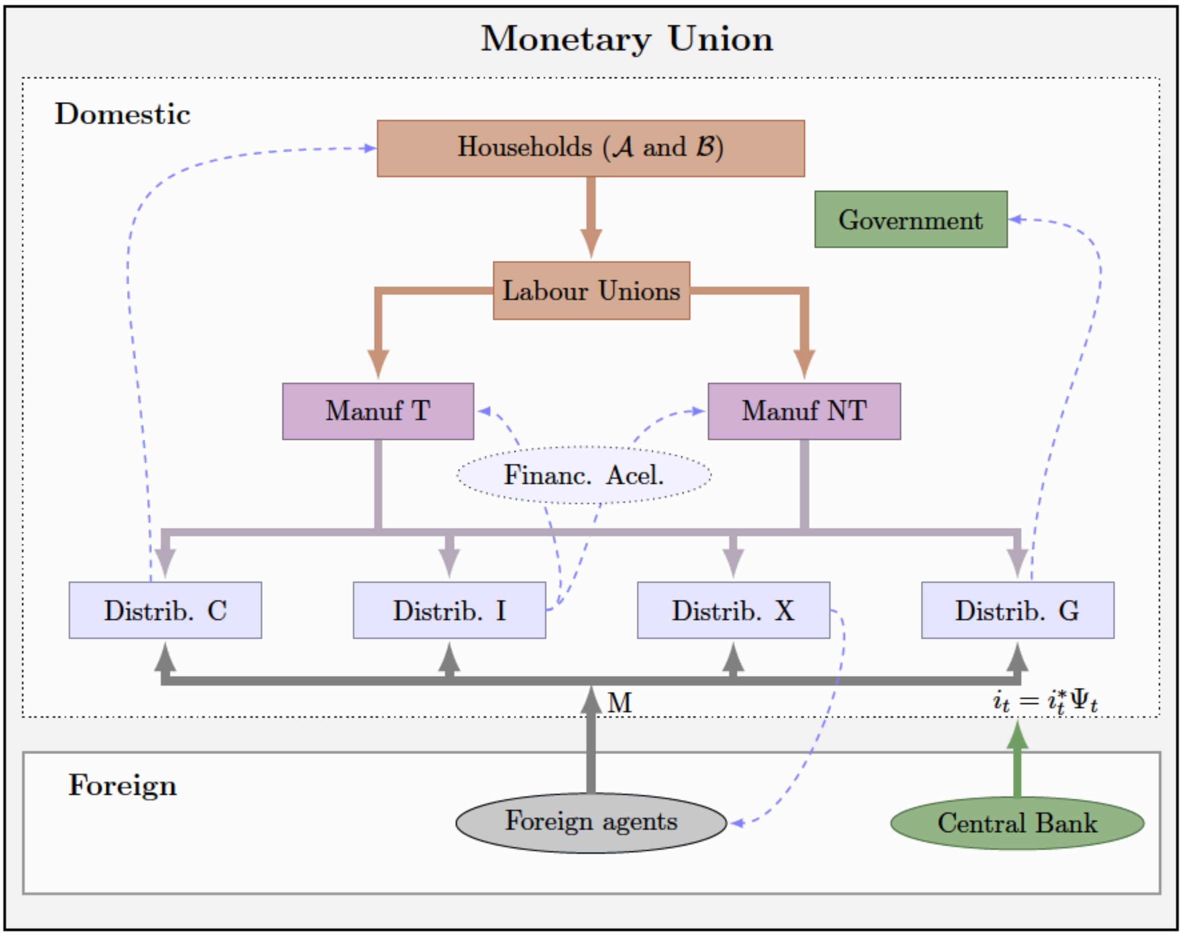
This article contains a detailed description of a large-scale dynamic general equilibrium model for a small economy integrated in a monetary union, developed at Banco de Portugal. The structure of the domestic economy draws fundamentally from Kumhof and Laxton (2007, 2009a,b), Kumhof et al. (2009) and Kumhof et al. (2010). The model is calibrated to match Portuguese and euro area data, and is termed *PESSOA*—**P**ortuguese **E**conomy **S**tructural **S**mall **O**pen **A**nalytical model. It can however be applied to any small economy integrated in a monetary union. *PESSOA* has been used in several policy evaluation and simulation exercises, ranging from the effects of structural reforms to fiscal stimulus and consolidation (Almeida, Castro, and Félix, 2009, 2010; Almeida et al., 2010a,b, 2013). The current version of *PESSOA* contemplates financial frictions, which are increasingly important since the triggering of the 2008 financial crisis, along the lines of Bernanke, Gertler, and Gilchrist (1999b). The research agenda includes plans to augment the model in several dimensions, for instance to have a more prominent banking sector and a richer labor market.

This article has the following structure. Section 2 provides an overview of *PESSOA* and its transmission mechanisms. Section 3 presents a complete description of the model with financial frictions, alongside with a brief motivation of the framework. We derive the optimality conditions for each type of agent, the dynamic equations describing the behavior of economic aggregates, and the corresponding stationary and steady-state formulations. The key equations that characterize the model's solution are clearly identified throughout the text, and listed in the appendix for convenience. Section 4 briefly outlines the version of the model without financial frictions. Section 5 explains how shocks are introduced, turning the model into a powerful tool for policy analysis and simulation. Section 6 presents the calibration. Finally, Section 7 provides a brief overview of the major simulation exercises performed with *PESSOA*.

2 Overview of *PESSOA*

PESSOA is a New-Keynesian DSGE model for a small-open economy integrated in a monetary union. Its basic structure builds mainly on the IMF's large scale multi-country model, the Global Integrated Monetary and Fiscal model (GIMF) Kumhof and Laxton (2007, 2009a,b); Kumhof et al. (2009, 2010). *PESSOA* features non-Ricardian characteristics, a multi-sectoral production structure, imperfect market competition, and a number of nominal, financial and real rigidities that allow for realistic short-run dynamics and create room for welfare improving stabilization policy. Figure 1 depicts the flows of goods between agents in *PESSOA*.

Figure 1: Flows of goods between agents in *PESSOA*



In *PESSOA*, monetary policy is set by a monetary authority, *viz* the European Central Bank (ECB). Since the domestic economy is small, domestic idiosyncratic shocks have no effect on the monetary union's aggregates, and thus on monetary policy decisions. This constitutes an important simplification, since the foreign economy does not need to be modeled explicitly. In addition, the nominal exchange rate *vis-à-vis* the rest of the monetary union is irrevocably set to unity. For tractability, trade and financial flows are restricted to euro area countries, and thus competitiveness gains or losses are confined to price changes against the rest of the monetary union. *PESSOA* also includes an exogenous risk-premium on the euro area interest rate, allowing the assessment of imperfect risk-sharing in case of tense financing conditions.

The economy is composed by nine types of agents: households, labor unions, capital goods producers, entrepreneurial firms, banks, intermediate goods producers, final goods producers, the Government, and the foreign economy (the rest of the euro area). Intermediate goods producers are hereinafter termed as manufacturers, and final goods producers as distributors. We use the terms "entrepreneurial firms" and "entrepreneurs" interchangeably throughout the article.

Households follow the Blanchard-Yaari-Buiter-Weill overlapping generations framework (Blanchard, 1985; Yaari, 1965; Buiter, 1988; Weil, 1989), wherein they live a finite number of periods and the number of newborns equals that of dying agents in every period. The overlapping generations framework is coupled to a life insurance scheme along the lines in Yaari (1965), which enforces net wealth transfers from succumbing households to those that survive. This setup generates a strong non-Ricardian outcome whereby households prefer debt financing to tax financing of public expenditures, since future generations will bear some fraction of the tax burden (Buiter, 1988). Households also face a declining lifetime productivity of labor which adds to the non-Ricardian properties of the model, shifting their proneness towards paying taxes latter, when labor income is lower, rather then sooner.

We consider two types of households: those who perform inter-temporal optimization by accessing asset markets to smooth consumption, and those who do not have access to asset markets and therefore can only optimize intra-temporally. These latter agents, termed hand-to-mouth households, further enhance the non-Ricardian features of the model, since they are unable to shift consumption over time. Hence, any shock that impacts their budget constraint or changes their share on total population has an immediate, direct, impact on macroeconomic aggregates. Both household types derive utility from consumption and leisure, and are subject to external habit formation.

Households rent labor services to heterogeneous labor unions, receiving a given wage and paying the corresponding labor income taxes. Furthermore, the typical household receives its fair share of labor union's dividends, as well as lump-sum transfers from the government and from abroad. In addition, financially unconstrained agents receive dividends from firms and earn/pay interest on asset/debt holdings. They are also remunerated for services in the bankruptcy monitoring of firms, which they perform at no cost and with no effort whenever an entrepreneurial firm goes bankrupt.

Labor unions rent labor services from households and sell them to manufacturers, charging a markup over the wage paid to households. Market power arises from the fact that labor unions supply differentiated, imperfectly substitutable labor services. Dividends are fully transferred to households. This modeling strategy—widely used in DSGE models—implies that households are rewarded for labor services in excess of their marginal rate of substitution between consumption and leisure. Unions face adjustment costs on wage changes in order to mimic the dynamics of sticky wage growth.

Manufacturers—the intermediate goods producer—combine capital, rented from entrepreneurs, and labor services, rented from labor unions, to produce two types of differentiated goods, tradable and non-tradable. These are used by distributors as inputs in different stages of their production process. We obtain staggered price adjustment by imposing adjustment costs on price changes. Adjusting the fraction of time worked by households is also costly, so that hours worked adjust sluggishly. Manufacturers pay social security taxes on their payroll and capital income taxes on profits. After-tax profits are distributed to financially unconstrained households as dividends.

Distributors—the final goods producer—produce four types of differentiated goods, each acquired by a unique type of costumer: consumption goods are acquired by households, investment goods by capital goods producers, government consumption goods by the public sector, and export goods by foreign distributors. Final goods are produced in a two-stage process. In the first stage, the distributor obtains assembled goods by combining domestic tradable goods with imported goods. This stage determines also the imports of the domestic economy. In the second stage, the distributor combines assembled goods with domestic non-tradable goods, obtaining the final good. Analogously to manufacturers, price adjustment costs lead to price staggering. Distributors pay capital income taxes on profits, and distribute dividends to financially unconstrained households.

The baseline model includes a financial transmission mechanism along the lines of Bernanke, Gertler, and Gilchrist (1999a), Christiano, Motto, and Rostagno (2010), and Kumhof et al. (2010), whereby financial frictions affect the after tax return on capital and therefore capital demand. Before each production cycle, capital goods producers buy the undepreciated capital stock from entrepreneurs, combining it with investment goods bought from distributors to produce new installed capital. New capital is then sold to entrepreneurs, which will own it during the next production cycle.

Entrepreneurs do not have access to sufficient internal resources to finance desired capital purchases, but can borrow the difference from banks at a cost. Entrepreneurs are however risky: each faces an idiosyncratic shock that changes the value of the firm after decisions have been made. All entrepreneurs have the same *a priori* expectation about the shock, but the posterior distribution may be quite different. If hit by a severe shock, the value of assets collapse, and the entrepreneur may be forced to declare bankruptcy, handing over the value of the firm to the bank. Contrarily, if hit by a propitious shock, the value of entrepreneur's assets rise, and net worth increases as a result.

Banks operate in a perfectly competitive environment, making zero-profits at all times. They are pure financial intermediaries, with the sole mission of borrowing funds from financially unconstrained households, lending them to entrepreneurs. When lending to an entrepreneur that goes bankrupt, the bank must pay monitoring costs to be able to recover the value of the firm. Since entrepreneurs are risky, so are the loans of banks, who therefore charge a spread over the risk free interest rate to cover for the losses incurred in the mass of entrepreneurs that declares bankruptcy. The existence of identical *a priori* expectations on the idiosyncratic shock implies that the credit spread is identical to all entrepreneurs. Albeit the riskness of individual loans, the aggregate portfolio of banks is risk free, since each bank recovers through the credit spread what is lost to the bankrupt entrepreneurs. Households loans are therefore risk free, and thus they lend to banks at the risk free interest rate.

The financial accelerator mechanism magnifies economic fluctuations, by creating an extra channel through which shocks are transmitted and propagated to the real economy. Any shock decreasing aggregate demand also negatively impacts the price of capital, therefore increasing the number of entrepreneurs in financial distress and reducing the value of net worth for those that survive. As risk increases, so does the credit spread. With lower internal resources and more expensive credit, entrepreneurs acquire less physical capital. Investment is reduced and credit slumps, magnifying the fall in output and employment. Therefore, the model implies, realistically, a countercyclical credit spread, and procyclical investment and stock market value, in addition to procyclical consumption, inflation, and employment already featuring the model without a financial sector. In the model version with no financial accelerator, capital goods producers sell capital directly to manufacturers.

The government buys from distributors government consumption goods, and performs income transfers across households. These activities are financed through direct and indirect taxation—namely labor income, capital income, and consumption taxes—and also through transfers from the monetary union. The government issues one-period bonds to finance excess expenditures, paying an interest rate on public debt that is assumed in general to be equal to the monetary union's financing cost. However, a differential is allowed to exist if an exogenous risk premium on domestic bonds emerges in financial markets, for instance due to credibility issues or to a financial turbulence. To ensure that debt follows an non-explosive path, we impose a fiscal rule linking the government-debt-to-GDP ratio to a pre-determined target. Hence, deviations from that target are followed by tax adjustments or changes in public expenditures, that restore the long-run government debt to a sustainable path. Government consumption goods have no particular use, nor they provide any direct welfare benefit.

The foreign economy in *PESSOA* corresponds to the rest of the monetary union. The domestic economy interacts with the foreign economy *via* goods and financial markets. In the goods market, domestic distributors buy imported goods from abroad to be used in the assemblage stage. Likewise for foreign distributors, who buy export goods to domestic distributors for the same purpose. In the financial market, financially unconstrained domestic households can trade assets with the foreign economy to smooth out consumption over time. Contrary to most DSGE models that feature infinitely lived households, the Blanchard-Yaari framework of *PESSOA* allows to pin down the net foreign asset position endogenously (refer to Frenkel and Razin, 1996 and Harrison et al., 2005 for additional details).

Some parameters in *PESSOA*—mostly related with key economic outcomes such as labor and capital shares or the labor allocation between the tradables and the non-tradables—are calibrated to match Portuguese data from the national accounts, while others rely on the literature mainstream and on estimates for Portugal whenever available. Adding financial frictions to the model does not change the model's parameters or key ratios, with some notable exceptions. In particular, the net foreign position of the economy deteriorates when financial frictions are incorporated. As households desire to hold exactly the same amount of assets (relative to GDP) in both versions of the model, corporate bonds—which are introduced along with financial frictions—draw funds from abroad on a one-to one basis. Finally, physical capital stock is also lower when financial frictions are incorporated, motivated by the existence of a steady-state external finance premium.

3 The model

The model is set in discrete time. The domestic and the foreign economy experience both a constant technological growth rate of $g = T_t/T_{t-1}\ \forall t$, where T_t is the level of labor augmenting technology. We follow the convention of representing time as subscripts, and any other identifier, such as sector identifiers, as superscripts. Time subscripts refer to the period when the quantity is used, and may differ from the period when production or consumption decisions take place. The model's real variables, say X_t, endowed with a steady-state growth rate of g, are converted to stationary form after dividing by T_t. We denote these new variables by $\check{X}_t = X_t/T_t$. The real counterpart of nominal variables, say Z_t, are given by $z_t = Z_t/P_t$, and their stationary form by $\check{z}_t = Z_t/(P_tT_t)$, where P_t is the after tax price level of the consumption good, selected as the *numéraire* of the economy. We use the notation $p_t^X = P_t^X / P_t$ to refer to X's relative price *vis-à-vis* the consumption good.

3.1 Households: general features

This section clarifies the overlapping generations scheme, the utility function, and the labor productivity profile of households.

3.1.1 Overlapping generations scheme

All households evolve according to the overlapping generations scheme *à la* Blanchard (1985). They are subject to stochastic finite lifetimes and face an instant probability of death of $1-\theta \in [0,1]$, independent of age. The overall size of the population N is constant, implying that $N(1-\theta)$ households die in each period t and that the same number are born.

The household's probability of dying at period t equals the probability of staying alive until $t-1$ times the probability of dying at t, $\theta^{t-1}(1-\theta)$. The average life expectancy at any time is constant at $(1-\theta)^{-1}$

$$\begin{aligned} 1(1-\theta)\theta^0 + 2(1-\theta)\theta^1 + 3(1-\theta)\theta^2 + \ldots &= (1-\theta)\sum_{t=1}^{\infty} t\theta^{t-1} = (1-\theta)\sum_{t=1}^{\infty}\theta^{t-1}\sum_{t=1}^{\infty}\theta^{t-1} \\ &= (1-\theta)(1-\theta)^{-1}(1-\theta)^{-1} \\ &= (1-\theta)^{-1} \end{aligned}$$

Instead of biological death, $1-\theta$ can also be interpreted as the relevant economic horizon behind agents' decisions, *i.e.* the probability of "economic death" or an indicator of the degree of "myopia" (Blanchard, 1985; Frenkel and Razin, 1996; Harrison et al., 2005; Bayoumi and Sgherri, 2006). In other words, the future can be seen as a period of lower economic relevance. In this case, $(1-\theta)^{-1}$ is interpreted as the "average planning horizon."

Two types of households coexist in each and every period: asset holders, identified as type-$\mathcal{A}$ households and assumed to represent $N(1-\psi)$ of total population; and "hand-to-mouth households," termed type-$\mathcal{B}$ households and assumed to represent $N\psi$ of total population. Type-$\mathcal{A}$ households smooth out their lifetime consumption by trading assets, whilst type-$\mathcal{B}$ households do not have access to financial markets and therefore consume all their income in each and every period (they are hand-to-mouth households *à la* Galí, López-Salido, and Vallés, 2007).

At time t, the new cohorts being born and the existing cohorts that die are consistent with total population N. Thus, the size of a new generation of type-$\mathcal{A}$ households is $N(1-\psi)(1-\theta)$, and of type-$\mathcal{B}$ households is $N\psi(1-\theta)$. At age a, the size of these cohorts is $N(1-\psi)(1-\theta)\theta^a$ in the case of type-$\mathcal{A}$ households and $N\psi(1-\theta)\theta^a$ in the case of type-$\mathcal{B}$ households, implying an aggregate figure of $N(1-\theta)\theta^a$. At period t, total population is given by all those that were born at t, $t-1$, $t-2$, ... and are still alive

$$N(1-\theta)\theta^0 + N(1-\theta)\theta^1 + N(1-\theta)\theta^2 + \ldots = N(1-\theta)\sum_{s=0}^{\infty}\theta^s = N$$

Each cohort is assumed to be large enough so that $1-\theta$ is also the rate at which the generation size decreases through time. Thus, although each individual is uncertain about the time of death, the size of a cohort declines deterministically through time.

3.1.2 Utility function

A representative household of type $H \in \{\mathcal{A}, \mathcal{B}\}$ with age a derives utility from consumption, $C^H_{a,t}$, and leisure, $1 - L^H_{a,t}$. The term $L^H_{a,t}$ stands for hours worked as a fraction of total time endowment. The expected lifetime utility function is

$$\mathrm{E}_t \sum_{s=0}^{\infty} (\beta\theta)^s U^H_{a+s,t+s} \tag{1}$$

where E_t is the expectation operator and $0 \leq \beta \leq 1$ stands for the standard discount factor. Instantaneous utility is given by the following Constant Relative Risk Aversion (CRRA) specification

$$U^H_{a,t} = \frac{1}{1-\gamma} \left[\left(\frac{C^H_{a,t}}{Hab^H_t} \right)^{\eta^H} (1 - L^H_{a,t})^{1-\eta^H} \right]^{1-\gamma}$$

where $\gamma > 0$ is the risk aversion coefficient, $\eta^H \in [0,1]$ is a distribution parameter, and Hab^H_t stands for type-H households external habits. Aggregation across generations is feasible under multiplicative habits; however, these generate lower consumption persistence as compared to additive habits.

In equation (1), households attach an extra value to the present and over-discount the future, as they account for the probability θ of being dead (Harrison et al., 2005), in addition to the standard discount factor β. Households do not draw utility from holding money, which is consistent with a cashless limit (Woodford, 2003). Moreover, they ignore intergenerational transfers that could change utility levels of not-yet-born cohorts, *i.e.* there is no bequest motive.

Households aggregate consumption, $C^H_{a,t}$ is a bundle of different varieties of consumption goods c, $C^H_{a,t}(c)$, obtained according to the following Constant Elasticity of Substitution (CES) aggregator

$$C^H_{a,t} = \left(\int_0^1 C^H_{a,t}(c)^{\frac{\sigma^{\mathcal{C}}-1}{\sigma^{\mathcal{C}}}} \mathrm{d}c \right)^{\frac{\sigma^{\mathcal{C}}}{\sigma^{\mathcal{C}}-1}}$$

where $\sigma^{\mathcal{C}}_t \geq 0$ stands for the elasticity of substitution between different varieties of the consumption good. Consumption goods are bought from the final goods producers of consumption goods—the distributors. The demand for each variety, $C^H_{a,t}(c)$, is obtained by minimizing consumption expenditure subject to the definition of $C^H_{a,t}$

$$\min_{C^H_{a,t}(c)} \int_0^1 P^{\mathcal{C}}_t(c) C^H_{a,t}(c) \mathrm{d}c \qquad \text{s.t.} \qquad C^H_{a,t} = \left(\int_0^1 C^H_{a,t}(c)^{\frac{\sigma^{\mathcal{C}}-1}{\sigma^{\mathcal{C}}}} \mathrm{d}c \right)^{\frac{\sigma^{\mathcal{C}}}{\sigma^{\mathcal{C}}-1}}$$

where $P_t^{\mathcal{C}}(c)$ is the price of variety c. The solution steps are presented in Box 1. This problem implies that the aggregate price level before consumption taxes $P_t^{\mathcal{C}}$ is a combination of individual prices

$$P_t^{\mathcal{C}} = \left(\int_0^1 P_t^{\mathcal{C}}(c)^{1-\sigma^{\mathcal{C}}} \mathrm{d}c\right)^{\frac{1}{1-\sigma^{\mathcal{C}}}}$$

In addition, the demand for variety c is

$$C_{a,t}^H(c) = \left(\frac{P_t^{\mathcal{C}}(c)}{P_t^{\mathcal{C}}}\right)^{-\sigma^{\mathcal{C}}} C_{a,t}^H$$

Consumption habits are a function of lagged type-H households aggregate consumption, C_{t-1}^H, and thus are not affected by current decisions or by the age of each cohort. In addition, they are measured relative to the respective population size. More specifically

$$Hab_t^{\mathcal{A}} = \left(\frac{C_{t-1}^{\mathcal{A}}}{N(1-\psi)}\right)^{v}$$

$$Hab_t^{\mathcal{B}} = \left(\frac{C_{t-1}^{\mathcal{B}}}{N\psi}\right)^{v}$$

where $v \in [0,1]$ parameterizes the degree of habit persistence.

3.1.3 Labor productivity

According to the life-cycle theory, households labor income increases throughout adulthood and decreases at the retirement age. However, modeling such a profile makes aggregation unfeasible. The life-cycle income profile used herein is based on a simpler pattern according to which labor productivity declines over lifetime at a constant rate χ (Blanchard, 1985). More precisely, labor productivity of a type-H household with age a is

$$\Phi_a = k\chi^a, \ \forall t$$

where $\chi \in [0,1]$ is the rate of decay. This function is depicted in Figure 2.

Population average productivity is normalized to one, implying

$$(1-\theta)\sum_{a=0}^{\infty} \theta^a \ \Phi_a = 1 \Leftrightarrow (1-\theta)k\,(1-\theta\chi)^{-1} = 1 \Leftrightarrow k = \frac{1-\theta\chi}{1-\theta}$$

Since the productivity profile is the same for both household types within the same cohort and the population structure is constant through time, the average productivity of type-$\mathcal{A}$ and of type-$\mathcal{B}$ households must also equal one. Recall that, at each period t, $N(1-\psi)(1-\theta)$ type-$\mathcal{A}$ households are born and $N(1-\psi)(1-\theta)\theta^a$ households of the same type with age a still live. For the case of type-$\mathcal{B}$ households, the equivalent expressions are $N\psi(1-\theta)$ and $N\psi(1-\theta)\theta^a$,

Box 1: The standard cost minimization problem.

A common problem in *PESSOA* is a minimization problem with the following structure

$$\min_{C_{a,t}(c)} \int_0^1 P_t^{\mathcal{C}}(c) C_{a,t}(c) \mathrm{d}c \qquad \text{s.t.} \qquad C_{a,t} = \left(\int_0^1 C_{a,t}(c)^{\frac{\sigma^{\mathcal{C}}-1}{\sigma^{\mathcal{C}}}} \mathrm{d}c \right)^{\frac{\sigma^{\mathcal{C}}}{\sigma^{\mathcal{C}}-1}}$$

where $P_t^{\mathcal{C}}(c)$ is the price of variety c and is taken as given by households. Superscript $H \in \{\mathcal{A}, \mathcal{B}\}$ has been omitted for simplicity, as the problem is identical for both household types. Parameter $\sigma^{\mathcal{C}} \geq 0$ is the elasticity of substitution between varieties. The Lagrangian for this problem is

$$\mathcal{L}(\cdot) = \int_0^1 P_t^{\mathcal{C}}(c) C_{a,t}(c) \mathrm{d}c - \lambda_t \left[\left(\int_0^1 C_{a,t}(c)^{\frac{\sigma^{\mathcal{C}}-1}{\sigma^{\mathcal{C}}}} \mathrm{d}c \right)^{\frac{\sigma^{\mathcal{C}}}{\sigma^{\mathcal{C}}-1}} - C_{a,t} \right]$$

where λ_t is the Lagrange multiplier, which equals the *marginal cost* of acquiring one extra unit of $C_{a,t}$, *i.e.* $\lambda_t = P_t^{\mathcal{C}}$. The following steps solve the problem in the context of households, but generalizing to other agents is straightforward. Making $\partial \mathcal{L}(\cdot)/\partial C_{a,t}(c) = 0$ yields

$$P_t^{\mathcal{C}}(c) - P_t^{\mathcal{C}} \frac{\sigma^{\mathcal{C}}}{\sigma^{\mathcal{C}}-1} \left(\int_0^1 C_{a,t}(c)^{\frac{\sigma^{\mathcal{C}}-1}{\sigma^{\mathcal{C}}}} \mathrm{d}c \right)^{\frac{1}{\sigma^{\mathcal{C}}-1}} \frac{\sigma^{\mathcal{C}}-1}{\sigma^{\mathcal{C}}} C_{a,t}(c)^{\frac{\sigma^{\mathcal{C}}-1}{\sigma^{\mathcal{C}}}-1} = 0 \Leftrightarrow$$

$$\Leftrightarrow P_t^{\mathcal{C}}(c) - P_t^{\mathcal{C}} \left[\left(\int_0^1 C_{a,t}(c)^{\frac{\sigma^{\mathcal{C}}-1}{\sigma^{\mathcal{C}}}} \mathrm{d}c \right)^{\frac{\sigma^{\mathcal{C}}}{\sigma^{\mathcal{C}}-1}} \right]^{\frac{1}{\sigma^{\mathcal{C}}}} C_{a,t}(c)^{-\frac{1}{\sigma^{\mathcal{C}}}} = 0 \Leftrightarrow$$

$$\Leftrightarrow P_t^{\mathcal{C}}(c) - P_t^{\mathcal{C}} \cdot (C_{a,t})^{\frac{1}{\sigma^{\mathcal{C}}}} C_{a,t}(c)^{-\frac{1}{\sigma^{\mathcal{C}}}} = 0 \Leftrightarrow$$

$$\Leftrightarrow C_{a,t}(c) = \left(\frac{P_t^{\mathcal{C}}(c)}{P_t^{\mathcal{C}}} \right)^{-\sigma^{\mathcal{C}}} C_{a,t} \tag{1.1}$$

The aggregate price index $P_t^{\mathcal{C}}$ is obtained after replacing equation (1.1) in the definition of $C_{a,t}$

$$C_{a,t} = \left[\int_0^1 \left(\left(\frac{P_t^{\mathcal{C}}(c)}{P_t^{\mathcal{C}}} \right)^{-\sigma^{\mathcal{C}}} C_{a,t} \right)^{\frac{\sigma^{\mathcal{C}}-1}{\sigma^{\mathcal{C}}}} \mathrm{d}c \right]^{\frac{\sigma^{\mathcal{C}}}{\sigma^{\mathcal{C}}-1}} \Leftrightarrow$$

$$\Leftrightarrow C_{a,t} = \left[\int_0^1 \left(P_t^{\mathcal{C}}(c)^{-\sigma^{\mathcal{C}}} \right)^{\frac{\sigma^{\mathcal{C}}-1}{\sigma^{\mathcal{C}}}} \mathrm{d}c \right]^{\frac{\sigma^{\mathcal{C}}}{\sigma^{\mathcal{C}}-1}} \frac{C_{a,t}}{(P_t^{\mathcal{C}})^{-\sigma^{\mathcal{C}}}} \Leftrightarrow$$

$$\Leftrightarrow (P_t^{\mathcal{C}})^{-\sigma^{\mathcal{C}}} = \left(\int_0^1 P_t^{\mathcal{C}}(c)^{1-\sigma^{\mathcal{C}}} \mathrm{d}c \right)^{\frac{\sigma^{\mathcal{C}}}{\sigma^{\mathcal{C}}-1}} \Leftrightarrow$$

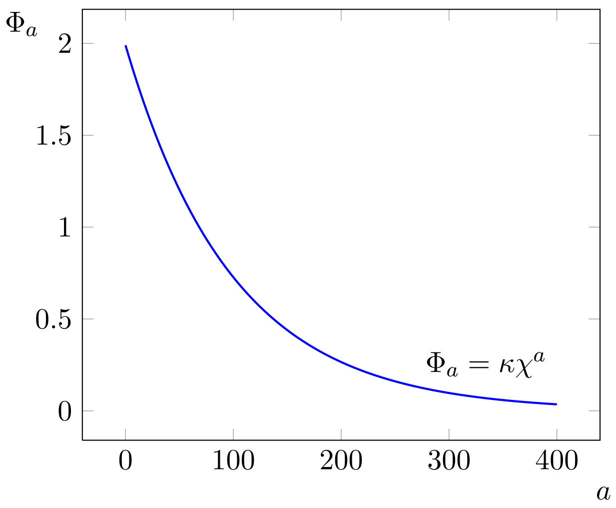
$$\Leftrightarrow P_t^{\mathcal{C}} = \left(\int_0^1 P_t^{\mathcal{C}}(c)^{1-\sigma^{\mathcal{C}}} \mathrm{d}c \right)^{\frac{1}{1-\sigma^{\mathcal{C}}}} \tag{1.2}$$

Equations (1.1) and (1.2) will be recalled throughout this technical appendix whenever a particular agent—household, firm or union—faces an equivalent cost minimization problem.

respectively. The average productivity of type-$\mathcal{A}$ households is therefore

$$\frac{N(1-\psi)(1-\theta)\sum_{a=0}^{\infty} \theta^a \, \Phi_a}{N(1-\psi)} = 1 \Leftrightarrow N(1-\psi)(1-\theta) \sum_{a=0}^{\infty} \theta^a \, \Phi_a = N(1-\psi) \tag{2}$$

Figure 2: Labor productivity of age group a



And for type-$\mathcal{B}$ households

$$\frac{N\psi(1-\theta)\sum_{a=0}^{\infty}\theta^a\ \Phi_a}{N\psi} = 1 \Leftrightarrow N\psi(1-\theta)\sum_{a=0}^{\infty}\theta^a\ \Phi_a = N\psi \tag{3}$$

Equations (2) and (3) imply that labor productivity of type-$\mathcal{A}$ and type-$\mathcal{B}$ households equals the total number of individuals, respectively $N(1-\psi)$ and $N\psi$.

3.2 Households with access to financial markets (type-$\mathcal{A}$ households)

This section presents and solves the maximization problem of Overlapping Generation (type-$\mathcal{A}$) households, *i.e.* those with access to financial markets.

3.2.1 Budget constraint and maximization problem

On the expenditure side, a type-$\mathcal{A}$ household with age a buys a consumption bundle worth $P_t C^{\mathcal{A}}_{a,t}$, where $P_t = (1+\tau^{\mathcal{C}}_t)P^{\mathcal{C}}_t$ is the after-tax price of the consumption bundle, $P^{\mathcal{C}}_t$ is the price paid to distributors, and $\tau^{\mathcal{C}}_t$ is an *ad-valorem* consumption tax. The after tax price, P_t, is taken as *numéraire*, and thus the relative price of final consumption goods is

$$p^{\mathcal{C}}_t = \frac{1}{1+\tau^{\mathcal{C}}_t} \tag{4}$$

where $p^{\mathcal{C}}_t = P^{\mathcal{C}}_t/P_t$. This equation establishes the link between the price paid by households for the final good and the price received by consumption goods distributors. In the steady state

$$p^{\mathcal{C}} = \frac{1}{1+\tau^{\mathcal{C}}} \tag{5}$$

Type-$\mathcal{A}$ households may buy domestic and foreign financial assets. There are two types of domestic bonds: those issued by the national Government, $B_{a,t}$, and those issued by Banks, which act as financial intermediaries by lending to entrepreneurs operating in the tradable sector, $B^{\mathcal{T}}_{a,t}$, and in the nontradable sector, $B^{\mathcal{N}}_{a,t}$. For brevity, let $\widehat{B}_{a,t} \equiv B_{a,t} + {B_{a,t}}^{\mathcal{T}} + B^{\mathcal{N}}_{a,t}$ denote all domestically issued bonds held by type-$\mathcal{A}$ households with age a at time t. Foreign bonds are denoted by $B^*_{a,t}$. Domestic government bonds cannot be held by foreigners, *i.e.* there is a complete home bias in domestic government debt. Thus, markets are incomplete in this model.

All bonds are denominated in local currency, implying that $\widehat{B}_{a,t}$ and $B^*_{a,t}$ could in principle be denominated in distinct domestic and foreign currency, respectively. In this case, foreign assets could be converted into domestic currency through the indirect quotation of the nominal exchange rate *vis-á-vis* the rest of the world, ε_t (and foreign assets would worth $\varepsilon_t B^*_{a,t}$ in domestic currency). In the special case of a Monetary Union, however, the nominal exchange rate equals one, and there is no difference in the unit of account of $\widehat{B}_{a,t}$ and $B^*_{a,t}$. Domestic bonds and foreign bonds held between period t and period $t+1$ pay a gross nominal interest rate of i_t and i^*_t, respectively, at the beginning of period $t+1$. The interest rate i is allowed to differ from i^* given that type-$\mathcal{A}$ households demand a risk premium Ψ for holding domestic assets. It is common to set a steady-state value of $\Psi = 1$ in the case of a Monetary Union, which implies no deviation between domestic and foreign interest rates.

A type-$\mathcal{A}$ household with age a has three sources of income: labor, capital, and interest. The latter is associated with financial wealth derived from bond holdings.

Households supply labor services to labor unions, paying taxes to the government on their labor income, and receiving an after-tax amount of $(1 - \tau^{\mathcal{L}}_t) W_t \Phi_a L^{\mathcal{A}}_{a,t}$, where $W_t \Phi_a$ is the productivity-adjusted wage rate and $\tau^{\mathcal{L}}_t$ is the employees labor income tax. The wage rate W_t is taken as given.

There exists a competitive life insurance company which guarantees that households do not implement intergenerational wealth transfers. Given that there is individual uncertainty about the time of death, in the absence of this insurance company households could die leaving either unintended positive bequests, if they die as creditors, or unintended negative bequests, if they die indebted. The gap between individual uncertainty and no aggregate uncertainty is filled by the insurance company, who collects the wealth of $1-\theta$ agents who did not survive between two consecutive periods, and distributes it to the θ agents who survived. More precisely, each household with age a receives an additional $(1-\theta)/\theta$ for each unit of financial wealth. Thus, financial wealth is multiplied by a factor $1 + (1-\theta)/\theta = 1/\theta > 1$, becoming

$$\frac{1}{\theta}\Big(i_{t-1}\widehat{B}_{a-1,t-1} + i^*_{t-1}\Psi\varepsilon_t B^*_{a-1,t-1}\Big)$$

The assumption of a large population turnover ensures that income receipts and the payout of the insurance company match in every period.

Another source of revenue of type-$\mathcal{A}$ households is the remuneration for services in the bankruptcy monitoring of firms. In real terms this equals $rbr_{a,t} = p^{\mathcal{N}}_t rbr^{\mathcal{N}}_{a,t} + p^{\mathcal{T}}_t rbr^{\mathcal{T}}_{a,t}$, where $rbr^{\mathcal{N}}_{a,t}$ and $rbr^{\mathcal{T}}_{a,t}$ are the remuneration for services in the nontradable and tradable sectors, respectively, and $p^{\mathcal{N}}_t$ and $p^{\mathcal{T}}_t$ are the relative prices, measured against the *numérarie* of the economy, P_t.

Type-$\mathcal{A}$ households also receive dividends from firm/union i operating in sector x, denoted as $D^{\mathcal{A},x}_{a,t}(i)$, $x \in \{\mathcal{T}, \mathcal{N}, \mathcal{U}, \mathcal{C}, \mathcal{G}, \mathcal{I}, \mathcal{X}, \mathcal{K}, \mathcal{EP}\}$. In the intermediate goods sector, dividends originate from tradable ($\mathcal{T}$) and nontradable ($\mathcal{N}$) manufacturers. In the final goods sector, dividends originate from distributors of private consumption ($\mathcal{C}$), government consumption ($\mathcal{G}$), investment ($\mathcal{I}$) and export ($\mathcal{X}$) goods. The remaining dividends originate from capital producers ($\mathcal{K}$), unions ($\mathcal{U}$) and, in the financial accelerator context, from entrepreneurs ($\mathcal{EP}$). Finally, households receive lump-sum transfers from the government, $TRG^{\mathcal{A}}_{a,t}$, and from abroad, $TRX^{\mathcal{A}}_{a,t}$. The latter is measured in local currency, implying that the amount effectively received is $\varepsilon_t TRX^{\mathcal{A}}_{a,t}$.

The nominal budget constraint embodying that expenditures cannot exceed revenues is

$$\begin{aligned} P_t C^{\mathcal{A}}_{a,t} + \widehat{B}_{a,t} + \varepsilon_t B^*_{a,t} \leq & \frac{1}{\theta}\left[i_{t-1}\widehat{B}_{a-1,t-1} + i^*_{t-1}\Psi\varepsilon_t B^*_{a-1,t-1}\right] \\ & + (1-\tau^{\mathcal{L}}_t) W_t \Phi_a L^{\mathcal{A}}_{a,t} + P_t rbr_{a,t} \\ & + \sum_{\substack{x\in\{\mathcal{T},\mathcal{N},\mathcal{U},\mathcal{C},\mathcal{G} \\ \mathcal{I},\mathcal{X},\mathcal{K},\mathcal{EP}\}}} \int_0^1 D^{\mathcal{A},x}_{a,t}(i)\mathrm{d}i + TRG^{\mathcal{A}}_{a,t} + \varepsilon_t TRX^{\mathcal{A}}_{a,t} \end{aligned} \tag{6}$$

The optimization problem of a type-$\mathcal{A}$ household consists in maximizing expected lifetime utility (1) with respect to $\{C^{\mathcal{A}}_{a+s,t+s}, L^{\mathcal{A}}_{a+s,t+s}, B_{a+s,t+s}, B^*_{a+s,t+s}, B^{\mathcal{T}}_{a+s,t+s}, B^{\mathcal{N}}_{a+s,t+s}\}^{\infty}_{s=0}$, subject to (6). The problem and the first-order conditions are presented in detail in Box 2.

Combining equations (2.2) and (2.3) from Box 2, we obtain the consumption/labor supply choice

$$\begin{aligned} C^{\mathcal{A}}_{a,t} &= \frac{\eta^{\mathcal{A}}}{1-\eta^{\mathcal{A}}}(1-\tau^{\mathcal{L}}_t) w_t \Phi_a (1-L^{\mathcal{A}}_{a,t}) \Leftrightarrow \\ \Leftrightarrow\ 1-L^{\mathcal{A}}_{a,t} &= \frac{1-\eta^{\mathcal{A}}}{\eta^{\mathcal{A}}}\frac{C^{\mathcal{A}}_{a,t}}{(1-\tau^{\mathcal{L}}_t) w_t \Phi_a} \end{aligned} \tag{7}$$

where $w_t = W_t/P_t$ is the real wage.
Aggregate consumption of type-$\mathcal{A}$ households at time t is

$$C^{\mathcal{A}}_t = N(1-\psi)(1-\theta)\sum_{a=0}^{\infty}\theta^a C^{\mathcal{A}}_{a,t} \tag{8}$$

and effective labor supply is

$$L^{\mathcal{A}}_t = N(1-\psi)(1-\theta)\sum_{a=0}^{\infty}\theta^a \Phi_a L^{\mathcal{A}}_{a,t} \tag{9}$$

Aggregation takes into account the size of each cohort at the time of birth, $N(1-\psi)(1-\theta)$, and the size of the remaining generations, $N(1-\psi)(1-\theta)\theta^a$. In equation (9), individual labor supply is weighted by labor productivity to obtain the effective labor supply. Using (7) in (8)

$$C^{\mathcal{A}}_t = N(1-\psi)(1-\theta)\sum_{a=0}^{\infty}\theta^a \left[\frac{\eta^{\mathcal{A}}}{1-\eta^{\mathcal{A}}}(1-\tau^{\mathcal{L}}_t) w_t \Phi_a (1-L^{\mathcal{A}}_{a,t})\right]$$

Box 2: Type-$\mathcal{A}$ households maximization problem.

Type-$\mathcal{A}$ households maximize expected lifetime utility (1) with respect to

$$\{C^{\mathcal{A}}_{a+s,t+s}, L^{\mathcal{A}}_{a+s,t+s}, B_{a+s,t+s}, B^{*}_{a+s,t+s}, B^{\mathcal{T}}_{a+s,t+s}, B^{\mathcal{N}}_{a+s,t+s}\}_{s=0}^{\infty}$$

subject to (6). The Lagrangian for the maximization problem is

$$\begin{aligned}
\mathcal{L}(\cdot) \quad = \quad & \mathrm{E}_t \sum_{s=0}^{\infty} (\beta\theta)^s \left\{ \frac{1}{1-\gamma} \left[\left(\frac{C^{\mathcal{A}}_{a+s,t+s}}{\left(\frac{C^{\mathcal{A}}_{t+s-1}}{N(1-\psi)}\right)^{v}} \right)^{\eta^{\mathcal{A}}} (1 - L^{\mathcal{A}}_{a+s,t+s})^{1-\eta^{\mathcal{A}}} \right]^{1-\gamma} \right. \\
+ \quad & \frac{\lambda^{\mathcal{A}}_{a+s,t+s}}{P_{t+s}} \left[\frac{1}{\theta} \left[i_{t+s-1} (B_{a+s-1,t+s-1} + B^{\mathcal{T}}_{a+s-1,t+s-1} + B^{\mathcal{N}}_{a+s-1,t+s-1}) \right. \right. \\
+ \quad & \left. i^{*}_{t+s-1} \Psi \varepsilon_{t+s} B^{*}_{a+s-1,t+s-1} \right] + (1 - \tau^{\mathcal{L}}_{t+s}) W_{t+s} \Phi_{a+s} L^{\mathcal{A}}_{a+s,t+s} \\
+ \quad & P_{t+s} rbr_{a+s,t+s} + \sum_{\substack{x \in \{\mathcal{T},\mathcal{N},\mathcal{U},\mathcal{C},\mathcal{G} \\ \mathcal{I},\mathcal{X},\mathcal{K},\mathcal{EP}\}}} \int_0^1 D^{\mathcal{A},x}_{a+s,t+s}(i) \mathrm{d}i + TRG^{\mathcal{A}}_{a+s,t+s} + \varepsilon_t \, TRX^{\mathcal{A}}_{a+s,t+s} \\
- \quad & \left. \left. P_{t+s} C^{\mathcal{A}}_{a+s,t+s} - (B_{a+s,t+s} + B^{\mathcal{T}}_{a+s,t+s} + B^{\mathcal{N}}_{a+s,t+s}) - \varepsilon_{t+s} B^{*}_{a+s,t+s} \right] \right\}
\end{aligned}$$

where $\lambda^{\mathcal{A}}_{a+s,t+s} = \Lambda^{\mathcal{A}}_{a+s,t+s} P_{t+s}$ is the marginal utility of an extra unit of the consumption bundle and $\Lambda^{\mathcal{A}}_{a+s,t+s}$ the original lagrange multiplier.
Let

$$u^{\mathcal{A}}_{a,t}(C^{\mathcal{A}}_{a,t}, L^{\mathcal{A}}_{a,t}) = \left[\frac{C^{\mathcal{A}}_{a,t}}{\left(\frac{C^{\mathcal{A}}_{t-1}}{N(1-\psi)}\right)^{v}} \right]^{\eta^{\mathcal{A}}} (1 - L^{\mathcal{A}}_{a,t})^{1-\eta^{\mathcal{A}}} \tag{2.1}$$

and notice that the first derivatives with respect to $C^{\mathcal{A}}_{a,t}$ and $L^{\mathcal{A}}_{a,t}$ are

$$\frac{\partial u^{\mathcal{A}}_{a,t}}{\partial C^{\mathcal{A}}_{a,t}} = (1 - L^{\mathcal{A}}_{a,t})^{1-\eta^{\mathcal{A}}} \left(\frac{C^{\mathcal{A}}_{t-1}}{N(1-\psi)} \right)^{-v\eta^{\mathcal{A}}} \eta^{\mathcal{A}} (C^{\mathcal{A}}_{a,t})^{\eta^{\mathcal{A}}-1} = u^{\mathcal{A}}_{a,t} \frac{\eta^{\mathcal{A}}}{C^{\mathcal{A}}_{a,t}}$$

$$\frac{\partial u^{\mathcal{A}}_{a,t}}{\partial L^{\mathcal{A}}_{a,t}} = - \left(\frac{C^{\mathcal{A}}_{a,t}}{\left(\frac{C^{\mathcal{A}}_{t-1}}{N(1-\psi)}\right)^{v}} \right)^{\eta^{\mathcal{A}}} (1 - \eta^{\mathcal{A}}) (1 - L^{\mathcal{A}}_{a,t})^{-\eta^{\mathcal{A}}} = -u^{\mathcal{A}}_{a,t} \frac{1 - \eta^{\mathcal{A}}}{1 - L^{\mathcal{A}}_{a,t}}$$

The first-order conditions are as follows.

1. Optimal consumption

$$\frac{\partial \mathcal{L}(\cdot)}{\partial C^{\mathcal{A}}_{a,t}} = 0 \Leftrightarrow \lambda^{\mathcal{A}}_{a,t} = \frac{\partial u^{\mathcal{A}}_{a,t}}{\partial C^{\mathcal{A}}_{a,t}} (u^{\mathcal{A}}_{a,t})^{-\gamma} \Leftrightarrow \lambda^{\mathcal{A}}_{a,t} = \frac{\eta^{\mathcal{A}} (u^{\mathcal{A}}_{a,t})^{1-\gamma}}{C^{\mathcal{A}}_{a,t}} \tag{2.2}$$

2. Labor supply

$$\frac{\partial \mathcal{L}(\cdot)}{\partial L^{\mathcal{A}}_{a,t}} = 0 \Leftrightarrow \frac{\lambda^{\mathcal{A}}_{a,t}}{P_t}(1-\tau^{\mathcal{L}}_t)W_t\Phi_a = \frac{\partial u^{\mathcal{A}}_{a,t}}{\partial L^{\mathcal{A}}_{a,t}}(u^{\mathcal{A}}_{a,t})^{-\gamma} \Leftrightarrow$$
$$\Leftrightarrow \lambda^{\mathcal{A}}_{a,t}(1-\tau^{\mathcal{L}}_t)w_t\Phi_a = (1-\eta^{\mathcal{A}})\frac{(u^{\mathcal{A}}_{a,t})^{1-\gamma}}{1-L^{\mathcal{A}}_{a,t}} \Leftrightarrow$$
$$\Leftrightarrow \lambda^{\mathcal{A}}_{a,t} = \frac{1-\eta^{\mathcal{A}}}{(1-\tau^{\mathcal{L}}_t)w_t\Phi_a}\frac{(u^{\mathcal{A}}_{a,t})^{1-\gamma}}{1-L^{\mathcal{A}}_{a,t}} \tag{2.3}$$

where $w_t = W_t/P_t$ is the real wage.

3. Optimal domestic government bond holdings

$$\frac{\partial \mathcal{L}(\cdot)}{\partial B_{a,t}} = 0 \Leftrightarrow \frac{\lambda^{\mathcal{A}}_{a,t}}{P_t} = \beta\theta \mathrm{E}_t \frac{\lambda^{\mathcal{A}}_{a+1,t+1}}{P_{t+1}}\frac{i_t}{\theta} \Leftrightarrow \lambda^{\mathcal{A}}_{a,t} = \beta \mathrm{E}_t \lambda^{\mathcal{A}}_{a+1,t+1}\frac{i_t}{\pi_{t+1}} \tag{2.4}$$

where $\pi_{t+1} = P_{t+1}/P_t$ corresponds to the (gross) inflation rate between period t and period $t+1$.

4. Optimal foreign bond holdings

$$\frac{\partial \mathcal{L}(\cdot)}{\partial B^{*}_{a,t}} = 0 \Leftrightarrow \frac{\lambda^{\mathcal{A}}_{a,t}}{P_t}\varepsilon_t = \beta\theta \mathrm{E}_t \frac{\lambda^{\mathcal{A}}_{a+1,t+1}}{P_{t+1}}\frac{{i_t}^{*}\Psi\varepsilon_{t+1}}{\theta} = 0 \Leftrightarrow$$
$$\Leftrightarrow \lambda^{\mathcal{A}}_{a,t} = \beta \mathrm{E}_t \lambda^{\mathcal{A}}_{a+1,t+1}\frac{{i_t}^{*}\Psi\frac{\varepsilon_{t+1}}{\varepsilon_t}}{\pi_{t+1}} \tag{2.5}$$

5. Optimal private domestic bond holdings

The optimal conditions associated with $\partial\mathcal{L}(\cdot)/\partial B^{\mathcal{T}}_{a,t} = \partial\mathcal{L}(\cdot)/\partial B^{\mathcal{N}}_{a,t} = 0$ are identical to $\partial\mathcal{L}(\cdot)/\partial B_{a,t} = 0$ and are therefore omitted.

$$= \frac{\eta^{\mathcal{A}}}{1-\eta^{\mathcal{A}}}(1-\tau^{\mathcal{L}}_t)w_t N(1-\psi)(1-\theta)\left(\sum_{a=0}^{\infty}\theta^a\Phi_a - \sum_{a=0}^{\infty}\theta^a\Phi_a L^{\mathcal{A}}_{a,t}\right)$$
$$= \frac{\eta^{\mathcal{A}}}{1-\eta^{\mathcal{A}}}(1-\tau^{\mathcal{L}}_t)w_t[N(1-\psi) - L^{\mathcal{A}}_t]$$

The simplification in the last step is due to (2) and (9). The above equation can be rewritten as

$$\frac{C^{\mathcal{A}}_t}{N(1-\psi)-L^{\mathcal{A}}_t} = \frac{\eta^{\mathcal{A}}}{1-\eta^{\mathcal{A}}}(1-\tau^{\mathcal{L}}_t)w_t \tag{10}$$

which clarifies how changes in real wages w_t or in the labor income tax $\tau^{\mathcal{L}}_t$ affect the consumption/labor supply choice and confirms the distortionary nature of labor taxes. In stationary form (*i.e.* after rescaling trend variables by technology) this equation becomes

$$\frac{\check{C_t}^{\mathcal{A}}}{N(1-\psi)-L^{\mathcal{A}}_t} = \frac{\eta^{\mathcal{A}}}{1-\eta^{\mathcal{A}}}(1-\tau^{\mathcal{L}}_t)\check{w}_t \tag{11}$$

In the steady state

$$\frac{\check{C}^{\mathcal{A}}}{N(1-\psi)-L^{\mathcal{A}}} = \frac{\eta^{\mathcal{A}}}{1-\eta^{\mathcal{A}}}(1-\tau^{\mathcal{L}})\check{w} \tag{12}$$

Combining equations (2.4) and (2.5) results in the no-arbitrage condition in financial markets—the uncovered interest rate parity

$$i_t = i_t^* \Psi \frac{\varepsilon_{t+1}}{\varepsilon_t} \tag{13}$$

where we ignored the expected value operator. This equation is already in stationary form, since none of the variables has a deterministic trend. In the steady state

$$i = i^* \Psi \tag{14}$$

3.2.2 Aggregate consumption and aggregate wealth

A key behavioral equation of type-$\mathcal{A}$ households is the aggregate consumption equation, linking optimal aggregate consumption to wealth. To obtain this equation, one needs to establish the optimal link between consumption and wealth for each generation a, and then aggregate across generations. It is useful to break this exercise in five steps: (I) *Euler equations* for each generation a; (II) the *no-Ponzi game condition* and the *subjective discount factor*; (III) *human wealth*, stemming from *labor-based* and from *capital-based* income; (IV) *financial wealth*, associated with bond holdings; and (V) *marginal propensity to consume out of wealth.*

(I) Euler equations for each generation a. An Euler equation for each generation a is obtained by replacing (2.3) in (2.4) (we ignore the expected value operator for simplicity)

$$\frac{1-\eta^{\mathcal{A}}}{(1-\tau_t^{\mathcal{L}})w_t\Phi_a}\frac{(u_{a,t})^{1-\gamma}}{1-L_{a,t}^{\mathcal{A}}} = \beta\frac{1-\eta^{\mathcal{A}}}{(1-\tau_{t+1}^{\mathcal{L}})w_{t+1}\Phi_{a+1}}\frac{(u_{a+1,t+1})^{1-\gamma}}{1-L_{a+1,t+1}^{\mathcal{A}}}\frac{i_t}{\pi_{t+1}} \tag{15}$$

where $\pi_{t+1} = P_{t+1}/P_t$ stands for the gross inflation rate of final consumer goods and $u_{a,t}$ is defined in (2.1). This equation can be expressed as

$$C_{a+1,t+1}^{\mathcal{A}} = j_t C_{a,t}^{\mathcal{A}} \tag{16}$$

where the new object

$$j_t = \left(\frac{C_t^{\mathcal{A}}}{C_{t-1}^{\mathcal{A}}}\right)^{v\eta^{\mathcal{A}}\left(1-\frac{1}{\gamma}\right)} \left(\frac{1-\tau_{t+1}^{\mathcal{L}}}{1-\tau_t^{\mathcal{L}}}\frac{w_{t+1}}{w_t}\right)^{\left(1-\eta^{\mathcal{A}}\right)\left(1-\frac{1}{\gamma}\right)} \left(\beta\frac{i_t}{\pi_{t+1}}\right)^{\frac{1}{\gamma}} \chi^{\left(1-\eta^{\mathcal{A}}\right)\left(1-\frac{1}{\gamma}\right)} \tag{17}$$

is introduced to simplify notation. The derivation is presented in Box 3. Notice that equation (17) is not generation specific, a fact that allows aggregation across generations. In stationary form j_t becomes

$$j_t = \left(\frac{\check{C}_t^{\mathcal{A}} g}{\check{C}_{t-1}^{\mathcal{A}}} \right)^{v\eta^{\mathcal{A}}\left(1-\frac{1}{\gamma}\right)} \left(\frac{1-\tau_{t+1}^{\mathcal{L}}}{1-\tau_t^{\mathcal{L}}} \frac{\check{w}_{t+1} g}{\check{w}_t} \right)^{\left(1-\eta^{\mathcal{A}}\right)\left(1-\frac{1}{\gamma}\right)} \left(\beta \frac{i_t}{\pi_{t+1}} \right)^{\frac{1}{\gamma}} \chi^{\left(1-\eta^{\mathcal{A}}\right)\left(1-\frac{1}{\gamma}\right)} \tag{18}$$

where g is the deterministic growth rate of the (labor-augmenting) technological progress. In the steady state

$$j = g^{\left(1+\eta^{\mathcal{A}}(v-1)\right)\left(1-\frac{1}{\gamma}\right)} \left(\beta \frac{i}{\pi} \right)^{\frac{1}{\gamma}} \chi^{\left(1-\eta^{\mathcal{A}}\right)\left(1-\frac{1}{\gamma}\right)} \tag{19}$$

(II) The no-Ponzi game condition and the subjective discount factor. The following no-Ponzi game condition is assumed to hold

$$\lim_{s\to\infty} \mathrm{E}_t \tilde{R}_{t,s} [\widehat{B}_{a+s,t+s} + \varepsilon_{t+s} B^*_{a+s,t+s}] = 0, \ \forall a$$

where $\tilde{R}_{t,s}$ is the nominal subjective discount factor

$$\tilde{R}_{t,s} = \begin{cases} \prod_{l=1}^{s} \frac{\theta}{i_{t+l-1}} & \text{for } s > 0 \\ 1 & \text{for } s = 0 \end{cases} \tag{20}$$

This condition states that households cannot engage in Ponzi schemes, which would allow them to attained infinite consumption by rolling-over debt over lifetime (recall that the no-Ponzi game condition is a constraint that prevents overaccumulation of debt, whereas the transversality condition is an optimality condition that rules out overaccumulation of wealth). Let

$$r_t = \frac{i_t}{\pi_{t+1}} \tag{21}$$

denote the (gross) real interest rate. In the steady state

$$r = \frac{i}{\pi} \tag{22}$$

Equation (20) for $s > 0$ can thus be rewritten as

$$\tilde{R}_{t,s} = \prod_{l=1}^{s} \left[\frac{\theta}{r_{t+l-1}} (\pi_{t+l})^{-1} \right] = \tilde{r}_{t,s} \prod_{l=1}^{s} (\pi_{t+l})^{-1} \tag{23}$$

Box 3: Generation a Euler equation.

To derive $C^{\mathcal{A}}_{a+1,t+1} = j_t C^{\mathcal{A}}_{a,t}$, notice that equation (15) can be rewritten as

$$\frac{(1-\tau^{\mathcal{L}}_{t+1})w_{t+1}\Phi_{a+1}}{(1-\tau^{\mathcal{L}}_{t})w_{t}\Phi_{a}}\frac{\pi_{t+1}}{i_t}\frac{1}{\beta} = \left(\frac{u_{a+1,t+1}}{u_{a,t}}\right)^{1-\gamma}\frac{1-L^{\mathcal{A}}_{a,t}}{1-L^{\mathcal{A}}_{a+1,t+1}} \tag{3.1}$$

Using (2.1) and (7), the right-hand side simplifies to

$$\left[\frac{\left(\frac{C^{\mathcal{A}}_{a+1,t+1}}{\left(C^{\mathcal{A}}_{t}/N(1-\psi)\right)^{v}}\right)^{\eta^{\mathcal{A}}}(1-L^{\mathcal{A}}_{a+1,t+1})^{1-\eta^{\mathcal{A}}}}{\left(\frac{C^{\mathcal{A}}_{a,t}}{\left(C^{\mathcal{A}}_{t-1}/N(1-\psi)\right)^{v}}\right)^{\eta^{\mathcal{A}}}(1-L^{\mathcal{A}}_{a,t})^{1-\eta^{\mathcal{A}}}}\right]^{1-\gamma}\frac{1-L^{\mathcal{A}}_{a,t}}{1-L^{\mathcal{A}}_{a+1,t+1}}$$

$$=\left(\frac{C^{\mathcal{A}}_{a+1,t+1}/C^{\mathcal{A}}_{a,t}}{\left(C^{\mathcal{A}}_{t}/C^{\mathcal{A}}_{t-1}\right)^{v}}\right)^{\eta^{\mathcal{A}}(1-\gamma)}\left(\frac{1-L^{\mathcal{A}}_{a+1,t+1}}{1-L^{\mathcal{A}}_{a,t}}\right)^{(1-\eta^{\mathcal{A}})(1-\gamma)-1}$$

$$=\left(\frac{C^{\mathcal{A}}_{a+1,t+1}/C^{\mathcal{A}}_{a,t}}{\left(C^{\mathcal{A}}_{t}/C^{\mathcal{A}}_{t-1}\right)^{v}}\right)^{\eta^{\mathcal{A}}(1-\gamma)}\left(\frac{\frac{1-\eta^{\mathcal{A}}}{\eta^{\mathcal{A}}}\frac{C^{\mathcal{A}}_{a+1,t+1}}{(1-\tau^{\mathcal{L}}_{t+1})w_{t+1}\Phi_{a+1}}}{\frac{1-\eta^{\mathcal{A}}}{\eta^{\mathcal{A}}}\frac{C^{\mathcal{A}}_{a,t}}{(1-\tau^{\mathcal{L}}_{t})w_{t}\Phi_{a}}}\right)^{(1-\eta^{\mathcal{A}})(1-\gamma)-1}$$

$$=\left(\frac{C^{\mathcal{A}}_{a+1,t+1}/C^{\mathcal{A}}_{a,t}}{\left(C^{\mathcal{A}}_{t}/C^{\mathcal{A}}_{t-1}\right)^{v}}\right)^{\eta^{\mathcal{A}}(1-\gamma)}\left(\frac{C^{\mathcal{A}}_{a+1,t+1}}{C^{\mathcal{A}}_{a,t}}\frac{(1-\tau^{\mathcal{L}}_{t})w_{t}\Phi_{a}}{(1-\tau^{\mathcal{L}}_{t+1})w_{t+1}\Phi_{a+1}}\right)^{(1-\eta^{\mathcal{A}})(1-\gamma)-1}$$

Using this result, equation (3.1) can be expanded to

$$\left(\frac{(1-\tau^{\mathcal{L}}_{t+1})w_{t+1}\Phi_{a+1}}{(1-\tau^{\mathcal{L}}_{t})w_{t}\Phi_{a}}\right)^{(1-\eta^{\mathcal{A}})(1-\gamma)}\frac{\pi_{t+1}}{\beta\cdot i_t}=\left(\frac{\frac{C^{\mathcal{A}}_{a+1,t+1}}{C^{\mathcal{A}}_{a,t}}}{\left(\frac{C^{\mathcal{A}}_{t}}{C^{\mathcal{A}}_{t-1}}\right)^{v}}\right)^{\eta^{\mathcal{A}}(1-\gamma)}\left(\frac{C^{\mathcal{A}}_{a+1,t+1}}{C^{\mathcal{A}}_{a,t}}\right)^{(1-\eta^{\mathcal{A}})(1-\gamma)-1}\Leftrightarrow$$

$$\Leftrightarrow\left(\frac{(1-\tau^{\mathcal{L}}_{t+1})w_{t+1}\Phi_{a+1}}{(1-\tau^{\mathcal{L}}_{t})w_{t}\Phi_{a}}\right)^{(1-\eta^{\mathcal{A}})(1-\gamma)}\frac{\pi_{t+1}}{\beta\cdot i_t}=\left(\frac{C^{\mathcal{A}}_{t}}{C^{\mathcal{A}}_{t-1}}\right)^{-v\eta^{\mathcal{A}}(1-\gamma)}\left(\frac{C^{\mathcal{A}}_{a+1,t+1}}{C^{\mathcal{A}}_{a,t}}\right)^{-\gamma}\Leftrightarrow$$

$$\Leftrightarrow\left(\frac{C^{\mathcal{A}}_{a+1,t+1}}{C^{\mathcal{A}}_{a,t}}\right)^{-\gamma}=\left(\frac{C^{\mathcal{A}}_{t}}{C^{\mathcal{A}}_{t-1}}\right)^{v\eta^{\mathcal{A}}(1-\gamma)}\left(\frac{(1-\tau^{\mathcal{L}}_{t+1})w_{t+1}\Phi_{a+1}}{(1-\tau^{\mathcal{L}}_{t})w_{t}\Phi_{a}}\right)^{(1-\eta^{\mathcal{A}})(1-\gamma)}\frac{\pi_{t+1}}{\beta\cdot i_t}\Leftrightarrow$$

$$\Leftrightarrow C^{\mathcal{A}}_{a+1,t+1}=\left(\frac{C^{\mathcal{A}}_{t}}{C^{\mathcal{A}}_{t-1}}\right)^{v\eta^{\mathcal{A}}(1-\frac{1}{\gamma})}\left(\frac{(1-\tau^{\mathcal{L}}_{t+1})w_{t+1}\Phi_{a+1}}{(1-\tau^{\mathcal{L}}_{t})w_{t}\Phi_{a}}\right)^{(1-\eta^{\mathcal{A}})(1-\frac{1}{\gamma})}\left(\beta\frac{i_t}{\pi_{t+1}}\right)^{\frac{1}{\gamma}}C^{\mathcal{A}}_{a,t}\Leftrightarrow$$

$$\Leftrightarrow C^{\mathcal{A}}_{a+1,t+1}=\left(\frac{C^{\mathcal{A}}_{t}}{C^{\mathcal{A}}_{t-1}}\right)^{v\eta^{\mathcal{A}}(1-\frac{1}{\gamma})}\left(\frac{(1-\tau^{\mathcal{L}}_{t+1})w_{t+1}}{(1-\tau^{\mathcal{L}}_{t})w_{t}}\right)^{(1-\eta^{\mathcal{A}})(1-\frac{1}{\gamma})}\left(\beta\frac{i_t}{\pi_{t+1}}\right)^{\frac{1}{\gamma}}\chi^{(1-\eta^{\mathcal{A}})(1-\frac{1}{\gamma})}C^{\mathcal{A}}_{a,t}$$

The last step follows from $\Phi_a = k\chi^a$. Letting

$$j_t=\left(\frac{C^{\mathcal{A}}_{t}}{C^{\mathcal{A}}_{t-1}}\right)^{v\eta^{\mathcal{A}}\left(1-\frac{1}{\gamma}\right)}\left(\frac{(1-\tau^{\mathcal{L}}_{t+1})w_{t+1}}{(1-\tau^{\mathcal{L}}_{t})w_{t}}\right)^{\left(1-\eta^{\mathcal{A}}\right)\left(1-\frac{1}{\gamma}\right)}\left(\beta\frac{i_t}{\pi_{t+1}}\right)^{\frac{1}{\gamma}}\chi^{\left(1-\eta^{\mathcal{A}}\right)\left(1-\frac{1}{\gamma}\right)}$$

equation (16) follows immediately.

where $\tilde{r}_{t,s}$ is the real subjective discount factor

$$\tilde{r}_{t,s} = \begin{cases} \prod_{l=1}^{s} \frac{\theta}{r_{t+l-1}} & \text{for } s > 0 \\ 1 & \text{for } s = 0 \end{cases} \tag{24}$$

For future reference, notice that, for $s > 0$

$$\tilde{r}_{t,s+1} = \frac{\theta}{r_t} \prod_{l=1}^{s} \frac{\theta}{r_{t+l}}$$

and

$$\tilde{r}_{t+1,s} = \prod_{l=1}^{s} \frac{\theta}{r_{t+l}}$$

implying that

$$\tilde{r}_{t,s+1} = \frac{\theta}{r_t} \tilde{r}_{t+1,s} \tag{25}$$

(III) Human wealth. Human wealth stems from labor and capital. Generation a's contemporaneous nominal income at t is

$$Inc_{a,t} = (1 - \tau_t^{\mathcal{L}}) W_t \Phi_a + \left[\sum_{\substack{x \in \{\mathcal{T},\mathcal{N},\mathcal{U},\mathcal{C}, \\ \mathcal{G},\mathcal{I},\mathcal{X},\mathcal{K},\mathcal{EP}\}}} \int_0^1 D_{a,t}^{\mathcal{A},x}(i) \mathrm{d}i + TRG_{a,t}^{\mathcal{A}} + \varepsilon_t TRX_{a,t}^{\mathcal{A}} + P_t rbr_{a,t} \right]$$

The labor-based income is the value of households entire time endowment. Notice that households use a fraction $1 - L_{a,t}^{\mathcal{A}}$ of labor-based income to buy leisure, and the remaining to buy consumption goods. Capital-based income comprises dividends, transfers, and the remuneration from services in the bankruptcy monitoring of firms.

Lifetime human wealth of a household with age a in period t, $HW_{a,t}$, is the present discounted value of all future incomes. This can be split in a labor component and a capital component

$$\begin{aligned} HW_{a,t} = \sum_{s=0}^{\infty} \tilde{R}_{t,s} Inc_{a+s,t+s} = & \underbrace{\sum_{s=0}^{\infty} \tilde{R}_{t,s} (1 - \tau_{t+s}^{\mathcal{L}}) W_{t+s} \Phi_{a+s}}_{HW_{a,t}^{\mathcal{L}}} \\ & + \underbrace{\sum_{s=0}^{\infty} \tilde{R}_{t,s} \left[\sum_x \int_0^1 D_{a+s,t+s}^{\mathcal{A},x}(i) \mathrm{d}i + TRG_{a+s,t+s}^{\mathcal{A}} + \varepsilon_{t+s} TRX_{a+s,t+s}^{\mathcal{A}} + P_{t+s} rbr_{a+s,t+s} \right]}_{HW_{a,t}^{\mathcal{K}}} \end{aligned}$$

Aggregating across generations yields

$$HW_t = HW_t^{\mathcal{L}} + HW_t^{\mathcal{K}} = \left[N(1-\psi)(1-\theta) \sum_{a=0}^{\infty} \theta^a HW_{a,t}^{\mathcal{L}} \right] + \left[N(1-\psi)(1-\theta) \sum_{a=0}^{\infty} \theta^a HW_{a,t}^{\mathcal{K}} \right] \tag{26}$$

Equation (26) can be expressed in real terms by dividing both sides by P_t, yielding $hw_t = hw_t^{\mathcal{L}} + hw_t^{\mathcal{K}}$. Further dividing by technological progress T_t one obtains aggregate lifetime human wealth in stationary form

$$\check{hw}_t = \check{hw}_t^{\mathcal{L}} + \check{hw}_t^{\mathcal{K}} \tag{27}$$

In the steady state

$$\check{hw} = \check{hw}^{\mathcal{L}} + \check{hw}^{\mathcal{K}} \tag{28}$$

It remains to determine the complete expressions for $\check{hw}_t^{\mathcal{L}}$ and $\check{hw}_t^{\mathcal{K}}$. Nominal labor-based aggregate lifetime human wealth simplifies to

$$\begin{aligned}
HW_t^{\mathcal{L}} &= N(1-\psi)(1-\theta)\sum_{a=0}^{\infty}\theta^a HW_{a,t}^{\mathcal{L}} \\
&= N(1-\psi)(1-\theta)\sum_{a=0}^{\infty}\theta^a \sum_{s=0}^{\infty}\tilde{R}_{t,s}(1-\tau_{t+s}^{\mathcal{L}})W_{t+s}\Phi_{a+s} \\
&= N(1-\psi)(1-\theta)\sum_{a=0}^{\infty}\theta^a \sum_{s=0}^{\infty}\tilde{R}_{t,s}(1-\tau_{t+s}^{\mathcal{L}})W_{t+s}k\chi^{a+s} \\
&= N(1-\psi)(1-\theta)\left(\sum_{a=0}^{\infty}\theta^a k\chi^a\right)\sum_{s=0}^{\infty}\tilde{R}_{t,s}(1-\tau_{t+s}^{\mathcal{L}})W_{t+s}\chi^{s} \\
&= N(1-\psi)\sum_{s=0}^{\infty}\tilde{R}_{t,s}(1-\tau_{t+s}^{\mathcal{L}})W_{t+s}\chi^{s} \\
&= N(1-\psi)\sum_{s=0}^{\infty}(1-\tau_{t+s}^{\mathcal{L}})W_{t+s}\chi^{s}\tilde{r}_{t,s}\prod_{l=1}^{s}(\pi_{t+l})^{-1}
\end{aligned} \tag{29}$$

Equation (29) incorporates the relationship in (2), according to which $(1-\theta)\sum_{a=0}^{\infty}\theta^a k\chi^a = 1$, and the link between nominal and real subjective discount factors in (23). Dividing by P_t and letting $hw_t^{\mathcal{L}}$ denote real labor-based aggregate human wealth yields

$$\begin{aligned}
hw_t^{\mathcal{L}} &= N(1-\psi)\sum_{s=0}^{\infty}\tilde{r}_{t,s}\chi^s(1-\tau_{t+s}^{\mathcal{L}})W_{t+s}\frac{1}{P_t}\prod_{l=1}^{s}(\pi_{t+l})^{-1} \\
&= N(1-\psi)\sum_{s=0}^{\infty}\tilde{r}_{t,s}\chi^s\frac{(1-\tau_{t+s}^{\mathcal{L}})W_{t+s}}{P_{t+s}} \\
&= N(1-\psi)\sum_{s=0}^{\infty}\tilde{r}_{t,s}\chi^s(1-\tau_{t+s}^{\mathcal{L}})w_{t+s}
\end{aligned} \tag{30}$$

since $P_{t+s}\prod_{l=1}^{s}(\pi_{t+l})^{-1} = P_t$. Equation (30) can be further decomposed as follows

$$hw_t^{\mathcal{L}} = N(1-\psi)(1-\tau_t^{\mathcal{L}})w_t + N(1-\psi)\sum_{s=1}^{\infty}\tilde{r}_{t,s}\chi^s(1-\tau_{t+s}^{\mathcal{L}})w_{t+s}$$

$$= N(1-\psi)(1-\tau_t^{\mathcal{L}})w_t + N(1-\psi)\sum_{s=0}^{\infty}\tilde{r}_{t,s+1}\chi^{s+1}(1-\tau_{t+s+1}^{\mathcal{L}})w_{t+s+1}$$

$$= N(1-\psi)(1-\tau_t^{\mathcal{L}})w_t + N(1-\psi)\frac{\theta\cdot\chi}{r_t}\sum_{s=0}^{\infty}\tilde{r}_{t+1,s}\chi^{s}(1-\tau_{t+1+s}^{\mathcal{L}})w_{t+1+s}$$

$$= N(1-\psi)(1-\tau_t^{\mathcal{L}})w_t + \frac{\theta\cdot\chi}{r_t}hw_{t+1}^{\mathcal{L}}$$

where we used the result in (25) stating that $\tilde{r}_{t,s+1} = (\theta/r_t)\tilde{r}_{t+1,s}$. This equation has the following stationary form

$$\check{hw}_t^{\mathcal{L}} = N(1-\psi)(1-\tau_t^{\mathcal{L}})\check{w}_t + g\frac{\theta\cdot\chi}{r_t}\check{hw}_{t+1}^{\mathcal{L}} \tag{31}$$

In the steady state

$$\check{hw}^{\mathcal{L}} = N(1-\psi)(1-\tau^{\mathcal{L}})\check{w} + g\frac{\theta\cdot\chi}{r}\check{hw}^{\mathcal{L}}$$

and therefore

$$\check{hw}^{\mathcal{L}} = \frac{N(1-\psi)(1-\tau^{\mathcal{L}})\check{w}}{1-g\frac{\theta\cdot\chi}{r}} \tag{32}$$

According to these equations, agents discount future labor-based income streams at a higher rate when the probability of death or the productivity decay rate are higher.

Nominal capital-based aggregate lifetime human wealth is

$$HW_t^{\mathcal{K}} = N(1-\psi)(1-\theta)\sum_{a=0}^{\infty}\theta^a HW_{a,t}^{\mathcal{K}} = N(1-\psi)(1-\theta)\sum_{a=0}^{\infty}\theta^a \times$$
$$\times\sum_{s=0}^{\infty}\tilde{R}_{t,s}\left(\sum_x\int_0^1 D_{a+s,t+s}^{\mathcal{A},x}(i)\mathrm{d}i + TRG_{a+s,t+s}^{\mathcal{A}} + \varepsilon_{t+s}TRX_{a+s,t+s}^{\mathcal{A}} + P_{t+s}rbr_{a+s,t+s}\right)$$

All firms within a given sector are identical and thus all pay equal dividends in equilibrium, implying that $\int_0^1 D_{a,t}^{\mathcal{A},x}(i)\mathrm{d}i = D_{a,t}^{\mathcal{A},x}$. Assuming that dividends, remuneration payments for the bankruptcy monitoring of firms, and net transfers received by each type-$\mathcal{A}$ household are the same for all cohorts, irrespective of their age, and letting $D_t^{\mathcal{A}}$, rbr_t, $TRG_t^{\mathcal{A}}$ and $TRX_t^{\mathcal{A}}$ denote aggregate figures at time t, the previous expression simplifies to

$$HW_t^{\mathcal{K}} = \sum_{s=0}^{\infty}\tilde{R}_{t,s}\left(\sum_x D_{t+s}^{\mathcal{A},x} + TRG_{t+s}^{\mathcal{A}} + \varepsilon_{t+s}TRX_{t+s}^{\mathcal{A}} + P_{t+s}rbr_{t+s}\right)$$
$$= \sum_{s=0}^{\infty}\tilde{r}_{t,s}\prod_{l=1}^{s}(\pi_{t+l})^{-1}\left(\sum_x D_{t+s}^{\mathcal{A},x} + TRG_{t+s}^{\mathcal{A}} + \varepsilon_{t+s}TRX_{t+s}^{\mathcal{A}} + P_{t+s}rbr_{t+s}\right)$$

where we used the result in (23). Dividing by P_t and letting $hw_t^{\mathcal{K}}$ denote real capital-based aggregate human wealth yields

$$
\begin{aligned}
hw_t^{\mathcal{K}} &= \sum_{s=0}^{\infty} \tilde{r}_{t,s} \prod_{l=1}^{s} \frac{(\pi_{t+l})^{-1}}{P_t} \left(\sum_x D_{t+s}^{\mathcal{A},x} + TRG_{t+s}^{\mathcal{A}} + \varepsilon_{t+s} TRX_{t+s}^{\mathcal{A}} + P_{t+s} rbr_{t+s} \right) \\
&= \sum_{s=0}^{\infty} \tilde{r}_{t,s} \frac{1}{P_{t+s}} \left(\sum_x D_{t+s}^{\mathcal{A},x} + TRG_{t+s}^{\mathcal{A}} + \varepsilon_{t+s} TRX_{t+s}^{\mathcal{A}} + P_{t+s} rbr_{t+s} \right) \\
&= \sum_{s=0}^{\infty} \tilde{r}_{t,s} \left(\sum_x d_{t+s}^{\mathcal{A},x} + trg_{t+s}^{\mathcal{A}} + \frac{\varepsilon_{t+s} P_{t+s}^*}{P_{t+s}} trx_{t+s}^{\mathcal{A}} + rbr_{t+s} \right)
\end{aligned}
$$

since $P_{t+s} \prod_{l=1}^{s} (\pi_{t+l})^{-1} = P_t$. Following our convention, $d_t^{\mathcal{A},x}$, $trg_t^{\mathcal{A}}$, and $trx_t^{\mathcal{A}}$ represent respectively the real value of dividends, net transfers from the government, and net transfers from abroad for type-$\mathcal{A}$ households. Net transfers from abroad at t are deflated by the foreign price P_t^*. Let

$$
\epsilon_t = \frac{\varepsilon_t P_t^*}{P_t} \tag{33}
$$

denote the period t real exchange rate (thus, an increase in ϵ_t represents a real depreciation). Real capital-based aggregate lifetime human wealth can be further decomposed as

$$
\begin{aligned}
hw_t^{\mathcal{K}} &= \sum_x d_t^{\mathcal{A},x} + trg_t^{\mathcal{A}} + \epsilon_t trx_t^{\mathcal{A}} + rbr_t + \sum_{s=1}^{\infty} \tilde{r}_{t,s} \left(\sum_x d_{t+s}^{\mathcal{A},x} + trg_{t+s}^{\mathcal{A}} + \epsilon_{t+s} trx_{t+s}^{\mathcal{A}} + rbr_{t+s} \right) \\
&= \sum_x d_t^{\mathcal{A},x} + trg_t^{\mathcal{A}} + \epsilon_t trx_t^{\mathcal{A}} + rbr_t + \sum_{s=0}^{\infty} \tilde{r}_{t,s+1} \left(\sum_x d_{t+s+1}^{\mathcal{A},x} + trg_{t+s+1}^{\mathcal{A}} + \epsilon_{t+s+1} trx_{t+s+1}^{\mathcal{A}} + rbr_{t+s+1} \right) \\
&= \sum_x d_t^{\mathcal{A},x} + trg_t^{\mathcal{A}} + \epsilon_t trx_t^{\mathcal{A}} + rbr_t + \frac{\theta}{r_t} hw_{t+1}^{\mathcal{K}}
\end{aligned}
$$

since, from equation (25), $\tilde{r}_{t,s+1} = (\theta/r_t)\tilde{r}_{t+1,s}$. The previous equation has the following stationary form

$$
\check{hw}_t^{\mathcal{K}} = \sum_{\substack{x \in \{\mathcal{T},\mathcal{N},\mathcal{U},\mathcal{C},\mathcal{G} \\ \mathcal{I},\mathcal{X},\mathcal{K},\mathcal{EP}\}}} \check{d}_t^{\mathcal{A},x} + \check{trg}_t^{\mathcal{A}} + \epsilon_t \check{trx}_t^{\mathcal{A}} + \check{rbr}_t + g \frac{\theta}{r_t} \check{hw}_{t+1}^{\mathcal{K}} \tag{34}
$$

In the steady state,

$$
\check{hw}^{\mathcal{K}} = \sum_{\substack{x \in \{\mathcal{T},\mathcal{N},\mathcal{U},\mathcal{C},\mathcal{G} \\ \mathcal{I},\mathcal{X},\mathcal{K},\mathcal{EP}\}}} \check{d}^{\mathcal{A},x} + \check{trg}^{\mathcal{A}} + \epsilon \check{trx}^{\mathcal{A}} + \check{rbr} + g \frac{\theta}{r} \check{hw}^{\mathcal{K}}
$$

and therefore

$$
\check{hw}^{\mathcal{K}} = \frac{1}{1 - g\frac{\theta}{r}} \left(\sum_{\substack{x \in \{\mathcal{T},\mathcal{N},\mathcal{U},\mathcal{C},\mathcal{G} \\ \mathcal{I},\mathcal{X},\mathcal{K},\mathcal{EP}\}}} \check{d}^{\mathcal{A},x} + \check{trg}^{\mathcal{A}} + \epsilon \check{trx}^{\mathcal{A}} + \check{rbr} \right) \tag{35}
$$

Equations (34) and (35) were derived under the deliberate omission of the exact functional forms for dividends and transfers. In practice they can take several forms. A useful possibility that can be used is to allow for income transfers from type-$\mathcal{A}$ to type-$\mathcal{B}$ households.

(IV) Financial wealth. Financial wealth of a type-$\mathcal{A}$ household with age a at time t is

$$FW_{a,t} = \frac{1}{\theta}\left[i_{t-1}\widehat{B}_{a-1,t-1} + i^*_{t-1}\Psi\varepsilon_t B^*_{a-1,t-1}\right]$$

Summing up across all type-$\mathcal{A}$ households yields nominal aggregate financial wealth

$$\begin{aligned} FW_t &= N(1-\psi)(1-\theta)\sum_{a=0}^{\infty}\theta^a\frac{1}{\theta}\left[i_{t-1}\widehat{B}_{a-1,t-1} + i^*_{t-1}\Psi\varepsilon_t B^*_{a-1,t-1}\right] \\ &= N(1-\psi)(1-\theta)\left(i_{t-1}\sum_{a=0}^{\infty}\theta^{a-1}\widehat{B}_{a-1,t-1} + i^*_{t-1}\Psi\varepsilon_t\sum_{a=0}^{\infty}\theta^{a-1}B^*_{a-1,t-1}\right) \\ &= i_{t-1}\widehat{B}_{t-1} + i^*_{t-1}\Psi\varepsilon_t B^*_{t-1} \end{aligned} \tag{36}$$

where $\widehat{B}_{t-1}$ and B^*_{t-1} are nominal aggregate values of domestic and foreign bond holdings, respectively. Aggregation takes into account that the wealth of $1-\theta$ agents that did not survive is distributed to the θ agents that did survive. Furthermore, with no heirs and no bequests, $\widehat{B}_{-1,t-1} = 0$. In real terms equation (36) becomes

$$fw_t = \frac{1}{\pi_t}\left[i_{t-1}\widehat{b}_{t-1} + i^*_{t-1}\Psi\varepsilon_t\frac{P^*_{t-1}}{P_{t-1}}b^*_{t-1}\right] = \frac{1}{\pi_t}\left[i_{t-1}\widehat{b}_{t-1} + i^*_{t-1}\Psi\frac{\varepsilon_t}{\varepsilon_{t-1}}\epsilon_{t-1}b^*_{t-1}\right]$$

where $\widehat{b}_t = \widehat{B}_t/P_t$ and $b^*_t = B^*_t/P^*_t$. The above equation uses the fact $P_t = \pi_t P_{t-1}$ and the definition of real exchange rate in (33). Employing the equivalence $\widehat{b}_t \equiv b_t + b^{\mathcal{T}}_t + b^{\mathcal{N}}_t$, its stationary form is

$$\check{fw}_t = \frac{1}{g\cdot\pi_t}\left[i_{t-1}(\check{b}_{t-1} + \check{b}^{\mathcal{T}}_{t-1} + \check{b}^{\mathcal{N}}_{t-1}) + i^*_{t-1}\Psi\frac{\varepsilon_t}{\varepsilon_{t-1}}\epsilon_{t-1}\check{b}^*_{t-1}\right] \tag{37}$$

In the steady state

$$\check{fw} = \frac{1}{g\cdot\pi}\left[i\left(\check{b} + \check{b}^{\mathcal{T}} + \check{b}^{\mathcal{N}}\right) + i^*\Psi\epsilon\check{b}^*\right] \tag{38}$$

(V) Marginal propensity to consume out of wealth. The condition that associates consumption with wealth is derived from the nominal budget constraint of a representative OLG household with age a, given in (6). Rewrite the after-tax wage income of a type-$\mathcal{A}$ household as

$$(1-\tau^{\mathcal{L}}_t)W_t\Phi_a L^{\mathcal{A}}_{a,t} = (1-\tau^{\mathcal{L}}_t)W_t\Phi_a - (1-\tau^{\mathcal{L}}_t)W_t\Phi_a(1-L^{\mathcal{A}}_{a,t})$$

$$= (1-\tau_t^{\mathcal{L}})W_t\Phi_a - \frac{(1-\eta^{\mathcal{A}})}{\eta^{\mathcal{A}}}P_tC_{a,t}^{\mathcal{A}}$$

The simplification in the last step is due to (7). The type-$\mathcal{A}$ household budget constraint can thus be restated as

$$P_tC_{a,t}^{\mathcal{A}} + \widehat{B}_{a,t} + \varepsilon_t B_{a,t}^* \leq -\frac{(1-\eta^{\mathcal{A}})}{\eta^{\mathcal{A}}}P_tC_{a,t}^{\mathcal{A}} + \underbrace{\frac{1}{\theta}\left[i_{t-1}\widehat{B}_{a-1,t-1} + i_{t-1}^*\Psi\varepsilon_t B_{a-1,t-1}^*\right]}_{FW_{a,t}}$$

$$\underbrace{+ (1-\tau_t^{\mathcal{L}})W_t\Phi_a + \sum_{\substack{x\in\{\mathcal{T},\mathcal{N},\mathcal{U},\mathcal{C},\mathcal{G}\\ \mathcal{I},\mathcal{X},\mathcal{K},\mathcal{EP}\}}}\int_0^1 D_{a,t}^{\mathcal{A},x}(i)\mathrm{d}i + TRG_{a,t}^{\mathcal{A}} + \varepsilon_t TRX_{a,t}^{\mathcal{A}} + P_t rbr_{a,t}}_{Inc_{a,t}}$$

or equivalently

$$P_tC_{a,t}^{\mathcal{A}} + \widehat{B}_{a,t} + \varepsilon_t B_{a,t}^* = -\frac{1-\eta^{\mathcal{A}}}{\eta^{\mathcal{A}}}P_tC_{a,t}^{\mathcal{A}} + FW_{a,t} + Inc_{a,t} \Leftrightarrow$$

$$\Leftrightarrow \frac{P_tC_{a,t}^{\mathcal{A}}}{\eta^{\mathcal{A}}} + \widehat{B}_{a,t} + \varepsilon_t B_{a,t}^* = FW_{a,t} + Inc_{a,t} \tag{39}$$

We now need to rewrite (39) in terms of human and financial wealth. This requires successive forward substitution of this equation and the use of the no-arbitrage condition, yielding

$$\sum_{s=0}^{\infty}\tilde{R}_{t,s}\frac{P_{t+s}C_{a+s,t+s}^{\mathcal{A}}}{\eta^{\mathcal{A}}} = HW_{a,t} + FW_{a,t} \tag{40}$$

The detailed derivation is presented in Box 4. Using the definition of $\tilde{r}_{t,s}$ in (24), the relationship in (16) according to which $C_{a+1,t+1} = j_tC_{a,t}$, and the fact $P_{t+s}\prod_{l=1}^{s}(\pi_{t+l})^{-1} = P_t$, the left-hand side of equation (40) can be expressed as

$$\sum_{s=0}^{\infty}\tilde{R}_{t,s}\frac{P_{t+s}C_{a+s,t+s}^{\mathcal{A}}}{\eta^{\mathcal{A}}} = C_{a,t}^{\mathcal{A}}\sum_{s=0}^{\infty}\prod_{l=1}^{s}j_{t+l-1}\tilde{R}_{t,s}\frac{P_{t+s}}{\eta^{\mathcal{A}}} = C_{a,t}^{\mathcal{A}}\sum_{s=0}^{\infty}\prod_{l=1}^{s}j_{t+l-1}\frac{\tilde{r}_{t,s}}{\pi_{t+l}}\frac{P_{t+s}}{\eta^{\mathcal{A}}}$$
$$= P_tC_{a,t}^{\mathcal{A}}\sum_{s=0}^{\infty}\prod_{l=1}^{s}j_{t+l-1}\frac{\tilde{r}_{t,s}}{\eta^{\mathcal{A}}} = \Theta_tP_tC_{a,t}^{\mathcal{A}}$$

where the new variable Θ_t is defined as

$$\Theta_t = \sum_{s=0}^{\infty}\prod_{l=1}^{s}j_{t+l-1}\frac{\tilde{r}_{t,s}}{\eta^{\mathcal{A}}}$$

Equation (40) thus becomes

$$\Theta_tP_tC_{a,t}^{\mathcal{A}} = HW_{a,t} + FW_{a,t} \tag{41}$$

The expression for aggregate consumption at time t as a function of lifetime human wealth and

Box 4: Type-$\mathcal{A}$ household budget constraint: successive forward substitution.

Let us omit the expected value operator. Writing equation (39) for period $t+1$ yields

$$\frac{P_{t+1}C^{\mathcal{A}}_{a+1,t+1}}{\eta^{\mathcal{A}}} + \widehat{B}_{a+1,t+1} + \varepsilon_{t+1}B^*_{a+1,t+1} = Inc_{a+1,t+1} + \frac{1}{\theta}\left[i_t\widehat{B}_{a,t} + i^*_t\Psi\varepsilon_{t+1}B^*_{a,t}\right] \Leftrightarrow$$

$$\Leftrightarrow \widehat{B}_{a,t} = \frac{\theta}{i_t}\left[\frac{P_{t+1}C^{\mathcal{A}}_{a+1,t+1}}{\eta^{\mathcal{A}}} + \widehat{B}_{a+1,t+1} + \varepsilon_{t+1}B^*_{a+1,t+1} - Inc_{a+1,t+1} - \frac{1}{\theta}i^*_t\Psi\varepsilon_{t+1}B^*_{a,t}\right] \tag{4.1}$$

Replacing (4.1) in (39) and re-arranging

$$\begin{aligned}&\frac{P_tC^{\mathcal{A}}_{a,t}}{\eta^{\mathcal{A}}} + \frac{\theta}{i_t}\frac{P_{t+1}C^{\mathcal{A}}_{a+1,t+1}}{\eta^{\mathcal{A}}} + \frac{\theta}{i_t}\left[\widehat{B}_{a+1,t+1} + \varepsilon_{t+1}B^*_{a+1,t+1}\right] - \frac{i^*_t}{i_t}\Psi\varepsilon_{t+1}B^*_{a,t} + \varepsilon_tB^*_{a,t}\\ &= Inc_{a,t} + \frac{\theta_t}{i_t}Inc_{a+1,t+1} + FW_{a,t}\end{aligned} \tag{4.2}$$

The no-arbitrage condition in (13) implies that $-(i^*_t/i_t)\Psi\varepsilon_{t+1}B^*_{a,t} + \varepsilon_tB^*_{a,t} = 0$. Writing (39) for $t+2$ and inserting into (4.2) yields

$$\begin{aligned}&\frac{P_tC^{\mathcal{A}}_{a,t}}{\eta^{\mathcal{A}}} + \frac{\theta}{i_t}\frac{P_{t+1}C^{\mathcal{A}}_{a+1,t+1}}{\eta^{\mathcal{A}}} + \frac{\theta}{i_t}\frac{\theta}{i_{t+1}}\left[\frac{P_{t+2}C^{\mathcal{A}}_{a+2,t+2}}{\eta^{\mathcal{A}}} + \widehat{B}_{a+2,t+2} + \varepsilon_{t+2}B^*_{a+2,t+2} - Inc_{a+2,t+2}\right.\\ &\left.-\frac{1}{\theta}i^*_{t+1}\Psi\varepsilon_{t+2}B^*_{a+1,t+1}\right] + \frac{\theta}{i_t}\varepsilon_{t+1}B^*_{a+1,t+1} = Inc_{a,t} + \frac{\theta}{i_t}Inc_{a+1,t+1} + FW_{a,t}\end{aligned}$$

This simplifies to

$$\begin{aligned}&\frac{P_tC^{\mathcal{A}}_{a,t}}{\eta^{\mathcal{A}}} + \frac{\theta}{i_t}\frac{P_{t+1}C^{\mathcal{A}}_{a+1,t+1}}{\eta^{\mathcal{A}}} + \frac{\theta^2}{i_ti_{t+1}}\frac{P_{t+2}C^{\mathcal{A}}_{a+2,t+2}}{\eta^{\mathcal{A}}} + \frac{\theta^2}{i_ti_{t+1}}\left[\widehat{B}_{a+2,t+2} + \varepsilon_{t+2}B^*_{a+2,t+2}\right]\\ &= Inc_{a,t} + \frac{\theta}{i_t}Inc_{a+1,t+1} + \frac{\theta^2}{i_ti_{t+1}}Inc_{a+2,t+2} + FW_{a,t}\end{aligned} \tag{4.3}$$

where we used the fact

$$-\frac{\theta^2}{i_ti_{t+1}}\left[\frac{1}{\theta}i^*_{t+1}\Psi\varepsilon_{t+2}B^*_{a+1,t+1}\right] + \frac{\theta}{i_t}\varepsilon_{t+1}B^*_{a+1,t+1} = 0$$

due to the no-arbitrage condition $i_{t+1} = i^*_{t+1}\Psi\varepsilon_{t+2}/\varepsilon_{t+1}$. Using successive forward substitution, one can express equation (4.3) as

$$\begin{aligned}&\frac{P_tC^{\mathcal{A}}_{a,t}}{\eta^{\mathcal{A}}} + \sum_{s=1}^{\infty}\left(\prod_{l=1}^{s}\frac{\theta}{i_{t+l-1}}\right)\frac{P_{t+s}C^{\mathcal{A}}_{a+s,t+s}}{\eta^{\mathcal{A}}} + \lim_{T\to\infty}\left[\left(\prod_{l=1}^{T}\frac{\theta}{i_{t+l-1}}\right)\left(\widehat{B}_{a+T,t+T} + \varepsilon_{t+T}B^*_{a+T,t+T}\right)\right]\\ &= Inc_{a,t} + \sum_{s=1}^{\infty}\left(\prod_{l=1}^{s}\frac{\theta}{i_{t+l-1}}\right)Inc_{a+s,t+s} + FW_{a,t}\end{aligned} \tag{4.4}$$

Using the definition of the nominal subjective discount factor in (20) and the no-Ponzi game condition

$$\lim_{s\to\infty}\tilde{R}_{t,s}[\widehat{B}_{a+s,t+s} + \varepsilon_{t+s}B^*_{a+s,t+s}] = 0$$

equation (4.4) reduces to

$$\begin{aligned}\sum_{s=0}^{\infty}\tilde{R}_{t,s}\frac{P_{t+s}C^{\mathcal{A}}_{a+s,t+s}}{\eta^{\mathcal{A}}} &= \sum_{s=0}^{\infty}\tilde{R}_{t,s}Inc_{a+s,t+s} + FW_{a,t}\\ &= HW_{a,t} + FW_{a,t}\end{aligned}$$

financial wealth can be obtained by aggregating (41) over different age groups

$$\Theta_t P_t C_t^{\mathcal{A}} = HW_t + FW_t$$

In real terms, this equation has the following stationary form

$$\check{C}_t^{\mathcal{A}} = (\Theta_t)^{-1}(\check{hw}_t + \check{fw}_t) \tag{42}$$

In the steady state

$$\check{C}^{\mathcal{A}} = \Theta^{-1}(\check{hw} + \check{fw}) \tag{43}$$

The key behavioral equation of type-$\mathcal{A}$ households linking optimal aggregate consumption to aggregate wealth is therefore

$$\check{C}_t^{\mathcal{A}} = mpc_t \left(\check{hw}_t + \check{fw}_t\right) = mpc_t(\check{hw}_t^{\mathcal{L}} + \check{hw}_t^{\mathcal{K}} + \check{fw}_t)$$

where $mpc_t \equiv (\Theta_t)^{-1}$ is the time-varying marginal propensity to consume of out of wealth.

Finally, one can write Θ_t recursively as

$$\begin{aligned}\Theta_t = \frac{1}{\eta^{\mathcal{A}}} + \sum_{s=1}^{\infty}\prod_{l=1}^{s} j_{t+l-1}\frac{\tilde{r}_{t,s}}{\eta^{\mathcal{A}}} &= \frac{1}{\eta^{\mathcal{A}}} + j_t \sum_{s=0}^{\infty}\prod_{l=1}^{s} j_{t+1+l-1}\frac{\tilde{r}_{t,s+1}}{\eta^{\mathcal{A}}} \\ &= \frac{1}{\eta^{\mathcal{A}}} + \frac{\theta\cdot j_t}{r_t}\sum_{s=0}^{\infty}\prod_{l=1}^{s} j_{t+1+l-1}\frac{\tilde{r}_{t+1,s}}{\eta^{\mathcal{A}}}\end{aligned}$$

yielding

$$\Theta_t = \frac{1}{\eta^{\mathcal{A}}} + \frac{\theta\cdot j_t}{r_t}\Theta_{t+1} \tag{44}$$

In the steady state $\Theta_t = \Theta_{t+1} = \Theta$ and therefore

$$\Theta = \frac{1/\eta^{\mathcal{A}}}{1-(\theta\cdot j)/r} \tag{45}$$

Using (11), aggregate effective labor supply for type-$\mathcal{A}$ households is

$$L_t^{\mathcal{A}} = N(1-\psi) - (\Theta_t)^{-1}\cdot(\check{hw}_t^{\mathcal{L}} + \check{hw}_t^{\mathcal{K}} + \check{fw}_t)\left(\frac{1-\eta^{\mathcal{A}}}{\eta^{\mathcal{A}}}\frac{1}{(1-\tau_t^{\mathcal{L}})\check{w}_t}\right)$$

3.3 Households with no access to financial markets (type-$\mathcal{B}$ households)

This section presents and solves the maximization problem of hand-to-mouth (type-$\mathcal{B}$) households, *i.e.* those with no access to financial markets.

3.3.1 Budget constraint and maximization problem

On the expenditure side, a type-$\mathcal{B}$ household with age a buys a consumption bundle worth $P_t C^{\mathcal{B}}_{a,t}$. These households are hired by labor unions, receiving an after-tax labor income of $(1-\tau^{\mathcal{L}}_t)W_t\Phi_a L^{\mathcal{B}}_{a,t}$. They also receive transfers from the government, $TRG^{\mathcal{B}}_{a,t}$, and from abroad, $TRX^{\mathcal{B}}_{a,t}$, and dividends from unions, $D^{\mathcal{B},\mathcal{U}}_{a,t}(i)$. The nominal budget constraint is

$$P_t C^{\mathcal{B}}_{a,t} \;\leq\; (1-\tau^{\mathcal{L}}_t)W_t\Phi_a L^{\mathcal{B}}_{a,t} + \int_0^1 D^{\mathcal{B},\mathcal{U}}_{a,t}(i)\mathrm{d}i + TRG^{\mathcal{B}}_{a,t} + \varepsilon_t TRX^{\mathcal{B}}_{a,t} \tag{46}$$

The optimization problem of a type-$\mathcal{B}$ household consists in maximizing expected lifetime utility (1) with respect to $\{C^{\mathcal{B}}_{a+s,t+s}, L^{\mathcal{B}}_{a+s,t+s}\}^{\infty}_{s=0}$, subject to (46). The first-order conditions are similar to those from a type-$\mathcal{A}$ household and are therefore omitted. The consumption/labor supply choice is similar to (7), with $\mathcal{B}$ replacing $\mathcal{A}$

$$\begin{aligned} C^{\mathcal{B}}_{a,t} &= \frac{\eta^{\mathcal{B}}}{1-\eta^{\mathcal{B}}}\big(1-\tau^{\mathcal{L}}_t\big)w_t\Phi_a\big(1-L^{\mathcal{B}}_{a,t}\big) \Leftrightarrow \\ \Leftrightarrow\; 1-L^{\mathcal{B}}_{a,t} &= \frac{1-\eta^{\mathcal{B}}}{\eta^{\mathcal{B}}}\frac{C^{\mathcal{B}}_{a,t}}{\big(1-\tau^{\mathcal{L}}_t\big)w_t\Phi_a} \end{aligned} \tag{47}$$

Aggregation is also immediate. The counterpart of equation (10) for type-$\mathcal{B}$ households is

$$\frac{C^{\mathcal{B}}_t}{N\psi - L^{\mathcal{B}}_t} = \frac{\eta^{\mathcal{B}}}{1-\eta^{\mathcal{B}}}(1-\tau^{\mathcal{L}}_t)w_t$$

In stationary form this equation becomes

$$\frac{\check{C}^{\mathcal{B}}_t}{N\psi - L^{\mathcal{B}}_t} = \frac{\eta^{\mathcal{B}}}{1-\eta^{\mathcal{B}}}(1-\tau^{\mathcal{L}}_t)\check{w}_t \tag{48}$$

In the steady state

$$\frac{\check{C}^{\mathcal{B}}}{N\psi - L^{\mathcal{B}}} = \frac{\eta^{\mathcal{B}}}{1-\eta^{\mathcal{B}}}(1-\tau^{\mathcal{L}})\check{w} \tag{49}$$

3.3.2 Aggregate consumption

Type-$\mathcal{B}$ households aggregate consumption is solely a function of contemporaneous aggregate income, since they are unable to undertake any type of intertemporal smoothing. Using the budget constraint in (46), real aggregate consumption of type-$\mathcal{B}$ households is

$$\begin{aligned} C_t^{\mathcal{B}} &= N\psi(1-\theta)\sum_{a=0}^{\infty}\theta^a C_{a,t}^{\mathcal{B}} \\ &= N\psi(1-\theta)\left[\sum_{a=0}^{\infty}\theta^a(1-\tau_t^{\mathcal{L}})w_t\Phi_a L_{a,t}^{\mathcal{B}} + \sum_{a=0}^{\infty}\theta^a\left(d_{a,t}^{\mathcal{B},\mathcal{U}} + trg_{a,t}^{\mathcal{B}} + \epsilon_t trx_{a,t}^{\mathcal{B}}\right)\right] \\ &= (1-\tau_t^{\mathcal{L}})w_t L_t^{\mathcal{B}} + d_t^{\mathcal{B},\mathcal{U}} + trg_t^{\mathcal{B}} + \epsilon_t trx_t^{\mathcal{B}} \end{aligned} \tag{50}$$

The term $L_t^{\mathcal{B}}$ corresponds to the effective labor supply

$$L_t^{\mathcal{B}} = N\psi(1-\theta)\sum_{a=0}^{\infty}\theta^a\Phi_a L_{a,t}^{\mathcal{B}}$$

As for type-$\mathcal{A}$, we assume that dividends and net transfers received by each type-$\mathcal{B}$ household are the same for all cohorts, irrespective of age. Equation (50) has the following stationary form

$$\boxed{\check{C}_t^{\mathcal{B}} = (1-\tau_t^{\mathcal{L}})\check{w}_t L_t^{\mathcal{B}} + \check{d}_t^{\mathcal{B},\mathcal{U}} + \check{trg}_t^{\mathcal{B}} + \epsilon_t \check{trx}_t^{\mathcal{B}}} \tag{51}$$

In the steady state

$$\boxed{\check{C}^{\mathcal{B}} = (1-\tau^{\mathcal{L}})\check{w} L^{\mathcal{B}} + \check{d}^{\mathcal{B},\mathcal{U}} + \check{trg}^{\mathcal{B}} + \epsilon \check{trx}^{\mathcal{B}}} \tag{52}$$

As in the cases of (34) and (35), equations (51) and (52) were obtained under the deliberate omission of exact functional forms for dividends and transfers. In a more general setup, type-$\mathcal{B}$ households could receive income transfers from type-$\mathcal{A}$ households.

Equations (51) and (52) can be further detailed by using the consumption/labor supply choice. Plugging equation (47) in the budget constraint, and simplifying (note that $(1-\tau_t^{\mathcal{L}})W_t\Phi_a L_{a,t}^{\mathcal{B}} = (1-\tau_t^{\mathcal{L}})W_t\Phi_a - (1-\tau_t^{\mathcal{L}})W_t\Phi_a(1-L_{a,t}^{\mathcal{B}})$), one obtains

$$C_{a,t}^{\mathcal{B}} = \eta^{\mathcal{B}}(1-\tau_t^{\mathcal{L}})w_t + \eta^{\mathcal{B}}\left(\int_0^1 d_{a,t}^{\mathcal{B},\mathcal{U}}(i)\mathrm{d}i + trg_{a,t}^{\mathcal{B}} + \epsilon_t trx_{a,t}^{\mathcal{B}}\right)$$

Aggregating across a and dividing by technological progress yields

$$\check{C}_t^{\mathcal{B}} = \eta^{\mathcal{B}}N\psi(1-\tau_t^{\mathcal{L}})\check{w}_t + \eta^{\mathcal{B}}\left(\check{d}_t^{\mathcal{B},\mathcal{U}} + \check{trg}_t^{\mathcal{B}} + \epsilon_t\check{trx}_t^{\mathcal{B}}\right)$$

The term $\eta^{\mathcal{B}}$ captures the fact that type-$\mathcal{B}$ households use the extra dividends and transfers to increase consumption by a fraction $\eta^{\mathcal{B}}$. The remaining fraction $1-\eta^{\mathcal{B}}$ is spent on leisure. Using (48), aggregate effective labor supply for type-$\mathcal{B}$ households follows immediately

$$L_t^{\mathcal{B}} = N\psi - \check{C}_t^{\mathcal{B}}\left(\frac{1-\eta^{\mathcal{B}}}{\eta^{\mathcal{B}}}\frac{1}{(1-\tau_t^{\mathcal{L}})\check{w}_t}\right)$$

3.4 Labor unions

Labor unions hire labor services from households and sell them to manufacturers operating in the intermediate goods market. Labor unions are perfectly competitive in the input market and monopolistically competitive in the output market—they charge a markup to manufacturers, therefore creating a wedge between the wage paid by these firms and the wage received by households. Unions' profits are distributed to households in the form of dividends.

More specifically, there exists a continuum $h \in [0,1]$ of labor unions supplying labor to a continuum $j \in [0,1]$ of manufacturers operating in the tradable sector ($\mathcal{T}$) and to an identical continuum of manufacturers operating in the nontradable sector ($\mathcal{N}$). Intermediate goods sectors are indexed by $J \in \{\mathcal{T}, \mathcal{N}\}$. Each labor union supplies a specific variety of labor. Each manufacturer j operating in sector J demands some quantity of each labor variety from union h, $U_t^J(h,j)$, and aggregate varieties to form an homogeneous labor input, $U_t^J(j)$, according to the following CES specification

$$U_t^J(j) = \left(\int_0^1 U_t^J(h,j)^{\frac{\sigma^{\mathcal{U}}-1}{\sigma^{\mathcal{U}}}} \mathrm{d}h \right)^{\frac{\sigma^{\mathcal{U}}}{\sigma^{\mathcal{U}}-1}}$$

where $\sigma^{\mathcal{U}} \geq 0$ is the elasticity of substitution between labor varieties.
Manufacturers select the demand for each labor variety h by minimizing the cost of acquiring each one of them, subject to the fact that they must attain an overall labor quantity of $U_t^J(j)$. Letting $V_t(h)$ denote the wage charged by union h, each manufacturer solves

$$\min_{U_t^J(h,j)} \int_0^1 V_t(h) U_t^J(h,j) \mathrm{d}h \qquad \text{s.t.} \qquad U_t^J(j) = \left(\int_0^1 U_t^J(h,j)^{\frac{\sigma^{\mathcal{U}}-1}{\sigma^{\mathcal{U}}}} \mathrm{d}h \right)^{\frac{\sigma^{\mathcal{U}}}{\sigma^{\mathcal{U}}-1}}$$

The Lagrange multiplier of this problem corresponds to the manufacturer marginal cost of acquiring an extra unit of the labor input, *i.e.* the Lagrange multiplier equals V_t. The solution steps are similar to those presented in Box 1, yielding

$$U_t^J(h,j) = \left(\frac{V_t(h)}{V_t} \right)^{-\sigma^{\mathcal{U}}} U_t^J(j) \tag{53}$$

$$V_t = \left(\int_0^1 V_t(h)^{1-\sigma^{\mathcal{U}}} \mathrm{d}h \right)^{\frac{1}{1-\sigma^{\mathcal{U}}}}$$

The demand for labor variety h, $U_t(h)$, is obtained by integrating (53) over j and then summing across J, yielding

$$U_t(h) = \left(\frac{V_t(h)}{V_t} \right)^{-\sigma^{\mathcal{U}}} U_t \tag{54}$$

where U_t is aggregate labor demand.

We obtain a sluggish wage adjustment capturing the short-run dynamics present in the data by imposing quadratic adjustment costs with the form

$$\Gamma_t^V(h) = \frac{\phi_{\mathcal{U}}}{2} T_t U_t \left(\frac{\frac{V_t(h)}{V_{t-1}(h)}}{\frac{V_{t-1}}{V_{t-2}}} - 1 \right)^2 \tag{55}$$

where $\phi_{\mathcal{U}}$ is a sector specific scaling factor. The level of technology T_t enters as an economy-wide scaling factor so that costs do not become insignificant over time. Since all labor unions are identical, they solve the same optimization problem and set the same pricing rule in equilibrium (see Box 5). Hence $V_t(h) = V_t$, $\forall h$. Letting $\pi_t^V = V_t/V_{t-1}$ denote the (gross) rate of wage inflation, equation (55) becomes in stationary form

$$\check{\Gamma}_t^V = \frac{\phi_{\mathcal{U}}}{2} U_t \left(\frac{\pi_t^V}{\pi_{t-1}^V} - 1 \right)^2 \tag{56}$$

In the steady state wage inflation is constant and therefore

$$\check{\Gamma}^V = 0 \tag{57}$$

Let $\check{v}_t$ denote the relative price charged by labor unions scaled by productivity, $\check{v}_t = V_t/(P_t T_t)$. Scaling by productivity is necessary—which was already the case with $\check{w}_t$—since the relative price of labor grows at rate g in the steady state. This contrasts with the remaining relative prices, namely those charged by intermediate and final goods producers, which are stationary and not scaled by productivity. Wage inflation can thus be written as

$$\pi_t^V = \frac{\check{v}_t}{\check{v}_{t-1}} \frac{P_t T_t}{P_{t-1} T_{t-1}} \pi_t = \frac{\check{v}_t}{\check{v}_{t-1}} \pi_t g$$

or equivalently

$$\frac{\check{v}_t}{\check{v}_{t-1}} = \frac{\pi_t^V}{\pi_t g} \tag{58}$$

In the steady-state $\check{v}_t = \check{v}_{t-1}$ and hence

$$\pi^V = \pi \cdot g \tag{59}$$

Equation (59) shows that the steady-state wage inflation π^V is determined by the final consumer price inflation π and by the technological growth rate g. This contrasts with inflation rates of intermediate and final goods, which are solely a function of the final consumer price inflation.

The dividends of labor union h are

$$D_t^{\mathcal{U}}(h) = (1 - \tau_t^{\mathcal{L}})\big[\big(V_t(h) - W_t\big)U_t(h) - P_t \Gamma_t^V(h)\big] \tag{60}$$

Notice that nominal adjustment costs are measured in terms of the *numéraire*. In equilibrium the indexer h can be dropped, since all unions behave identically. Dividing (60) by $P_t T_t$ yields real dividends in stationary form

$$\check{d}_t^{\mathcal{U}} = (1 - \tau_t^{\mathcal{L}})\big[(\check{v}_t - \check{w}_t)U_t - \check{\Gamma}_t^V\big] \tag{61}$$

In the steady state

$$\check{d}^{\mathcal{U}} = (1 - \tau^{\mathcal{L}})(\check{v} - \check{w})U \tag{62}$$

Labor unions select the wage profile $\{V_{t+s}(h)\}_{s=0}^{\infty}$ that maximizes the present discounted value of the dividends stream, subject to the constraints imposed by demand and adjustment costs. This problem is presented and solved in Box 5.

Equation (5.1), which defines unions optimal pricing rule, mapping wages paid to households w_t to wages charged by unions v_t, has the following stationary form

$$\frac{\sigma^{\mathcal{U}}}{\sigma^{\mathcal{U}} - 1}\check{w}_t - \check{v}_t = \frac{\phi_{\mathcal{U}}}{\sigma^{\mathcal{U}} - 1}\left[\left(\frac{\pi_t^V}{\pi_{t-1}^V} - 1\right)\frac{\pi_t^V}{\pi_{t-1}^V} - \frac{1 - \tau_{t+1}^{\mathcal{L}}}{1 - \tau_t^{\mathcal{L}}}\frac{\theta \cdot g}{r_t}\frac{U_{t+1}}{U_t}\left(\frac{\pi_{t+1}^V}{\pi_t^V} - 1\right)\frac{\pi_{t+1}^V}{\pi_t^V}\right] \tag{63}$$

In the steady state wage inflation is constant and thus

$$\check{v} = \frac{\sigma^{\mathcal{U}}}{\sigma^{\mathcal{U}} - 1}\check{w} \tag{64}$$

This equation makes clear that, in the steady state where adjustment costs are zero, the wage charged by unions is simply a markup $\sigma^{\mathcal{U}}/(\sigma^{\mathcal{U}} - 1)$ over the marginal cost. Outside the steady state one must add adjustment costs to the pricing rule.

3.5 Firms

This section describes capital goods producers, entrepreneurial firms (also denominated as entrepreneurs), banks, manufacturers, and distributors.

The intermediate good sector is composed by tradable goods and non-tradable goods manufacturers. Each manufacturer combines labor services, rented from labor unions, with capital, rented from entrepreneurs. Intermediate goods are sold to distributors to be combined with imported goods, yielding four types of differentiated final goods: private consumption ($\mathcal{C}$), investment goods ($\mathcal{I}$), government consumption ($\mathcal{G}$) and export goods ($\mathcal{X}$).

Box 5: Unions maximization problem.

The labor union selects the wage profile $\{V_{t+s}(h)\}_{s=0}^{\infty}$ that maximizes the present discounted value of the dividends stream, subject to the constraints imposed by demand in (54) and adjustment costs in (55). The objective function for the union problem is

$$\mathcal{L}(\cdot) = \mathrm{E}_t \sum_{s=0}^{\infty} \tilde{R}_{t,s}(1-\tau^{\mathcal{L}}_{t+s})$$
$$\times \left\{ \big(V_{t+s}(h) - W_{t+s}\big) \left(\frac{V_{t+s}(h)}{V_{t+s}}\right)^{-\sigma^{\mathcal{U}}} U_{t+s} - P_{t+s}T_{t+s}U_{t+s}\frac{\phi_{\mathcal{U}}}{2}\left(\frac{\frac{V_{t+s}(h)}{V_{t+s-1}(h)}}{\frac{V_{t+s-1}}{V_{t+s-2}}} - 1\right)^2 \right\}$$

Dropping the expected value operator, the first-order condition yields

$$\frac{\partial \mathcal{L}(\cdot)}{\partial V_t(h)} = 0 \Leftrightarrow \tilde{R}_{t,0}(1-\tau^{\mathcal{L}}_t)\left[\left(\frac{V_t(h)}{V_t}\right)^{-\sigma^{\mathcal{U}}} U_t - \sigma^{\mathcal{U}}\big(V_t(h)-W_t\big)\left(\frac{V_t(h)}{V_t}\right)^{-\sigma^{\mathcal{U}}-1}\frac{U_t}{V_t}\right.$$
$$\left. - P_tT_tU_t\phi_{\mathcal{U}}\left(\frac{\frac{V_t(h)}{V_{t-1}(h)}}{\frac{V_{t-1}}{V_{t-2}}} - 1\right)\left(\frac{\frac{V_t(h)}{V_{t-1}(h)}}{\frac{V_{t-1}}{V_{t-2}}}\right)\frac{1}{V_t(h)}\right]$$
$$+ \tilde{R}_{t,1}(1-\tau^{\mathcal{L}}_{t+1})P_{t+1}T_{t+1}U_{t+1}\phi_{\mathcal{U}}\left(\frac{\frac{V_{t+1}(h)}{V_t(h)}}{\frac{V_t}{V_{t-1}}} - 1\right)\left(\frac{\frac{V_{t+1}(h)}{V_t(h)}}{\frac{V_t}{V_{t-1}}}\right)\frac{1}{V_t(h)} = 0$$

This equation can be simplified by recalling that $\tilde{R}_{t,0} = 1$ and $\tilde{R}_{t,1} = \theta/i_t$. Moreover, all labor unions solve the same problem in equilibrium, and so $V_t(h) = V_t$, $\forall h$. The previous equation thus collapses to

$$(1-\tau^{\mathcal{L}}_t)\left[U_t - \sigma^{\mathcal{U}}(V_t - W_t)\frac{U_t}{V_t} - T_tU_t\phi_{\mathcal{U}}\frac{P_t}{V_t}\left(\frac{\pi^V_t}{\pi^V_{t-1}} - 1\right)\frac{\pi^V_t}{\pi^V_{t-1}}\right]$$
$$+(1-\tau^{\mathcal{L}}_{t+1})T_{t+1}U_{t+1}\frac{\theta\cdot\phi_{\mathcal{U}}}{i_t}\frac{P_{t+1}}{V_t}\left(\frac{\pi^V_{t+1}}{\pi^V_t} - 1\right)\frac{\pi^V_{t+1}}{\pi^V_t} = 0$$

Multiplying by $V_t/(U_tP_t)$ we obtain

$$(1-\tau^{\mathcal{L}}_t)\left[\frac{V_t}{P_t} - \sigma^{\mathcal{U}}\frac{V_t - W_t}{P_t} - T_t\phi_{\mathcal{U}}\left(\frac{\pi^V_t}{\pi^V_{t-1}} - 1\right)\frac{\pi^V_t}{\pi^V_{t-1}}\right]$$
$$+(1-\tau^{\mathcal{L}}_{t+1})\frac{\theta\cdot\phi_{\mathcal{U}}}{i_t}\frac{P_{t+1}}{P_t}\frac{T_{t+1}U_{t+1}}{U_t}\left(\frac{\pi^V_{t+1}}{\pi^V_t} - 1\right)\frac{\pi^V_{t+1}}{\pi^V_t} = 0$$

Using the definitions $\pi_t = P_t/P_{t-1}$ and $i_t = r_t \cdot \pi_{t+1}$, the previous expression simplifies to

$$v_t - \sigma^{\mathcal{U}}(v_t - w_t) - T_t\phi_{\mathcal{U}}\left(\frac{\pi^V_t}{\pi^V_{t-1}} - 1\right)\frac{\pi^V_t}{\pi^V_{t-1}} + \frac{1-\tau^{\mathcal{L}}_{t+1}}{1-\tau^{\mathcal{L}}_t}\frac{\theta\cdot\phi_{\mathcal{U}}}{r_t}\frac{T_{t+1}U_{t+1}}{U_t}\left(\frac{\pi^V_{t+1}}{\pi^V_t} - 1\right)\frac{\pi^V_{t+1}}{\pi^V_t} = 0$$

Rearranging

$$\frac{\sigma^{\mathcal{U}}}{\sigma^{\mathcal{U}}-1}w_t - v_t = \frac{\phi_{\mathcal{U}}}{\sigma^{\mathcal{U}}-1}\left[T_t\left(\frac{\pi^V_t}{\pi^V_{t-1}} - 1\right)\frac{\pi^V_t}{\pi^V_{t-1}} - \frac{1-\tau^{\mathcal{L}}_{t+1}}{1-\tau^{\mathcal{L}}_t}\frac{\theta}{r_t}\frac{T_{t+1}U_{t+1}}{U_t}\left(\frac{\pi^V_{t+1}}{\pi^V_t} - 1\right)\frac{\pi^V_{t+1}}{\pi^V_t}\right] \quad (5.1)$$

Capital is produced by capital goods producers and acquired by entrepreneurial firms, who are the owners of the capital stock during each production cycle. Entrepreneurial firms have insufficient internal resources to finance capital acquisitions, but are allowed to borrow additional funds from banks at a given cost. In each period, entrepreneurial firms face an idiosyncratic risk shock changing the value of the capital stock (or equivalently the return on the capital stock) and making their activity risky. Banks operate in a perfectly competitive environment. Each bank borrows resources from households, lending them to many entrepreneurs, implying a zero *ex-post* profit at all times. They charge to entrepreneurs a spread over the risk free rate—which depends on the entrepreneurs' probability of default, leverage, return on assets, and monitoring costs incurred by banks—on the funds borrowed, to compensate for the losses incurred on those entrepreneurs who default on their debt.

Banks and capital goods producers are perfectly competitive in both input and output markets. All remaining firms operate in a monopolistically competition environment in the output market—therefore charging a markup over the marginal cost—and are perfectly competitive in the input market.

3.5.1 Capital goods producers

Capital is exclusively produced by capital goods producers. It is thereafter bought by entrepreneurs and rented afterwards to manufacturers for usage in the production process. Box 6 summarizes the relevant framework and notation used to characterize the capital goods sector. Capital goods producers are price takers in input and output markets.

More specifically, there is a continuum of capital goods producers indexed by $i \in [0, 1]$, for each manufacturing sector $J \in \{\mathcal{T}, \mathcal{N}\}$, where $\mathcal{T}$ stands for the Tradable sector and $\mathcal{N}$ stands for the Non-tradable sector. In each period, capital goods producers supplying sector J purchase previously installed capital from entrepreneurs $\widetilde{K}_t^J(i)$ and investment goods from investment goods producers $I_t^J(i)$ to produce new installed capital $\widetilde{K}_{t+1}^J(i)$, according to the following law of motion

$$\widetilde{K}_{t+1}^J(i) = \widetilde{K}_t^J(i) + \zeta_t^{\mathcal{I}} I_t^J(i) \tag{65}$$

where $\zeta_t^{\mathcal{I}}$ is an investment efficiency shock. We impose a sluggish pattern for investment, consistent with the data, by assuming quadratic adjustment costs with the form

$$\Gamma_t^{\mathcal{I}J}(i) = \frac{\phi_{\mathcal{I}J}}{2} I_t^J \left(\frac{I_t^J(i)/g}{I_{t-1}^J(i)} - 1 \right)^2 \tag{66}$$

where I_t^J is sector J's overall investment at period t. Since the problem faced by each capital goods producer is identical, the indexer i can be dropped when referring to equilibrium conditions. Thus, in stationary form (66) becomes

$$\check{\Gamma}_t^{\mathcal{I}J} = \frac{\phi_{\mathcal{I}J}}{2} \check{I}_t^J \left(\frac{\check{I}_t^J}{\check{I}_{t-1}^J} - 1 \right)^2 \tag{67}$$

Box 6: Framework and notation related with capital utilization and accumulation.

$\bar{K}^J_{t+1}$	–	Represents the total stock of physical capital that can be used at period $t+1$. This quantity is produced by capital goods producers and bought by entrepreneurs at the end of period t. It respects the standard law of motion $\bar{K}^J_{t+1} = (1-\delta^J)\bar{K}^J_t + \zeta^{\mathcal{I}}_t I^J_t$, where δ^J is the depreciation rate, $\zeta^{\mathcal{I}}_t$ is an investment efficiency shock, and I^J_t represents the investment level, which is set by capital goods producers.
K^J_t	–	Represents the stock of capital that is actually used by sector J's manufacturers in period t, $K^J_t = u^J_t \bar{K}^J_t$. This quantity is influenced by capital utilization u^J_t, which is optimally selected by entrepreneurs.
$\widetilde{K}^J_{t+1}$	–	Represents "new installed capital" at period t. This quantity is set by capital goods producers, and evolves according to the following law of motion: $\widetilde{K}^J_{t+1} = \widetilde{K}^J_t + \zeta^{\mathcal{I}}_t I^J_t$. The quantity $\widetilde{K}^J_{t+1}$ is sold to entrepreneurs at the end of period t, and thus $\widetilde{K}^J_{t+1} = \bar{K}^J_{t+1}$.
$\widetilde{K}^J_t$	–	Represents "previously installed capital" at period t. This quantity is bought by capital goods producers from entrepreneurs at the end of period t. Since capital depreciates during the production cycle, $\widetilde{K}^J_t = (1-\delta^J)\bar{K}^J_t$. The quantity $\widetilde{K}^J_t$ can be seen as the "undepreciated fraction of physical capital that has been used in the period t production cycle" (Christiano, Motto, and Rostagno, 2010).

According to these conventions, at the end of period t entrepreneurs receive back from manufacturers utilized capital $K^J_t = u^J_t \bar{K}^J_t$. During period t, the existing stock of capital, $\bar{K}^J_t$, faces a depreciation of δ^J_t, regardless of whether it has been used in the production cycle. Therefore, physical capital at the end of period t is $\widetilde{K}^J_t = (1-\delta^J)\bar{K}^J_t$. This amount, termed "previously installed capital," is sold to capital goods producers, who combine them with investment goods to produce "new installed capital," $\widetilde{K}^J_{t+1} = \widetilde{K}^J_t + \zeta^{\mathcal{I}}_t I^J_t$. At the end of period t, entrepreneurs buy at price P^K_t the new installed capital, $\bar{K}^J_{t+1} = \widetilde{K}^J_{t+1}$, and rent it, partially or entirely, to manufacturers, receiving an unitary rent of $R^{\mathcal{K}J}_t$.

In the steady state investment in stationary form is constant and therefore

$$\check{\Gamma}^{\mathcal{I}J} = 0 \tag{68}$$

Dividends of capital goods producer i supplying sector J are

$$\begin{aligned} D^{\mathcal{K}J}_t(i) &= P^{\mathcal{K}J}_t \widetilde{K}^J_{t+1}(i) - P^{\mathcal{K}J}_t \widetilde{K}^J_t(i) - P^{\mathcal{I}}_t\left[I^J_t(i) + \Gamma^{\mathcal{I}J}_t(i)\right] \\ &= P^{\mathcal{K}J}_t\left[\widetilde{K}^J_t(i) + \zeta^{\mathcal{I}}_t I^J_t(i)\right] - P^{\mathcal{K}J}_t \widetilde{K}^J_t(i) - P^{\mathcal{I}}_t\left[I^J_t(i) + \Gamma^{\mathcal{I}J}_t(i)\right] \\ &= P^{\mathcal{K}J}_t \zeta^{\mathcal{I}}_t I^J_t(i) - P^{\mathcal{I}}_t\left[I^J_t(i) + \Gamma^{\mathcal{I}J}_t(i)\right] \end{aligned} \tag{69}$$

The second step uses the law of motion of capital in (65). The term $P^{\mathcal{K}J}_t$ is simultaneously the price of previously and newly installed capital at the end of period t, since the marginal rate of transformation between the two is assumed to be one. From equation (69) follows that capital goods producers select the investment level I^J_t, which adds to the existing capital stock $\widetilde{K}^J_t$ to yield new installed capital $\widetilde{K}^J_{t+1}$. In the process they pay for investment goods plus the adjustment cost. Taxes on capital are paid by entrepreneurs, since they are the capital holders in this economy. The stationary specification of (69) is obtained after dropping the indexer i

and dividing by $P_t T_t$

$$\check{d}_t^{\mathcal{K}J} = p_t^{\mathcal{K}J} \zeta_t^{\mathcal{I}} \check{I}_t^J - p_t^{\mathcal{I}} \left[\check{I}_t^J + \check{\Gamma}_t^{\mathcal{I}J} \right] \tag{70}$$

where $p_t^{\mathcal{K}J} = P_t^{\mathcal{K}J}/P_t$ is the relative price of sector J's capital goods and $p_t^{\mathcal{I}} = P_t^{\mathcal{I}}/P_t$ is the relative price of investment goods. In the steady state adjustment costs are zero and shocks are absent. Thus

$$\check{d}^{\mathcal{K}J} = \left(p^{\mathcal{K}J} - p^{\mathcal{I}} \right) \check{I}^J \tag{71}$$

Capital goods producers select the intertemporal profile $\{I_{t+s}^J(i)\}_{s=0}^{\infty}$ that maximizes the net present value of the dividends stream, subject to adjustment costs in (66), and taking $P_t^{\mathcal{K}J}$ and $P_t^{\mathcal{I}}$ as given. The problem is

$$\max_{I_{t+s}^J(i)} \mathrm{E}_t \sum_{s=0}^{\infty} \tilde{R}_{t,s} \left\{ P_{t+s}^{\mathcal{K}J} \zeta_{t+s}^{\mathcal{I}} I_{t+s}^J(i) - P_{t+s}^{\mathcal{I}} \left[I_{t+s}^J(i) + \frac{\phi_{\mathcal{I}J}}{2} I_{t+s}^J \left(\frac{I_{t+s}^J(i)}{g \cdot I_{t+s-1}^J(i)} - 1 \right)^2 \right] \right\}$$

Using the fact $\tilde{R}_{t,1} = \theta / i_t$, the first-order condition defining optimal investment demand is (we drop the expected value operator)

$$P_t^{\mathcal{K}J} \zeta_t^{\mathcal{I}} = P_t^{\mathcal{I}} \left[1 + \phi_{\mathcal{I}J} \left(\frac{I_t^J(i)}{g \cdot I_{t-1}^J(i)} - 1 \right) \frac{I_t^J}{g \cdot I_{t-1}^J(i)} - \frac{\theta \cdot \phi_{\mathcal{I}J}}{i_t} \frac{P_{t+1}^{\mathcal{I}}}{P_t^{\mathcal{I}}} I_{t+1}^J \left(\frac{I_{t+1}^J(i)}{g \cdot I_t^J(i)} - 1 \right) \frac{I_{t+1}^J(i)}{g \cdot \left(I_t^J(i) \right)^2} \right]$$

Dropping the indexer i, using the definition of real interest rate $r_t = i_t / \pi_t$, and dividing by P_t we obtain

$$p_t^{\mathcal{K}J} \zeta_t^{\mathcal{I}} = p_t^{\mathcal{I}} \left[1 + \phi_{\mathcal{I}J} \left(\frac{I_t^J}{g \cdot I_{t-1}^J} - 1 \right) \frac{I_t^J}{g \cdot I_{t-1}^J} - \frac{\theta \cdot \phi_{\mathcal{I}J}}{g \cdot r_t} \frac{p_{t+1}^{\mathcal{I}}}{p_t^{\mathcal{I}}} \left(\frac{I_{t+1}^J}{g \cdot I_t^J} - 1 \right) \left(\frac{I_{t+1}^J}{I_t^J} \right)^2 \right] \tag{72}$$

In stationary form condition (72) becomes

$$p_t^{\mathcal{K}J} \zeta_t^{\mathcal{I}} = p_t^{\mathcal{I}} \left[1 + \phi_{\mathcal{I}J} \left(\frac{\check{I}_t^J}{\check{I}_{t-1}^J} - 1 \right) \frac{\check{I}_t^J}{\check{I}_{t-1}^J} - \frac{g \cdot \theta \cdot \phi_{\mathcal{I}J}}{r_t} \frac{p_{t+1}^{\mathcal{I}}}{p_t^{I}} \left(\frac{\check{I}_{t+1}^J}{\check{I}_t^J} - 1 \right) \left(\frac{\check{I}_{t+1}^J}{\check{I}_t^J} \right)^2 \right] \tag{73}$$

In the steady state shocks and adjustment costs are absent, and therefore

$$p^{\mathcal{K}J} = p^{\mathcal{I}} \tag{74}$$

Hence, the price of capital equals the marginal cost (there is no markup) and dividends equal zero in equilibrium. This result follows directly from the fact that capital goods producers are perfectly competitive in input and output markets. Outside the steady state there is a wedge between the price of capital and the price of investment goods, induced by investment efficiency shocks and adjustment costs.

3.5.2 Entrepreneurs and banks

There is a continuum of infinitely lived entrepreneurial firms $l \in [0,1]$ for each manufacturing sector $J \in \{\mathcal{T}, \mathcal{N}\}$. At the beginning of period $t+1$ entrepreneurs have an aggregate physical capital stock of (we use the expressions "entrepreneurs" and "entrepreneurial firms" interchangeably)

$$\bar{K}_{t+1}^{J} = (1-\delta_t^J)\bar{K}_t^J + \zeta_t^{\mathcal{I}} I_t^J \tag{75}$$

where δ_t^J is sector J's capital depreciation rate (which may also be subject to a capital destroying shock), $\bar{K}_t^J$ represents the total stock of physical capital at t, and $\zeta_t^{\mathcal{I}}$ is the time t investment efficiency shock previously introduced. Dividing (75) by T_t yields the stationary specification for capital accumulation

$$\check{\bar{K}}_{t+1}^{J} = \frac{1}{g}\left[(1-\delta_t^J)\check{\bar{K}}_t^J + \zeta_t^{\mathcal{I}} \check{I}_t^J\right] \tag{76}$$

In the steady state this equation simplifies to

$$\check{\bar{K}}^{J} = \frac{1}{g}\left[(1-\delta^J)\check{\bar{K}}^J + \check{I}^J\right]$$

which can be written as

$$\frac{\check{I}^J}{\check{\bar{K}}^J} = g + \delta^J - 1 \tag{77}$$

Thus, in the steady-state the investment to capital ratio equals the net growth rate plus the depreciation rate. Below, we clarify the idiosyncratic risky environment in which entrepreneurs select capital purchases and characterize the renting activity. For convenience, we solve the model backwards, *i.e.* we first present the renting activity and only thereafter characterize capital purchases. However, the ordering is irrelevant since these two stages are independent.

Capital renting

The entrepreneurial firm l operating in sector J selects the capital utilization rate, $u_t^J(l)$, after observing the idiosyncratic shock $\omega_t(l)$. To ease the exposition, we drop the indexer l from the idiosyncratic shock variable ω_t. We adopt the following functional form for the costs of capital utilization

$$a\big(u_t^J(l)\big) = \frac{1}{2}\phi_a^J\sigma_a^J\big(u_t^J(l)\big)^2 + \phi_a^J\big(1-\sigma_a^J\big)u_t^J(l) + \phi_a^J\left(\frac{\sigma_a^J}{2} - 1\right) \tag{78}$$

Box 7: The adjustment cost function $a(u_t^J)$.

Let us drop the indexer l. As in Christiano, Motto, and Rostagno (2010), the adjustment cost function $a(u_t^J)$ takes the form

$$a(u_t^J) = \frac{1}{2}\phi_a^J \sigma_a^J \left(u_t^J\right)^2 + \phi_a^J \left(1 - \sigma_a^J\right) u_t^J + \phi_a^J \left(\frac{\sigma_a^J}{2} - 1\right)$$

where ϕ_a^J and σ_a^J are parameters that govern the behavior of $a(u_t^J)$. In particular, $\phi_a^J > 0$ is selected to ensure that capital utilization costs equal one in the steady state, whereas $\sigma_a^J > 0$ controls the degree of convexity. The first and second order derivatives with respect to u_t^J are

$$\frac{\mathrm{d}a}{\mathrm{d}u_t^J} = \phi_a^J \sigma_a^J u_t^J + \phi_a^J \left(1 - \sigma_a^J\right)$$

$$\frac{\mathrm{d}^2 a}{\mathrm{d}\left(u_t^J\right)^2} = \phi_a^J \sigma_a^J > 0$$

The sign of the first derivative depends on the magnitude of σ_a^J. In the steady-state $u_t^J = 1$ implying that adjustment costs are zero

$$a(1) = \frac{1}{2}\phi_a^J \sigma_a^J + \phi_a^J(1 - \sigma_a^J) + \phi_a^J \left(\frac{\sigma_a^J}{2} - 1\right) = \phi_a^J \sigma_a^J + \phi_a^J - \phi_a^J \sigma_a^J - \phi_a^J = 0$$

With financial frictions, capital utilization rate u_t^J is selected by entrepreneurs, whereas without financial frictions it is selected by capital goods producers. In both cases, the first-order condition of the optimization problem is the same, determining the real rental rate of capital paid by manufacturers in equilibrium

$$r_t^{\mathcal{K}J} = \frac{\mathrm{d}a}{\mathrm{d}u_t^J} = \phi_a^J \sigma_a^J u_t^J + \phi_a^J \left(1 - \sigma_a^J\right)$$

In the steady state with $u_t^J = 1$

$$r^{\mathcal{K}J} = \phi_a^J \sigma_a^J + \phi_a^J(1 - \sigma_a^J) = \phi_a^J$$

Hence, ϕ_a^J pins down $r^{\mathcal{K}J}$ uniquely in the steady state. Finally, notice that

$$\left.\frac{\mathrm{d}^2 a/\mathrm{d}u_t^J}{\mathrm{d}a/\mathrm{d}u_t^J}\right|_{u_t^J=1} = \frac{\phi_a^J \sigma_a^J}{\phi_a^J \sigma_a^J + \phi_a^J(1 - \sigma_a^J)} = \sigma_a^J$$

Thus, σ_a^J establishes the curvature of $a(u_t^J)$. An increase in the curvature of the adjustment cost function (an increase in σ_a^J) implies higher costs for a given change in the capital utilization rate.

where $\phi_a^J > 0$ is calibrated to ensure that the capital utilization rate equals one in the steady state and $\sigma_a^J > 0$ is a parameter that controls the curvature of $a(u_t^J)$. The parameter ϕ_a^J plays a key role in (78), since it pins down the real rental rate of capital in the steady state. The function $a(u_t^J)$ is analyzed in detail in Box 7. In equilibrium all entrepreneurs set the same utilization rate and thus adjustment costs are

$$a(u_t^J) = \frac{1}{2}\phi_a^J \sigma_a^J \left(u_t^J\right)^2 + \phi_a^J(1 - \sigma_a^J)u_t^J + \phi_a^J \left(\frac{\sigma_a^J}{2} - 1\right) \tag{79}$$

In the steady state $u_t^J = 1$ (since $\phi_a^J > 0$ is calibrated accordingly), and

$$a(1) = 0 \tag{80}$$

The capital stock effectively used in production is, in stationary form

$$\check{K}_t^J = u_t^J \check{\bar{K}}_t^J \tag{81}$$

In the steady state

$$\check{K}^J = \check{\bar{K}}^J \tag{82}$$

The after-tax profits of entrepreneur l operating in sector J arising from the rental activity are

$$\Pi_t^{\mathcal{EP}J,RA}(l) = (1 - \tau_t^{\mathcal{K}})\left[R_t^{\mathcal{K}J} u_t^J(l) - P_t a\left(u_t^J(l)\right)\right]\omega_t^J \bar{K}_t^J(l) \tag{83}$$

where $\tau_t^{\mathcal{K}}$ is the capital income tax rate and $R_t^{\mathcal{K}J}$ is the nominal rental rate of capital charged to intermediate goods producers. Entrepreneurs are price takers, and thus $R_t^{\mathcal{K}J}$ is taken as given. The component ω_t^J is the time t idiosyncratic shock that changed physical capital from $\bar{K}_t^J$ into $\omega_t^J \bar{K}_t^J$. Entrepreneurs select the capital utilization rate profile $\{u_{t+s}^J(l)\}_{s=0}^{\infty}$ that maximizes the present discounted value of after-tax profits related with the rental activity. Since there are no intertemporal interactions (contrary to other firms), this is equivalent to maximize (83) in each period. The first-order condition yields

$$\frac{\mathrm{d}\Pi_t^{\mathcal{EP}J,RA}(l)}{\mathrm{d}u_t^J(l)} = 0 \Leftrightarrow (1 - \tau_t^{\mathcal{K}})\left[R_t^{\mathcal{K}J} - P_t \frac{\mathrm{d}a}{\mathrm{d}u_t^J(l)}\right]\omega_t^J \bar{K}_t^J(l) = 0 \Leftrightarrow R_t^{\mathcal{K}J} = P_t \frac{\mathrm{d}a}{\mathrm{d}u_t^J(l)}$$

The idiosyncratic shock plays no role in the entrepreneur decision. Moreover, the utilization rate of capital is independent of the capital stock. In stationary form and using the result presented in Box 7, the first-order condition can be restated as

$$r_t^{\mathcal{K}J} = \phi_a^J \sigma_a^J u_t^J + \phi_a^J (1 - \sigma_a^J) \tag{84}$$

Thus, the real rental rate of capital equals the cost of using one additional unit of installed capital. In the steady state the parameter ϕ_a^J pins down uniquely the real rental rate of capital

$$r^{\mathcal{K}J} = \phi_a^J \tag{85}$$

The resource cost associated with capital utilization—a key element of the intermediate goods market clearing condition as it measures real resources that were not used in production—is

$$RCU_t^J = P_t a(u_t^J)\bar{K}_t^J$$

It is useful to evaluate this resource cost in terms of P_t^J and in stationary form

$$r\check{c}u_t^J = \frac{a(u_t^J)\check{\bar{K}}_t^J}{p_t^J} \tag{86}$$

Dividing by P_t^J is crucial for the market clearing condition, as the resource cost becomes measured in the same unit of intermediate goods. In the steady state $a(1) = 0$ and thus

$$r\check{c}u^J = 0 \tag{87}$$

Idiosyncratic risky environment and optimal capital purchases

At the end of period t each entrepreneur buys a nominal amount of capital $P_t^{\mathcal{K}J}\bar{K}_{t+1}^J(l)$ from capital goods producers. The element $\bar{K}_{t+1}^J(l)$ represents the total quantity of capital that can be used in the period $t+1$ production cycle. To purchase the new amount of capital, the entrepreneur has to respect the following balance sheet

$$P_t^{\mathcal{K}J}\bar{K}_{t+1}^J(l) = B_t^J(l) + N_t^J(l)$$

where $B_t^J(l)$ is an external finance component (bank loans) and $N_t^J(l)$ is an internal finance component (net worth), both evaluated at the end of period t.

Bank loans represent an important liability with potentially large macroeconomic effects, since the entrepreneur is forced to declare bankruptcy if he is unable to paid them back. The balance sheet equation can be rearranged to focus on external finance needs

$$B_t^J(l) = P_t^{\mathcal{K}J}\bar{K}_{t+1}^J(l) - N_t^J(l) \tag{88}$$

This equation can be expressed in terms of the leverage ratio $B_t^J(l)/N_t^J(l)$

$$\frac{B_t^J(l)}{N_t^J(l)} = \frac{P_t^{\mathcal{K}J}\bar{K}_{t+1}^J(l)}{N_t^J(l)} - 1$$

Aggregating the balance sheet equation across all entrepreneurs and dividing by $P_t T_t$ yields

$$\check{b}_t^J = p_t^{\mathcal{K}J}\check{\bar{K}}_{t+1}^J \cdot g - \check{n}_t^J \tag{89}$$

The steady-state version is

$$\check{b}^J = p^{\mathcal{K}J} \check{\bar{K}}^J \cdot g - \check{n}^J \tag{90}$$

After purchasing new capital from capital goods producers (but before selecting the utilization rate), entrepreneurs experience an idiosyncratic shock ω_{t+1}^J that changes the capital stock from $\bar{K}_{t+1}^J(l)$ to $\omega_{t+1}^J \bar{K}_{t+1}^J(l)$. This shock takes place at the beginning of period $t+1$, creating a risky environment. In the extreme case of a severe shock, the entrepreneur experiences a loss that makes impossible to meet his debt obligations, forcing him to declare bankruptcy. More specifically, there exists an endogenous threshold level for the idiosyncratic shock, $\bar{\omega}_{t+1}^J$, separating two distinct outcomes: if $\omega_{t+1}^J \geq \bar{\omega}_{t+1}^J$ the entrepreneur is able to payoff his debts and is therefore solvent, whereas if $\omega_{t+1}^J < \bar{\omega}_{t+1}^J$ the entrepreneur cannot meet his debt obligations and is forced to declare bankruptcy. The value for the threshold $\bar{\omega}_{t+1}^J$ is optimally set by the entrepreneur when selecting optimal capital purchases.

The idiosyncratic shock is directly observed by the entrepreneur at no cost. On the contrary, banks can only observe the shock if they pay a unitary monitoring cost μ_{t+1} over the firm value. This asymmetric information setup is in line with the "costly state verification" literature, pioneered by Townsend (1979). Monitoring costs include all bankruptcy costs (auditing costs, accounting and legal costs, asset liquidation, business interruption effects, among others). The monitoring activity is undertaken by type-$\mathcal{A}$ households at no cost and with no effort, and thus the remuneration for monitoring work performance paid by banks corresponds to a pure income effect.

The random variable ω_{t+1}^J follows a log-normal distribution with a mean of unity,

$$\ln \omega_t^J \sim \mathcal{N}\left(-\frac{1}{2}\left(\sigma_t^{\mathcal{E}J}\right)^2, \left(\sigma_t^{\mathcal{E}J}\right)^2\right)$$

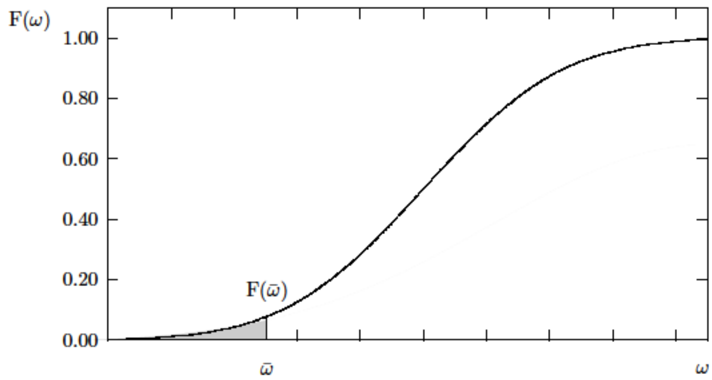
distributed independently over time and across entrepreneurs. Let $\mathfrak{F}_t^J(x) = \Pr[\omega_{t+1}^J < x]$ denote the cumulative distribution function and $\mathfrak{f}_t(x)$ the corresponding probability density function of ω_{t+1}^J. The standard deviation $\sigma_t^{\mathcal{E}J}$ is itself a stochastic process, known to entrepreneurs when deciding on capital purchases (contrary to ω_t^J, which is only observed afterwards). Figure 3 depicts the unit mean cumulative log-normal distribution $\mathfrak{F}(\omega)$.

Entrepreneurs *ex-ante* after-tax return to capital, $Ret_t^{\mathcal{K}J}$, is

$$\begin{aligned}
Ret_t^{\mathcal{K}J} &= \mathrm{E}_t \frac{(1-\tau_{t+1}^{\mathcal{K}})\left[R_{t+1}^{\mathcal{K}J} u_{t+1}^J - P_{t+1} a(u_{t+1}^J)\right] + (1-\delta_{t+1}^J) P_{t+1}^{\mathcal{K}J} + \tau_{t+1}^{\mathcal{K}} \delta_{t+1}^J P_{t+1}^{\mathcal{K}J}}{P_t^{\mathcal{K}J}} \\
&= \mathrm{E}_t \frac{(1-\tau_{t+1}^{\mathcal{K}})\left[r_{t+1}^{\mathcal{K}J} u_{t+1}^J - a(u_{t+1}^J)\right] + (1-\delta_{t+1}^J) p_{t+1}^{\mathcal{K}J} + \tau_{t+1}^{\mathcal{K}} \delta_{t+1}^J p_{t+1}^{\mathcal{K}J}}{\frac{P_t^{\mathcal{K}J}}{P_t} \frac{P_t}{P_{t+1}}} \\
&= \mathrm{E}_t \pi_{t+1} \frac{(1-\tau_{t+1}^{\mathcal{K}})\left[r_{t+1}^{\mathcal{K}J} u_{t+1}^J - a(u_{t+1}^J)\right] + (1-\delta_{t+1}^J) p_{t+1}^{\mathcal{K}J} + \tau_{t+1}^{\mathcal{K}} \delta_{t+1}^J p_{t+1}^{\mathcal{K}J}}{p_t^{\mathcal{K}J}}
\end{aligned}$$

This element is identical for all entrepreneurs, since in equilibrium $u_{t+1}^J(l) = u_{t+1}^J$, $\forall l$. The

Figure 3: The cdf of a lognormal distribution



return to capital is an expected value, reflecting $t+1$ events conditional on the information set at t. More specifically, it reflects $t+1$ after-tax profits per unit of capital arising from the renting activity—see equation (83)—evaluated at $\mathrm{E}_t(\omega_{t+1}) = 1$, plus the $t+1$ unitary value of the undepreciated fraction of capital and the tax deduction on capital depreciation. The expected value of entrepreneur l's assets at t (the expected value of the entrepreneurial firm) is therefore $Ret_t^{\mathcal{K}J} P_t^{\mathcal{K}J} \bar{K}_{t+1}^J(l)$.

The return to capital can be alternatively expressed as

$$Ret_t^{\mathcal{K}J} = \pi_{t+1} ret_t^{\mathcal{K}J}$$

where $\pi_{t+1} = P_{t+1}/P_t$ is the *numéraire* inflation and

$$ret_t^{\mathcal{K}J} = \left(p_t^{\mathcal{K}J}\right)^{-1} \left[(1-\tau_{t+1}^{\mathcal{K}})(r_{t+1}^{\mathcal{K}J} u_{t+1}^J - a(u_{t+1}^J)) + (1-\delta_{t+1}^J) p_{t+1}^{\mathcal{K}J} + \tau_{t+1}^{\mathcal{K}} \delta_{t+1}^J p_{t+1}^{\mathcal{K}J} \right] \tag{91}$$

The steady-state version of equation (91) is

$$ret^{\mathcal{K}J} = \left(p^{\mathcal{K}J}\right)^{-1} \left[(1-\tau^{\mathcal{K}}) r^{\mathcal{K}J} + (1-\delta^J) p^{\mathcal{K}J} + \tau^{\mathcal{K}} \delta^J p^{\mathcal{K}J} \right] \tag{92}$$

After setting $\bar{K}_{t+1}^J(l)$, the entrepreneur faces an idiosyncratic shock ω_{t+1}^J that changes the value of capital from $P_t^{\mathcal{K}J} \bar{K}_{t+1}^J(l)$ into $\omega_{t+1}^J P_t^{\mathcal{K}J} \bar{K}_{t+1}^J(l)$. Therefore, the value of the entrepreneurial firm also changes to $\omega_{t+1}^J Ret_t^{\mathcal{K}J} P_t^{\mathcal{K}J} \bar{K}_{t+1}^J(l)$. This amount is comprised between zero and infinity, since $\omega_{t+1}^J \in [0, \infty)$.

Net worth is insufficient to finance all capital purchases at the end of period t and thus entrepreneurs are forced to borrow from banks. Otherwise, leverage would be zero and the model would collapse to the benchmark case with no financial frictions (latter we impose a set of aggregate shocks on net worth and dividends to ensure that $P_t^{\mathcal{K}J} \bar{K}_{t+1}^J - N_t^J > 0$ holds in every period). Each entrepreneur l signs a standard one-period debt contract with the bank in each period t, defining the total amount borrowed $B_t^J(l)$ and the gross interest rate $i_{t+1}^{BJ}(l)$. At period $t+1$ the entrepreneur has to pay to the bank a nominal amount of $i_{t+1}^{BJ}(l) B_t^J(l)$, otherwise he

Box 8: The consequences of the idiosyncratic shock ω_{t+1}.

The cut-off level $\bar{\omega}^J_{t+1}$ must satisfy the following condition

$$\bar{\omega}^J_{t+1} Ret^{\mathcal{K}J}_t P^{\mathcal{K}J}_t \bar{K}^J_{t+1}(l) = i^{BJ}_{t+1}(l) B^J_t(l)$$

which states that, if $\omega^J_{t+1} = \bar{\omega}^J_{t+1}$, the value of the firm is just enough to keep up with financial obligations, implying a net worth value of zero. For a given draw ω_{t+1}, this condition bears the following consequences

$$\text{If} \begin{cases} \omega_{t+1} < \bar{\omega}_{t+1} \Rightarrow \begin{cases} \text{the bank pays monitoring cost } \mu_{t+1}\omega^J_{t+1} Ret^{\mathcal{K}J}_t P^{\mathcal{K}J}_t \bar{K}^J_{t+1}(l) \\ \quad \text{and receives } (1-\mu_{t+1})\omega^J_{t+1} Ret^{\mathcal{K}J}_t P^{\mathcal{K}J}_t \bar{K}^J_{t+1}(l) \\ \\ \text{the entrepreneur receives 0 and goes bankrupt} \end{cases} \\ \\ \omega_{t+1} \geq \bar{\omega}_{t+1} \Rightarrow \begin{cases} \text{the bank receives } i^{BJ}_{t+1}(l) B^J_t(l) \text{ from the entrepreneur} \\ \\ \text{the entrepreneur receives } \omega^J_{t+1} Ret^{\mathcal{K}J}_t P^{\mathcal{K}J}_t \bar{K}^J_{t+1}(l) - i^{BJ}_{t+1}(l) B^J_t(l) \geq 0, \\ \quad \text{which is the quantity to be subject to a maximization procedure} \end{cases} \end{cases}$$

goes bankrupt. The key point of this framework is that, if the idiosyncratic shock is above the threshold value $\bar{\omega}^J_{t+1}$, the entrepreneur is able to honor the debt contract paying to the bank $i^{BJ}_{t+1}(l)B^J_t(l)$, whereas in the opposite case he is forced to declare bankruptcy. The cut-off level $\bar{\omega}^J_{t+1}$ has to satisfy the following condition

$$\bar{\omega}^J_{t+1} Ret^{\mathcal{K}J}_t P^{\mathcal{K}J}_t \bar{K}^J_{t+1}(l) = i^{BJ}_{t+1}(l) B^J_t(l) \tag{93}$$

which states that, if $\omega^J_{t+1} = \bar{\omega}^J_{t+1}$, the value of the entrepreneurial firm is just enough to keep up with financial obligations, implying an *ex-post* net worth value of zero. Given the decision to buy capital worth $P^{\mathcal{K}J}_t \bar{K}^J_{t+1}(l)$ at the end of period t, equation (93) sets liquidity needs, and establishes a critical link between $\bar{\omega}^J_{t+1}$, $Ret^{\mathcal{K}J}_t$ and $i^{BJ}_{t+1}(l)$.

If the entrepreneur goes bankrupt, the bank keeps only a fraction $1-\mu_{t+1}$ of the value of the entrepreneurial firm (the recovery rate)

$$(1-\mu_{t+1})\omega^J_{t+1} Ret^{\mathcal{K}J}_t P^{\mathcal{K}J}_t \bar{K}^J_{t+1}(l)$$

The remaining fraction μ_{t+1} represent monitoring costs paid to type-$\mathcal{A}$ households for monitoring services. If the entrepreneur is able to satisfy the debt contract, the bank receives the borrowed amount $i^{BJ}_{t+1}(l)B^J_t(l)$, while the entrepreneur, as the residual claimant, obtains

$$\begin{aligned} &\omega^J_{t+1} Ret^{\mathcal{K}J}_t P^{\mathcal{K}J}_t \bar{K}^J_{t+1}(l) - i^{BJ}_{t+1}(l) B^J_t(l) \\ =&\omega^J_{t+1} Ret^{\mathcal{K}J}_t P^{\mathcal{K}J}_t \bar{K}^J_{t+1}(l) - \bar{\omega}^J_{t+1} Ret^{\mathcal{K}J}_t P^{\mathcal{K}J}_t \bar{K}^J_{t+1}(l) \\ =&\left(\omega^J_{t+1} - \bar{\omega}^J_{t+1}\right) Ret^{\mathcal{K}J}_t P^{\mathcal{K}J}_t \bar{K}^J_{t+1}(l) \end{aligned}$$

where we used equation (93). All these possible outcomes are summarized in Box 8.

Each entrepreneur l maximizes

$$\int_0^{\infty} \widetilde{N}^{\mathcal{EP}J}_{t+1}(l)\mathfrak{f}(\omega^J_{t+1})\mathrm{d}\omega^J_{t+1}$$

where

$$\widetilde{N}^{\mathcal{EP}J}_{t+1}(l) = \begin{cases} \left(\omega^J_{t+1} - \bar{\omega}^J_{t+1}\right) Ret^{\mathcal{K}J}_t P^{\mathcal{K}J}_t \bar{K}^J_{t+1}(l) & \text{, if } \omega^J_{t+1} \geq \bar{\omega}^J_{t+1} \\ 0 & \text{, otherwise} \end{cases}$$

is the $t+1$ net worth before dividends are paid out and prior to aggregate shocks. This is naturally equivalent to maximize $\widetilde{N}^{\mathcal{EP}J}_{t+1}(l)$ over the non-default region $\omega^J_{t+1} \geq \bar{\omega}^J_{t+1}$

$$\int_{\bar{\omega}^J_{t+1}}^{\infty} \widetilde{N}^{\mathcal{EP}J}_{t+1}(l)\mathfrak{f}(\omega^J_{t+1})\mathrm{d}\omega^J_{t+1} \tag{94}$$

We now turn to the restrictions faced by entrepreneurs, which include the terms of the debt contract and account for the environment where banks operate.

Banks are assumed to make zero *ex-ante* and *ex-post* profits at all times. Their sole activity is to borrow resources from households in order to finance entrepreneurs capital acquisitions. We make the simplifying assumption that households are exclusively involved in risk-free activities, and thus they only lend to banks if they are indifferent against receiving the non-state contingent rate of return i_t. That is, the terms of the contract must state that households receive a non-contingent gross amount of $i_{t+1}B^J_t(l)$ at $t+1$, given a loan of $B^J_t(l)$ at t, regardless of the shock ω^J_{t+1}. Hence, entrepreneurial activity and bank loans are risky, but household loans are not—the major role of banks is to transform risky borrowing from entrepreneurs into riskless lending from households. To guarantee that households contractual interest rate is effectively verified, the debt contract has to be state contingent to account for all unexpected perturbations, implying that both $\bar{\omega}^J_{t+1}$ and $i^{BJ}_{t+1}(l)$ have to be functions of time $t+1$ idiosyncratic shocks ω^J_{t+1}. The debt contract has therefore to satisfy the following participation constraint

$$\underbrace{[1-\mathfrak{F}(\bar{\omega}^J_{t+1})]}_{\substack{\text{No default}\\ \text{probability}}}\underbrace{i^{BJ}_{t+1}(l)B^J_t(l)}_{\substack{\text{Bank revenues}\\ \text{in case of}\\ \text{no default}}} + \underbrace{(1-\mu_{t+1})}_{\substack{\text{Recovery}\\ \text{rate}}}\underbrace{\int_0^{\bar{\omega}^J_{t+1}} \omega^J_{t+1} Ret^{\mathcal{K}J}_t P^{\mathcal{K}J}_t \bar{K}^J_{t+1}(l)\mathfrak{f}(\omega^J_{t+1})\mathrm{d}\omega^J_{t+1}}_{\substack{\text{Average value of capital}\\ \text{in case of default } \omega^J_{t+1}<\bar{\omega}^J_{t+1}}} = \underbrace{i_t B^J_t(l)}_{\substack{\text{risk-free}\\ \text{amount}}} \tag{95}$$

This condition implies that households always receive a gross amount of $i_t B^J_t(l)$ for lending to entrepreneur l, regardless of the shock ω^J_{t+1}. Banks, in turn, receive $i^{BJ}_{t+1}(l)B^J_t(l)$ if no default occurs, and the value of the firm—which depends on ω^J_{t+1}—if a default occurs. Hence, the bank demands a risk premium to the entrepreneur whenever the probability of default is positive, *i.e.* $i^{BJ}_{t+1}(l) \geq i_t$. Moreover, a bank can make a profit or loss for a given entrepreneur (for a given realization of ω^J_{t+1}), even though on expectation profits are zero. In addition, we assume that each bank lends to many entrepreneurs (each facing a different idiosyncratic shock ω^J_{t+1}), so that *ex-post* profits are also zero by the law of large numbers.

As discussed by Levin, Natalucci, and Zakrajsek (2004), $Ret^{\mathcal{K}J}_t = i_t$ if $\mu_{t+1} = 0$ (the frictionless case), but $i^{BJ}_{t+1} - i_t > 0$ as long as the above probability of default is positive. In any case, the degree of financial frictions is always conditional on μ_{t+1}. Bernanke, Gertler, and

Gilchrist (1999b) defined the ratio of default costs to quantity borrowed, reflecting the premium for external finance, as

$$EFP_t = \frac{\mu_{t+1} \int_0^{\bar{\omega}^J_{t+1}} \omega^J_{t+1} Ret^{\mathcal{K}J}_t P^{\mathcal{K}J}_t \bar{K}^J_{t+1}(l) \mathfrak{f}(\omega^J_{t+1}) \mathrm{d}\omega^J_{t+1}}{B^J_t(l)} \tag{96}$$

This quantity EFP_t will be our measure of the "External Finance Premium". Plugging equation (93) into (95) yields

$$\left[1 - \mathfrak{F}(\bar{\omega}^J_{t+1})\right]\left[\bar{\omega}^J_{t+1} Ret^{\mathcal{K}J}_t P^{\mathcal{K}J}_t \bar{K}^J_{t+1}(l)\right] + (1-\mu_{t+1}) \int_0^{\bar{\omega}^J_{t+1}} \omega^J_{t+1} Ret^{\mathcal{K}J}_t P^{\mathcal{K}J}_t \bar{K}^J_{t+1}(l) \mathfrak{f}(\omega^J_{t+1}) \mathrm{d}\omega^J_{t+1} = i_t B^J_t(l)$$

Notice that the interest rate required by the bank increases to compensate for the higher probability of default following an increase in the threshold $\bar{\omega}^J_{t+1}$. Using equation (88) and rearranging

$$\left[[1 - \mathfrak{F}(\bar{\omega}^J_{t+1})]\bar{\omega}^J_{t+1} + (1-\mu_{t+1}) \int_0^{\bar{\omega}^J_{t+1}} \omega^J_{t+1} \mathfrak{f}(\omega^J_{t+1}) \mathrm{d}\omega^J_{t+1} \right] Ret^{\mathcal{K}J}_t P^{\mathcal{K}J}_t \bar{K}^J_{t+1}(l) = i_t \left[P^{\mathcal{K}J}_t \bar{K}^J_{t+1}(l) - N^J_t(l)\right] \tag{97}$$

In each period, entrepreneurs select $\{\bar{\omega}^J_{t+1}, \bar{K}^J_{t+1}(l)\}$ that maximizes the period $t+1$ net worth in (94), subject to the terms of the debt contract summarized in the banks participation constraint. This problem can be restated as (see Box 9)

$$\max_{\substack{\bar{\omega}^J_{t+1} \\ \bar{K}^J_{t+1}(l)}} \left[1 - \Gamma(\bar{\omega}^J_{t+1})\right] Ret^{\mathcal{K}J}_t P^{\mathcal{K}J}_t \bar{K}^J_{t+1}(l) \tag{98}$$

$$\text{s.t. } \left[\Gamma(\bar{\omega}^J_{t+1}) - \mu_{t+1} G(\bar{\omega}^J_{t+1})\right] Ret^{\mathcal{K}J}_t P^{\mathcal{K}J}_t \bar{K}^J_{t+1}(l) = i_t \left[P^{\mathcal{K}J}_t \bar{K}^J_{t+1}(l) - N^J_t(l)\right]$$

where

$$\underbrace{\Gamma(\bar{\omega}^J_{t+1})}_{\substack{\text{Banks capital} \\ \text{earnings gross share}}} \equiv \underbrace{\int_0^{\bar{\omega}^J_{t+1}} \omega^J_{t+1} \mathfrak{f}(\omega^J_{t+1}) \mathrm{d}\omega^J_{t+1}}_{\text{If } \omega^J_{t+1} < \bar{\omega}^J_{t+1}} + \underbrace{\bar{\omega}^J_{t+1} \int_{\bar{\omega}^J_{t+1}}^{\infty} \mathfrak{f}(\omega^J_{t+1}) \mathrm{d}\omega^J_{t+1}}_{\text{If } \omega^J_{t+1} \geq \bar{\omega}^J_{t+1}}$$

$$\underbrace{1 - \Gamma(\bar{\omega}^J_{t+1})}_{\substack{\text{Entrepreneurs capital} \\ \text{earnings net share}}} \equiv \underbrace{0}_{\text{If } \omega^J_{t+1} < \bar{\omega}^J_{t+1}} + \underbrace{\int_{\bar{\omega}^J_{t+1}}^{\infty} (\omega^J_{t+1} - \bar{\omega}^J_{t+1}) \mathfrak{f}(\omega^J_{t+1}) \mathrm{d}\omega^J_{t+1}}_{\text{If } \omega^J_{t+1} \geq \bar{\omega}^J_{t+1}}$$

and

$$G(\bar{\omega}^J_{t+1}) \equiv \int_0^{\bar{\omega}^J_{t+1}} \omega^J_{t+1} \mathfrak{f}(\omega^J_{t+1}) \mathrm{d}\omega^J_{t+1}$$

Banks capital earnings net share is $\Gamma(\bar{\omega}^J_{t+1}) - \mu_{t+1} G(\bar{\omega}^J_{t+1})$.

The entrepreneur problem has been reduced so far to selecting $\bar{K}_{t+1}(l)$ and a schedule $\bar{\omega}^J_{t+1}$ (as a function of the realized values of $Ret^{\mathcal{K}J}_t$). The distribution of aggregate and idiosyncratic risks, the price of capital $P^{\mathcal{K}J}_t$, and net worth $N^J_t(l)$, are taken as given. The solution to the

Box 9: Banks and entrepreneurs capital earnings share.

The entrepreneurs maximization problem

$$\max_{\substack{\bar{\omega}^J_{t+1} \\ \bar{K}^J_{t+1}(l)}} \int_{\bar{\omega}^J_{t+1}}^{\infty} (\omega^J_{t+1} - \bar{\omega}^J_{t+1}) Ret^{\mathcal{K}J}_t P^{\mathcal{K}J}_t \bar{K}^J_{t+1}(l) \mathfrak{f}(\omega^J_{t+1}) \mathrm{d}\omega^J_{t+1} \qquad \text{s.t.} \qquad (97)$$

can be simplified by noticing that capital earnings are shared between banks and entrepreneurs. Ignoring time subscripts and the superscript J, total density can be decomposed as

$$1 = \underbrace{\int_0^{\bar{\omega}} \omega \mathfrak{f}(\omega) \mathrm{d}\omega}_{\substack{\text{Region associated with} \\ \text{the default area}}} + \underbrace{\int_{\bar{\omega}}^{\infty} \omega \mathfrak{f}(\omega) \mathrm{d}\omega}_{\substack{\text{Region associated with} \\ \text{the non-default area}}} \qquad (9.1)$$

Equation (9.1) can be manipulated to account for alternative outcomes

$$\begin{aligned} 1 &= \int_0^{\bar{\omega}} \omega \mathfrak{f}(\omega) \mathrm{d}\omega + \int_{\bar{\omega}}^{\infty} \omega \mathfrak{f}(\omega) \mathrm{d}\omega + \bar{\omega} \int_{\bar{\omega}}^{\infty} \mathfrak{f}(\omega) \mathrm{d}\omega - \bar{\omega} \int_{\bar{\omega}}^{\infty} \mathfrak{f}(\omega) \mathrm{d}\omega \\ &= \underbrace{\underbrace{\int_0^{\bar{\omega}} \omega \mathfrak{f}(\omega) \mathrm{d}\omega}_{\substack{\text{The entrepreneur} \\ \text{goes bankrupt}}} + \underbrace{\bar{\omega} \int_{\bar{\omega}}^{\infty} \mathfrak{f}(\omega) \mathrm{d}\omega}_{\substack{\text{The entrepreneur} \\ \text{does not go bankrupt}}}}_{\substack{\text{Gross share of banks} \\ \text{(before paying monit. costs)}}} + \underbrace{\underbrace{\int_{\bar{\omega}}^{\infty} (\omega - \bar{\omega}) \mathfrak{f}(\omega) \mathrm{d}\omega}_{\substack{\text{The entrepreneur} \\ \text{does not go bankrupt}}}}_{\text{Share of entrepreneurs}} \\ &= \Gamma(\bar{\omega}) + 1 - \Gamma(\bar{\omega}) \end{aligned} \qquad (9.2)$$

Equation (9.2) splits total density $\int_0^{\infty} \omega \mathfrak{f}(\omega) \mathrm{d}\omega = 1$ in banks gross share, $\Gamma(\bar{\omega})$, and entrepreneurs gross share, $1 - \Gamma(\bar{\omega})$. Entrepreneurs gross share equals their net share, since they observe ω at no cost. On the contrary, banks have to pay monitoring costs $\mu G(\bar{\omega})$ if the entrepreneur goes bankrupt, where

$$G(\bar{\omega}) \equiv \int_0^{\bar{\omega}} \omega \mathfrak{f}(\omega) \mathrm{d}\omega$$

Hence, the banks net share is $\Gamma(\bar{\omega}) - \mu G(\bar{\omega})$.
The entrepreneurs maximization problem can therefore be restated as

$$\max_{\substack{\bar{\omega}^J_{t+1} \\ \bar{K}^J_{t+1}(l)}} \left[1 - \Gamma(\bar{\omega}^J_{t+1})\right] Ret^{\mathcal{K}J}_t P^{\mathcal{K}J}_t \bar{K}^J_{t+1}(l)$$

$$\text{s.t. } \left[\Gamma(\bar{\omega}^J_{t+1}) - \mu_{t+1} G(\bar{\omega}^J_{t+1})\right] Ret^{\mathcal{K}J}_t P^{\mathcal{K}J}_t \bar{K}^J_{t+1}(l) = i_t \left[P^{\mathcal{K}J}_t \bar{K}^J_{t+1}(l) - N^J_t(l)\right]$$

entrepreneur problem is presented in Box 10. Notice that, in the current setup, entrepreneurs face an additional balance sheet effect taht is absent from models without financial frictions: a marginal increase in capital raises the entrepreneur leverage, and therefore the costs of external finance. Thus, *ceteris paribus*, entrepreneurs set a relatively lower level of physical capital.

Even though all entrepreneurs face the same decision rule, they may have different net worth levels and set distinct capital purchases. Dropping the indexer l when aggregating for the overall

Box 10: Entrepreneurs maximization problem.

The entrepreneur selects the profile $\{\bar{\omega}^J_{t+1}, \bar{K}^J_{t+1}(l)\}$ that solves problem (98). The Lagrangian is

$$\begin{aligned}\mathcal{L}(\cdot) =& \big[1-\Gamma(\bar{\omega}^J_{t+1})\big] Ret^{\mathcal{K}J}_t P^{\mathcal{K}J}_t \bar{K}^J_{t+1}(l) \\ &+\lambda_t\Big[\big(\Gamma(\bar{\omega}^J_{t+1})-\mu_{t+1}G(\bar{\omega}^J_{t+1})\big)Ret^{\mathcal{K}J}_t P^{\mathcal{K}J}_t \bar{K}^J_{t+1}(l) - i_t\big(P^{\mathcal{K}J}_t \bar{K}^J_{t+1}(l) - N^J_t(l)\big)\Big]\end{aligned}$$

where λ_t is the Lagrange multiplier. Let

$$\Gamma^J_{t+1} \equiv \Gamma(\bar{\omega}^J_{t+1}) \qquad \text{with} \qquad \big(\Gamma^J_{t+1}\big)' \equiv \frac{\mathrm{d}\Gamma^J_{t+1}}{\mathrm{d}\bar{\omega}^J_{t+1}} = 1-\mathfrak{F}(\bar{\omega}^J_{t+1})$$

$$G^J_{t+1} \equiv G(\bar{\omega}^J_{t+1}) \qquad \text{with} \qquad \big(G^J_{t+1}\big)' \equiv \frac{\mathrm{d}G^J_{t+1}}{\mathrm{d}\bar{\omega}^J_{t+1}} = \bar{\omega}^J_{t+1}\mathfrak{f}(\bar{\omega}^J_{t+1})$$

The first-order conditions are as follows.

1. **The optimal cutoff $\bar{\omega}^J_{t+1}$**

$$\frac{\partial\mathcal{L}(\cdot)}{\partial\bar{\omega}^J_{t+1}} = 0 \Leftrightarrow \big(\Gamma^J_{t+1}\big)' Ret^{\mathcal{K}J}_t P^{\mathcal{K}J}_t \bar{K}^J_{t+1}(l) = \lambda_t\Big(\big(\Gamma^J_{t+1}\big)' - \mu_{t+1}\big(G^J_{t+1}\big)'\Big) Ret^{\mathcal{K}J}_t P^{\mathcal{K}J}_t \bar{K}^J_{t+1}(l) \Leftrightarrow$$

$$\Leftrightarrow \lambda_t = \frac{\big(\Gamma^J_{t+1}\big)'}{\big(\Gamma^J_{t+1}\big)' - \mu_{t+1}\big(G^J_{t+1}\big)'} \tag{10.1}$$

2. **Optimal capital purchases**

$$\frac{\partial\mathcal{L}(\cdot)}{\partial\bar{K}^J_{t+1}(l)} = 0 \Leftrightarrow \big(1-\Gamma^J_{t+1}\big) Ret^{\mathcal{K}J}_t P^{\mathcal{K}J} + \lambda_t\Big[\big(\Gamma^J_{t+1}-\mu_{t+1}G^J_{t+1}\big) Ret^{\mathcal{K}J}_t P^{\mathcal{K}J} - i_t P^{\mathcal{K}J}_t\Big] = 0$$

Dividing by $i_t P^{\mathcal{K}J}_t$ and noting that

$$\frac{Ret^{\mathcal{K}J}_t}{i_t} = \frac{Ret^{\mathcal{K}J}_t}{r_t\pi_{t+1}} = \frac{ret^{\mathcal{K}J}_t}{r_t}$$

yields

$$\big(1-\Gamma^J_{t+1}\big)\frac{ret^{\mathcal{K}J}_t}{r_t} + \lambda_t\Big[\big(\Gamma^J_{t+1}-\mu_{t+1}G^J_{t+1}\big)\frac{ret^{\mathcal{K}J}_t}{r_t} - 1\Big] = 0 \tag{10.2}$$

which implies that all entrepreneurs set the same cutoff $\bar{\omega}^J_{t+1}$

Finally, notice that nominal capital purchases can be expressed as

$$P^{\mathcal{K}J}_t \bar{K}^J_{t+1}(l) = \frac{i_t N^J_t(l)}{i_t - \big(\Gamma(\bar{\omega}^J_{t+1}) - \mu_{t+1}G(\bar{\omega}^J_{t+1})\big)Ret^{\mathcal{K}J}_t}$$

and therefore each entrepreneur sets capital purchases according to his net worth. Nevertheless, all entrepreneurs select the same leverage ratio

$$\frac{B^J_t(l)/N^J_t(l)}{1+B^J_t(l)/N^J_t(l)} = \frac{\big(\Gamma(\bar{\omega}^J_{t+1}) - \mu_{t+1}G(\bar{\omega}^J_{t+1})\big)Ret^{\mathcal{K}J}_t}{i_t}$$

economy and pugging (10.1) into (10.2) yields

$$\big(1-\Gamma^J_{t+1}\big)\frac{ret^{\mathcal{K}J}_t}{r_t} + \left(\frac{\big(\Gamma^J_{t+1}\big)'}{\big(\Gamma^J_{t+1}\big)' - \mu_{t+1}\big(G^J_{t+1}\big)'}\right)\Big[\big(\Gamma^J_{t+1}-\mu_{t+1}G^J_{t+1}\big)\frac{ret^{\mathcal{K}J}_t}{r_t} - 1\Big] = 0 \tag{99}$$

This equation—which is already in stationary form—establishes the link between $ret_t^{\mathcal{K}J}$ and $\bar{\omega}_{t+1}^J$. The steady-state version is

$$\left(1-\Gamma^J\right)\frac{ret^{\mathcal{K}J}}{r}+\left(\frac{\left(\Gamma^J\right)'}{\left(\Gamma^J\right)'-\mu\left(G^J\right)'}\right)\left[\left(\Gamma^J-\mu G^J\right)\frac{ret^{\mathcal{K}J}}{r}-1\right]=0 \tag{100}$$

The constraint in problem (98) defining the zero-profit condition can be written in terms of the assets-to-net worth ratio $P_t^{\mathcal{K}J}\bar{K}_{t+1}^J/N_t$ and of the corporate risk spread $Ret_t^{\mathcal{K}J}/i_t$. Dropping the indexer l and dividing by i_tN_t yields

$$\left[\Gamma_{t+1}^J-\mu_{t+1}G_{t+1}^J\right]\frac{Ret_t^{\mathcal{K}J}}{i_t}\frac{P_t^{\mathcal{K}J}\bar{K}_{t+1}^J}{N_t}=\frac{P_t^{\mathcal{K}J}\bar{K}_{t+1}^J}{N_t}-1$$

In stationary form

$$\left[\Gamma_{t+1}^J-\mu_{t+1}G_{t+1}^J\right]\frac{Ret_t^{\mathcal{K}J}}{r_t\pi_{t+1}}\frac{\frac{P_t^{\mathcal{K}J}\bar{K}_{t+1}^J}{P_tT_{t+1}}}{\frac{N_t}{P_tT_tg}}=\frac{\frac{P_t^{\mathcal{K}J}\bar{K}_{t+1}^J}{P_tT_{t+1}}}{\frac{N_t}{P_tT_tg}}-1\Leftrightarrow$$

$$\Leftrightarrow\left[\Gamma_{t+1}^J-\mu_{t+1}G_{t+1}^J\right]\frac{ret_t^{\mathcal{K}J}}{r_t}\frac{p_t^{\mathcal{K}J}\check{\bar{K}}_{t+1}^Jg}{\check{n}_t}=\frac{p_t^{\mathcal{K}J}\check{\bar{K}}_{t+1}^Jg}{\check{n}_t}-1 \tag{101}$$

This equation states that *ex-ante* bank profits are zero, but is silent about *ex-post* events. However, given that each bank lends to many entrepreneurs, *ex-post* profits are also zero. Banks will thus make positive profits on some entrepreneurs and negative profits on others, but on average profits are zero. Following Kumhof et al. (2010), the period t *ex-post* version of (101) becomes

$$\left[\Gamma_t^J-\mu_tG_t^J\right]\frac{ret_{m1,t}^{\mathcal{K}J}}{r_{m1,t}}\frac{p_{t-1}^{\mathcal{K}J}\check{\bar{K}}_t^Jg}{\check{n}_{t-1}}=\frac{p_{t-1}^{\mathcal{K}J}\check{\bar{K}}_t^Jg}{\check{n}_{t-1}}-1 \tag{102}$$

where $r_{m1,t}$ is the *ex-post* real interest rate

$$r_{m1,t}=\frac{i_{t-1}}{\pi_t} \tag{103}$$

and $ret_{m1,t}^{\mathcal{K}J}$ is the *ex-post* after-tax return on capital

$$ret_{m1,t}^{\mathcal{K}J}=\left(p_{t-1}^{\mathcal{K}J}\right)^{-1}\left[\left(1-\tau_t^{\mathcal{K}}\right)\left(u_t^Jr_t^{\mathcal{K}J}-a(u_t^J)\right)+(1-\delta_t^J)p_t^{\mathcal{K}J}+\tau_t^{\mathcal{K}}\delta_t^Jp_t^{\mathcal{K}J}\right] \tag{104}$$

Notice that, even though $ret_{m1,t}^{\mathcal{K}J}$ has the same formulation as $ret_{t-1}^{\mathcal{K}J}$, they are conceptually different. The former is the *ex-post* return on capital at t, after the shock ω_t^J is known. The latter is the *ex-ante* return on capital at $t-1$, before the shock ω_t is known. For latter reference, notice that $Ret^{\mathcal{K}J}_{m1,t} = \pi_t ret^{\mathcal{K}J}_{m1,t}$. The steady-state version of (102) is

$$\left[\Gamma^J - \mu G^J\right] \frac{ret^{\mathcal{K}J}_{m1}}{r_{m1}} \frac{p^{\mathcal{K}J} \check{\bar{K}}^J g}{\check{n}} = \frac{p^{\mathcal{K}J} \check{\bar{K}}^J g}{\check{n}} - 1 \tag{105}$$

and of (103) and (104) are respectively

$$r_{m1} = \frac{i}{\pi} \tag{106}$$

and

$$ret^{\mathcal{K}J}_{m1} = \left(p^{\mathcal{K}J}\right)^{-1} \left[(1-\tau^{\mathcal{K}})\left(u^J r^{\mathcal{K}J} - a(u^J)\right) + (1-\delta^J) p^{\mathcal{K}J} + \tau^{\mathcal{K}} \delta^J p^{\mathcal{K}J}\right] \tag{107}$$

Since ω^J_t follows a unit mean log-normal distribution, the terms Γ^J_t, G^J_t, $(\Gamma^J_t)'$, and $(G^J_t)'$ can be computed by resorting to the following auxiliary variable

$$\bar{z}^J_t = \frac{\ln(\bar{\omega}^J_t)}{\sigma^{\mathcal{E}J}_t} + \frac{1}{2}\sigma^{\mathcal{E}J}_t$$

Solving for $\bar{\omega}_t$ yields

$$\bar{\omega}^J_t = \exp\left[\bar{z}^J_t \sigma^{\mathcal{E}J}_t - \frac{1}{2}\left(\sigma^{\mathcal{E}J}_t\right)^2\right] \tag{108}$$

In the steady state

$$\bar{\omega}^J = \exp\left[\bar{z}^J \sigma^{\mathcal{E}J} - \frac{1}{2}\left(\sigma^{\mathcal{E}J}\right)^2\right] \tag{109}$$

Let $\Phi(x)$ denote the cumulative distribution function of the standard normal distribution. The following results hold in stationary form

$$\Gamma^J_t = \Phi\left(\bar{z}^J_t - \sigma^{\mathcal{E}J}_t\right) + \bar{\omega}^J_t\left[1 - \Phi\left(\bar{z}^J_t\right)\right] \tag{110}$$

$$G^J_t = \Phi\left(\bar{z}^J_t - \sigma^{\mathcal{E}J}_t\right) \tag{111}$$

$$\left(\Gamma_t^J\right)' = 1 - \Phi\left(\bar{z}_t^J\right) \tag{112}$$

$$\left(G_t^J\right)' = \bar{\omega}_t^J \mathfrak{f}\left(\bar{\omega}_t^J\right) \tag{113}$$

where

$$\mathfrak{f}\left(\bar{\omega}_t^J\right) = \frac{1}{\sqrt{2\pi}\bar{\omega}_t^J \sigma_t^{\mathcal{E}J}} \exp\left\{-\frac{1}{2}(\bar{z}_t^J)^2\right\} \tag{114}$$

The derivation is clarified in Box 11. The steady-state versions are respectively

$$\Gamma^J = \Phi\left(\bar{z}^J - \sigma^{\mathcal{E}J}\right) + \bar{\omega}^J\left[1 - \Phi\left(\bar{z}^J\right)\right] \tag{115}$$

$$G^J = \Phi\left(\bar{z}^J - \sigma^{\mathcal{E}J}\right) \tag{116}$$

$$\left(\Gamma^J\right)' = 1 - \Phi\left(\bar{z}^J\right) \tag{117}$$

$$\left(G^J\right)' = \bar{\omega}^J \mathfrak{f}\left(\bar{\omega}^J\right) \tag{118}$$

where

$$\mathfrak{f}\left(\bar{\omega}^J\right) = \frac{1}{\sqrt{2\pi}\bar{\omega}^J \sigma^{\mathcal{E}J}} \exp\left\{-\frac{1}{2}\left(\bar{z}^J\right)^2\right\} \tag{119}$$

To close the entrepreneurial sector one needs to describe in more precise terms how aggregate net worth evolves over time. *Ex-post* aggregate returns to capital are $Ret_{m1,t}^{\mathcal{K}J} P_{t-1}^{\mathcal{K}J} \bar{K}_t^J$. The difference between this amount net of bankruptcy costs $\mu_t^J G_t Ret_{m1,t}^{\mathcal{K}J} P_{t-1}^{\mathcal{K}J} \bar{K}_t^J$ and the gross amount $i_{t-1}^{BJ} B_{t-1}^J$ paid for bank loans goes entirely to entrepreneurs, accumulating over time possibly up to the point where external finance is no longer needed. If this is the case, leverage converges to zero, and the financial accelerator setup collapses to the benchmark case with no financial frictions. To prevent this, we impose some restrictions on the time path of aggregate net worth.

Box 11: Using the lognormal distribution in the financial accelerator set up.

The random variable ω_t^J follows a lognormal distribution

$$\ln\omega_t^J \sim \mathcal{N}(-1/2(\sigma_t^{\mathcal{E}J})^2, (\sigma_t^{\mathcal{E}J})^2)$$

Consequently

$$\mathrm{E}(\omega_t^J) = 1$$
$$\mathrm{Var}(\omega_t^J) = \left(\sigma_t^{\mathcal{E}J}\right)^2$$

The density function of ω_t^J is

$$\mathfrak{f}(\omega_t^J) = \frac{1}{\sqrt{2\pi}\omega_t^J\sigma_t^{\mathcal{E}J}} \exp\left\{-\frac{1}{2}\left(\frac{\ln(\omega_t^J)}{\sigma_t^{\mathcal{E}J}} + \frac{1}{2}\sigma_t^{\mathcal{E}J}\right)^2\right\}$$

Let us omit the time subscript and the sector superscript. Furthermore, consider the following auxiliary variables and results

$$\bar{z} = \frac{\ln(\bar{\omega})}{\sigma^{\mathcal{E}}} + \frac{1}{2}\sigma^{\mathcal{E}}$$

$$y = \frac{\ln(\omega)}{\sigma^{\mathcal{E}}} + \frac{1}{2}\sigma^{\mathcal{E}} \Rightarrow \begin{cases} \omega = \exp\left\{y\sigma^{\mathcal{E}} - \frac{1}{2}\left(\sigma^{\mathcal{E}}\right)^2\right\} \Rightarrow \mathrm{d}\omega = \sigma^{\mathcal{E}}\exp\left\{y\sigma^{\mathcal{E}} - \frac{1}{2}\left(\sigma^{\mathcal{E}}\right)^2\right\}\mathrm{d}y \\ y - \sigma^{\mathcal{E}} = \frac{\ln(\omega)}{\sigma^{\mathcal{E}}} - \frac{1}{2}\sigma^{\mathcal{E}} \quad \Rightarrow (y-\sigma^{\mathcal{E}})^2 = \left(\frac{\ln(\omega)}{\sigma^{\mathcal{E}}}\right)^2 - \ln(\omega) + \left(\frac{1}{2}\sigma^{\mathcal{E}}\right)^2 \end{cases}$$

We first show that $\int_{\bar{\omega}}^{\infty}\mathfrak{f}(\omega)\mathrm{d}\omega = 1 - \Phi(\bar{z})$, where $\Phi(\cdot)$ is the cumulative distribution function of the standard normal distribution

$$\begin{aligned}
\int_{\bar{\omega}}^{\infty}\mathfrak{f}(\omega)\mathrm{d}\omega &= \int_{\bar{\omega}}^{\infty}\frac{1}{\sqrt{2\pi}\omega\sigma^{\mathcal{E}}}\exp\left\{-\frac{1}{2}\left(\frac{\ln(\omega)}{\sigma^{\mathcal{E}}} + \frac{1}{2}\sigma^{\mathcal{E}}\right)^2\right\}\mathrm{d}\omega \\
&= \int_{\bar{z}}^{\infty}\frac{1}{\sqrt{2\pi}\omega\sigma^{\mathcal{E}}}\exp\left\{-\frac{1}{2}y^2\right\}\sigma^{\mathcal{E}}\exp\left\{y\sigma^{\mathcal{E}} - \frac{1}{2}\left(\sigma^{\mathcal{E}}\right)^2\right\}\mathrm{d}y \\
&= \int_{\bar{z}}^{\infty}\frac{1}{\sqrt{2\pi}\omega}\exp\left\{-\frac{1}{2}(y^2 - 2y\sigma^{\mathcal{E}} + \left(\sigma^{\mathcal{E}}\right)^2)\right\}\mathrm{d}y \\
&= \int_{\bar{z}}^{\infty}\frac{1}{\sqrt{2\pi}}\exp\left\{-\ln(\omega)\right\}\exp\left\{-\frac{1}{2}(y-\sigma^{\mathcal{E}})^2\right\}\mathrm{d}y \\
&= \int_{\bar{z}}^{\infty}\frac{1}{\sqrt{2\pi}}\exp\left\{-\ln(\omega) - \frac{1}{2}\left(\frac{\ln(\omega)}{\sigma^{\mathcal{E}}} - \frac{1}{2}\sigma^{\mathcal{E}}\right)^2\right\}\mathrm{d}y \\
&= \int_{\bar{z}}^{\infty}\frac{1}{\sqrt{2\pi}}\exp\left\{-\frac{1}{2}\left[\left(\frac{\ln(\omega)}{\sigma^{\mathcal{E}}}\right)^2 + 2\ln(\omega) - \ln(\omega) + \left(\frac{1}{2}\sigma^{\mathcal{E}}\right)^2\right]\right\}\mathrm{d}y \\
&= \int_{\bar{z}}^{\infty}\frac{1}{\sqrt{2\pi}}\exp\left\{-\frac{1}{2}\left(\frac{\ln(\omega)}{\sigma^{\mathcal{E}}} + \frac{1}{2}\sigma^{\mathcal{E}}\right)^2\right\}\mathrm{d}y = \int_{\bar{z}}^{\infty}\frac{1}{\sqrt{2\pi}}\exp\left\{-\frac{1}{2}y^2\right\}\mathrm{d}y \\
&= 1 - \int_{-\infty}^{\bar{z}}\frac{1}{\sqrt{2\pi}}\exp\left\{-\frac{1}{2}y^2\right\}\mathrm{d}y = 1 - \Phi(\bar{z})
\end{aligned}$$

Consider now the following auxiliary variables and results

$$\tilde{z} = \frac{\ln(\bar{\omega})}{\sigma^{\mathcal{E}}} - \frac{1}{2}\sigma^{\mathcal{E}} \Rightarrow \tilde{z} = \bar{z} - \sigma^{\mathcal{E}}$$

$$\tilde{y} = \frac{\ln(\omega)}{\sigma^{\mathcal{E}}} - \frac{1}{2}\sigma^{\mathcal{E}} \Rightarrow \begin{cases} \omega = \exp\left\{\tilde{y}\sigma^{\mathcal{E}} + \frac{1}{2}\left(\sigma^{\mathcal{E}}\right)^2\right\} \Rightarrow \mathrm{d}\omega = \sigma^{\mathcal{E}} \exp\left\{\tilde{y}\sigma^{\mathcal{E}} + \frac{1}{2}\left(\sigma^{\mathcal{E}}\right)^2\right\}\mathrm{d}\tilde{y} \\ \tilde{y} + \sigma^{\mathcal{E}} = \frac{\ln(\omega)}{\sigma^{\mathcal{E}}} + \frac{1}{2}\sigma^{\mathcal{E}} \quad \Rightarrow (\tilde{y} + \sigma^{\mathcal{E}})^2 = \left(\frac{\ln(\omega)}{\sigma^{\mathcal{E}}}\right)^2 + \ln(\omega) + \left(\frac{1}{2}\sigma^{\mathcal{E}}\right)^2 \end{cases}$$

We show that $\int_{\bar{\omega}}^{\infty} \omega \mathfrak{f}(\omega)\mathrm{d}\omega = 1 - \Phi(\bar{z} - \sigma^{\mathcal{E}})$

$$\begin{aligned}
\int_{\bar{\omega}}^{\infty} \omega \mathfrak{f}(\omega)\mathrm{d}\omega &= \int_{\bar{\omega}}^{\infty} \frac{\omega}{\sqrt{2\pi}\omega\sigma^{\mathcal{E}}} \exp\left\{-\frac{1}{2}\left(\frac{\ln(\omega)}{\sigma^{\mathcal{E}}} + \frac{1}{2}\sigma^{\mathcal{E}}\right)^2\right\}\mathrm{d}\omega \\
&= \int_{\tilde{z}}^{\infty} \frac{1}{\sqrt{2\pi}\sigma^{\mathcal{E}}} \exp\left\{-\frac{1}{2}(\tilde{y} + \sigma^{\mathcal{E}})^2\right\} \sigma^{\mathcal{E}} \exp\left\{\tilde{y}\sigma^{\mathcal{E}} + \frac{1}{2}\left(\sigma^{\mathcal{E}}\right)^2\right\} \mathrm{d}\tilde{y} \\
&= \int_{\tilde{z}}^{\infty} \frac{1}{\sqrt{2\pi}} \exp\left\{-\frac{1}{2}(\tilde{y}^2 + 2\tilde{y}\sigma^{\mathcal{E}} + \left(\sigma^{\mathcal{E}}\right)^2) + \tilde{y}\sigma^{\mathcal{E}} + \frac{1}{2}\left(\sigma^{\mathcal{E}}\right)^2\right\}\mathrm{d}\tilde{y} \\
&= \int_{\tilde{z}}^{\infty} \frac{1}{\sqrt{2\pi}} \exp\left\{-\frac{1}{2}\tilde{y}^2\right\}\mathrm{d}\tilde{y} = 1 - \int_{-\infty}^{\tilde{z}} \frac{1}{\sqrt{2\pi}} \exp\left\{-\frac{1}{2}\tilde{y}^2\right\}\mathrm{d}\tilde{y} \\
&= 1 - \Phi(\tilde{z}) = 1 - \Phi(\bar{z} - \sigma^{\mathcal{E}})
\end{aligned}$$

Replacing these results in the definitions of Γ_t^J, $\mathfrak{G}_t^J$, $(\Gamma_t^J)'$, and $(\mathfrak{G}_t^J)'$ yields

$$\Gamma_t^J = \int_0^{\bar{\omega}_t^J} \omega_t^J \mathfrak{f}(\omega_t^J)\mathrm{d}\omega_t^J + \bar{\omega}_t^J \int_{\bar{\omega}_t^J}^{\infty} \mathfrak{f}(\omega_t^J)\mathrm{d}\omega_t^J = \Phi(\bar{z}_t^J - \sigma_t^{\mathcal{E}J}) + \bar{\omega}_t^J \left[1 - \Phi(\bar{z}_t^J)\right]$$

$$G_t^J = \int_0^{\bar{\omega}_t^J} \omega_t^J \mathfrak{f}(\omega_t^J)\mathrm{d}\omega_t^J = \Phi(\bar{z}_t^J - \sigma_t^{\mathcal{E}J})$$

$$\left(\Gamma_t^J\right)' = \bar{\omega}_t^J \mathfrak{f}(\bar{\omega}_t^J) + \int_{\bar{\omega}_t^J}^{\infty} \mathfrak{f}(\omega_t^J)\mathrm{d}\omega_t^J - \bar{\omega}_t^J \mathfrak{f}(\bar{\omega}_t^J) = 1 - \Phi(\bar{z}_t^J)$$

$$\left(G_t^J\right)' = \bar{\omega}_t^J \mathfrak{f}(\bar{\omega}_t^J)$$

More specifically, we assume that entrepreneurs pay out dividends to households on a regular basis, worth $D_t^J = P_t^J div_t^J$ (sector J's dividends are measured in real units of output of the respective sector). Additionally, aggregate net worth accumulation may also be hindered by output–destroying shocks, valued at $P_t^J S_t^{J,y}$. Sector J's entrepreneurial firms net worth therefore evolves according to

$$\begin{aligned}
N_t^J &= \left(Ret_{m1,t}^{\mathcal{K}J} P_{t-1}^{\mathcal{K}J} \bar{K}_t^J - \mu_t^J G_t Ret_{m1,t}^{\mathcal{K}J} P_{t-1}^{\mathcal{K}J} \bar{K}_t^J - i_{t-1} B_{t-1}^J\right) - P_t^J div_t^J - P_t^J S_t^{J,y} \\
&= \left(Ret_{m1,t}^{\mathcal{K}J} P_{t-1}^{\mathcal{K}J} \bar{K}_t^J (1 - \mu_t^J G_t) - i_{t-1} B_{t-1}^J\right) - P_t^J \left(div_t^J + S_t^{J,y}\right) \\
&= \left(Ret_{m1,t}^{\mathcal{K}J} P_{t-1}^{\mathcal{K}J} \bar{K}_t^J (1 - \mu_t^J G_t) - i_{t-1}\left(P_{t-1}^{\mathcal{K}J} \bar{K}_t^J - N_{t-1}^J\right)\right) - P_t^J \left(div_t^J + S_t^{J,y}\right) \\
&= \left(i_{t-1} N_{t-1}^J + P_{t-1}^{\mathcal{K}J} \bar{K}_t^J \left(Ret_{m1,t}^{\mathcal{K}J}(1 - \mu_t^J G_t) - i_{t-1}\right)\right) - P_t^J \left(div_t^J + S_t^{J,y}\right)
\end{aligned}$$

where we used the aggregate version of (88), $B_{t-1}^J = P_{t-1}^{\mathcal{K}J}\bar{K}_t^J - N_{t-1}^J$. In stationary form this equation becomes

$$\check{n}_t^J = \left(\frac{r_{m1,t}}{g}\check{n}_{t-1}^J + p_{t-1}^{\mathcal{K}J}\check{\bar{K}}_t^J\left(ret_{m1,t}^{\mathcal{K}J}(1-\mu_t^J G_t) - r_{m1,t}\right)\right) - p_t^J\left(\check{div}_t^J + \check{S}_t^{J,y}\right) \tag{120}$$

The steady-state version is

$$\check{n}^J = \frac{g}{g-r_{m1}}\left(p^{\mathcal{K}J}\check{\bar{K}}^J\left(ret_{m1}^{\mathcal{K}J}(1-\mu^J G) - r_{m1}\right) - p^J\left(\check{div}^J + \check{S}_t^{J,y}\right)\right) \tag{121}$$

Dividends $\check{div}_t^J$ evolve according to a structure present in Kumhof et al. (2010). First, let entrepreneurial income $p_t^J \check{inc}_t^J$ be defined as a fraction $S_t^{J,d}$ of net worth gross return

$$\begin{aligned} p_t^J\check{inc}_t^J &= S_t^{J,d}\left[\check{n}_t^J + p_t^J\left(\check{div}_t^J + \check{S}_t^{J,y}\right)\right] \\ &= S_t^{J,d}\left(\frac{r_{m1,t}}{g}\check{n}_{t-1}^J + p_{t-1}^{\mathcal{K}J}\check{\bar{K}}_t^J\left(ret_{m1,t}^{\mathcal{K}J}(1-\mu_t^J G_t) - r_{m1,t}\right)\right) \end{aligned}$$

The element $S_t^{J,d}$ is a dividend related net worth shock (with $\mathrm{E}_t S_t^{J,d}$ typically in a range between 0 and 0.05) affecting the share of gross returns on net worth that is distributed to households. Hence, a shock to $S_t^{J,d}$ originates a pure redistribution effect between entrepreneurs and households, without direct resource implications. Moreover, let $\check{inc}_t^{J,ma}$ and $\check{n}_t^{J,ma}$ denote moving averages of $\check{inc}_t^J$ and $\check{n}_t^J$, respectively (moving averages are assumed to contain only lagged terms). Dividends $\check{div}_t^J$ evolve according to

$$\check{div}_t^J = \check{inc}_t^{J,ma} + \theta_{nw}^J(\check{n}_t^J - \check{n}_t^{J,ma}) \tag{122}$$

where θ_{nw}^J is a parameter measuring the change in dividends if net worth raises or decreases against its long-run value. This parameter is typically set between 0 and 0.05 (Kumhof et al., 2010). In the steady state $\check{n}_t^J = \check{n}_t^{J,ma}$, and the above equation becomes

$$\check{div}^J = \check{inc}^{J,ma} \tag{123}$$

This setup deserves some clarification. To prevent aggregate net worth from increasing indefinitely over time, we assume that entrepreneurs face output destroying shocks and pay regular dividends to households. The remaining amount are simply retained earnings, used to finance entrepreneurs activity in the following period (in complement with external finance). Dividends, in turn, are a function of current and past income of entrepreneurs, but also depend on the deviation of current and past net worth value from the long-run value. Therefore, dividends are lower if net worth is below the steady-state value and *vice-versa*. Moving averages enhance the dynamic properties of the model, allowing adjustments to occur smoothly through time.

In each period t, a fraction $\mathfrak{F}(\bar{\omega}_{t+1}^{J})$ of entrepreneurs declares bankruptcy and goes out of business. To ensure that entrepreneurial firms have a mass of one at all times, we assume that the same fraction of new businesses starts in every period, *viz* the most successful entrepreneurs use a portion of their net worth to set a new business. This setup implies that aggregate net worth is not affected by new entrants, thus ensuring consistency between the path for aggregate net worth and individual decisions. Moreover, given that the decision rule is the same for all entrepreneurs, they all set the same leverage ratio in equilibrium, even though they have different net worth values and therefore purchase different amounts of capital. Thus, aggregate figures evolve according to a pre-determined path, even though in equilibrium entrepreneurs face distinct outcomes.

Finally, sector J's real bankruptcy monitoring costs paid by banks are

$$\check{rbr}_t^{J} = \frac{1}{p_t^J}\check{\bar{K}}_t^J ret_{m1,t}^{\mathcal{K}J} p_{t-1}^{\mathcal{K}J}\mu_t^J G_t^J \tag{124}$$

This is not a physical resource cost but a remuneration for monitoring work performance, paid to type-$\mathcal{A}$ households in a lump-sum fashion. In the steady state

$$\check{rbr}^{J} = \frac{1}{p^J}\check{\bar{K}}^J ret_{m1}^{\mathcal{K}J} p^{\mathcal{K}J}\mu^J G^J \tag{125}$$

3.5.3 Manufacturers

Manufacturers combine labor services (hired from unions) with capital goods (rented from entrepreneurs) to produce an intermediate good, which is then sold to distributors. They are perfectly competitive in the input market and monopolistically competitive in the output market, charging a markup to distributors. Profits are distributed to households in the form of dividends.

More specifically, there is a continuum of manufacturing firms $j \in [0, 1]$ in each sector $J \in \{\mathcal{T}, \mathcal{N}\}$, where $\mathcal{T}$ stands for the Tradable sector and $\mathcal{N}$ stands for the Non-tradable sector. Each firm produces a specific variety of the intermediate good, which is bought by a continuum of distributor firms $f \in [0, 1]$ operating in sectors $F \in \{\mathcal{C}, \mathcal{G}, \mathcal{I}, \mathcal{X}\}$. Let $Z_t^{JF}(j, f)$ stand for the time t quantity of variety j produced by firm j operating in sector J and purchased by distributor f operating in sector F. Distributors buy intermediate goods from many manufacturers, bundling them together in a homogeneous intermediate good of type J, $Z_t^{JF}(f)$, to be used in the final goods production. The bundling technology is given by the familiar CES aggregator

$$Z_t^{JF}(f) = \left(\int_0^1 Z_t^{JF}(j, f)^{\frac{\sigma^J - 1}{\sigma^J}} \mathrm{d}j\right)^{\frac{\sigma^J}{\sigma^J - 1}}$$

where $\sigma^J \geq 0$ is the elasticity of substitution between varieties of sector J intermediate good.

The demand for intermediate goods faced by firm j results from the standard cost minimization problem of distributers, in which they select $Z_t^{JF}(j,f)$ so as to minimize the costs of acquiring intermediate goods, given that the bundle $Z_t^{JF}(f)$ must be achieved. The problem is

$$\min_{Z_t^{JF}(j,f)} \int_0^1 P_t^J(j) Z_t^{JF}(j,f) \mathrm{d}j \qquad \text{s.t.} \qquad Z_t^{JF}(f) = \left(\int_0^1 Z_t^{JF}(j,f)^{\frac{\sigma^J - 1}{\sigma^J}} \mathrm{d}j \right)^{\frac{\sigma^J}{\sigma^J - 1}}$$

where $P_t^J(j)$ is the price charged by the intermediate goods producer j operating in sector J. Since distributors are perfectly competitive in the input market, they take the price $P_t^J(j)$ as given. As usual, the Lagrange multiplier corresponds to the buyers' marginal cost of acquiring an extra unit of input, *i.e.*, it equals the aggregate input price, in this case P_t^J. Following the same steps as in Box 1 we obtain

$$Z_t^{JF}(j,f) = \left(\frac{P_t^J(j)}{P_t^J} \right)^{-\sigma^J} Z_t^{JF}(f) \tag{126}$$

$$P_t^J = \left(\int_0^1 P_t^J(j)^{1-\sigma^J} \mathrm{d}j \right)^{\frac{1}{1-\sigma^J}}$$

Integrating equation (126) over all distributors f and summing across F yields the demand for variety j

$$Z_t^J(j) = \left(\frac{P_t^J(j)}{P_t^J} \right)^{-\sigma^J} Z_t^J \tag{127}$$

where Z_t^J is the aggregate demand for sector J intermediate good.

Each manufacturing firm j operating in sector J combines labor services $U_t^J(j)$ with capital $K_t^J(j)$ according to the following production function

$$\begin{aligned} Z_t^J(j) &\equiv \mathcal{F}_t^J \left(K_t^J(j), U_t^J(j) \right) = \\ &= \left(\left(1 - \alpha_U^J\right)^{\frac{1}{\xi_J}} \left(K_t^J(j) \right)^{\frac{\xi_J - 1}{\xi_J}} + \left(\alpha_U^J \right)^{\frac{1}{\xi_J}} \left(T_t A_t^J U_t^J(j) \right)^{\frac{\xi_J - 1}{\xi_J}} \right)^{\frac{\xi_J}{\xi_J - 1}} \end{aligned} \tag{128}$$

where $\xi_J \geq 0$ is the elasticity of substitution between capital an labor for a firm operating in sector J and $0 \leq \alpha_U^J \leq 1$ is a distribution parameter. The element A_t^J is a temporary labor-augmenting technology shock specific to sector J. The marginal productivity of labor and capital are respectively

$$\frac{\partial \mathcal{F}_t^J}{\partial U_t^J(j)} = \left(\alpha_U^J \right)^{\frac{1}{\xi_J}} \left(Z_t^J(j) \right)^{\frac{1}{\xi_J}} \left(T_t A_t^J U_t^J(j) \right)^{\frac{\xi_J - 1}{\xi_J} - 1} T_t A_t^J = T_t A_t^J \left(\frac{\alpha_U^J Z_t^J(j)}{T_t A_t^J U_t^J(j)} \right)^{\frac{1}{\xi_J}}$$

$$\frac{\partial \mathcal{F}_t^J}{\partial K_t^J(j)} = \left(1 - \alpha_U^J\right)^{\frac{1}{\xi_J}} \left(Z_t^J(j) \right)^{\frac{1}{\xi_J}} \left(K_t^J(j) \right)^{\frac{\xi_J - 1}{\xi_J} - 1} = \left(\frac{\left(1 - \alpha_U^J\right) Z_t^J(j)}{K_t^J(j)} \right)^{\frac{1}{\xi_J}}$$

Since all manufacturing firms in sector J solve the same optimization problem, the equilibrium is symmetric and one can drop the indexer j when solving for the equilibrium. Equation (128) in stationary form boils down to

$$\check{Z}_t^J \equiv \check{\mathcal{F}}_t^J\left(\check{K}_t^J, U_t^J\right) = \left[\left(1-\alpha_U^J\right)^{\frac{1}{\xi_J}}\left(\check{K}_t^J\right)^{\frac{\xi_J-1}{\xi_J}} + \left(\alpha_U^J\right)^{\frac{1}{\xi_J}}\left(A_t^J U_t^J\right)^{\frac{\xi_J-1}{\xi_J}}\right]^{\frac{\xi_J}{\xi_J-1}} \tag{129}$$

In the steady state

$$\check{Z}^J \equiv \check{\mathcal{F}}^J\left(\check{K}^J, U^J\right) = \left[\left(1-\alpha_U^J\right)^{\frac{1}{\xi_J}}\left(\check{K}^J\right)^{\frac{\xi_J-1}{\xi_J}} + \left(\alpha_U^J\right)^{\frac{1}{\xi_J}}\left(A^J U^J\right)^{\frac{\xi_J-1}{\xi_J}}\right]^{\frac{\xi_J}{\xi_J-1}} \tag{130}$$

Moreover, in a symmetric equilibrium, the stationary form of labor and capital marginal productivities are

$$\left(\check{\mathcal{F}}_t^{UJ}\right)' \equiv \frac{\partial \check{\mathcal{F}}_t^J}{\partial U_t^J} = A_t^J\left(\frac{\alpha_U^J \check{Z}_t^J}{A_t^J U_t^J}\right)^{\frac{1}{\xi_J}} \tag{131}$$

$$\left(\check{\mathcal{F}}_t^{\mathcal{K}J}\right)' \equiv \frac{\partial \check{\mathcal{F}}_t^J}{\partial \check{K}_t^J} = \left(\frac{\left(1-\alpha_U^J\right)\check{Z}_t^J}{\check{K}_t^J}\right)^{\frac{1}{\xi_J}} \tag{132}$$

The steady-state versions are

$$\left(\check{\mathcal{F}}^{UJ}\right)' \equiv \frac{\partial \check{\mathcal{F}}^J}{\partial U^J} = \bar{A}^J\left(\frac{\alpha_U^J \check{Z}^J}{\bar{A}^J U^J}\right)^{\frac{1}{\xi_J}} \tag{133}$$

$$\left(\check{\mathcal{F}}^{\mathcal{K}J}\right)' \equiv \frac{\partial \check{\mathcal{F}}^J}{\partial \check{K}^J} = \left(\frac{\left(1-\alpha_U^J\right)\check{Z}^J}{\check{K}^J}\right)^{\frac{1}{\xi_J}} \tag{134}$$

where $\bar{A}^J$ is the steady-state value of the labor-augmenting technology shock. Following Ireland (2001) and Laxton and Pesenti (2003), price inflation of intermediate goods is subject to Rotemberg-type quadratic adjustment costs, which generate inflation persistence

$$\Gamma_t^{PJ}(j) = \frac{\phi_{PJ}}{2} Z_t^J \left(\frac{\frac{P_t^J(j)}{P_{t-1}^J(j)}}{\frac{P_{t-1}^J}{P_{t-2}^J}} - 1\right)^2 \tag{135}$$

where ϕ_{PJ} is a sector specific scaling factor determining the magnitude of price adjustment costs for firms operating in sector J. In stationary form equation (135) becomes

$$\check{\Gamma}_t^{PJ} = \frac{\phi_{PJ}}{2}\check{Z}_t^J\left(\frac{\pi_t^J}{\pi_{t-1}^J} - 1\right)^2 \tag{136}$$

where $\pi_t^J = P_t^J/P_{t-1}^J$ is sector J (gross) inflation rate. In the steady state prices are constant and therefore

$$\check{\Gamma}^{PJ} = 0 \tag{137}$$

On the real side, a sluggish adjustment of hours worked is also ensured through quadratic adjustment costs

$$\Gamma_t^{UJ}(j) = \frac{\phi_{UJ}}{2}U_t^J\left(\frac{U_t^J(j)}{U_{t-1}^J(j)} - 1\right)^2 \tag{138}$$

where ϕ_{UJ} is a sector specific scaling factor determining the magnitude of adjustment costs. In stationary form and after dropping the indexer j equation (138) collapses to

$$\Gamma_t^{UJ} = \frac{\phi_{UJ}}{2}U_t^J\left(\frac{U_t^J}{U_{t-1}^J} - 1\right)^2 \tag{139}$$

In the steady state hours worked are constant and therefore

$$\Gamma^{UJ} = 0 \tag{140}$$

Capital is accumulated by entrepreneurs and rented to manufacturers at a unitary nominal rental rate of $R_t^{\mathcal{K}J}$. Thus, taxes on capital holdings are paid by entrepreneurs. Manufacturers pay a profit tax $\tau_t^{\mathcal{K}}$, identical to the one faced by entrepreneurs, and a social security tax, τ_t^{SP}. Dividends of manufacturer j operating in sector J are therefore

$$\begin{aligned} D_t^J(j) =& (1-\tau_t^{\mathcal{K}})\Big[P_t^J(j)Z_t^J(j) - R_t^{\mathcal{K}J}K_t^J(j) \\ & -(1+\tau_t^{SP})V_t\left(U_t^J(j) + \Gamma_t^{UJ}(j)\right) - P_t^J\left(\Gamma_t^{PJ}(j) + T_t\varpi^J\right)\Big] \end{aligned} \tag{141}$$

where $P_t^J T_t \varpi^J$ is a quasi-fixed cost paid by manufacturers that counteracts the steady-state economic profits arising from monopolistic competition and faciliates the calibration of the model (particularly the depreciation rate and the ratio of investment to GDP).

Let $p_t^J = P_t^J/P_t$ denote the relative price of sector J intermediate good and $r_t^{\mathcal{K}J}$ denote the real rental rate of capital. The stationary form of equation (141) in real terms (*i.e.* after dropping the indexer j and dividing by $P_t T_t$) is

$$\check{d}_t^J = \left(1-\tau_t^{\mathcal{K}}\right)\left[p_t^J \check{Z}_t^J - r_t^{\mathcal{K}J}\check{K}_t^J - (1+\tau_t^{SP})\check{v}_t\left(U_t^J + \Gamma_t^{UJ}\right) - p_t^J\left(\check{\Gamma}_t^{PJ} + \varpi^J\right)\right] \tag{142}$$

In the steady state adjustment costs are zero and thus

$$\check{d}^J = \left(1-\tau^{\mathcal{K}}\right)\left[p^J \check{Z}^J - r^{\mathcal{K}J}\check{K}^J - (1+\tau^{SP})\check{v}\cdot U^J - p^J\varpi^J\right] \tag{143}$$

The manufacturer operating in sector J and producing variety j sets labor demand $U_t^J(j)$, capital demand $K_{t+1}^J(j)$, and the price $P_t^J(j)$ in each period in order to maximize the present discounted value of the dividends stream, subject to variety j demand in (127), to the production technology in (128), and to adjustment costs in (135) and (138) (notice that at date t capital $K_t^J(j)$ is already installed and cannot be changed). The problem is presented and solved in Box 12.

A useful relationship in what follows is the ratio of sector J's inflation to final consumer goods' inflation

$$\frac{\pi_t^J}{\pi_t} = \frac{p_t^J}{p_{t-1}^J} \tag{144}$$

In the steady state relative prices are constant and the relationship becomes

$$\pi^J = \pi \tag{145}$$

Dividing equation (13.1) by $\left(1-\tau_t^{\mathcal{K}}\right)\left(1-\sigma^J\right)Z_t^J$ and using the definition of real interest rate $r_t = i_t/\pi_{t+1}$ one obtains the optimal pricing rule

$$\frac{\sigma^J}{\sigma^J-1}\frac{\lambda_t^J}{p_t^J} - 1 = \frac{\phi_{PJ}}{\sigma^J-1}\left[\left(\frac{\pi_t^J}{\pi_{t-1}^J}-1\right)\frac{\pi_t^J}{\pi_{t-1}^J} - \frac{1-\tau_{t+1}^{\mathcal{K}}}{1-\tau_t^{\mathcal{K}}}\frac{\theta}{r_t}\frac{\pi_{t+1}^J}{\pi_{t+1}}\frac{Z_{t+1}^J}{Z_t^J}\left(\frac{\pi_{t+1}^J}{\pi_t^J}-1\right)\frac{\pi_{t+1}^J}{\pi_t^J}\right]$$

The term λ_t^J captures the real cost of producing one additional unit of the intermediate good. Using (144), the stationary form of this equation becomes

$$\frac{\sigma^J}{\sigma^J-1}\frac{\lambda_t^J}{p_t^J} - 1 = \frac{\phi_{PJ}}{\sigma^J-1}\left[\left(\frac{\pi_t^J}{\pi_{t-1}^J}-1\right)\frac{\pi_t^J}{\pi_{t-1}^J} - \frac{1-\tau_{t+1}^{\mathcal{K}}}{1-\tau_t^{\mathcal{K}}}\frac{\theta\cdot g}{r_t}\frac{p_{t+1}^J}{p_t}\frac{\check{Z}_{t+1}^J}{\check{Z}_t^J}\left(\frac{\pi_{t+1}^J}{\pi_t^J}-1\right)\frac{\pi_{t+1}^J}{\pi_t^J}\right] \tag{146}$$

Box 12: Manufacturer maximization problem

The manufacturer selects the triplet $\{P^J_{t+s}(j), U^J_{t+s}(j), K^J_{t+s+1}(j)\}_{s=0}^{\infty}$ that maximizes the present discounted value of the dividends stream, subject to variety j demand in (127), to the production technology in (128), and to adjustment costs in (135) and (138). The Lagrangian is

$$\mathcal{L}(\cdot) = \mathrm{E}_t \sum_{s=0}^{\infty} \tilde{R}_{t,s} \left(1-\tau^{\mathcal{K}}_{t+s}\right) \Bigg\{ P^J_{t+s}(j) \left(\frac{P^J_{t+s}(j)}{P^J_{t+s}}\right)^{-\sigma^J} Z^J_{t+s} - R^{\mathcal{K}J}_{t+s} K^J_{t+s}(j)$$

$$-(1+\tau^{SP}_{t+s}) V_{t+s} \left(U^J_{t+s}(j) + \frac{\phi_{UJ}}{2} U^J_{t+s} \left(\frac{U^J_{t+s}(j)}{U^J_{t+s-1}(j)} - 1 \right)^2 \right)$$

$$-P^J_{t+s} \left(\frac{\phi_{PJ}}{2} Z^J_{t+s} \left(\frac{P^J_{t+s}(j)}{P^J_{t+s-1}(j)} \div \frac{P^J_{t+s-1}}{P^J_{t+s-2}} - 1 \right)^2 + T_{t+s} \varpi^J \right)$$

$$-P_{t+s} \lambda^J_{t+s}(j) \left[\left(\frac{P^J_{t+s}(j)}{P^J_{t+s}} \right)^{-\sigma^J} Z^J_{t+s} - \mathcal{F}^J_{t+s} \left(K^J_{t+s}(j), U^J_{t+s}(j) \right) \right] \Bigg\}$$

where $P_{t+s}\lambda^J_{t+s}(j)$ reflects the (nominal) marginal cost of producing an additional unit of the intermediate good and $\mathcal{F}^J_{t+s}\left(K^J_{t+s}(j), U^J_{t+s}(j)\right)$ is the production function defined in (128). The first-order conditions are as follows (we drop the expected value operator).

1. Optimal pricing rule

$$\frac{\partial \mathcal{L}(\cdot)}{\partial P^J_t(j)} = 0 \Leftrightarrow \left(1-\tau^{\mathcal{K}}_t\right) \left[\left(1-\sigma^J\right) \left(\frac{P^J_t(j)}{P^J_t}\right)^{-\sigma^J} Z^J_t - \phi_{PJ} Z^J_t \left(\frac{\frac{P^J_t(j)}{P^J_{t-1}(j)}}{\frac{P^J_{t-1}}{P^J_{t-2}}} - 1 \right) \left(\frac{\frac{P^J_t}{P^J_{t-1}(j)}}{\frac{P^J_{t-1}}{P^J_{t-2}}} \right) \right]$$

$$+P_t \lambda^J_t(j) \left(1-\tau^{\mathcal{K}}_t\right) \sigma^J \frac{\left(P^J_t(j)\right)^{-\sigma^J-1}}{(P^J_t)^{-\sigma^J}} Z^J_t$$

$$+\frac{\theta}{i_t} \left(1-\tau^{\mathcal{K}}_{t+1}\right) P^J_{t+1} \phi_{PJ} Z^J_{t+1} \left(\frac{\frac{P^J_{t+1}(j)}{P^J_t(j)}}{\frac{P^J_t}{P^J_{t-1}}} - 1 \right) \left(\frac{\frac{P^J_{t+1}(j)}{\left(P^J_t(j)\right)^2}}{\frac{P^J_t}{P^J_{t-1}}} \right) = 0$$

where we used the fact $\tilde{R}_{t,1} = \theta / i_t$. As the equilibrium is symmetric the indexer j can be dropped and the previous expression collapses to

$$\left(1-\tau^{\mathcal{K}}_t\right) \left[\left(1-\sigma^J\right) Z^J_t - \phi_{PJ} Z^J_t \left(\frac{\pi^J_t}{\pi^J_{t-1}} - 1 \right) \frac{\pi^J_t}{\pi^J_{t-1}} \right] + \frac{\lambda^J_t}{p^J_t} \left(1-\tau^{\mathcal{K}}_t\right) \sigma^J Z^J_t$$

$$+\frac{\theta}{i_t} \left(1-\tau^{\mathcal{K}}_{t+1}\right) \pi^J_{t+1} \phi_{PJ} Z^J_{t+1} \left(\frac{\pi^J_{t+1}}{\pi^J_t} - 1 \right) \frac{\pi^J_{t+1}}{\pi^J_t} = 0 \tag{13.1}$$

where we used the facts $p^J_t = P^J_t / P_t$ and $\pi^J_t = P^J_t / P^J_{t-1}$.

2. Labor demand

$$\frac{\partial \mathcal{L}(\cdot)}{\partial U^J_t(j)} = 0 \Leftrightarrow -\left(1-\tau^{\mathcal{K}}_t\right) \left\{ \left(1+\tau^{SP}_t\right) V_t \left[1 + \phi_{UJ} \left(\frac{U^J_t(j)}{U^J_{t-1}(j)} - 1 \right) \frac{U^J_t}{U^J_{t-1}(j)} \right] - P_t \lambda^J_t(j) \frac{\partial \mathcal{F}^J_t}{\partial U^J_t(j)} \right\}$$

$$+\frac{\theta}{i_t} \left(1-\tau^{\mathcal{K}}_{t+1}\right) \left(1+\tau^{SP}_{t+1}\right) V_{t+1} U^J_{t+1} \phi_{UJ} \left(\frac{U^J_{t+1}(j)}{U^J_t(j)} - 1 \right) \frac{U^J_{t+1}(j)}{\left(U^J_t(j)\right)^2} = 0$$

since $\tilde{R}_{t,1} = \theta / i_t$. In equilibrium the above expression can be rearranged to yield

$$- \left(1 - \tau_t^{\mathcal{K}}\right)\left(1 + \tau_t^{SP}\right) V_t \left[1 + \phi_{UJ}\left(\frac{U_t^J}{U_{t-1}^J} - 1\right)\frac{U_t^J}{U_{t-1}^J}\right] + \left(1 - \tau_t^{\mathcal{K}}\right) P_t \lambda_t^J \frac{\partial \mathcal{F}_t^J}{\partial U_t^J}$$

$$+ \frac{\theta}{i_t}\left(1 - \tau_{t+1}^{\mathcal{K}}\right)\left(1 + \tau_{t+1}^{SP}\right) V_{t+1} \phi_{UJ}\left(\frac{U_{t+1}^J}{U_t^J} - 1\right)\left(\frac{U_{t+1}^J}{U_t^J}\right)^2 = 0 \tag{13.2}$$

3. Capital demand

$$\frac{\partial \mathcal{L}(\cdot)}{\partial K_{t+1}^J(j)} = 0 \Leftrightarrow -(1 - \tau_{t+1}^{\mathcal{K}})\left(R_{t+1}^{\mathcal{K}J} - P_{t+1}\lambda_{t+1}^J(j)\frac{\partial \mathcal{F}_{t+1}^J}{\partial K_{t+1}^J(j)}\right) = 0$$

$$\Leftrightarrow R_{t+1}^{\mathcal{K}J} = P_{t+1}\lambda_{t+1}^J(j)\frac{\partial \mathcal{F}_{t+1}^J}{\partial K_{t+1}^J(j)}$$

given that $\tau_{t+1}^{\mathcal{K}} < 1$. In equilibrium

$$R_{t+1}^{\mathcal{K}J} = P_{t+1}\lambda_{t+1}^J \frac{\partial \mathcal{F}_{t+1}^J}{\partial K_{t+1}^J} \tag{13.3}$$

In the steady state adjustment costs are zero and thus

$$p^J = \frac{\sigma^J}{\sigma^J - 1}\lambda^J \tag{147}$$

These equations stress that sector J prices are a markup over the marginal cost. Outside the steady state, the markup is influenced by price adjustment costs.

Dividing (13.2) by $\left(1 - \tau_t^{\mathcal{K}}\right)\left(1 + \tau_t^{SP}\right) V_t$, and using the facts $r_t = i_t/\pi_{t+1}$, $v_t = V_t/P_t$, and $P_{t+1} = \pi_{t+1} P_t$, one obtains the equation defining labor demand

$$\frac{\lambda_t^J}{\left(1 + \tau_t^{SP}\right) v_t}\left(\mathcal{F}_t^{UJ}\right)' - 1 = \phi_{UJ}\left(\frac{U_t^J}{U_{t-1}^J} - 1\right)\frac{U_t^J}{U_{t-1}^J}$$
$$- \frac{\theta \cdot \phi_{UJ}}{r_t}\frac{1 - \tau_{t+1}^{\mathcal{K}}}{1 - \tau_t^{\mathcal{K}}}\frac{1 + \tau_{t+1}^{SP}}{1 + \tau_t^{SP}}\frac{v_{t+1}}{v_t}\left(\frac{U_{t+1}^J}{U_t^J} - 1\right)\left(\frac{U_{t+1}^J}{U_t^J}\right)^2$$

where $(\mathcal{F}_t^{UJ})' \equiv \partial \mathcal{F}_t^J / \partial U_t^J$. Using (58) leaded one period we obtain the following stationary form for labor demand

$$\frac{\lambda_t^J \cdot \left(\check{\mathcal{F}}_t^{UJ}\right)'}{\left(1 + \tau_t^{SP}\right)\check{v}_t} - 1 = \phi_{UJ}\left[\left(\frac{U_t^J}{U_{t-1}^J} - 1\right)\frac{U_t^J}{U_{t-1}^J} - \frac{\theta}{r_t}\frac{1 - \tau_{t+1}^{\mathcal{K}}}{1 - \tau_t^{\mathcal{K}}}\frac{1 + \tau_{t+1}^{SP}}{1 + \tau_t^{SP}}\frac{\pi_{t+1}^V}{\pi_{t+1}}\left(\frac{U_{t+1}^J}{U_t^J} - 1\right)\left(\frac{U_{t+1}^J}{U_t^J}\right)^2\right] \tag{148}$$

In the steady state adjustment costs are zero and the previous equation collapses to

$$\left(1 + \tau^{SP}\right)\check{v} = \lambda^J \left(\check{\mathcal{F}}^{UJ}\right)' \tag{149}$$

This equation states that labor demand is a function of its price, $\left(1+\tau^{SP}\right)\check{v}$, the intermediate goods marginal cost, λ^J, and the intermediate goods demand, captured implicitly by $(\check{\mathcal{F}}^{UJ})'$. An increase in the relative price of labor *vis-á-vis* the marginal cost must be compensated by an increase in labor marginal productivity, which is achieved by substituting capital for labor. Outside the steady state one must also take into account quantity adjustment costs. Finally, the stationary form of equation (13.3) defining manufacturers capital demand is

$$r_t^{\mathcal{K}J} = \lambda_t^J \left(\check{\mathcal{F}}_t^{\mathcal{K}J}\right)' \tag{150}$$

The steady-state version is

$$r^{\mathcal{K}J} = \lambda^J \left(\check{\mathcal{F}}^{\mathcal{K}J}\right)' \tag{151}$$

The interpretation of these two equations are analogous to that of (148) and (149), except that outside the steady state manufacturers do not face capital adjustment costs.

3.5.4 Distributors

Distributors produce four types of final goods: Consumption goods ($\mathcal{C}$), Investment goods ($\mathcal{I}$), Government consumption goods ($\mathcal{G}$), and Export goods ($\mathcal{X}$). For each type of final good $F \in \{\mathcal{C},\mathcal{G},\mathcal{I},\mathcal{X}\}$ there is a continuum of distributors $f \in [0,1]$, each producing a specific variety of the good. Distributors use the same technology irrespective of the sector where they operate. Production is divided into a two stage process. In the first stage, a distributor producing variety f of the final good F combines domestic tradable goods $Z_t^{\mathcal{T}F}(f)$ with a bundle of differentiated imported goods $M_t^F(f)$, through a CES technology. This stage yields a differentiated tradable good $Y_t^{AF}(f)$, henceforth termed assembled good. In the second stage, the same distributor combines the assembled good with domestic nontradable goods $Z_t^{\mathcal{N}F}(f)$ to obtain the variety f of type-F final good, $Y_t^F(f)$. Distributors are perfectly competitive in the input market and monopolistic competitive in the output market, charging a markup to final costumers. Profits are distributed to households in the form of dividends.

Each type of final good is demanded by a unique type of costumer: private consumption goods are demanded by households, government consumption goods by the government, investment goods by capital goods producers, and export goods by foreign agents. For each costumer type $E \in \{\mathcal{C},\mathcal{G},\mathcal{I},\mathcal{X}\}$ there is a continuum of agents $e \in [0,1]$, demanding many varieties of the good. Let $Y_t^F(f,e)$ stand for the time t quantity of variety f from the final good F purchased by costumer e. Each agent bundles the different varieties of the final good together to form an homogeneous consumption good $Y_t^F(e)$ according to the CES specification

$$Y_t^F(e) = \left(\int_0^1 Y_t^F(f,e)^{\frac{\sigma^F-1}{\sigma^F}}\mathrm{d}f\right)^{\frac{\sigma^F}{\sigma^F-1}} \tag{152}$$

where $\sigma^F \geq 0$ is the elasticity of substitution between varieties of type-F final good. The demand for each variety by a particular costumer, $Y_t^F(f,e)$, is obtained by minimizing the costs of acquiring final goods, subject to the fact that the bundle in (152) must be achieved

$$\min_{Y_t^F(f,e)} \int_0^1 P_t^F(f) Y_t^F(f,e) \mathrm{d}f \qquad \text{s.t.} \qquad Y_t^F(e) = \left(\int_0^1 Y_t^F(f,e)^{\frac{\sigma^F-1}{\sigma^F}} \mathrm{d}f \right)^{\frac{\sigma^F}{\sigma^F-1}}$$

The element $P_t^F(f)$ is the price charged by the distributer of variety f operating in sector F. Following the same steps as in Box 1 yields

$$Y_t^F(f,e) = \left(\frac{P_t^F(f)}{P_t^F} \right)^{-\sigma^F} Y_t^F(e) \tag{153}$$

$$P_t^F = \left(\int_0^1 P_t^F(f)^{1-\sigma^F} \mathrm{d}f \right)^{\frac{1}{1-\sigma^F}}$$

This same optimization problem was already solved for households, although with a different notation. Therein, $C_{a,t}^H(c)$ corresponds to the consumption of variety c by a type-H household with age a at time t, which is exactly equivalent to $Y_t^{\mathcal{C}}(f,e)$. In this case, $f = c$, and e indexes the household.

Integrating (153) over e yields the demand for variety f faced by a distributor operating in sector F

$$Y_t^F(f) = \left(\frac{P_t^F(f)}{P_t^F} \right)^{-\sigma^F} Y_t^F \tag{154}$$

where Y_t^F is the aggregate demand for sector F final good.

The production process is divided in two interdependent stages: the assemblage stage and the distribution stage. In the assemblage stage, domestic tradable goods are combined with imported goods, yielding a bundle of assembled goods. In the second stage, assembled goods are combined with domestic nontradable goods to obtain the final good.

The assemblage stage. In this stage, the distributor uses a CES technology to combine domestic tradable goods $Z_t^{\mathcal{T}F}(f)$ with differentiated imported goods $M_t^F(f)$. This process yields a composite differentiated good—the assembled good.

We obtain a realistic pattern for the import content by imposing quadratic adjustment costs with the following specification

$$\Gamma_t^{AF}(f) = \frac{\phi_{AF}}{2} \frac{\left(\mathcal{A}_t^{AF}(f) - 1\right)^2}{1 + \left(\mathcal{A}_t^{AF}(f) - 1\right)^2} \tag{155}$$

where ϕ_{AF} is a sector specific scaling factor and

$$\mathcal{A}_t^{AF}(f) = \frac{M_t^F(f)/Y_t^{AF}(f)}{M_{t-1}^F/Y_{t-1}^{AF}}$$

Given that all distributors in sector F are identical and face the same decision problem (see Box 13), the equilibrium is symmetric. Dropping the indexer f and dividing trend variables by the level of technology yields adjustment costs in stationary form

$$\check{\Gamma}_t^{AF} = \frac{\phi_{AF}}{2} \frac{\left(\check{\mathcal{A}}_t^{AF} - 1\right)^2}{1 + \left(\check{\mathcal{A}}_t^{AF} - 1\right)^2} \tag{156}$$

with $\check{\mathcal{A}}_t^{AF}$ given by

$$\check{\mathcal{A}}_t^{AF} = \frac{\check{M}_t^F / \check{Y}_t^{AF}}{\check{M}_{t-1}^F / \check{Y}_{t-1}^{AF}} \tag{157}$$

These equations state that adjustment costs at t increase with the change in the share of the import content in the assembled good. In the steady state the import share is constant and therefore

$$\check{\Gamma}^{AF} = 0 \tag{158}$$

$$\check{\mathcal{A}}_t^{AF} = 0 \tag{159}$$

Each distributor f operating in sector F combines domestic tradable goods with imported goods according to the following production function

$$\begin{aligned} Y_t^{AF}(f) \equiv & \mathcal{F}_t^{AF}\left(Z_t^{\mathcal{T}F}(f), M_t^F(f), \Gamma_t^{AF}(f)\right) = \Bigg(\left(\alpha_Z^{AF}\right)^{\frac{1}{\xi_{AF}}} \left(Z_t^{\mathcal{T}F}(f)\right)^{\frac{\xi_{AF}-1}{\xi_{AF}}} + \\ & + \left(1 - \alpha_Z^{AF}\right)^{\frac{1}{\xi_{AF}}} \left[M_t^F(f)\left(1 - \Gamma_t^{AF}(f)\right)\right]^{\frac{\xi_{AF}-1}{\xi_{AF}}} \Bigg)^{\frac{\xi_{AF}}{\xi_{AF}-1}} \end{aligned} \tag{160}$$

where $\xi_{AF} \geq 0$ is the elasticity of substitution between domestic tradable goods and imported good for a distributor operating in sector F and $0 \leq \alpha_Z^{AF} \leq 1$ is the home bias parameter. In stationary form this equation boils down to

$$\check{Y}_t^{AF} = \left[\left(\alpha_Z^{AF}\right)^{\frac{1}{\xi_{AF}}} \left(\check{Z}_t^{\mathcal{T}F}\right)^{\frac{\xi_{AF}-1}{\xi_{AF}}} + \left(1 - \alpha_Z^{AF}\right)^{\frac{1}{\xi_{AF}}} \left[\check{M}_t^F\left(1 - \check{\Gamma}_t^{AF}\right)\right]^{\frac{\xi_{AF}-1}{\xi_{AF}}}\right]^{\frac{\xi_{AF}}{\xi_{AF}-1}} \tag{161}$$

In the steady state

$$\check{Y}^{AF} = \left[\left(\alpha_Z^{AF}\right)^{\frac{1}{\xi_{AF}}} \left(\check{Z}^{\mathcal{T}F}\right)^{\frac{\xi_{AF}-1}{\xi_{AF}}} + \left(1 - \alpha_Z^{AF}\right)^{\frac{1}{\xi_{AF}}} \left[\check{M}^F\left(1 - \check{\Gamma}^{AF}\right)\right]^{\frac{\xi_{AF}-1}{\xi_{AF}}}\right]^{\frac{\xi_{AF}}{\xi_{AF}-1}} \tag{162}$$

Each distributor selects the quantities $\{Z_t^{\mathcal{T}F}(f), M_t^F(f)\}$ in each period so as to minimize the cost of producing the assembled good, subject to adjustment costs in (155) and to the production technology in (160). The problem is presented and solved in Box 13.

Dividing equation (14.1) by T_t and dropping the indexer f we obtain the demand for domestic tradable goods in stationary form

$$\check{Z}_t^{\mathcal{T}F} = \alpha_Z^{AF} \left(\frac{p_t^{\mathcal{T}}}{\lambda_t^{AF}} \right)^{-\xi_{AF}} \check{Y}_t^{AF} \tag{163}$$

where $p_t^{\mathcal{T}} = P_t^{\mathcal{T}}/P_t$ is the relative price of the tradable good and λ_t^{AF} stands for the real marginal cost. This equation states that the demand for tradable domestic goods is a function of the relative price of this input *vis-á-vis* the marginal cost. The steady-state version is

$$\check{Z}^{\mathcal{T}F} = \alpha_Z^{AF} \left(\frac{p^{\mathcal{T}}}{\lambda^{AF}} \right)^{-\xi_{AF}} \check{Y}^{AF} \tag{164}$$

A similar procedure for equation (14.2) yields the demand for imported goods in stationary form

$$\check{M}_t^F (1 - \check{\Gamma}_t^{AF}) = (1 - \alpha_Z^{AF}) \left(\frac{e_t}{\lambda_t^{AF} \cdot \check{\iota}_t^{AF}} \right)^{-\xi_{AF}} \check{Y}_t^{AF} \tag{165}$$

where the element $\check{\iota}_{A,F}$ is

$$\check{\iota}_t^{AF} = 1 - \check{\Gamma}_t^{AF} - \phi_{AF} \frac{(\check{\mathcal{A}}_t^{AF} - 1)\check{\mathcal{A}}_t^{AF}}{\left[1 + (\check{\mathcal{A}}_t^{AF} - 1)^2\right]^2} \tag{166}$$

The interpretation of (165) is analogous to that of (163), except that one must also take into account the costs of adjusting the import content of the assembled good. In the steady state

$$\check{M}^F = (1 - \alpha_Z^{AF}) \left(\frac{e}{\lambda^{AF}} \right)^{-\xi_{AF}} \check{Y}^{AF} \tag{167}$$

and

$$\check{\iota}^{AF} = 1 \tag{168}$$

The distribution stage. In the second stage, the distributor combines assembled goods $Y_t^{AF}(f)$ with nontradable goods $Z_t^{\mathcal{N}F}(f)$ bought from manufacturers to produce a final good

Box 13: The distributor assemblage stage

The distributor selects the quantities $\{Z_t^{\mathcal{T}F}(f), M_t^F(f)\}$ in each period that minimize the cost of producing a given amount of the assembled good, subject to adjustment costs in (155) and to the production technology in (160). Let

$$\Gamma_t^{AF}(f) = \Gamma_t^{AF}\Big(\mathcal{A}_t^{AF}\big(M_t^F(f), Y_t^{AF}(f)\big)\Big) \qquad \text{and} \qquad \mathcal{A}_t^{AF}(f) = \mathcal{A}_t^{AF}\big(M_t^F(f), Y_t^{AF}(f)\big)$$

The Lagrangian is

$$\begin{aligned}\mathcal{L}(\cdot) =& P_t^{\mathcal{T}} Z_t^{\mathcal{T}F}(f) + P_t^* \varepsilon_t M_t^F(f) - P_t \lambda_t^{AF}(f) \Bigg[\Bigg(\big(\alpha_Z^{AF}\big)^{\frac{1}{\xi_{AF}}} \big(Z_t^{\mathcal{T}F}(f)\big)^{\frac{\xi_{AF}-1}{\xi_{AF}}} \\ &+ \big(1-\alpha_Z^{AF}\big)^{\frac{1}{\xi_{AF}}} \big[M_t^F(f)\big(1-\Gamma_t^{AF}(f)\big)\big]^{\frac{\xi_{AF}-1}{\xi_{AF}}}\Bigg)^{\frac{\xi_{AF}}{\xi_{AF}-1}} - Y_t^{AF}(f)\Bigg]\end{aligned}$$

where P_t^* is the price of the imported good in foreign currency and $\lambda_t^{AF}(f)$ stands for the real cost of producing an extra unit of type-F assembled good. In what follows, we treat $\Gamma_t^{AF}(f)$ separately.
For later reference we compute the marginal product of domestic tradable goods and imported goods

$$\frac{\partial \mathcal{F}_t^{AF}}{\partial Z_t^{\mathcal{T}F}(f)} = \big(Y_t^{AF}(f)\big)^{\frac{1}{\xi_{AF}}} \big(\alpha_Z^{AF}\big)^{\frac{1}{\xi_{AF}}} \big(Z_t^{\mathcal{T}F}(f)\big)^{\frac{\xi_{AF}-1}{\xi_{AF}}-1} = \big(\alpha_Z^{AF}\big)^{\frac{1}{\xi_{AF}}} \left(\frac{Y_t^{AF}(f)}{Z_t^{\mathcal{T}F}(f)}\right)^{\frac{1}{\xi_{AF}}}$$

$$\begin{aligned}\frac{\partial \mathcal{F}_t^{AF}}{\partial M_t^F(f)} =& \big(Y_t^{AF}(f)\big)^{\frac{1}{\xi_{AF}}} \big(1-\alpha_Z^{AF}\big)^{\frac{1}{\xi_{AF}}} \big[M_t^F(f)\big(1-\Gamma_t^{AF}(f)\big)\big]^{\frac{\xi_{AF}-1}{\xi_{AF}}-1} \iota_t^{AF}(f) \\ =& \big(1-\alpha_Z^{AF}\big)^{\frac{1}{\xi_{AF}}} \left(\frac{Y_t^{AF}(f)}{M_t^F(f)\big[1-\Gamma_t^{AF}(f)\big]}\right)^{\frac{1}{\xi_{AF}}} \iota_t^{AF}(f)\end{aligned}$$

where

$$\iota_t^{AF}(f) = 1 - \Gamma_t^{AF}(f) - M_t^F(f)\left(\frac{\partial \Gamma_t^{AF}}{\partial M_t^F(f)}\right)$$

Noticing that $\partial \mathcal{A}_t^{AF}/\partial M_t^F(f) = \mathcal{A}_t^{AF}(f) \cdot \big(M_t^F(f)\big)^{-1}$, the derivative of adjustment costs with respect to imported goods is

$$\begin{aligned}\frac{\partial \Gamma_t^{AF}}{\partial M_t^F(f)} =& \phi_{AF} \frac{\big(\mathcal{A}_t^{AF}(f)-1\big)\mathcal{A}_t^{AF}(f)\cdot\big(M_t^F(f)\big)^{-1}\Big[1+\big(\mathcal{A}_t^{AF}(f)-1\big)^2\Big]}{\Big[1+\big(\mathcal{A}_t^{AF}(f)-1\big)^2\Big]^2} \\ &- \frac{\big(\mathcal{A}_t^{AF}(f)-1\big)^2\big(\mathcal{A}_t^{AF}(f)-1\big)\mathcal{A}_t^{AF}(f)\cdot\big(M_t^F(f)\big)^{-1}}{\Big[1+\big(\mathcal{A}_t^{AF}(f)-1\big)^2\Big]^2} \\ =& \frac{\phi_{AF}}{M_t^F(f)} \frac{\big(\mathcal{A}_t^{AF}(f)-1\big)\mathcal{A}_t^{AF}(f)}{\Big[1+\big(\mathcal{A}_t^{AF}(f)-1\big)^2\Big]^2}\end{aligned}$$

The first-order conditions of the optimization problem are as follows.

1. Domestic tradable goods demand

$$\frac{\partial \mathcal{L}(\cdot)}{\partial Z_t^{\mathcal{T}F}(f)} = 0 \Leftrightarrow P_t^{\mathcal{T}} - P_t \lambda_t^{AF}(f) \frac{\partial \mathcal{F}_t^{AF}}{\partial Z_t^{\mathcal{T}F}(f)} = 0 \Leftrightarrow \frac{\partial \mathcal{F}_t^{AF}}{\partial Z_t^{\mathcal{T}F}(f)} = \frac{p_t^{\mathcal{T}}}{\lambda_t^{AF}(f)} \Leftrightarrow$$

$$\Leftrightarrow \left(\alpha_Z^{AF}\right)^{\frac{1}{\xi_{AF}}} \left(\frac{Y_t^{AF}(f)}{Z_t^{\mathcal{T}F}(f)}\right)^{\frac{1}{\xi_{AF}}} = \frac{p_t^{\mathcal{T}}}{\lambda_t^{AF}(f)} \Leftrightarrow Z_t^{\mathcal{T}F}(f) = \alpha_Z^{AF} \left(\frac{p_t^{\mathcal{T}}}{\lambda_t^{AF}(f)}\right)^{-\xi_{AF}} Y_t^{AF}(f) \tag{14.1}$$

where $p_t^{\mathcal{T}} = P_t^{\mathcal{T}}/P_t$ is the relative price of the tradable good.

2. Imported goods demand

$$\frac{\partial \mathcal{L}(\cdot)}{\partial M_t^F(f)} = 0 \Leftrightarrow \varepsilon_t P_t^* - P_t \lambda_t^{AF}(f) \frac{\partial \mathcal{F}_t^{AF}}{\partial M_t^F(f)} = 0 \Leftrightarrow \frac{\partial \mathcal{F}_t^{AF}}{\partial M_t^F(f)} = \frac{e_t}{\lambda_t^{AF}(f)} \Leftrightarrow$$

$$\Leftrightarrow \left(1-\alpha_Z^{AF}\right)^{\frac{1}{\xi_{AF}}} \left(\frac{Y_t^{AF}(f)}{M_t^F(f)\left[1-\Gamma_t^{AF}(f)\right]}\right)^{\frac{1}{\xi_{AF}}} \iota_t^{AF}(f) = \frac{e_t}{\lambda_t^{AF}(f)}$$

We used the definition of real exchange rate $e_t = \varepsilon_t P_t^*/P_t$. The first-order condition can be further simplified to

$$\left[M_t^F(f)\left(1-\Gamma_t^{AF}(f)\right)\right]^{-\frac{1}{\xi_{AF}}} = \frac{e_t}{\lambda_t^{AF}(f) \cdot \iota_t^{AF}(f)} \left(1-\alpha_Z^{AF}\right)^{-\frac{1}{\xi_{AF}}} \left(Y_t^{AF}(f)\right)^{-\frac{1}{\xi_{AF}}} \Leftrightarrow$$

$$\Leftrightarrow M_t^F(f)\left[1-\Gamma_t^{AF}(f)\right] = \left(1-\alpha_Z^{AF}\right) \left(\frac{e_t}{\lambda_t^{AF}(f) \cdot \iota_t^{AF}(f)}\right)^{-\xi_{AF}} Y_t^{AF}(f) \tag{14.2}$$

$Y_t^F(f)$, according to the following CES technology

$$\begin{aligned} Y_t^F(f) =& \mathcal{F}_t^F\left(Y_t^{AF}(f), Z_t^{\mathcal{N}F}(f)\right) \\ =& \left(\left(1-\alpha_Z^F\right)^{\frac{1}{\xi_F}} \left(Y_t^{AF}(f)\right)^{\frac{\xi_F-1}{\xi_F}} + \left(\alpha_Z^F\right)^{\frac{1}{\xi_F}} \left(Z_t^{\mathcal{N}F}(f)\right)^{\frac{\xi_F-1}{\xi_F}}\right)^{\frac{\xi_F}{\xi_F-1}} \end{aligned} \tag{169}$$

where $\xi_F \geq 0$ is the elasticity of substitution between assembled goods and nontradable goods and $0 \leq \alpha_Z^F \leq 1$ is the nontradable goods bias parameter. The final good $Y_t^F(f)$ is thereafter sold at the price $P_t^F(f)$. In stationary form this production technology boils down to

$$\check{Y}_t^F = \left(\left(1-\alpha_Z^F\right)^{\frac{1}{\xi_F}} \left(\check{Y}_t^{AF}\right)^{\frac{\xi_F-1}{\xi_F}} + \left(\alpha_Z^F\right)^{\frac{1}{\xi_F}} \left(\check{Z}_t^{\mathcal{N}F}\right)^{\frac{\xi_F-1}{\xi_F}}\right)^{\frac{\xi_F}{\xi_F-1}} \tag{170}$$

The steady-state version is

$$\check{Y}^F = \left(\left(1-\alpha_Z^F\right)^{\frac{1}{\xi_F}} \left(\check{Y}^{AF}\right)^{\frac{\xi_F-1}{\xi_F}} + \left(\alpha_Z^F\right)^{\frac{1}{\xi_F}} \left(\check{Z}^{\mathcal{N}F}\right)^{\frac{\xi_F-1}{\xi_F}}\right)^{\frac{\xi_F}{\xi_F-1}} \tag{171}$$

Distributors must pay adjustment costs when updating prices, according to the following quadratic specification

$$\Gamma_t^{PF}(f) = \frac{\phi_{PF}}{2} Y_t^F \left(\frac{\frac{P_t^F(f)}{P_{t-1}^F(f)}}{\frac{P_{t-1}^F}{P_{t-2}^F}} - 1 \right)^2 \tag{172}$$

where ϕ_{PF} is a sector specific scaling factor determining the magnitude of price adjustment costs for firms operating in sector F. These adjustment costs create a nominal rigidity that leads to price stickiness in final goods. In stationary form adjustment costs are

$$\check{\Gamma}_t^{PF} = \frac{\phi_{PF}}{2} \check{Y}_t^F \left(\frac{\pi_t^F}{\pi_{t-1}^F} - 1 \right)^2 \tag{173}$$

where $\pi_t^F = P_t^F / P_{t-1}^F$ is sector F (gross) inflation rate. In the steady state final goods inflation is constant and therefore

$$\check{\Gamma}^{PF} = 0 \tag{174}$$

Analogously to manufacturers, distributors pay a quasi-fixed cost totaling $P_t^F T_t \varpi^F$, which ensures that economic profits arising from monopolistic competition are zero in the steady state. Dividends of distributor f operating in sector F are

$$D_t^F(f) = (1 - \tau_t^D) \Big[P_t^F(f) Y_t^F(f) - P_t \lambda_t^{AF}(f) Y_t^{AF}(f) \\ - P_t^{\mathcal{N}} Z_t^{\mathcal{N}F}(f) - P_t^F \left(\Gamma_t^{PF}(f) + T_t \varpi^F \right) \Big] \tag{175}$$

Notice that distributors pay for the assembled good that they produced in the assemblage stage and for the nontradable good bought from manufacturers—in addition to adjustment and quasi-fixed costs. The real marginal cost of the assembled good corresponds to λ_t^{AF}. Let $p_t^F = P_t^F / P_t$ denote the relative price of sector F final good and recall that the element $p_t^{\mathcal{N}} = P_t^{\mathcal{N}} / P_t$, introduced in Section 3.5.3, is the relative price of the nontradable good. The stationary form of (175) in real terms (*i.e.* after dropping the indexer f and dividing by $P_t T_t$) thus becomes

$$\check{d}_t^F = (1 - \tau_t^D) \left[p_t^F \check{Y}_t^F - \lambda_t^{AF} \check{Y}_t^{AF} - p_t^{\mathcal{N}} \check{Z}_t^{\mathcal{N}F} - p_t^F \left(\check{\Gamma}_t^{PF} + \varpi^F \right) \right] \tag{176}$$

In the steady state adjustment costs are zero and hence

$$\check{d}^F = (1 - \tau^D) \left[p^F \check{Y}^F - \lambda^{AF} \check{Y}^{AF} - p^{\mathcal{N}} \check{Z}^{\mathcal{N}F} - p^F \varpi^F \right] \tag{177}$$

Distributor f operating in sector F sets assembled goods utilization $Y_t^{AF}(f)$, nontradable goods demand $Z_t^{\mathcal{N}F}(f)$, and final goods price $P_t^F(f)$ in each period in order to maximize the present discounted value of the dividends stream, subject to the constraints imposed by variety f demand in (154), by the production technology in (169), and by adjustment costs in (172). The problem is presented and solved in Box 14.

A useful relationship in what follows is the ratio of sector F inflation to final consumer goods inflation

$$\frac{\pi_t^F}{\pi_t} = \frac{p_t^F}{p_{t-1}^F} \tag{178}$$

In the steady state prices are constant and the relationship becomes

$$\pi^F = \pi \tag{179}$$

Dividing equation (15.1) by $\left(1-\tau_t^D\right)\left(1-\sigma^F\right)Y_t^F$ and using the definition of real interest rate $r_t = i_t/\pi_{t+1}$ one obtains the optimal pricing rule

$$\frac{\sigma^F}{\sigma^F-1}\frac{\lambda_t^F}{p_t^F} - 1 = \frac{\phi_{PF}}{\sigma^F-1}\left[\left(\frac{\pi_t^F}{\pi_{t-1}^F}-1\right)\frac{\pi_t^F}{\pi_{t-1}^F} - \frac{1-\tau_{t+1}^D}{1-\tau_t^D}\frac{\theta}{r_t}\frac{\pi_{t+1}^F}{\pi_{t+1}}\frac{Y_{t+1}^F}{Y_t^F}\left(\frac{\pi_{t+1}^F}{\pi_t^F}-1\right)\frac{\pi_{t+1}^F}{\pi_t^F}\right]$$

The term λ_t^F captures the real cost of producing one additional unit of the final good. Using (178), the stationary form of this equation becomes

$$\frac{\sigma^F}{\sigma^F-1}\frac{\lambda_t^F}{p_t^F} - 1 = \frac{\phi_{PF}}{\sigma^F-1}\left[\left(\frac{\pi_t^F}{\pi_{t-1}^F}-1\right)\frac{\pi_t^F}{\pi_{t-1}^F} - \frac{1-\tau_{t+1}^D}{1-\tau_t^D}\frac{\theta\cdot g}{r_t}\frac{p_{t+1}^F}{p_t}\frac{\check{Y}_{t+1}^F}{\check{Y}_t^F}\left(\frac{\pi_{t+1}^F}{\pi_t^F}-1\right)\frac{\pi_{t+1}^F}{\pi_t^F}\right] \tag{180}$$

In the steady state adjustment costs are zero and thus

$$p^F = \frac{\sigma^F}{\sigma^F-1}\lambda^F \tag{181}$$

These equations stress that sector F prices are a markup over the marginal cost. Outside the steady state, the markup is influenced by price adjustment costs.

Equation (15.2) in stationary form becomes

$$\check{Y}_t^{AF} = \left(1-\alpha_Z^F\right)\left(\frac{\lambda_t^{AF}}{\lambda_t^F}\right)^{-\xi_F}\check{Y}_t^F \tag{182}$$

This equation states that the assembled good utilization is a function of the relative (marginal) cost of the assembled good *vis-à-vis* the final good, $\lambda_t^{AF}/\lambda_t^F$, and the final goods demand $\check{Y}_t^F$. An increase in the relative cost of the assembled good leads to a substitution effect towards

Box 14: Distributor maximization problem

The distributor selects the triplet $\{P^F_{t+s}(f), Y^{AF}_{t+s}(f), Z^{\mathcal{N}F}_{t+s}(f)\}_{s=0}^{\infty}$ that maximizes the present discounted value of the dividends stream, subject to variety h demand in (154), to technology in (169), and to adjustment costs in (172). The Lagrangian can be written as

$$\begin{aligned}\mathcal{L}(\cdot) = & \mathrm{E}_t \sum_{s=0}^{\infty} \tilde{R}_{t,s}\left(1-\tau^D_{t+s}\right)\Bigg\{ P^F_{t+s}(f)\left(\frac{P^F_{t+s}(f)}{P^F_{t+s}}\right)^{-\sigma^F} Y^F_{t+s} - P_{t+s}\lambda^{AF}_{t+s}(f) Y^{AF}_{t+s}(f) \\ & - P^{\mathcal{N}}_{t+s} Z^{\mathcal{N}F}_{t+s}(f) - P^F_{t+s}\left(\frac{\phi_{PF}}{2} Y^F_{t+s}\left(\frac{P^F_{t+s}(f)}{P^F_{t+s-1}(f)} \div \frac{P^F_{t+s-1}}{P^F_{t+s-2}} - 1\right)^2 + T_{t+s}\varpi^F\right) \\ & - P_{t+s}\lambda^F_{t+s}(f)\left[\left(\frac{P^F_{t+s}(f)}{P^F_{t+s}}\right)^{-\sigma^F} Y^F_{t+s} - \mathcal{F}^F_{t+s}\left(Y^{AF}_{t+s}(f), Z^{\mathcal{N}F}_{t+s}(f)\right)\right]\Bigg\}\end{aligned}$$

where $P_{t+s}\lambda^F_{t+s}(f)$ reflects the (nominal) marginal cost of producing an additional unit of the final good and $\mathcal{F}^F_{t+s}(Y^{AF}_{t+s}(f), Z^{\mathcal{N}F}_{t+s}(f))$ is the production function defined in (169). For latter reference we compute the marginal product of assembled goods and nontradable goods

$$\frac{\partial \mathcal{F}^F_t}{\partial Y^{AF}_t(f)} = \left(\frac{\left(1-\alpha^F_Z\right) Y^F_t(f)}{Y^{AF}_t(f)}\right)^{\frac{1}{\xi_F}}$$

$$\frac{\partial \mathcal{F}^F_t}{\partial Z^{\mathcal{N}F}_t(f)} = \left(\frac{\alpha^F_Z Y^F_t(f)}{Z^{\mathcal{N}F}_t(f)}\right)^{\frac{1}{\xi_F}}$$

The first-order conditions are as follows (we drop the expected value operator).

1. Optimal pricing rule

$$\begin{aligned}\frac{\partial \mathcal{L}(\cdot)}{\partial P^F_t(f)} = 0 \Leftrightarrow & \left(1-\tau^D_t\right)\left[\left(1-\sigma^F\right)\left(\frac{P^F_t(f)}{P^F_t}\right)^{-\sigma^F} Y^F_t - \phi_{PF} Y^F_t \left(\frac{\frac{P^F_t(f)}{P^F_{t-1}(f)}}{\frac{P^F_{t-1}}{P^F_{t-2}}} - 1\right)\left(\frac{\frac{P^F_t}{P^F_{t-1}(f)}}{\frac{P^F_{t-1}}{P^F_{t-2}}}\right)\right] \\ & + P_t\lambda^F_t(f)\left(1-\tau^D_t\right)\sigma^F \frac{\left(P^F_t(f)\right)^{-\sigma^F-1}}{\left(P^F_t\right)^{\sigma^F}} Y^F_t \\ & + \frac{\theta}{i_t}\left(1-\tau^D_{t+1}\right) P^F_{t+1}\phi_{PF} Y^F_{t+1}\left(\frac{\frac{P^F_{t+1}(f)}{P^F_t(f)}}{\frac{P^F_t}{P^F_{t-1}}} - 1\right)\left(\frac{\frac{P^F_{t+1}(f)}{\left(P^F_t(f)\right)^2}}{\frac{P^F_t}{P^F_{t-1}}}\right) = 0\end{aligned}$$

where we used the fact that $\tilde{R}_{t,1} = \theta/i_t$. In equilibrium the indexer f can be dropped and the previous expression collapses to

$$\begin{aligned}&\left(1-\tau^D_t\right)\left[\left(1-\sigma^F\right) Y^F_t - \phi_{PF} Y^F_t\left(\frac{\pi^F_t}{\pi^F_{t-1}} - 1\right)\frac{\pi^F_t}{\pi^F_{t-1}}\right] + \frac{\lambda^F_t}{p^F_t}\left(1-\tau^D_t\right)\sigma^F Y^F_t \\ &\qquad + \frac{\theta}{i_t}\left(1-\tau^D_{t+1}\right)\pi^F_{t+1}\phi_{PF} Y^F_{t+1}\left(\frac{\pi^F_{t+1}}{\pi^F_t} - 1\right)\frac{\pi^F_{t+1}}{\pi^F_t} = 0 \qquad (15.1)\end{aligned}$$

where we used the facts $p^F_t = P^F_t/P_t$ and $\pi^F_t = P^F_t/P^F_{t-1}$.

2. Assembled good utilization

$$\frac{\partial \mathcal{L}(\cdot)}{\partial Y_t^{AF}(f)} = 0 \Leftrightarrow \lambda_t^{AF}(f) = \lambda_t^F(f)\frac{\partial \mathcal{F}_t^F}{\partial Y_t^{AF}(f)} \Leftrightarrow \frac{\lambda_t^{AF}(f)}{\lambda_t^F(f)} = \left(\frac{\left(1-\alpha_Z^F\right)Y_t^F(f)}{Y_t^{AF}(f)}\right)^{\frac{1}{\xi_F}} \Leftrightarrow$$

$$\Leftrightarrow Y_t^{AF}(f) = (1-\alpha_Z^F)\left(\frac{\lambda_t^{AF}(f)}{\lambda_t^F(f)}\right)^{-\xi_F} Y_t^F(f)$$

In equilibrium

$$Y_t^{AF} = (1-\alpha_Z^F)\left(\frac{\lambda_t^{AF}}{\lambda_t^F}\right)^{-\xi_F} Y_t^F \tag{15.2}$$

3. Nontradable good demand

$$\frac{\partial \mathcal{L}_t^F}{\partial Z_t^{\mathcal{N}F}(f)} = 0 \Leftrightarrow P_t^{\mathcal{N}} = P_t\lambda_t^F(f)\frac{\partial \mathcal{F}_t^F}{\partial Z_t^{\mathcal{N}F}(f)} \Leftrightarrow \frac{p_t^{\mathcal{N}}}{\lambda_t^F(f)} = \left(\frac{\alpha_Z^F Y_t^F(f)}{Z_t^{\mathcal{N}F}(f)}\right)^{\frac{1}{\xi_F}} \Leftrightarrow$$

$$\Leftrightarrow Z_t^{\mathcal{N}F}(f) = \alpha_Z^F\left(\frac{p_t^{\mathcal{N}}}{\lambda_t^F(f)}\right)^{-\xi_F} Y_t^F(f)$$

In equilibrium

$$Z_t^{\mathcal{N}F} = \alpha_Z^F\left(\frac{p_t^{\mathcal{N}}}{\lambda_t^F}\right)^{-\xi_F} Y_t^F \tag{15.3}$$

nontradable goods. In the steady state

$$\check{Y}^{AF} = \left(1-\alpha_Z^F\right)\left(\frac{\lambda^{AF}}{\lambda^F}\right)^{-\xi_F} \check{Y}^F \tag{183}$$

Equation (15.3) in stationary form is

$$\check{Z}_t^{\mathcal{N}F} = \alpha_Z^F\left(\frac{p_t^{\mathcal{N}}}{\lambda_t^F}\right)^{-\xi_F} \check{Y}_t^F \tag{184}$$

The interpretation of (184) is analogous to that of (182). In the steady state

$$\check{Z}^{\mathcal{N}F} = \alpha_Z^F\left(\frac{p^{\mathcal{N}}}{\lambda^F}\right)^{-\xi_F} \check{Y}^F \tag{185}$$

3.6 Fiscal and monetary policy authorities

The government buys from distributors a particular consumption good, G_t, and performs lump-sum transfers to households, TRG_t. To finance expenditures, the government levies taxes on labor income, $\tau_t^{\mathcal{L}}$ and $\tau_t^{\mathcal{SP}}$, households' consumption, $\tau_t^{\mathcal{C}}$, and firms' dividends, $\tau_t^{\mathcal{K}}$ and $\tau_t^{\mathcal{D}}$; and receives transfers from abroad, TRE_t. The government issues debt to finance expenditures not covered by revenues, according to a fiscal rule that binds debt to a stationary path.

Government consumption operates as a pure inefficient good that does not affect agent decisions or welfare. This simplifying setup naturally ignores many of the government roles in the economy. First, the role as an employer is neglected. Second, the government is not allowed to invest in public goods or in capital goods. The latter might complement private capital, interacting with the production function of manufacturers. Third, the absence of unemployment precludes the existence of unemployment benefits. These are important simplifications to keep the model tractable, but they suffice to generate important debt dynamics and to analyze the response of public debt to shocks.

Several government variables—namely government consumption, lump-sum transfers to households and tax rates—follow autoregressive processes with similar specifications. Naturally, net government transfers to both household types must add total government transfers, *i.e.* $\check{trg}_t = \check{trg}_t^{\mathcal{A}} + \check{trg}_t^{\mathcal{B}}$. It should be noted that *PESSOA* may consider transfers from type-$\mathcal{A}$ to type-$\mathcal{B}$ households.

Government revenues arise from three major sources: consumption, labor, and dividends. The real revenue accruing to the government from consumption taxes is, in stationary form

$$\check{rv}_t^{\mathcal{C}} = \tau_t^{\mathcal{C}} \cdot (p_t^{\mathcal{C}} \check{C}_t) \tag{186}$$

and in the steady state

$$\check{rv}^{\mathcal{C}} = \tau^{\mathcal{C}} \cdot (p^{\mathcal{C}} \cdot \check{C}) \tag{187}$$

Revenues associated with employees labor taxes arise from two sources: households and labor unions. The households' tax base is $\check{w}_t U_t$, whereas the unions' tax base is $(\check{v} - \check{w})U_t - \check{\Gamma}_t^V$. Adding these two amounts yields

$$\check{rv}_t^{\mathcal{L}} = \tau_t^{\mathcal{L}} \cdot (\check{v}_t U_t - \check{\Gamma}_t^V) \tag{188}$$

Since labor unions' profits are distributed to households, the tax base equals the overall wage received by households for their labor supply U_t net of adjustment costs paid by unions. In the steady state with zero adjustment costs the employees labor tax revenue is

$$\check{rv}^{\mathcal{L}} = \tau^{\mathcal{L}} \cdot (\check{v} \cdot U) \tag{189}$$

Labor taxes also include manufacturers' social security contributions

$$r\check{v}_t^{\mathcal{SP}} = \tau_t^{\mathcal{SP}} \cdot (\check{v}_t U_t) \tag{190}$$

The steady-state version is

$$r\check{v}^{\mathcal{SP}} = \tau^{\mathcal{SP}} \cdot (\check{v} \cdot U) \tag{191}$$

In stationary form, tax revenues accruing from capital taxes on manufacturers are

$$\tau_t^{\mathcal{K}} \sum_{J \in \{\mathcal{T},\mathcal{N}\}} \Big[p_t^J \big(\check{Z}_t^J - \check{\Gamma}_t^{PJ} - \varpi^J \big) - r_t^{\mathcal{K}J} \check{K}_t^J - (1 + \tau_t^{\mathcal{SP}}) \check{v}_t \big(U_t^J + \Gamma_t^{\mathcal{U}J} \big) \Big]$$

and from entrepreneurs

$$\tau_t^{\mathcal{K}} \sum_{J \in \{\mathcal{T},\mathcal{N}\}} \Big[r_t^{\mathcal{K}J} \check{K}_t^J - \big(a(u_t^J) + \delta_t^J p_t^{\mathcal{K}J} \big) \check{\bar{K}}_t^J \Big]$$

This yields a revenue from taxes on capital of

$$r\check{v}_t^{\mathcal{K}} = \tau_t^{\mathcal{K}} \sum_{J \in \{\mathcal{T},\mathcal{N}\}} \Big[p_t^J \big(\check{Z}_t^J - \check{\Gamma}_t^{PJ} - \varpi^J \big) - (1 + \tau_t^{\mathcal{SP}}) \check{v}_t \big(U_t^J + \Gamma_t^{\mathcal{U}J} \big) - \big(a(u_t^J) + \delta_t^J p_t^{\mathcal{K}J} \big) \check{\bar{K}}_t^J \Big] \tag{192}$$

In the steady state with zero adjustment costs

$$r\check{v}^{\mathcal{K}} = \tau^{\mathcal{K}} \sum_{J \in \{\mathcal{T},\mathcal{N}\}} \Big[p^J (\check{Z}^J - \varpi^J) - (1 + \tau^{\mathcal{SP}}) \check{v} \cdot U^J - \delta_t^J p^{\mathcal{K}J} \check{\bar{K}}_t^J \Big] \tag{193}$$

Finally, from equation (176) it follows that tax revenues accruing from distributors are

$$r\check{v}_t^{\mathcal{D}} = \tau_t^{\mathcal{D}} \sum_{F \in \{\mathcal{C},\mathcal{I},\mathcal{G},\mathcal{X}\}} \Big[p_t^F \big(\check{Y}_t^F - \check{\Gamma}_t^{PF} - \varpi^F \big) - p_t^{\mathcal{T}} \check{Z}_t^{\mathcal{T}F} - \epsilon_t \check{M}_t^F - p_t^{\mathcal{N}} \check{Z}_t^{\mathcal{N}F} \Big] \tag{194}$$

The steady-state version is

$$r\check{v}^{\mathcal{D}} = \tau^{\mathcal{D}} \sum_{F \in \{\mathcal{C},\mathcal{I},\mathcal{G},\mathcal{X}\}} \Big[p^F (\check{Y}^F - \varpi^F) - p^{\mathcal{T}} \check{Z}^{\mathcal{T}F} - \epsilon \check{M}^F - p^{\mathcal{N}} \check{Z}^{\mathcal{N}F} \Big] \tag{195}$$

Transfers from abroad, TRE_t, are assumed fully exogenous. Government's revenue can therefore be expressed in stationary form as

$$\check{rv}_t = \sum_{\substack{x \in \{\mathcal{C},\mathcal{L},\\ \mathcal{SP},\mathcal{K},\mathcal{D}\}}} \check{rv}_t^x + \check{tre}_t \tag{196}$$

and in the steady state

$$\check{rv} = \sum_{\substack{x \in \{\mathcal{C},\mathcal{L},\\ \mathcal{SP},\mathcal{K},\mathcal{D}\}}} \check{rv}^x + \check{tre} \tag{197}$$

The government's budget constraint states that the amount of government expenditures not covered by revenues must be financed by issuing debt. In nominal terms this can be represented by

$$B_t = i_{t-1} B_{t-1} + P_t^{\mathcal{G}} G_t + TRG_t - RV_t$$

Dividing by $P_t T_t$ yields the government's budget constraint in stationary form

$$\check{b}_t = \frac{i_{t-1}}{\pi_t \cdot g} \check{b}_{t-1} + p_t^{\mathcal{G}} \check{G}_t + \check{trg}_t - \check{rv}_t \tag{198}$$

Using $i = \pi \cdot r$, the steady-state version can be expressed as

$$\check{b} = \frac{r}{g} \check{b}_{t-1} + p_t^{\mathcal{G}} \check{G}_t + \check{trg}_t - \check{rv}_t \tag{199}$$

To ensure that public debt follows a non-explosive path, we introduce a fiscal rule, imposing a convergence of the government surplus–to–GDP ratio, $\check{sg}_t / \check{GDP}_t$, and consequently of the debt–to–GDP ratio, $\check{b}_t / \check{GDP}_t$, to a predetermined (target) steady-state level. The fiscal rule implies that at least one fiscal instrument must adjust endogenously. This choice is purely arbitrary and any variable fully controlled by the government can play this role, but the literature tends to set the labor income tax as the endogenous fiscal instrument (Harrison et al., 2005; Kilponen and Ripatti, 2006; Kumhof and Laxton, 2007; Kumhof et al., 2010). In line with Kumhof and Laxton (2009a), the following fiscal rule in stationary form is considered

$$\frac{\check{sg}_t}{\check{GDP}_t} = \left(\frac{\check{sg}_t}{\check{GDP}_t}\right)^{\text{target}} + d_1 \left(\frac{\check{rv}_t - \check{rv}_t^{\text{ss}}}{\check{GDP}_t}\right) + d_2 \left(\frac{\check{b}_t}{\check{GDP}_t} - \left(\frac{\check{b}_t}{\check{GDP}_t}\right)^{\text{target}}\right) + d_3 \, ln \left(\frac{GDP_t}{GDP_t^\star}\right) \tag{200}$$

where $\check{rv}_t^{\text{ss}}$ is the structural overall tax revenue—collected when tax bases stand at their steady-state levels—in stationary form; $\check{GDP}_t$ and $\check{GDP}_t^{\text{ss}}$ are respectively observed and steady-state GDP levels in stationary form, and $GDP_t^\star$ represents potential output, defined as a moving average of past realizations. In the steady state with $\check{rv}_t = \check{rv}_t^{\text{ss}}$, $\check{b}_t = \check{b}_t^{\text{target}}$, and $GDP_t = GDP_t^{\star}$ the fiscal rule collapses to

$$\frac{\check{sg}}{G\check{D}P} = \left(\frac{\check{sg}}{G\check{D}P}\right)^{\text{target}} \tag{201}$$

The fiscal policy rule takes into consideration government debt stabilization and business cycle smoothness, adjusting automatically the endogenous tax rate to bind debt to a stationary path. The parameter d_1 reflects how the government adjusts the surplus-to-GDP ratio to cyclical economic fluctuations. For instance, if $d_1 = 1$, the fiscal rule has an intrinsic structural nature. During a boom, when tax revenues are above the long run level, the government increases surplus above target, paying off debt. During a slump, debt increases to allow for a government surplus below the long run value. In this case, the fiscal rule mimics the automatic stabilizer mechanism, minimizing the variability of the tax instrument and reducing output fluctuations throughout the cycle *vis-á-vis* a balance budget rule ($d_1 = 0$). Setting $d_1 < 0$ yields a counter-cyclical fiscal policy rule (*i.e.* a pro-cyclical government deficit), in which the government decreases surplus during booms and *vice-versa*. On the opposite direction, setting $d_1 > 1$ yields an aggressive stabilization policy in which surplus is adjusted to minimize cyclical fluctuations. The parameter d_2 reflects the degree of aversion to deviations of the debt-to-GDP ratio from the target value. Finally, parameter d_3 determines the aggressiveness of the fiscal rule to the output gap.

Monetary policy is fully exogenous, since interest rates are set by an external monetary authority and the home economy is assumed small enough to affect macroeconomic aggregates in a MU.

3.7 Rest of the world

The home country imports differentiated goods M_t^F at the price P_t^*, to be used in the distributor's assemblage stage. The imported goods' inflation rate, $\pi_t^* = P_t^*/P_{t-1}^*$, is assumed to follow an exogenous process.

In domestic currency the distributor pays $\epsilon_t = p_t^* \varepsilon_t$ for each unit of the imported good relative to the *numeráire* of the economy, which equals the real exchange rate by definition. Thus, the growth rate of imported goods' inflation can be expressed as

$$\frac{\epsilon_t}{\epsilon_{t-1}} = \frac{p_t^*}{p_{t-1}^*} \frac{\varepsilon_t}{\varepsilon_{t-1}}$$

or equivalently

$$\frac{\epsilon_t}{\epsilon_{t-1}} = \frac{\pi_t^*}{\pi_t} \frac{\varepsilon_t}{\varepsilon_{t-1}} \tag{202}$$

In the steady-state

$$\pi^* = \pi \tag{203}$$

Exports are essentially determined by foreign demand and price competitiveness. In a multi-country model, exports demand is endogenously determined, depending on relative comparative advantages amongst countries. In a single country small open economy model the rest of the world is not explicitly modeled, and thus one must find alternate strategies to determine exports demand. We follow closely Adolfson et al. (2007a) and assume that in the rest of the world there exists a continuum of distributors $m \in [0, 1]$, who demand $Y_t^{\mathcal{X}}(m)$ units of the final good from domestic distributors operating in the exports sector, to be used in their assemblage stage. This good is thereafter combined with foreign tradable goods $Z_t^{\mathcal{T}*}(m)$ according to the following production function

$$Y_t^{A*}(m) \equiv \mathcal{F}\big(Z_t^{\mathcal{T}*}, Y_t^{\mathcal{X}}(m)\big) = \Bigg(\big(\alpha_Y^{A*}\big)^{\frac{1}{\xi_{A*}}} \big(Y_t^{\mathcal{X}}(m)\big)^{\frac{\xi_{A*}-1}{\xi_{A*}}} + \big(1-\alpha_Y^{A*}\big)^{\frac{1}{\xi_{A*}}} \big(Z_t^{\mathcal{T}*}(m)\big)^{\frac{\xi_{A*}-1}{\xi_{A*}}} \Bigg)^{\frac{\xi_{A*}}{\xi_{A*}-1}} \tag{204}$$

yielding $Y_t^{A*}(m)$ units of the assembled good. The home economy is assumed to be sufficiently small so that domestic shocks do not affect the rest of the world. The element ξ_{A*} is the elasticity of substitution between rest of the world tradable goods and domestic exports, and α^{A*} is the rest of the world home bias parameter.

Each foreign distributor selects the quantities $\{Y_t^{\mathcal{X}}(m), Z_t^{\mathcal{T}*}(m)\}$ in each period so as to minimize the cost of producing the assembled good, subject to the production function in (204). The problem is solved in Box 15, yielding the following demand for domestic goods in stationary form

$$Y_t^{\mathcal{X}} = \alpha_Y^{A*} \left(\frac{p_t^{\mathcal{X}}}{\epsilon_t} \right)^{-\xi_{A*}} Y_t^{A*} \tag{205}$$

In the steady state

$$Y^{\mathcal{X}} = \alpha_Y^{A*} \left(\frac{p^{\mathcal{X}}}{\epsilon} \right)^{-\xi_{A*}} Y^{A*} \tag{206}$$

An important condition resulting from interactions with the rest of the world is the net foreign asset position

$$\varepsilon_t B_t^* = i_{t-1}^* \Psi_{t-1} \varepsilon_{t-1} B_{t-1}^* + P_t^{\mathcal{X}} X_t - \varepsilon_t P_t^* M_t + \varepsilon_t \big(TRE_t + TRX_t \big)$$

Box 15: The foreign distributor's demand for export goods.

The foreign distributor selects the quantities $\{Y_t^{\mathcal{X}}(m), Z_t^{\mathcal{T}*}(m)\}$ in each period that minimize the cost of producing a given amount of the assembled good

$$\min_{Y_t^{\mathcal{X}}(m), Z_t^{\mathcal{T}*}(m)} P_t^{\mathcal{T}*} Z_t^{\mathcal{T}*}(m) + \frac{P_t^{\mathcal{X}}}{\varepsilon_t} Y_t^{\mathcal{X}}(m)$$

subject the production technology in (204). The Lagrangian is

$$\mathcal{L}(\cdot) = P_t^{\mathcal{T}*} Z_t^{\mathcal{T}*}(m) + \frac{P_t^{\mathcal{X}}}{\varepsilon_t} Y_t^{\mathcal{X}}(m) - \lambda_t^{A*}(m) \Bigg[\Bigg(\left(\alpha_Y^{A*}\right)^{\frac{1}{\xi_{A*}}} \left(Y_t^{\mathcal{X}}(m)\right)^{\frac{\xi_{A*}-1}{\xi_{A*}}} + \left(1 - \alpha_Y^{A*}\right)^{\frac{1}{\xi_{A*}}} \left(Z_t^{\mathcal{T}*}(m)\right)^{\frac{\xi_{A*}-1}{\xi_{A*}}} \Bigg)^{\frac{\xi_{A*}}{\xi_{A*}-1}} \Bigg]$$

where $\lambda_t^{A*}(m)$ stands for foreign distributer's marginal cost of producing an extra unit of the assembled good. We assume that foreign distributors work under perfect competition, and thus the marginal cost equals the price they charge for their product, *i.e.* $\lambda_t^{A*}(m) = P^*$, $\forall m$. The first-order condition of the optimization problem with respect to $Y_t^{\mathcal{X}}(m)$ is

$$\frac{\partial \mathcal{L}(\cdot)}{\partial Y_t^{\mathcal{X}}(m)} = 0 \Leftrightarrow \frac{P_t^{\mathcal{X}}}{\varepsilon_t} = P_t^* \left(\alpha_Y^{A*}\right)^{\frac{1}{\xi_{A*}}} \left(\frac{Y_t^{A*}(m)}{Y_t^{\mathcal{X}}(m)} \right)^{\frac{1}{\xi_{A*}}} \Leftrightarrow Y_t^{\mathcal{X}}(m) = \alpha_Y^{A*} \left(\frac{P_t^{\mathcal{X}}}{\varepsilon_t P_t^*} \right)^{-\xi_{A*}} Y_t^{A*}(m)$$

The stationary form of the net foreign asset position is

$$\epsilon_t \check{b}_t^* = \frac{i_{t-1}^* \Psi_{t-1} \epsilon_{t-1}}{\pi_t \cdot g} \check{b}_{t-1}^* + p_t^{\mathcal{X}} \check{X}_t - \epsilon_t \check{M}_t + \epsilon_t (\check{tre}_t + \check{trx}_t) \tag{207}$$

and the steady-state version is

$$\check{b}^* = \frac{p^{\mathcal{X}} X - \epsilon \check{M} + \epsilon(\check{tre} + \check{trx})}{\epsilon\left(1 - \frac{i^* \Psi}{\pi \cdot g}\right)} \tag{208}$$

3.8 Economic aggregates and market clearing conditions

We close the model by presenting economic aggregates and market clearing conditions.

Aggregate consumption. To obtain aggregate consumption, we add the respective optimal quantities of the different consumer types

$$\check{C}_t = \check{C}_t^{\mathcal{A}} + \check{C}_t^{\mathcal{B}} \tag{209}$$

In the steady state

$$\check{C} = \check{C}^{\mathcal{A}} + \check{C}^{\mathcal{B}} \tag{210}$$

Aggregate labor supply and demand. Similarly to aggregate consumption, aggregate labor supply is

$$L_t = L_t^{\mathcal{A}} + L_t^{\mathcal{B}} \tag{211}$$

In the steady state

$$L = L^{\mathcal{A}} + L^{\mathcal{B}} \tag{212}$$

In turn, aggregate labor demand is determined by manufacturers operating in the tradable and non-tradable sectors. Outside the steady state, one as to take into consideration manufacturers' adjustment costs, which are measured in terms of worked hours, even though they do not affect production (examples include training and lay-offs), and unions' adjustment costs

$$U_t = U_t^{\mathcal{T}} + U_t^{\mathcal{N}} + \Gamma_t^{\mathcal{U}\mathcal{T}} + \Gamma_t^{\mathcal{U}\mathcal{N}} + \check{\Gamma}_t^{V} \tag{213}$$

In the steady state adjustment costs are zero and thus

$$U = U^{\mathcal{T}} + U^{\mathcal{N}} \tag{214}$$

Labor market clearing condition. The labor market clearing condition is

$$L_t = U_t \tag{215}$$

which in the steady state becomes

$$L = U \tag{216}$$

Intermediate goods market clearing conditions. In the intermediate goods' market, the output produced by manufacturers net of adjustment and fixed costs, and of the entrepreneurs' output destroying net worth shock, must equal the demand from distributors

$$\check{Z}_t^J - \check{\Gamma}_t^{PJ} - r\check{c}u_t^J - \varpi^J - S_t^{J,y} = \check{Z}_t^{J\mathcal{C}} + \check{Z}_t^{J\mathcal{I}} + \check{Z}_t^{J\mathcal{G}} + \check{Z}_t^{J\mathcal{X}},\ J \in \{\mathcal{T},\mathcal{N}\} \tag{217}$$

Taking into account adjustment costs is necessary, since they correspond to real resources that are consumed in the production process. Likewise for the resource cost associated with capital utilization and the entrepreneurs' output destroying net worth shock. In the steady state

$$\check{Z}^J - \varpi^J - S^{J,y} = \check{Z}^{J\mathcal{C}} + \check{Z}^{J\mathcal{I}} + \check{Z}^{J\mathcal{G}} + \check{Z}^{J\mathcal{X}},\ J \in \{\mathcal{T},\mathcal{N}\} \tag{218}$$

where $S^{J,y}$ is the steady-state value for the entrepreneurs' output destroying net worth shock.

Final goods market clearing conditions. In the final goods sector, the output produced by distributors net of adjustment costs must equal costumers' demand

$$\check{Y}_t^F - \Gamma_t^{PF} - \mathbf{1}_{\mathcal{I}}(F)\big(\Gamma_t^{\mathcal{T}\mathcal{I}} + \Gamma_t^{\mathcal{N}\mathcal{I}}\big) - \varpi^F = \mathbf{1}_{\mathcal{C}}(F)\check{C}_t + \mathbf{1}_{\mathcal{I}}(F)\check{I}_t + \mathbf{1}_{\mathcal{G}}(F)\check{G}_t + \mathbf{1}_{\mathcal{X}}(F)\check{X}_t \tag{219}$$

where $F \in \{\mathcal{C},\mathcal{I},\mathcal{G},\mathcal{X}\}$, $I = I^{\mathcal{T}} + I^{\mathcal{N}}$, and $\mathbf{1}_x(F)$ is an indicator function which takes the value of 1 if $F \in x$ and 0 otherwise. Recall that capital goods producers face specific investment adjustment costs, whereas the remaining agents do not face similar costs. In the steady state

$$\check{Y}_t^F - \varpi^F = \mathbf{1}_{\mathcal{C}}(F)\check{C} + \mathbf{1}_{\mathcal{I}}(F)\check{I} + \mathbf{1}_{\mathcal{G}}(F)\check{G} + \mathbf{1}_{\mathcal{X}}(F)\check{X},\ F \in \{\mathcal{C},\mathcal{I},\mathcal{G},\mathcal{X}\} \tag{220}$$

Aggregate imports. To obtain total imports, we add the corresponding figures respecting final goods distributors

$$\check{M}_t = \check{M}_t^{\mathcal{C}} + \check{M}_t^{\mathcal{I}} + \check{M}_t^{\mathcal{G}} + \check{M}_t^{\mathcal{X}} \tag{221}$$

In the steady state

$$\check{M} = \check{M}^{\mathcal{C}} + \check{M}^{\mathcal{I}} + \check{M}^{\mathcal{G}} + \check{M}^{\mathcal{X}} \tag{222}$$

Final output. GDP on the expenditure side, measured in terms of P_t, can be simply defined as

$$G\check{D}P_t = \check{C}_t + p_t^{\mathcal{G}} \check{G}_t + p_t^{\mathcal{I}} \check{I}_t + p_t^{\mathcal{X}} \check{X}_t - \epsilon_t \check{M}_t \tag{223}$$

In the steady state, this equation becomes

$$G\check{D}P = \check{C} + p^{\mathcal{G}} \check{G} + p^{\mathcal{I}} \check{I} + p^{\mathcal{X}} \check{X} - \epsilon \check{M} \tag{224}$$

The definition of GDP is however not unique and alternative weighting schemes can be followed, for instance using Fisher weights.

4 The model without financial frictions

This section pinpoints the major differences between the model with and the model without financial accelerator and financial frictions. Without financial frictions, entrepreneurs and banks can be drop altogether, implying the following adjustments to the model.

4.1 Households and labor unions

Type-$\mathcal{A}$ households do not lend money to banks, and therefore $\check{b}_t^{\mathcal{T}} = \check{b}_t^{\mathcal{N}} = 0,\ \forall t$. Additionally, there are no bankruptcy monitoring services and entrepreneurs' dividends are absent, implying that $r\check{b}r_t = \check{d}_t^{\mathcal{A},\mathcal{EP}} = 0,\ \forall t$.

Labor unions behavior is identical to the model with financial frictions.

4.2 Firms

Taxes for holding capital are transferred to capital goods producers, who now produce, accumulate and rent capital to manufacturers. The remaining firms behave identically.

In addition to investment, capital goods producers now also decide capital utilization. As before, they are perfectly competitive in input and output markets, but now they pay the capital-based income tax, which was previously paid by entrepreneurs. In each period, capital goods producers supplying sector J purchase previously installed capital from manufacturing firms $\bar{K}_t^J$ and investment goods from investment goods producers I_t^J to produce new installed capital $\bar{K}_{t+1}^J(i)$, according to the same aggregate law of motion presented in (75). In equilibrium, capital accumulation is given by (76), and in the steady state by (77).

Capital goods producers face the same investment adjustment costs as before, presented in equation (66). In stationary form, these are given by (67), and in the steady state by (68). Since they also select the capital utilization rate, they face the same costs of capital utilization as entrepreneurs in the version with financial frictions, given by (78). In stationary form and in the steady state these are given respectively by (79) and (80). The capital stock effectively used in production is given by (81) in stationary form, and by (82) in the steady state.

The after-tax dividends of capital goods producers operating in sector J are defined as the difference between operational cashflows and net operational profits

$$D_t^{\mathcal{K}J}(i) = \underbrace{\left[R_t^{\mathcal{K}J} u_t^J(i) - P_t a\big(u_t^J(i)\big)\right] \bar{K}_t^J(i) - P_t^{\mathcal{I}}\left[I_t^J(i) + \Gamma_t^{\mathcal{I}J}(i)\right]}_{\text{Operational cashflows}}$$
$$-\tau_t^{\mathcal{K}}\Big\{\underbrace{\left[R_t^{\mathcal{K}J} u_t^J(i) - P_t a\big(u_t^J(i)\big) - \delta_t^J P_t^{\mathcal{K}J}\right] \bar{K}_t^J(i) - P_t^{\mathcal{I}}\Gamma_t^{\mathcal{I}J}(i)}_{\text{Net operational profits}}\Big\} \tag{225}$$

where $\tau_t^{\mathcal{K}}$ is the capital income tax rate, $P_t^{\mathcal{I}}$ is the price of investment goods, and $P_t^{\mathcal{K}J}$ is the price of capital. The price of old and new capital is the same, since the marginal rate of transformation between the two is assumed to be one.

The expression in (225) is designed to mimic the Portuguese framework. Operational cashflows include the overall revenue minus the overall expenditure. However, for fiscal purposes, the relevant concept is net operational profits, which take into consideration that investment is not considered for the tax base and that depreciation is tax deductible. In real terms, the stationary form of equation (225) is

$$\check{d}_t^{\mathcal{K}J} = (1-\tau_t^{\mathcal{K}})\left[\big(r_t^{\mathcal{K}J} u_t^J - a(u_t^J)\big)\check{\bar{K}}_t^J - p_t^{\mathcal{I}}\Gamma_t^{\mathcal{I}J}\right] - p_t^{\mathcal{I}}\check{I}_t^J + \tau_t^{\mathcal{K}}\delta_t^J p_t^{\mathcal{K}J}\check{\bar{K}}_t^J \tag{226}$$

In the steady-state, with $u_t^J = 1$ and $a(1) = \Gamma_t^{\mathcal{I}J} = 0$, dividends become

$$\check{d}^{\mathcal{K}J} = (1-\tau^{\mathcal{K}}) r^{\mathcal{K}J}\check{\bar{K}}^J - p^{\mathcal{I}}\check{I}^J + \tau^{\mathcal{K}}\delta^J p^{\mathcal{K}J}\check{\bar{K}}^J \tag{227}$$

Capital goods producers select the intertemporal profile $\{I_{t+s}^J(i), \bar{K}_{t+s+1}^J(i), u_t^J(i)\}_{s=0}^{\infty}$ that maximizes the present discounted value of the dividends stream, subject to investment adjustment costs in (66), to the law of motion of capital in (75), and to capital utilization costs in (78). The problem is presented and solved in Box 16.

The stationary form of equation (17.1) is

$$p_t^{\mathcal{K}J} = \frac{\theta}{r_t}\left[(1-\tau_{t+1}^{\mathcal{K}})\Big(r_{t+1}^{\mathcal{K}J} u_{t+1}^J - a\big(u_{t+1}^J\big)\Big) + p_{t+1}^{\mathcal{K}J}\Big(1-\delta_{t+1}^J(1-\tau_{t+1}^{\mathcal{K}})\Big)\right] \tag{228}$$

where we used the definition of real interest rate $r_t = i_t/\pi_{t+1}$ and the expressions for relative prices. In the steady state $u_{t+1}^J = 1$ and $a(1) = 0$. Additionally, investment goods inflation equals total inflation. Therefore, equation (228) simplifies to

$$p^{\mathcal{K}J}\left[1 - \frac{\theta}{r}\Big(1-\delta^J\big(1-\tau^{\mathcal{K}}\big)\Big)\right] = \frac{\theta}{r}\big(1-\tau^{\mathcal{K}}\big) r^{\mathcal{K}J}$$

Box 16: Capital goods producers maximization problem

The capital goods producer select the triplet $\{I^J_{t+s}(i), \bar{K}^J_{t+s+1}(i), u^J_t(i)\}_{s=0}^{\infty}$ that maximizes the present discounted value of the dividends stream, subject to investment adjustment costs in (66), to the law of motion of capital in (75), and to capital utilization costs in (78). The Lagrangian can be written as

$$\mathcal{L}(\cdot) = \mathrm{E}_t \sum_{s=0}^{\infty} \tilde{R}_{t,s} \Bigg\{ (1-\tau^{\mathcal{K}}_{t+s}) \Bigg\{ R^{\mathcal{K}J}_{t+s} u^J_{t+s}(i) \bar{K}^J_{t+s}(i) - P_{t+s} a\big(u^J_{t+s}(i)\big) \bar{K}^J_{t+s}(i)$$
$$-P^{\mathcal{I}}_{t+s} \left[\frac{\phi_{\mathcal{I}J}}{2} I^J_{t+s} \left(\frac{I^J_{t+s}(i)/g}{I^J_{t+s-1}(i)} - 1 \right)^2 \right] \Bigg\} - P^{\mathcal{I}}_{t+s} I^J_{t+s}(i) + \tau^{\mathcal{K}}_{t+s} \delta^J_{t+s} P^{\mathcal{K}J}_{t+s} \bar{K}_{t+s}(i) -$$
$$-\lambda^J_{t+s} \left(\bar{K}^J_{t+s+1}(i) - (1-\delta^J_{t+s}) \bar{K}^J_{t+s}(i) - \zeta^{\mathcal{I}}_{t+s} I^J_{t+s}(i) \right) \Bigg\}$$

where the Lagrange multiplier λ^J_{t+s} corresponds to the marginal cost of buying an extra unit of capital, *i.e.* $\lambda^J_{t+s} = P^{\mathcal{K}J}_{t+s}$. The first-order conditions are as follows (we drop the expected value operator).

1. **Capital supply**

$$\frac{\partial \mathcal{L}_t(\cdot)}{\partial \bar{K}^J_{t+1}(i)} = 0 \Leftrightarrow P^{\mathcal{K}J}_t = \frac{\theta}{i_t}(1-\tau^{\mathcal{K}}_{t+1}) \big[R^{\mathcal{K}J}_{t+1} u^J_{t+1}(i) - P_{t+1} a\big(u^J_{t+1}(i)\big) \big] + \tau^{\mathcal{K}}_{t+1} \delta^J_{t+1} P^{\mathcal{K}J}_{t+1} + P^{\mathcal{K}J}_{t+1}(1-\delta^J_{t+1}) \tag{17.1}$$

where we used the facts $\tilde{R}_{t,1} = \theta / i_t$ and $\lambda^J_{t+s} = P^{\mathcal{K}J}_{t+s}$.

2. **Investment demand**

$$\frac{\partial \mathcal{L}_t(\cdot)}{\partial I^J_t(i)} = 0 \Leftrightarrow P^{\mathcal{K}J}_t \zeta^I_t = P^{\mathcal{I}}_t + (1-\tau^{\mathcal{K}}_t) P^{\mathcal{I}}_t \phi_{\mathcal{I}J} I^J_t \left(\frac{I^J_t(i)/g}{I^J_{t-1}(i)} - 1 \right) \left(\frac{1/g}{I^J_{t-1}(i)} \right) - \frac{\theta}{i_t}(1-\tau^{\mathcal{K}}_{t+1}) P^{\mathcal{I}}_{t+1} \phi_{\mathcal{I}J} I^J_{t+1} \left(\frac{I^J_{t+1}(i)/g}{I^J_t(i)} - 1 \right) \left(\frac{I^J_{t+1}(i)/g}{\big(I^J_t(i)\big)^2} \right) \tag{17.2}$$

3. **Optimal capacity utilization**

$$\frac{\partial \mathcal{L}_t(\cdot)}{\partial u^J_t(i)} = 0 \Leftrightarrow R^{\mathcal{K}J}_t = P_t \frac{\partial a}{\partial u^J_t(i)} \Leftrightarrow R^{\mathcal{K}J}_t = P_t \big[\phi^J_a \sigma^J_a u^J_t(i) + \phi^J_a (1-\sigma^J_a) \big] \tag{17.3}$$

where we used the expression for $\mathrm{d}a/\mathrm{d}u^J_t$ presented in Box 7.

and hence

$$p^{\mathcal{K}J} = \frac{\big(1-\tau^{\mathcal{K}}\big) r^{\mathcal{K}J}}{r/\theta - \big(1 - \delta^J(1-\tau^{\mathcal{K}})\big)} \tag{229}$$

Equation (17.2) in stationary form becomes

$$\frac{p^{\mathcal{K}J}_t \zeta^{\mathcal{I}}_t}{p^{\mathcal{I}}} = 1 + (1-\tau^{\mathcal{K}}_t) \phi_{\mathcal{I}J} \left(\frac{\check{I}^J_t}{\check{I}^J_{t-1}} - 1 \right) \left(\frac{\check{I}^J_t}{\check{I}^J_{t-1}} \right) - \frac{\theta \cdot g}{r_t} (1-\tau^{\mathcal{K}}_{t+1}) \phi_{\mathcal{I}J} \frac{\pi^I_{t+1}}{\pi_{t+1}} \left(\frac{\check{I}^J_{t+1}}{\check{I}^J_t} - 1 \right) \left(\frac{\check{I}^J_{t+1}}{\check{I}^J_t} \right)^2 \tag{230}$$

where we used the relationship in (178) forwarded one period. Investment is constant in the steady state and hence

$$p^{\mathcal{K}} = p^{\mathcal{I}} \tag{231}$$

The investment demand equation relates the real price of capital with investment specific shocks and with investment adjustment costs. As $p_t^{\mathcal{K}J}$ deviates from $p_t^{\mathcal{I}}$, capital goods producers become more prone to adjust capital and face the corresponding adjustment costs. More specifically, a value of $p_t^{\mathcal{K}J}/p_t^{\mathcal{I}}$ above (below) unity means that new capital goods are relatively cheaper (more expensive) *vis-á-vis* existing capital, and thus capital investment—and consequently the investment to capital ratio—should increase (decrease).

Equation (17.3) in stationary form becomes

$$r_t^{\mathcal{K}J} = \phi_a^J \sigma_a^J u_t^J + \phi_a^J (1 - \sigma_a^J) \tag{232}$$

Thus, the rate of return on capital equals the marginal cost of using an additional unit of the installed capital. In the steady-state $u^J = 1$ and

$$r^{\mathcal{K}J} = \phi_a^J \tag{233}$$

Finally, the resource cost associated with variable capital utilization is given by (86) in stationary form, and by (87) in the steady state.

4.3 Fiscal and monetary policy authorities

The government's behavior is identical to the model with financial friction, with the exception of capital tax revenues, which now incorporate investment adjustment costs faced by capital goods producers

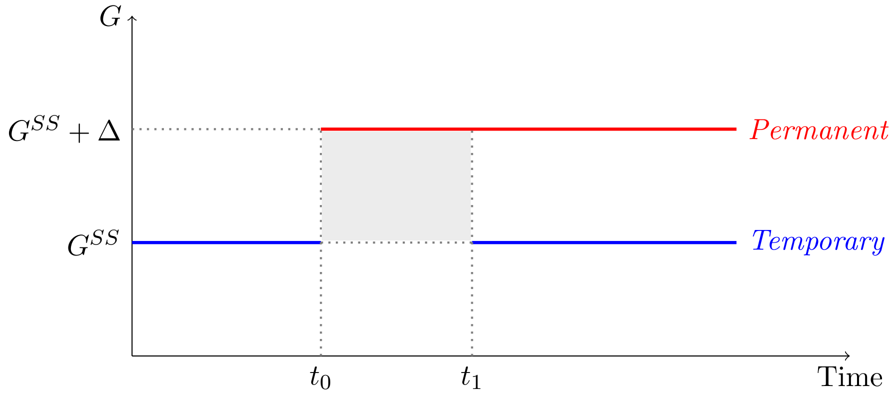
$$r\check{v}_t^{\mathcal{K}} = \tau_t^{\mathcal{K}} \sum_{J \in \{\mathcal{T},\mathcal{N}\}} \Big[p_t^J \big(\check{Z}_t^J - \check{\Gamma}_t^{PJ} - \varpi^J \big) - (1 + \tau_t^{\mathcal{SP}}) \check{v}_t \big(U_t^J + \Gamma_t^{\mathcal{U}J} \big) - \big(a(u_t^J) + \delta_t^J p_t^{\mathcal{K}J} \big) \check{\bar{K}}_t^J - p_t^{\mathcal{I}} \Gamma_t^{\mathcal{I}\mathcal{J}} \Big] \tag{234}$$

In the steady state with zero adjustment costs revenues collapse to

$$r\check{v}^{\mathcal{K}} = \tau^{\mathcal{K}} \sum_{J \in \{\mathcal{T},\mathcal{N}\}} \Big[p^J (\check{Z}^J - \varpi^J) - (1 + \tau^{\mathcal{SP}}) \check{v} \cdot U^J - \delta_t^J p^{\mathcal{K}J} \check{\bar{K}}_t^J \Big] \tag{235}$$

which is identical to (193).

Figure 4: Fiscal stimulus based on G



4.4 Market clearing conditions

Market clearing conditions are identical to the model without financial frictions, with the exception that output destroying net worth shocks, $S_t^{J,y}$, are absent and hence they must be dropped from equations (217) and (218).

5 Shocks

Any exogenous parameter of a dynamic general equilibrium model can be turned into a stochastic variable, through an appropriate exogenous process. One possibility behind the law of motion of a shock is to consider autoregressive formulations. In this case we allow for two different possibilities, depending on the parameter. The first is based on the level of the variable, say x_t

$$x_t = (1-\rho)\bar{x} + \rho x_{t-1} + e_t^x$$

where $\bar{x}$ is the steady-state value, ρ is the persistence parameter, and e_t^x stands for a time t i.i.d. zero-mean innovation. This specification is used when variables are expressed as percentages (*e.g.* tax rates), or when innovations are better interpreted as shocks to the level variable (*e.g.* elasticities of substitution or entrepreneurs' riskiness). The second is based on the logarithm of the variable, say $\log x_t$

$$\log x_t = (1-\rho)\log\bar{x} + \rho\log x_{t-1} + e_t^x$$

This specification is preferred for variables expressed in levels where innovations should be interpreted as percentage changes (*e.g.* labour augmenting technological shocks). In the appendix we present the specific formulation for several shocks that can be introduced in *PESSOA*.

Another possibility defining the exogenous process of a shock is to consider pre-determined paths that are not captured by autoregressive formulations. This is exemplified in Figure 4. In this case, we consider both a temporary and permanent increase in government consumption G. The starting date t_0 is the period when the shock is initiated; t_1 is the ending date if the stimulus is temporary; G^{SS} is the steady-state level of government consumption; and Δ is the amplitude of the shock. The macroeconomic impacts of these exogenous processes have been discussed in Almeida, Castro, Félix, and Maria (2010a).

6 Calibration

PESSOA is calibrated to match Portuguese and euro-area economies. Some parameters are exogenously set by taking into consideration common options in the literature, available historical data or empirical evidence. Others are endogenously determined within the model, with the objective of matching desired features, for instance the consumption- or investment-to-GDP ratios. Table 1 in the appendix reports the main steady-state ratios, whilst Table 2 and Table 3 present the model's non-financial and financial key parameters, respectively. A comparison with actual data is included when available.

The annual growth rate of the labor-augmenting productivity is set to 2 percent, which is a plausible estimate for potential output growth in both Portugal and the euro area (Almeida and Félix, 2006; Musso and Westermann, 2005; Proietti and Musso, 2007). Steady-state inflation stands at 2 percent per year and the euro area nominal interest rate at 4.5 percent (Coenen, McAdam, and Straub, 2007). The risk premium is nil, implying $\Psi = 1$. The elasticity of substitution between rest of the world tradable goods and domestic exports is assumed to be large.

Households parameters are largely based on Fagan, Gaspar, and Pereira (2004), Harrison et al. (2005), Kumhof and Laxton (2007) and Kumhof et al. (2010). Consumption shares, $\eta^{\mathcal{A}}$ and $\eta^{\mathcal{B}}$, are calibrated to ensure a unitary elasticity of labor supply to real wage. The intertemporal elasticity of substitution is set to 0.5, implying a coefficient of relative risk aversion of 2. The degree of habit persistence is 0.7 and the discount factor roughly 0.99. The instant probability of death, θ, and the productivity decay rate, χ, are assumed to be identical, implying an average lifetime and an expected working life of 25 years. The share of hand-to-mouth households is 40 percent, broadly in line with the estimates for Portugal presented in Castro (2006).

The steady-state wage markup of labor unions is 25 percent, which is relatively high for the euro area, but in line with the figures for Portugal. Wage adjustment costs imply that wages take roughly 5 quarters to adjust to the new equilibrium, a value in line with euro area estimates published in Coenen, McAdam, and Straub (2007).

The depreciation rate of capital is assumed to be identical in the tradable and non-tradable sectors, and is calibrated by taking into account actual data on the investment-to-GDP ratio. The unitary elasticity of substitution between capital and labor in the production function takes into account the actual labor income share. The steady-state price markup of tradable and non-tradable goods is calibrated using OECD product market regulation indicators, as well as the correlation between tradable and non-tradable goods markups and product market regulation indicators found in Høj et al. (2007). The price markup of the non-tradable goods is set at 20 percent, to capture low competition levels in this sector, whilst for the tradable sector the price markup is 10 percent. Investment adjustment costs and labor adjustment costs are parameterized so as to ensure plausible investment and labor dynamics, respectively, while adjustment costs in price changes are calibrated to match reasonable average price adjustment time spans. In particular, price adjustment in the non-tradable goods sector (4.5 quarters) is slightly slower than in the tradable goods sector (roughly 3 quarters). Production functions are calibrated in line with the National Accounts import contents and the non-tradable goods content of each type of final good.

The elasticity of substitution between domestic tradable goods and imported goods is assumed to be identical across firms and set above unity (Coenen, McAdam, and Straub, 2007; Harrison et al., 2005; Erceg, Henderson, and Levin, 2000; Kumhof et al., 2010). The degree of monopolistic competition amongst distributors is lower than among manufacturers, with the steady-state markup being set to 5 percent, except in the case of exporters, where fiercer competition justifies a lower markup. In terms of price stickiness, price adjustment costs imply an adjustment to equilibrium in roughly 2 quarters for consumption goods and investment goods. Prices in the export sector adjust faster, in slightly more than 1 quarter, while prices of government consumption goods take around 4 quarters to adjust. With the exception of government goods, where the sizable wage share justifies a slower adjustment, these parameterizations lie within the plausible range of price adjustment costs reported in Keen and Wang (2007). Adjustment costs in import contents are set to ensure that they move plausibly with real exchange rate fluctuations. The combination of assembled goods with non-tradable goods in the production of final goods is assumed to feature a low substitutability, in line with Mendoza (2005) and Kumhof et al. (2010).

Steady-state tax rates, transfers from the rest of the euro area, government consumption, and government transfers are calibrated to match actual data. The fiscal policy rule is parameterized to ensure smooth adjustments—the exact calibration depending on the simulation (see Almeida et al., 2013 for alternative configurations). The target debt is set to 60 percent of GDP, in line with fiscal targets set in the European Union. This implies a steady-state fiscal balance of -2.4 percent.

In the financial sector of the model, tradable and non-tradable sectors have identical calibrations. The leverage ratio of entrepreneurs, B/N, the probability of default, $\mathfrak{F}(\bar{\omega})$, and the return on capital, $Ret^{\mathcal{K}}$, are approximated with aggregate Portuguese historical features. The leverage ratio is 100 percent. The same value is used in Bernanke, Gertler, and Gilchrist (1999b) or Kumhof et al. (2010). The probability of default—8 percent—is relatively close to the exit rates reported in Mata, Antunes, and Portugal (2010), and in line with the value found in Kumhof et al. (2010). The return on capital is approximately 9 percent, a spread of 4 percentage points over the risk free rate. These figures imply a non-default loan rate, i^B, of 6.5 percent, which is close to the "debt cost ratio" published by the *Banco de Portugal*. This figure embodies a spread of 1.7 percentage points over the risk free rate. The steady-state monitoring cost parameter is 11 percent, implying that on aggregate banks recover 89 percent of the value of bankrupt firms. The monitoring cost level is relatively close to the "average unconditional loss" estimated by Bonfim, Dias, and Richmond (2012) for the 1995–2008 period. Finally, the standard deviation of idiosyncratic risk shock of 0.32 stands between the benchmark figures used by Christiano, Motto, and Rostagno (2013) and Bernanke, Gertler, and Gilchrist (1999b), of 0.26 and 0.53 respectively.

One must stress that the model's parameters reflect our best assessment of the economic environment. They are naturally subject to uncertainty and may be updated if necessary to reflect new or better information, or changes in the underlying environment. Introducing new extensions into the model may also lead to parameter revisions. For instance, the incorporation of financial frictions into the model leads to a sharp deterioration of the net foreign position position of the economy if calibration remains unchanged. As households desire to hold exactly the same amount of assets (relative to GDP), corporate bonds—which are introduced along with financial frictions—draw funds from abroad on a one-to-one basis. Structural reforms may justify also a re-calibration of the model. An effective labor market reform, for instance may imply a lower steady-state wage markup in the medium run. The model's parameters may also be conditional on the specific issue that is being addressed. Castro et al. (2013) consider two distinct alternative states of the economy, *viz* "normal times" and "crisis times." The former is based on the standard calibration of the model, while the latter embodies some changes in parameters that may better reflect the underlying economic environment.

7 Economic analysis and policy simulations

PESSOA has been used to illustrate a few number of issues, most of them for the Portuguese economy. In Almeida, Castro, and Félix (2009), the model is used to assess the impact of a number of shocks that played an important role in Portuguese economic developments, namely: a slowdown in total factor productivity, a decline in interest rates and a reduction in liquidity constraints, a decrease of exports non-price competitiveness, and a short-lived fiscal boom and subsequent fiscal consolidation.

In Almeida, Castro, and Félix (2010), the authors provide well-grounded support for structural reforms in Portugal, by showing that substantial increases in competition in the non-tradable goods sector and in the labor market could induce important international competitiveness gains. These structural reforms could be valuable instruments in promoting necessary adjustments for a small open economy integrated in a monetary union.

Almeida, Castro, Félix, and Maria (2010a) evaluate the impact of fiscal stimulus in a small open economy within a monetary union, and conclude that increases in permanent government expenditures should be avoided due to their negative welfare effects, as opposed to temporary stimulus. This result creates room for welfare improving stabilization policies. Nevertheless, a temporary stimulus which triggers a hike in the country's risk premium due to high indebtedness levels may have a negligible effect on welfare.

In Almeida, Castro, Félix, and Maria (2013), *PESSOA* is used to show that a fiscal consolidation strategy based on a permanent reduction in Government expenditure increases the long-run level of output, private consumption and welfare, at the cost of short-run welfare losses and output reduction.

Finally, Castro, Félix, Júlio, and Maria (2013) evaluate the size of short-run fiscal multipliers associated with fiscal consolidation under two distinct alternative scenarios, *viz* "normal times" and "crisis times." The authors find that first year fiscal multipliers can be around 60-70 percent larger in crisis times vis-à-vis normal times for a government consumption-based fiscal consolidation, and around 40-60 percent larger for a revenue-based fiscal consolidation.

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Appendices

List of Equations: the stationary model

List of Equations: the steady-state model

List of shocks (including steady-state values)

1 Population size.

$$\log N_t = (1-\rho_N)\log \bar{N} + \rho_N \log N_{t-1} + e_t^N$$

In the steady state: $N_t = \bar{N}$ 83

2 Share of liquidity constrained households.

$$\log \psi_t = (1-\rho_\psi)\log \bar{\psi} + \rho_\psi \log \psi_{t-1} + e_t^\psi$$

In the steady state: $\psi_t = \bar{\psi}$ 83

3 Elasticity of substitution between varieties of the consumption good.

$$\sigma_t^{\mathcal{C}} = (1-\rho_{\sigma_\mathcal{C}})\bar{\sigma}^{\mathcal{C}} + \rho_{\sigma_\mathcal{C}}\sigma_{t-1}^{\mathcal{C}} + e_t^{\sigma_\mathcal{C}}$$

In the steady state: $\sigma_t^{\mathcal{C}} = \bar{\sigma}^{\mathcal{C}}$ 83

4 Elasticity of substitution between varieties of the intermediate good.

$$\sigma_t^J = (1-\rho_{\sigma_J})\bar{\sigma}^J + \rho_{\sigma_J}\sigma_{t-1}^J + e_t^{\sigma_J}$$

In the steady-state: $\sigma_t^J = \bar{\sigma}^J$ 83

5 Elasticity of substitution between varieties of the final good.

$$\sigma_t^F = (1-\rho_{\sigma_F})\bar{\sigma}^F + \rho_{\sigma_F}\sigma_{t-1}^F + e_t^{\sigma_F}$$

In the steady-state: $\sigma_t^F = \bar{\sigma}^F$ 83

6 Elasticity of substitution between varieties of labor.

$$\sigma_t^{\mathcal{U}} = (1-\rho_{\sigma_\mathcal{U}})\bar{\sigma}^{\mathcal{U}} + \rho_{\sigma_\mathcal{U}}\sigma_{t-1}^{\mathcal{U}} + e_t^{\sigma_\mathcal{U}}$$

In the steady-state $\sigma_t^{\mathcal{U}} = \bar{\sigma}^{\mathcal{U}}$ 83

7 Risk premium on domestic bonds.

$$\log \Psi_t = (1-\rho_\Psi)\log \bar{\Psi} + \rho_\Psi \log \Psi_{t-1} + e_t^\Psi$$

In the steady-state: $\Psi_t = \bar{\Psi}$ 83

8 Investment efficiency shock.

$$\zeta_t^{\mathcal{I}} = (1-\rho_{\mathcal{I}}) + \rho_{\mathcal{I}}\zeta_{t-1}^{\mathcal{I}} + e_t^{\mathcal{I}}$$

In the steady state: $\zeta^{\mathcal{I}} = 1$ 83

9 Labor augmenting thehnology shock.

$$\log A_t^J = (1-\rho_{A_J})\log \bar{A}^J + \rho_{A_J}\log A_{t-1}^J + e_t^{A_J}$$

In the steady-state: $A_t^J = \bar{A}^J$ 83

10 Government consumption.

$\log \check{G}_t = (1 - \rho_G) \log \check{\bar{G}} + \rho_G \log \check{G}_{t-1} + e_t^G$

11 Lump-sum transfers to households.

$\log \check{trg}_t = (1 - \rho_{trg}) \log \check{\bar{trg}} + \rho_{trg} \log \check{trg}_{t-1} + e_t^{trg}$

12 Tax rates; $z \in \{\mathcal{C}, \mathcal{SP}, \mathcal{K}, \mathcal{D}\}$.

$\tau_t^z = (1 - \rho_{\tau_z})\bar{\tau}^z + \rho_{\tau_z}\tau_{t-1}^z + e_t^{\tau_z}$

13 Imported goods' inflation rate.

$\log \pi_t^* = (1 - \rho_{\pi^*}) \log \bar{\pi}^* + \rho_{\pi^*} \log \pi_{t-1}^* + e_t^{\pi^*}$

14 Entrepreneurs' riskiness.

$\sigma_t^{\mathcal{E}J} = (1 - \rho_\sigma) \log \bar{\sigma}^{\mathcal{E}J} + \rho_\sigma \log \sigma_{t-1}^{\mathcal{E}J} + e_t^\sigma$

15 Bankruptcy monitoring costs.

$\mu_t^J = (1 - \rho_\mu) \log \bar{\mu}^J + \rho_\mu \log \mu_{t-1}^J + e_t^\mu$

16 Entrepreneurs' dividend net worth shock.

$S_t^{J,d} = (1 - \rho_{S_d}) \log \bar{S}^{J,d} + \rho_{S_d} \log S_{t-1}^{J,d} + e_t^{S_d}$

Calibration: key ratios

Table 1: Key ratios

	Model	**Data**
Expenditure (ratio to GDP)		
Private consumption	0.60	0.64
Government consumption and GFCF	0.23	0.22
Private investment	0.21	0.21
Exports	0.29	0.29
Imports	0.33	0.37
Labor income share (ratio to overall income)	0.57	0.57
Tradable sector	0.55	0.54
Non-tradable sector	0.59	0.58
Capital (ratio to GDP)	2.33	*NA*
Tradable sector	2.51	*NA*
Non-tradable sector	2.21	*NA*
Government (ratio to GDP)		
Public debt	0.60	0.57
Fiscal surplus	-0.02	-0.05
Total revenues	0.43	0.40
Total expenditure	0.46	0.44
External account (as a % of GDP)		
Net foreign assets		-0.50
Without Financial accelerator	-0.34	
With Financial accelerator	-1.29	
Current and capital accounts		-0.06
Without Financial accelerator	-0.01	
With Financial accelerator	-0.05	
Trade balance	-0.04	-0.08

Sources: *Banco de Portugal* quarterly database, National accounts data, and authors' own calculations.
Notes: The figures for Portugal are for the 1999–2007 period.

Calibration: main parameters

Table 2: Main parameters

	Parameter	Value
Monetary union parameters		
Interest rate (A)	i^*	1.045
Labor-augmenting prod. growth (A)	g	1.02
Inflation target (A)	π^*	1.02
Risk Premium	Ψ	1.00
EoS bt. ROW tradable goods and domestic exports	ξ_{A*}	2.50
Households and Unions		
Discount rate	β	0.996
Intertemporal elasticity of substitution	$1/\gamma$	0.50
Instant probability of death (A)	$1-\theta$	0.04
Productivity decay rate (A)	$1-\chi$	0.04
Habit persistence	ν	0.70
Consumption share—Type-$\mathcal{A}$ households	$\eta^{\mathcal{A}}$	0.73
Consumption share—Type-$\mathcal{B}$ households	$\eta^{\mathcal{B}}$	0.65
Share of type-$\mathcal{B}$ households	ψ	0.40
Wage markup (SS)	$\sigma^{\mathcal{U}}/(\sigma^{\mathcal{U}}-1)$	1.25
Wage adjustment cost	$\phi_{\mathcal{U}}$	125
Manufacturers and capital goods producers		
EoS bt. capital and labor	ξ_J	0.99
Price markup—tradable sector (SS)	$\sigma^{\mathcal{T}}/(\sigma^{\mathcal{T}}-1)$	1.10
Price markup—non-tradable sector (SS)	$\sigma^{\mathcal{N}}/(\sigma^{\mathcal{N}}-1)$	1.20
Labor adjustment cost	$\phi_{\mathcal{U}J}$	5.00
Price adjustment cost	ϕ_{PT}; ϕ_{PN}	125
Quasi labor income share—tradable sector	$\alpha_{\mathcal{U}}^{\mathcal{T}}$	0.56
Quasi labor income share—non-tradable sector	$\alpha_{\mathcal{U}}^{\mathcal{N}}$	0.60
Depreciation rate (A)	δ^J	0.09
Investment adjustment cost	$\phi_{\mathcal{I}J}$	10.0
Distributors		
EoS bt. domestic tradable and imported goods	ξ_{AF}	1.50
EoS bt. assembled and non-tradable goods	ξ_F	0.50
Price markup of distributors (SS)	$\sigma^F/(\sigma^F-1),\ F \notin \{\mathcal{X}\}$	1.05
Price markup of exporters (SS)	$\sigma^{\mathcal{X}}/(\sigma^{\mathcal{X}}-1)$	1.03
Adjustment cost of import content	ϕ_{AF}	2.00
Price adjustment cost (except for government goods)	ϕ_{AC}; ϕ_{AI}; ϕ_{AX}	125
Price adjustment cost for government goods	ϕ_{AG}	400
Government		
Consumption tax rate	$\tau^{\mathcal{C}}$	0.31
Capital income tax rate	$\tau^{\mathcal{K}}$	0.17
Dividends tax rate	$\tau^{\mathcal{D}}$	0.15
Employers' payroll tax rate	τ^{SP}	0.19
Sensibility to deviations from target		
Revenues	d_1	0.0
Debt-to-GDP ratio	d_2	0.0
GDP	d_3	0.5

Sources: *Banco de Portugal* quarterly database, National accounts data, several studies on the Portuguese and euro area economies, and authors' own calculations.
Notes: A—Annualized; EoS—Elasticity of Substitution; SS—Steady state; ROW—Rest of the World; $J \in \{\mathcal{T},\mathcal{N}\}$; $F \in \{\mathcal{C},\mathcal{G},\mathcal{I},\mathcal{X}\}$. The labor income tax rate is the default endogenous instrument in the fiscal rule and is thus omitted. The reported parameters d_i, $i = 1, 2, 3$, represent default values, however they can take any alternative configuration. The model is quarterly and parameters are not annualized, unless otherwise indicated.

Calibration: financial sector parameters

Table 3: Financial sector parameters

	Parameter	Model	Data, PT	
			Interval	Average
Leverage (%)	B/N	100.0	95.0–130.2	104.9
Probability of default (%)	$\mathfrak{F}(\bar{\omega})$	8.0	8.7–10.6	9.4
Return on capital (%)	$Ret^{\mathcal{K}}$	9.1	8.0–10.0	9.4
Return on cap. *spread* (pp)	$Ret^{\mathcal{K}} - i$	4.0	n.a.	n.a.
Non-default loan rate (%)	i^B	6.5	5.9–8.5	6.7
Loan rate *spread* (pp)	$i^B - i$	1.7	n.a.	n.a.
Borrowers Riskiness	σ	0.32	n.a.	n.a.
Monitoring costs	μ	0.11	n.a.	n.a.

Sources: Banco de Portugal, Eurostat, Mata, Antunes, and Portugal (2010), and authors' own calculations.
Notes: Average figures for the 2002–2008 period, except the probability of default, which is for the 1996–1998 period. Leverage is defined as the debt-to-equity ratio of non-financial corporations, where debt includes debt securities issued by non-financial corporations as well as loans (consolidated data). The values reported for the probability of default consider the total exit rate for the Portuguese economy, as defined in Mata, Antunes, and Portugal (2010). More recent data from *Quadros de Pessoal* for the 2002–2007 period shows an exit rate comprised between 8.1 and 9.2 percent. The return on capital is calibrated to match the definition of "return on investment" used by the *Banco de Portugal*, the ratio between ordinary profit plus interest costs, against other shares and other equity plus financial debt. The non-default loan rate is proxied by the "debt cost ratio" used by *Banco de Portugal*, the ratio between interest costs and financial debt.

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