WORKING PAPERS
9 | 2012

MARKET PERCEPTION OF FISCAL SUSTAINABILITY: AN APPLICATION TO THE LARGEST EURO AREA ECONOMIES

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March 2012

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EUROSYSTEM

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Edition

Economics and Research Department

Pre-press and Distribution

Administrative Services Department

Documentation, Editing and Museum Division

Editing and Publishing Unit

Printing

Administrative Services Department

Logistics Division

Lisbon, March 2012

Number of copies

80

ISBN 978-989-678-123-1

ISSN 0870-0117 (print)

ISSN 2182-0422 (online)

Legal Deposit no. 3664/83

Market perception of fiscal sustainability:
An application to the largest euro area economies

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(March 2012)

Abstract: Debt intolerance may rule out fiscal trajectories which otherwise appear to be sustainable. If fiscal policy lacks credibility, the interest on the sovereign debt may rise sharply and the country may lose market access. Indicators for assessing the market perception of fiscal sustainability should complement the conventional empirical sustainability analysis. I propose an approach for extracting information from sovereign bond data, which provides snapshots of market sentiment. It is based on a multi-borrower default-intensity pricing model, allowing for the cross-section estimation (under a risk-neutral probability measure) of the term-structure of the unobservable default-free interest rates, as well as (for all sovereigns included in the sample) of the probabilities of default (for any horizon deemed relevant) and the associated recovery rates given default. The approach is illustrated by the estimation of the model for Germany, France, Italy and Spain for every Friday from October 2, 2009 to November 25, 2011.

Keywords: Sovereign debt, European debt crisis, fiscal sustainability, risk-neutral probability, default intensity, recovery rate.

JEL: C58, G12, H63, H68

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♦ The views expressed in this paper do not necessarily reflect those of Banco de Portugal or the Eurosystem. I thank Rui Carvalho, Joaquim Martinez, José Pedro Braga, Jorge Marçal and Hélia Mendes, of the Markets and Reserve Management Department of Banco de Portugal, for providing the data used in the paper and for very helpful comments. I also thank the comments and suggestions made by participants in the Ecomod 2011 Conference, hosted by the University of Azores, where I presented a very preliminary version of the paper. The usual disclaimer applies.

1. INTRODUCTION

Conventional fiscal sustainability assessments are empirically performed either by testing the government's present-value borrowing constraint or by computing tax-gap indicators. The analytical apparatus used by the first approach is based on unit root tests of the stock of public debt and on cointegration tests of primary expenditure and revenue. This literature was initiated by Hamilton and Flavin (1986), and the early contributors include Wilcox (1989), Hakkio and Rush (1991), Haug (1991), Trehan and Walsh (1991) and MacDonald (1992), among others[1]. The main limitation of this approach is that the tests are based on past fiscal data. Besides its backward-looking nature, the timeliness of the conclusions obtained from these tests is also hampered by the creative accounting to which many countries have been resorting to in different degrees (for example, the accounting of public-private partnerships – the so-called PPP's , public owned firms, transfers of pension funds to public social security, etc.).

The second empirical approach consists of computing synthetic tax-gap indicators. They were introduced by Blanchard (1990) and Blanchard et al. (1990) in three versions, according to the time horizon: short-term (one year), medium term (five years) and long-term (around 40 years). They were defined as the gap between: (i) the tax rate that (if kept constant) allows the public debt to stabilize at a given level over the chosen horizon; and (ii) the current tax rate. These indicators, and in particular the long-term version, were designed to assess the extent to which the government can maintain the current tax and spending programs without experiencing a continued increase in the public debt ratio.

Since 2006, the European Commission publishes regularly for all member-states two variants of the long-term tax-gap indicator, the so-called S1 and S2 sustainability indicators[2]. They aim at measuring the size of the permanent discretionary adjustment in the primary balance required to: (i) in the case of the S1 indicator, reaching by 2050 the target level of 60% for the ratio of public debt to GDP; and (ii) in the case of the S2 indicator, fulfilling the government's present-value borrowing constraint (over an

[1] For a recent paper with an application to the EU-15 using panel data techniques and allowing for structural breaks, see Afonso and Rault (2010).
[2] See European Commission (2006).

infinite horizon, without a specific terminal condition for the debt ratio). When computing these indicators, the European Commission makes use of its medium-term forecasts, as well as of long-term projections of age-related expenditure (social security transfers and health care expenditures).

Although forward-looking, these sustainability indicators reflect partial equilibrium analysis. Beyond the medium term, interest rates are typically assumed constant over the horizon, and the projection of GDP per capita is rather simplistic. Therefore, the consequences of existing macroeconomic imbalances (for instance, large current account deficits in some countries) or non-age-related long-term financing pressures on the public sector (associated with PPP's or with public owned firms) are largely ignored. Moreover, and critically, the sustainability indicators do not take into account that if the fiscal programs are not credible (for instance, because they are insufficiently front-loaded), interest rates on public debt charged by private investors may rise sharply and the countries may experience a loss of market access. In other words, in the terminology of Reinhart et al. (2003), there may be a phenomenon of debt intolerance, which is ignored by the typical sustainability assessments.

In the current context of the euro area debt crisis, one should complement the standard sustainability analysis by monitoring the credibility of the sovereign borrowers. With that purpose in mind, in this paper I suggest a way to extract information from sovereign bond market data using a multi-issuer default intensity-based pricing model, built upon the financial modeling framework developed by Jarrow and Turnbull (1995), Lando (1998), Duffie and Singleton (1999), Duffie, Pan and Singleton (2000), Duffie and Gârleanu (2001), and Mortensen (2006), among others. For any given moment of time, if a cross-section of market quotes for bonds issued in the same currency by a set of sovereigns is available, the model proposed in this paper allows the joint estimation of the unobservable term-structure of the default-free interest rate, as well as of risk-neutral default probabilities (at any horizon deemed relevant) and the associated recovery rates given default, for each country in the data set.

There is an extensive literature on modeling sovereign default intensities, but most contributions so far have developed the pricing equations in a univariate setting (i.e. single issuer), like *inter alia* Duffie, Pedersen and Singleton (2003) and Pan and Singleton (2008). In the literature on corporate default, there are several examples of multi-issuer settings, in some cases designed for very large cross-sections (also *inter alia* Mortensen, 2006, and Eckner, 2009). Recently, Ang and Longstaff (2011) suggested a multi-borrower model for sovereign debt designed for panel data on CDS spreads, with most parameters time-invariant, and with given default-free interest rates based on LIBOR and swap rates.

By focusing on cross-section data, one may interpret the estimation results as snapshots of market sentiment relative to the sovereign borrowers at a given moment of time. These snapshots will be very sensitive to changes of the market mood, without estimation inertia transmitted from previous periods. The price, in my view worth paying, is that sometimes one may observe some instability in the results. In most of situations, the instability simply reflects the market instability itself and so it has informational value.

The option for using cross-section data ruled out the use of sovereign CDS spreads because at any moment of time there are only few quotes available for each denomination of the notional. Instead, I opted for government bond data and restricted the analysis to the four largest larger euro area countries (Germany, France, Italy and Spain). For these countries I jointly estimated the model by nonlinear least squares for every Friday from October 2, 2009 to November 25, 2011. Notice that the initial date is just before the Greek general election of October 4, 2009. After the election, estimates for the Greek general government and the public debt were substantially revised upwards, triggering the euro area debt crisis.

The remainder of the paper is organized as follows. In section 2, I introduce the main concepts and the intuition behind the multi-issuer pricing model. In section 3, a formal specification of the model is provided. The data and the estimation results are presented and discussed in section 4. I conclude in section 5.

2. INFORMAL PRESENTATION OF MAIN CONCEPTS AND MODEL FEATURES

When specifying the pricing equations, I will consider a risk-neutral probability measure, which means that all assets are priced in accordance with expected discounted values of cash-flows. Intuitively, the risk-neutral probability measure reflects the beliefs that a risk neutral investor would have to hold in order to be in agreement with observed market prices. Hence, all estimates that will be presented in the paper, including the expected recovery rates and the probabilities of default, should be interpreted in this way and not as “real-world” probabilities and recovery rates. If risk aversion leads investors to avoid unfavorable states of nature, asset prices will reflect such behavior. Therefore, a risk-neutral probability measure over-weights the probability mass attached to the unfavorable states of nature, implying that risk-neutral probabilities of default overestimate the investors’ probabilities of default and risk-neutral recovery rates underestimate the investors’ expected recovery rates.

I will abstract from official borrowing and lending, considering that the sovereign borrowers interact with private lenders through the bond market. Default is absent when the sovereign debtor fulfills all its contractual obligations fully and timely. If the debtor fails to meet interest or principal payments at the specified dates, default occurs. Default may be complete or partial. In the case of complete default, the investor recovers nothing. In historical experience, complete defaults of sovereign debt are very rare events. More frequent are cases of debt restructuring, in which the investors recover a part of the principal and/or suffer a change in the time profile of the debt service (rescheduling).

As an initial step towards a more realistic analytical framework, one may consider default modeled as the first occurrence of a homogeneous Poisson process with constant intensity λ. Let *t* be the continuous time index such that $t = 0$ corresponds to the reference time, i.e. the moment of time the market data is collected. In this simple framework, time to default follows an exponential distribution with parameter λ, implying that the expected time to default is given by λ^{-1} and the probability of default before some date *t* can be computed as $1-\exp(-\lambda t)$, where $\exp(-\lambda t)$ is simply the probability that no default has occurred before date *t*. The parameter λ is known as the (constant) intensity of default, owing to the fact that, for a small time interval Δt, the probability of a default occurring in $(t, t+\Delta t)$ is approximately $\lambda \Delta t$. This expression warrants the above-mentioned interpretation of λ as the intensity of default.

The pricing-model formally presented in the next section is based on a generalization of this default-intensity approach. Instead of treating λ as a fixed parameter, I let it vary over time as a function of some latent factors. These driving factors may be either common to all borrowers or idiosyncratic. For a given sovereign debtor g, the default intensity is assumed to be

$$\lambda_{g,1} r_t + \lambda_{g,2} z_t + u_{g,t}$$

where $\lambda_{g,1}$ and $\lambda_{g,2}$ are non-negative coefficients, r_t is the default-free instantaneous interest rate, z_t is another common factor of default intensity (independent of r_t) and $u_{g,t}$ is an idiosyncratic factor (independent of r_t, z_t and the idiosyncratic components of other sovereign debtors). Note that the first term of the intensity is associated with the default-free instantaneous interest rate, which allows for positive correlation between the default-intensities and the default-free interest rate.

This specification of default-intensity allows default correlation amongst different debtors, because the common factors r_t and z_t may enter the expression of the default intensities of all debtors (if the respective coefficients are strictly positive). A specification in which the default correlation arises exclusively from the common factors is said to verify the conditional independence assumption, because the actual default of one debtor does not impact *per se* on the likelihood of default of other debtors. Using cross-section data, it is impossible to go beyond conditional independence.

For parameter parsimony and computational simplification, in most default intensity pricing models for corporate securities applied to advanced economies, it is assumed that government debt is default-free. For a model dealing with sovereign debt of different euro area countries, a similar assumption would consist of considering *a* *priori* the German government bonds as the default-free benchmark. However, in my view, sovereign default risk has become very salient in markets and no sovereign bond should *a priori* be assumed to be default-free, in particular when mutual financial assistance is among the relevant possibilities. Therefore, I opted for a formulation that allows the data to determine the default-free interest rate process.

3. MODEL SPECIFICATION

3.1 Model setup

Let *t* denote the (continuous) time index such that $t = 0$ corresponds to the reference time, i.e. the moment which the data refer to. The number of bond issuers included in the analysis is G and, for simplification and without loss of generality, I assume that all bonds have a unitary face value. I also assume that issuer g $(g = 1, \cdots, G)$ may default on her bonds sometime in the future and that: (i) the default will affect simultaneously all her bonds; (ii) the creditors will recover the amount b_g per bond immediately upon the event. These recovery rates are stochastic, and assumed to be independent of the time of default and identical for all bonds issued by *g*.

I assume that a probability space $(\Omega, \mathcal{F}, \mathbb{Q})$ is given, large enough to support the $\mathbb{R}^D$-valued stochastic process $x = \{x_t\}_{t \geq 0}$ (hereafter, the state vector), with *cadlag* (i.e. right-continuous with left limits) non-negative mutually independent components.

The filtration of the σ-algebra $\mathcal{F}$ generated by *x* is denoted $\{\mathcal{F}_t^x\}_{t \geq 0}$ and satisfies the "usual hypotheses"[3]. The default time of issuer *g* will be denoted τ_g (> 0) and is defined as the first jump time of a Cox process with intensity $\{\lambda_g \cdot x_t\}_{t \geq 0}$:

$$\tau_g = \inf \left\{ t \mid \int_0^t \lambda_g \cdot x_s ds \geq \varepsilon_g \right\}$$

[3] See, for instance, Protter (2005, Ch. I).

where λ_g is a *D*-dimensional column-vector of non-negative constants, $\lambda_g \cdot x_t$ is the dot product of vectors λ_g and x_t, and ε_g is (under $\mathbb{Q}$) an exponential random variable with unitary mean and independent of $\left\{\mathcal{F}_t^x\right\}_{t\geq 0}$ and ε_h for all $h \neq g$. Thus,

$$\mathbb{Q}_0\left(\tau_g > t\right) = E_0^{\mathbb{Q}}\left[\mathbf{1}\left(\tau_g > t\right)\right] = E_0^{\mathbb{Q}}\left[\exp\left(-\int_0^t \lambda_g \cdot x_s ds\right)\right]$$

where $\mathbb{Q}_0(\cdot)$ and $E_0^{\mathbb{Q}}(\cdot)$ denote the probability and the expected value, respectively, both under $\mathbb{Q}$ and conditional on $\mathcal{F}_0^x$, and $\mathbf{1}(A)$ stands for the indicator function that takes the value one if the condition A is fulfilled and zero otherwise[4]. As mentioned in the previous section, this specification of default intensity allows default correlation amongst different bond issuers through the sharing of the state vector, the so-called "conditional independence" assumption. The first component of the state-vector will be identified as the (non-observable) default-free spot instantaneous interest rate *r*, i.e. $x_{1,t} = r_t$ for all *t*.

I think of $\mathbb{Q}$ as a risk-neutral probability measure[5], under which all securities are priced as expected discounted values. The value at time 0 of a default-free zero-coupon bond maturing at time $t > 0$ is $E_0^{\mathbb{Q}}\left[\exp\left(-\int_0^t r_s ds\right)\right]$ (or, equivalently, $E_0^{\mathbb{Q}}\left[\exp\left(-\int_0^t x_{1,s} ds\right)\right]$). On the other hand, if the zero-coupon bond is issued by *g* (and therefore defaultable), its value is (see Appendix A):

$$E_0^{\mathbb{Q}}\left[\exp\left(-\int_0^t r_s ds\right)\mathbf{1}\left(\tau_g > t\right)\right] + E_0^{\mathbb{Q}}\left[b_g \exp\left(-\int_0^{\tau_g} r_s ds\right)\right] =$$

$$= \prod_{d=1}^{D} \delta_{g,d}\left(t\right) + \rho_g\left[\frac{\lambda_{g,1}}{1+\lambda_{g,1}} \prod_{d=2}^{D} \delta_{g,d}\left(\tilde{\tau}_{g,1}\right) + \sum_{d=2}^{D} \prod_{i=1, i\neq d}^{D} \delta_{g,i}\left(\tilde{\tau}_{g,d}\right)\right] \qquad (1)$$

---

[4] In general, let $\mathbb{Q}_s(\cdot)$ denote the risk-neutral probability conditional on $\mathcal{F}_s^x$ ($s \geq 0$). Note that $\mathbb{Q}_0\left(\tau_g > t\right) = E_0^{\mathbb{Q}}\left[\mathbf{1}\left(\tau_g > t\right)\right] = E_0^{\mathbb{Q}}\left\{E_t^{\mathbb{Q}}\left[\mathbf{1}\left(\tau_g > t\right)\right]\right\} = E_0^{\mathbb{Q}}\left[\mathbb{Q}_t\left(\tau_g > t\right)\right]$ and, directly from the definition of τ_g (taking into account that ε_g is exponential with mean 1 and is independent of $\left\{\mathcal{F}_t^x\right\}_{t\geq 0}$), $\mathbb{Q}_t\left(\tau_g > t\right) = \mathbb{Q}_t\left(\varepsilon_g > \int_0^t \lambda_g \cdot x_s ds\right) = \exp\left(-\int_0^t \lambda_g \cdot x_s ds\right)$.

[5] Up to some purely technical assumptions, and in the absence of arbitrage, a risk-neutral measure $\mathbb{Q}$ exists and is equivalent to the "real world" probability measure. See Harrison and Kreps (1979), Harrison and Pliska (1981) and Delbaen and Schachermayer (1994).

where $\delta_{g,d}(t)$ and ρ_g denote $E_0^{\mathbb{Q}}\left[\exp\left(-\int_0^t \tilde{x}_{g,d,s}ds\right)\right]$ and $E_0^{\mathbb{Q}}\left(b_g\right)$, respectively, $\tilde{\tau}_{g,d}$ is the time of the first jump of a Cox process with intensity $\left\{\tilde{x}_{g,d,t}\right\}$ and

$$\tilde{x}_{g,d,t}=\begin{cases}\left(1+\lambda_{g,1}\right)r_t & \text{if } d=1\\ \lambda_{g,d}x_{d,t} & \text{if } d>1\end{cases}$$

3.2 Pricing equations

I now tackle the pricing of bullet bonds. For a given bond *n* issued by *g*, hereafter referred to as bond (g,n), $c_{g,n}$ and $T_{g,n}$ are the coupon annual rate[6] and the residual time to maturity, respectively. The $M_{g,n}$ coupon payment dates until maturity for bond (g,n) are denoted $\left\{t_{g,n,1},\cdots,t_{g,n,m},\cdots,t_{g,n,M_{g,n}}\right\}$, with $t_{g,n,1}>0$ and $t_{g,n,M_{g,n}}=T_{g,n}$ for all *g* and *n*. For algebraic convenience, I add $t_{g,n,0}=0$ to the latter set (with associated zero payment).

Let $p_{g,n}^0$ denote the "theoretical price" at the reference time for bond (g,n) according to the intensity model laid-out in the previous sub-section. One has the following generalization of (1) (see Appendix B):

$$p_{g,n}^0=\sum_{m=1}^{M_{g,n}}\left[c_{g,n}\left(t_{g,n,m}-t_{g,n,m-1}\right)+\mathbf{1}\left(m=M_{g,n}\right)\right]E_0^{\mathbb{Q}}\left[\exp\left(-\int_0^{t_{g,n,m}}r_s ds\right)\mathbf{1}\left(\tau_g>t_{g,n,m}\right)\right]+$$

$$+E_0^{\mathbb{Q}}\left[b_g\exp\left(-\int_0^{\tau_g}r_s ds\right)\right]=$$

$$=\sum_{m=1}^{M_{g,n}}\left[c_{g,n}\left(t_{g,n,m}-t_{g,n,m-1}\right)+\mathbf{1}\left(m=M_{g,n}\right)\right]\prod_{d=1}^{D}\delta_{g,d}\left(t_{g,n,m}\right)+$$

$$+\rho_g\left[\frac{\lambda_{g,1}}{1+\lambda_{g,1}}\prod_{d=2}^{D}\delta_{g,d}\left(\tilde{\tau}_{g,1}\right)+\sum_{d=2}^{D}\prod_{i=1,i\neq d}^{D}\delta_{g,i}\left(\tilde{\tau}_{g,d}\right)\right] \quad (2)$$

Note that, according to the latter expression, in order to compute the theoretical bond price, one needs to evaluate $\delta_{g,i}\left(\tilde{\tau}_{g,d}\right)=E_0^{\mathbb{Q}}\left[\exp\left(-\int_0^{\tilde{\tau}_{g,d}}\tilde{x}_{g,i,s}ds\right)\right]$ for all pairs (i,d) with $i\neq d$. But

[6] With payments computed according to the ISMA day-count convention actual/actual.

computing $\delta_{g,i}\left(\tilde{\tau}_{g,d}\right)$ for some $\left(g,d,i\right)$ is more complex than evaluating $\delta_{g,d}\left(t\right)$ for any fixed t because the first-jump time $\tilde{\tau}_{g,d}$ is a random variable. The simplest way to overcome the difficulty is to resort to numerical quadrature.

Let each time interval between coupon payments of bond $\left(g,n\right)$ be partitioned into K_g subintervals. For instance, for the interval $\left(t_{g,n,m-1},t_{g,n,m}\right)$, I consider the partition

$$\left\{t_{g,n,m-1},t_{g,n,m-1}+\Delta_g,\cdots,t_{g,n,m-1}+k\Delta_g,\cdots,t_{g,n,m-1}+\left(K_g-1\right)\Delta_g,t_{g,n,m}\right\}$$

where $\Delta_g=\left(t_{g,n,m}-t_{g,n,m-1}\right)/K_g$. By making K_g sufficiently large, one can approximate $\delta_{g,i}\left(\tilde{\tau}_{g,d}\right)$ to any required degree of precision, by (see Appendix C):

$$\sum_{m=1}^{M_{g,n}}\sum_{k=1}^{K_g}\delta_{g,i}\left(t_{g,n,m-1}+\left(k-1/2\right)\Delta_g\right)\left[\delta_{g,d}\left(t_{g,n,m-1}+\left(k-1\right)\Delta_g\right)-\delta_{g,d}\left(t_{g,n,m-1}+k\Delta_g\right)\right] \qquad (3)$$

Hence, replacing $\delta_{g,i}\left(\tilde{\tau}_{g,d}\right)$ in (2) by (3), the pricing equation for the unitary face valued bullet bond $\left(g,n\right)$ may be written approximately as:

$$\begin{aligned}
p_{g,n}^{0}\simeq&\sum_{m=1}^{M_{g,n}}\left[c_{g,n}\left(t_{g,n,m}-t_{g,n,m-1}\right)+\mathbf{1}\left(m=M_{g,n}\right)\right]\prod_{d=1}^{D}\delta_{g,d}\left(t_{g,n,m}\right)+\\
&+\rho_g\left\{\frac{\lambda_{g,1}}{1+\lambda_{g,1}}\prod_{d=2}^{D}\sum_{m=1}^{M_{g,n}}\sum_{k=1}^{K_g}\delta_{g,d}\left(t_{g,n,m-1}+\left(k-1/2\right)\Delta_g\right)\cdot\right.\\
&\qquad\cdot\left[\delta_{g,1}\left(t_{g,n,m-1}+\left(k-1\right)\Delta_g\right)-\delta_{g,1}\left(t_{g,n,m-1}+k\Delta_g\right)\right]+\\
&+\sum_{d=2}^{D}\prod_{i=1,i\neq d}^{D}\sum_{m=1}^{M_{g,n}}\sum_{k=1}^{K_g}\delta_{g,i}\left(t_{g,n,m-1}+\left(k-1/2\right)\Delta_g\right)\cdot\\
&\qquad\left.\cdot\left[\delta_{g,d}\left(t_{g,n,m-1}+\left(k-1\right)\Delta_g\right)-\delta_{g,d}\left(t_{g,n,m-1}+k\Delta_g\right)\right]\right\}
\end{aligned} \qquad (4)$$

3.3 Basic affine diffusions and CIR processes

In order to obtain simple closed-form expressions for $\delta_{g,d}\left(t\right)$, I assume that the state variables are at most basic affine diffusions. The non-negative mutually independent univariate stochastic processes $x_{d,t}$ $(d=1,\cdots,D)$ are basic affine diffusions on the filtered probability space $\left(\Omega,\mathcal{F},\left\{\mathcal{F}_t^x\right\}_{t\geq 0},\mathbb{Q}\right)$ if they satisfy the stochastic differential equations

$$dx_{d,t}=\kappa_d\left(\theta_d-x_{d,t}\right)dt+\sigma_d\sqrt{x_{d,t}}dw_{d,t}+dj_{d,t}\quad (d=1,\cdots,D)$$

with initial condition $x_{d,0}=\chi_d$ and where $w_{d,t}$ and $j_{d,t}$ $(d=1,\cdots,D)$ are mutually independent Brownian motions and compound Poisson processes, respectively, the latter with jump intensities l_d and exponentially distributed jump sizes with means μ_d. The vector $(\chi_d,\kappa_d,\theta_d,\sigma_d,l_d,\mu_d)$ comprises the six parameters on which depend the generating process of $x_{d,t}$. Besides χ_d, l_d and μ_d, which were already introduced above, κ_d is the parameter regulating the speed of adjustment to the long-term deterministic trend, defined by $\theta_d t$, and σ_d is the parameter regulating the volatility of the stochastic term associated with the Brownian motion. If $l_d=0$ (i.e. if there are no jumps)[7], one obtains the so-called CIR process (named after Cox, Ingersoll and Ross, 1985). If in addition one also has $\sigma_d=0$, the basic affine diffusion reduces to a simple deterministic mean-reverting process.

The basic affine diffusions are special cases of affine-jump diffusions. Duffie, Pan and Singleton (2000) characterized this wider class of processes and proposed solutions for transforms (in terms of systems of ordinary differential equations) which appear in the pricing equation of many securities (and in particular derivatives). For our purposes, the relevant transform and corresponding solution are the following[8]:

$$E_0^{\mathbb{Q}}\left[\exp\left(-\int_0^t x_{d,s}ds\right)\right]=\exp\left[A\left(t;\kappa_d,\theta_d,\sigma_d,l_d,\mu_d\right)+\chi_d B\left(t;\kappa_d,\sigma_d\right)\right] \qquad (5)$$

where

$$A\left(t;\kappa_d,\theta_d,\sigma_d,l_d,\mu_d\right)=\frac{\kappa_d\theta_d\gamma_d}{\beta_d\xi_{d,1}\eta_{d,1}}\log\left(\frac{\xi_{d,1}+\eta_{d,1}\exp\left(\beta_d t\right)}{-\gamma_d}\right)+\frac{\kappa_d\theta_d}{\xi_{d,1}}t+$$

[7] Note that if $l_d=0$ then μ_g is undetermined and may be set to zero.

[8] This transform and solution are presented e.g. in Mortensen (2006). It is a slightly restricted version of the transforms/solutions proposed by Duffie and Gârleanu (2001) and Duffie and Singleton (2003, Appendix A.5), which were derived as specialized versions of the formulas suggested in Duffie, Pan and Singleton (2000) for affine jump-diffusions.

$$+\frac{l_d\left(\eta_{d,1}\xi_{d,2}/\xi_{d,1}-\eta_{d,2}\right)}{\beta_d\xi_{d,2}\eta_{d,2}}\log\left(\frac{\xi_{d,2}+\eta_{d,2}\exp\left(\beta_d t\right)}{\xi_{d,2}+\eta_{d,2}}\right)+\frac{l_d\left(1-\xi_{d,2}\right)}{\xi_{d,2}}t$$

$$B\left(t;\kappa_d,\sigma_d\right)=\frac{1-\exp\left(\beta_d t\right)}{\xi_{d,1}+\eta_{d,1}\exp\left(\beta_d t\right)}$$

with $\gamma_d=\sqrt{\kappa_d^2+2\sigma_d^2}$, $\xi_{d,1}=-\left(\gamma_d+\kappa_d\right)/2$, $\eta_{d,1}=\xi_{d,1}+\kappa_d$, $\xi_{d,2}=1-\mu_d/\xi_{d,1}$, $\eta_{d,2}=\left(\eta_{d,1}+\mu_d\right)/\xi_{d,1}$ e $\beta_d=\eta_{d,1}+\left(\kappa_d\xi_{d,1}-\sigma_d^2\right)/\gamma_d$.

A useful property of basic affine processes is that if $\left\{x_{d,t}\right\}$ belongs to this class with parameters $\left(\chi_d,\kappa_d,\theta_d,\sigma_d,l_d,\mu_d\right)$, then also does the process $\left\{\alpha_{g,d}x_{d,t}\right\}$ with parameters $\left(\alpha_{g,d}\chi_d,\kappa_d,\alpha_{g,d}\theta_d,\sqrt{\alpha_{g,d}}\sigma_d,l_d,\alpha_{g,d}\mu_d\right)$, for any constant $\alpha_{g,d}>0$. Thus, from (5),

$$\delta_{g,d}(t)=E_0^{\mathbb{Q}}\left[\exp\left(-\int_0^t\tilde{x}_{g,d,s}ds\right)\right]=$$

$$=\exp\left[A\left(t;\kappa_d,\alpha_{g,d}\theta_d,\sqrt{\alpha_{g,d}}\sigma_d,l_d,\alpha_{g,d}\mu_d\right)+\alpha_{g,d}\chi_d B\left(t;\kappa_d,\sqrt{\alpha_{g,d}}\sigma_d\right)\right] \qquad (6)$$

where

$$\alpha_{g,d}=\begin{cases}\left(1+\lambda_{g,1}\right) & \text{if } d=1\\ \lambda_{g,d} & \text{if } d>1\end{cases}$$

Replacing (6) into (4), one obtains the expression for the bond price $\left(g,n\right)$ as a function of the parameters $\left(\chi_d,\kappa_d,\theta_d,\sigma_d,l_d,\mu_d\right)$ $\left(d=1,\cdots,D\right)$ and the vector of coefficients λ_g.

3.4 Identification of the state variables

Given G and D, and before any *a priori* identifying restrictions, the number of unknown parameters in the system of pricing equations for all the G issuers is $\left[\left(G+6\right)D+G\right]$ or $\left[\left(G+4\right)D+G\right]$, depending on assuming that the state variables are basic affine or CIR processes, respectively. Moreover, for $D>1$, not all of the parameters are identified.

As explained in section 2, I consider that the bonds of all issuers are defaultable, implying that the default-free interest rate process is a function of the state variables. In order to simplify the identification, I assumed that the future behavior of the instantaneous default-free interest rate is well described by a single state variable (the first one, i.e. $\{x_{1,t}\} = \{r_t\}$, as mentioned), instead of favoring much less parsimonious multi-factor specifications. The default intensities of bond issuer *g* will be positively correlated with the default-free interest rate (the first state variable) whenever the associated (non-negative) coefficient is strictly positive.

Also for parsimony, I assume that, besides the first state variable identified as the default-free interest rate, there is a single common factor of default intensity $\{x_{2,t}\} = \{z_t\}$, potentially shared by all issuers (the degree of sharing being determined by the coefficients associated with this common factor). In order to ensure the identification of the parameters of this latent common factor, as well as the identification of the associated coefficients $\lambda_{g,2}$ $(g = 1, \cdots, G)$, I will restrict *a priori* one of the latter coefficients to be equal to one.

Finally, I assume that there are other G state variables identified as (latent) idiosyncratic components of the default intensities, one and only one for each issuer ($\{x_{2+g,t}\} = \{u_{g,t}\}$ for $g = 1, \cdots, G$). These identification restrictions are enforced by imposing *a priori* that, for $d > 2$, $\lambda_{g,d} = 1$ if $d = g + 2$ and $\lambda_{g,d} = 0$ otherwise.

Summing-up, and as indicated in section 2, I specified the default intensity of issuer *g* as

$$\lambda_g \cdot x_t = \lambda_{g,1} r_t + \lambda_{g,2} z_t + u_{g,t} \quad (g = 1, \cdots, G)$$

where r_t is the default-free interest rate, z_t is the other common default intensity factor (independent of r_t) and $u_{g,t}$ $(g = 1, \cdots, G)$ are idiosyncratic factors (independent of r_t and z_t). For the sake of model identifiability, and without loss of generality, the estimation results presented in section 4 were obtained imposing that the coefficient of the common factor for Spain is unitary.

3.5 Estimation

Let $y_{g,n}$ represent the observed quote of the annual yield to maturity at the reference time for bond (g,n). Given the relevant parameters, the theoretical price $p^0_{g,n}$ of this bond is given by formula (4), complemented by (6), and the associated "theoretical yield to maturity" $y^0_{g,n}$ can be determined by solving the equation

$$\sum_{m=1}^{M_{g,n}} \left[c_{g,n} \left(t_{g,n,m} - t_{g,n,m-1} \right) + \mathbf{1}\left(m = M_{g,n} \right) \right] \left(1 + y^0_{g,n} \right)^{-t_{g,n,m}} = p^0_{g,n}$$

I estimated the model by nonlinear least squares, i.e. I minimized the mean squared error

$$MSE(\omega) = \frac{1}{\sum_{g=1}^{G} N_g} \sum_{g=1}^{G} \sum_{n=1}^{N_g} \left[y_{g,n} - y^0_{g,n}(\omega) \right]^2$$

where ω and N_g denote the vector of all the model parameters and the number of bonds issued by g included in the sample, respectively. The minimization was performed using the Levenberg-Marquardt algorithm.

In all estimations, the number K_g of subintervals in the partitions of periods between coupon payments referred to in section 3.2 was set at 12 for all issuers but Italy, for which it was set at 6. The sovereign bullet bonds issued by the latter country have half-annual coupon payments, while in all the other countries the payments are annual. These choices of K_g correspond to considering time subintervals of about one month when using the quadrature method. The results are rather insensitive to larger choices of K_g.

4. EMPIRICAL RESULTS

As mentioned in the introductory section, the model was estimated for every Friday from October 2, 2009 to November 25, 2011 (113 cross-section estimations). For each Friday, the data set was collected from Bloomberg and consists of information on sovereign bullet bonds of Germany, France, Italy and Spain with a residual maturity of at least six months. The number of bonds in the sample depends on the Friday and on average is 166 (on average 47 bonds for Germany and Italy, 42 bonds for France and 30 bonds for Spain). Each observation comprises the maturity date, the coupon rate, the frequency of coupon payments, and the end of day bid quote for the yield to maturity.

The base case for the estimation assumed CIR processes (excluding intensity jumps) for the state variables and unknown recovery rates. A variant with intensity jumps (i.e. assuming basic affine processes for the state variables) was also estimated but the improvement of the fit (measured by the root mean squared error - RMSE) relative to the base case proved to be negligible. In a second estimation variant I considered CIR processes as in the base case but restricted the recovery rates for all bonds to be 40%. This restriction implied significant deteriorations of the RMSE for many Fridays, especially in 2011. Therefore, given the large amount of estimations for each variant, I will only present the estimation results for the base case.

The RMSEs for the overall sample and by country are presented in Figure 1. On average of the 113 estimations, the overall RMSE is 6.1 basis points (b.p.) and by country it is 5.4 b.p. for Germany, 4.3 b.p. for France, 7.8 b.p. for Italy, and 5.2 b.p. for Spain. I interpret these values as suggestive of a reasonable ability of the proposed model to fit sovereign bond yield quotes. For most Fridays the overall RMSE is at around 5 basis points (b.p.). The exceptions are the period from late April to the end of June of 2010, in the aftermath of the request of financial assistance by the Greek authorities, and the period after early August 2011. In some of these especially troubled Fridays, the overall RMSE reached 10 b.p. and on November 25, 2011, the last Friday in the period under analysis, it almost reached 15 b.p. (associated, in particular, with a RMSE for Italy of 21.5 b.p.).

Estimated default-free yields to maturity are shown in Figure 2. They are low relative to the corresponding values that one obtains for instance from swap rates. For the 10-year maturity they are about 1.4-1.6% until mid-2010, with a strong perturbation in April-May of that year, then decline throughout the second half of 2010, and then experience a second period of instability during the first five months of 2011, before stabilizing at 1.0-1.2% afterwards. As pointed out for the RMSE, the first episode of instability in the estimates of the default-free yields corresponds to the period of largely dysfunctional bond markets in the aftermath of the request of financial assistance by Greece. As of the second period of instability, it is associated with a sharp deterioration of the Greek financial situation, as well as with the period during which Portugal also requested financial assistance (officially it was requested on April 6, 2011, and granted on May 16).

Figure 3 presents the estimated expected risk-neutral recovery rates by country and Figures 4 to 7 include the estimated risk-neutral probabilities for horizons of 1, 3, 5 and 10 years, respectively. Up to April of 2010 and the Greek request of financial assistance, there is not a significant differentiation of the four largest euro area countries in terms of probabilities of default and recovery rates, although one can notice since December 2010 slightly lower probabilities of default for Germany and France, on the one hand, than for Italy and Spain, on the other hand. In this earlier period of the European debt crisis, the estimated risk-neutral expected recovery rates are all around 30-50%.

In April-May of 2010 the recovery rates showed a sharp increase for all countries, but with a visible decoupling: estimates for Germany and France became visibly higher than those for Italy and Spain. At the same time, one also observes a decoupling in terms of probabilities of default. The two countries with lower probabilities of default are the countries with higher recovery rates. It is also worth mentioning that the estimated probabilities of default for Germany and France did even show some decline in April-May 2010, unlike the case of Italy and Spain.

After a relative stabilization of the bond markets during the summer months of 2010, on October 18 the French-German Deauville Declaration was released, where explicit mention was made to private sector involvement, and a request of financial assistance by the Irish authorities became increasingly likely (and eventually happened on November 21 and was granted on November 28). In this juncture, by the end of October, there was an increase in the probabilities of default of Spain relative to those of Italy. This difference was gradually eroded in the following months, during which (until May 2011) one observes an upward trend in the probabilities of default of the three other countries. Simultaneously, and consistently, there is a declining trend in the expected recovery rates.

After May 2011, and in particular since mid-summer, the signs of instability in the European sovereign debt markets multiplied (coincidently with the above mentioned sharp increase in the RMSEs). In this period the expected recovery rates of Germany rose to very high levels (close to 100%) and the probabilities of default decreased significantly, to levels lower than those before the debt crisis. At the same time, there seems to be a decoupling of France relative to Germany, with a smaller decrease of probabilities of default for France in May-August, and afterwards a rise. As of Italy and Spain, the estimated probabilities rose during the period, sharply so in October and November. It is also noticeable that in the latter months the difference between Italy and Spain became reversed, with Italy's estimates of probabilities of default exceeding those of Spain. On November 25, 2011, the estimated risk-neutral probabilities of default for a 10-years horizon were 13.6%, 28.7%, 47.0% and 54.9%, respectively for Germany, France, Spain and Italy. Even taking into account that these are risk-neutral probabilities which overestimate the real-world probabilities implicit in the markets behavior, such shocking numbers highlight the salience of sovereign risk as an important driver in the current juncture of the European bond markets.

5. CONCLUDING REMARKS

Adapting to fiscal sustainability the famous utterance of Alan Greenspan on price stability, one could say that a fundamental task of policy-makers entrusted with the responsibility of managing public finances is to ensure that there is no ground for sovereign default risk to be a relevant consideration in investors' decisions. Unfortunately, in the current juncture, this utterance is not fulfilled for many euro area countries, including some of the largest.

Hence, it is the case that indicators focusing on market perceptions, such as those used in this paper, should be monitored on a regular basis, complementing more traditional tools for assessing fiscal sustainability and credibility. The proposed model can be applied to sovereign bond market data, providing snapshots of the market sentiment towards the sovereign borrowers included in the sample.

The major limitation of my approach is that it can only be applied to euro area countries for which there are reasonable numbers of sovereign bonds traded in the market. This concern about degrees of freedom excludes most of the smaller euro area countries, including those currently under assistance programs. In the case of the latter, an additional problem is that they are being supported by large amounts of official financing and the turnover in the markets of their sovereign bonds has decreased substantially, much reducing the informational content of the available bond yield quotes. Nevertheless, even restricting the analysis to the four largest euro area countries, as I did in this paper, it provides very relevant insights, in particular since the second half of 2011, when the euro area debt crisis could not be contained to some of the smaller countries and was clearly spreading to Italy and Spain.

APPENDIX

A. Identity in (1)

The first term of the left-hand side of (1) may be written as[9]:

$$E_0^{\mathbb{Q}}\left[\exp\left(-\int_0^t r_s ds\right)\mathbf{1}\left(\tau_g > t\right)\right] = E_0^{\mathbb{Q}}\left[\exp\left(-\int_0^t \left(r_s + \lambda_g \cdot x_s\right) ds\right)\right] = \prod_{d=1}^{D} \delta_{g,d}\left(t\right) \qquad \text{(A.1)}$$

As for the second term, one has[10]:

$$E_0^{\mathbb{Q}}\left[b_g \exp\left(-\int_0^{\tau_g} r_s ds\right)\right] = \rho_g E_0^{\mathbb{Q}}\left[\exp\left(-\int_0^{\tau_g} r_s ds\right)\right] = \rho_g E_0^{\mathbb{Q}}\left[\exp\left(-\int_0^{\tau_g} r_s ds\right)\right] =$$

(taking into account expression (3.3) of Proposition 3.1 in Lando (1998))

$$= \rho_g E_0^{\mathbb{Q}}\left[\int_0^t \lambda_g \cdot x_s \exp\left(-\int_0^s \left(r_u + \lambda_g \cdot x_u\right) du\right) ds\right] =$$

(owing to the independence of the state variables)

$$= \rho_g \left\{ \frac{\lambda_{g,1}}{1+\lambda_{g,1}} E_0^{\mathbb{Q}}\left[\int_0^t \tilde{x}_{g,1,s} \exp\left(-\int_0^s \left(\sum_{d=1}^{D} \tilde{x}_{g,d,u}\right) du\right) ds\right] + \right.$$

$$\left. + \sum_{d=2}^{D} E_0^{\mathbb{Q}}\left[\int_0^t \tilde{x}_{g,d,s} \exp\left(-\int_0^s \left(\sum_{i=1}^{D} \tilde{x}_{g,i,u}\right) du\right) ds\right] \right\} =$$

(using again expression (3.3) of Proposition 3.1 in Lando (1998)[11])

---

[9] As for the first identity, see e.g. expression (3.1) of Proposition 3.1 in Lando (1998), noting that the default times are strictly positive (i.e. it is assumed that no issuer has yet defaulted at the reference time). The second identity follows directly from the assumption of independence of the state variables.

[10] Recall that b_g is independent of $\left\{\mathcal{F}_t^x\right\}_{t\geq 0}$

$$= \rho_g \left\{ \frac{\lambda_{g,1}}{1+\lambda_{g,1}} E_0^{\mathbb{Q}} \left[\exp\left(-\int_0^{\tilde{\tau}_{g,1}} \left(\sum_{d=2}^{D} \tilde{x}_{g,d,s} \right) ds \right) \right] + \right.$$

$$\left. + \sum_{d=2}^{D} E_0^{\mathbb{Q}} \left[\exp\left(-\int_0^{\tau_{g,d}} \left(\sum_{i=1, i\neq d}^{D} \tilde{x}_{g,i,u} \right) du \right) \right] \right\} =$$

$$= \rho_g \left[\frac{\lambda_{g,1}}{1+\lambda_{g,1}} \prod_{d=2}^{D} \delta_{g,d}\left(\tilde{\tau}_{g,1}\right) + \sum_{d=2}^{D} \prod_{i=1, i\neq d}^{D} \delta_{g,i}\left(\tilde{\tau}_{g,d}\right) \right] \qquad \text{(A.2)}$$

B. Last expression in (2)

The last expression in (2) follows directly from (A.1) and (A.2).

C. Approximation (3)

$$\delta_{g,i}(\tilde{\tau}_{g,d}) = E_0^{\mathbb{Q}} \left[\exp\left(-\int_0^{\tilde{\tau}_{g,d}} \tilde{x}_{g,i,s} ds \right) \right] \simeq$$

$$\simeq E_0^{\mathbb{Q}} \left[\sum_{m=1}^{M_{g,n}} \sum_{k=1}^{K_g} \exp\left(-\int_0^{t_{g,n,m-1}+(k-1/2)\Delta_g} \tilde{x}_{g,i,s} ds \right) \mathbf{1}\left(t_{g,n,m-1} + (k-1)\Delta_g < \tilde{\tau}_{g,d} \leq t_{g,n,m-1} + k\Delta_g \right) \right] =$$

(Owing to the mutual independence of the state variables)

$$= \sum_{m=1}^{M_{g,n}} \sum_{k=1}^{K_g} \delta_{g,i}\left(t_{g,n,m-1} + (k-1/2)\Delta_g \right) \cdot$$

$$\cdot \left\{ E_0^{\mathbb{Q}} \left[\mathbf{1}\left(\tilde{\tau}_{g,d} > t_{g,n,m-1} + (k-1)\Delta_g \right) \right] - E_0^{\mathbb{Q}} \left[\mathbf{1}\left(\tilde{\tau}_{g,d} > t_{g,n,m-1} + k\Delta_g \right) \right] \right\} =$$

$$= \sum_{m=1}^{M_{g,n}} \sum_{k=1}^{K_g} \delta_{g,i}\left(t_{g,n,m-1} + (k-1/2)\Delta_g \right) \left[\delta_{g,d}\left(t_{g,n,m-1} + (k-1)\Delta_g \right) - \delta_{g,d}\left(t_{g,n,m-1} + k\Delta_g \right) \right]$$

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[11] For example, $E_0^{\mathbb{Q}} \left[\int_0^t \tilde{x}_{g,1,s} \exp\left(-\int_0^s \left(\sum_{d=1}^{D} \tilde{x}_{g,d,u} \right) du \right) ds \right] = E_0^{\mathbb{Q}} \left[\int_0^t \tilde{x}_{g,1,s} \exp\left(-\int_0^s \left(\varphi_{g,1,u} + \tilde{x}_{g,1,u} \right) du \right) ds \right] =$ $= E_0^{\mathbb{Q}} \left[\exp\left(-\int_0^{\tilde{\tau}_{g,1}} \varphi_{g,1,s} ds \right) \right]$, where $\varphi_{g,1,t} \equiv \sum_{d=2}^{D} \tilde{x}_{g,d,t}$ and the last equality follows from expression (3.3) of Lando's Proposition (3.1), with r_t and λ_t (his notation) replaced by $\varphi_{g,1,t}$ and $\tilde{x}_{g,1,t}$, respectively.

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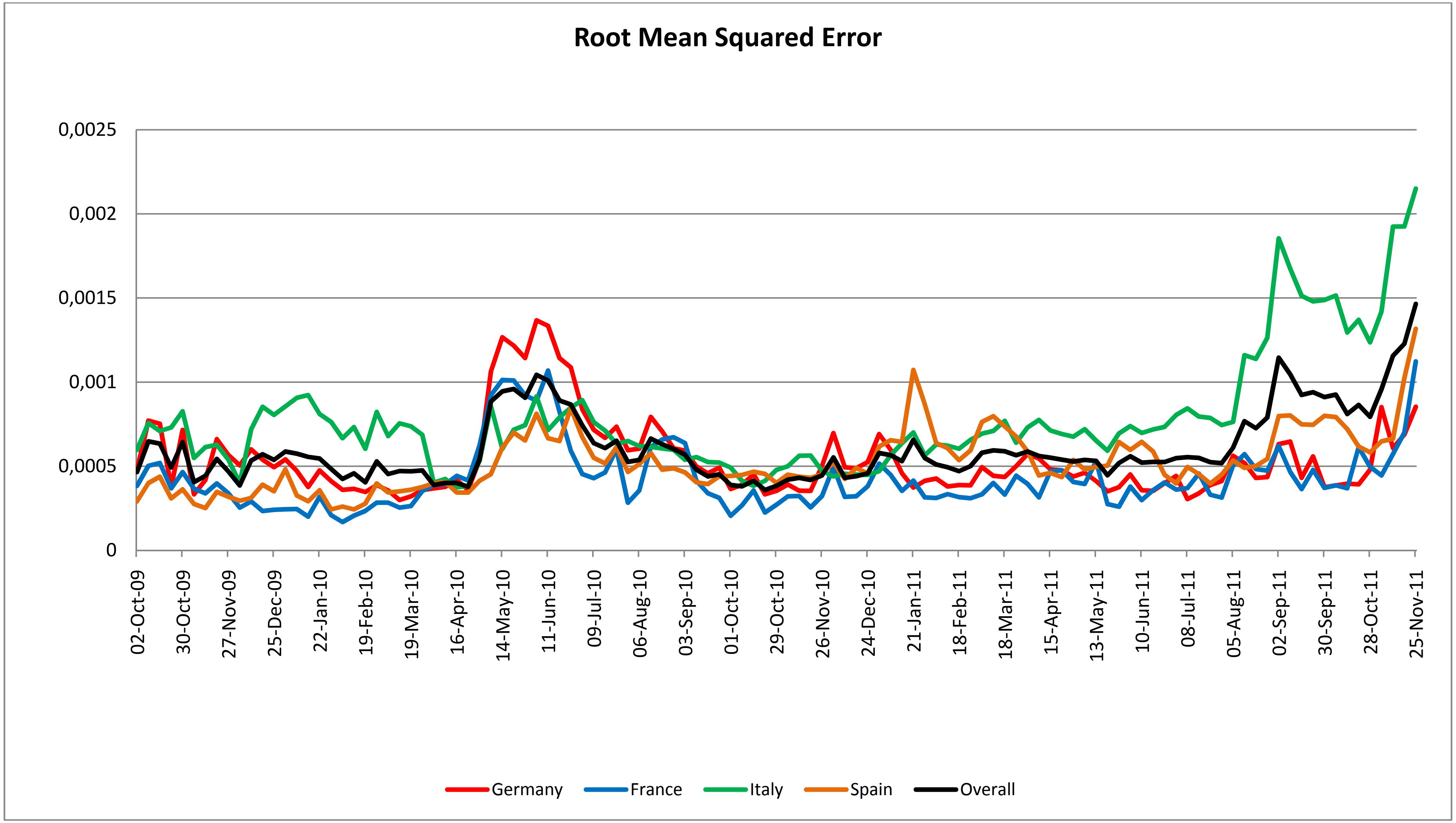
Figure 1



Root Mean Squared Error
0,0025
0,002
0,0015
0,001
0,0005
0
02-Oct-09
30-Oct-09
27-Nov-09
25-Dec-09
22-Jan-10
19-Feb-10
19-Mar-10
16-Apr-10
14-May-10
11-Jun-10
09-Jul-10
06-Aug-10
03-Sep-10
01-Oct-10
29-Oct-10
26-Nov-10
24-Dec-10
21-Jan-11
18-Feb-11
18-Mar-11
15-Apr-11
13-May-11
10-Jun-11
08-Jul-11
05-Aug-11
02-Sep-11
30-Sep-11
28-Oct-11
25-Nov-11
Germany
France
Italy
Spain
Overall

Figure 2

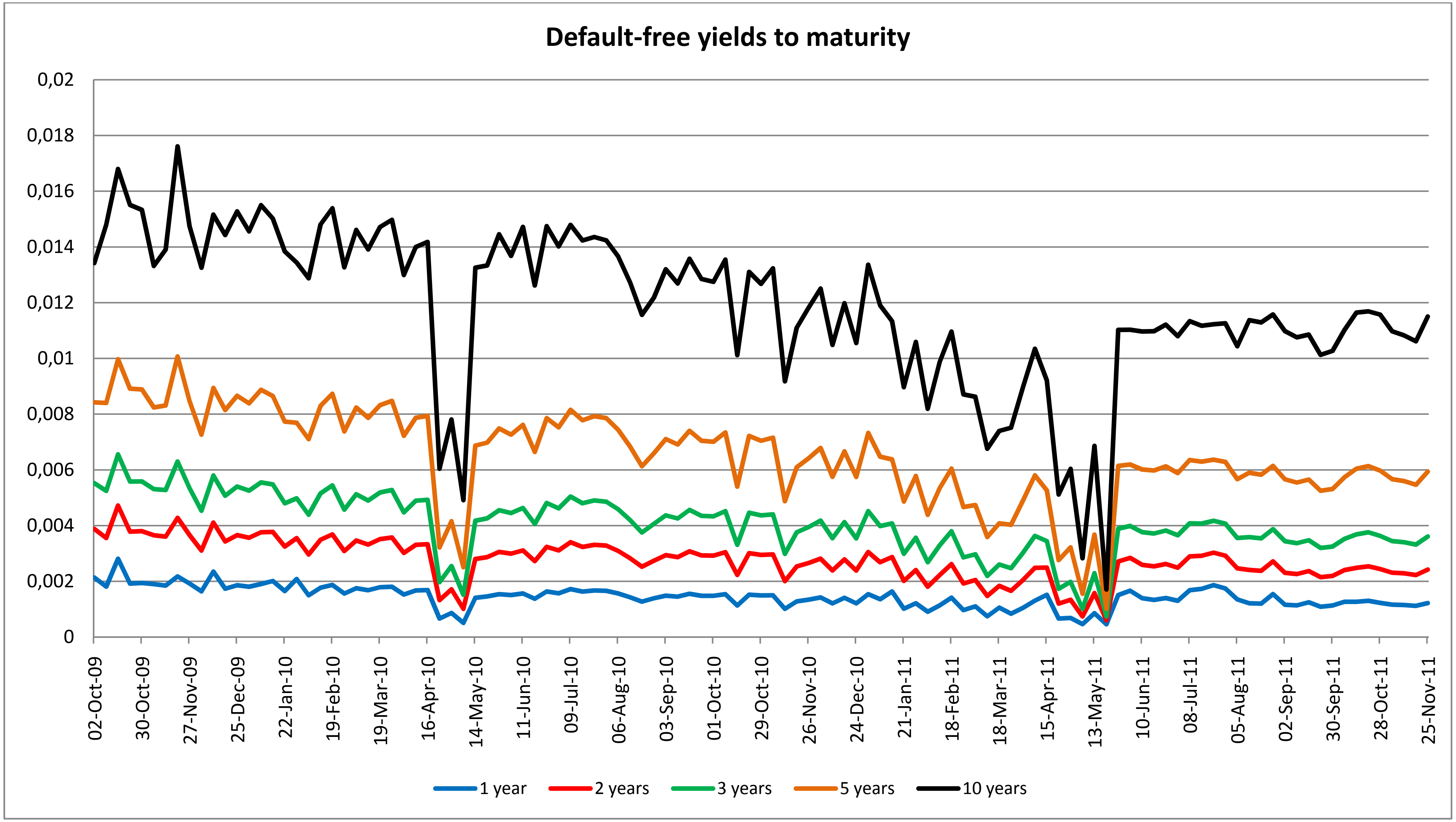
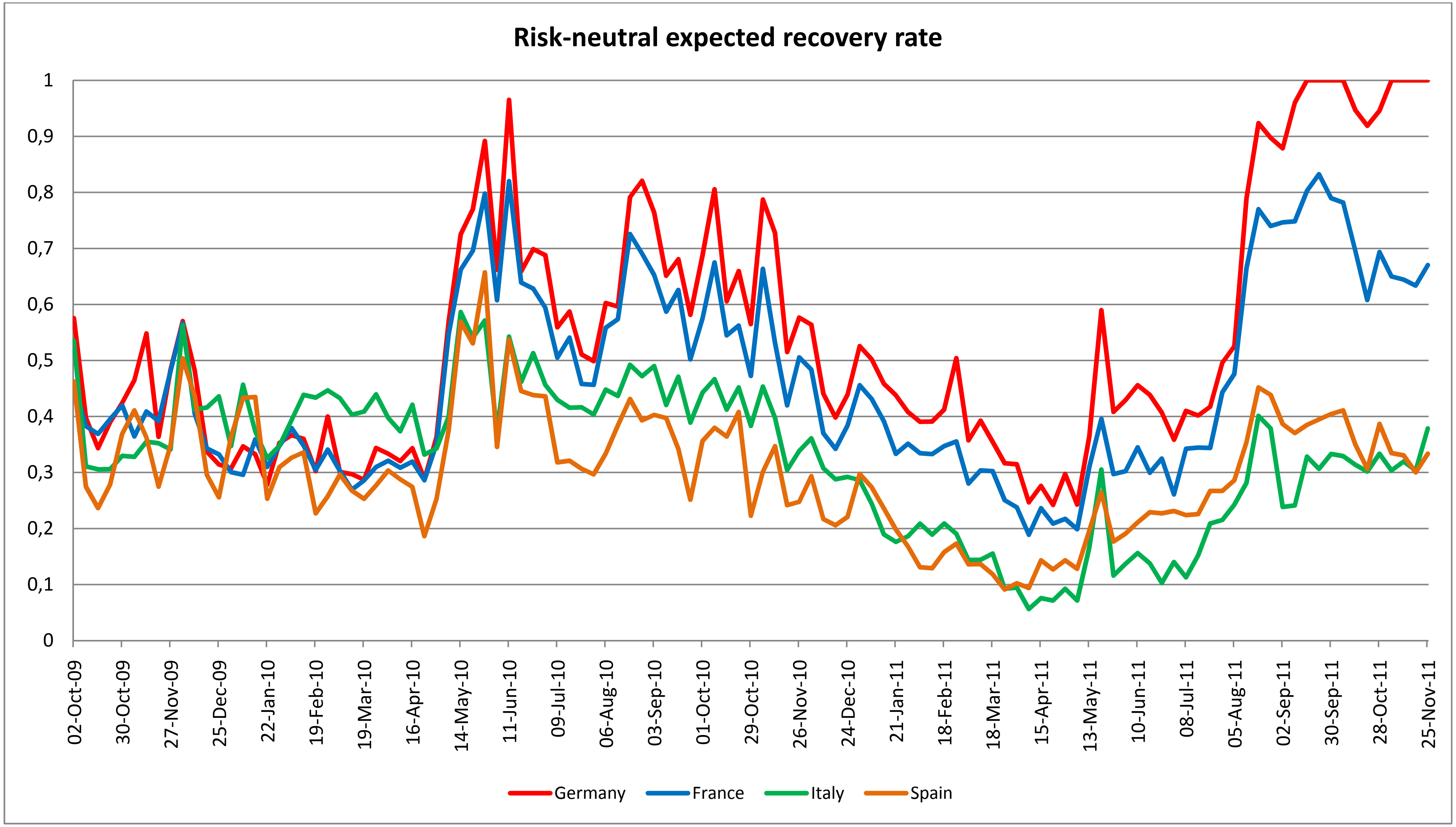

Default-free yields to maturity
0,02
0,018
0,016
0,014
0,012
0,01
0,008
0,006
0,004
0,002
0
02-Oct-09
30-Oct-09
27-Nov-09
25-Dec-09
22-Jan-10
19-Feb-10
19-Mar-10
16-Apr-10
14-May-10
11-Jun-10
09-Jul-10
06-Aug-10
03-Sep-10
01-Oct-10
29-Oct-10
26-Nov-10
24-Dec-10
21-Jan-11
18-Feb-11
18-Mar-11
15-Apr-11
13-May-11
10-Jun-11
08-Jul-11
05-Aug-11
02-Sep-11
30-Sep-11
28-Oct-11
25-Nov-11
1 year
2 years
3 years
5 years
10 years

Figure 3



Risk-neutral expected recovery rate
1
0,9
0,8
0,7
0,6
0,5
0,4
0,3
0,2
0,1
0
02-Oct-09
30-Oct-09
27-Nov-09
25-Dec-09
22-Jan-10
19-Feb-10
19-Mar-10
16-Apr-10
14-May-10
11-Jun-10
09-Jul-10
06-Aug-10
03-Sep-10
01-Oct-10
29-Oct-10
26-Nov-10
24-Dec-10
21-Jan-11
18-Feb-11
18-Mar-11
15-Apr-11
13-May-11
10-Jun-11
08-Jul-11
05-Aug-11
02-Sep-11
30-Sep-11
28-Oct-11
25-Nov-11
Germany
France
Italy
Spain

Figure 4

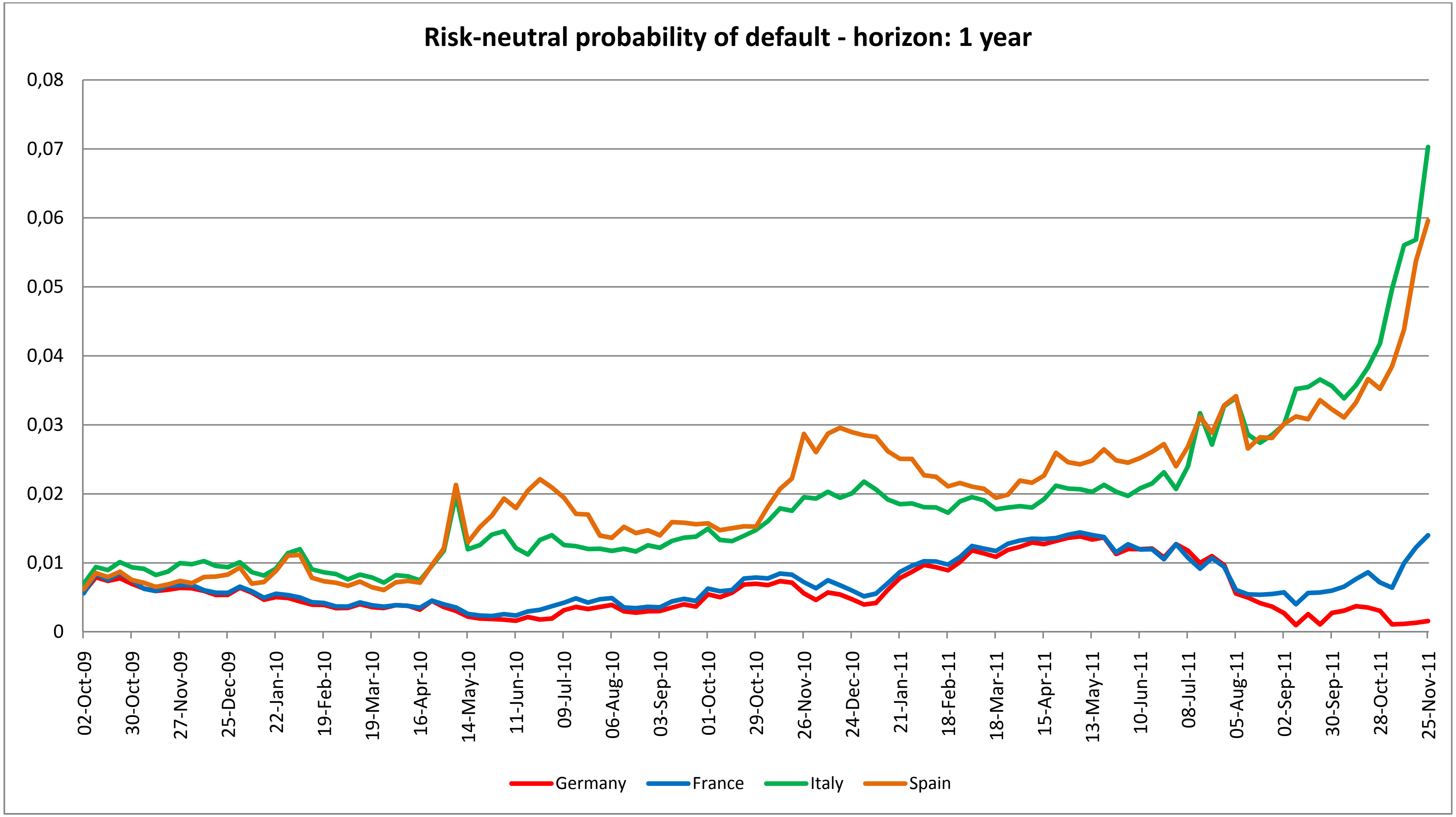
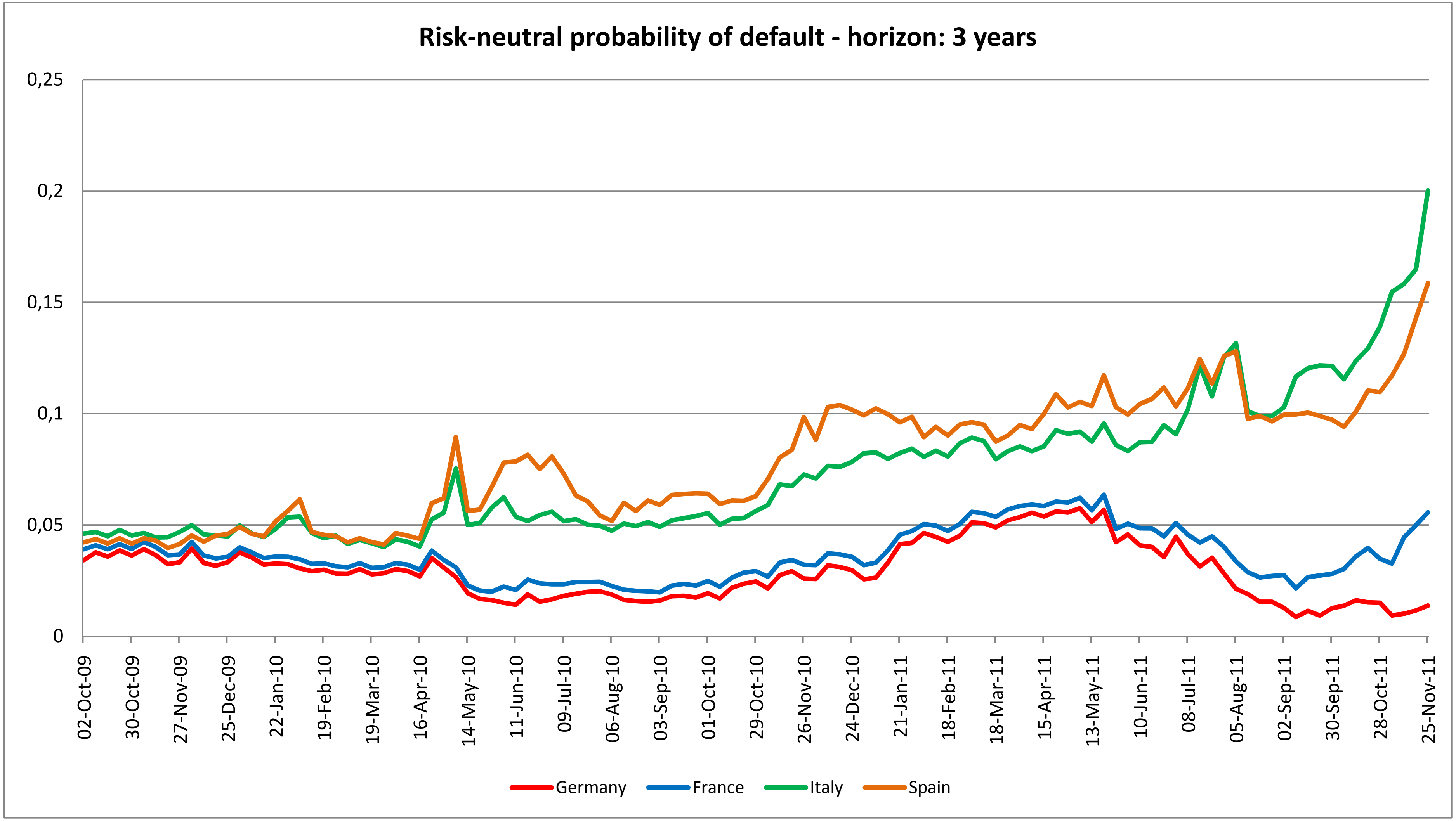

Risk-neutral probability of default - horizon: 1 year
0,08
0,07
0,06
0,05
0,04
0,03
0,02
0,01
0
02-Oct-09
30-Oct-09
27-Nov-09
25-Dec-09
22-Jan-10
19-Feb-10
19-Mar-10
16-Apr-10
14-May-10
11-Jun-10
09-Jul-10
06-Aug-10
03-Sep-10
01-Oct-10
29-Oct-10
26-Nov-10
24-Dec-10
21-Jan-11
18-Feb-11
18-Mar-11
15-Apr-11
13-May-11
10-Jun-11
08-Jul-11
05-Aug-11
02-Sep-11
30-Sep-11
28-Oct-11
25-Nov-11
Germany
France
Italy
Spain

Figure 5



Risk-neutral probability of default - horizon: 3 years
0,25
0,2
0,15
0,1
0,05
0
02-Oct-09
30-Oct-09
27-Nov-09
25-Dec-09
22-Jan-10
19-Feb-10
19-Mar-10
16-Apr-10
14-May-10
11-Jun-10
09-Jul-10
06-Aug-10
03-Sep-10
01-Oct-10
29-Oct-10
26-Nov-10
24-Dec-10
21-Jan-11
18-Feb-11
18-Mar-11
15-Apr-11
13-May-11
10-Jun-11
08-Jul-11
05-Aug-11
02-Sep-11
30-Sep-11
28-Oct-11
25-Nov-11
Germany
France
Italy
Spain

Figure 6

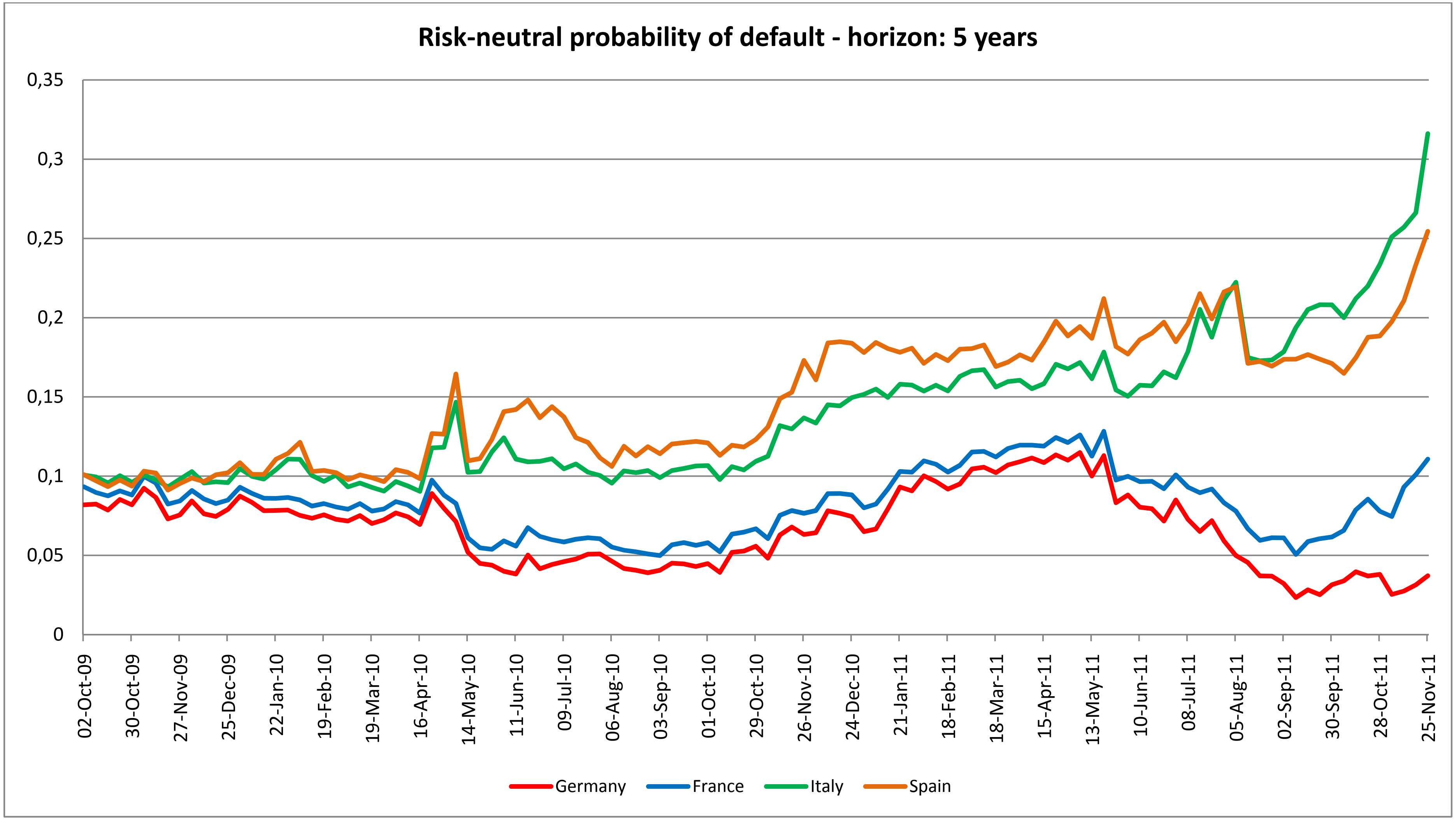
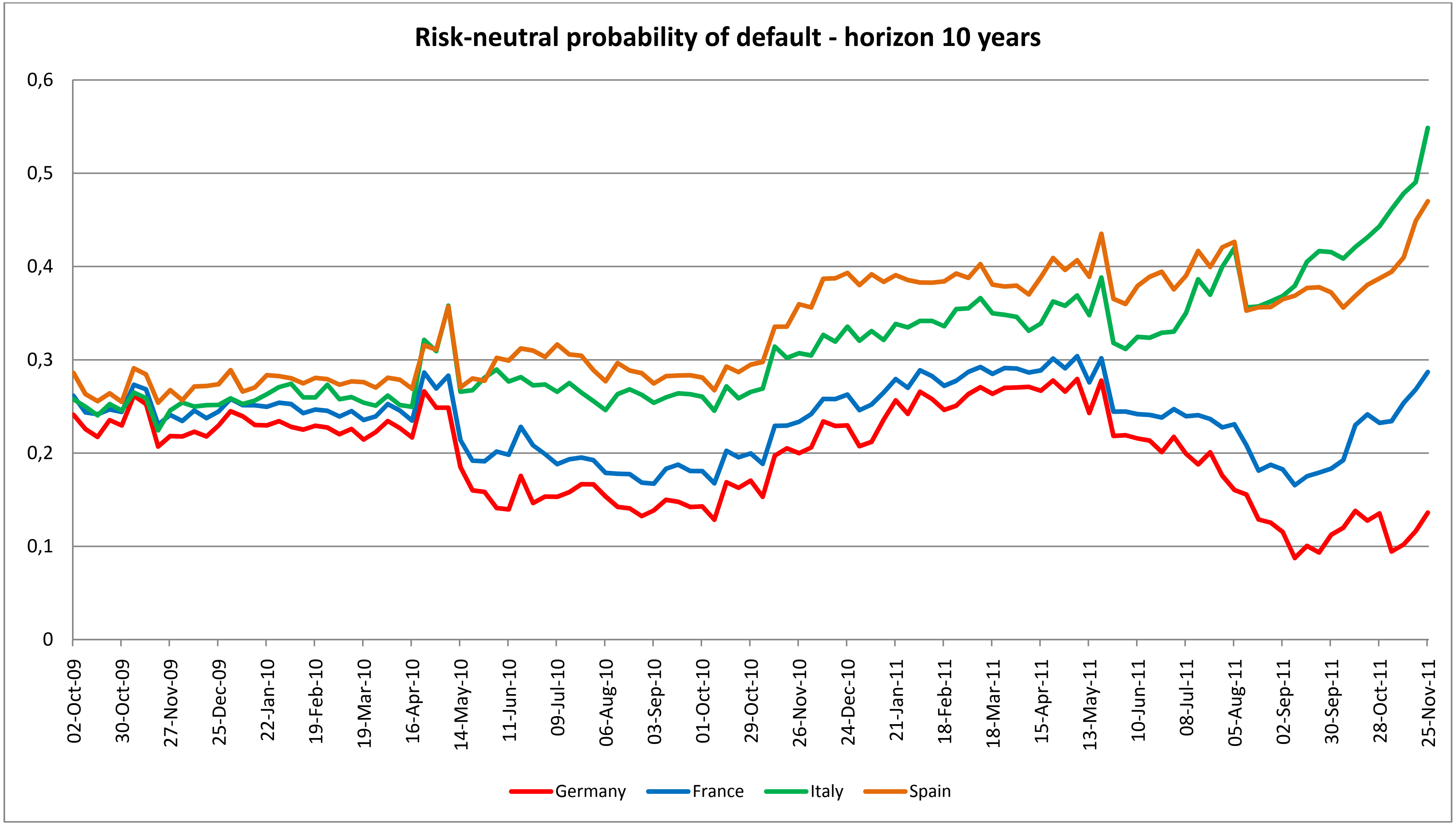

Risk-neutral probability of default - horizon: 5 years
0,35
0,3
0,25
0,2
0,15
0,1
0,05
0
02-Oct-09
30-Oct-09
27-Nov-09
25-Dec-09
22-Jan-10
19-Feb-10
19-Mar-10
16-Apr-10
14-May-10
11-Jun-10
09-Jul-10
06-Aug-10
03-Sep-10
01-Oct-10
29-Oct-10
26-Nov-10
24-Dec-10
21-Jan-11
18-Feb-11
18-Mar-11
15-Apr-11
13-May-11
10-Jun-11
08-Jul-11
05-Aug-11
02-Sep-11
30-Sep-11
28-Oct-11
25-Nov-11
Germany
France
Italy
Spain

Figure 7


Risk-neutral probability of default - horizon 10 years
0,6
0,5
0,4
0,3
0,2
0,1
0
02-Oct-09
30-Oct-09
27-Nov-09
25-Dec-09
22-Jan-10
19-Feb-10
19-Mar-10
16-Apr-10
14-May-10
11-Jun-10
09-Jul-10
06-Aug-10
03-Sep-10
01-Oct-10
29-Oct-10
26-Nov-10
24-Dec-10
21-Jan-11
18-Feb-11
18-Mar-11
15-Apr-11
13-May-11
10-Jun-11
08-Jul-11
05-Aug-11
02-Sep-11
30-Sep-11
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