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Modeling and Forecasting Interval Time Series with Threshold Models: An Application to S&P500 Index Returns*

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Abstract

Over recent years several methods to deal with high-frequency data (economic, financial and other) have been proposed in the literature. An interesting example is for instance interval-valued time series described by the temporal evolution of high and low prices of an asset. In this paper a new class of threshold models capable of capturing asymmetric effects in interval-valued data is introduced as well as new forecast loss functions and descriptive statistics of the forecast quality proposed. Least squares estimates of the threshold parameter and the regression slopes are obtained; and forecasts based on the proposed threshold model computed. A new forecast procedure based on the combination of this model with the k nearest neighbors method is introduced. To illustrate this approach, we report an application to a weekly sample of S&P500 index returns. The results obtained are encouraging and compare very favorably to available procedures.

Keywords: Interval Time Series, Forecasting, Threshold Model, Forecast Accuracy Measures.
JEL: C12, C22, C52, C53.

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1 Introduction

Modeling and forecasting interval-valued time series (ITS, hereafter) has received considerable attention in recent literature. ITS analysis has been introduced as a new field related to multivariate analysis and pattern recognition. One of the main advantages of using this new type of data is that it avoids problems associated with neglecting data variability in large data sets. In economic and financial settings, one may consider data registered in an almost continuous way, such as, for instance, exchange rate fluctuations, stock prices and returns, and electricity demand. For instance, the daily or weekly highest and lowest prices of assets may be regarded as boundary values of an interval and therefore ITS modeling and forecasting techniques, such as the ones presented in this paper may provide useful tools for analysis.

Over recent years different approaches have been introduced for the analysis of interval valued data. In particular, exponential smoothing methods, pattern recognition and multivariate models are among the most influential methodologies. For instance, Billard and Diday (2000, 2003) and Lima and Carvalho (2010) consider regression analysis of interval-valued data as part of symbolic data methods; and Hu and He (2007) and Han *et al.* (2008) propose linear interval methods as a counterpart to point based econometrics. ITS models are used in these latter papers to analyze stock market indices and the sterling-dollar exchange rate. Moreover, Garcia-Ascanio and Maté (2010) consider the use of ITS methods as a risk management tool in power systems planning. Some important contributions in the area of nonparametric forecast methods include Muñoz *et al.* (2007), Arroyo *et al.* (2010) and Maia and Carvalho (2011). A more detailed review of ITS forecasting methods will be presented below in Section 3.

The limited economic and financial application of linear (interval) models when interest lies in the analysis of regime dependence or asymmetric behavior of the series over the business cycle has lead to the development of a large number of nonlinear models. One class of nonlinear models that has proven successful in the literature are threshold autoregressive (TAR) models. For instance, Tong (1990) developed TAR models and applied them to predict stock price movements. Henry *et al.* (2001) present evidence of threshold nonlinearity in the Australian real exchange rate, and Duaker *et al.* (2007) develop a contemporaneous TAR model for the bonds market. In the context of ITS several works have also stressed the importance of nonlinearities (see, *e.g.*, Muñoz *et al.*, 2007 Maia *et al.,* 2008, Maia and Carvalho, 2011). These studies present evidence of the low accuracy of linear approaches to forecast ITS with nonlinear characteristics and introduce the application of neural networks and hybrid methods using the multi-layer perceptron algorithm adapted to interval data. However, these procedures aim to produce forecasts without explicitly modelling the nonlinear characteristics of the data and to the best of our knowledge there have been no studies in the ITS context that attempt to model and explain nonlinear features. Furthermore, no empirical results on regime dependent ITS forecasts are available so far.

In this paper, econometric methods for regime switching threshold models are adapted to ITS characterized by their center and radius. In our analysis the center-radius representation of interval-valued data is favoured since it is known from literature that the range (or radius of an interval) is a suitable measure of the variability of a random variable. Several empirical applications in the context of financial data use the range, in particular, for the estimation of volatilities (see, *e.g.*, Parkinson, 1980, Beckers, 1983, Chou, 2005 and Brandt and Jones, 2002). Moreover, the choice of this representation is also based on the available evidence of the good predictive power of the interval range (see, *e.g.*, Brandt and Jones, 2002 and Arroyo *et al.,* 2011) and its superiority when compared to the lower/upper bound representation.

In relation to the forecasts from the threshold model proposed in this paper, it is well known that despite the natural appeal of nonlinear models and their potentially useful features for analyzing state-dependent relationships, the development of forecasting techniques is far from trivial (see, *e.g.*, Clements and Smith, 1999) and several difficulties emerge in empirical applications (see, *e.g.*, De Gooijer and Kumar, 1992, Potter, 1995). In this paper we focus on one step ahead forecasts based on the proposed interval threshold model and its combination with the k-nearest neighbors method (k-NN, hereafter) in a similar manner as the hybrid system proposed by Zhang (2003). The new forecasting algorithm with the given k-NN correction seems to be more robust to possible changes in the series and consequently able to improve the forecasting performance of the interval threshold model.

Accuracy measures also play an important role in the context of ITS analysis since as indicated by Chatfield (1996) the error made by an inappropriate interval prediction method can be more severe than that made by a simple point prediction. Some of the important contributions in this field are Ichino and Yaguchi (1994), Nguyen and Wu (2006) Arroyo and Maté (2006) and Hsy and Wu (2008). Ichino and Yaguchi (1994) present simple and convenient generalized Minkowski metrics in the multidimensional feature space which can be generalized to the ITS context. Nguyen and Wu (2006) introduce the basic foundational aspects for a theory of statistics with fuzzy data, Arroyo and Maté (2006) analyse accuracy measures for ITS based on Hausdorff and Ichino-Yaguchi distances, and Hsy and Wu (2008) compare several approaches to evaluate interval forecasts. In line with these works, in this paper we also introduce interval quality measures, as well as additional forecast descriptive statistics (such as efficiency and coverage rates) in order to provide more information for an objective decision regarding the interval forecast performance.

This paper is structured as follows. Section 2 introduces the theoretical framework of the interval threshold model proposed and discusses accuracy measures and estimation;

Section 3 provides a brief overview of available ITS forecasting methods and presents new forecasting approaches; Section 4 discusses an empirical application to the S&P500 index returns and section 5 concludes the paper. An appendix collects some important auxiliary results.

2 Threshold Models and Accuracy Measures

2.1 The Threshold Model

Consider the observed interval time series $\{Y_t\}_{t=1}^T$, with $Y_t = 0$ for $t \leq 0$, such that each observation Y_t in the sample has an interval structure and can be represented either by its lower and upper bounds (*i.e.*, $Y_t = [l_t, u_t]$) or equivalently by its center and radius (*i.e.*, $Y_t = \langle c_t, r_t \rangle$).

The model considered in this paper is a two regime center-radius self-exiting threshold model (CR-SETAR hereafter) for ITS. Although the focus is only on a two regime CR-SETAR model, the results presented can be extended to more general models. However, for the sake of simplicity and given that in the empirical application, our interest is mainly in detecting periods of high volatility, it is reasonable to consider only a two regime specification.

The general two-regime CR-SETAR model that we propose takes the form,

$$\begin{aligned} c_t = (\mu_1 + \sum_{i=1}^{p_c} \alpha_{1i} c_{t-i} + \sum_{j=1}^{p_r} \beta_{1j} r_{t-j}) \times I_{\{z_{t-d} \leq \gamma\}} + \\ (\mu_2 + \sum_{i=1}^{p_c} \alpha_{2i} c_{t-i} + \sum_{j=1}^{p_r} \beta_{2j} r_{t-j}) \times I_{\{z_{t-d} > \gamma\}} + \varepsilon_t, \end{aligned} \tag{2.1}$$

$$\begin{aligned} r_t = (\lambda_1 + \sum_{i=1}^{q_c} \varphi_{1i} c_{t-i} + \sum_{j=1}^{q_r} \phi_{1j} r_{t-j}) \times I_{\{z_{t-d} \leq \gamma\}} + \\ (\lambda_2 + \sum_{i=1}^{q_c} \varphi_{2i} c_{t-i} + \sum_{j=1}^{q_r} \phi_{2j} r_{t-j}) \times I_{\{z_{t-d} > \gamma\}} + \xi_t, \end{aligned} \tag{2.2}$$

where $I_{\{\cdot\}}$ is an indicator function, $z_{t-d} = z(\mathbf{F}_{t-d}^p)$ is a known function of the data available at time $t-d$ (*i.e.*, $\mathbf{F}_{t-d}^p = \{Y_{t-d}, Y_{t-d-1}, ..., Y_1\}$) and $p = \max\{p_c, p_r, q_c, q_r\}$. The parameters p_c, p_r, q_c, and q_r represent the different lag orders of the components entering (2.1) and (2.2), and γ denotes the threshold parameter which takes values in $\Gamma = [\underline{\gamma}, \overline{\gamma}]$, a bounded subset of $\mathbb{R}$. The parameter vectors $\boldsymbol{\theta}_{1C} = (\mu_1, \alpha_{11}, ..., \alpha_{1p_c}, \beta_{11}, ..., \beta_{1p_r})$ and $\boldsymbol{\theta}_{1R} = (\lambda_1, \varphi_{11}, ..., \varphi_{1q_c}, \phi_{11}, ..., \phi_{1q_r})$ correspond to the slopes when $z_{t-d} \leq \gamma$ and $\boldsymbol{\theta}_{2C} = (\mu_2, \alpha_{21}, ..., \alpha_{2p_c}, \beta_{21}, ..., \beta_{2p_r})$ and $\boldsymbol{\theta}_{2R} = (\lambda_2, \varphi_{21}, ..., \varphi_{2q_c}, \phi_{21}, ..., \phi_{2q_r})$ are the slopes when $z_{t-d} > \gamma$. The errors ε_t and ξ_t are assumed to be Martingale difference sequences with respect to the past history of c_t and r_t, respectively. In principle, we would like to allow for different autoregressive orders for c_t and r_t in equations (2.1) and

(2.2), but for simplicity of exposition and without loss of generality we will assume that $p_c = p_r = q_c = q_r = p$.

An alternative representation of (2.1)-(2.2) will be considered for the derivation of the estimators. Thus, defining $\boldsymbol{\theta}_C = (\boldsymbol{\theta}'_{1C}, \boldsymbol{\theta}'_{2C})'$, $\boldsymbol{\theta}_R = (\boldsymbol{\theta}'_{1R}, \boldsymbol{\theta}'_{2R})'$, $X_{C,t-1} = (1\ c_{t-1}\ \cdots\ c_{t-p}\ r_{t-1}\ \cdots\ r_{t-p})'$, $X_{R,t-1} = (1\ c_{t-1}\ \cdots\ c_{t-p}\ r_{t-1}\ \cdots\ r_{t-q})'$,

$$X_{C,t-1}(\gamma) = \left(X'_{C,t-1} \times I_{\{z_{t-d} \leq \gamma\}} \quad X'_{C,t-1} \times I_{\{z_{t-d} > \gamma\}}\right)'$$

and

$$X_{R,t-1}(\gamma) = \left(X'_{R,t-1} \times I_{\{z_{t-d} \leq \gamma\}} \quad X'_{R,t-1} \times I_{\{z_{t-d} > \gamma\}}\right)',$$

it follows that (2.1)-(2.2) can equivalently be written as

$$\begin{cases} c_t = X_{C,t-1}(\gamma)'\boldsymbol{\theta}_C + \varepsilon_t \\ r_t = X_{R,t-1}(\gamma)'\boldsymbol{\theta}_R + \xi_t \end{cases}. \tag{2.3}$$

To proceed with the estimation of this CR-SETAR model we will first introduce necessary loss functions and accuracy measures for ITS. These will provide us with useful tools for the analysis of the CR-SETAR model.

2.2 Accuracy Measures[1]

Consider the realized interval value Y_t and its forecast or fitted value $\widehat{Y}_t$. Classical time series approaches, typically used to measure the error between Y_t and $\widehat{Y}_t$, based on the difference, $Y_t - \widehat{Y}_t$, are not adequate accuracy measures for ITS (see, for instance, Arroyo *et al.*, 2010). Thus, in what follows, two alternative convenient approaches to construct loss functions for interval valued data are introduced.

One uses a generalization of the Minkowski-type loss function of order $\varrho \geq 1$ (see, *e.g.*, Arroyo and Mate, 2006), defined as,

$$d_{L_\varrho}(Y_t, \widehat{Y}_t) = \sqrt[\varrho]{\left|l_t - \widehat{l}_t\right|^\varrho + |u_t - \widehat{u}_t|^\varrho} = \sqrt[\varrho]{|\widetilde{c}_t - \widetilde{r}_t|^\varrho + |\widetilde{c}_t + \widetilde{r}_t|^\varrho}, \tag{2.4}$$

where $\widetilde{c}_t = c_t - \widehat{c}_t$ and $\widetilde{r}_t = r_t - \widehat{r}_t$. Some difficulties with (2.4) arise when $\varrho \to \infty$. It appears that, $d_{L_\infty}(Y_t, \widehat{Y}_t) = \lim_{\varrho \to \infty} d_{L_\varrho}(Y_t, \widehat{Y}_t) = \max\{\left|l_t - \widehat{l}_t\right|, |u_t - \widehat{u}_t|\}$, cannot distinguish between some sets of intervals, because it does not take into account the nearness of the inner bound of the pair of intervals considered. Therefore, application of $d_{L_\varrho}(.,.)$ is restricted to finite ϱ. In the following sections of this paper we use the Euclidean-type loss function (which corresponds to (2.4) with $\varrho = 2$ and which we define as d_{L_2}) due to its intuitive and mathematical appeal.

[1] For a better understanding of some of the properties presented throughout this paper consider Definition A.1 provided in the appendix, which collects relevant interval operations.

The other approach that can be used to construct a loss function for ITS is based on the normalized symmetric difference (NSD hereafter) of intervals, *i.e.*,

$$d_{NSD}(Y_t, \widehat{Y}_t) = \frac{w(Y_t \triangle \widehat{Y}_t)}{w(Y_t \sqcup \widehat{Y}_t)}, \tag{2.5}$$

where $w(\cdot)$ indicates the width of the argument as given in (A.5), Δ corresponds to the symmetric difference operator as discussed in (A.4) and $\sqcup$ denotes the interval hull presented in (A.3) of Definition A.1 in the appendix, respectively. Several modifications of (2.5) appear in related literature, each of which with advantages and disadvantages. For instance, Hsu and Wu (2008) considered the d_{NSD} loss function normalized to the width of the realized interval Y_t $\left(i.e.,\ d^1_{NSD}(Y_t, \widehat{Y}_t) = \frac{w(Y_t \triangle \widehat{Y}_t)}{w(Y_t)}\right)$; Ichino-Yaguchi (1994) and De Carvalho (1996) proposed a loss function for a multidimensional space of mixed variables given as,

$$d^2_{NSD}(Y_t, \widehat{Y}_t, \theta) = \frac{d_{IY}(Y_t, \widehat{Y}_t, \theta)}{w(Y_t \cup \widehat{Y}_t)}, \tag{2.6}$$

where $d_{IY}(Y_t, \widehat{Y}_t, \theta)$ is the Ichino-Yaguchi distance, and $\theta \in [0, 0.5]$ is an exogenous parameter that controls the effect of the inner-bound and outer-bound nearness between Y_t and $\widehat{Y}_t$. However, care needs to be taken when using the d_{NSD}, d^1_{NSD} or d^2_{NSD} loss-functions due to the possible difficulties in distinguishing the nearness of intervals with empty intersection (cases of d_{NSD} and d^1_{NSD}) and given the need to select a value for the exogenous parameter θ (case of d^2_{NSD}). Hence, given the possible drawbacks of these NSD loss functions, we suggest a modification which presents interesting features. The modification consists of correcting the width of the symmetric difference in (2.5) with an additional term that provides information on the shortest distance between two intervals with empty intersection, *i.e.*,

$$w(Y_t \triangle^* \widehat{Y}_t) = w(Y_t \sqcup \widehat{Y}_t) - w(Y_t \cap \widehat{Y}_t) + w([Y_t \sqcup \widehat{Y}_t]/[Y_t \cup \widehat{Y}_t]) I_{Y_t \cap \widehat{Y}_t = \emptyset} \tag{2.7}$$

where $\triangle^*$ denotes the corrected symmetric difference. Thus, using the properties of set operations the following expression is obtained

$$d^*_{NSD}(Y_t, \widehat{Y}_t) = \frac{w(Y_t \triangle^* \widehat{Y}_t)}{w(Y_t \sqcup \widehat{Y}_t)} = 2 - \frac{w(Y_t) + w(\widehat{Y}_t)}{w(Y_t \sqcup \widehat{Y}_t)}. \tag{2.8}$$

The usefulness of the metric proposed in (2.8) results from its scale-independence and simplicity of application. From (2.8) we observe that the range of this statistic is $[0, 2]$. In particular, note that,

- $d^*_{NSD}(Y_t, \widehat{Y}_t) = 2$ if and only if the considered intervals are degenerated[2] and not equal;

[2] An interval is called degenerated when $l_t = u_t$.

- $1 < d^*_{NSD}(Y_t, \widehat{Y}_t) < 2$ if the intersection of the nondegenerated intervals is empty;
- $d^*_{NSD}(Y_t, \widehat{Y}_t) = 1$ for any pair of tangent intervals or if one of the intervals is degenerated and is contained in the second one;
- and $d^*_{NSD}(Y_t, \widehat{Y}_t) < 1$ if and only if $Y_t \cap \widehat{Y}_t \neq \emptyset$.

The assessment of dissimilarities between ITS will proceed through the choice of a loss function. In order to quantify the overall accuracy of the fitted or forecasted ITS, $\{\widehat{Y}_i\}$, the *mean distance error* (MDE) of intervals is used, *i.e.,*

$$MDE_d\left(\{Y_i\}, \{\widehat{Y}_i\}\right) = \frac{\sum_{i=1}^{N} d(Y_i, \widehat{Y}_i)}{N}, \tag{2.9}$$

where $d(Y_i, \widehat{Y}_i)$ denotes either the loss function proposed in (2.4) with $\varrho = 2$ or in (2.8), $\{\widehat{Y}_i\} = \{\widehat{Y}_t\}_{t=1}^{T}$ if $\widehat{Y}_i$ denotes the fitted values of Y_i and $\{\widehat{Y}_i\} = \{\widehat{Y}_t\}_{t=T+1}^{T+h}$ if $\widehat{Y}_i$ denotes the forecast of Y_i, and consequently $N = T$ or $N = h$, respectively. Furthermore, for competitive and accuracy analysis of interval forecasts we also introduce a set of descriptive statistics defined as,

- coverage rate
$$R_C = \frac{1}{N}\sum_{i=1}^{N} \frac{w(Y_i \cap \widehat{Y}_i)}{w(Y_i)}, \tag{2.10}$$
- efficiency rate
$$R_E = \frac{1}{N}\sum_{i=1}^{N} \frac{w(Y_i \cap \widehat{Y}_i)}{w(\widehat{Y}_i)}, \tag{2.11}$$
- residual rate
$$R_R = \frac{1}{N}\sum_{i=1}^{N} \frac{w(\widehat{Y}_i \Delta Y_i)}{w(Y_i)}. \tag{2.12}$$

The main purpose of considering the R_C, R_E and R_R statistics presented in (2.10), (2.11) and (2.12) is to provide additional information on what part of the realized ITS is covered by its forecast (*i.e.*, coverage rate), what part of the forecast covers the realized ITS (*i.e.*, efficiency rate) and finally the part of the realized ITS that is not covered by the forecast plus the part of the forecats that is redundant (*i.e.*, residual rate). It should however be noted that these statistics need to be considered jointly, given that if analysed individually wrong conclusions may be drawn. For instance, if the realized intervals are fully enclosed in the predicted intervals then the coverage rate will be 100%. However, the efficiency rate may be less then 100% and reveal the fact that the forecasted ITS is actually wider then the realized ITS. Note that the reverse may also be observed[3]. Hence,

[3] We thank an anonymous referee for highlighting this point.

only when the coverage and efficiency rates are reasonably high and the difference between them is small will these provide indication of a good forecast.

Given the results in (2.9), (2.10), (2.11) and (2.12) we are now able to proceed to the discussion of the estimation of the CR-SETAR model, the ITS forecast methods and their respective evaluation.

2.3 Estimation of CR-SETAR models

The parameters of interest are $\boldsymbol{\theta}_C$, $\boldsymbol{\theta}_R$, and γ. Since, model (2.3) is nonlinear in parameters and discontinuous, sequential conditional least squares (see, *e.g.*, Tong, 1990, and Hansen, 1996) is used to obtain the parameter estimates. For any given value of γ the LS estimates of $\boldsymbol{\theta}_C$ and $\boldsymbol{\theta}_R$ are respectively,

$$\widehat{\boldsymbol{\theta}}_C(\gamma) = \left(\sum_{t=1}^{T} X_{C,t-1}(\gamma)X_{C,t-1}(\gamma)'\right)^{-1}\left(\sum_{t=1}^{T} X_{C,t-1}(\gamma)c_t\right)$$

and

$$\widehat{\boldsymbol{\theta}}_R(\gamma) = \left(\sum_{t=1}^{T} X_{R,t-1}(\gamma)X_{R,t-1}(\gamma)'\right)^{-1}\left(\sum_{t=1}^{T} X_{R,t-1}(\gamma)r_t\right).$$

Consequently, the estimated residuals for the center and radius equations, are $\widehat{\varepsilon}_t(\gamma) = c_t - X_{C,t-1}(\gamma)'\widehat{\boldsymbol{\theta}}_C(\gamma)$ and $\widehat{\xi}_t(\gamma) = r_t - X_{R,t-1}(\gamma)'\widehat{\boldsymbol{\theta}}_R(\gamma)$, respectively, and the corresponding estimated residual variances $\widehat{\sigma}^2_{C,T}(\gamma) = \frac{1}{T}\sum_{t=1}^{T}\widehat{\varepsilon}_t(\gamma)^2$ and $\widehat{\sigma}^2_{R,T}(\gamma) = \frac{1}{T}\sum_{t=1}^{T}\widehat{\xi}_t(\gamma)^2$.

Hence, given that typically γ is unknown, in order to obtain the model parameter estimates one requires an estimate of this threshold parameter. Considering that γ is the value that optimizes the fit of the model, its estimation can be carried out in two alternative ways:

(i) through the minimization of the sum of squared residuals from both equations in (2.3), or equivalently,

$$\widehat{\gamma} = \arg\min_{\gamma\in\Gamma} \widehat{S}^2_T(\gamma), \tag{2.13}$$

where Γ is the set of all possible thresholds and $\widehat{S}^2_T(\gamma)$ denotes the sum of the variances $\widehat{\sigma}^2_{C,T}(\gamma)$ and $\widehat{\sigma}^2_{R,T}(\gamma)$;

(ii) through the minimization of the mean distance error introduced in (2.9), *i.e.*,

$$\widehat{\gamma} = \arg\min_{\gamma\in\Gamma} MDE_d\left(\{Y_t\}_{t=1}^{T}, \{\widehat{Y}_t(\gamma)\}_{t=1}^{T}\right) = \arg\min_{\gamma\in\Gamma} MDE_d(\gamma) \tag{2.14}$$

where $\widehat{Y}_t(\gamma) = \langle \widehat{c}_t(\gamma), \widehat{r}_t(\gamma)\rangle$, $\widehat{c}_t(\gamma)$ and $\widehat{r}_t(\gamma)$ are the fitted values of (2.3) for a given γ.

Once $\widehat{\gamma}$ is obtained, the slope estimates are $\widehat{\boldsymbol{\theta}}_C = \widehat{\boldsymbol{\theta}}_C(\widehat{\gamma})$ and $\widehat{\boldsymbol{\theta}}_R = \widehat{\boldsymbol{\theta}}_R(\widehat{\gamma})$, and the LS residuals $\widehat{\varepsilon}_t = \widehat{\varepsilon}_t(\widehat{\gamma})$ and $\widehat{\xi}_t = \widehat{\xi}_t(\widehat{\gamma})$, with sample variances $\widehat{\sigma}^2_{C,T} = \widehat{\sigma}^2_{C,T}(\widehat{\gamma})$ and $\widehat{\sigma}^2_{R,T} = \widehat{\sigma}^2_{R,T}(\widehat{\gamma})$.

The minimizations in (2.13) and (2.14) can be performed by direct search. Since $\widehat{S}^2_T(\gamma)$ and $MDE_d(\gamma)$ depend on γ through the indicator function $I_{\{z_{t-d}\leq\gamma\}}$, these two functions are step functions with at most T steps, with steps occurring at distinct values of the observed threshold variable z_{t-d}. Thus, the minimization problems in (2.13) and (2.14) can be reduced to searching over values of γ equalling the distinct values of z_{t-d} in the sample, *i.e.*,

$$\widehat{\gamma} = \arg\min_{z_{t-d}\in\Gamma} \widehat{S}^2_T(z_{t-d}), \tag{2.15}$$

and

$$\widehat{\gamma} = \arg\min_{z_{t-d}\in\Gamma} MDE_d(z_{t-d}). \tag{2.16}$$

It is useful to note that estimates of the threshold γ which allocate a small number of observations into one regime are undesirable. This possibility can be excluded by restricting the search in (2.15) and (2.16) to values of γ such that a minimum number of observations in each regime is ensured.

3 Interval Time Series Forecasting Methods

Research on ITS forecasting methods has recently received considerable attention in related literature. This section reviews available procedures and their contributions to ITS forecasting, and contributes with new methods.

Most of the proposed ITS forecasting methods can be classified into one of three groups. The first group is based on adaptations of smoothing techniques to ITS. For instance, Arroyo *et al.* (2011) and Maia and Carvalho (2011) consider adaptations of exponential smoothing (ES) methods and ES with damped additive trend. The second group considers nonparametric pattern recognition methods. The most substantial and influential approaches in this group are the k-nearest neighbors (k-NN) and the multi-layer perceptions (MLP) as representative of artificial neural networks (ANN). The former is introduced by Yakowitch (1987) to classical time series and adapted by Arroyo *et al.* (2010) to ITS. For an extensive review of the latter for interval MLP see e.g., Zhang (1998) and Munoz *et al.* (2007). The third group focuses on multivariate models for ITS which are based on classical linear time series approaches such as ARIMA, VAR and VEC models (see, *e.g.*, Fiess and MacDonald, 2002, Cheung, 2007, Hu and He, 2007, and Arroyo *et al.*, 2010). Some regression problems are stressed in Billard and Diday (2000, 2003) and Lima and Carvalho (2010).

It is however, consensual in the forecasting literature that no single method is superior in every situation (see, *e.g.*, Chatfield, 1988, Makridakis, 1989, and Zou and Yang, 2004) motivating researchers in the ITS context to combine methods from the different groups described above (forecast combinations in the conventional time series literature are also frequently considered, see, for instance, Clemen, 1989 and Diebold and Lopez, 1996, for overviews). For instance, Zhang (2003) proposes a hybrid approach based on ARIMA models and MLP, Maia *et al.* (2008) apply this method to independent series of lower and upper bounds, and Maia and Carvalho (2011) consider combining Holt's smoothing and interval MLP with accuracy measures based on differencing intervals.

The results of these studies reveal several important insights. First, evidence suggests that approaches that independently forecast the lower and upper bounds of ITS obtained the lowest rank among the various models analyzed. This conclusion agrees with the results obtained in He and Hu (2009), Arroyo *et al.* (2011) and Maia and Carvalho (2011). Second, multivariate forecasting methods and methods that treat intervals as a whole using interval arithmetics are reported to have similar properties and perform globally better than naïve ITS methods. Third, the performance of the models from the first group becomes less effective in terms of interval accuracy if the underlying data displays nonlinear features. To solve this problem the second group of approaches (ANN type procedures) was suggested since it has shown interesting potential in classical time series forecasting applications (see, *e.g.,* Hill *et al.* 1994, and Bishop, 1995). However, the reported evidence of ANN adaptations to ITS is still ambiguous. For instance, Maia *et al.* (2008) and Maia and Carvalho (2011) present results favoring univariate MLP, ARIMA-MLP and ES- interval MLP, respectively, whereas Arroyo *et al.* (2011) reveal that MLP and ARIMA MLP as well as interval MLP and ES are not superior to other interval methods. Hence, forecasting nonlinearities in ITS still remains an important issue.

Due to the fact that real world systems are often nonlinear (see, *e.g.*, Granger and Terasvita, 1993) it is reasonable to develop mechanisms capable of explaining nonlinearities and forecast them. In this paper, we focus our attention on forecasting with the CR-SETAR model introduced in Section 2. It is important to note that for classical (SE)TAR models, the overall evidence of relative forecasting performance when compared to traditional linear forecast methods is mixed (see, *e.g.*, Rothman, 1998, Montgomery *et al.*, 1998, and Clements and Smith, 1997, 2000). Consequently, in order to improve the properties of the CR-SETAR ITS forecasts we consider combining the CR-SETAR forecasts with the k-NN method. The combination of these two approaches aims to build forecasts based on the specified models for ITS and correct these forecasts using the k-NN mechanism. In what follows we review briefly the k-NN algorithm, the computation of one step ahead CR-SETAR forecasts and discuss details of their combination.

3.1 k - Nearest Neighbors Method

The k-NN method is used in the literature to forecast ITS. It is a pattern recognition procedure used to classify objects based on the closest neighbor in the feature space. The method can be used for conventional time series forecasting (see Yakowitz, 1987) as well as for ITS forecasting (see Arroyo *et al.*, 2010).

To briefly review the k-NN method consider the ITS, $\{Y_t\}_{t=1}^{T}$, and the q-dimensional interval vector $\{Y_t\}^q = (Y_t, Y_{t-1}, ..., Y_{t-(q-1)})'$, where $q \in \mathbb{N}$ is the number of lags and $t = q, ..., T-1$. The k-nearest neighbors of the most recent (feature) interval vector $\{Y_T\}^q$ can be found by using the mean distance measure (2.9), *i.e.*,

$$MDE_d\left(\{Y_T\}^q, \{Y_t\}^q\right) = \frac{\sum\limits_{i=0}^{q-1} d(Y_{T-i}, Y_{t-i})}{q}. \tag{3.1}$$

Once the dissimilarities for each $\{Y_t\}^q$, $t = q, ..., T-1$, are computed, the k - closest vectors to $\{Y_T\}^q$ are selected[4]. Denote these k vectors as $\{Y_{T_1}\}^q, ..., \{Y_{T_k}\}^q$, with $T_i < T$, $i = 1, ..., k$. Hence, to obtain a forecast, a weighted average of the subsequent values, $Y_{T_1+1}, ..., Y_{T_k+1}$, is considered, *i.e.*,

$$\widehat{Y}_{T+1} = \sum_{i=1}^{k} \omega_i \cdot Y_{T_i+1}, \tag{3.2}$$

where ω_i is the weight assigned to neighbor i, with $\omega_i \geq 0$ and $\sum\limits_{i=1}^{k} \omega_i = 1$. The weights can be the same for all neighbors ($\omega_i = \frac{1}{k}$ for all i) or inversely proportional to the distance between $\{Y_T\}^q$ and $\{Y_j\}^q$, $j = T_1, ..., T_k$, *i.e.*,

$$\omega_i = \frac{\psi_i}{\sum\limits_{m=1}^{k} \psi_m},$$

with $\psi_i = (MDE_d(\{Y_T\}^q, \{Y_{T_i}\}^q) + \varepsilon)^{-1}$, where $\varepsilon \geq 0$ is a constant used to avoid division by zero when the distance between two sequences is zero.

3.2 The CR-SETAR Forecasts and Forecast Corrections

In this paper, we focus on one step ahead forecasts from the CR-SETAR model. This will simplify the derivation of the forecasts and the exposition of the correction suggested.

The method provided can however be extended for multi-step ahead forecasting. Exact analytical solutions to obtain multi-period forecasts using threshold type nonlinear models

[4]Note that the number, k, of closest vectors to be used can be obtained by minimizing the mean distance between $\{Y_T\}^q$ and $\{\bar{Y}^k\}^q$, where $\{\bar{Y}^k\}^q = \sum\limits_{i=1}^{k} \omega_i \cdot \{Y_{T_i}\}^q$.

are generally not available (see, *e.g.*, De Gooijer and Kumar, 1992). Often simulation methods such as Monte Carlo or Bootstrap approaches are employed to construct these forecasts (see, *e.g.*, Tiao and Tsay, 1994, and Clements and Smith, 1997) and are reported to perform reasonably well. As an alternative, one may also use the normal forecast error method proposed by Al-Qassam and Lane (1989) and adapted by De Gooijer and De Bruin (1997) to forecast SETAR models. Another possibility is an exact method involving numerical integration based on Chapman-Kolmogorov relations (see, *e.g.*, De Gooijer and De Bruin, 1997).

The one step ahead forecasts from the CR-SETAR model, defined as $\widehat{Y}_{T+1} \equiv E[Y_{T+1}|\mathbf{F}_T]$, where $\mathbf{F}_T = \{Y_t\}_1^T$, is straightforwardly obtained from (2.3) as,

$$\begin{aligned} \widehat{Y}_{T+1} &= < \widehat{c}_{T+1}, \widehat{r}_{T+1} > \\ &= < E[g_C(c_T) + \varepsilon_{T+1}|\mathbf{F}_T], E[g_R(r_T) + u_{T+1}|\mathbf{F}_T] > \\ &= < g_C(c_T), g_R(r_T) >, \end{aligned} \tag{3.3}$$

where $g_C(.)$ and $g_R(.)$ are nonlinear functions that satisfy model (2.3), *i.e.*,

$$\begin{cases} g_C(c_t) = X_{C,t}(\gamma)'\boldsymbol{\theta}_C \\ g_R(r_t) = X_{R,t}(\gamma)'\boldsymbol{\theta}_R \end{cases}.$$

As stated above, in empirical applications, SETAR models do frequently not display suitable forecasting performance, and the same is true for interval valued data. Therefore, the k-NN corrections of the nonlinear forecasts may prove useful for practical purposes, due to the fact that different aspects of the underlying patterns may be additionally captured and therefore forecasts improved.

Essentially, the idea is to compute the CR-SETAR forecasts as suggested in (3.3) and to correct them using the k-NN approach. The forecast correction procedure proposed is similar to the idea of the hybrid system proposed by Zhang (2003). To illustrate the procedure, let $\widehat{Y}_{T+1} =< \widehat{c}_{T+1}, \widehat{r}_{T+1} >$ and $\widetilde{Y}_{T+1}$ denote the CR-SETAR and corrected CR-SETAR forecasts, respectively. For the latter, the following principle is used

$$\widetilde{Y}_{T+1} =< \widehat{c}_{T+1} + \widehat{\varepsilon}_{T+1}, \widehat{r}_{T+1} + \widehat{\xi}_{T+1} >, \tag{3.4}$$

where $\widehat{\varepsilon}_{T+1}$ and $\widehat{\xi}_{T+1}$ are the correction terms. In order to adjust the forecast $\widehat{Y}_{T+1}$ the terms $\widehat{\varepsilon}_{T+1}^{k-NN}$ and $\widehat{\xi}_{T+1}^{k-NN}$ are computed from the CR-SETAR residuals using k-NN as,

$$\begin{cases} \widehat{\varepsilon}_{T+1}^{k-NN} = \sum_{i=1}^{k} \omega_i \cdot \widehat{\varepsilon}_{T_i+1} \\ \widehat{\xi}_{T+1}^{k-NN} = \sum_{i=1}^{k} \omega_i \cdot \widehat{\xi}_{T_i+1} \end{cases}.$$

The time pattern T_i, $i = 1, ..., k$, necessary to compute these residuals is obtained through the application of the k-NN method to the fitted values $\left\{\widehat{Y}_t\right\}_{t=1}^{T}$, and the residuals, $\widehat{\varepsilon}_{T_i+1}$

and $\widehat{\xi}_{T_i+1}$, computed as

$$\widehat{\varepsilon}_{T_i+1} = c_{T_i+1} - \widehat{c}_{T_i+1},\ \widehat{\xi}_{T_i+1} = r_{T_i+1} - \widehat{r}_{T_i+1}.$$

In summary, the proposed methodology for CR-SETAR forecast corrections consists of two steps. First, the CR-SETAR model is estimated to capture the nonlinear behavior of the underlying ITS, and in the second step, the k-NN method is used to find the pattern of model fit and failure. If the CR-SETAR cannot reflect some structure of the data, the residuals will contain information about the model failure and the k-NN method can be used to exploit this pattern and predict the correction terms necessary to adjust the CR-SETAR forecasts.

4 An Empirical Example Using The S&P500 index

To illustrate the methods introduced in this paper, in this section we provide an empirical application to S&P500 index returns. It will be of interest to verify whether the CR-SETAR model can provide reference values that help detect high-volatility periods (*e.g.* this can be important information for investment strategies). Hence, the objective of this analysis is twofold. One is to illustrate the potential of the CR-SETAR model in explaining and forecasting ITS and the other to compare the proposed procedures with other methods available in the literature. This aspect is covered by comparing the one-step ahead forecasting performance of the k-NN, the CR-SETAR and the corrected CR-SETAR given in (3.2), (3.3) and (3.4), respectively, with the naive forecast $\widehat{Y}_{T+1} = Y_T$ which is used as a benchmark. Our comparison analysis is based on the accuracy measures discussed in Section 2.2.

Our sample consists of weekly S&P 500 interval returns observed from January 3, 1997 to November 12, 2010. This sample is divided into two parts: the first subsample, from January 3, 1997 to December 28, 2007 (574 weeks), is used for estimation purposes and the second subsample, from January 4, 2008 to November 12, 2010 (150 weeks), to compute the ITS forecasts and analyze their quality. Table 1 presents the chronology of major events that affected the stock market in the period under analysis. Note that the forecast period includes the financial crises that started in 2008.

[PLEASE INSERT TABLE 1]

Since the objective of this analysis is to analyze the evolution of returns, the starting point for building our data set is the adequate definition of interval returns. In what follows, two types of interval returns are considered:

(i) *The Expanded returns* - which consist of intervals that contain all possible differences between index price levels that occur in two neighbor trading periods, *i.e.*

$$\Delta Y_t^E = \{y_t - y_{t-1} : y_t \in [l_t, u_t] \wedge y_{t-1} \in [l_{t-1}, u_{t-1}]\}, \tag{4.5}$$

where $[l_t, u_t]$ is an interval of logarithmic prices. Note that, (4.5) satisfies the definition of interval subtraction (see, *e.g.* Moore, 2009), thus

$$\Delta Y_t^E = Y_t - Y_{t-1}.$$

(ii) *The Centered returns* - which are intervals of returns for the current period based on the center of the price index levels for the previous period, *i.e.*

$$\Delta Y_t^C = \{y_t - y_{C,t-1} : y_t \in [l_t, u_t]\}, \tag{4.6}$$

where $y_{C,t-1}$ is the center of Y_{t-1}. Figure 1 illustrates the different representations of the S&P 500 returns in the upper panel and presents the weekly ranges in the lower panel. Note that the following relation between ΔY_t^E and ΔY_t^C can be considered. Considering $\Delta Y_t^E =< c_t^E, r_t^E >$ and $\Delta Y_t^C =< c_t^C, r_t^C >$, then intervals of expanded and centered returns have the same center (*i.e.*, $c_t^E = c_t^C$) and the radius of the expanded returns is the aggregation of the contemporaneous and lagged centered returns radius *(i.e.*, $r_t^E = r_t^C + r_{t-1}^C$).

[PLEASE INSERT FIGURE 1]

4.1 The CR-SETAR Model

Due to the relation between the two types of interval returns defined in (4.5) and (4.6) in what follows we will only consider the model for centered returns since for the expanded returns it will follow along similar lines. Thus,

$$\begin{aligned} c_t = {} & (\mu_1 + \sum_{i=1}^{8} \alpha_{1i} c_{t-i} + \sum_{j=1}^{8} \beta_{1j} r_{t-j}) \times I_{\{z_{t-d} \leq \gamma\}} + \\ & (\mu_2 + \sum_{i=1}^{8} \alpha_{2i} c_{t-i} + \sum_{j=1}^{8} \beta_{2j} r_{t-j}) \times I_{\{z_{t-d} > \gamma\}} + \varepsilon_t, \end{aligned} \tag{4.7}$$

$$\begin{aligned} r_t = {} & (\lambda_1 + \sum_{i=1}^{8} \varphi_{1i} c_{t-i} + \sum_{j=1}^{8} \phi_{1j} r_{t-j}) \times I_{\{z_{t-d} \leq \gamma\}} + \\ & (\lambda_2 + \sum_{i=1}^{8} \varphi_{2i} c_{t-i} + \sum_{j=1}^{8} \phi_{2j} r_{t-j}) \times I_{\{z_{t-d} > \gamma\}} + \xi_t, \end{aligned} \tag{4.8}$$

where c_t denotes the center of the centered returns interval and r_t denotes the radius. For convenience of notation we drop the index "C" in c_t^C and r_t^C, so that in what follows, unless otherwise stated, $< c_t, r_t >=< c_t^C, r_t^C >$. Note that eight lags of c_t and r_t where considered in both equations as this appeared to be the minimum necessary to adequately describe the short-run dynamics.

In order to analyze the presence of nonlinearities and threshold effects in the S&P500 returns we considered a standard lag of the radius, $z_{t-d} = r_{t-d}$ for $1 \leq d \leq 8$, as the threshold variable, but we also investigated other potential threshold variable candidates, such as lags of the centered returns, $z_{t-d} = c_{t-d}$, and other stationary transformations of the price levels. In what follows we report only estimation results when $z_{t-d} = r_{t-d}$ is used as a threshold variable as this provided the best results. The model fit in terms of the sum of squared residuals, the interval MDE with different loss functions and the sum of squared residuals from (4.7) and (4.8) modelled independently are used to select the adequate estimate of d. Table 2 reports these results. We find that the model with threshold variable $z_{t-1} = r_{t-1}$ provides the best adjustment of all models considered.

[PLEASE INSERT TABLE 2]

Hence, setting $\widehat{d} = 1$, minimization of (2.13) and (2.14) is used to estimate the threshold parameter γ. The results obtained show that $\widehat{\gamma}_{SSE} = 0.0214$, $\widehat{\gamma}_{L_2} = 0.0207$, and $\widehat{\gamma}_{NSD} = 0.0210$. Asymptotic valid confidence intervals for $\widehat{\gamma}$ obtained through (2.13) can be constructed (see, *e.g.*, Hansen, 2000) to verify whether the obtained threshold parameter estimates are statistically significant. At the 95% confidence level the asymptotic confidence interval is $[0.0204, 0.0215]$ and contains all threshold estimates, suggesting that all three methods provide statistically valid results for the S&P500 return's data used. Without loss of generality $\widehat{\gamma}_{NSD}$ will be considered for further analysis.

The estimate, $\widehat{\gamma}_{NSD} = 0.021$, indicates that the CR-SETAR model in (4.7)-(4.8) classifies the sample observations into one of the two regimes depending on wether the S&P500 radius of interval returns has been higher or lower than 0.021 over the previous period (week). Of the 565 observations in the fitted sample, 137 fall in the high-volatility regime ($r_{t-1} > 0.021$). Heuristically, we can think of this regime as market swings or volatile periods, since a high range means wider distribution of possible values of returns over trading periods. Illustration of the regime splitting is presented in the following section jointly with the forecast splitting. Note that the large historical recession period of the S&P500 ("dot com" aftermath) is mostly associated with the high-volatility period observing the highest peak in March 2000 when the "dot com" bubble burst occurred.

Tables 3 and 4 present the estimation outputs of model (4.7)-(4.8). We report parameter estimates, standard errors, absolute parameter changes over regimes, the number of observations in each regime and the threshold estimates. According to the general-to-specific modelling strategy, insignificant regressors were eliminated if the corresponding heteroskedasticity robust Wald test presented a p-value that exceeded the 10% significance level in both regimes (see, *e.g.*, Li, 2006). To assess the dynamics of the point estimates obtained for the autoregressions in both regimes we also employ spectral analysis. Figure 2 plots the spectral density functions corresponding to the autoregressive coefficients from the two regimes of equation (4.7) in the left panel and equation (4.8) in the right panel. Regime 1 ($r_{t-1} \leq \widehat{\gamma}$) of c_t has one large peak corresponding to a cycle with a period of 2-3 weeks whereas regime 2 ($r_{t-1} > \widehat{\gamma}$) of c_t has two peaks. One corresponding to a cycle with a period of $\approx$ 1 week and the second corresponding to a business cycle with a 8-9 years period. The latter result reflects the recession frequency given in Table 1. For the returns' radius equation a different picture is observed. According to this Figure, the radius in the high volatility regime ($r_{t-1} > \widehat{\gamma}$) picks up the $\approx$1-2 weeks cycle and has business cycle frequency in the low-volatility regime.

[PLEASE INSERT FIGURE 2]
[PLEASE INSERT TABLE 3 AND 4]

4.2 The Relative Performance of CR-SETAR in Forecasting

In this section we analyze the performance of CR-SETAR forecasts and compare results with those from the naive method ($\widehat{Y}_{T+1} = Y_T$), the k-NN method and the corrected CR-SETAR approach. Two ITS forecasts are obtained from each model which correspond to forecasts of centered and expanded S&P500 returns given in (4.5) and (4.6). The main quality measure (and the performance ranking) is based on the MDE discussed in Section 2.2. The descriptive statistics given in (2.10), (2.11) and (2.12) are also used to analyze the relative performance.

[PLEASE INSERT TABLE 5]

Table 5 quantifies the performance of the forecasts obtained from the different approaches. The k-NN method has been implemented with inversely proportional weights, since this allows us to neglect the effects of insignificant neighbors on the forecasts. The length of the feature interval vector was set equal to eight (*i.e.,* the same length as the short-run dynamics in the CR-SETAR model). The estimated number of neighbors is $\widehat{k} = 20$. To obtain the CR-SETAR forecasts and their corrections we employ the procedures discussed in Section 3.2 (in particular (3.3) and (3.4)).

According to both MDE measures (using d_{L_2} and d^*_{NSD}), the CR-SETAR model and the k-NN method show better performance than the naive approach, which is usually hard to beat in financial time series. The results obtained also show that the CR-SETAR model and the k-NN approach have nearly identical forecast performance. It is also observed that the corrected CR-SETAR forecasts are superior to all methods considered in this paper. This method presents the highest coverage rates, R_C, (62.9% and 81.2% for centered and expanded returns, respectively) and forecast efficiency rates, R_E, (63.46% and 79.57%, respectively). As previously discussed, the closeness of the results of these two statistics can be taken as an indicator of the quality of the forecast.

One conclusion that can be drawn from these results is that (as in the conventional (SE)TAR context) the CR-SETAR model is limited in forecasting. This shortcoming can be caused by the selected forecasting period which contains a lot of uncertainty (e.g., 2008 crisis) and the fact that the underlying nonlinear pattern may not be strong enough in this period. However, the results in Table 5 indicate that the forecast correction based on the nonparametric k-NN method can remedy this drawback. Figure 3 shows the regime splitting for the estimation period (1997-2007) and for the forecast period (2008- 2010). According to the estimate of γ most of the high volatility periods (as expected) have been found during recessions, the "dot-com" aftermath and the 2008 crisis.

[PLEASE INSERT FIGURE 3]

5 Conclusions

This paper develops new methods for the estimation and forecasting of interval-valued time series. We introduced a threshold-type model for the center-radius representation of ITS and shown that the model is rather straightforward to estimate using sequential conditional LS (as in the conventional time series context). We reviewed available forecasting methods for ITS and introduced new approaches based on the nonparametric k-NN algorithm and the CR-SETAR model.

The methods are applied to S&P500 index returns for the period from 1997 to 2010. We find that CR-SETAR and k-NN exhibit satisfactory performance in forecasting ITS by beating the naive method, which is usually hard to outperform in financial settings. Thus, both approaches seem to be useful for forecasting ITS. However, the corrected CR-SETAR forecasts achieved the best performance based on the proposed accuracy measures. This is an important result since it shows that CR-SETAR may be limited in the explicit explanation of the underlying nonlinear pattern but the combination of the two methods may find constructive use in applications. Another important point to highlight from the results obtained is that the model is potentially useful in the analysis of volatility dynamics. In particular distinguishing high volatile periods based on the behavior of the returns' radius.

Several extensions of the methods presented will be pursued in future work. First, it will be interesting to compare our results with those of other nonlinear models (such as, *e.g.*, Markov switching models, smooth transition models, and ANN). Second, it will also be important to investigate how well the radius component of ITS does in forecasting volatility.

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A Appendix

A.1 Interval operations

Definition A.1 *Considering the intervals* $\mathcal{A} = [a_L, a_U]$ *and* $\mathcal{B} = [b_L, b_U]$ *the following interval operations can be stated:*

1. *The* ***intersection*** *of* $\mathcal{A}$ *and* $\mathcal{B}$ *is empty if either* $a_U < b_L$ *or* $b_U < a_L$*. Otherwise, it is define as*

$$\mathcal{A} \cap \mathcal{B} = \{z : z \in \mathcal{A} \textit{ and } z \in \mathcal{B}\} = [\max\{a_L, b_L\}, \min\{a_U, b_U\}]. \tag{A.1}$$

2. *The* ***union*** *of* $\mathcal{A}$ *and* $\mathcal{B}$ is a set of all ellements that are in $\mathcal{A}$ *and* $\mathcal{B}$, *i.e.*,

$$\mathcal{A} \cup \mathcal{B} = \{z : z \in \mathcal{A} \textit{ or } z \in \mathcal{B}\}. \tag{A.2}$$

3. *The* ***interval hull*** *of* $\mathcal{A}$ *and* $\mathcal{B}$ *is also an interval since,*

$$\mathcal{A} \mathbin{\underline{\cup}} \mathcal{B} = [\min\{a_L, b_L\}, \max\{a_U, b_U\}]. \tag{A.3}$$

4. *The* ***symmetric difference*** *of* $\mathcal{A}$ *and* $\mathcal{B}$ *(denoted as* $\mathcal{A} \triangle \mathcal{B}$*) is the set of real values which are in one of the intervals, but not in both. If* $\mathcal{A} \cap B = 0$ *then* $\mathcal{A} \triangle \mathcal{B} = \mathcal{A} \mathbin{\underline{\cup}} \mathcal{B}$ *else*

$$\mathcal{A} \triangle \mathcal{B} = (A \cup B)/(A \cap B). \tag{A.4}$$

5. *The* ***width*** *(or diameter) of* $\mathcal{A}$ *is defined as*

$$w(\mathcal{A}) = a_U - a_L. \tag{A.5}$$

Furthermore, the width of the symmetric difference is given as,

$$w(A \triangle B) = \begin{cases} w(A \cup B) - w(A \cap B) & \textit{if } A \cap B \neq \emptyset \\ w(A \mathbin{\underline{\cup}} B) & \textit{if } A \cap B = \emptyset \end{cases}. \tag{A.6}$$

TABLES

Table 1: Chronology of Stock Market Events

Date	Event
1997-1999	Late 90s. Development of the "dot-com" bubble in the financial markets
2000-2002	Bubble burst. ≈50% loss of its value in a two-year bear market
2003-2007	Sustainable period
2007-2008	Second bear market of the 21st century
2009-2010	Exit from the crisis and return to economic growth

Table 2: Fit of the model (4.7)-(4.8) with $z_{t-d} = r_{t-d}$

d	$SSE_{Center} \times 10^3$	$SSE_{Radius} \times 10^3$	$SSE_{Total} \times 10^3$	MDE_{NSD}	MDE_{L_2}
1	0.287797	0.061789	0.349587	0.512242	0.019820
2	0.318009	0.066868	0.384878	0.521692	0.020619
3	0.315095	0.06535	0.380445	0.521945	0.020478
4	0.313835	0.064676	0.378511	0.526834	0.020793
5	0.311634	0.060816	0.372449	0.526995	0.020663
6	0.316956	0.065794	0.38275	0.522063	0.020599
7	0.316377	0.066105	0.382482	0.530048	0.020852
8	0.322871	0.067857	0.390728	0.530630	0.020842

Table 3: Estimation Results for the Center Model in (4.8)

	Regime 1		Regime 2			Absolute Change
	Coeff.	$SE \times 10^2$	Coeff.	$SE \times 10^2$	Wald test	over regimes
Constant	-0.001	0.001	-0.025	0.003	11.631 (0.003)	0.024
r_{t-1}	0.135	5.397	0.710	2.077	7.621 (0.022)	0.575
r_{t-2}	-0.378	1.198	0.802	3.643	20.756 (0.000)	1.180
r_{t-4}	0.274	2.455	-0.387	1.252	14.023 (0.001)	0.661
r_{t-5}	0.285	2.188	0.191	1.920	7.052 (0.029)	0.094
r_{t-6}	-0.201	1.944	-0.320	1.821	6.436 (0.040)	0.119
c_{t-1}	0.149	0.418	0.213	0.395	7.704 (0.021)	0.064
c_{t-5}	0.089	0.321	0.145	0.562	4.816 (0.090)	0.056
γ	0.021					
N	428		137			

Note: Regime 1 is specified as $R_{t-1} \leq 0.021$ and Regime 2 as $R_{t-1} > 0.021$.

Table 4: Estimation Results for the Radius Model in (4.9)

	Regime 1		Regime 2			Absolute Change
	Coeff.	$SE \times \times 10^3$	Coeff.	$SE \times \times 10^3$	Wald test	over regimes
Constant	0.005	0.001	0.014	0.01	87.813 (0.000)	0.009
r_{t-2}	0.188	3.04	-0.040	9.09	28.159 (0.000)	0.228
r_{t-3}	0.259	5.32	0.290	4.39	35.539 (0.000)	0.031
r_{t-6}	0.068	4.05	0.123	4.24	6.512 (0.039)	0.055
r_{t-7}	0.125	4.16	-0.034	3.27	8.005 (0.018)	0.159
r_{t-8}	0.077	2.28	0.054	7.35	3.500 (0.174)	0.023
c_{t-1}	-0.106	0.85	-0.082	0.72	22.219 (0.000)	0.024
c_{t-2}	-0.075	0.79	-0.021	1.22	12.985 (0.002)	0.054
c_{t-3}	-0.062	0.73	-0.016	0.92	14.091 (0.001)	0.046
c_{t-4}	0.007	0.71	-0.077	0.76	6.352 (0.042)	0.084
c_{t-5}	0.004	0.63	-0.120	1.02	5.224 (0.073)	0.124
c_{t-8}	0.016	0.61	-0.082	1.06	5.290 (0.071)	0.098
γ	0.021					
N	428		137			

Note: Regime 1 is specified as $R_{t-1} \leq 0.021$ and Regime 2 as $R_{t-1} > 0.021$.

Table 5: Competitive analysis of the model forecast performance (2008 -2010)

	CR-SETAR		K-NN		CR-SETAR corrected		Naive	
MDE/Forecast	Centered	Expanded	Centered	Expanded	Centered	Expanded	Centered	Expanded
$MDE_{d_{L_2}}$	0.03330	0.03765	0.03278	0.03924	**0.02923**	**0.03261**	0.04067	0.04142
$MDE_{d^*_{NSD}}$	0.55388	0.35952	0.55140	0.37130	**0.50572**	**0.32233**	0.62360	0.38772
Efficiency rate	58.31%	76.99%	59.93%	78.75%	**63.46%**	**79.57%**	52.50%	75.10%
Coverage rate	59.75%	79.51%	59.91%	77.32%	**62.91%**	**81.19%**	51.56%	74.85%
Residual rate	55.48%	37.88%	54.64%	37.00%	**50.29%**	**32.23%**	61,62%	38.77%

FIGURES

Figure 1: Lower-upper limits of expanded and centered returns.

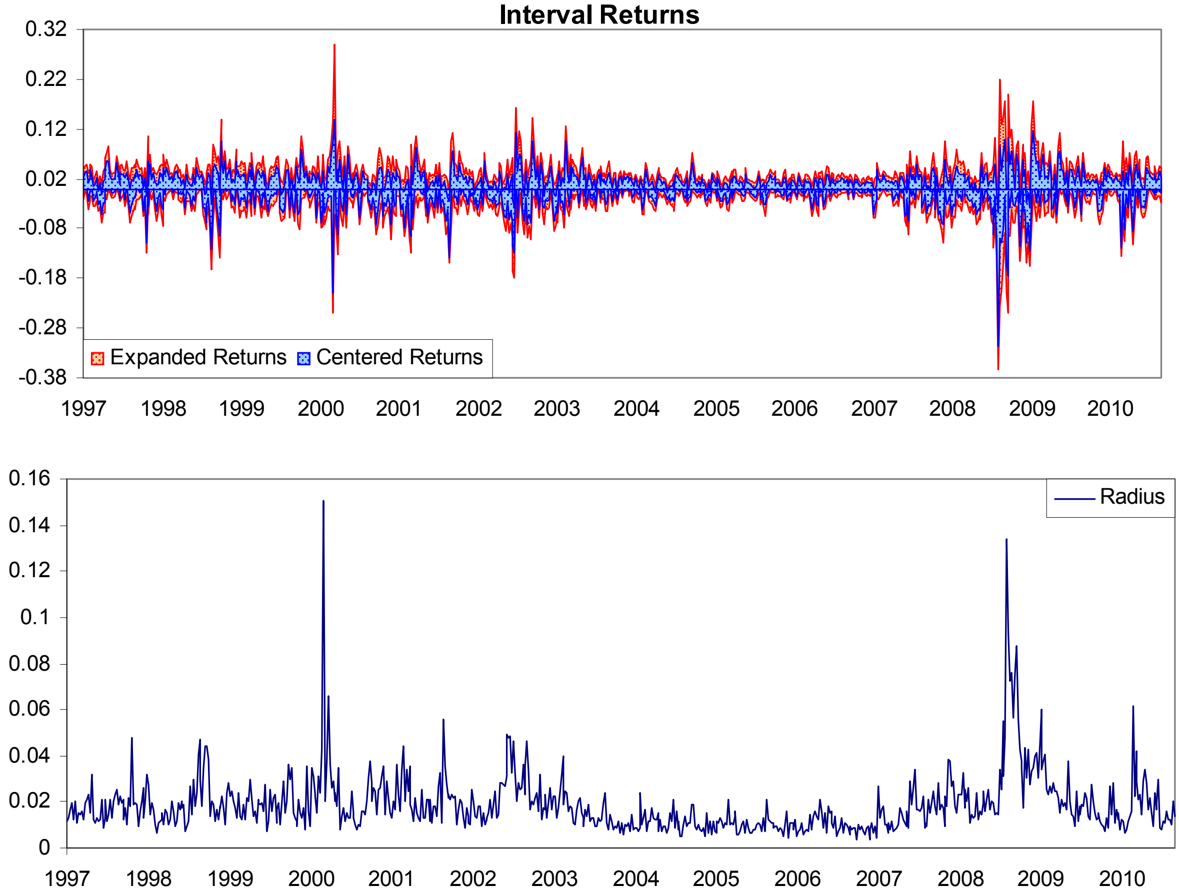


Figure 2: Spectral Density by Regime for c_t and r_t

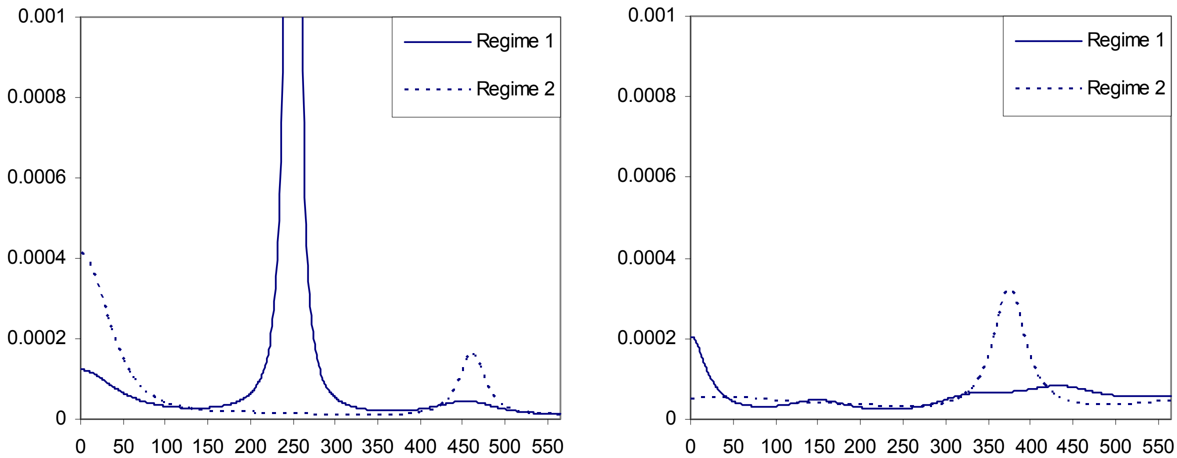
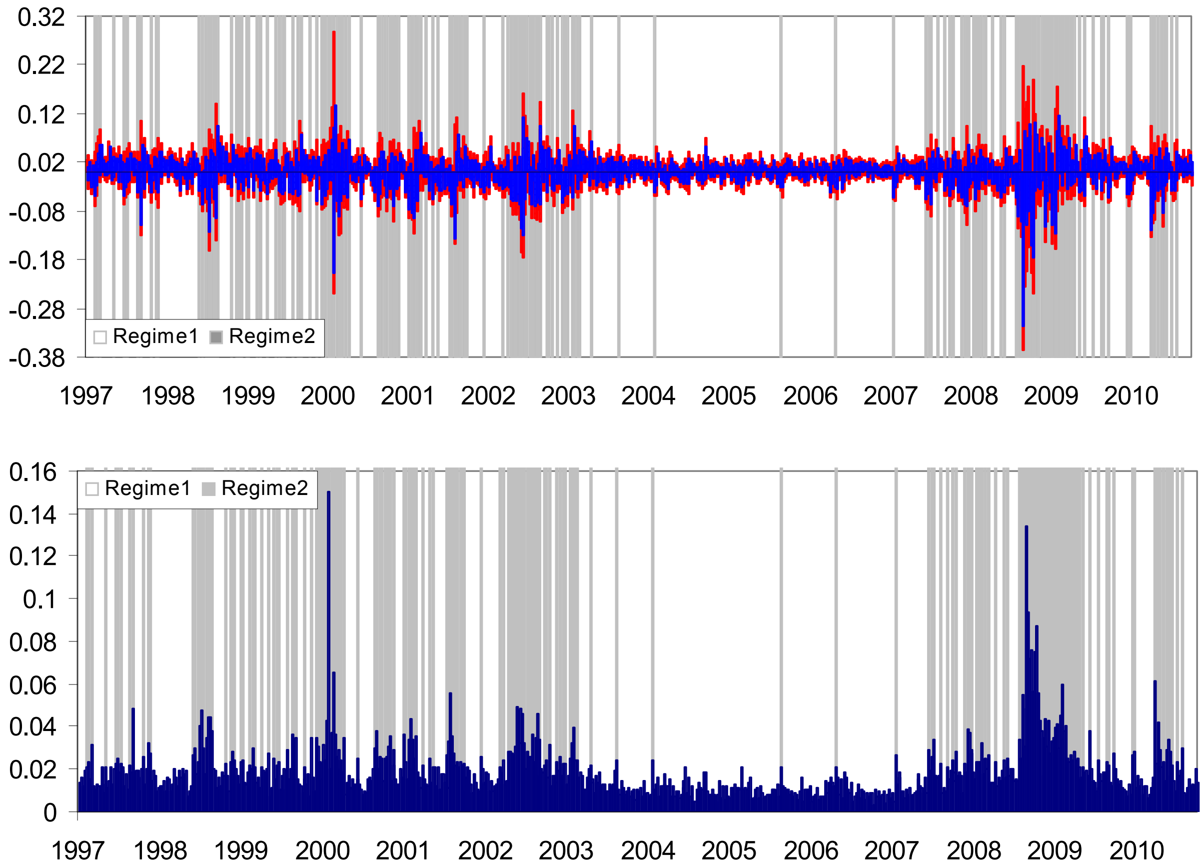


Figure 3 : Regime splitting for forecasts and fitted values of returns.



Note: The top panel refers to the interval returns and the bottom panel to the radius.

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