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Valerio Ercolani
João Valle e Azevedo

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Please address correspondence to
Valerio Ercolani
Banco de Portugal, Economics and Research Department
Av. Almirante Reis 71, 1150-012 Lisboa, Portugal;
Tel.: 351 21 313 0065, email: vercolani@bportugal.pt

Banco de Portugal
EUROSYSTEM

BANCO DE PORTUGAL

Av. Almirante Reis, 71

1150-012 Lisboa

www.bportugal.pt

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The Effects of Public Spending Externalities

Valerio Ercolani*

Bank of Portugal

João Valle e Azevedo

Bank of Portugal

Nova School of Business and Economics

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Abstract

We take to the data an RBC model with two salient features. First, we allow government consumption to directly affect the marginal utility of consumption. Second, we allow public capital to affect the productivity of private factors. On the one hand, private and government consumption are estimated to be substitute goods. As a consequence, the estimated response of private consumption to a government consumption shock is negative, as in models with separable government consumption, but such response is much stronger. Further, substitutability makes labor supply to react less, so the estimated output multiplier is lower than in models with separabilities, peaking - on impact - at 0.39. On the other hand, non-defense public investment enhances mildly or negligibly, depending on the specification, the productivity of private factors. In those specifications where non-defense public investment is found to be productive, a non-defense investment shock generates the following estimated responses (after several quarters): a positive reaction for private consumption, Tobin's q, private investment and real wages. Unlike models with unproductive government investment, the estimated output multiplier builds up over time, starting well below one on impact, then reaching 0.93 after three years and 1.44 after six.

JEL classification: E32, E62

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*Corresponding author: E-mail: valerio.ercolani@gmail.com.

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1 Introduction

Assessing the mechanisms through which government spending affects the private sector occupies a large portion of the macroeconomic literature (see Ramey 2011a for a review of the leading theories on the effects of government spending). We focus on channels resulting from "externalities" produced by government spending and, importantly, we aim at understanding and measuring the effects of such externalities. By externality we mean that government consumption can affect households' marginal utility of consumption and, therefore, the level of consumption itself. This occurs if some items of government spending act as substitutes or complements for private consumption. For example, public health care can reduce the need for private health services, or, public education services can reduce the need for private tutors and schools but, on the other hand, increase the demand for textbooks or personal computers. These potential relations between private consumption and different items of public spending make government consumption, on the aggregate, to be either substitute or complement for private consumption. Thus, omitting a priori the potential channel of substitutability/complementarity can produce a bias in the estimates of the response of private consumption to a government consumption shock. Also, government investment can create externalities for the private sector. Specifically, public capital can act as a shifter of the productivity of private factors, such that an increase in public investment has the potential to create a positive wealth effect, affecting the dynamics of private investment as well as of consumption. For example, an efficient system of public highways built in place of an old route can enhance the productivity of private factors for firms operating in that area (e.g. by fostering within-country trade). Again, omitting this role for government investment can bias estimates of the effects of such investment.

The standard hypothesis of the bulk of RBC (e.g. Baxter and King 1993) and new Keynesian models (e.g. Smets and Wouters 2007) is that private and government consumption are separable in preferences or that government consumption is a pure waste of resources.[1] Moreover, even public capital is often omitted since it is assumed to be "unproductive". Within these models, the so called *negative wealth effect* is the main driver of spending shocks. If government spending increases, then the present discounted value of taxes to be paid by households also increases and so permanent income is lower. The well known consequence of this effect is the negative correlation between public spending and private consumption conditional on government spending shocks.[2] The negative wealth effect positively impacts on labor supply which, in turn,

[1]However, it is worth noting that already Bailey (1971) and Barro (1981) identified the substitutability relation between private and government consumption as a possible mechanism through which public spending exerts its effects on private decisions.

[2]Notable exceptions can be found in the literature. For example, Galí, Lopez-Salido and Vallés (2007) introduce a market imperfection in a new Keynesian model, namely, a share of the population cannot borrow or lend. Because of this, aggregate consumption can increase after a government spending shock. Following a different route, Linnemann (2006) builds a neoclassical

generates an increase in output and a decrease in real wages.[3] Finally, private investment usually falls in response to a (temporary) government spending shock.

Against this background, it is clear that the externalities we explore have the potential to reverse the usual sign of the model variables' reactions to government spending shocks or, even if the sign is correct, to assess the likely bias of that response. For example, evidence of substitutability (complementarity) between private and government consumption leads, ceteris paribus, to a negative (positive) response of private consumption to a government consumption shock. Also, allowing for the possibility that public capital affects the productivity of private factors may induce government investment shocks to create positive wealth effects in the economy, hence generating positive reactions for private consumption, investment, and real wages, ceteris paribus. Last but not least, if one is interested on the magnitude of the output effect of fiscal stimulus, the responses obtained in Uhlig (2010) - focusing on distortionary taxes - or in Christiano et al. (2011) - focusing on the zero lower bound - or in Monacelli, Perotti and Trigari (2010) - focusing on the labor market - can likewise be affected by the government spending externalities.

Our objective is thus answering three main questions: does the standard hypothesis of separability between private and government consumption hold in the data? Is there evidence that public capital affects the productivity of private factors? What are the effects produced by these externalities? To answer these questions we add the two "externalities channels" into an otherwise standard RBC model. First, we allow government consumption to directly affect the marginal utility of consumption. Second, we allow public capital to affect the productivity of private factors. Further, the model is augmented with features that have proven useful to fit the data well: external habit formation in consumption, monopolistic competition in factor markets, investment adjustment costs, costs of adjusting capacity utilization and (possibly) distortionary taxation. Uncertainty arises from six orthogonal shocks: a preference shock, total factor productivity, investment adjustment, wage markup (wedge) as well as public consumption and public investment shocks. We will mainly consider public investment split into defense and non-defense items, which results in the addition of another shock. We estimate the model using standard Bayesian techniques, using U.S. data from 1969 through to 2008.

Estimation of various versions of the model indicates that government and private consumption are substitute goods. As a consequence, the estimated response of private consumption to a government consumption shock is negative, as in models with separable government spending, but the response is much

model in which leisure and consumption are not separable in the utility function. This type of non-separability can allow consumption to react positively to government spending shocks.

[3] Real wages surely react negatively within an RBC model since the labor demand schedule remains unchanged. Under specific versions of sticky prices models, instead, real wages can happen to increase in response to a government spending shock (for details, see Linnemann and Schabert 2003).

stronger. Substitutability makes labor supply to react less to a government consumption shock, so the estimated output multiplier is lower (approximately one third) than the one in models with separabilities, peaking - on impact - at 0.39. This value is directly comparable with the output multipliers found by Mountford and Uligh (2009), which range between 0.44 and 0.23 during the first 12 quarters. Also, non-defense public investment enhances mildly or negligibly, depending on the specification, the productivity of private factors while investment in defense appears not to have any such impact. In our benchmark specification, the mean of the output elasticity to non-defense public capital productivity is estimated to be 0.09, though the associated 90% posterior interval contains values close to zero. Conditional on this specification, a non-defense investment shock generates the following estimated responses: a positive reaction for private consumption, Tobin's q, private investment and real wages. Note that these positive responses tend to manifest themselves after several quarters. Even looking at the multipliers, the long-run effect of a non-defense public investment shock emerges as a crucial characteristic. For example, the estimated output multiplier builds up over time, starting well below one on impact, then reaching 0.93 after three years and 1.44 after six. Note that the output multiplier measured from the corresponding model with unproductive government investment, behaves in the opposite way. It starts higher on impact, i.e. above one, then it falls off over time. Our multipliers partially overlap with the ones found by Baxter and King (1993), Perotti (2004) and Leeper et al. (2010), see sections 2 and 4.4 for details.

The remainder of the paper is organized as follows: Section 2 gives an overview of the empirical evidence and relates it to our work. Section 3 outlines the model while Section 4 describes the estimation exercise and results. In Section 5, concluding remarks and future extensions are presented.

2 Review of the empirical evidence

To the best of our knowledge, all the estimates of the degree of substitutability/complementarity between private consumption and government consumption, except for Bouakez and Rebei (2007), are obtained within partial equilibrium models and often based on Euler equations. The empirical evidence obtained by estimating Euler equations is not conclusive. Aschauer (1985) finds a significant degree of substitutability between the two variables of interest in the case of the U.S. whereas Amano and Wirjanto (1998) find weak complementarity. Focusing on the U.K., Ahmed (1986) finds substitutability while Karras (1994), examining the relationship between private and public consumption across thirty countries, finds that the two types of goods are best described as complementary (but often unrelated). Fiorito and Kollitznas (2004) split government consumption in two groups named "public goods" and "merit goods". The first includes spending in defense, security forces and judicial system, the second contains health, education and other services that can be provided privately. Using dynamic panel methods motivated by Euler equations they show that, for twelve European countries, public goods slightly substitute while merit goods complement private consumption. Using general equilibrium models represents a contribution mainly because relevant omitted variables problems can arise in partial equilibrium, since, for instance, changes in agent's permanent income are not accounted for. More precisely, it's likely that the estimate of the degree of substitutability/complementarity could be spoiled by the *negative wealth effect* if correct instruments are not used in the estimation of Euler equations.

Focusing on general equilibrium models, Bouakez and Rebei (2007) estimate an RBC model and find complementarity between private and government consumption. Then, as a result of a calibration exercise, they find a positive reaction of private consumption to a government consumption shock. We differ from their work in several aspects other than in the results. First, unlike us, they don't use public spending data throughout the estimation. Second, they fix the parameter measuring the weight of private consumption in the effective consumption aggregator (see section 3.1 for details) whereas we estimate it since this parameter is fundamental to establish whether or not government consumption affects the welfare of agents. Third, we consider not only the externalities arising from government consumption but also the ones coming from the capital part, i.e. we allow public capital to be "productive". Fourth, other than habit in consumption, we augment the RBC model with features that help fitting the data well: monopolistic competition in product and labor markets, costs of adjusting investment as well as capacity utilization and (possibly) distortionary taxation. Finally, we produce impulse responses to spending shocks that are an internal outcome of the estimation procedure.

The last class of papers focuses on the importance of public capital in boosting output growth. Aschauer (1989) estimates an aggregate production for the U.S. economy, with inputs being labor, private capital but also public capital, finding that the output elasticity of government capital is 0.39. Following a similar approach, Finn (1993) estimates much lower output elasticities of various items of government capital (the biggest is 0.16 for highways) and surrounded by great uncertainty. The implication of these two papers is that public capital is an important explanatory factor for changes in the productivity of the economy. Other authors like Tatom (1991) find, instead, that the best estimate for the mentioned elasticity is zero. Belo and Yu (2011) report movements in stock returns compatible with a specification, very similar to ours, where public investment is directly productive. Unlike all these papers, we estimate the productivity shift caused by public capital within a general equilibrium model. As a consequence, we can study the effect of a government investment shock, controlling for general equilibrium effects. There are virtually no estimates of the effects of a government investment shock within an estimated general equilibrium model. However, it is worth noting the paper by Leeper et al. (2010) which analyses, within an estimated general equilibrium model, scenarios with different values for the output elasticity of public capital. Conditional on choosing the value of 0.1 for this elasticity, they find the mean of the output multiplier ranging from 0.90 to 1.14 within the first three years. Furthermore, Baxter and King (1993), within a fully calibrated framework, find a long-run output multiplier equal to 4.12, conditional on choosing 0.1 for the output elasticity of public capital. Straub and Tchakarov (2007) conduct a calibration exercise for the Euro area within a general equilibrium framework, finding that under reasonable parameter values both permanent and temporary public investment shocks generate a much larger multiplier than the one obtained upon exogenous increases in government consumption. Finally, turning to the analysis of VARs, Perotti (2004) finds that a government investment shock creates an output multiplier ranging between 0.17 and 1.68 within the first five years.

3 The model

We now describe our model economy, making clear the problems solved by households and firms. We also describe the behavior of the government, or fiscal authority. In a nutshell, we will be looking at an otherwise standard RBC model which crucially includes two ingredients aimed at assessing the role of government consumption and investment on private decisions. Specifically, government consumption is allowed to affect the marginal utility of consumption and public investment is allowed to enhance the productivity of private factors by entering the final goods' production function. Further, we borrow from the literature ingredients that have proven useful to fit the data: external habit formation in consumption, monopolistic competition in factor markets, investment adjustment costs, costs of adjusting capacity utilization and (possibly) distortionary taxation. Uncertainty arises from six orthogonal shocks: a preference shock, total factor productivity, investment adjustment, wage markup (wedge) as well as public consumption and public investment shocks. Most often we break public investment into defense and non-defense items which results in the addition of another shock.

3.1 Households

The economy is populated by a continuum of households. We assume that the representative household derives utility from effective consumption, $\tilde{C}_t$, and disutility from working, L_t, in each quarter t. Effective consumption is assumed to be an Armington aggregator of private consumption, C_t, and government consumption, G_t:

$$\tilde{C}_t = \left[\phi \left(C_t\right)^{\frac{v-1}{v}} + \left(1-\phi\right) G_t^{\frac{v-1}{v}}\right]^{\frac{v}{v-1}}, \tag{1}$$

where ϕ is the weight of private consumption in the effective consumption aggregator, and $\nu \in (0;\infty)$ is the elasticity of substitution between C_t and G_t.[4] Note that if $\phi = 1$ then $\tilde{C}_t = C_t$ and the standard hypothesis of considering G_t as pure waste of public resources emerges. The lifetime expected utility is given by:

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta^k e^{\varepsilon_t^b} \left[\frac{\left(\tilde{C}_t - h\tilde{C}_{t-1}^A\right)^{1-\sigma_c}}{1-\sigma_c} - \chi \frac{1}{1+\sigma_L} \left(L_t\right)^{1+\sigma_L} \right] \right\}, \tag{2}$$

where σ_c denotes the degree of relative risk aversion, σ_L is the inverse of the Frisch elasticity of labor supply. The parameter $h \in (0;1)$ measures the degree of habit formation in effective consumption whereas $\beta \in (0,1)$ is the subjective discount factor. The χ is a positive parameter. $\tilde{C}_{t-1}^A$ is the aggregate level of effective consumption at time $t-1$ which creates external habit formation in consumption. ε_t^b represents a preference shock, assumed to follow a first-order autoregressive process with an i.i.d.-normal error term: $\varepsilon_t^b = \rho_b \varepsilon_{t-1}^b + \eta_t^b$. We impose separability between consumption and labor for tractability while ensuring that steady-state hours is constant along the steady-state growth path. Given the assumption of separability we are required to use log utility in consumption ($\sigma_c = 1$), so as to guarantee the existence of a steady-state growth path, given the fact that we consider growth of technology (or of the economy) and the production function is neoclassical.[5]

Household maximize their lifetime expected utility by choosing consumption, C_t, labor supply, L_t, next period's physical capital stock, K_{t+1}, the level of investment, I_t, and the intensity with which the installed capital stock is utilized, u_t. Further, in order to justify the existence of a representative agent we complete the markets by making agents able to trade a full set of one period state-contingent claims, paying x_{t+1}^h at $t+1$ at the cost $E_t[r_{t,t+1}x_{t+1}^h]$, where $r_{t,t+1}$ is a stochastic discount factor. Here, we present the version of the model with distortionary taxation on labor, consumption and capital, with marginal rates given, respectively, by τ^w, τ^c and τ^k. The agents thus face the following budget constraint (expressed in real terms):

[4] When $\nu = 0$, we have a "Leontief" aggregator, i.e. C_t and G_t become perfect complements. When $\nu = 1$, we have a "Cobb-Douglas" aggregator of the form $\tilde{C}_t = C_t^{\phi} G_t^{(1-\phi)}$. As $\nu \to \infty$, we have a linear aggregator of the form $\tilde{C}_t = \phi C_t + (1-\phi) G_t$, the two goods are perfect substitutes.

[5] KPR preferences, see King, Plosser and Rebelo (1988), could achieve the same objective while not imposing separability between consumption and labor.

$$(1+\tau^c)C_t + I_t + E_t[r_{t,t+1}x_{t+1}^h] = x_t^h + (1-\tau^w)W_tL_t \tag{3}$$
$$+(1-\tau^k)\left[r_t^k u_t - a\left(u_t\right)\right] K_t + D_t - T_t,$$

where r_t^k is the net return on capital, W_tL_t is labor income, $a\left(u_t\right) = \gamma_1\left(u_t - 1\right) + \frac{\gamma_2}{2}\left(u_t - 1\right)^2$ represents the cost of using capital at intensity u_t (see, e.g., Schmitt-Grohé and Uribe 2006), D_t are the dividends paid by household-owned firms while T_t are lump-sum taxes/transfers to/from the government. Since Ricardian equivalence holds in the model, we abstract from government debt and assume that the government balances its budget.

The capital stock evolves according to the following equation:

$$K_{t+1} = \left(1-\delta_k\right) K_t + I_t \left[1 - S\left(e^{\varepsilon_t^I}\frac{I_t}{I_{t-1}}\right)\right], \tag{4}$$

where δ_k is the depreciation rate and the function $S(.)$ introduces investment adjustment costs à la Christiano, Eichenbaum, and Evans (2005). Specifically, $S(.) = \frac{\kappa}{2}\left(e^{\varepsilon_t^I}\frac{I_t}{I_{t-1}} - e^{\overline{\gamma}}\right)^2$, where ε_t^I is a shock to the investment cost function assumed to follow a first-order autoregressive process with an i.i.d.-normal error term ($\varepsilon_t^I = \rho_I\varepsilon_{t-1}^I + \eta_t^I$), and $\overline{\gamma}$ is the steady state growth rate of productivity (see next section for details).

3.2 Firms

We assume there is a continuum of monopolistically competitive firms indexed by $j \in [0,1]$ each of which produces a single variety of final goods, $Y_{j,t}$. They sell $Y_{j,t}$ at price $P_{j,t}$ to the final goods competitive firms, which combine the differentiated final goods $Y_{j,t}$ in the same way households would choose, using a standard Dixit-Stiglitz aggregator:

$$Y_t = \left[\int_0^1 \left(Y_{j,t}\right)^{\frac{1}{1+\lambda_{p,ss}}} dj\right]^{1+\lambda_{p,ss}}, \tag{5}$$

where $\frac{1+\lambda_{p,ss}}{\lambda_{p,ss}}$ represents the elasticity of substitution across goods varieties. The competitive final goods firm takes $P_{j,t}$ as given and supplies goods Y_t at price P_t to the households and government, for which it has to pay a total cost equal to $\int_0^1 P_{j,t}Y_{j,t}dj$. The profit maximization conditions in the final goods sector generate the following demand schedule for the varieties of final goods:

$$Y_{j,t} = \left(\frac{P_{j,t}}{P_t}\right)^{-\frac{1+\lambda_{p,ss}}{\lambda_{p,ss}}} Y_t, \tag{6}$$

while zero profit makes the price of the final good (which we normalize to 1) equal to:

$$P_t = \left[\int_0^1 (P_{j,t})^{-\frac{1}{\lambda_{p,ss}}} dj \right]^{-\lambda_{p,ss}} = 1. \tag{7}$$

Each of the monopolistically competitive firms produces a single variety of final goods, $Y_{j,t}$, using as inputs capital services, $K_{j,t}$ and labor services, $N_{j,t}$ from competitive suppliers. Moreover, we augment the standard production function with K_t^G, representing the "productivity" of public capital. The production function is given by:

$$Y_{j,t} = \max(A_t K_{j,t}^{\alpha} L_{j,t}^{1-\alpha} (K_t^G)^{\theta_g} - \Phi, 0), \tag{8}$$

where A_t is a productivity shock such that the process for $\ln(A_t)$ has a unit-root and evolves according to:

$$\ln(A_t) = \overline{\gamma} + \ln(A_{t-1}) + \varepsilon_t^a,$$

where $\overline{\gamma}$ is the steady-state growth rate of productivity (and hence of the economy) and $\varepsilon_t^a = \rho_a \varepsilon_{t-1}^a + \eta_t^a$, where η_t^a is an i.i.d.-normal sequence. The parameter Φ represents a fixed cost of production while $\theta_g \in (0; \infty)$ is the output elasticity to public capital productivity, measuring the effect of public capital productivity on firm's output and thus on the productivity of private factors.

The productivity of public capital is assumed to evolve according to:

$$K_{t+1}^G = (1 - \delta_{Kg}) K_t^G + \xi_t^{ig}, \tag{9}$$

where δ_{Kg} is the depreciation rate and ξ_t^{ig} is the public investment rate (in our case, public investment, I_t^g, over total output, i.e. $\xi_t^{ig} = I_t^g / Y_t$). We will later specify how ξ_t^{ig} evolves. For now, we refer that ξ_t^{ig} follows a stationary process (which seems consistent with the data), implying that K_t^G is stationary. This is convenient for technical reasons (see Belo and Yu 2011 and references therein for a similar specification and reasoning) and avoids keeping track of - poorly measured - public capital.

The existence of an economy-wide competitive factor market implies that all firms producing final goods varieties pay the same rental rate, r_t^k, and the same real wage, W_t, while taking into account the demand for their product. Cost minimization subject to the production technology (8), assuming output is positive, yields first order conditions for inputs which can be expressed as relative factor demands:

$$\frac{K_t}{N_t} = \frac{\alpha}{1-\alpha}\frac{W_t}{r_t^k}, \tag{10}$$

where we omit the index j for firms.

Finally, these firms take aggregate output Y_t and the price level P_t as given while setting $P_{j,t}$ so as to maximize the present value of the flow of profits. This results as usual in $P_{j,t}$ as a markup, equal to $\lambda_{p,ss}$, over marginal cost.

We will mainly consider in the empirical application versions of the model with public investment split into defense and non-defense items. The production function for final goods varieties producers becomes $Y_{j,t} = A_t K_{j,t}^{\alpha} N_{j,t}^{1-\alpha} (K_t^G)^{\theta_g} (K_t^{G,def})^{\theta_{g,def}} - \Phi$, where the productivity of defense capital, $K_t^{G,def}$, is assumed to evolve according to $K_{t+1}^{G,def} = (1-\delta_{Kg,def})K_t^{G,def} + \xi_t^{ig,def}$ and $\xi_t^{ig,def}$ is defense investment over output, i.e. $\xi_t^{ig,def} = I_t^{g,def}/Y_t$. Public investment, I_t^g, is then assumed to exclude defense items.

3.3 Labor market

There is a continuum of monopolistically competitive households, indexed by $i \in [0;1]$, which set their wage rate $W_{i,t}$ and supply labor hours $L_{i,t}$. They sell labor services to a competitive labor aggregator sector which combines differentiated labor hours in the same way intermediate goods firms would choose, in a Dixit-Stiglitz form:

$$L_t = \left[\int_0^1 (L_{i,t})^{\frac{1}{1+\lambda_{w,t}}} di\right]^{1+\lambda_{w,t}}, \tag{11}$$

where $\frac{1+\lambda_{w,t}}{\lambda_{w,t}}$ represents the elasticity of substitution across labor varieties. Note that the stochastic parameter $\lambda_{w,t}$ evolves as $\lambda_{w,t} = (1-\rho_w)\lambda_{w,ss} + \rho_w \lambda_{w,t-1} + \eta_t^w$, where η_t^w is an i.i.d.-normal sequence. The competitive labor aggregator takes $W_{i,t}$ as given and supplies labor services L_t at wage rate W_t to the final goods varieties's firms, for which it has to pay a total cost equal to $\int_0^1 W_{i,t}L_{i,t}di$. This generates the following demand schedule by the labor aggregator:

$$L_{i,t} = \left(\frac{W_{i,t}}{W_t}\right)^{\frac{1+\lambda_{w,t}}{\lambda_{w,t}}} L_t. \tag{12}$$

Households exploit the demand for $L_{i,t}$ in order to set $W_{i,t}$ so as to maximize (2), taking aggregate labor demand, L_t, and aggregate nominal wage, W_t, as given. This results in $W_{i,t}$ as a markup, equal to $\lambda_{w,t}$, over the marginal rate of substitution between consumption and leisure. Finally, zero profits in the aggregator sector guarantee that in equilibrium the aggregate wage rate W_t is given by:

$$W_t = \left[\int_0^1 (W_{i,t})^{-\frac{1}{\lambda_{w,t}}} di\right]^{-\lambda_{w,t}}. \quad (13)$$

3.4 Government

First, we specify the evolution of public consumption, G_t, and public investment, I_t^g. These can always be expressed as a varying fraction of output:

$$G_t = \xi_t^g Y_t \ ; \quad I_t^g = \xi_t^{ig} Y_t.$$

We further specify ξ_t^g and ξ_t^{ig} as follows:

$$\xi_t^g = \exp(\varepsilon_t^g + ss^g)/(1 + \exp(\varepsilon_t^g + ss^g)) \quad ; \quad \xi_t^{ig} = \exp(\varepsilon_t^{ig} + ss^{ig})/(1 + \exp(\varepsilon_t^{ig} + ss^{ig})).$$

This formulation (basically a reparametrization) ensures ξ_t^g and ξ_t^{ig} are always between 0 and 1. The exogenous shocks ε_t^g and ε_t^{ig} to government consumption and investment, respectively, follow an autoregressive process:

$$\varepsilon_t^g = \rho_g \varepsilon_{t-1}^g + \eta_t^g \quad (14)$$

$$\varepsilon_t^{ig} = \rho_{ig} \varepsilon i_{t-1}^{ig} + \eta_t^{ig}, \quad (15)$$

where η_t^g and η_t^{ig} are normal i.i.d. and mutually independent with mean zero. Note that ss^g and ss^{ig} fix the average (or steady-state) levels of ξ_t^g and ξ_t^{ig} (denoted, respectively, by $\xi^{g,ss}$ and $\xi^{ig,ss}$). Therefore:

$$ss^g = \log(\xi^{g,ss}/(1 - \xi^{g,ss})) \ \ ; \ \ ss^{ig} = \log(\xi^{ig,ss}/(1 - \xi^{ig,ss})).$$

Since we often split public investment into defense and non-defense items, the model is augmented with the equations:

$$\begin{aligned} I_t^{g,def} &= \xi_t^{ig,def} Y_t \\ \xi_t^{g,def} &= \exp(\varepsilon_t^{g,def} + ss^{g,def})/(1 + \exp(\varepsilon_t^{g,def} + ss^{g,def})) \\ \varepsilon_t^{ig,def} &= \rho_{ig,def} \varepsilon i_{t-1}^{ig,def} + \eta_t^{ig,def}, \end{aligned}$$

where the superscript def refers to defense items, $I_t^{g,def}$ denotes defense investment, $ss^{g,def} = \log(\xi^{g,def,ss}/(1-\xi^{g,def,ss}))$ and $\eta_t^{ig,def}$ is a normal i.i.d. defense investment shock with mean zero. Given this formulation, the paths of ξ_t^g, ξ_t^{ig} and $\xi_t^{ig,def}$ are assumed to be fully exogenous while the paths for G_t, I_t^g or $I_t^{g,def}$ are not. For example, if there is a fall in total factor productivity and output falls government consumption and investment will on average fall. Thus, in our benchmark specification, we rule out the role of automatic stabilizers.

Since the government balances its budget, the following constraint holds for the government:

$$\tau^c C_t + \tau^w W_t L_t + \tau^k \left[r_t^k u_t - a(u_t) \right] K_t + T_t = G_t + I_t^g + I_t^{g,def}. \tag{16}$$

3.5 Market clearing

The labor market is in equilibrium when labor demanded by firms, $\int_0^1 L_{j,t} dj$, equals the differentiated labor services supplied by households, $\int_0^1 L_{i,t} di = L_t$, at the aggregate wage rate W_t. The market for capital is in equilibrium when the demand for capital services by the firms equals the capital supplied by households at the market rental rate r_t^k. The market for one period state-contingent claims is in equilibrium when net supply is zero at market prices.

Finally, the final goods market is in equilibrium when the supply by firms equals the demand by households and government:

$$Y_t = C_t + I_t + a(u_t) K_t + G_t + I_t^g + I_t^{g,def}. \tag{17}$$

3.6 The "externalities" mechanisms

Here, we further clarify the functioning of the two non-standard "externalities" channels under analysis: the substitutability/complementarity mechanism and the one for the productivity of public investment.

To analyze the first channel, we look at the derivative of the instant marginal utility of consumption with respect to government consumption. Given a steady-state level of consumption, this is given, in log-linearized form, by:

$$U_{cg} = (1-\phi) \left(G/\widetilde{C} \right)^{\frac{v-1}{v}} \left(\frac{1}{v} - \frac{\sigma_c}{(1-h)} \right), \tag{18}$$

where G and $\tilde{C}$ are the steady state levels of government consumption and effective consumption, respectively. The parameters which are important in delivering the sign to U_{cg} are the elasticity of substitution between private and government consumption, v, the coefficient of relative risk aversion, σ_c, and the level of habit persistence, h. When U_{cg} is greater than 0, private and government consumption are defined to be complements, whereas, when U_{cg} is less than 0, private and government consumption are defined to be substitutes. When U_{cg} is equal to 0, private and government consumption are not related in preferences. Obviously, if we set ϕ equal to 1 government consumption does not enter the utility function and U_{cg} collapse to zero. For values of ϕ less than one, U_{cg} could be either positive or negative depending on the other parameters in U_{cg}. In particular, if we use log utility (or $\sigma_c = 1$), U_{cg} is strictly positive if $v < 1 - h$ and negative otherwise. Since $0 \leq h < 1$, $v > 1$ guarantees that U_{cg} is negative.[6]

The potential role for the productivity of public capital is clearly visible in the production function $Y_{j,t} = A_t K_{j,t}^{\alpha} L_{j,t}^{1-\alpha} (K_t^G)^{\theta_g} - \Phi$, with $\theta_g \in (0;\infty)$. Conditional on $\theta_g > 0$, K_t^G has a direct effect on firm's output and, as a consequence of this, a positive influence on the productivity of the private factors. The higher θ_g is, the more effective is K_t^G in boosting firm's output and private productivities. To be noted that if $\theta_g = 0$, the standard production function pops up and K_t^G doesn't produce any externality effects.

3.7 Discussion

There are usually two polar positions when discussing the implications of estimated general equilibrium models. One points to the hopeless misspecification of the model, to the unreasonableness of the specified "structural" shocks or features. The other passes lightly over these issues, interprets every shock and formulation as truly structural and argues forcefully on the "speaking of the data" aspect of the exercises. We combine the two positions. We certainly acknowledge the difficulty of interpreting the preference shock, total factor productivity, investment adjustment and wage markup shocks as truly structural shocks, invariant to monetary or fiscal policies. For instance, we accept that movements about the wage markup can be just shifts of relevant margins (wedges, in the analysis of Chari et al. 2009) in the decisions of agents. If this is the case, we retain the formulation of the labor market as a set of equilibrium conditions (resulting in a wedge between the marginal rate of substitution and the wage rate) could be derived from other models. At the same time, we give up on the interpretation of shocks to the wedge as wage markup shocks.

With regards to our main objective in this paper, i.e. to measure the aggregate effects of government spending given specific externalities, it should be noted that we are not in a position to discuss or measure whether the level of public consumption or public investment is far from a social optimum. What we observe in the data are deviations of public spending from a roughly constant fraction of output. It can be that if public investment is below some threshold, the productivity of private factors is seriously affected, but

[6] Note that the described implications hold even if we consider the non linearized version of U_{cg} (available upon request).

we have not observed that in our sample. Or, if it is above, it becomes an inefficient shift of resources in the economy. Similar arguments apply in the case of government consumption. We are in a position to measure the effects of (arguably small) deviations of public investment and consumption from their historical averages, which we interpret as their steady-state values.

3.8 Solution

We start by deriving the first order conditions associated with the households' and firms' problems, combining them with market clearing conditions and exogenous processes while recognizing that all firms and households are ex-ante identical. We then stationarize the variables C_t, I_t, K_t, W_t , Tax_t, T_t, G_t, I_t^g and $I_t^{g,def}$ dividing them by the level of technology, A_t. The same treatment is required for the Lagrange multipliers associated with the budget constraint and the capital accumulation equation, respectively Λ_t and Q_t (Tobin's q). We then rewrite all the equilibrium conditions in terms of the standardized variables.

For example, the Euler equation induced by a risk-free bond, paying gross return R_t^b would be given by:

$$\beta E_t \left[\frac{\Lambda_{t+1}}{\Lambda_t} R_t^b \right] = 1,$$

which becomes $\beta E_t \left[\frac{\Lambda_{t+1}/A_{t+1}}{\Lambda_t/A_t} \frac{A_{t+1}}{A_t} R_t^b \right] = 1$. Given the evolution of A_t, this becomes:

$$\beta E_t \left[\frac{\lambda_{t+1}}{\lambda_t} \exp(-(\overline{\gamma} + \varepsilon_{t+1}^a)) R_t^b \right] = 1,$$

where λ_t is the standardized Lagrange multiplier.

All the equilibrium conditions can be found in the Appendix. Before proceeding to the estimation, we log-linearize the model equations around the deterministic steady state. The exception to log-linearization occurs with the variables ξ_t^g , ξ_t^{ig} and $\xi_t^{ig,def}$, which are fractions of total output.

4 Estimation

We estimate the model described in the previous section using Bayesian techniques, taking as observables (denoted with a superscript *obs*) log differences of quarterly real output (GDP), real consumption, real investment and real wages as well as a particular transformation of public consumption and public investment (the latter split into defense and non-defense items), covering the period 1969Q1-2008Q3, see the Data Appendix for a detailed description of the dataset. We believe that using data until 2011 could open (further) issues of misspecification in our empirical model. For example, the absence of financial frictions and the lack of consideration of the zero lower bound on nominal interest rates could be sources of serious misspecification.

For the first four observables, the mapping of data to variables in the model is made through measurement equations that take into account the fact that the solution of the model is in (log) deviations of the stationarized variables from the steady state, e.g., $\log(Y_t) - \log(Y_{t-1}) = \log(A_t y_t) - \log(A_{t-1} y_{t-1}) = \log(A_t/A_{t-1}) + (\log(y_t) - \log(y^{ss})) - (\log(y_{t-1}) - \log(y^{ss})) = \overline{\gamma} + \varepsilon_t^a + \widehat{y}_t - \widehat{y}_{t-1}$ where hats denote log deviations from the steady-state. Thus:

$$\log(Y_t^{obs}) - \log(Y_{t-1}^{obs}) = \widehat{y}_t - \widehat{y}_{t-1} + \overline{\gamma} + \varepsilon_t^a \quad \text{(real GDP)} \tag{19}$$

$$\log(C_t^{obs}) - \log(C_{t-1}^{obs}) = \widehat{c}_t - \widehat{c}_{t-1} + \overline{\gamma} + \varepsilon_t^a \quad \text{(real Consumption)} \tag{20}$$

$$\log(I_t^{obs}) - \log(I_{t-1}^{obs}) = \widehat{i}_t - \widehat{i}_{t-1} + \overline{\gamma} + \varepsilon_t^a \quad \text{(real Investment)} \tag{21}$$

$$\log(W_t^{obs}) - \log(W_{t-1}^{obs}) = \widehat{w}_t - \widehat{w}_{t-1} + \overline{\gamma} + \varepsilon_t^a \quad \text{(real Wage).} \tag{22}$$

As for public consumption and investment as a fraction of output, the following measurement equations are employed:

$$\log(\xi_t^{g,obs}/(1-\xi_t^{g,obs})) = \varepsilon_t^g + \log(\xi^{g,ss}/(1-\xi^{g,ss})) \quad \text{(public Consumption)} \tag{23}$$

$$\log(\xi_t^{ig,obs}/(1-\xi_t^{ig,obs})) = \varepsilon_t^{ig} + \log(\xi^{ig,ss}/(1-\xi^{ig,ss})) \quad \text{(public Investment),} \tag{24}$$

after recognizing we have specified ξ_t^g and ξ_t^{ig} as:

$$\xi_t^g = \exp(\varepsilon_t^g + ss^g)/(1+\exp(\varepsilon_t^g + ss^g)) \quad ; \quad \xi_t^{ig} = \exp(\varepsilon_t^{ig} + ss^{ig})/(1+\exp(\varepsilon_t^{ig} + ss^{ig})),$$

where $ss^g = \log(\xi^{g,ss}/(1-\xi^{g,ss}))$ and $ss^{ig} = \log(\xi^{ig,ss}/(1-\xi^{ig,ss}))$. Also, since public investment will be split, most of the times, into defense and non-defense items, the following measurement equation is added:

$$\log(\xi_t^{ig,def,obs}/(1-\xi_t^{ig,def,obs})) = \varepsilon_t^{ig,def} + \log(\xi^{ig,def,ss}/(1-\xi^{ig,def,ss})) \quad \text{(public Investment, defense),}$$

recalling that $\xi_t^{ig,def} = \exp(\varepsilon_t^{ig,def} + ss^{ig,def})/(1+\exp(\varepsilon_t^{ig,def} + ss^{ig,def}))$ and $ss^{ig,def} = \log(\xi^{ig,def,ss}/(1-\xi^{ig,def,ss}))$.

Bayesian estimation entails specifying prior distributions for the parameters that are not fixed. Let $P(\theta|m)$ be the prior distribution of the parameter vector $\theta \in \Theta$ for some model $m \in M$ and $L(Y_T|\theta, m)$

be the likelihood function for the observed data $Y_T = \{y_t\}_{t=1}^{T}$, conditional on the parameter vector θ and model m. The likelihood is computed starting from the solution to the log-linear approximation of the model. The solution can be cast in state-space form which makes easy the application of the Kalman filter and thus computation of the likelihood. The posterior distribution of the parameter vector θ for model m, $P(\theta|Y_T, m)$, is then obtained combining the likelihood function for X_T with the prior distribution of θ:

$$P(\theta|Y_T, m) \propto L(Y_T|\theta, m)P(\theta|m), \tag{25}$$

$P(\theta|Y_T, m)$ can be numerically maximized to obtain the mode of the posterior distribution, which is often seen as a point estimate of the parameter vector θ. Use of the Metropolis-Hastings algorithm allows to obtain numerically the distribution $P(\theta|Y_T, m)$ as well as distributions of functions of the parameter vector θ (e.g., impulse response functions), see An and Schorfheide (2007). As discussed in Geweke (1999), Bayesian inference also provide tools to compare the fit of various models. For a given model m, the marginal likelihood is:

$$L(Y_T|m) = \int_{\theta\in\Theta} L(Y_T|m)P(\theta|m)d\theta, \tag{26}$$

which gives an indication of the overall likelihood of a model conditional on observed data. Below we discuss the priors employed in our estimation, the calibrated parameters and an analysis of the posterior distribution.

4.1 Calibration and Prior distributions

First, and as common in the literature, we fix (calibrate) several parameters. We set β to 0.995. This value, together with the average growth rate of productivity, $\overline{\gamma}$, which is around 0.004 (or 0.4% per quarter), implies an annual steady-state real interest rate around 4%. The depreciation rate of private capital, δ_k, and the depreciation of the productivity of public investment, δ_{kg} (as well as $\delta_{kg,def}$, when defense items are analyzed separately), are set at 0.025, implying a common annual depreciation rate of 10% (see Christiano et al. 2005). In versions of the model with distortionary taxation we follow Leeper et al. (2009) in fixing $\tau_l = 0.223$, $\tau_k = 0.184$ and $\tau_c = 0.028$. There are two other distortions in this economy, the wage and price markups, whose steady-state values are set to $\lambda_{p,ss} = 0.20$ and $\lambda_{w,ss} = 0.05$, following Christiano et al. (2005). Finally, we will keep the steady-state level of hours fixed at 0.31. This implies writing the parameter χ as a function of other parameters. Thus, χ varies throughout the estimation but guarantees that steady-state hours are fixed at 0.31. We proceed similarly with γ_1 which is set equal to the real return on capital as this must occur in equilibrium. We gather these remarks in table I.

Table I: Calibrated Parameters

Parameter	Value	Justification
β	0.995	Real interest rate (yearly) $\approx 4\%$
δ_k	0.025	Depreciation rate (yearly) $= 10\%$
δ_{kg}	0.025	”
$\delta_{kg,def}$	0.025	”
τ^w	0.223	Leeper et al. (2009)
τ^k	0.184	Leeper et al. (2009)
τ^c	0.028	Leeper et al. (2009)
λ_p	0.20	Christiano et al. (2005)
λ_w	0.05	Christiano et al. (2005)
χ	Varying	$s.t.\ n\_ss = 0.31$
γ_1	Varying	r^k_{ss}, eq'm. relation

Concerning the choice of the priors, they are independent and we keep them mostly uninformative, very much so in the case of the parameters related to the effects of government spending. The priors for the parameters related to potential externalities of government spending are very loose. The utility parameter ϕ follows a uniform distribution with support in $[0, 1]$. Concerning the parameter $\nu \in (0; \infty)$ we decide to reparametrize it such that $\nu = \exp(\nu\_b)$, where now $\nu\_b \in (-\infty; \infty)$. Then, in assigning prior to ν_b we want to be as agnostic as possible[7], so we decide again for a uniform distribution with support in $[-4, 20]$ (meaning that ν is in the range [0.018, almost perfect substitutes] , say), which covers a wide range of possibilities in the complementarity/separability space. Regarding the choice for the prior mean of θ_g (or $\theta_{g,def}$) we hold to uniform distributions with support in $[0, 4]$, which is never restrictive across Metropolis-Hastings iterations. Regarding the preferences parameters, for σ_L (inverse of Frisch elasticity) we specify a Normal distribution with mean 2 and standard deviation of 0.5, thus covering a wide range of admissible values in the literature, whereas for h (habit formation) we specify a Beta distribution with mean 0.7, around what is employed in the literature, and standard deviation 0.2. For the parameters related to the constant terms in the measurement equations , i.e. $\overline{\gamma}$, $\xi^{g,ss}$, $\xi^{ig,ss}$ and $\xi^{ig,def,ss}$, we specify a Normal distribution with mean

[7]The aim is to avoid providing prior information in favour either of substitutability or of complementarity .

equal to the sample average found in the data (1969Q1-2008Q3) and a reasonably high standard deviation. All the standard deviations of the shocks follow an Inverse Gamma distribution with mean equal to 0.1 and a standard deviation of 2 while the autoregressive parameters follow a Beta distribution with mean 0.5 and standard deviation of 0.2. The priors for the parameters Φ (fixed cost), κ (adjustment cost parameter), γ_2 (capacity utilization adjustment) and α (capital share), have loose priors centered around values close to (or derived from) those in Schmitt-Grohé and Uribe (2006). Tables III and IV, containing also estimation results, summarize these remarks.

4.2 Estimation Results

This section presents the estimation results, computational details can be found in the Appendix. We analyze various versions of the model, focusing on the following variations:

- Use of distortionary taxation and lump-sum taxation or lump-sum taxation only

- Full-sample, i.e. 1969Q1 through 2008Q3 or only the so-called "Great Moderation", i.e. 1984Q1 through 2008Q3

- Restricted models: without public spending channels ($\phi = 1$, $\theta_g = 0$, $\theta_{g,def} = 0$), with only the utility channel ($\theta_g = 0$, $\theta_{g,def} = 0$), with only the production function channel ($\phi = 1$) and, within this one, consideration of no output effects of defense investment ($\phi = 1$, $\theta_{g,def} = 0$)

We start by discussing the results of the (arguably more realistic) distortionary taxation model. Table II reports virtually all the estimated specifications with the associated values for the marginal data density and the externalities' parameters, both for the full sample and for the post '84 one. The model with the highest marginal data density is, irrespective of the sample, the one with the productivity of public investment's channel closed, i.e. θ_g and $\theta_{g,def}$ restricted to 0. This specification reveals that government consumption affects the marginal utility of consumption, since ϕ is estimated to be less than 1, and that government and private consumption are substitute goods because of the high estimated value for ν_b, which implies a very high elasticity of substitution, ν. Focusing on those specifications where θ_g and $\theta_{g,def}$ are estimated, we appreciate the following facts. First, the estimates for ϕ and ν_b are very close to ones obtained in the version where θ_g and $\theta_{g,def}$ were restricted to 0. Second, whenever estimated, $\theta_{g,def}$ is 0. Third, in some specifications the posterior mean of θ_g is bigger than zero, ranging from 0.05 to 0.59. Finally, models with $\theta_{g,def}$ restricted to 0 are very clearly preferred to models with $\theta_{g,def}$ left unrestricted.[8] Given these findings,

[8]We should refer that we have also estimated versions of the model with public investment not split into defense and non-defense items. In this case, estimates of the posterior mode and mean of the parameter measuring the productivity of public investment, θ_g, were almost always exactly 0. This indicates perhaps the increased difficulty of finding public investment to act as a shifter of private factors' productivity once defense and non-defense items are considered jointly.

Table II Distortionary Taxation, Posterior Mean and Mode of externalities parameters

POSTERIOR Post 1969

	$\theta_{g,def}=0$						$\theta_{g,def}$ Unrestr.				No Channels
	$\phi=1$		$\theta_g=0$		Unrestr.		$\phi=1$		Unrestr.		
Parameter	Mode	Mean	Mode	Mean	Mode	Mean	Mode	Mean	Mode	Mean	
ν_b	-	-	7.9	10.2	7.6	10.7	-	-	8.1	10.5	
ϕ	-	-	0.65	0.63	0.65	0.63	-	-	0.65	0.61	
θ_g	0.0	0.0	-	-	0.0	0.09	0.0	0.28	0.0	0.0	
$\theta_{g,def}$	-	-	-	-	-	-	0.0	0.0	0.0	0.0	
Laplace Log D Dens.	2522.3		**2544.8**		2539.6		2518.9		2518.5		2541.4
Log D Dens.	2534.9		**2555.1**		2551.6		2534.7		2533.3		2548.3

POSTERIOR Post 1984

	$\theta_{g,def}=0$						$\theta_{g,def}$ Unrestr.				No Channels
	$\phi=1$		$\theta_g=0$		Unrestr.		$\phi=1$		Unrestr.		
Parameter	Mode	Mean	Mode	Mean	Mode	Mean	Mode	Mean	Mode	Mean	
ν_b	-	-	8.9	14.3	7.7	11.4	-	-	12.1	10.8	
ϕ	-	-	0.66	0.47	0.66	0.51	-	-	0.66	0.52	
θ_g	0.0	0.05	-	-	0.0	0.0	0.0	0.59	0.0	0.39	
$\theta_{g,def}$	-	-	-	-	-	-	0.0	0.0	0.0	0.0	
Laplace Log D Dens.	1713.9		**1722.3**		1710.4		1688.2		1711.6		1715.7
Log D Dens.	1725.3		**1734.3**		1721.7		1709.8		1728.3		1727.9

we present in more detail results for only two specifications among the ones of Table II, labelled as the "Preferred" version, i.e. the one with θ_g and $\theta_{g,def}$ restricted to 0, and the "Productive Investment" version, i.e. the one with $\theta_{g,def}$ restricted to 0, both estimated on the full sample.

Tables III and IV present the prior distributions along with the estimated mode and percentiles of the posterior distribution of the parameters of both the *Preferred* and *Productive Investment* specifications. As for the parameters of greatest interest for us, i.e. those related to the potential public spending externalities, ϕ has, in both variations, a mode of 0.65 and mean of 0.63, indicating that government spending does affect the welfare of agents. More interestingly, the mode of ν_b is 7.9 in the *Preferred* version (7.56 in the *Productive investment* one) while the mean is 10.16 (respectively 10.69) indicating strong substitutability between private and government consumption. To be noted that conditional on the estimated value for ν_b (both mode and mean), U_{cg} in equation (18) is unambiguosly negative. Regarding θ_g in the *Productive Investment* specification, the mode is zero but the mean is positive, equal to 0.09, with an associated 90% posterior interval ranging from 0.00 to 0.20. This indicates a role for non-defense public investment in boosting private productivity. Focusing on the other parameters, we note that those related to constant terms in the measurement equation have a mode very close to the mean of the specified prior, which is, we recall, simply the sample average of the corresponding observed variables. The posterior for these parameters

Table III Priors and Posteriors of estimated parameters 1969Q1-2008Q3

Distortionary taxation. *Preferred* specification ($\theta_g = \theta_{g,def} = 0$) vs. *Productive Investment* ($\theta_{g,def} = 0$)

Parameter		PRIOR		POSTERIOR				POSTERIOR			
				$\theta_g = \theta_{g,def} = 0$				$\theta_{g,def} = 0$			
	Distr.	Mean	St. Dev.	Mode	Mean	5%	95%	Mode	Mean	5%	95%
A. Utility function											
h	*Beta*	0.7	0.1	0.77	0.81	0.73	0.89	0.77	0.80	0.72	0.87
σ_L	*Normal*	2	0.5	0.32	0.98	0.49	1.43	0.31	0.86	0.21	1.32
ν_b	*Uniform*	$[-5, 20]$		7.90	10.16	0.65	17.98	7.56	10.75	2.87	19.99
ϕ	*Uniform*	$[0, 1]$		0.65	0.63	0.52	0.70	0.65	0.63	0.52	0.74
B. Production function											
Φ	*Normal*	0.05	0.02	0.023	0.023	0.00	0.043	0.020	0.019	0.00	0.038
α	*Normal*	0.3	0.02	0.38	0.39	0.38	0.40	0.38	0.39	0.38	0.41
θ_g	*Uniform*	$[0, 4]$		-	-	-	-	0.0	0.09	0.00	0.20
$\theta_{g,def}$	*Uniform*	$[0, 1]$		-	-	-	-	-	-	-	-
C. Investment Adj. costs											
$\kappa/100$	*Normal*	4	0.5	4.74	4.73	4.08	5.36	4.74	4.79	4.12	5.45
γ_2	*Normal*	0.0685	0.002	0.063	0.064	0.061	0.066	0.063	0.063	0.060	0.065
D. Constant terms											
$\overline{\gamma}/100$	*Normal*	0.4	0.02	0.40	0.40	0.37	0.43	0.40	0.40	0.37	0.43
$\xi^{g,ss}$	*Normal*	0.16	0.01	0.169	0.169	0.160	0.178	0.169	0.169	0.161	0.178
$\xi^{ig,ss}$	*Normal*	0.025	0.001	0.026	0.026	0.024	0.028	0.027	0.027	0.025	0.028
$\xi^{g,def,ss}$	*Normal*	0.008	0.001	0.009	0.009	0.007	0.010	0.009	0.009	0.007	0.010

is tight around the mode. Most structural parameters not related to public spending have coverage from the intervals found in the literature and have a posterior distribution generally concentrated around the mode, cf. the $5\% - 95\%$ percentile for h, α or κ and γ_2. The parameter surrounded by greater uncertainty is the inverse of the Frisch elasticity of labor supply σ_L, whose mode is away from its mean in the two specification and has a looser posterior distribution.[9] As for the shocks parameters, we note the strong persistence (high autoregressive parameter) of all shocks except for the mild persistence of the preference shock and the low persistence of the productivity shock (see Table IV).

Now we focus on results obtained with lump-sum taxation. As it is visible from Table V, the results under lump-sum taxation confirm the qualitative findings obtained in the model with distortionary taxation. That is, models with θ_g and $\theta_{g,def}$ restricted to 0 are usually the preferred ones. Further, ϕ is always estimated to be less than 1, although it is usually higher than in the distortionary taxation case. The mean and mode of v_b fall always in the substitutability region, ranging from 4.1 to 12.7. The mode of θ_g is always nil, although in some instances its posterior mean is away from zero, ranging from 0.01 and 0.26.

[9] This lack of robustness is further confirmed when looking at results for the post 1984 sample only (see the appendix), where the distribution of σ_L is concentrated around low values that seem to be compensated by low values of ρ_w, the auto-regressive parameter of the wage wedge (or markup) shock, and high values of its standard deviation, σ_w. This suggest problems in the joint identification of σ_L and ρ_w, σ_w.

Table IV Priors and Posteriors of Shocks parameters 1969Q1-2008Q3

Distortionary taxation. *Preferred* specification ($\theta_g = \theta_{g,def} = 0$) vs. *Productive Investment* ($\theta_{g,def} = 0$)

Parameter	PRIOR			POSTERIOR				POSTERIOR			
				$\theta_g = \theta_{g,def} = 0$				$\theta_{g,def} = 0$			
	Distr.	Mean	St. Dev.	Mode	Mean	5%	95%	Mode	Mean	5%	95%
A. Autoregressive Parameters											
ρ_b	*Beta*	0.5	0.2	0.73	0.72	0.67	0.76	0.73	0.73	0.68	0.77
ρ_a	*Beta*	0.5	0.2	0.08	0.11	0.02	0.18	0.08	0.10	0.02	0.17
ρ_I	*Beta*	0.5	0.2	0.95	0.95	0.92	0.98	0.95	0.95	0.92	0.98
ρ_w	*Beta*	0.5	0.2	0.96	0.95	0.92	0.98	0.96	0.95	0.92	0.99
ρ_g	*Beta*	0.5	0.2	0.97	0.97	0.96	0.99	0.97	0.98	0.96	0.99
ρ_{ig}	*Beta*	0.5	0.2	0.94	0.95	0.91	0.98	0.94	0.94	0.91	0.98
$\rho_{ig,def}$	*Beta*	0.5	0.2	0.95	0.95	0.93	0.98	0.95	0.95	0.93	0.98
B. Standard deviation of shocks											
σ_b	*Inv Gamma*	0.1	2.0	3.64	3.57	3.13	3.98	3.63	3.61	3.16	4.03
σ_a	*Inv Gamma*	0.1	2.0	0.024	0.024	0.022	0.027	0.024	0.024	0.022	0.026
σ_I	*Inv Gamma*	0.1	2.0	0.039	0.040	0.030	0.052	0.039	0.040	0.030	0.050
σ_w	*Inv Gamma*	0.1	2.0	0.023	0.034	0.021	0.047	0.023	0.032	0.020	0.044
σ_g	*Inv Gamma*	0.1	2.0	0.015	0.015	0.014	0.017	0.015	0.015	0.014	0.017
σ_{ig}	*Inv Gamma*	0.1	2.0	0.030	0.030	0.028	0.033	0.030	0.030	0.028	0.033
$\sigma_{ig,def}$	*Inv Gamma*	0.1	2.0	0.086	0.087	0.079	0.095	0.086	0.087	0.080	0.096

Table V Lump-Sum Taxation, Posterior Mean and Mode of externalities parameters

	POSTERIOR Post 1969										
	$\theta_{g,def} = 0$						$\theta_{g,def}$ Unrestr.				No Channels
	$\phi = 1$		$\theta_g = 0$		Unrestr.		$\phi = 1$		Unrestr.		
Parameter	Mode	Mean	Mode	Mean	Mode	Mean	Mode	Mean	Mode	Mean	
ν_b	-	-	4.5	12.7	4.1	9.7	-	-	7.2	5.0	
ϕ	-	-	0.73	0.76	0.72	0.74	-	-	0.65	0.61	
θ_g	0.0	0.26	-	-	0.0	0.0	0.0	0.0	0.0	0.0	
$\theta_{g,def}$	-	-	-	-	-	-	0.01	0.08	0.0	0.1	
Laplace Log D Dens.	2458.9		**2511.6**		2482.4		2395.4		2471.5		2420.4
Log D Dens.	2467.8		2483.0		**2492.8**		2422.1		2489.3		2443.5

	POSTERIOR Post 1984										
	$\theta_{g,def} = 0$						$\theta_{g,def}$ Unrestr.				No Channels
	$\phi = 1$		$\theta_g = 0$		Unrestr.		$\phi = 1$		Unrestr.		
Parameter	Mode	Mean	Mode	Mean	Mode	Mean	Mode	Mean	Mode	Mean	
ν_b	-	-	8.0	10.2	8.9	10.0	-	-	6.5	10.2	
ϕ	-	-	0.75	0.78	0.77	0.76	-	-	0.78	0.76	
θ_g	0.0	0.01	-	-	0.0	0.0	0.0	0.15	0.0	0.02	
$\theta_{g,def}$	-	-	-	-	-	-	0.0	0.0	0.0	0.0	
Laplace Log D Dens..	1629.3		**1678.6**		1657.2		1612.7		1640.2		1638.9
Log D Dens.	1638.4		**1674.8**		1665.8		1625.8		1656.3		1643.8

All in all, the results suggest clear evidence for strong substitutability between public consumption and private consumption and mixed evidence on the positive effects of non-defense public investment on private sector productivity.

Full results concerning all the variations are available upon request. The Appendix contains some results and comparisons for the case of distortionary taxation and including the post 1984 sample. We notice an ample stability of most parameters across the two samples but, in general, a lower standard deviation of most shocks in the post 1984 sample. Further, the conclusions from Tables III and IV seem to carry over to the post 1984 sample with distortionary taxation, although a positive posterior mean of θ_g only pops out if $\theta_{g,def}$ is left unrestricted (as seen in Table II).

Next, we turn to the analysis of the response of our *Preferred* and *Productive Investment* economy to government spending shocks, comparing these responses to what obtains in the absence of public spending externalities.

4.3 Impulse-response analysis

In this section we analyze the dynamic effects of government consumption and non-defense investment shocks on selected model variables. Figures 1 and 2 embed the estimated model variables' reactions to a government consumption shock (within the *Preferred* economy) and to a government investment shock (within the *Productive Investment* economy), respectively.[10] The size of the shock is normalized so that on impact government consumption or investment increase by one percentage point of steady state output. Within the Figures, each plot presents two lines, a black and a dark grey one. The first one is the posterior mean of the estimated responses obtained from the *Preferred* model (Figure 1) and the *Productive Investment* model (Figure 2), and is named "Posterior Mean". The dark grey line is the reaction obtained by fixing all the parameters at the respective posterior mean, with the difference that the "externality" parameters, ϕ, θ_g and $\theta_{g,def}$ are set to 1, 0 and 0, respectively. The latter summarizes the reactions when the externality channels are shut down, and is labelled "No Channels".[11] The shaded area within each plot draws the 80% Bayesian posterior credibility interval associated with the "Posterior Mean" line. The impulse response functions are expressed either as percentage points of steady-state output or as percentage points steady-state deviations.

Focusing on the behavior of the variables in Figure 1, we can say that, because of substitutability, the increase in government consumption lowers the marginal utility of consumption, leading households

[10]We note that in the case of a government consumption shock, the reported responses obtained with the *Preferred* model are very similar to what obtains in the *Productive Investment* model.

[11]Hence, this restricted model is not estimated, in order to guarantee that the only parameters changing are those related to public spending externalities. However, we should note that the estimated posterior mean of the impulse response functions obtained with the imposed restrictions are very similar to the reported ones.

to substitute part of their private consumption with the newly available government consumption. As a consequence, private consumption (black line) decreases more than private consumption in the *No Channels* specification (dark grey line). That is, in the *Preferred* model, both the negative wealth effect and the substitutability effect sum up. Households work less in the *Preferred* framework since - for given negative wealth effect - the marginal utility of consumption is lower than in the *No Channels* case. As a result, wages decrease less in the *Preferred* case. To accommodate the new path for private consumption, the real interest rate increases by less on impact, so that investment is crowed-out less than in the *Preferred* specification. The impact on output is smaller in the *Preferred* model because of the lower increase in labor supply. Quantitatively, the following numbers associated with the *Preferred* specification are worth referring: the impact (mean) response of output amounts to 0.39 percentage points of steady state output, the one of consumption reaches -0.79 percentage points of steady-state output after 3 quarters and the investment reaction is virtually null.

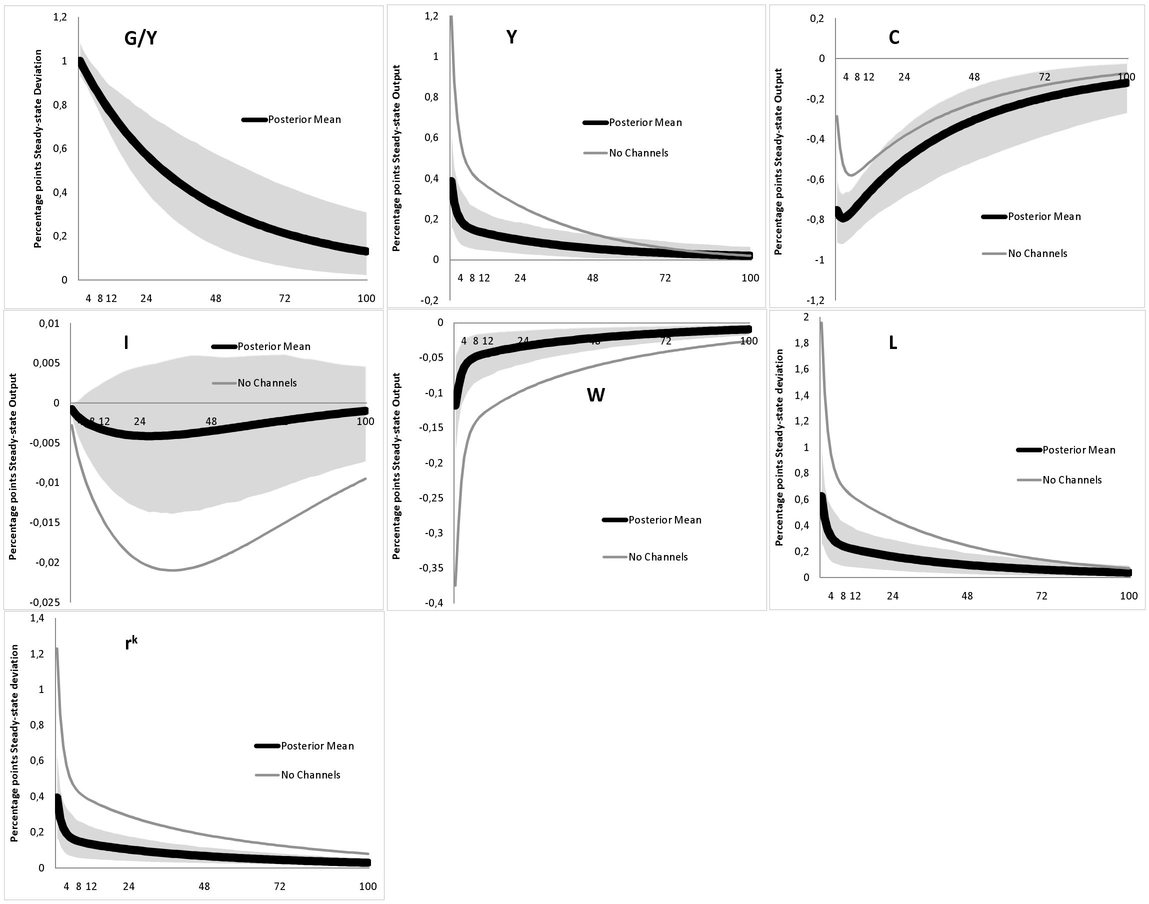

Figure 1

Impulse Response Functions (IRFs) of the *Preferred* model: effects on output (Y), consumption (C), investment (I), wages (W), hours (L) and return on capital (r^k) of a government consumption shock that on impact increases government consumption by 1 p.p. of steady-state output. The black line is the posterior mean of the IRF and the grey area is the associated 80% Bayesian credibility interval. The dark grey line is the IRF obtained with the "externalities" channels closed ($\phi = 1$, $\theta_g = 0$, $\theta_{g,def} = 0$) and the other parameters set to the posterior mean obtained from the *Preferred* model (denoted by *No Channels*)

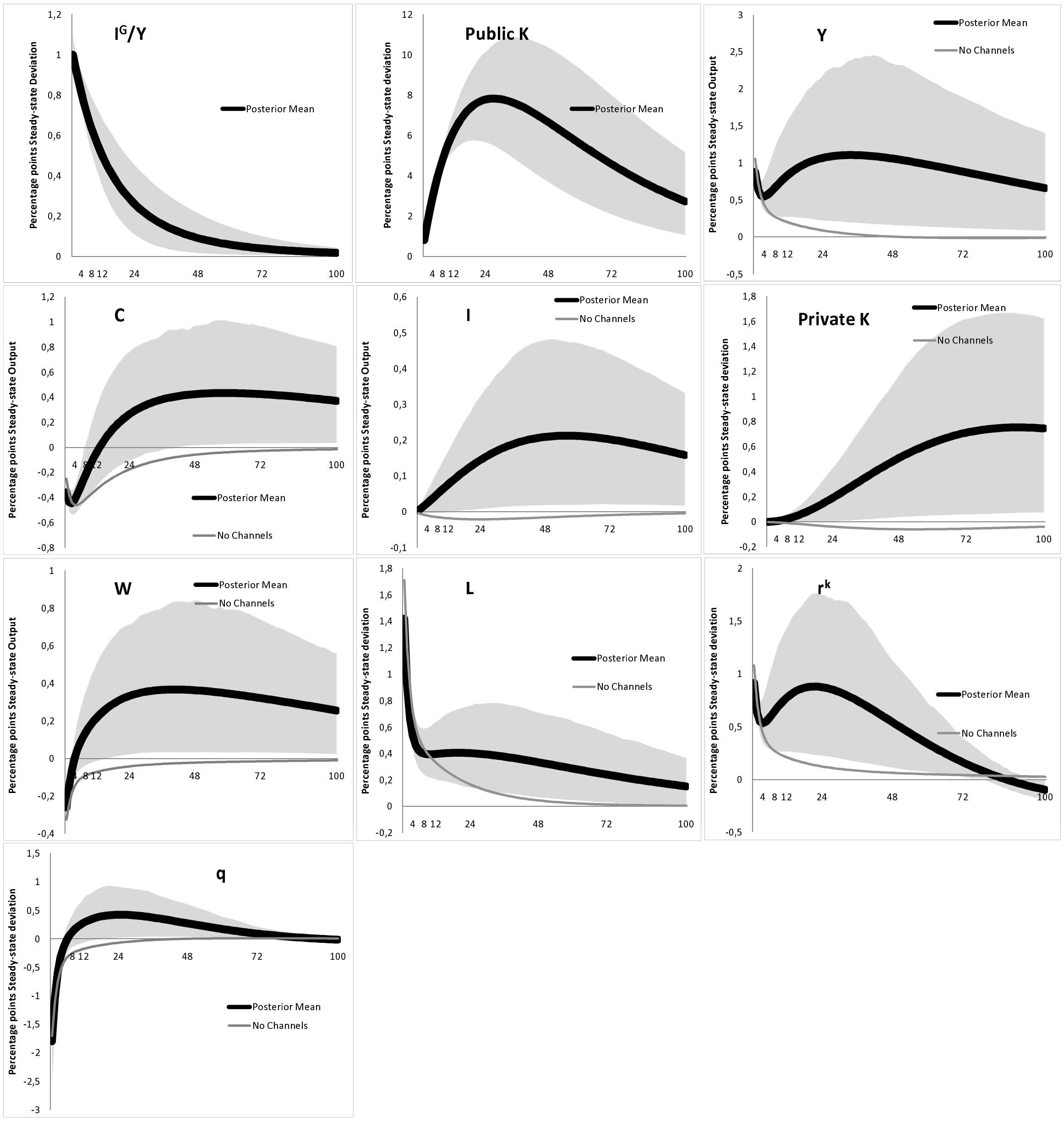

Figure 2

Impulse Response Functions (IRFs) of the *Productive Investment* model: effects on output (Y), consumption (C), investment (I), wages (W), hours (L), return on capital (r^k), Tobin's q (q), Private Capital (Private K) and Public Capital Productivity (Public K) of a government investment shock that on impact increases government investment by 1 p.p. of steady-state output. The black line is the posterior mean of the IRF and the grey area is the associated 80% Bayesian credibility interval. The dark grey line is the IRF obtained with the "externalities" channels closed ($\phi = 1$, $\theta_g = 0$, $\theta_{g,def} = 0$) and the other parameters set to the posterior mean obtained from the *Productive Investment* model (denoted by *No Channels*)

Turning the focus to Figure 2, we recall that, conditional on a positive value for θ_g, a non-defense government investment shock contributes to enhance the productivity of private factors, creating a positive wealth effect in the economy, ceteris paribus. As a result, in the *Productive Investment* economy the market value of firms (labelled q) tends to increase (at least for some quarters), so firms are willing to invest more. The positive wealth effect also fosters private consumption. Note that initially both consumption and Tobin's q go down on impact, due to the tension between two opposite forces operating in the economy: the negative wealth effect created by government's financing needs and the mentioned positive wealth effect. A rise in private factors productivity entails a rise in labor input as the labor demand schedule shifts outwards. As a consequence, both real wage and labor go up, ceteris paribus. However, note that the positive wealth effect tends to lower labor supply. In our specific case, labor remains at a higher level in the *Productive Investment* specification compared to the *No Channels* case. The response of output is positive on impact. Furthermore, in the *Productive Investment* model, output tends to remain at a higher level with respect to its steady state since public capital (productivity) and private capital build up and directly sustain output growth. Note that in the *No Channels* specification output converges much sooner to the steady-state. The quantitative analysis of the responses associated to the *Productive Investment* model reinforces the view that the government investment shock releases its effects in the long run. The (mean) output response peaks at 1.11 after 34 quarters, the one of consumption reaches 0.43 in quarter 59, and the investment response peaks at 0.21 after 56 quarters. The long run properties of these effects are tightly linked to the estimated persistence for the public investment process, which directly sustains the increase in the productivity of public capital for many periods. Anyway, it is worth noting the width of the posterior interval, meaning that the reactions can credibly take a large spectrum of values. For example, at the lower bound of the credibility interval, output, consumption and investment reach the values of 0.18, 0.03 and 0.02, respectively.

The next section completes the quantitative analysis, resorting to the analysis of dynamic multipliers induced by the public spending externalities.

4.4 Dynamic Multipliers

We analyze here public spending multipliers associated with the estimated effects of government spending shocks on output, consumption and investment. We use the notion of present value multipliers formulated in Mountford and Uhlig (2009); the present value multiplier of output, say, t quarters after an increase in government consumption (or investment) is:

$$\varphi_t = \frac{\sum_{k=0}^{t} (1+r^{ss})^{-k} \widehat{y}_k}{\sum_{k=0}^{t} (1+r^{ss})^{-k} g_k}$$

where $\widehat{y}_k$ represents the deviation of output from its steady-state and g_k is government consumption (or investment) measured as a fraction of steady-state output. r^{ss} is the steady-state real interest rate (net of depreciation). The expression generalizes for the case of consumption and investment, in which case deviations are expressed in units of steady-state output.

Tables VI and VII show the present value multipliers for Y, C and I and for various periods in response to a government consumption and a government investment shock, conditional on the respective specification. We look at the posterior mean of the multipliers and also at an 80% Bayesian credibility interval.

Looking at the multipliers produced by a government consumption shock (i.e. Table VI), we see that the one related to output reaches its maximum, i.e. 0.39, on impact and then slowly decreases. The 80% multiplier's posterior interval contains most of the values found by Mountford and Uligh (2009), but are lower than those found by Blanchard and Perotti (2002). Indeed, the first paper finds a multiplier's range between 0.44 and 0.23, the second one between 0.90 and 0.66, during the first 12 quarters, both within a VAR framework. It is worth noting that the output multiplier calculated within the *No Channels* specification results to be 1.21 on impact, i.e. three times bigger than the one in our *Preferred* economy. This discrepancy holds quantitatively even if we directly estimate the *No Channel* specification; in this case, the posterior mean of the output multiplier turns out to be 1.25. As expected, the multipliers for consumption and investment are negative, though the 80% posterior interval for investment contains zero. In the case of consumption the multipliers are clearly below those obtained in the *No Channels* model, especially at short horizons, whereas for investment they are above, but still negative.[12]

Looking at the government investment multipliers, we underline their long-run characteristics, bearing in mind that the strong estimated persistence of the government investment process is important to what obtains. The output multiplier starts below 1, at 0.87, similar to the *No Channels* multiplier, but increases as public capital productivity builds up, reaching 0.93 after three years and 1.44 after six. These values partially overlap with the multipliers found by Perotti (2004), Leeper (2010) and Baxter and King (1993). The first paper finds a multiplier ranging between 0.17 and 1.68 within the first five years. Leeper (2010)'s paper,

[12]It is worth noting that many of the analysis of the effects of government spending focus on military spending, instead of government consumption, as it is unlikely that this type of spending interacts with the business cycle. Among others, Barro and Redlick (2010) estimate an output multiplier ranging from 0.6 to 0.7 at the median unemployment rate (reaching 1.0 when the unemployment rate is around 12%); also, they find a crowding out for investment and net exports. Further, Hall (2009)'s range for the output multiplier is 0.7-1.0. Finally, Ramey (2011b), using news shocks obtained with a narrative approach, finds output multipliers in the range 0.6-1.2 (at peak GDP) and slightly negative consumption multipliers.

Table VI - Dynamic Multipliers, *Preferred* vs *No Channels* model, Government **Consumption** Shock

Quarters	1	2	4	8	12	24	48	72	100
Y *Preferred*	0.39 (0.15,0.62)	0.34 (0.13,0.54)	0.28 (0.11,0.47)	0.24 (0.09,0.41)	0.22 (0.08,0.38)	0.20 (0.07,0.35)	0.19 (0.06,0.34)	0.19 (0.05,0.35)	0.19 (0.05,0.0.36)
Y *No Channels*	1.21	1.01	0.88	0.72	0.66	0.59	0.54	0.52	0.51
C *Preferred*	−0.75 (−0.91,−0.60)	−0.77 (−0.92,−0.64)	−0.81 (−0.95,−0.67)	−0.83 (−0.97,−0.70)	−0.84 (−0.99,−0.70)	−0.86 (−1.04,0.−0.68)	−0.86 (−1.12,−0.63)	−0.87 (−1.17,−0.60)	−0.87 (−1.21,−0.58)
C *No Channels*	−0.29	−0.37	−0.47	−0.56	−0.59	−0.62	−0.63	−0.63	−0.63
I *Preferred*	−0.001 (−0.002,0.000)	−0.001 (−0.002,0.000)	−0.001 (−0.004,0.000)	−0.002 (−0.005,0.001)	−0.003 (−0.007,0.001)	−0.004 (−0.011,0.003)	−0.005 (−0.017,0.006)	−0.006 (−0.019,0.007)	−0.006 (−0.021,0.008)
I *No Channels*	−0.003	−0.004	−0.005	−0.009	−0.011	−0.017	−0.025	−0.029	−0.031

80% Bayesian credibility interval in parenthesis

Table VII - Dynamic Multipliers, *Productive Investment* vs *No Channels* model, Government **Investment** Shock

Quarters	1	2	4	8	12	24	48	72	100
Y *Productive I.*	0.87 (0.76,1.02)	0.78 (0.66,0.95)	0.72 (0.55,0.93)	0.79 (0.49,1.18)	0.93 (0.48,1.52)	1.44 (0.53,2.62)	2.30 (0.63,4.56)	2.84 (0.68,5.75)	3.18 (0.72,6.47)
Y *No Channels*	1.06	0.91	0.73	0.57	0.48	0.38	0.30	0.27	0.25
C *Productive I.*	−0.35 (−0.41,−0.25)	−0.41 (−0.47,−0.29)	−0.45 (−0.54,−0.33)	−0.44 (−0.59,−0.25)	−0.38 (−0.60,−0.10)	−0.16 (−0.57,0.41)	0.23 (−0.51,1.22)	0.50 (−0.47,1.82)	0.69 (−0.45,2.23)
C *No Channels*	−0.25	−0.32	−0.39	−0.44	−0.45	−0.42	−0.37	−0.35	−0.33
I *Productive I.*	0.01 (0.00,0.01)	0.01 (0.00,0.02)	0.01 (0.00,0.04)	0.05 (0.00,0.08)	0.13 (0.00,0.12)	0.16 (0.00,0.29)	0.29 (0.01,0.66)	0.42 (0.03,0.94)	0.50 (0.04,1.12)
I *No Channels*	0.00	0.00	−0.01	−0.01	−0.01	−0.02	−0.03	−0.03	−0.03

80% Bayesian credibility interval in parenthesis

conditional on choosing 0.1 for the output elasticity of public capital, finds the mean of the output multiplier ranging from 0.90 to 1.14 within the first 3 years. The last paper finds long-run multipliers equal to 4.12, conditional on setting the output elasticity of public capital to 0.1. Even the investment multiplier builds up over time, reaching, for instance, 0.16 after six years. The consumption multiplier starts at negative values similar to the *No Channels* case, and then becomes positive after six years. We must nonetheless refer that whenever the consumption multiplier is positive the associated posterior interval contains zero, contrary to what occurs with investment. In any case, the 80% posterior intervals of the government investment multipliers have a noticeable width, especially after 12 quarters.

5 Conclusions and Extensions

This paper has posed attention on the potential externalities produced by public expenditures on the private sector, focusing on how these externalities affect the response of the economy to two government spending shocks: a government consumption shock and a government investment shock. To this effect, we have built an otherwise standard RBC model extended with two important features. First, we have allowed government consumption to affect the welfare of agents, by entering directly the households' utility function. Second, we have allowed public capital to affect the productivity of private factors, entering in the firms' production function.

Our results show that the standard hypothesis of separability between private and government consumption does not hold in the data, and that the tho goods are robustly estimated to be substitutes. As a consequence, the estimated response of private consumption to a government consumption shock is negative, as in models where government investment is considered a pure waste (e.g. Baxter and King 1993 or Smets and Wouters 2003), but the response is much stronger. Because of substitutability labor supply reacts less to a government consumption shock, so the estimated output multiplier is lower (approximately one third) than the one measured in models with separable government consumption. On the other hand, we find that non-defense public investment enhances mildly, in some specifications, the productivity of private factors while investment in defense appears not to have any such impact. When non-defense public investment is found to shift the production frontier, shocks to it generate responses that manifest themselves only after several quarters: a positive reaction for private consumption, Tobin's q, private investment and real wages. Further, the estimated output multiplier builds up over time, in contrast with what obtains in the corresponding model with unproductive government investment.

One potential extension of the present paper amounts to verifying whether our measures are affected by several important features of fiscal policy, such as varying distortionary taxes, debt smoothing details, implementation delays, or the zero lower bound on nominal interest rates. On other front, another significant extension is related to the potential "composition" effect coming from the government consumption side. Resorting to disaggregate government consumption data from 1947 onwards (Bureau of Economic Analysis), it would be interesting to identify how each single category (e.g., administrative services, justice, health, education and so on) is related - in preferences - with private consumption. Then, it would also be worth assessing whether these potential relations vary over the business cycle.

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Appendix: Further Results

Sub-Samples Comparison: Distortionary Taxation, Unrestricted versions

Table I - A: Priors and Posteriors of selected parameters 1969Q1-2008Q3
Distortionary taxation, Unrestricted

Parameter	PRIOR			POSTERIOR 1969-2008				POSTERIOR 1984-2008			
	Distr.	Mean	St. Dev.	Mode	Mean	5%	95%	Mode	Mean	5%	95%
A. Utility function											
h	*Beta*	0.7	0.1	0.76	0.83	0.77	0.89	0.86	0.96	0.94	0.97
σ_L	*Normal*	2	0.5	0.31	1.12	0.73	1.49	0.38	0.16	0.01	0.34
ν_b	*Uniform*	$[-5, 20]$		8.12	7.40	-0.16	17.46	12.07	10.83	3.58	19.93
ϕ	*Uniform*	$[0, 1]$		0.65	0.68	0.53	0.88	0.66	0.52	0.38	0.65
B. Production function											
Φ	*Normal*	0.05	0.02	0.020	0.018	0.00	0.037	0.007	0.022	0.00	0.042
α	*Normal*	0.3	0.02	0.38	0.39	0.39	0.40	0.34	0.33	0.31	0.35
θ_g	*Uniform*	$[0, 4]$		0.00	0.00	0.00	0.002	0.00	0.39	0.00	0.74
$\theta_{g,def}$	*Uniform*	$[0, 1]$		0.00	0.002	0.00	0.003	0.00	0.01	0.00	0.008
C. Investment Adj. costs											
$\kappa/100$	*Normal*	4	0.5	4.73	4.49	3.77	5.19	4.48	4.48	3.86	5.21
γ_2	*Normal*	0.0685	0.002	0.063	0.064	0.062	0.066	0.062	0.062	0.059	0.065
D. Constant terms											
$\overline{\gamma}/100$	*Normal*	0.4	0.02	0.40	0.40	0.36	0.43	0.41	0.42	0.39	0.44
$\xi^{g,ss}$	*Normal*	0.16	0.01	0.169	0.170	0.161	0.178	0.159	0.158	0.149	0.166
$\xi^{ig,ss}$	*Normal*	0.025	0.001	0.026	0.026	0.024	0.029	0.025	0.025	0.024	0.026
$\xi^{g,def,ss}$	*Normal*	0.008	0.001	0.009	0.009	0.007	0.010	0.008	0.008	0.007	0.009

Table II - A: Priors and Posteriors of Shocks parameters 1969Q1-2008Q3
Distortionary taxation, Unrestricted

Parameter	PRIOR			POSTERIOR 1969-2008				POSTERIOR 1984-2008			
	Distr.	Mean	St. Dev.	Mode	Mean	5%	95%	Mode	Mean	5%	95%
A. Autoregressive Parameters											
ρ_b	*Beta*	0.5	0.2	0.73	0.72	0.68	0.76	0.71	0.69	0.63	0.75
ρ_a	*Beta*	0.5	0.2	0.078	0.10	0.02	0.18	0.10	0.13	0.02	0.24
ρ_I	*Beta*	0.5	0.2	0.95	0.96	0.93	0.99	0.96	0.95	0.93	0.99
ρ_w	*Beta*	0.5	0.2	0.96	0.95	0.92	0.98	0.88	0.27	0.10	0.45
ρ_g	*Beta*	0.5	0.2	0.97	0.98	0.96	0.99	0.97	0.97	0.95	0.99
ρ_{ig}	*Beta*	0.5	0.2	0.94	0.94	0.90	0.98	0.89	0.89	0.83	0.97
$\rho_{ig,def}$	*Beta*	0.5	0.2	0.95	0.95	0.92	0.98	0.98	0.97	0.96	0.99
B. Standard deviation of shocks											
σ_b	*Inv Gamma*	0.1	2.0	3.64	3.55	3.14	4.00	2.74	2.83	2.42	3.23
σ_a	*Inv Gamma*	0.1	2.0	0.024	0.025	0.023	0.027	0.021	0.023	0.019	0.025
σ_I	*Inv Gamma*	0.1	2.0	0.039	0.044	0.031	0.06	0.031	0.033	0.023	0.044
σ_w	*Inv Gamma*	0.1	2.0	0.022	0.039	0.028	0.05	0.037	0.089	0.068	0.11
σ_g	*Inv Gamma*	0.1	2.0	0.015	0.015	0.014	0.017	0.015	0.014	0.012	0.015
σ_{ig}	*Inv Gamma*	0.1	2.0	0.030	0.030	0.028	0.034	0.022	0.022	0.019	0.025
$\sigma_{ig,def}$	*Inv Gamma*	0.1	2.0	0.086	0.087	0.080	0.095	0.054	0.055	0.049	0.062

Post-1984 Sample: ***Preferred*** ($\theta_g = \theta_{g,def} = 0$) vs. Unrestricted specification

Table I - B Priors and Posteriors of selected parameters 1984Q1-2008Q3
Preferred specification vs. Unrestricted

Parameter	PRIOR			POSTERIOR				POSTERIOR			
				$\theta_g = \theta_{g,def} = 0$				Unrestricted			
	Distr.	Mean	St. Dev.	Mode	Mean	5%	95%	Mode	Mean	5%	95%
A. Utility function											
h	*Beta*	0.7	0.1	0.85	0.94	0.93	0.96	0.86	0.96	0.94	0.97
σ_L	*Normal*	2	0.5	0.35	0.17	0.01	0.37	0.38	0.16	0.01	0.34
ν_b	*Uniform*	$[-5, 20]$		8.91	14.3	2.44	26.5	12.07	10.83	3.58	19.93
ϕ	*Uniform*	$[0, 1]$		0.66	0.47	0.36	0.58	0.66	0.52	0.38	0.65
B. Production function											
Φ	*Normal*	0.05	0.02	0.007	0.014	0.00	0.028	0.007	0.022	0.00	0.042
α	*Normal*	0.3	0.02	0.34	0.33	0.31	0.35	0.34	0.33	0.31	0.35
θ_g	*Uniform*	$[0, 4]$		-	-	-	-	0.00	0.39	0.00	0.74
$\theta_{g,def}$	*Uniform*	$[0, 1]$		-	-	-	-	0.00	0.01	0.00	0.008
C. Investment Adj. costs											
$\kappa/100$	*Normal*	4	0.5	4.50	4.84	4.16	5.51	4.48	4.48	3.86	5.21
γ_2	*Normal*	0.0685	0.002	0.062	0.061	0.058	0.064	0.062	0.062	0.059	0.065
D. Constant terms											
$\overline{\gamma}/100$	*Normal*	0.4	0.02	0.41	0.41	0.37	0.44	0.41	0.42	0.39	0.44
$\xi^{g,ss}$	*Normal*	0.16	0.01	0.159	0.158	0.150	0.166	0.159	0.158	0.149	0.166
$\xi^{ig,ss}$	*Normal*	0.025	0.001	0.025	0.025	0.024	0.026	0.025	0.025	0.024	0.026
$\xi^{g,def,ss}$	*Normal*	0.008	0.001	0.008	0.008	0.007	0.009	0.008	0.008	0.007	0.009

Table II - B Priors and Posteriors of Shocks parameters 1984Q1-2008Q3
Preferred specification vs. Unrestricted

Parameter	PRIOR			POSTERIOR				POSTERIOR			
				$\theta_g = \theta_{g,def} = 0$				Unrestricted			
	Distr.	Mean	St. Dev.	Mode	Mean	5%	95%	Mode	Mean	5%	95%
A. Autoregressive Parameters											
ρ_b	*Beta*	0.5	0.2	0.71	0.71	0.65	0.76	0.71	0.69	0.63	0.75
ρ_a	*Beta*	0.5	0.2	0.10	0.13	0.02	0.22	0.10	0.13	0.02	0.24
ρ_I	*Beta*	0.5	0.2	0.96	0.95	0.91	0.99	0.96	0.95	0.93	0.99
ρ_w	*Beta*	0.5	0.2	0.88	0.35	0.10	0.57	0.88	0.27	0.10	0.45
ρ_g	*Beta*	0.5	0.2	0.97	0.97	0.95	0.99	0.97	0.97	0.95	0.99
ρ_{ig}	*Beta*	0.5	0.2	0.89	0.90	0.83	0.97	0.89	0.89	0.83	0.97
$\rho_{ig,def}$	*Beta*	0.5	0.2	0.98	0.98	0.96	0.99	0.98	0.97	0.96	0.99
B. Standard deviation of shocks											
σ_b	*Inv Gamma*	0.1	2.0	2.74	2.77	2.36	3.21	2.74	2.83	2.42	3.23
σ_a	*Inv Gamma*	0.1	2.0	0.021	0.022	0.019	0.024	0.021	0.023	0.019	0.025
σ_I	*Inv Gamma*	0.1	2.0	0.031	0.034	0.022	0.047	0.031	0.033	0.023	0.044
σ_w	*Inv Gamma*	0.1	2.0	0.034	0.070	0.055	0.083	0.037	0.089	0.068	0.11
σ_g	*Inv Gamma*	0.1	2.0	0.013	0.014	0.013	0.014	0.015	0.014	0.012	0.015
σ_{ig}	*Inv Gamma*	0.1	2.0	0.022	0.022	0.019	0.024	0.022	0.022	0.019	0.025
$\sigma_{ig,def}$	*Inv Gamma*	0.1	2.0	0.054	0.055	0.048	0.062	0.054	0.055	0.049	0.062

Appendix: Equilibrium conditions with transformed variables

We restric attention to the model with distortionary taxation. The version with lump-sum taxation obtains with all marginal tax rates equal to zero. Equilibrium conditions follow from the first order conditions (F.O.C.s) of households' and firms' problems while imposing symmetry, fiscal policy equations, market clearing conditions and processes for the exogenous processes. We then stationarize the variables C_t, I_t, K_t, W_t, Tax_t, T_t, G_t, I_t^g and $I_t^{g,def}$ dividing them by the level of technology, A_t. Lower case letters indicates transformed variables. The same treatment is required for the Lagrange multipliers associated with the budget constraint and the capital accumulation equation, respectively λ_t and q_t (Tobin's q).

- *Aggregator (consumption):*

$$\widetilde{c}_t = \left[\phi \left(c_t\right)^{\frac{v-1}{v}} + (1-\phi)\left(g_t\right)^{\frac{v-1}{v}}\right]^{\frac{v}{v-1}} \tag{27}$$

- *Consumption F.O.C.:*

$$(1+\tau^c)\lambda_t \exp(-(\overline{\gamma}+\varepsilon_t^a)) = e^{\varepsilon_t^b}\left[\phi\left(\frac{\widetilde{c}_t}{c_t}\right)^{\frac{1}{v}}\left[\left(\widetilde{c}_t \exp(\overline{\gamma}+\varepsilon_t^a) - \gamma \widetilde{c}_{t-1}\right)^{-1}\right]\right] \tag{28}$$

- *Labor supply F.O.C.:*

$$\lambda_t = \frac{\chi L_t^{\sigma_n}}{(1-\tau^w)w_t} \tag{29}$$

- *Investment F.O.C.:*

$$\lambda_t = \lambda_t q_t E_t \left\{\left[1 - \frac{\kappa}{2}\left(e^{\varepsilon_t^I}\frac{i_t \exp(\overline{\gamma}+\varepsilon_t^a)}{i_{t-1}} - e^{\overline{\gamma}}\right)^2\right] - \frac{i_t}{i_{t-1}}\exp(\overline{\gamma}+\varepsilon_t^a)\kappa e^{\varepsilon_t^I}\left(e^{\varepsilon_t^I}\frac{i_t \exp(\overline{\gamma}+\varepsilon_t^a)}{i_{t-1}} - e^{\overline{\gamma}}\right)\right\} + \tag{30}$$

$$+\beta E_t\left[\lambda_{t+1}q_{t+1}\exp(-(\overline{\gamma}+\varepsilon_{t+1}^a))\left(e^{\varepsilon_{t+1}^I}\frac{i_{t+1}\exp(\overline{\gamma}+\varepsilon_{t+1}^a)}{i_t}\right)^2 \kappa e^{\varepsilon_{t+1}^I}\left(e^{\varepsilon_{t+1}^i}\frac{i_{t+1}\exp(\overline{\gamma}+\varepsilon_{t+1}^a)}{i_t} - e^{\overline{\gamma}}\right)\right]$$

- *Next period capital F.O.C.:*

$$\lambda_t q_t = E_t \left[\beta \lambda_{t+1} \exp(-(\overline{\gamma} + \varepsilon^a_{t+1})) r^k_{t+1} u_{t+1} - a\left(u_{t+1}\right)](1-\tau^k) + q_{t+1}\left(1-\delta\right) \right] \tag{31}$$

where $a\left(u_t\right) = \gamma_1 \left(u_t - 1\right) + \frac{\gamma_2}{2} \left(u_t - 1\right)^2$ represents the cost of using capital at intensity u_t.

- *Capital law of motion:*

$$k_{t+1} = \left(1-\delta\right) k_t \exp(-(\overline{\gamma} + \varepsilon^a_t)) + i_t \left[1 - \frac{\kappa}{2} \left(e^{\varepsilon^I_t} \frac{i_t \exp(\overline{\gamma} + \varepsilon^a_t)}{i_{t-1}} - e^{\overline{\gamma}} \right)^2 \right] \tag{32}$$

- *Capacity utilization F.O.C.:*

$$r^k_t = a'\left(u_t\right) = \gamma_1 + \gamma_2 \left(u_t - 1\right) \tag{33}$$

- *Marginal rate of substitution consumption/labor:*

$$mrs_t = e^{\varepsilon^b_t} \frac{\chi L_t^{\sigma_n}}{(1+\tau^c)\lambda_t} \tag{34}$$

- *Wage markup:*

$$1 + \lambda_{w,t} = \frac{w_t(1-\tau^w)}{mrs_t} \tag{35}$$

- *Production function:*

$$y_t = \exp(-\alpha(\overline{\gamma} + \varepsilon^a_t)) k^\alpha_t (L_t)^{1-\alpha} (K^G_t)^{\theta_g} (K^{G,def}_t)^{\theta_{g,def}} - \Phi \tag{36}$$

- *Factor demands:*

$$(1-\alpha) \frac{y_t}{L_t(1+\lambda_{p,ss})} = w_t \tag{37}$$

$$\alpha \frac{y_t}{k_t(1+\lambda_{p,ss})} \exp(\overline{\gamma} + \varepsilon^a_t) = r^k_t \tag{38}$$

- *Marginal cost:*

$$MC_t = \frac{\left(r_t^k\right)^{\alpha} w_t^{1-\alpha}}{\alpha^{\alpha}\left(1-\alpha\right)^{1-\alpha}} * \frac{1}{(K_t^G)^{\theta_g}(K_t^{G,def})^{\theta_{g,def}}} \tag{39}$$

- *Price markup:*

$$\frac{1}{MC_t} = 1 + \lambda_{p,ss} \tag{40}$$

- *Evolution of the productivity of public capital:*

$$K_{t+1}^G = (1-\delta_{Kg})K_t^G + \xi_t^{ig} \tag{41}$$

where $\xi_t^{ig} = i_t^g/y_t = \exp(\varepsilon_t^{ig} + ss^{ig})/(1+\exp(\varepsilon_t^{ig} + ss^{ig}))$ and $ss^{ig} = \log(\xi^{ig,ss}/(1-\xi^{ig,ss}))$

$$K_{t+1}^{G,def} = (1-\delta_{Kg,def})K_t^{G,def} + \xi_t^{ig,def} \tag{42}$$

where $\xi_t^{ig,def} = i_t^{g,def}/y_t = \exp(\varepsilon_t^{ig,def} + ss^{ig,def})/(1+\exp(\varepsilon_t^{ig,def} + ss^{ig,def}))$ and $ss^{ig,def} = \log(\xi^{ig,def,ss}/(1-\xi^{ig,def,ss}))$

- *Government consumption:*

$$\xi_t^g = g_t/y_t = \exp(\varepsilon_t^g + ss^g)/(1+\exp(\varepsilon_t^g + ss^g)) \tag{43}$$

where $ss^g = \log(\xi^{g,ss}/(1-\xi^{g,ss}))$

- *Balanced government budget:*

$$\tau^c c_t + \tau^w w_t L_t + \tau^k \left[r_t^k u_t - a\left(u_t\right)\right] k_t + t_t = \xi_t^g y_t + \xi_t^{ig} y_t + \xi_t^{ig,def} y_t \tag{44}$$

- *Shocks processes:*

$$\begin{aligned}
\varepsilon_t^b &= \rho_b \varepsilon_{t-1}^b + \eta_t^b \\
\varepsilon_t^I &= \rho_i \varepsilon_{t-1}^I + \eta_t^I \\
\varepsilon_t^a &= \rho_a \varepsilon_{t-1}^a + \eta_t^a \\
\lambda_{w,t} &= (1-\rho_w)\lambda_{w,ss} + \rho_w \lambda_{w,t-1} + \eta_t^w \\
\varepsilon_t^g &= \rho_g \varepsilon_{t-1}^g + \eta_t^g \\
\varepsilon_t^{ig} &= \rho_{ig} \varepsilon i_{t-1}^{ig} + \eta_t^{ig} \\
\varepsilon_t^{ig,def} &= \rho_{ig,def} \varepsilon i_{t-1}^{ig,def} + \eta_t^{ig,def}
\end{aligned} \tag{45}$$

Appendix: Bayesian inference

The following characterizes briefly the estimation procedure. The model is solved up to a log-linear approximation around the deterministic steady state. Once the solution is obtained, the model can be cast in state-space form, and the likelihood function can be computed using the Kalman filter. In every variation we start by finding the mode of the posterior distribution, or the parameter vector θ_M maximizing $P(\theta|Y_T, m)$. Initial values in the numerical algorithms are sometimes important so we repeated the procedure using various initial values in order to guarantee as much as possible that the mode is indeed found. Only then we ran the Metropolis-Hastings algorithm using 3.10^5 iterations while monitoring its convergence. Specifically, we use the Random-Walk Metropolis algorithm which generates Markov chains with stationary distributions that correspond to the the posterior distribution of interest. Practically, we strictly follow the procedure described in section 4.1 of An and Schorfheide (2007).

That said, the following information regarding our specific estimation is worth noting. First, the acceptation rate in the Metropolis-Hastings algorithm is around 25%. Second, we monitor the convergence of the Markov Chain generated by the Metropolis Hastings algorithm to the posterior distribution of interest by using the diagnostics proposed by Brooks and Gelman (1998). Intuitively, different Markov Chain sequences obtained with the algorithm should be similar once the effects of starting values vanish. Hence, second moments of the parameters calculated across sequences should not differ much from second moments calculated within a sequence. Using a rule-of-thumb that the two moments are equal if their difference is below 20%, should ensure convergence. If this is not the case, either the effect of the initial replications has not worn off, or the number of draws taken is too low. In our case, the number of draws we use is sufficient to achieve convergence.

Appendix: Data

We follow closely Smets and Wouters (2007) in treating the data. We use the September 30, 2010 vintage of data. The tables below clarify sources and transformations.

Table D1 - Data Sources

Variable	Designation	Source	CODE
Gross Domestic Product (Nominal)	GDP	US Dep. of Commerce - BEA	A191RC1
Personal Cons. Expenditures (Nominal)	C	US Dep. of Commerce - BEA	DPCERC1
Personal Cons.Expenditures - Durables (Nominal)	Durables	US Dep. of Commerce - BEA	DDURRC1
Private Fixed Domestic Investment (Nominal)	PFI	US Dep. of Commerce - BEA	A007RC1
Federal Cons. Expenditures (Nominal)	G_Federal	US Dep. of Commerce - BEA	A957RC1
State & Local Cons. Expenditures (Nominal)	G_StateLocal	US Dep. of Commerce - BEA	A991RC1
Federal Gross Investment, Non-Defense (Nominal)	IG_Federal	US Dep. of Commerce - BEA	A798RC1
Federal Gross Investment, Defense (Nominal)	IG_Defense	US Dep. of Commerce - BEA	A788RC1
State & Local Gross Investment (Nominal)	IG_StateLocal	US Dep. of Commerce - BEA	A799RC1
Gross Domestic Product Deflator	GDPDEF	US Dep. of Commerce - BEA	GDPDEF
Hourly Compensation, Non Farm Sector (Nominal)	Wages	Bureau of Labor Statistics	PRS85006103
Civilian noninstitutional population, 16 years and over	POPULATION	Bureau of Labor Statistics	LNU00000000Q
All series are seasonally adjusted at annual rates			

Table D2 - Observables for measurement equations

Y_t^{obs} =(GDP/GDPDEF)/POPULATION
C_t^{obs} =((C-Durables)/GDPDEF)/POPULATION
I_t^{obs} =((PFI+Durables)/GDPDEF)/POPULATION
W_t^{obs} =(Wages/GDPDEF)/POPULATION
$\xi_t^{g,obs}$ =(G_Federal+G_StateLocal)/GDP
ξ_t^{ig} =(IG_Federal+IG_StateLocal)/GDP
$\xi_t^{ig,def,obs}$ =IG_Defense/GDP

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— *Paulo M.M. Rodrigues, Antonio Rubia*

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— *Paulo M. M. Rodrigues, Nazarii Salish*

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— *Paulo Soares Esteves*

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— *Martín Saldías*

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— *Fernando A. F. Ferreira, Sérgio P. Santos, Paulo M. M. Rodrigues, Ronald W. Spahr*

2012

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2/12 ASSET PRICING WITH A BANK RISK FACTOR
— *João Pedro Pereira, António Rua*

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— *António Rua, Luis C. Nunes*

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— *António Rua, Artur Silva Lopes*

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— *Mário Centeno, Álvaro A. Novo*

6/12 THE DYNAMICS OF CAPITAL STRUCTURE DECISIONS
— *Paula Antão, Diana Bonfim*

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— *Uwe Hassler, Paulo M. M. Rodrigues, Antonio Rubia*