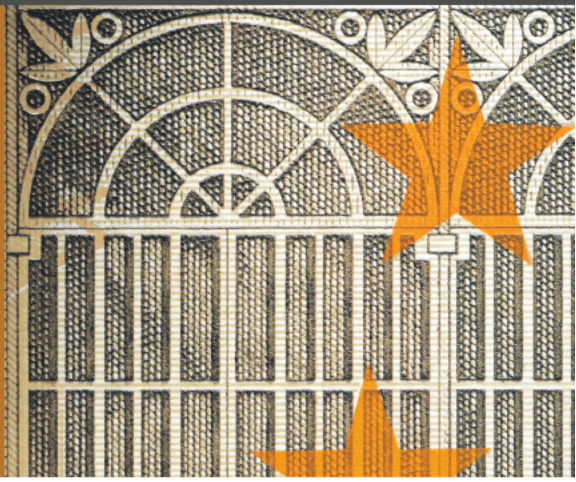

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September 2013

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Characterizing economic growth paths based on new structural change tests*

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Abstract

One of the prevalent topics in the economic growth literature is the debate between neoclassical, semi-endogenous, and endogenous growth theories regarding the model that best describes the data. An important part of this discussion can be summarized in three mutually exclusive hypotheses: the "constant trend", the "level shift", and the "slope shift" hypotheses. In this paper we propose the characterization of a country's economic growth path according to these break hypotheses. We address the problem in two steps. First, the number and timing of trend breaks is determined using new structural change tests that are robust to the presence, or not, of unit roots, surpassing technical and methodological concerns of previous empirical studies. Second, conditional on the estimated number of breaks, break dates, and coefficients, a statistical framework is introduced to test for general linear restrictions on the coefficients of the suggested linear disjoint broken trend model. We further show how the aforementioned hypotheses, regarding the economic growth path, can be analysed by a test of linear restrictions on the parameters of the breaking trend model. We apply the methodology to historical per capita GDP for an extensive list of countries. The results support the three alternative hypotheses for different sets of countries. (JEL C22, F43, O40)

Keywords: Structural Change, Long Run Economic Growth, Restricted Trend Breaks, Stationarity, Unit Roots

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1 INTRODUCTION

Determining the nature of the trend in per capita output, *i.e.* whether it is deterministic or stochastic, and whether structural breaks are present, has been extensively debated in the literature. These two important and interrelated features have very important macroeconomic and econometric implications.

On the one hand, as put forward by Nelson and Plosser (1982), if per capita output has a unit root, implying a stochastic trend, then real shocks are likely to be the most important source of macroeconomic fluctuations as opposed to disturbances with only a transitory impact (such as monetary and other demand-side shocks), being consistent with the real business cycle theory. Conversely, if the trend in per capita output is deterministic then one should observe only short-run fluctuations primarily determined by demand shocks with a transitory impact (in such cases, monetary shocks explain a large fraction of business cycle fluctuations). Moreover, the interpretation and usefulness of linear regression models involving the output variable critically depend on the nature of the trend, as OLS may produce spurious results in the presence of a stochastic trend (Granger and Newbold, 1974, Phillips, 1986).

On the other hand, studying the stability of the output growth rate is an important topic for the debate between neoclassical, semi-endogenous, and endogenous growth theories for the model that best describes what we observe in the data. Jones (1995a, 2002, 2005) contrasted the substantial and permanent rise of investment in human capital and R&D with the remarkable stability of U.S. per capita output. Based on these models we should have observed permanent positive shifts in the rate of economic growth, according to the endogenous growth literature, or, at least, short run increases and long run "level effects" according to the neoclassical and semi-endogenous growth models. However, the growth rate of U.S. per capita output has been remarkably stable since the end of the 19th century. Moreover, Jones (1995b) documents that several variables that should lead to permanent changes in the long run growth rate or at least originate "level effects", exhibited large, persistent movements, generally in the "growth-increasing" direction in OECD economies, at least, since World War II. Based on the documented increase of these variables, Papell and Prodan (2005) classified several countries according to three mutually exclusive hypotheses, each compatible with a certain class of economic growth models:

(a) The "Summer-Weil-Jones" or "constant trend" hypothesis, originally suggested by David Weil and Lawrence Summers and subsequently considered in Jones (1995b), postulate that a simple time trend with slope equal to the average growth rate should describe the log of per capita output accurately. Some temporary departures from the trend are allowed, corresponding to large exogenous shocks to the economy and its subsequent recovery, but the linear trend should return to its original path.

(b) The "Jones-Solow" or "level shift" hypothesis, favors the neoclassical (Solow, 1956) and the Jones' (Jones, 1995a, 2005) semi-endogenous growth theories. It states that, after policy changes (such as a rise in human capital or R & D investment), output growth may change in the short run but should return to its original value in the long run. However, these changes should lead to long-run increases in the level of per capita GDP.

(c) The "Romer" or "slope shift" hypothesis postulated by Romer (1986) suggests that policy changes should alter the growth rate of per capita output permanently.

Considering the hypotheses previously indicated (*i.e.* the "constant trend", the "level shift" and the "slope shift" hypotheses), the objective of this paper is to analyze which of these better characterizes the growth path of per capita output. The literature closely related to the topic addressed in this paper is Ben-David and Papell (1995) who pre-tested the unit root hypothesis with the Zivot and Andrews (1992) approach and then used the Vogelsang (1997) test, with critical values selected according to the resultant order of integration, to search for evidence for one break in the trend function. Papell and

Prodan (2005, 2011) pre-tested for the existence of a unit root with an ADF test discussed in Papell and Prodan (2007) which allows for two endogenous break points, but with the second break restricted to have only one slope shift. After filtering out the nonstationary countries, Papell and Prodan (2007) used a modification of the sequential procedure of Bai (1999), as suggested by Prodan (2008), to estimate the number of breaks. Finally, for countries with more than one break, Papell and Prodan (2007) formally tested the constant trend and level shift hypotheses with a standard F statistic. However, all these approaches have several limitations. First, the unit root pre-testing procedure imposes, but does not estimate the number of breaks in the trend function. Second, the unit root tests considered are based on search procedures under the alternative hypothesis and do not render pivotal asymptotic distributions in the presence of trend breaks under the null hypothesis. Third, it is well known that this sequence of pre-testing procedures can generate substantial size and power distortions (even asymptotically) specially if the first step statistics have poor finite sample properties.

The contribution of this paper is to use tests for structural changes which do not require specifying in advance whether per capita GDP is I(0) or I(1), in order to differentiate among the different economic growth theories. To categorize countries according to the "constant trend", "level shift" and "growth shift" hypotheses we first identify when large and exogenous shocks occurred for each country using the framework in Nunes and Sobreira (2012) (hereafter NS). The approach followed does not require any unit root pre-testing and is similar to that proposed in the recent literature on structural changes (see Perron and Yabu, 2009, Harvey et al., 2009, Kejriwal and Perron, 2010). Moreover, whenever two or more breaks are found, it becomes necessary to test additional restrictions on the trend function across regimes. This is done in our paper by a new statistic which we introduce to test for restrictions on the trend function extending the framework proposed in NS.

We note that Kejriwal and Lopez (2012) also took advantage of these recent econometric developments to test three hypotheses labeled with the same names as ours, but used different definitions for each hypothesis. For the "constant trend" hypothesis Kejriwal and Lopez (2012) do not allow a country to return to its original level of per capita GDP and GDP growth after the transitional period following a large shock, and for the "level shift" hypothesis they do not allow a country to return to its steady state value of GDP growth after the transitional period following a large shock.

We apply our procedure to long historical per capita GDP series for an extensive set of countries. Statistical evidence obtained in this paper supports the "constant trend" hypothesis for nine countries, the "level shift" hypothesis for six countries, and the "growth shift" hypothesis for eight countries.

The remaining of the paper is organized as follows. Section 2 describes the econometric approach and the proposed test statistics. Section 3 presents and discusses the empirical results and provides a categorization of the countries analyzed. Section 4 provides some concluding remarks.

2 ASSUMPTIONS AND METHODOLOGY

In the first step of our approach we follow NS to test for the existence, number, and timing of the trend breaks. Consequently, and contrary to e.g. Papell and Prodan (2005, 2011), we do not need to pre-test the unit root hypothesis since these tests are robust as to whether the underlying process is I(0) or I(1).

If the statistical evidence does not indicate the existence of at least one break in per capita GDP growth rate then our empirical analysis will stop, since no evidence for the existence of trend breaks favors the "constant trend" hypothesis. Of course, as in any statistical hypothesis test, one should be careful not to interpret a non-rejection of the null hypothesis of no structural change as evidence against the alternative hypothesis of structural change.

On the other hand, if there is evidence for the presence of one break in trend then this favors a changing steady-state growth rate pattern which is only compatible with the "slope shift" hypothesis. Finally, if the testing procedure detects the presence of two or more breaks in the trend function, then several situations may occur. A first possibility is that after the first large shock (which typically coincides with the World Wars or The Great Depression) the output growth rate deviated from its steady state value but, after enough time has passed, transition dynamics returned the economy to its steady state growth path. This reasoning is in line with the "constant trend" hypothesis which defends that not only the steady-state growth rate, but also the trend function as a whole should be equal except in the transition period. A second possibility, compatible with the "level shift" hypothesis, occurs when only the steady state growth rates remain the same before the first break and after transitional dynamics. As a final possibility, we may consider that, after recovery from a shock, the economy enters a new and different steady state growth path which is compatible with the "slope shift" hypothesis. These three different and mutually exclusive behaviors of the long-run trend and growth rates are associated with specific linear restrictions on the breaking trend model will be described below in Section 2.3.

2.1 Econometric Model and Assumptions

The most general setup to model the behavior of long-term per capita output is the disjoint broken linear trend model discussed in NS and Kejriwal and Perron (2010). We will use their framework to test for additional restrictions on the trend breaks coefficients. The log of real per capita GDP, denoted by y_t $(t = 1, \ldots, T)$, is assumed to be generated by the following equation that includes a constant, a linear trend and m structural breaks in the trend function which may occur at dates $T_1^*, \ldots, T_m^*$:

$$y_t = \alpha + \beta t + \sum_{j=1}^{m} \delta_j DU_t\left(\tau_j^*\right) + \sum_{j=1}^{m} \gamma_j DT_t\left(\tau_j^*\right) + u_t \qquad t = 1, ..., T, \tag{1}$$

where $DU_t\left(\tau_j^*\right) := \mathbb{1}\left(t > T_j^*\right)$ and $DT_t\left(\tau_j^*\right) := \mathbb{1}\left(t > T_j^*\right)\left(t - T_j^*\right)$ capture the eventual j^{th} break, in the level and slope, respectively, occurring at date $T_j^* := \lfloor \tau_j^* T \rfloor$ for $j =$

$1, ..., m$, with $\mathbb{1}(.)$ denoting indicator functions and $\lfloor . \rfloor$ corresponds to the integer part of the argument. Notice that the first differenced form of equation (1) is given by:

$$\Delta y_t = \beta + \sum_{j=1}^{m} \delta_j D_t\left(\tau_j^*\right) + \sum_{j=1}^{m} \gamma_j DU_t\left(\tau_j^*\right) + v_t \qquad t = 2, ..., T, \tag{2}$$

where $D_t\left(\tau_j^*\right) = \mathbb{1}\left(t = T_j^* + 1\right)$. From (1) and (2), it is readily seen that the slope coefficient is the long-run or steady state growth rate. Hence, in this unrestricted version of the model we allow for different steady state growth rates across regimes. Until the occurrence of the first structural break at T_1^*, the slope coefficient is equal to β. After T_1^* the long-run growth rate changes from β to $\beta + \gamma_1$ and the level shifts by δ_1. At break point T_2^* the steady-state growth rate changes from $\beta + \gamma_1$ to $\beta + \gamma_1 + \gamma_2$ and the level shifts by δ_2. Generally, in period T_j^* the slope coefficient changes from $\beta + \sum_{i=1}^{j-1} \gamma_i$ to $\beta + \sum_{i=1}^{j} \gamma_i$ while the level shifts by δ_j. Note that whenever $\delta_j \neq 0$, the trend function becomes discontinuous at the break date T_j^*.

The disturbance term u_t in (1) is assumed to have an AR(1) representation,

$$u_t = \rho u_{t-1} + \varepsilon_t, \qquad t = 2, ..., T, \tag{3}$$

where $u_1 = \varepsilon_1$ and $\{\varepsilon_t\}$ in (3) satisfies the following assumption:

Assumption 1. *The stochastic process* $\{\varepsilon_t\}$ *is such that* $\varepsilon_t = C(L)\eta_t$, *where* $C(L) = \sum_{i=0}^{\infty} c_i L^i$, $[C(1)]^2 > 0$ *and* $\sum_{i=0}^{\infty} i|c_i| < \infty$, *and where* η_t *is a martingale difference sequence with unit conditional variance and* $\sup_t E\left(\eta_t^4\right) < \infty$.

Notice that Assumption 1 is quite general. In particular, we allow for the presence of substantial serial correlation in the errors of the AR(1) representation of u_t. The autoregressive coefficient, ρ, is allowed to be either smaller or equal to 1 in absolute value so that real per capita output can be either I(0) or I(1), respectively.

Our goal is to classify countries according to the "constant trend", the "level shift" and the "slope shift" hypotheses. We approach this problem in two steps: first, we test for the existence of slope breaks in the trend function and estimate both the number and the timing of the change points. This is done unrestrictedly using the methods proposed by NS which are briefly discussed in the next section. Second, conditional on the estimated number of breaks, break dates, and coefficients, we build a statistical framework to test for general linear restrictions on the coefficients of the linear disjoint broken trend model as described below in Section 2.3. Considering the $2m+1$ vector $\Phi = (\delta_1, \ldots, \delta_m, \beta, \gamma_1, \ldots, \gamma_m)'$, this amounts to testing the null hypothesis $H_0 : R\Phi = r$ against the two-sided alternative $H_A : R\Phi \neq r$ where R is a q by $2m+1$ matrix with rank q and r is a q-dimensional vector of constants. We do not include α (the intercept coefficient in (1)) in Φ because this parameter is not identified in the first-differenced model (2). These procedures are all made robust to whether $\{u_t\}$ is I(0) or I(1).

2.2 Detection and estimation of the number of breaks

To start the analysis, NS suggest the use of a *sup* F type test of no slope breaks against the alternative hypothesis that there are m slope breaks. The tests involve estimating equations (1) and (2) by OLS for all candidate break fractions $\tau^m = (\tau_1, \ldots, \tau_m)$, and are obtained from

$$\mathcal{F}_0^* (m|0) := \sup_{\tau^m \in \Lambda_m} \mathcal{F}_0 (\tau^m) \tag{4}$$

and

$$\mathcal{F}_1^* (m|0) := \sup_{\tau^m \in \Lambda_m} \mathcal{F}_1 (\tau^m) \tag{5}$$

where $\mathcal{F}_0 (\tau^m)$ and $\mathcal{F}_1 (\tau^m)$ denote, respectively, standard F statistics for testing $\gamma_1 = \ldots = \gamma_m = 0$ from the estimated equations (1) and (2). To account for general forms of serial correlation in the data, $\mathcal{F}_0 (\tau^m)$ and $\mathcal{F}_1 (\tau^m)$ are "standardized" by a Bartlett long run variance estimate. Λ_m specifies the dates allowed for the search of the structural breaks and is given by $\Lambda_m = \{(\tau_1, ..., \tau_m) : |\tau_{i+1} - \tau_i| \geq \eta, \tau_1 \geq \eta, \tau_m \leq 1 - \eta\}$. Note that for empirical purposes, given no knowledge of the change points, we follow the suggestion in Andrews (1993) and set $\eta = 0.15$. Basically, this rules out dates that are close to each other and/or close to the beginning/end of the sample to guarantee invertibility of the moments matrix and enough neighborhood observations to identify the true break points (see also Andrews and Ploberger, 1994, Bai and Perron, 1998, for more details). Finally, the break point estimators are the global maximizers of the objective functions: $\widehat{\tau}^m := \underset{\tau^m \in \Lambda_m}{\arg\sup} \mathcal{F}_0(\tau^m)$ and $\widetilde{\tau}^m := \underset{\tau^m \in \Lambda_m}{\arg\sup} \mathcal{F}_1(\tau^m)$.

Now, since $\mathcal{F}_0^*(m|0)$ and $\mathcal{F}_1^*(m|0)$ converge to a non degenerate asymptotic distribution, regardless of whether the data are I(0) or I(1) (see Theorem 5 of NS), these test statistics were weighted by a weight function which is asymptotically binary and ensures that, in the limit, $\mathcal{F}_0^*(m|0)$ is selected if $\{u_t\}$ is I(0) and $\mathcal{F}_1^*(m|0)$ is chosen when $\{u_t\}$ is I(1). Hence, this weighted $\mathcal{F}$ statistic, $\mathcal{F}_\lambda^*(m|0)$, is given by:

$$\mathcal{F}_\lambda^*(m|0) := \lambda(\widehat{\tau}^m, \widetilde{\tau}^m)\, \mathcal{F}_0^*(m|0) + b_\xi^m \left[1 - \lambda(\widehat{\tau}^m, \widetilde{\tau}^m)\right] \mathcal{F}_1^*(m|0) \tag{6}$$

where b_ξ^m is a constant that ensures that, for a given significance level ξ, the critical values of the asymptotic distribution of $\mathcal{F}_\lambda^*$ are the same in both I(0) and I(1) cases and $\lambda(\widehat{\tau}^m, \widetilde{\tau}^m) := \exp\left[-\{g_m S_0(\widehat{\tau}^m) S_1(\widetilde{\tau}^m)\}^6\right]$ where $g_m = 500 + 750 \times (m-1)$, and $S_0(\widehat{\tau}^m)$ and $S_1(\widetilde{\tau}^m)$ denote KPSS (Kwiatkowski et al., 1992) test statistics based on the residuals from the estimated regressions (1) and (2) with associated break fractions $\widehat{\tau}^m$ and $\widetilde{\tau}^m$. The $\mathcal{F}_\lambda^*(m|0)$ statistic can then be used to test the null of no slope breaks against the alternative hypothesis that there are m slope breaks without making any assumptions about the errors being I(0) or I(1) since $\lambda(\widehat{\tau}^m, \widetilde{\tau}^m) \xrightarrow{p} 1$ if $\{u_t\}$ is I(0) and $\lambda(\widehat{\tau}^m, \widetilde{\tau}^m) \xrightarrow{p} 0$ if $\{u_t\}$ is I(1). The final estimator for the vector of break fractions is obtained from $\lambda(\widehat{\tau}^m, \widetilde{\tau}^m)\, \widehat{\tau}^m + \left[1 - \lambda(\widehat{\tau}^m, \widetilde{\tau}^m)\right] \widetilde{\tau}^m$.

As in NS and Bai and Perron (1998) we adopt a sequential test statistic for testing the null hypothesis of l breaks against the alternative of $l+1$ breaks constructed from the maximum value of the *sup*F type statistics associated with testing for one break in the trend slope within each of the segments set by the estimated partitions $(\widehat{\tau}_1, \ldots, \widehat{\tau}_l)$

and $(\widetilde{\tau}_1, \ldots, \widetilde{\tau}_l)$. These statistics will be denoted as $F_0^*(l+1|l)$ and $F_1^*(l+1|l)$ in the model in levels and in first differences, respectively. For the exact same reasons outlined above for $F_0^*(m|0)$ and $F_1^*(m|0)$, the I(0)/I(1) dichotomy requires a weighted average of $F_0^*(l+1|l)$ and $F_1^*(l+1|l)$. The resulting weighted sequential F statistic, $F_\lambda^*(l+1|l)$, is then given by:

$$F_\lambda^*(l+1|l) := \lambda\left(\widehat{\tau}^{l+1}, \widetilde{\tau}^{l+1}\right) F_0^*(l+1|l) + b_\xi^{l+1|l}\left[1 - \lambda\left(\widehat{\tau}^{l+1}, \widetilde{\tau}^{l+1}\right)\right] F_1^*(l+1|l) \qquad (7)$$

where, as before, $b_\xi^{l+1|l}$ is the constant that ensures unique critical values.

The benchmark procedure starts with $l = 0$, by using $F_\lambda^*(1|0)$ to test for the presence of one break. If the null hypothesis is rejected, we set $l = 1$ and perform the $F_\lambda^*(2|1)$ test. The procedure is repeated until the $F_\lambda^*(l+1|l)$ test cannot reject the null hypothesis of l breaks.

In small samples, for some particular combinations of breaks in the trend slope, this sequential procedure may not perform well. For instance, in the presence of two breaks of opposite sign, the $F_\lambda^*(1|0)$ may have low power in identifying the two breaks, causing the sequential estimation procedure to stop too soon as can be observed in Table 4 of NS. To obviate this problem, NS recommended the use of the $F_\lambda^*(2|0)$ or a double maximum test, $UD\max F_\lambda^*$ or $WD\max F_\lambda^*$, whenever $F_\lambda^*(1|0)$ does not reject the null hypothesis of no break. If $F_\lambda^*(2|0)$ or a double maximum test does not reject H_0 then we conclude that there are no trend breaks. Otherwise, we proceed to $F_\lambda^*(3|2)$. NS called these sequential procedures $SeqF_\lambda^*(1|0)$, $SeqF_\lambda^*(2|0)$, $SeqUD\max F_\lambda^*$ and $SeqWD\max F_\lambda^*$. Figure 1 summarizes the necessary steps to implement each type of the sequential tests presented.

2.3 Testing for general linear restrictions on the trend function

Now, after establishing the regimes set by the partitions $\widehat{\tau}^m$ and $\widetilde{\tau}^m$ we are in a position to construct a statistic to test for general linear restrictions on the coefficients of the linear disjoint broken trend model, conditional on the estimated number of breaks, break dates and coefficients. This statistical test will then be used to categorize countries according to the "constant trend", the "level shift" and the "slope shift" hypotheses previously discussed. For notational convenience, we suppress the index m from $\widehat{\tau}^m$ and $\widetilde{\tau}^m$.

We still do not require any *a priori* knowledge as to whether the noise component is I(0) or I(1), since we construct a test procedure based on a weighted average of tests appropriate for the I(0) and I(1) environments.

To expose explicitly how the method works, it is useful to express equations (1) and (2) in matrix notation. We stack the regressors from the model in levels in a $2m+1$ vector $X_{DT,t}(\tau) = (DU_t(\tau_1), \ldots, DU_t(\tau_m), t, DT_t(\tau_1), \ldots, DT_t(\tau_m))'$, so that equation (1) can be written as,

$$y_t = \alpha + X_{DT,t}(\tau^*)'\Phi + u_t \qquad t = 1, ..., T. \tag{8}$$

Similarly, also the regressors from the model in first differences can be stacked in a $2m+1$ vector, as $X_{DU,t}(\tau) = (D_t(\tau_1), \ldots, D_t(\tau_m), 1, DU_t(\tau_1), \ldots, DU_t(\tau_m))'$ so that (2) can be rewritten as,

$$\Delta y_t = X_{DU,t}(\tau^*)'\Phi + \Delta u_t \qquad t = 2, ..., T. \tag{9}$$

Now suppose first that m and τ^* are known and $\{u_t\}$ is I(0). We want to build a statistical procedure to test for general linear restrictions on the coefficient vector Φ. This amounts to testing the null hypothesis $H_0 : R\Phi = r$ against the two-sided alternative $H_A : R\Phi \neq r$, where R is a q by $2m+1$ matrix with rank q and r is a q dimensional vector of constants. Then, the appropriate statistical inference method of testing H_0 against H_A rejects H_0 for large values of the F-statistic computed from (8) by OLS, which will be denoted as $\mathcal{F}_0^R$. Under these assumptions, it is well known that $q \cdot \mathcal{F}_0^R$ has a χ^2_q asymptotic distribution.

On the other hand, suppose now that m and τ^* continue to be known but $\{u_t\}$ is now I(1), that is $\rho = 1$ in (3). The appropriate statistical inference method for testing H_0 against H_A consists of estimating the coefficient vector Φ in equation (9) by OLS so that the noise component becomes I(0) and reject H_0 for large values of the corresponding F-statistic denoted as $\mathcal{F}_1^R$. Under these assumptions and normality of the errors, we have that $q \cdot \mathcal{F}_1^R$ also has a χ_q^2 asymptotic distribution.

In practice, the precise number of structural breaks and their dates are rarely known. Hence, to overcome this limitation we first use the sequential procedure described in Section 2.2 to estimate the number of breaks and replace τ^* in (8) and (9) by $\widehat{\tau}$ and $\widetilde{\tau}$, respectively. The next theorem shows that the asymptotic distribution of $\mathcal{F}_0^R$ and $\mathcal{F}_1^R$ is the same regardless of whether we use the true or the estimated break fractions.

Theorem 1. *Let the time series process $\{y_t\}$ be generated according to (1) and (3) with $\gamma_j \neq 0$, $j = 1, ..., m$ under $H_0 : R\Phi = r$ and let Assumption 1 hold with normality of the error term. If:*

(a) $\{u_t\}$ is I(0) then $q \cdot \mathcal{F}_0^R \xrightarrow{d} \chi_q^2$.

(b) $\{u_t\}$ is I(1) then $q \cdot \mathcal{F}_1^R \xrightarrow{d} \chi_q^2$.

As discussed in Remarks 1 and 5 in Perron and Yabu (2009) the normality of the noise component is needed only in case (b) of our Theorem because the level shift dummies, $DU_t(\tau^*)$, become impulse dummies, $D_t(\tau^*)$, with a single outlier at $T^* + 1$ when we apply first differences to Model (1). Consequently, if the linear restrictions to be tested do not involve the parameters $\delta_1, \ldots, \delta_m$ it is possible to rule out the normality assumption and still attain the chi-square asymptotic distribution.

Since the KPSS tests applied to the levels and first differenced data are invariant with respect to the values of the parameters $\alpha, \beta, \delta_1, \ldots, \delta_m, \gamma_1, \ldots, \gamma_m$ in (1) we propose a statistic to test for general linear restrictions on the trend function across regimes as an analogue of the $\mathcal{F}_\lambda^*$ statistics in (6) and (7) which is given by:

$$\mathcal{F}_\lambda^R = \lambda(\widehat{\tau}, \widetilde{\tau})\, \mathcal{F}_0^R + [1 - \lambda(\widehat{\tau}, \widetilde{\tau})]\, \mathcal{F}_1^R. \tag{10}$$

From the arguments presented above, we are now in a position to state the following corollary regarding the large sample behavior of the $\mathcal{F}_\lambda^R$ statistic:

Corollary 1. *Considering the conditions of Theorem 1 it follows that,*

(a) If $\{u_t\}$ *is I(0) then* $q \cdot \mathcal{F}_\lambda^R \xrightarrow{d} \chi_q^2$.

(b) If $\{u_t\}$ *is I(1) then* $q \cdot \mathcal{F}_\lambda^R \xrightarrow{d} \chi_q^2$.

From Corollary 1 we conclude that, regardless of whether $\{u_t\}$ is I(0) or I(1), $q \cdot \mathcal{F}_\lambda^R$ converges to a chi-square distribution with q degrees of freedom and so the two-sided test of H_0 against H_A is straightforward to implement using critical values from a chi-square distribution with degrees of freedom corresponding to the total number of restrictions being tested. The $\mathcal{F}_\lambda^R$ statistic is going to be a useful statistical tool to classify the countries according to the "linear trend", "level shift" and "growth shift" hypotheses.

Note that, as mentioned in the introductory note of Section 2, if we find evidence for the presence of two or more trend breaks this result is not sufficient to favor any of these three hypothesis. To support the "linear trend" hypothesis the deterministic trend following the last break has to be a linear projection of the trend function until the first break. This amounts to formally test the following two restrictions: one that imposes the slope of the trend function to be the same in the first and final regimes ($\gamma_1 + \cdots + \gamma_m = 0$) and the other that restricts the trend function from the last regime to be equal to the deterministic trend from the first regime ($\delta_1 + \ldots + \delta_m + \gamma_1 \left(T_m^* - T_1^*\right) + \ldots + \gamma_{m-1}\left(T_m^* - T_{m-1}^*\right) = 0$). This set of restrictions can be casted as $R\Phi = r$ if R and r are defined as,

$$R = \begin{bmatrix} 0 & \ldots & 0 & 0 & 1 & \ldots & 1 & 1 \\ 1 & \ldots & 1 & 0 & \left(T_m^* - T_1^*\right) & \ldots & \left(T_m^* - T_{m-1}^*\right) & 0 \end{bmatrix}, \qquad r = \begin{bmatrix} 0 \\ 0 \end{bmatrix}. \tag{11}$$

Since, in practice, we do not know $(T_1^*, \ldots, T_m^*)$ we replace these values in (11) by their estimates obtained from the first step procedure. If we perform the $\mathcal{F}_\lambda^R$ test and fail to reject this set of restrictions then we conclude that the corresponding country satisfies the 'linear trend" hypothesis. Rejection of the set of restrictions in (11) does not automatically imply favoring the endogenous growth theory. In fact, both Jones' semi-endogenous and Solow's neoclassical growth models allow for changing growth rates. Jones (1995a, 2002, 2005) documents that, at least since World War II, several policy variables exhibited large and persistent movements, generally in the "growth-increasing" direction, in several OECD countries. According to the semi-endogenous and neoclassical theories, per capita output should have deviated from the steady state level after these changes, inducing temporary higher than steady state growth rates. However, transitional dynamics should force a gradual decline in the growth rate until it attains its steady-state value. After these shocks, we should observe the same original steady state growth rate but a higher long run per capita output level. Hence, the "level shift" hypothesis is tested formally considering the first restriction in (11) *i.e.* that the slope coefficient before the first break and after the last break should be equal:

$$R = \begin{bmatrix} 0 & \dots & 0 & 0 & 1 & \dots & 1 \end{bmatrix}, \qquad r = 0 \tag{12}$$

The failure to reject (12) with the F_{λ}^{R} test is taken to imply that the "level shift" hypothesis holds for the country under analysis. Finally, if both sets of restrictions defined in (11) and in (12) are rejected this is can be interpreted as evidence compatible with the "growth shift" hypothesis.

Further details on the derivation of the limit results and finite sample performance of these tests can be obtained from the authors upon request.

3 ECONOMIC GROWTH HYPOTHESES TESTS

After describing the econometric methodology to be used we are now in a position to suggest a classification of the economic growth paths of the various countries according to the "linear trend", "level shift" and "growth shift" hypotheses.

We used data on per capita GDP from 1870 to 2008 for the following countries: Austria, Belgium, France, Germany, the Netherlands, Switzerland, Canada, the United States, Brazil, Chile, Uruguay, Sweden, Denmark, Finland, Norway, the United Kingdom, Japan, Sri Lanka, Australia, New Zealand, Italy, Portugal and Spain. This dataset was obtained from Maddison (2009).

3.1 Testing for Breaks in Steady State Growth

Our analysis starts by identifying which shocks have significantly affected real per capita GDP growth rates. Given that our dataset includes long historical time series for an extensive set of countries, by simple inspection of Economic History it is straightforward to write a large list of economic events that could have had a strong impact on the output growth path of each country. A data dependent algorithm is therefore needed to select the shocks that in fact had a statistically significant effect on the steady state growth rate and to specify exactly when the consequent change in trend occurred.

Hence, the first step tests for the existence of (one or multiple) structural breaks in the trend function without assuming any *a priori* knowledge of the candidate break points. Table 1 reports results from application of $F_\lambda^*(m|0)$ for $m = 1, 2, 3$, the $UD\max F_\lambda^*$ and $WD\max F_\lambda^*$ tests with $M = 3$ to per capita GDP series for various countries at the 10%, 5% and 1% significance levels. When the null is rejected at a 5% level, we present the estimated break dates in parentheses. All tests fail to reject the null of no trend break at all significance levels considered for Switzerland, Canada, the United States, Chile, Sweden and Australia. The $F_\lambda^*(3|0)$ rejects the no break in trend hypothesis for New Zealand at a nominal 10% level but not at a 5% level. Since all other tests fail to reject the null, we consider that there is not enough evidence to conclude that this country had any structural break in the slope of the trend function. Therefore, evidence favors the "constant trend" hypothesis for these countries.

In opposition, we reject the null of no trend break in all tests considered for Sri Lanka, Portugal, Spain (at all significance levels considered), Japan, Italy (at all significance levels considered except for $F_{\lambda}^{*}(1|0)$), Belgium, the Netherlands, Finland and Norway (at a 5% level or higher). Interestingly, for the United Kingdom the constant trend hypothesis is rejected when we apply the $F_{\lambda}^{*}(1|0)$ and $F_{\lambda}^{*}(2|0)$ tests even at a 1% significance level but is not rejected by either the $F_{\lambda}^{*}(3|0)$ or the double maximum tests for all significance levels considered. This may be explained by the loss of power due to allowing for more breaks than necessary as observed in Figures 1 to 3 of NS.

Since the implementation of $F_{\lambda}^{*}(m|0)$ requires the specification of the number of trend breaks under the alternative hypothesis and the double maximum tests do not estimate the break dates if the null is rejected, additional statistical procedures are needed to determine the exact number and timing of trend breaks. Hence, it is of practical relevance to implement the recursive methods described in Section 2.2 to estimate the number of structural breaks. Table 2 reports the number of breaks and the respective break dates estimated from the implementation of the sequential procedures to the per capita GDP series of the countries under analysis. Results for all sequential procedures in Table 2 show statistical evidence of two trend breaks for the Netherlands, Japan, the United Kingdom and Italy and one break in slope for Belgium, Finland, Norway, Sri Lanka, Portugal and Spain. Hence, our results seem to support the "growth shift" hypothesis for the second enumerated group of countries but are not conclusive for those countries where two breaks have been found. For the latter group, we still need to apply restricted structural change tests to classify them between the break hypotheses considered. The results of these tests are discussed in the next section.

We find ambiguous results for Uruguay as the decision of whether to reject the null hypothesis depends on the test implemented: we reject the null with $F_{\lambda}^{*}(1|0)$, $WD\max F_{\lambda}^{*}$ and $F_{\lambda}^{*}(3|0)$, but not with $F_{\lambda}^{*}(2|0)$ and $UD\max F_{\lambda}^{*}$ at a 5% significance level. To help solving this discrepancy we take advantage of the results from the sequential procedures in Table 2. Here the results are unanimous and identify one trend break for Uruguay which is supportive of the "growth shift" hypothesis and provide no evidence for breaks in France in line with the "constant trend" hypothesis.

The results for the remaining countries may also seem startling at first sight: the application of F_λ^* class of statistics to Austria, Germany and Brazil reject the null hypothesis against two and three trend breaks under the alternative, but surprisingly fails to reject against one trend break at a 5% significance level. The double maximum tests seem to confirm the $F_\lambda^*(2|0)$ and $F_\lambda^*(3|0)$ results as they always reject the no breaking trend hypothesis at a 5% significance level. France and Denmark again fail to reject the null against one trend break even at the 10% level, but the remaining tests show more ambiguous results: for Denmark the "constant trend" hypothesis is rejected if we use $F_\lambda^*(2|0)$ but is only rejected at a 10% significance level by $F_\lambda^*(3|0)$ and double maximum tests. For France $F_\lambda^*(3|0)$ and $WD\max F_\lambda^*$ find evidence for trend breaks at a 5% level but $F_\lambda^*(2|0)$ and $UD\max F_\lambda^*$ only reject the null at a 10% significance level.

This mixed evidence is also observed for the sequential procedures: $SeqF_\lambda^*(1|0)$ finds no evidence for trend breaks in total opposition to the two breaks evidence found by $SeqF_\lambda^*(2|0)$ except for France where two breaks are only detected if we use $SeqWD\max F_\lambda^*$. The $SeqUD\max F_\lambda^*$ and $SeqWD\max F_\lambda^*$ procedures reinforce the no breaks conclusion of $SeqF_\lambda^*(1|0)$ for Denmark and the two breaks conclusion of $SeqF_\lambda^*(2|0)$ for Austria, Germany and Brazil. We pursue our analysis with two trend breaks for these five countries and the battery of tests discussed in the next section provide insights on this conflicting evidence.

3.2 Restricted Structural Breaks and Economic Growth Hypotheses

After the first structural break, did per capita GDP's growth rate deviate from its steady state value but transition dynamics returned the economy to its steady state growth path? Or even in a stronger sense did per capita output trend returned to the no break counterfactual trend path? Or, contrarily, was there no transition dynamics and the economy continued on a new and different steady state growth path after the structural break?

The statistical answers to these questions are discussed in this section. Table 3 reports results for restricted structural change tests applied to countries that have shown evidence for 2 trend breaks. The second and third columns present F-statistics and p-values associated with testing whether steady state growth rates from the first and last regimes are equal. This amounts to testing the null hypothesis defined in (12) with $m = 2$. The F_{λ}^{R} fails to reject the null hypothesis at a 5% significance level for all listed countries except the UK, for which we conclude that evidence favors the "growth shift" hypothesis. These results also explain the disparate evidence as regards to the number of slope changes in the trend function for Austria, France, Germany, Brazil and Denmark: Prodan (2008) and NS document that it is likely that the standard sequential procedure cannot reject the null of no breaks in the presence of structural breaks of opposite sign. These countries represent the most problematic cases because not only is the direction the opposite, but statistical evidence shows that $\gamma_1 = -\gamma_2$, *i.e.*, the second structural break cancels the effect on the growth rate of the first structural break.

But can we say that not only the steady state growth rate but the whole trend function has been constant over time except during the period between the two estimated break dates? The fourth and fifth columns report F-statistics and p-values for testing whether the trend function from the last regime is a linear projection of the trend from the first regime. Here, the null hypothesis is given by (11) under the assumption that two breaks have occurred at times $\left(\widehat{T}_1, \widehat{T}_2\right)$ if the model is estimated in levels, or $\left(\widetilde{T}_1, \widetilde{T}_2\right)$ if the model is estimated in first differences. We fail to reject the null even at a 20% significance level for Austria and Germany. This result clearly supports the "constant trend" hypothesis. We obtain rejections at a 5% significant level for the Netherlands and Denmark and at a 1% significant level for France, Brazil, Japan and Italy and so we conclude in favor of the "weaker" "level shift" hypothesis for these countries.

Figures 2 to 6 plot the log of per capita GDP, for the countries analyzed. We superimposed the estimated break dates and the fitted values of the unrestricted model. For those countries with two statistically significant structural breaks we also superimposed the fitted values of the model restricted by the "level shift" and the "constant trend" hypotheses. From simple visual inspection, it seems that the estimated break dates adequatly capture the timings when the trend function's behavior changed in an important way. Also, for countries that did not reject the restrictions, the fitted restricted model seems to adjust well to the observed movements of the data.

In summary, according to the previous indepth econometric analysis we may divide the countries considered according to the different hypotheses in the following way:

Growth Hypotheses	Countries that best fit each hypothesis
"Constant trend"	Austria, Germany, Switzerland, Canada, US, Chile, Sweden, Australia, New Zealand
"Level shift"	France, the Netherlands, Brazil, Denmark, Japan, Italy
"Slope shift"	Belgium, Uruguay, Finland, Norway, UK, Sri Lanka, Portugal, Spain

4 CONCLUSION

In this paper we propose a classification of countries according to the growth path that best describes the behavior of their real per capita GDP. Our method is implemented in two steps: first, we select the number and timing of changes in the slope of the per capita output deterministic trend. However, this information may not be enough for proper classification because if we detect more than one trend break then different configurations of the slope changes may assign each country to different hypotheses. Hence, in the second step, given the estimated number and timing of trend breaks, we build a statistical framework to test for general linear restrictions on the level and slope of the linear trend function.

In the same spirit as Harvey et al. (2009), both tests are made robust to the I(0)/I(1) dichotomy via the use of weighted averages of two conventional F statistics, one appropriate for an I(0) environment and the other when the data are I(1). Hence, our approach surpasses technical and methodological limitations of previous approaches applied to the same research question.

The formulation of the hypotheses as linear restrictions on the parameters of the breaking trend model allows us to classify the countries into three different groups. We find evidence favoring the "constant trend" hypothesis for nine countries: Austria, Germany, Switzerland, Canada, the United States, Chile, Sweden, Australia and New Zealand. Furthermore, the results of our tests support the "level shift" hypothesis for six countries: France, the Netherlands, Brazil, Denmark, Japan and Italy, and finally, there is a third group of eight countries where statistical evidence favors the "growth shift" hypothesis: Belgium, Uruguay, Finland, Norway, the United Kingdom, Sri Lanka, Portugal and Spain.

We have focused on pre-testing slope changes in the deterministic trend function allowing for simultaneous breaks in level. If the test does not detect a change in slope this automatically assigned the country to the "constant trend" hypothesis. For those countries with no evidence of a significant change in slope, it would also be useful to apply the robust methods developed by Harvey et al. (2010) to detect level breaks while accommodating a deterministic linear trend. The level shifts may or may not prevent the linear trend following the last level shift to be strictly a linear projection of the trend preceding the first level shift. In spite of the invariant steady state growth rates across regimes, it is debatable as to whether the first case corresponds to the "level shift" hypothesis and so it would be interesting to accommodate this extension in our analysis. Finally, since the econometric framework analyzed is quite general it would be interesting to implement the two step econometric procedure to important economic sectors' datasets to infer about how soon the industry can recover from previous significantly negative shocks.

MATHEMATICAL APPENDIX

Proof of Theorem 1. According to Perron and Zhu (2005) $\widehat{\tau} = \tau^* + O_p\left(T^{-1}\right)$ if $\{u_t\}$ is I(0) and from Bai and Perron (1998) $\widetilde{\tau} = \tau^* + O_p\left(T^{-1}\right)$ if $\{u_t\}$ is I(1). Although the proof in Perron and Zhu (2005) is for the single break case, their results continue to hold for the multiple breaks case as argued by Kejriwal and Perron (2010). Using these results on the asymptotic properties of $\widehat{\tau}$ and $\widetilde{\tau}$ it is possible to show that $\Upsilon_0(\widehat{\Phi}(\widehat{\tau}) - \widehat{\Phi}(\tau^*)) \xrightarrow{p} 0$ and $\Upsilon_0\left(\widehat{V}(\widehat{\Phi}(\widehat{\tau})) - \widehat{V}(\widehat{\Phi}(\tau^*))\right) \xrightarrow{p} 0$ if $\{u_t\}$ is I(0). Similarly, for the model in differences we find that $\Upsilon_1(\widetilde{\Phi}(\widetilde{\tau}) - \widetilde{\Phi}(\tau^*)) \xrightarrow{p} 0$ and $\Upsilon_1\left(\widetilde{V}(\widetilde{\Phi}(\widetilde{\tau})) - \widetilde{V}(\widetilde{\Phi}(\tau^*))\right) \xrightarrow{p} 0$ if $\{u_t\}$ is I(1). Here Υ_0 and Υ_1 are the appropriate normalization matrices of the corresponding OLS estimators. Hence $\digamma_0^R(\widehat{\tau}) - \digamma_0^R(\tau^*) \xrightarrow{p} 0$ if $\{u_t\}$ is I(0) and $\digamma_1^R(\widetilde{\tau}) - \digamma_1^R(\tau^*) \xrightarrow{p} 0$ if $\{u_t\}$ is I(1). The rest of the proof now follows from the fact that $q \cdot \digamma_0^R(\tau^*) \xrightarrow{d} \chi_q^2$ if $\{u_t\}$ is I(0) and $q \cdot \digamma_1^R(\tau^*) \xrightarrow{d} \chi_q^2$ if $\{u_t\}$ is I(1). $\square$

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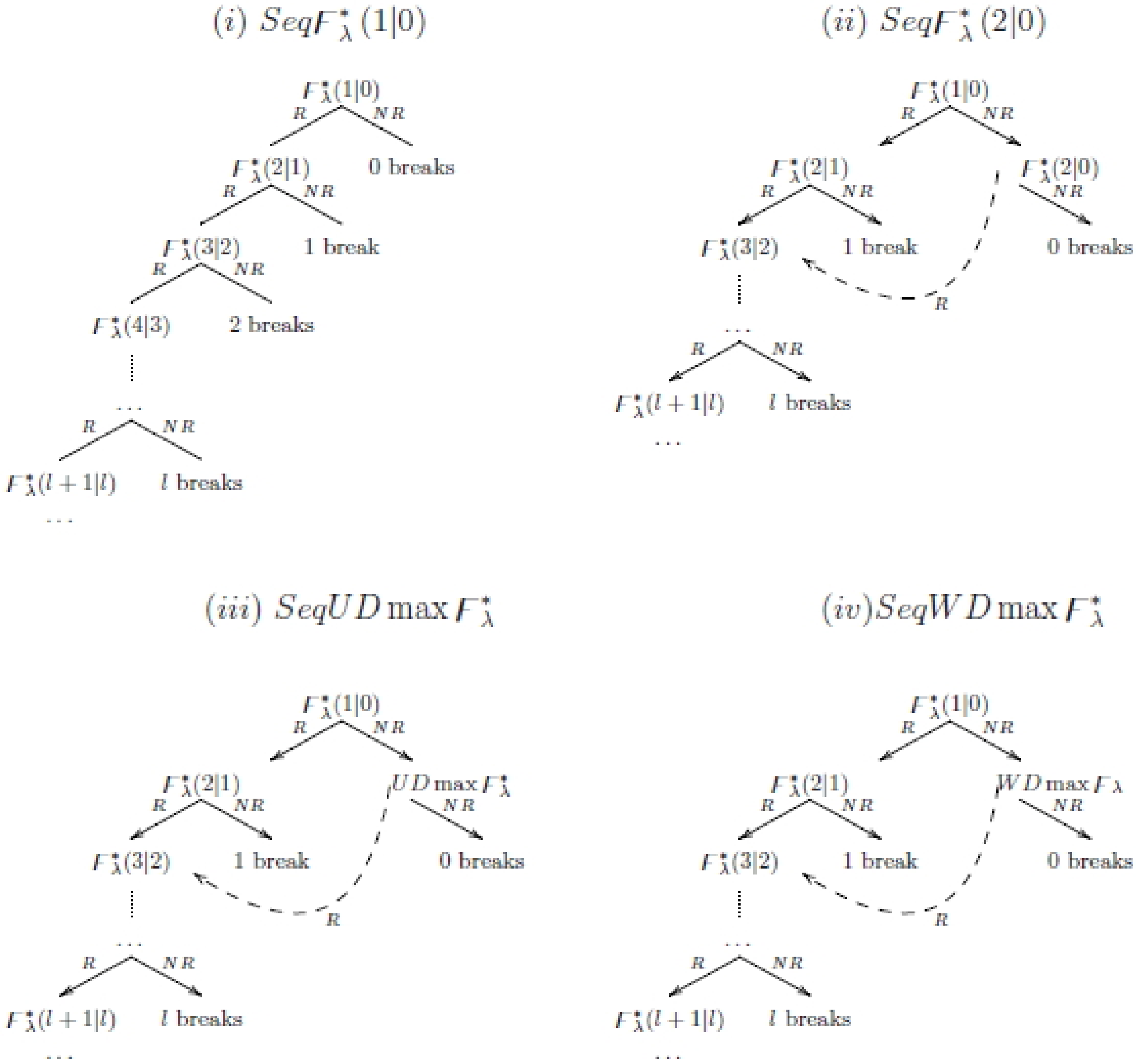

Figure 1: Sequential Tests procedure

Table 1: Empirical application of $\mathcal{F}_\lambda$ and $D\max\mathcal{F}_\lambda$ tests to log of real per capita GDP

Countries	$\mathcal{F}^*_\lambda(m\|0)$			$UD\max\mathcal{F}^*_\lambda$	$WD\max\mathcal{F}^*_\lambda$
	$m=1$	$m=2$	$m=3$		
Austria	7.98* (1944)	10.90*** (1943,1964)	8.62** (1919,1943,1964)	10.35**	11.78**
Belgium	11.30** (1941)	9.09** (1941,1973)	8.83*** (1920,1941,1973)	11.44**	11.24**
France	7.24	8.17* (1922,1943)	9.61*** (1922,1943,1972)	8.87*	12.23**
Germany	5.63	12.40*** (1944,1965)	10.09*** (1922,1944,1965)	11.78**	13.41**
Netherlands	10.85** (1943)	10.83*** (1923,1944)	10.45*** (1922,1943,1969)	10.99**	13.31**
Switzerland	2.37	6.18	4.87	5.87	6.68
Canada	2.41	4.75	4.11	4.51	5.23
United States	2.13	2.49	2.52	2.36	3.21
Brazil	8.50* (1892)	10.50** (1940,1979)	10.66*** (1917,1940,1979)	9.97**	13.57**
Chile	5.29	5.53	5.59	5.35	7.11
Uruguay	10.40** (1922)	1.71	8.19** (1906,1953,1968)	8.51*	10.71**
Sweden	3.49	5.10	5.55	5.13	7.07
Denmark	6.70	9.39** (1939,1972)	7.00* (1909,1939,1972)	8.92*	10.15*
Finland	11.72** (1916)	9.33** (1916,1937)	7.38** (1916,1937,1972)	11.87**	11.37**
Norway	12.00** (1943)	9.84** (1942,1979)	7.96** (1904,1942,1979)	12.15**	11.64**
United Kingdom	48.63*** (1935)	30.56*** (1902,1924)	4.01	6.04	5.78
Japan	10.93** (1943)	44.50*** (1943,1972)	36.14*** (1914,1943,1972)	42.26***	48.11***
Sri Lanka	17.57*** (1974)	11.14*** (1898,1974)	10.05*** (1898,1946,1974)	17.79***	17.04***
Australia	6.25	5.42	3.92	6.32	6.05
New Zealand	4.74	6.97	6.70* (1909,1931,1965)	6.62	8.53
Italy	11.98** (1943)	18.43*** (1943,1968)	16.77*** (1914,1943,1968)	17.51***	21.35***
Portugal	16.01*** (1940)	21.18*** (1950,1972)	18.95*** (1920,1950,1972)	20.12***	24.12***
Spain	15.88*** (1948)	13.84*** (1948,1973)	13.67*** (1927,1948,1973)	16.08***	17.40***

Notes: *, ** and *** refer to rejection at the 10%, 5% and 1% significance level, respectively. Where rejections are obtained for the $\mathcal{F}^*_\lambda(0|m)$ test at 5% significance level , the estimated break dates are reported in parentheses.

Table 2: Empirical application of sequential tests to log of real per capita GDP

Countries\Test	$SeqF^*_\lambda(1\|0)$	$SeqF^*_\lambda(2\|0)$	$SeqUD\max F^*_\lambda$	$SeqWD\max F^*_\lambda$
Austria	0	2 (1943,1964)	2 (1943,1964)	2 (1943,1964)
Belgium	1 (1941)	1 (1941)	1 (1941)	1 (1941)
France	0	0	0	2 (1922,1943)
Germany	0	2 (1944,1965)	2 (1944,1965)	2 (1944,1965)
Netherlands	2 (1923,1944)	2 (1923,1944)	2 (1923,1944)	2 (1923,1944)
Switzerland	0	0	0	0
Canada	0	0	0	0
United States	0	0	0	0
Brazil	0	2 (1940,1979)	2 (1940,1979)	2 (1940,1979)
Chile	0	0	0	0
Uruguay	1 (1922)	1 (1922)	1 (1922)	1 (1922)
Sweden	0	0	0	0
Denmark	0	2 (1939,1972)	0	0
Finland	1 (1916)	1 (1916)	1 (1916)	1 (1916)
Norway	1 (1943)	1 (1943)	1 (1943)	1 (1943)
United Kingdom	2 (1902,1924)	2 (1902,1924)	2 (1902,1924)	2 (1902,1924)
Japan	2 (1943,1972)	2 (1943,1972)	2 (1943,1972)	2 (1943,1972)
Sri Lanka	1 (1974)	1 (1974)	1 (1974)	1 (1974)
Australia	0	0	0	0
New Zealand	0	0	0	0
Italy	2 (1943,1968)	2 (1943,1968)	2 (1943,1968)	2 (1943,1968)
Portugal	1 (1940)	1 (1940)	1 (1940)	1 (1940)
Spain	1 (1948)	1 (1948)	1 (1948)	1 (1948)

Table 3: Restricted structural change tests

Countries\Test	"Level shift" hypothesis		"Constant trend" hypothesis	
	F_λ^R statistic	p-value	F_λ^R statistic	p-value
Austria	1.76	0.42	1.41	0.24
France	3.06*	0.22	5.36***	0.00
Germany	0.53	0.77	1.45	0.23
Netherlands	2.92*	0.23	3.32**	0.04
Brazil	0.06	0.97	6.30***	0.00
Denmark	0.50	0.78	4.42**	0.01
UK	53.18***	0.00	40.42***	0.00
Japan	0.33	0.85	10.38***	0.00
Italy	1.78	0.41	9.80***	0.00

Table 4: Estimated growth rates, in percentage terms, for the growth shift\level shift hypothesis

Countries\Growth rates	Unrestricted Model (growth shift)			Restricted Model (level shift)		
	1st regime	2nd regime	3rd regime	1st regime	2nd regime	3rd regime
Austria	1.07	3.00	2.65	1.65	3.00	1.65
France	1.26	-1.36	3.46	2.48	-1.36	2.48
Germany	1.62	3.55	1.89	1.72	3.55	1.72
Netherlands	0.98	-3.07	3.52	2.36	-3.07	2.36
Brazil	0.80	3.47	0.76	0.79	3.47	0.79
Denmark	1.59	3.04	1.62	1.60	3.04	1.60
UK	1.09	0.49	1.84	1.63	0.49	1.63
Japan	1.84	4.98	1.97	1.88	4.98	1.88
Italy	0.97	5.45	1.88	1.29	5.45	1.29

Table 5: Estimated growth rates, in percentage terms, for the growth shift\constant trend hypothesis

Countries\Growth rates	Unrestricted Model (growth shift)			Restricted Model (constant trend)		
	1st regime	2nd regime	3rd regime	1st regime	2nd regime	3rd regime
Austria	1.07	3.00	2.65	1.86	1.93	1.86
France	1.26	-1.36	3.46	1.79	2.25	1.79
Germany	1.62	3.55	1.89	1.76	3.34	1.76
Netherlands	0.98	-3.07	3.52	1.59	1.00	1.59
Brazil	0.80	3.47	0.76	1.59	1.50	1.59
Denmark	1.59	3.04	1.62	1.82	2.37	1.82
UK	1.09	0.49	1.84	1.45	1.39	1.45
Japan	1.84	4.98	1.97	2.49	2.81	2.49
Italy	0.97	5.45	1.88	1.87	2.91	1.87

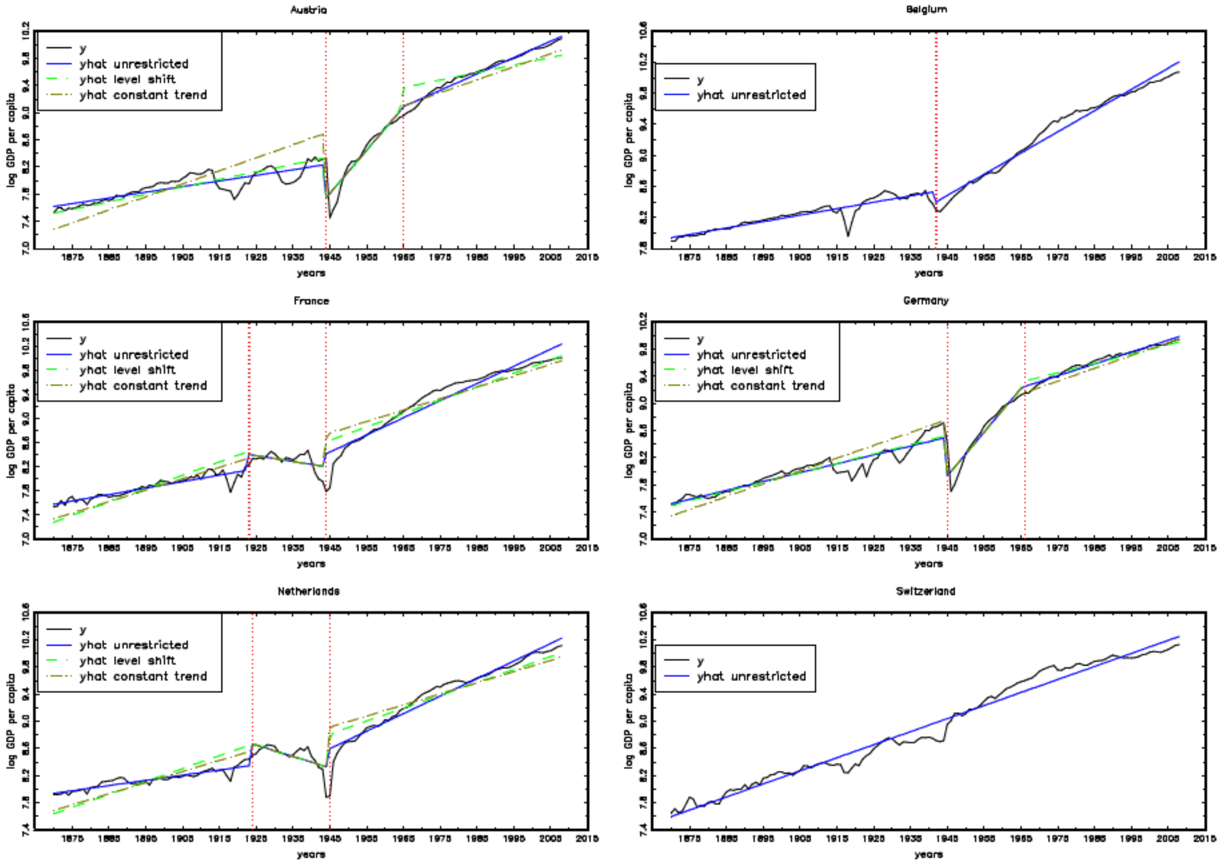

Figure 2: Log of real per capita GDP - Western Europe

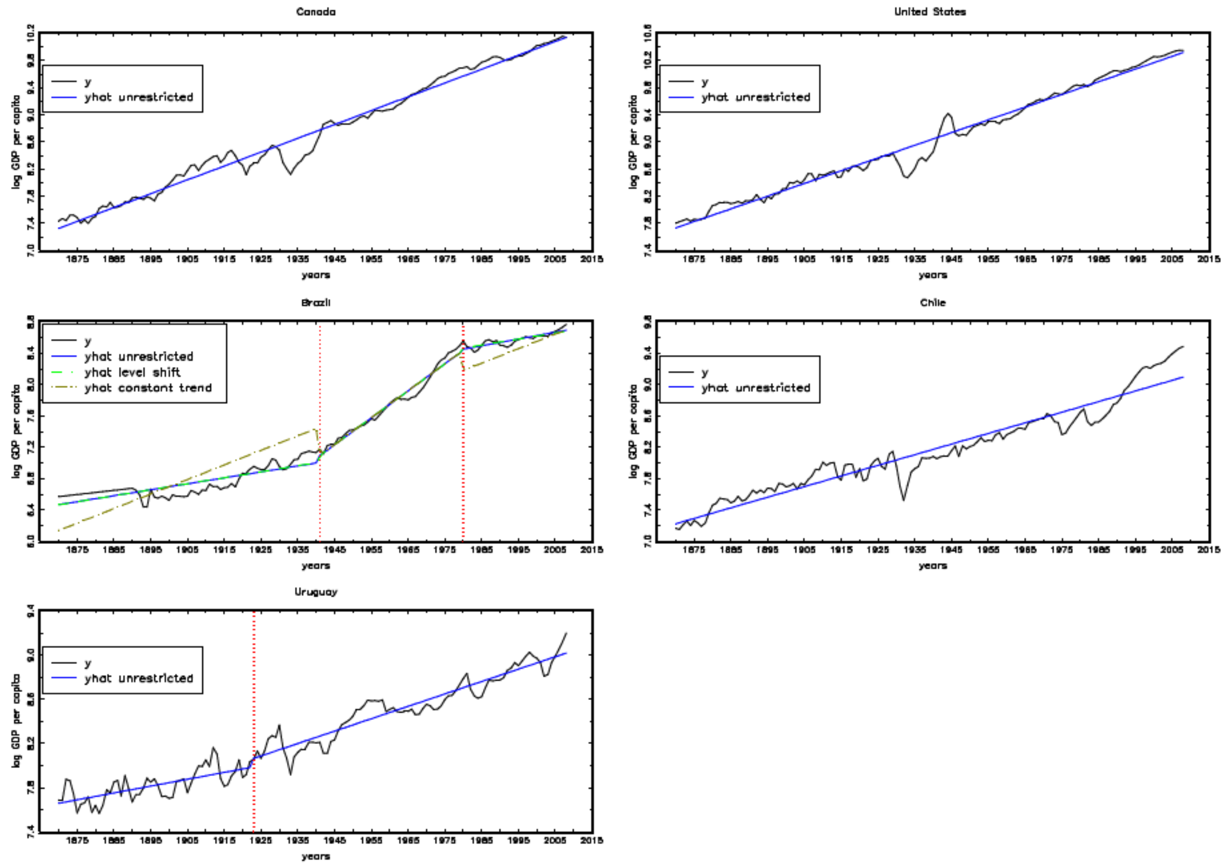

Figure 3: Log of real per capita GDP - North/South America

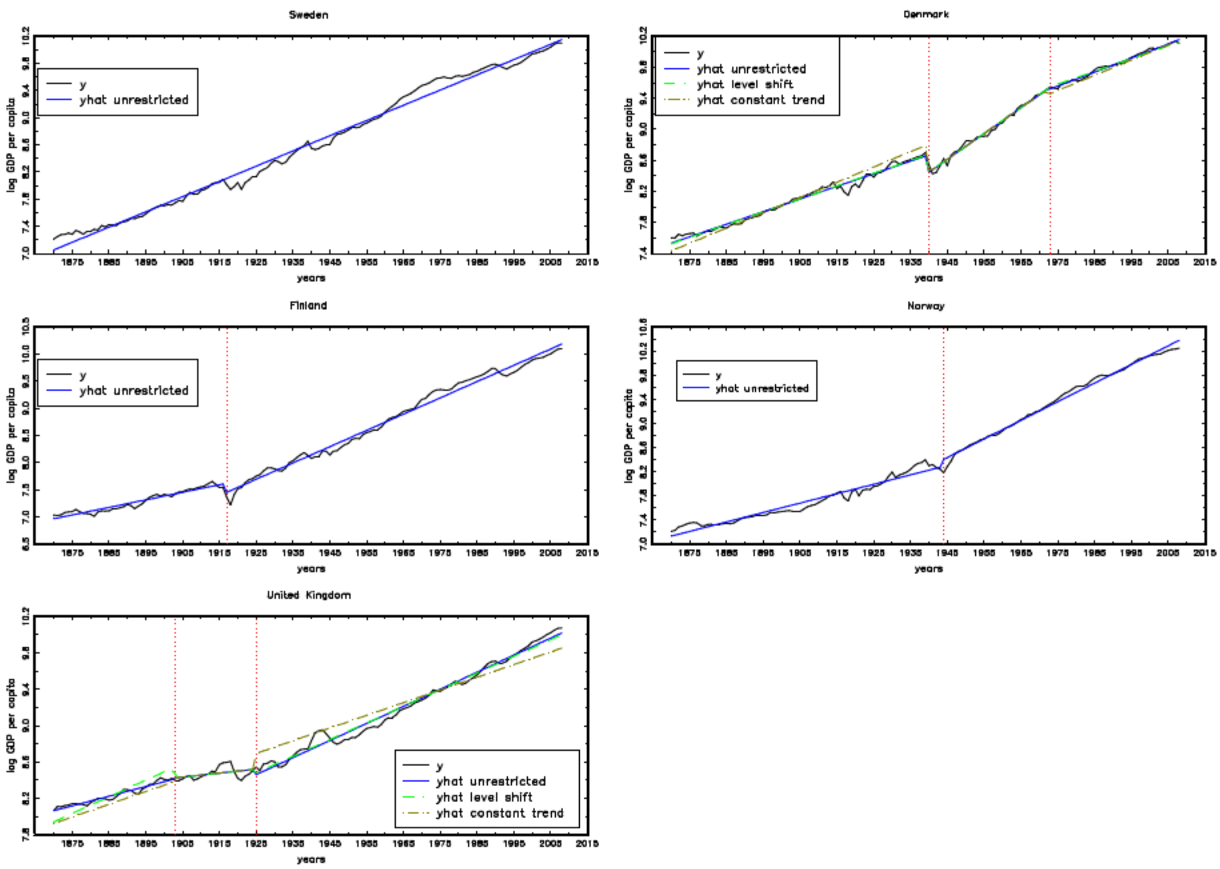

Figure 4: Log of real per capita GDP - Northern Europe

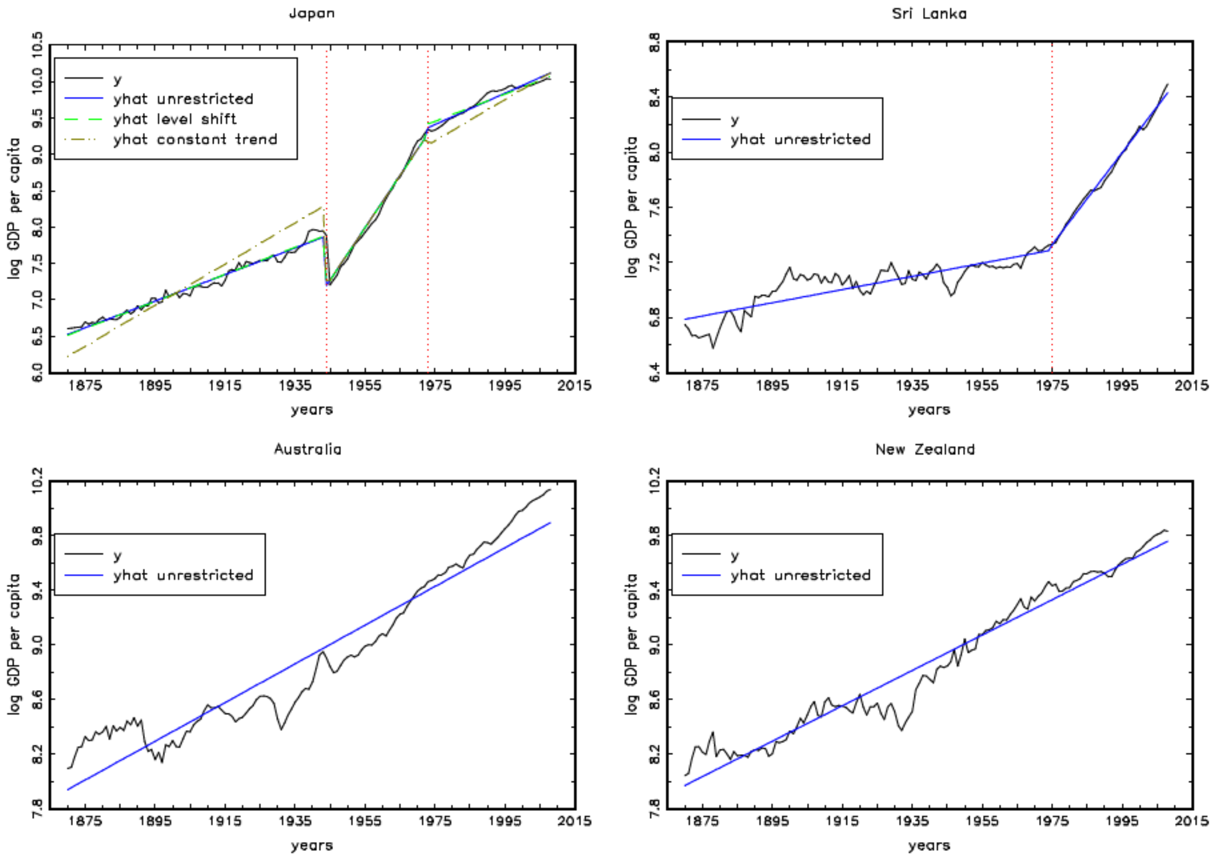

Figure 5: Log of real per capita GDP - Asia and Oceania

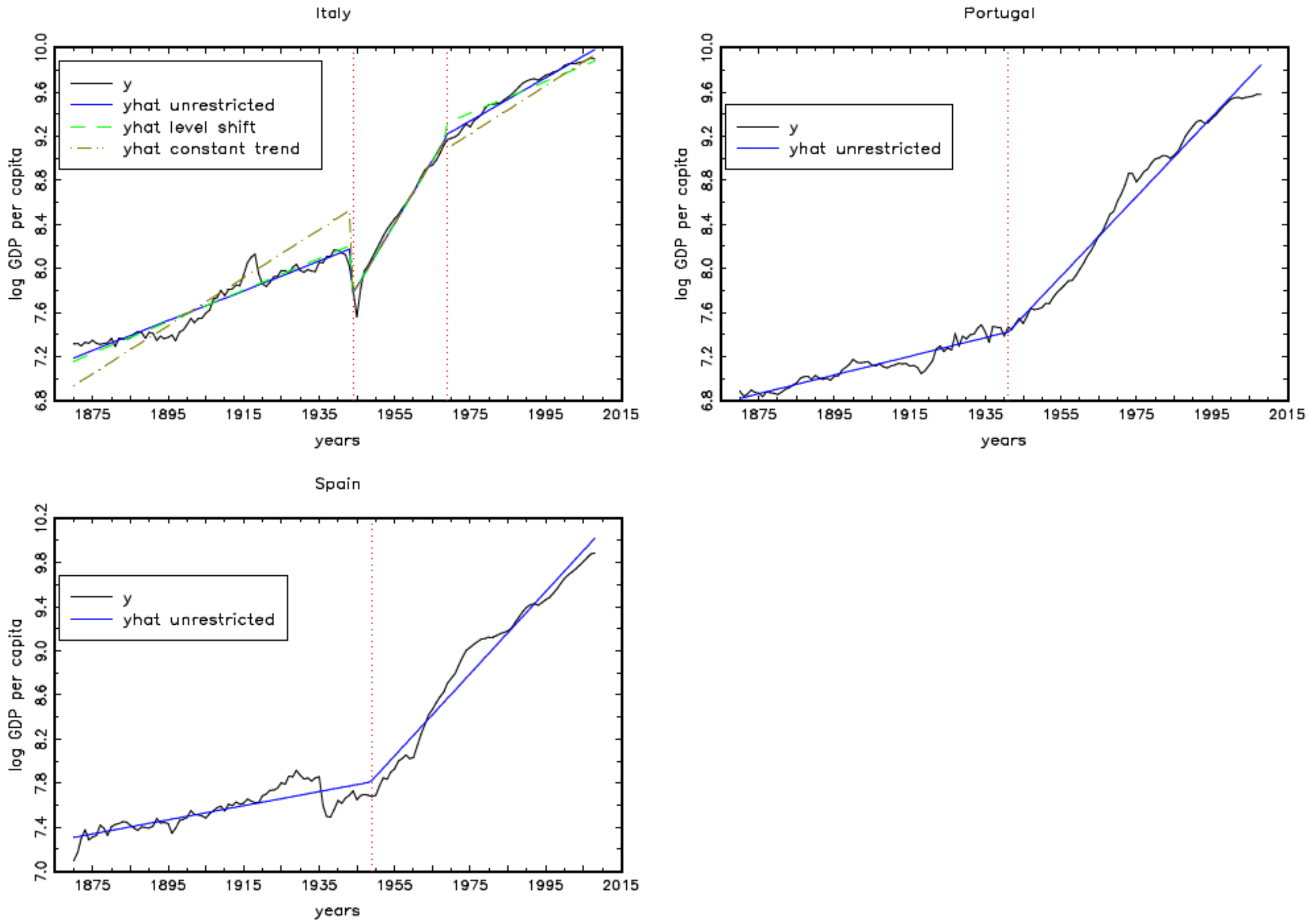

Figure 6: Log of real per capita GDP - Southern Europe

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