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Labor Immobility and the Transmission Mechanism of Monetary Policy in a Monetary Union

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Abstract

It is believed that a shock, common to a set of countries with identical fundamentals, has identical outcomes across countries. We show that in general, when specialization in production is such that a common shock creates a missing role for labor mobility across countries, the terms of trade of any country reacts to the shock. This is the case even if state contingent assets can be traded across countries. The transmission mechanism of a monetary shock in a monetary union has in this case an additional channel, the terms of trade. We also show that the country outcomes are significantly different, when compared with the effect of the shock on the union's aggregate. Monetary shocks impose cycles with higher volatility in "poor" countries relatively to the volatility of "richer" ones.

Key words: monetary union; transmission mechanism of monetary policy; labor immobility; idiosyncratic effects.

JEL: E31; E41; E58; E62

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1 Introduction

The widespread view in the profession is that common shocks in a set of countries or regions have no idiosyncratic effects if these countries or regions are identical. The standard hypotheses in the international macroeconomic literature imply that aggregate shocks do not affect the terms of trade when countries are identical. Therefore, relative consumptions, relative incomes and the current accounts do not change in response to such a shock. Contrary to this, in this paper we want to stress the effects of common shocks in identical countries, by not closing the potential role of the terms of trade and of the current account in the transmission of common shocks, and therefore allowing for different outcomes across identical countries. Countries in this paper are identical in the sense that they have identical preferences and technologies, even though they are specialized in the production of different tradable goods.

We focus on a monetary shock in a monetary union, but the conclusions extend to any other common shock. The main result of this paper conveys more importance to monetary policy because - by impacting on the terms of trade and on relative allocations - it is more powerful than in the traditional view where a common monetary policy cannot affect similar countries differently. In this sense, this paper represents a step forward in trying to understand the transmission mechanism of monetary policy in a monetary union. We consider a simple model where countries have an identical nominal rigidity and evaluate numerically the idiosyncratic effects of a monetary policy shock. The conclusion being that the asymmetric effects have the potential to be quantitatively significant.

Asset markets completeness is an important assumption in the open macroeconomics literature. The effects of idiosyncratic shocks may change substantially when this hypothesis is dropped, and therefore the non-existence of markets has non-trivial implications. The empirical plausibility of this assumption, is associated with the importance of changes in the current account in the transmission of shocks across countries. Although this assumption is clearly identified as determinant in the analysis of idiosyncratic shocks, or different exogenous transmission mechanisms, its importance for common shocks and identical transmissions mechanisms, to our knowledge, has not yet been explored in the literature.[1]

[1] The literature includes, among many others, Benigno (2004), Benigno and Benigno

Typically to avoid the indeterminacy of the aggregates in the steady-state when asset markets are incomplete, and the associated non-stationarity in the dynamics, it is necessary to introduce a modification to the standard models to induce stationarity (see Schmitt-Grohë and Uribe, 2003, for further details). In this paper, even with incomplete markets, we gain simplicity and tractability, because we consider a model that has well defined aggregates and is stationary at the union level, while it is non-stationary at the country level. That is, we have Gorman aggregation even when markets are incomplete[2]. The crucial market incompleteness is the labor immobility across countries. The existence of state-contingent asset markets across countries is unimportant for our results.

It is interesting that here the so much publicized role of the terms of trade as an insurance mechanism is reversed. It is exactly the endogenous response of the terms of trade to the common shock that leads to the asymmetric responses of the various economic variables across countries to the common shocks.

It is well documented that, in response to an idiosyncratic productivity shock, the country whose productivity increased the most will produce relatively more but the relative price of the bundle of goods it produces will decrease also. Thus, the terms of trade reaction in response to idiosyncratic shocks will determine a smaller dispersion of the relative income of any two countries. In the context of a simple model, Cole and Obstfeld (1991), demonstrated that the gains from completing the markets can be modest, as the terms of trade are a good insurance scheme for countries without state contingent asset markets. Even though, they provide full insurance only for a very small set of parameters, for a larger set of other realistic parameters they provide almost full insurance. Therefore, Cole and Obstfeld (1991) conclude that "the terms of trade may play an important role by automatically pooling national economic risks". More recently Ghironi (2006) showed these results may not be robust. The terms of trade can be a poor substitute for a full insurance scheme since, in more complex models, the transmission of idiosyncratic shocks has effects in an incomplete market framework that can be quantitatively fairly different from the ones obtained in a complete market set up. Our analysis is just on aggregate shocks, we show that the terms of

(2003, 2006), Benigno and Thoenissen (2006), Carlstrom, Fuerst, Ghironi and Hernandez (2006), Cole and Obstfeld (1991), Corsetti and Pesenti (2001, 2005), Corsetti, Luca and Leduc (2008), Dotsey and Duarte (2008), Gali and Monacelli (2006) and Ghironi (2006).

[2] See Adão and Correia (2009).

trade react to the common shock and therefore the outcomes of the shock differ across countries.

To develop the intuition for the change in the terms of trade, and to introduce the aggregation results, we consider first an economy where firms have no restrictions on the way they choose prices. For the monetary shock to have real effects in the flexible price economy we assume that money has a role in transactions. Latter, to get an idea of the quantitative importance of this effect, we consider an environment where firms set prices according to a Calvo mechanism, Calvo (1983). In this environment the model is solved numerically with log-linearization of the equilibrium equations. Contrary to what happens in most closed economy models, where with the first order approximation the behavior of relative prices is lost, here we want to stress that our result is due to the change in the terms of trade. This occurs because we assume non-homothetic preferences and, according with the literature, labor immobility across countries. These are our crucial assumptions. If instead we had assumed homothetic preferences for the households, the same result could be obtained if government expenditures were introduced, identical across countries but whose composition across goods did not coincide with the one of the households. In this way total demand (private and public) would be again non-homothetic, and the result would be preserved.

The paper proceeds as follows. In Section 2 we describe the basic two-country monetary union when firms have no price setting restrictions. We use this section to develop the intuition for the transmission mechanism of monetary shocks that creates asymmetric outcomes when countries have identical fundamentals. In Section 3 we show how the terms of trade can be computed analytically. The solution for the terms of trade is explored to identify how the common shock can create heterogeneous outcomes. In Section 4 we show that the result continues to hold even when the households preferences are homothetic. It suffices to introduce a standard public sector in the model. In Section 5 we use an environment with price-setting frictions a la Calvo, where the degree of stickiness is the same for every firm independently of the country, and we show numerically that the idiosyncratic effects are significative when compared with the union's wide effects of the monetary shock. Section 6 contains concluding remarks.

2 The Model

The model considered is a standard international macroeconomic model with monopolistic competition. The monetary union has two countries with identical tastes, technologies and initial assets. We denote the home country with H and the foreign country with F. The union is populated by a continuum of households, indexed by $j \in [0, 1]$. The households in the segment $[0, \theta]$ live in country H and the households in the segment $(\theta, 1]$ live in country F. There are economies of scale in the production of the final good and costless differentiation of the intermediate products. Each firm produces a distinct intermediate good and each good is identified with the firm that produces it. Firms use technologies that are linear in labor, and productivity is identical across goods and across countries. We assume that there is an initial sunk entry cost for each firm, which determines simultaneously the number of firms in each of the two countries, and a given pattern of trade and specialization between the two countries, as in Krugman (1980). The goods produced in the union are normalized to the unit interval, and indexed by $i \in [0, 1]$. The entry costs are such that the goods in the interval $[0, \theta]$ are produced in country H and the goods in the interval $(\theta, 1]$ are produced in country F.[3] As it is usual in the literature, it is assumed that there is no firm entry dynamics in response to monetary shocks.[4]

The monetary authority of the monetary union issues the common currency, that is distributed endogenously across countries in order to satisfy demand. Monetary policy is conducted by an interest rate rule, which is the instrument of monetary policy. We assume that seigniorage is transferred through lump sum transfers equitatively across countries.

There are union-wide markets for the goods but the market for labor is segmented across countries. Labor is homogeneous and perfectly mobile inside each country but immobile across countries.

The history of events up to period t, $(s_0, s_1, ..., s_t)$ is $s^t \in S^t$ and the initial realization s_0 is given. The aggregate productivity and nominal interest rate are the random variables indexed to these histories. Notice that we are not allowing idiosyncratic shocks across countries and firms. There is a state

[3] The number of goods produced in each country does not have to coincide with its size, however to simplify the analysis and the notation we assumed that the exogenous fixed cost in each of the countries is such that that happens.

[4] Few are the papers that allow for endogenous firm entry over the business cycle. See for instance Bilbiie et al (2007).

contingent nominal asset traded inside each country, and there is a non-state contingent nominal asset traded across countries.

The results of this paper would still hold if instead we had assumed that the nominal state contingent asset was traded across countries[5]. On the other hand, labor immobility across countries is a crucial assumption. If there was perfect mobility of labor the terms of trade channel of the monetary transmission mechanism would be closed.

We consider the monetary transmission mechanism in two environments, differentiated by the type of price setting of the firms. In this section every firm sets prices in every period contemporaneously and the model is solved analytically for prices and for the aggregate allocation. In section 5 we derive, numerically, the transmission mechanism when firms set prices à la Calvo.

2.1 Households

Given the described set-up, there are two representative households, one for each country. The preferences of the representative consumer in country H and of the representative consumer in country F are

$$U = E_0 \sum_{t=0}^{\infty} \beta^t u\left(C_t, N_t\right), \; 0 < \beta < 1 \tag{1}$$

and

$$U^* = E_0 \sum_{t=0}^{\infty} \beta^t u\left(C_t^*, N_t^*\right),$$

respectively, where E_0 is the expectation conditional on the information available at time 0, β is a discount factor, N_t is hours of labor of the representative household of country H and C_t is the composite consumption in excess of the subsistence level of the representative household of country H. The instantaneous utility function is non-homothetic, of the type Stone -Geary, and identical across these two consumers. The C_t is defined as:

$$C_t = \left[\int_0^{\theta} \widetilde{c}_{h,t}(j)^{\frac{\sigma-1}{\sigma}} dj + \int_{\theta}^{1} \widetilde{c}_{f,t}(i)^{\frac{\sigma-1}{\sigma}} di\right]^{\frac{\sigma}{\sigma-1}}, \tag{2}$$

[5] In the appendix we consider the case when the contingent nominal bond can be traded across countries. Notice that the existence of state contingent markets across countries when labor is immobile is in general not necessary nor sufficient for the existence of a representative household for the union.

with $\widetilde{c}_{h,t}(j) = c_{h,t}(j) - \overline{c}_{h,t}(j)$, and $\widetilde{c}_{f,t}(i) = c_{f,t}(i) - \overline{c}_{f,t}(i)$, for $\overline{c}_{h,t}(j) \geq 0$ and $\overline{c}_{f,t}(i) \geq 0$, which we interpret as subsistence levels, where $c_{h,t}(j)$ is the consumption of good j produced in country H, $c_{f,t}(i)$ is the consumption of good i produced in country F, and $\sigma > 1$ is the elasticity of substitution between the various goods. The variables concerning the foreign country are indexed with a star. Thus, N_t^* is hours of labor of the representative household of country F, and C_t^* is defined as:

$$C_t^* = \left[\int_0^{\theta} \widetilde{c}_{h,t}^*(j)^{\frac{\sigma-1}{\sigma}} dj + \int_{\theta}^{1} \widetilde{c}_{f,t}^*(i)^{\frac{\sigma-1}{\sigma}} di \right]^{\frac{\sigma}{\sigma-1}},$$

with $\widetilde{c}_{h,t}^*(j) = c_{h,t}^*(j) - \overline{c}_{h,t}(j)$, and $\widetilde{c}_{f,t}^*(i) = c_{f,t}^*(i) - \overline{c}_{f,t}(i)$, where $c_{h,t}^*(j)$ denotes the consumption by the representative household of country F of good j produced in country H, and $c_{f,t}^*(i)$ denotes consumption of good i produced in country F.

The home country has a composite subsistence level for the continuum of goods produced at home, $\overline{C}_{h,t}$ and a composite subsistence level for the continuum of goods produced in the foreign country, $\overline{C}_{f,t}$. These composite subsistence levels are defined

$$\overline{C}_{h,t} = \left[\left(\frac{1}{\theta} \right)^{\frac{1}{\sigma}} \int_0^{\theta} c_{h,t}(i)^{\frac{\sigma-1}{\sigma}} di \right]^{\frac{\sigma}{\sigma-1}}, \text{ and } \overline{C}_{f,t} = \left[\left(\frac{1}{1-\theta} \right)^{\frac{1}{\sigma}} \int_{\theta}^{1} c_{f,t}(j)^{\frac{\sigma-1}{\sigma}} dj \right]^{\frac{\sigma}{\sigma-1}}.$$

Similarly the foreign country has composite subsistence levels $\overline{C}_{h,t}^*$ and $\overline{C}_{f,t}^*$. We assume that the composite subsistence levels of the continuum of goods produced in each country are exogenous and the same across countries, i.e. $\overline{C}_{h,t} = \overline{C}_{h,t}^*$ and $\overline{C}_{f,t} = \overline{C}_{f,t}^*$. The minimization of the expenditure necessary to achieve these exogenous subsistence levels implies that the individual subsistence demands are such that:

$$\frac{\overline{c}_{h,t}(j)}{\overline{c}_{h,t}(k)} = \left(\frac{p_t(j)}{p_t(k)} \right)^{-\sigma}, \text{ for } j,k \in [0,\theta] \text{ and } \frac{\overline{c}_{f,t}(i)}{\overline{c}_{f,t}(m)} = \left(\frac{p_t(i)}{p_t(m)} \right)^{-\sigma}, \text{ for } i,m \in (\theta,1]. \quad (3)$$

During each period households make a sequence of choices in the various markets according with the Lucas timing. In each period the assets markets open first and close before the goods markets open. Thus, in the beginning of period t, households of the country H enter the financial markets and allocate the wealth they brought from the previous period plus the transfer made to them by the central bank, X_t, between state-contingent bonds, non-state contingent bonds, B_t, - remunerated at a gross interest rate R_t - and cash balances, M_t. After leaving the financial markets, the households enter in the goods and labor markets. They supply labor, demand goods produced in both countries and face a cash-in-advance constraint, stating that all nominal consumption must be purchased with their cash-balances. At the end of the period, the households receive wages and dividends.

Households in every country can trade state contingent assets, but cannot trade these assets with households of the other country. For this reason, in equilibrium, the net supplies, in each of the countries, of these assets are zero. We use this condition, by not including these assets, in the budget constraints of the representative household of the home country and of the foreign country.

Households of country H maximize utility (1) subject to cash-in-advance constraints, (4), and budget constraints, (5). The cash-in-advance constraints are

$$\int_0^\theta p_{h,t}(j)c_{h,t}(j)dj + \int_\theta^1 p_{f,t}(i)c_{f,t}(i)di \leq M_t, \text{ for all } t, \tag{4}$$

where $p_{h,t}(j)$ and $p_{f,t}(i)$ are the prices of goods j and i, for $j \in [0,\theta]$ and $i \in (\theta, 1]$. The budget constraints are

$$\begin{aligned} M_{t+1} + B_{t+1} - X_{t+1} &= M_t + B_t R_t + W_t N_t + D_t \\ &- \int_0^\theta p_{h,t}(j)c_{h,t}(j)dj - \int_\theta^1 p_{f,t}(i)c_{f,t}(i)di, \text{ for all } t, \end{aligned} \tag{5}$$

where W_t is the nominal wage and D_t are the dividends of the home country firms, which are assumed to be owned by the home country households. Foreign country households have a similar problem.

The first-order conditions of the households can be summarized in the following equations, which hold for all s^t and t:

$$\left(\frac{\widetilde{c}_{f,t}(i)}{\widetilde{c}_{h,t}(j)}\right)^{-\frac{1}{\sigma}} = \mathfrak{p}_t(i,j) = \left(\frac{\widetilde{c}^*_{f,t}(i)}{\widetilde{c}^*_{h,t}(j)}\right)^{-\frac{1}{\sigma}}, \text{ for } j \in [0,\theta],\ i \in (\theta, 1], \tag{6}$$

where $\mathfrak{p}_t(i,j) \equiv \frac{p_{f,t}(i)}{p_{h,t}(j)}$,

$$\frac{-u_{N_t}}{u_{\widetilde{c}_{h,t}(j)}} = \frac{W_t}{R_t p_{h,t}(j)}, \text{ for } j \in [0,\theta] \tag{7}$$

$$\frac{-u_{N_t^*}}{u_{\widetilde{c}_{f,t}^*(i)}} = \frac{W_t^*}{R_t p_{f,t}(i)}, \text{ for } i \in (\theta, 1] \tag{8}$$

$$\frac{1}{\beta R_t} = E_t\left(\frac{u_{\widetilde{c}_{h,t+1}(j)} p_{h,t}(j)}{u_{\widetilde{c}_{h,t}(j)} p_{h,t+1}(j)}\right) = E_t\left(\frac{u_{\widetilde{c}_{h,t+1}^*(j)} p_{h,t}(j)}{u_{\widetilde{c}_{h,t}^*(j)} p_{h,t+1}(j)}\right), \text{for } j \in [0, \theta] \tag{9}$$

$$= E_t\left(\frac{u_{\widetilde{c}_{f,t+1}(i)} p_{f,t}(i)}{u_{\widetilde{c}_{f,t}(i)} p_{f,t+1}(i)}\right) = E_t\left(\frac{u_{\widetilde{c}_{f,t+1}^*(i)} p_{f,t}(i)}{u_{\widetilde{c}_{f,t}^*(i)} p_{f,t+1}(i)}\right), \text{for } i \in (\theta, 1]. \tag{10}$$

Conditions (6) state that the relative consumptions (net of subsistence levels) of the goods produced in each country are inversely proportional to the relative price of the goods. Conditions (7) and (8) state that in each country the intratemporal marginal rate of substitution between leisure and consumption is equal to the relevant real wage times the inverse of the gross interest rate. In the terminology of Lucas and Stokey (1987), the interest rate introduces a wedge between the marginal rate of substitution and the relevant real wage paid by firms because leisure is a credit good and consumption is a cash good. Conditions (9) and (10) are the standard intertemporal conditions: the marginal utility at date t of one unit of money must be equal to the expected marginal utility at date $t+1$ of the proceeds that result from buying bonds at time t in the amount of one unit of money.

2.2 Firms

The production functions are identical across goods and use labor as its unique input. If good j is produced in the home country it has the following production technology,

$$y_t(j) = A_t n_t(j) \text{ , with } j \in [0, \theta], \tag{11}$$

where $y_t(j)$ is the production of good j, $n_t(j)$ is labor employed by the firm producing good j, and A_t is the technology level. Similarly for any good produced in the foreign country,

$$y_t(i) = A_t n_t^*(i) \text{ , with } i \in (\theta, 1], \tag{12}$$

where $y_t(i)$ is the production of good i and $n_t^*(i)$ is labor employed by the firm producing good i.

In each country, labor markets are competitive. However, there is no labor mobility between countries. Firms in each economy hire labor at a certain wage rate, W_t at home and W_t^* in the foreign country.

For all t and for all $j \in [0, \theta]$, firm j chooses $p_{h,t}(j)$ to maximize its profits subject to its production function and to the demand for its product, taking as given prices. The first-order condition of this problem implies that firms at home set their prices according to

$$p_{h,t}(j) = v\frac{W_t}{A_t}, \text{ for } j \in [0, \theta] \text{ and for all } t \tag{13}$$

i.e., prices are a constant mark-up, $v \equiv \frac{\sigma}{\sigma - 1}$, over marginal costs.

The price-setting behavior of the firms in the foreign country is symmetric and therefore,

$$p_{f,t}(i) = v\frac{W_t^*}{A_t} \text{ for } i \in (\theta, 1] \text{ and for all } t. \tag{14}$$

Given (13) and (14), then $p_{h,t}(j) = p_{h,t}$, $c_{h,t}(j) = c_{h,t}$ and $c_{h,t}^*(j) = c_{h,t}^*$, for $j \in [0, \theta]$ and for all t. Similar expressions hold for the goods produced in the other country, $p_{f,t}(i) = p_{f,t}$, $c_{f,t}(i) = c_{f,t}$ and $c_{f,t}^*(i) = c_{f,t}^*$, for $i \in (\theta, 1]$ and for all t. As a consequence $\mathfrak{p}_t(i, j) = \mathfrak{p}_t$ for $i \in (\theta, 1]$, $j \in [0, \theta]$ and for all t, $n_t(j) = n_t$ for $j \in [0, \theta]$ and for all t, and $n_t^*(i) = n_t^*$ for $i \in (\theta, 1]$ and for all t.

2.3 Monetary authority

The monetary union authority does two things: sets the interest rate, R_t, and injects money in the economy, through lump-sum transfers: X_t to the representative home household and X_t^* to the representative foreign household, so that money demand is satisfied. The money supply in the monetary union evolves according to $M_t^S = M_{t-1}^S + X_t + X_t^*$, where M_t^S is the total money supply in the union in period t.

2.4 Clearing conditions

In equilibrium, all markets clear. Since there are no government bonds, the stock of bonds held by every representative household coincides with the external assets held by the country[6]. The bond market clearing condition is therefore:

$$B_t + B_t^* = 0. \tag{15}$$

The labor markets clearing conditions are:

$$N_t = n_t, \tag{16}$$

and

$$N_t^* = n_t^*. \tag{17}$$

The clearing of the goods markets implies that consumption of all goods equals the respective production:

$$\theta c_{h,t} + (1-\theta)c_{h,t}^* = A_t n_t, \tag{18}$$

and

$$\theta c_{f,t} + (1-\theta)c_{f,t}^* = A_t n_t^*. \tag{19}$$

2.5 The Equilibrium with Labor Mobility

A competitive equilibrium is a sequence for each country of policies, allocations and prices such that the private agents (firms and households) solve their problems given the sequences of policies and prices, and markets clear.

When labor is mobile the two country economy is similar to a closed economy, with labor mobility. In this case $W = W^*$. Using the firms pricing conditions it is immediate that $p_{h,t} = p_{f,t}$, or that $\mathfrak{p}_t = 1$, for all t and all states. The non-existence of idiosyncratic shocks implies that markets are complete, even without state contingent bonds. Given the identical fundamentals, including identical initial net external asset positions and money holdings in each country, the equilibrium is identical in both countries and a monetary shock would have an identical effect in the two countries, namely on per capita aggregate consumption and hours of work.

Notice that labor mobility is crucial for those identical outcomes, since in general the per capita labor supply in a particular country does not coincide with the hours of work in each firm of that country. For $\mathfrak{p}_t = 1$, relative demand (net of the subsistence levels) in every country $\frac{\widetilde{c}_{f,t}}{\widetilde{c}_{h,t}}$ and $\frac{\widetilde{c}^*_{f,t}}{\widetilde{c}^*_{h,t}}$ are equal

[6] As said before, state-contingent assets market clearing in every country was already assumed to save on notation.

to one. However for $\overline{c}_{h,t} = \overline{c}^*_{h,t} \neq \overline{c}_{f,t} = \overline{c}^*_{f,t}$[7], the relative goods demand is not one and more labor is allocated to the firms (and the respective country) with the relatively higher demand.

This result shows that, when labor is immobile across countries, markets would be complete under aggregate shocks just in the particular case when households preferences are homothetic.

2.6 Equilibrium without Labor Mobility

Typically, the non existence of a global labor market for the union, implies incomplete markets. In this case equilibrium prices and aggregate allocations for the union cannot be computed independently of the allocations of each country. This loss of aggregation implies a more complex problem than the one of a closed economy with labor mobility. Even when there is a market for contingent assets, it is not possible in general to compute the equilibrium without keeping track of the country variables over time.

However, as shown in Adao and Correia (2009), there is a class of preferences, even with labor market segmentation, that allows the computation of the equilibrium prices and aggregate allocations independently of the distribution of the allocations across countries. That class of preferences is the GHH class proposed by Greenwood, Hercowitz and Huffman (1988). When preferences belong to the GHH class, an aggregation property of the equilibrium can be obtained. Even without complete markets we can solve for the aggregate quantities and prices without having to keep track on the distribution of the allocations across countries. This aggregation result comes from the fact that labor supply is independent of the wealth distribution.[8] We show that the path for the equilibrium terms of trade can be determined uniquely as a function of the nominal interest rate and aggregate productivity. The equilibrium terms of trade determines the aggregate labor supply in each country and production (and consumption) of every good in the union, for every date and state. Later, in a second stage, using the income level in each country, the path of equilibrium terms of trade and the interest rate we compute the consumptions in each country.

[7] Meaning that non-homothetiticity is relevant around the value of one for the terms of trade.

[8] Notice that most of the literature considers the opposite type of preferences. Preferences linear in leisure or labor, which implies zero wealth effects on aggregate consumption. Our assumption is easier to defend empirically than this one.

The instantaneous utility function of the representative home consumer is

$$u(C_t, N_t) = \frac{1}{1-\phi}\left(C_t - \epsilon\frac{(N_t)^{1+\chi}}{1+\chi}\right)^{1-\phi}, \phi > 0, \chi > 0.$$

In this case the intratemporal decisions, (7) and (8), as well as (6) can be used to obtain the supplies of labor,

$$N_t = \left\{ \frac{\left[\theta + (1-\theta)\left(\frac{1}{\mathfrak{p}_t}\right)^{\sigma-1}\right]^{\frac{1}{\sigma-1}}}{\epsilon R_t} \frac{W_t}{p_{h,t}} \right\}^{\frac{1}{\chi}} \tag{20}$$

and using identical preferences for the foreign country we obtain,

$$N_t^* = \left\{ \frac{\left[\theta \mathfrak{p}_t^{\sigma-1} + (1-\theta)\right]^{\frac{1}{\sigma-1}}}{\epsilon R_t} \frac{W_t^*}{p_{f,t}} \right\}^{\frac{1}{\chi}}. \tag{21}$$

As noted before, the main characteristic of this class of preferences is that there is no income or wealth effect on the supply of labor. The supply of labor in each country is a function of the interest rate, the terms of trade and the real wage in the production.

2.6.1 The aggregate equilibrium

In the flexible price environment, in each country, there is no heterogeneity across firms. Although they produce different goods, they have the same linear technology and face the same wage and demand elasticity. Therefore, the equilibrium relative price across goods produced inside each country is always one, and there is a representative firm in every country. However, as we show below, in general the relative price across goods produced in different countries, i.e. the terms of trade, is different from one.

We proceed by showing first that the terms of trade, $\mathfrak{p}_t$, in every state and date, is uniquely determined, and independent of the distribution of consumptions across countries. Given this relative price, hours per capita, productions and real wages across countries, $\left\{N_t, N_t^*, y_{h,t}, y_{f,t}, \frac{W_t}{p_{h,t}}, \frac{W_t^*}{p_{f,t}}\right\}$, are also determined in every state and date, independently of distributional considerations. The aggregate output in each country coincides, in equilibrium, with the aggregate consumption of the union.

The equilibrium vector $\left\{N_t, N_t^*, y_{h,t}, y_{f,t}, \frac{W_t}{p_{h,t}}, \frac{W_t^*}{p_{f,t}}\right\}$ satisfies a set of static equations for each date and state, and this lack of dynamics in the aggregate economy enables us to obtain a closed form solution for it. If that was not the case, for instance if capital was an input in production or prices were sticky, as in the next section, we would have to solve numerically for this equilibrium vector, but the level of complexity of such procedure would be similar to the one in a standard closed economy model, with a representative household.

For every date and state the equilibrium vector $\left\{N_t, N_t^*, y_{h,t}, y_{f,t}, \frac{W_t}{p_{h,t}}, \frac{W_t^*}{p_{f,t}}\right\}$ depends on the level of technology and the interest rate, at that state and date[9,10].

The equilibrium conditions described above, (6), (13), (14), (20) and (21), imply,

$$\frac{\widetilde{c}_{f,t}(i)}{\widetilde{c}_{h,t}(j)} = \frac{\widetilde{c}^*_{f,t}(i)}{\widetilde{c}^*_{h,t}(j)} = \mathfrak{p}_t^{\sigma}, \tag{22}$$

$$\frac{W_t}{p_{h,t}} = \frac{W_t^*}{p_{f,t}} = \frac{A}{v}, \tag{23}$$

$$N_t = \left\{ \frac{A}{\epsilon v R_t} \left[\theta + (1-\theta) \left(\frac{1}{\mathfrak{p}_t} \right)^{\sigma - 1} \right]^{\frac{1}{\sigma-1}} \right\}^{\frac{1}{\chi}}, \tag{24}$$

[9] It is well known in the literature, that setting an exogenous path for the interest rate does not, in the flexible price environment, determine uniquely the path of prices, or the path of inflation in a stochastic environment with monetary shocks. The indeterminacy is reflected in the initial price level, which given identical economies with zero stock of initial external assets, does not affect the real equilibrium. The distribution of the consumer price level across states is also indetermined for every date. However, given lump-sum taxes this indeterminacy does not affect the real allocations or the relative prices.

[10] To compute the distribution of consumptions across countries, we use the intertemporal budget constraints and intertemporal conditions for households in each country together with the realized values of this equilibrium vector, for every date and state. We followed a straightforward procedure that is described in the appendix.

and

$$N_t^* = \left\{ \frac{A}{\epsilon v R_t} \left[\theta \mathfrak{p}_t^{\sigma-1} + (1-\theta) \right]^{\frac{1}{\sigma-1}} \right\}^{\frac{1}{\chi}}. \tag{25}$$

Conditions (22) show that the relative consumption of any good produced in the foreign country depends negatively on the relative price of any good produced in the foreign country. Conditions (23) say that producer wages, measured in terms of the national goods, are a positive function of the common technology level. Via (24) and (25) we know that each labor supply depends negatively on the interest rate and positively on the technology level and the relative price of the good that uses that labor as input.

Using (24), (25) and the production functions, (11) and (12), we can determine the production of every good i produced in the home country,

$$y_{h,t} = A \left\{ \frac{A}{\epsilon v R_t} \left[\theta + (1-\theta) \left(\frac{1}{\mathfrak{p}_t} \right)^{\sigma-1} \right]^{\frac{1}{\sigma-1}} \right\}^{\frac{1}{\chi}}, \tag{26}$$

and the production of every good i produced in the foreign country,

$$y_{f,t} = A \left\{ \frac{A}{\epsilon v R_t} \left[\theta \mathfrak{p}_t^{\sigma-1} + (1-\theta) \right]^{\frac{1}{\sigma-1}} \right\}^{\frac{1}{\chi}}. \tag{27}$$

The market clearing condition of any good i produced in the home country, (18) implies,

$$\begin{aligned} & A \left\{ \frac{A}{\epsilon v R_t} \left[\theta + (1-\theta) \left(\frac{1}{\mathfrak{p}_t} \right)^{\sigma-1} \right]^{\frac{1}{\sigma-1}} \right\}^{\frac{1}{\chi}} - \theta \overline{c}_{h,t} - (1-\theta) \overline{c}_{h,t} \quad (28) \\ = \quad & \theta \widetilde{c}_{h,t} + (1-\theta) \widetilde{c}_{h,t}^*. \end{aligned}$$

We have a similar market clearing condition for any good i produced in the foreign country:

$$\begin{aligned} & A \left\{ \frac{A}{\epsilon v R_t} \left[\theta \left(\mathfrak{p}_t \right)^{\sigma-1} + (1-\theta) \right]^{\frac{1}{\sigma-1}} \right\}^{\frac{1}{\chi}} - \theta \overline{c}_{f,t} - (1-\theta) \overline{c}_{f,t} \\ = \quad & \theta \widetilde{c}_{f,t} + (1-\theta) \widetilde{c}_{f,t}^*. \end{aligned} \tag{29}$$

We obtain, making use of (22), that the ratio of these two clearing conditions, (28) and (29), is given by:

$$RNS_t \equiv \frac{\Upsilon\left[\theta+(1-\theta)\left(\frac{1}{\mathfrak{p}_t}\right)^{\sigma-1}\right]^{\frac{1}{\chi(\sigma-1)}}-\overline{c}_{h,t}}{\Upsilon\left[\theta(\mathfrak{p}_t)^{\sigma-1}+(1-\theta)\right]^{\frac{1}{\chi(\sigma-1)}}-\overline{c}_{f,t}} = (\mathfrak{p}_t)^{\sigma} \equiv RND_t \tag{30}$$

where $\Upsilon \equiv A\left(\frac{A}{\epsilon v R_t}\right)^{\frac{1}{\chi}}$.

The left hand side of (30) is the relative net supply, net of the subsistence level, (RNS_t) of each home good. The RNS_t depends negatively on the relative price of the foreign good. The right hand side of (30), RND_t, is the relative net demand, net of the subsistence level, of each home good. This ratio depends positively on the relative price of the foreign good. Market clearing implies that the relative net supply of each home good must equal the relative net demand of each home good.

Therefore, given the interest rate path, we can use (30) to compute the equilibrium path for the terms of trade[11] and use (26) and (27) to compute the output of every good. Labor supplies and real wages will be given by (24), (25), and (23).

When $\overline{c}_{h,t} = \overline{c}_{f,t}$, it is immediate to see that the equilibrium relative price is one, $\mathfrak{p}_t = 1$, as it satisfies (30). In this very particular case labor immobility is irrelevant.

For general preferences, for instance if $\overline{c}_{h,t} > \overline{c}_{f,t}$, it is easy to verify, using (30), that for a relative price equal to one, the RNS_t will be less than one, but the RND_t will be equal to one. As RNS_t is a negative function of $\mathfrak{p}_t$ and RND_t a positive function of $\mathfrak{p}_t$, the equilibrium relative price $\mathfrak{p}_t$ will have to be smaller than one. The quantity produced, and consumed, of each home good will be larger than the quantity produced of each foreign good. We state this result as a Proposition.

Proposition 1: In general, identical countries, have equilibrium terms of trade different from one, for any date and state. If $\overline{c}_{h,t} \gtreqless \overline{c}_{f,t}$ *then* $\mathfrak{p}_t \lesseqgtr 1$.

We have shown that if $\overline{c}_{h,t} > \overline{c}_{f,t}$, in equilibrium the supply of each good produced in the home economy is higher than the supply of each good produced in the foreign country. The per capita output will be higher in the home country than in the foreign country, since in each country the number

[11] Since technology and public expenditures are constant by assumption.

of goods is identical to the population. Moreover, the per capita hours of work will be relatively higher in the home country because productivity is identical in both economies. Thus, in a stationary equilibrium, the per capita total consumption is also relatively higher in the home country.

Corollary: If $\overline{c}_{h,t} > (<)\overline{c}_{f,t}$ *then, for a constant* R_t*, the home country will have higher (lower) consumption and production than the foreign country.*

The main result of this section goes against the intuition that two regions identical in per capita fundamentals should have identical per capita equilibrium allocations. As we saw above, this intuition would be correct if there was a global labor market for the whole monetary union. In this case allowing for trade across countries of contingent nominal assets would be irrelevant. However the inverse is not true. The presence of a market of state contingent nominal assets, when the labor market is segmented, will result in different per capita allocations among countries[12]. Thus, this main result is crucially driven by the labor immobility assumption.

The bias that the terms of trade different from one impose on the country specific equilibrium implies, as we describe in the next section, that a common shock will have asymmetric outcomes across countries.

How do Terms of Trade Respond to a Monetary Policy Shock? As we described, the differences across countries are related with the equilibrium terms of trade being different from one in equilibrium. Understanding how does the terms of trade react to a common shock is therefore key to understand how that shock can lead to different outcomes across countries. In the next section we will describe quantitatively these effects in a model with a price stickiness. Right now we explore the intuition behind the effects of the common shock on the terms of trade in the model with flexible prices, and try to establish which parameters affect its quantitative importance.

We saw that if $\overline{c}_{h,t} = \overline{c}_{f,t}$ the equilibrium relative price would be one. In this case a decline in the interest rate makes leisure relatively more expensive in both countries and as a result households supply more labor and productions increase. But, for $\mathfrak{p}_t = 1$, the relative production remains unchanged and the same happens with the relative demand. Thus, the equilibrium relative price does not change. In this particular case the monetary shock has

[12]We show this in the appendix.

identical effects across countries. However in general, we have $\overline{c}_{h,t} \neq \overline{c}_{f,t}$, and an aggregate monetary shock, a revision of the interest rate, changes the terms of trade. This happens because the change in the interest rate, for the initial equilibrium terms of trade, leads to a discrepancy between the relative demand and the relative supply. To show this we make use of Figure 1. Figure 1 shows how the relative price of the home good is affected by a decline of the interest rate. The relative net demand, RND_t, is a negative function of the relative price of the home good $(1/\mathfrak{p}_t)$, and is not a function of the interest rate. The relative net supply, RNS_t, is a positive function of the relative price of the home good, and also a function of the interest rate. When the interest rate decreases the the curve RNS_t moves to the right if $\overline{c}_{h,t} > \overline{c}_{f,t}$ and to the left if $\overline{c}_{h,t} < \overline{c}_{f,t}$. We prove this now. The RNS_t can be rewritten as $\frac{k-\frac{\overline{c}_{h,t}}{y_{f,t}}}{1-\frac{\overline{c}_{f,t}}{y_{f,t}}}$, where $k \equiv \frac{y_{h,t}}{y_{f,t}}$. Notice that k is not a function of the interest rate. Since $y_{f,t}$ increases when R_t decreases, the sign of the change in RNS_t due to a decrease in R_t is the sign of $\frac{\partial RNS_t}{\partial y_{f,t}}$,

$$\frac{\partial RNS_t}{\partial y_{f,t}} = \frac{\frac{\overline{c}_{h,t}}{y_{f,t}^2}\left(1-\frac{\overline{c}_{f,t}}{y_{f,t}}\right)-\frac{\overline{c}_{f,t}}{y_{f,t}^2}\left(k-\frac{\overline{c}_{f,t}}{y_{f,t}}\right)}{\left(1-\frac{\overline{c}_{f,t}}{y_{f,t}}\right)^2}.$$

Which implies that

$$sign\frac{\partial RNS_t}{\partial Y_{f,t}} = sign\left\{\overline{c}_{h,t} - k\overline{c}_{f,t}\right\}.$$

If $\overline{c}_{h,t} > \overline{c}_{f,t}$, then from (30) we get $\mathfrak{p}_t < 1$, $k > 1$, and

$$\frac{1-\frac{\overline{c}_{h,t}}{y_{h,t}}}{1-\frac{\overline{c}_{f,t}}{y_{f,t}}} = (\mathfrak{p}_t)^{\sigma}\frac{1}{k} < 1.$$

Using the inequality $(\mathfrak{p}_t)^{\sigma} < k$ and (30) we obtain

$$k - \frac{\overline{c}_{h,t}}{y_{f,t}} < k\left(1-\frac{\overline{c}_{f,t}}{y_{f,t}}\right).$$

Thus, $\overline{c}_{h,t} - k\overline{c}_{f,t} > 0$ and $\frac{\partial RNS_t}{\partial y_{f,t}} > 0$. This proves that $\frac{\partial RNS_t}{\partial Y_{f,t}} > 0$ (< 0) for any $\overline{c}_{h,t} > \overline{c}_{f,t}$ $(\overline{c}_{h,t} < \overline{c}_{f,t})$. Thus, after a decline in the interest rate the curve RNS_t moves to the right if $\overline{c}_{h,t} > \overline{c}_{f,t}$ and to the left if $\overline{c}_{h,t} < \overline{c}_{f,t}$. Using

Figure 1 it is straightforward to see that the relative price of the home good decreases (increases) for $\overline{c}_{h,t} >(<) \overline{c}_{f,t}$. This result is stated as a proposition:

Proposition 2: In general a positive monetary shock in a monetary union with two identical countries leads to an increase (decline) of the terms of trade, $\mathfrak{p}_t$*, when* $\overline{c}_{h,t} >(<) \overline{c}_{f,t}$.

Proposition 1 and 2 allow us to say that:

Corollary: Monetary shocks in a monetary union create cycles characterized by lower volatility of output for the country with higher trend output. The mechanism responsible for both the different trend and cycle is the path of the terms of trade.

The Corollary states that the country with higher output, in per capita terms, is also the one that is going to experience lower volatility of production and hours of work. When the shock is positive it is the richer country that benefits the less, while when the shock is negative it is the richer country that is harmed the less. There are two channels through which a change in the interest rate affects every national economy, in this flexible price model. In the cash-in-advance economy that we use here, the interest rate is a wedge between the marginal rate of substitution and the marginal rate of transformation. A reduction in the interest rate has a direct effect over production, increases the production of both goods, due to the decrease in the wedge. And has an indirect effect through its consequence on the terms of trade. Our emphasis on this channel comes from the fact that it is this indirect effect of the monetary shock that creates the asymmetric responses in each country. As we have seen in Proposition 1 there is a one to one relationship between the interest rate and the terms of trade. The decline of the interest rate, through its indirect effect on the terms of trade, will have a negative effect over the production in one country and a positive effect over the production of the other. This indirect effect affects with opposite signs the households' incomes of each country.

2.7 How do monetary shocks affect individual economies?

To determine the effect on aggregate consumption in every country it is necessary to take a position on the assets markets across countries. When the only asset traded across countries is the state non-contingent bond, the equilibrium consumption for each country can be computed after the determination of the aggregate equilibrium for the union. As we just described we can compute the aggregate allocations and terms of trade with no information about the distribution of consumption across countries. Given these equilibrium values, the consumption path of each country is pinned down using the remaining equilibrium conditions: the households' budget constraints and intertemporal equations and the non Ponzi game conditions. The construction of the intertemporal constraints for each country is straightforward but cumbersome. The appendix describes this construction as well as the determination of the consumption of each country.

Once understood that monetary shocks affect the terms of trade, it is immediate to see that monetary shocks can affect differently every national economy. For temporary shocks, and given the chosen GHH preferences, those different effects will be temporary for those aggregates which are stationary like labor and output. Consumption in each country of either good or of the aggregate consumption will be affected permanently. These permanent effects on consumption are associated with permanent effects on the position of each economy in external assets holdings. Given an initial position of zero net foreign debt, and conditional to the temporary monetary shock that we have been analyzing, the country that produces the good whose price is temporarily higher will have a consumption higher forever and a permanent balance of trade deficit, that will be financed by the assets accumulated during the periods when, given the higher terms of trade, the economy had a trade balance surplus with the rest of the union. The more pronounced the cumulative effect on the terms of trade, the higher will be the permanent effect on consumption and on the net asset position of each national economy.

The distribution of the inflation tax revenue is another way through which monetary policy could have asymmetric effects on the countries. We assumed that this seigniorage distribution is equitable to highlight the terms of trade channel.

Thus, we can conjecture, but we will quantify it in the next section that, without state contingent markets across countries, the welfare level of the rich country is higher not only because it has a higher stationary level of consumption and production but also because it has less volatile consumption and hours of work.

The results hold even with state contingent markets. Since the preferences used in this paper are not separable, the lower volatility of hours in the rich country will be transmitted in lower volatility of consumption when marginal utilities of aggregate consumption across countries are smoothed across states.

3 Public Sector and Homothetic Preferences

There are other environments where the results above continue to hold even if households have homothetic utility functions. We consider one of these environments in this section.

The environment is similar to the one we have been considering, except for two things. Now the subsistence levels of the households, $\overline{C}_{h,t}$ and $\overline{C}_{f,t}$, are zero, which implies homothetic utility functions for the households. Moreover there is a public sector in each country. The fiscal authority of each country makes government expenditures and raises revenues using lump-sum taxes.[13] The home fiscal authority determines consumptions of composite home goods and foreign goods, $G_{h,t}$ and $G_{f,t}$, and the foreign fiscal authority determines per capita consumptions of composite goods, $G^*_{h,t}$ and $G^*_{f,t}$. We assume that per-capita government expenditures in each country are exogenous and the same, that is $G_{h,t} = G^*_{h,t}$ and $G_{f,t} = G^*_{f,t}$. The per-capita government expenditures for the home country are:

$$G_{h,t} = \left[\left(\frac{1}{\theta} \right)^{\frac{1}{\sigma}} \int_0^{\theta} g_{h,t}(j)^{\frac{\sigma-1}{\sigma}} dj \right]^{\frac{\sigma}{\sigma-1}}, \text{ and } G_{f,t} = \left[\left(\frac{1}{1-\theta} \right)^{\frac{1}{\sigma}} \int_{\theta}^{1} g_{f,t}(i)^{\frac{\sigma-1}{\sigma}} di \right]^{\frac{\sigma}{\sigma-1}},$$

where $g_{h,t}(j)$ is the public consumption of good j produced in country H, and $c_{f,t}(i)$ is the public consumption of good i produced in country F. The per-capita government expenditures for the foreign country are similarly defined. The government minimization of costs implies that the public demands of individual goods are given by:

$$\frac{g_{h,t}(j)}{g_{h,t}(k)} = \left(\frac{p_t(j)}{p_t(k)} \right)^{-\sigma}, \text{ for } j,k \in [0,\theta] \text{ and } \frac{g_{f,t}(i)}{g_{f,t}(m)} = \left(\frac{p_t(i)}{p_t(m)} \right)^{-\sigma}, \text{ for } i,m \in (\theta,1]. \tag{31}$$

The firms' first order conditions, (13) and (14), continue to hold and imply $p_{h,t}(j) = p_{h,t}$, $c_{h,t}(j) = c_{h,t}$, $c^*_{h,t}(j) = c^*_{h,t}$, $g_{h,t}(j) = g_{h,t}$, and $g_{f,t}(i) = g_{f,t}$ for $j \in [0,\theta]$ and for all t. Similar expressions hold for the goods produced in

[13] Since taxes are lump-sum, we assume, without loss of generality, that government debt is zero.

the other country, $p_{f,t}(i) = p_{f,t}$, $c_{f,t}(i) = c_{f,t}$, $c^*_{f,t}(i) = c^*_{f,t}$, $g^*_{h,t}(i) = g_{h,t}$, and $g^*_{f,t}(i) = g_{f,t}$ for $i \in (\theta, 1]$ and for all t. The equilibrium equations associated with the households' first order conditions continue to be described by, (6), (13), (14), (20) and (21). The clearing conditions of the goods markets change to allow for public consumption. Now private consumption plus public consumption of each good must be equal to its production,

$$\theta c_{h,t} + (1-\theta)c^*_{h,t} + g_{h,t} = A_t n_t,$$

and

$$\theta c_{f,t} + (1-\theta)c^*_{f,t} + g_{f,t} = A_t n^*_t.$$

As we did before to obtain (30), here too, we can use the various equilibrium conditions to obtain (32). The expression (32) is similar to (30). They differ as the variables $\overline{c}_{h,t}$ and $\overline{c}_{f,t}$ are replaced with $g_{h,t}$ and $g_{f,t}$, respectively.

$$RNS_t \equiv \frac{\Upsilon\left[\theta+(1-\theta)\left(\frac{1}{\mathfrak{p}_t}\right)^{\sigma-1}\right]^{\frac{1}{\chi(\sigma-1)}} - g_{h,t}}{\Upsilon\left[\theta(\mathfrak{p}_t)^{\sigma-1}+(1-\theta)\right]^{\frac{1}{\chi(\sigma-1)}} - g_{f,t}} = (\mathfrak{p}_t)^{\sigma} \equiv RND_t \tag{32}$$

where $\Upsilon \equiv A\left(\frac{A}{\epsilon v R_t}\right)^{\frac{1}{\chi}}$.

It is trivial to verify that the equivalents of Proposition 1 and 2, which are Propositions 3 and 4 hold in this environment.

Proposition 3: In general, identical countries, have equilibrium terms of trade different from one, for any date and state. If $g_{h,t} \gtreqless g_{f,t}$ *then* $\mathfrak{p}_t \lesseqgtr 1$.

Proposition 4: In general a positive monetary shock in a monetary union with two identical countries leads to an increase (decline) of the terms of trade, $\mathfrak{p}_t$*, when* $g_{h,t} > (<) g_{f,t}$.

4 The Model with Calvo Prices

It remains to see whether the asymmetric effects coming from the terms of trade reaction to the monetary policy shocks are quantitatively significant. Since most recent literature stresses nominal rigidities as the main transmission of monetary shocks, we analyze whether the sort of arguments developed in the previous section can be extended to that type of environments, and we quantify the potential difference of outcomes across countries when there is nominal rigidities. Therefore, in this section we consider an extremely simplified model with the most used nominal rigidity, namely we impose that firms set prices according to Calvo (1983). We study the effects of a monetary shock in this environment and, as before, we investigate its transmission to the terms of trade over time, whether the response of this variable to a monetary shock is significantly different from zero. Once we get this response, the effects of this terms of trade path on the asymmetric transmission of the shock are similar to the ones described in the previous section. We evaluate the magnitude of this asymmetric shock for a basic calibration of the model, and compare quantitatively the idiosyncratic effects of the monetary shock with its union wide effect.

We begin by describing the changes introduced in the model described in section 2 to accommodate the sticky price friction. The behavior of households and central bank in the monetary union is the same as in the flexible prices economy. The agents that behave differently are the firms.

To take into account the possibility of heterogeneous price behavior by firms, we follow Calvo (1983) and assume that in each period only a fraction $(1-\xi_p)$ of firms is able to change prices optimally. Those firms that cannot re-optimize update their prices according to the lagged inflation in the continuum of goods produced in their country. The growth rates of $P_{h,t}$ and $P_{f,t}$, which are defined as

$$P_{h,t} = \left[\left(\frac{1}{\theta} \right)^{\sigma} \int_0^{\theta} p_{h,t}(i)^{1-\sigma} di \right]^{\frac{1}{1-\sigma}}, \tag{33}$$

and

$$P_{f,t} = \left[\left(\frac{1}{1-\theta} \right)^{\sigma} \int_{\theta}^{1} p_{f,t}(i)^{1-\sigma} di \right]^{\frac{1}{1-\sigma}}. \tag{34}$$

are denoted by $\pi_{h,t}$ and $\pi_{f,t}$, respectively. We maintain the identical countries assumption by using the same probability of revising prices across firms and across countries.

When a specific firm can re-optimize she chooses the price that maximizes expected profits. The problem of a firm j in the home country ($j \in [0, \theta]$), that can change the price at time t is the following:

$$\underset{\{p_{h,t}(j)\}}{Max} \, E_t \sum_{\tau=0}^{\infty} \left(\beta \xi_p\right)^{\tau} \lambda_{t+\tau} \left\{ \left(\prod_{s=1}^{\tau} \pi_{h,t+s-1} \frac{p_{h,t}(j)}{P_{h,t+\tau}} - \frac{W_{t+\tau}}{A_{t+\tau} P_{h,t+\tau}} \right) y_{t+\tau}(j) \right\} \tag{35}$$

subject to

$$y_{t+\tau}(j) = \left(\prod_{s=1}^{\tau} \pi_{h,t+s-1} \frac{p_{h,t}(j)}{P_{h,t+\tau}} \right)^{-\sigma} Y_{h,t+\tau}^{d}$$

where $Y^{d}_{h,t+\tau}$ is the total demand or the continuum of goods produced in country H. The firm uses the stochastic discount factor $\left(\beta\xi_p\right)^{\tau}\lambda_{t+\tau}$ to compute the value of profits. The term $\lambda_{t+\tau}$ is the marginal utility of the households' real income in period $t+\tau$, which is exogenous to the firms.

Log-linearizing the first order condition of the problem above, around the steady state, and aggregating the log-linearized equations for both optimizing and non-optimizing firms yields the following equation for the aggregate inflation of the goods produced in the home country,

$$\widehat{\pi}_{h,t}-\tfrac{\beta}{1+\beta}E_t\widehat{\pi}_{h,t+1}-\tfrac{1}{1+\beta}\widehat{\pi}_{h,t-1}-\tfrac{\left(1-\xi_p\right)\left(1-\xi_p\beta\right)}{\xi_p(1+\beta)}\left(\widehat{W}_t-\widehat{P}_{h,t}-\widehat{A}_t\right)=0, \quad (36)$$

where the variables with hat denote deviations from their steady state values. In this framework, inflation of the goods produced in the home country depends on lagged inflation, future inflation and current marginal costs of the goods produced in the home country.

The problem of each foreign firm that can choose the price is similar to the problem of the domestic firm that can choose the price. Similarly, those foreign firms that cannot re-optimize update their prices with the lagged inflation in the continuum of goods produced in their country. The equation for the inflation of goods produced in the foreign country,

$$\widehat{\pi}_{f,t}-\tfrac{\beta}{1+\beta}E_t\widehat{\pi}_{f,t+1}-\tfrac{1}{1+\beta}\widehat{\pi}_{f,t-1}-\tfrac{\left(1-\xi_p\right)\left(1-\xi_p\beta\right)}{\xi_p(1+\beta)}\left(\widehat{W}^{*}_t-\widehat{P}_{f,t}-\widehat{A}_t\right)=0,$$

is completely analogous to (36), with the variables $\widehat{\pi}_{f,t}$, $\widehat{W}^{*}_t$, and $\widehat{P}_{f,t}$ replacing $\widehat{\pi}_{h,t}$, $\widehat{W}_t$ and $\widehat{P}_{h,t}$, respectively.

As is standard in the literature the central bank conducts monetary policy through an interest rate rule that guarantees local determinacy. In its loglinearized form the simple rule followed by the central bank is

$$\widehat{R}_t=\rho_0\cdot\widehat{R}_{t-1}+\rho_1\cdot\widehat{\Pi}_t+\widehat{\varepsilon}_R \quad (37)$$

where Π_t is the inflation of the union, the growth level of P_t which is defined as

$$P_t=\left[\theta^{\sigma}P^{1-\sigma}_{h,t}+(1-\theta)^{\sigma}P^{1-\sigma}_{f,t}\right]^{\frac{1}{1-\sigma}}, \quad (38)$$

$\widehat{\varepsilon}_R$ is a random shock to the monetary policy and ρ_0 and ρ_1 coefficients.[14] What we want to study is the transmission mechanism of a monetary policy

[14] In its nonlinearized version the interest rate is $\frac{R_t}{R}=\left(\frac{R_{t-1}}{R}\right)^{\rho_0}\left(\frac{\Pi_t}{\Pi}\right)^{\rho_1}\left(\frac{\mathcal{Y}_t}{\mathcal{Y}}\right)^{\rho_2}\varepsilon_t$, where

Preferences,	$\beta = 0.993$	$\chi = 0.5$	$\upsilon = 1.2$
	$\phi = 1$	$\theta = 0.5$	$\epsilon = 1$
technology		$A = 1$	
government consumption	$\overline{c}_h = 0.2$	$\overline{c}_f = 0.1$	
price-setting frictions		$\xi_p = \xi_p^* = 0.67$	

Table 1: The benchmark calibration

shock, i.e. to determine the effects on the main variables of an innovation in $\widehat{\varepsilon}_R$.

The economy with flexible prices is identical to the economy with sticky prices, except for the way firms behave. Thus, the system of equations that determines the equilibrium in the flexible prices economy differs from the system of equations that determines the equilibrium in the sticky prices economy only on those equations associated with the behavior of firms. More specifically, the first order conditions (13) and (14) are replaced with the first order conditions of the firms' problems described in this section.

4.1 The Effects of a Monetary Shock

4.1.1 Calibration

The calibration of preferences and technology follows the literature, so we will not describe it in detail (see, for example, Christiano, Eichenbaum and Evans (2005)). Table 1 presents the calibration of all parameters. In the Calvo price setting environment we assume that firms change prices on average every 3 quarters. The firms' steady state mark-up is calibrated to be 1.2 and the inverse of the elasticity of labor supply to be 0.5. The countries are of equal size. Preferences are such that the consumption subsistence level is higher for goods produced in country H.

As said before the central bank follows an interest rate rule and the monetary shock is identified as a disturbance $\widehat{\varepsilon}_R$ in that interest rate policy rule.

R is the steady state interest rate, Π the steady state inflation rate of the union and $\mathcal{Y}$ the steady state output of the union. We do not discuss the optimality of this rule, as we do not assume that monetary policy aims at minimizing a specific loss function. Instead we assume that the interest rate rule is a good representation of the behavior of the monetary policy maker.

The specific interest rate rule considered was

$$\widehat{R}_t = 0.95 \cdot \widehat{R}_{t-1} + 1.5 \cdot \widehat{\Pi}_t + \widehat{\varepsilon}_R.$$

This rule satisfies fundamental requisites: the equilibrium interest rate obtained from it has a high degree of persistence as in the data, and has parameters that guarantee local determinacy of the equilibrium.

We assume that all revenue raised by the central bank from the inflation tax is redistributed back to each fiscal authority. To maintain neutrality we take that each government receives an identical per capita payment from the central bank.

4.1.2 Results

Figure 2 shows the shock and the persistent path of the monetary instrument, the nominal interest rate. All variables are measured in deviations from the steady state. For the chosen parameters, on impact the annualized interest rate declines 40 basis points. The magnitudes of the aggregate effects are roughly in line with the ones find in the literature, even though our model misses many details necessary to replicate the exact qualitative and quantitative characteristics of the monetary shock on the aggregate equilibrium. As usual the expansionary shock increases production of all goods, consumption and inflation. The dynamics are somewhat different from the ones found in more sophisticated models, namely the inverted U shape is missing since we do not have any real friction in this model.

Qualitatively the terms of trade under sticky prices appear to behave as they do under flexible prices, i.e. according with Proposition 2. An expansionary monetary shock implies, under sticky prices, a decline in the terms of trade for the country that has them higher in the steady-state.[15]

We want to use this numerical exercise to take a position on the magnitude of the asymmetrical effects on the aggregate consumption in every country. Therefore we construct, as described in the Appendix, the country level path of private consumption given the path of the nominal interest rate, the path of inflation, the path of the terms of trade and the path of income in every country. Figure 3 shows the paths in levels of the country variables. We compute the percentage deviations of aggregate consumption in the home

[15] This response of the terms of trade to a monetary shock is robust to changes in the parameters.

and in the foreign country and represent them in Figure 4. As we expected, the country variables response is not identical after the monetary shock. The monetary shock has two effects. It has a direct effect, as in the one good closed economy model, an identical increase in the consumption of each country. However, there is an additional indirect way of transmitting the monetary shock - through the terms of trade. This channel has opposite effects in each country and therefore the sum of both the direct and the indirect effect creates an asymmetry in the response of consumption in each country. For the calibration that is proposed in this example the asymmetry implies that on impact the consumption in the home country increases by 1.09 pp while in the foreign country it increases by 1.25 pp. This means that the differential is 0.16 pp. To take a position on whether this is a small or large number we use as metric the response, on impact to the shock, of the union aggregate consumption (Fig 2). It increases on impact by 1.17 pp. Therefore the differential across countries is 14% of this aggregate effect. According to this number the asymmetry is significative. In addition, as stated in Proposition 2, the effect in the output is lower for the "richer" country.

The Corollary to Proposition 2 is confirmed in this environment too. When there are monetary shocks, the lower volatility of the momentary utility and consumption, for the home country reinforces the higher stationary value of the utility and consumption in this country.

In a monetary union, a monetary policy shock affects differently countries with the same fundamentals. The shock is amplified in the poorer country and is restrained in the richer one. For instance, the effect of a contractionary monetary policy shock, which is negative on the aggregate, is moderated in the richer country and is augmented in the poorer country. Thus, although positive monetary shocks tend to make countries more similar in per capita terms, the opposite occurs with negative shocks and therefore they reinforce the welfare asymmetry that characterizes these countries in a stationary environment.

We described the effects of a common monetary shock but it is immediate to realize that these results can be extended to common technology shocks. Since monetary policy should be reacting to shocks, it is crucial to understand not only the monetary transmission but also the transmission of these other shocks.

4.2 Concluding Remarks

The conventional wisdom is that common shocks should be transmitted identically in a set of countries connected by trade and with no differences in fundamentals. In this paper we show that this conventional wisdom is not the general result under rather standard conditions. The crucial assumption is segmented labor markets. Under this assumption a common monetary policy shock, as any other common shock, has an additional channel for the transmission, that is asymmetric among countries. The terms of trade channel being operational depends not on different fundamentals like preferences, or technologies, but on non-homothetic preferences. In this case the specialization of production, implies that a shock will have different effects on producer prices across countries.

The automatic, partial or full, insurance mechanism of the terms of trade, that occurs with idiosyncratic shocks, is reversed when shocks are common. In this sense this paper can be regarded as complementary to the existent literature. As we showed the change of the terms of trade is the mechanism that makes the common shock have asymmetric country specific outcomes. Even when there is state contingent markets across countries, effects are asymmetric on national consumptions, unless consumption is additively separable in preferences.

We investigate whether this asymmetric effect is quantitatively significant, when compared with the aggregate effect of the monetary shock. For a very simple, but standard sticky price model, we conclude that the positive monetary policy shock is amplified in the country that has the worse terms of trade, the poorer country, while it is moderated in the country that has the better terms of trade, the richer country. The same happens when the shock is negative, it hurts more the poorer country. As a result the volatility of per capita consumption, and momentary utility, is larger in the poorer country than in the richer country. Thus, monetary policy shocks create different volatilities across countries with identical fundamentals in a monetary union.

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Appendix 1

Aggregate Consumption Across Countries

Here we show how the equilibrium consumption path of each country is determined. In section 2.6 we showed that once the path the interest rate is given, the equilibrium path of the variables $\left\{N_t, N_t^*, y_{h,t}, y_{f,t}, \frac{W_t}{p_{h,t}}, \frac{W_t^*}{p_{f,t}}, \mathfrak{p}_t\right\}$ is determined. The consumption path of each country is computed using this vector of variables together with the intertemporal budget constraints and intertemporal conditions for each country.

The intertemporal condition (9) implies

$$\left(\left[\theta+(1-\theta)\frac{\widetilde{c}_{f,t}(i)^{\frac{\sigma-1}{\sigma}}}{\widetilde{c}_{h,t}(j)^{\frac{\sigma-1}{\sigma}}}\right]^{\frac{\sigma}{\sigma-1}}\widetilde{c}_{h,t}(j)-\epsilon\frac{(N_t)^{1+\chi}}{1+\chi}\right)\left[\theta+(1-\theta)\frac{\widetilde{c}_{f,t}(i)^{\frac{\sigma-1}{\sigma}}}{\widetilde{c}_{h,t}(j)^{\frac{\sigma-1}{\sigma}}}\right]^{\frac{1}{1-\sigma}}$$
$$=\left(\beta R_t\frac{p_{h,t}}{p_{h,t+1}}\right)^{-\frac{1}{\phi}}\left[\theta+(1-\theta)\frac{\widetilde{c}_{f,t+1}(i)^{\frac{\sigma-1}{\sigma}}}{\widetilde{c}_{h,t+1}(j)^{\frac{\sigma-1}{\sigma}}}\right]^{\frac{1}{1-\sigma}}$$
$$\left(\left[\theta+(1-\theta)\frac{\widetilde{c}_{f,t+1}(i)^{\frac{\sigma-1}{\sigma}}}{\widetilde{c}_{h,t+1}(j)^{\frac{\sigma-1}{\sigma}}}\right]^{\frac{\sigma}{\sigma-1}}\widetilde{c}_{h,t+1}(j)-\epsilon\frac{(N_{t+1})^{1+\chi}}{1+\chi}\right),\ t=0,1,... \qquad (39)$$

There is a similar condition for the foreign country,

$$\left(\left[\theta+(1-\theta)\frac{\widetilde{c}_{f,t}(i)^{\frac{\sigma-1}{\sigma}}}{\widetilde{c}_{h,t}(j)^{\frac{\sigma-1}{\sigma}}}\right]^{\frac{\sigma}{\sigma-1}}\widetilde{c}^*_{h,t}(j)-\epsilon\frac{(N^*_t)^{1+\chi}}{1+\chi}\right)\left[\theta+(1-\theta)\frac{\widetilde{c}_{f,t}(i)^{\frac{\sigma-1}{\sigma}}}{\widetilde{c}_{h,t}(j)^{\frac{\sigma-1}{\sigma}}}\right]^{\frac{1}{1-\sigma}}$$
$$=\left(\beta R_t\frac{p_{h,t}}{p_{h,t+1}}\right)^{-\frac{1}{\phi}}\left[\theta+(1-\theta)\frac{\widetilde{c}_{f,t+1}(i)^{\frac{\sigma-1}{\sigma}}}{\widetilde{c}_{h,t+1}(j)^{\frac{\sigma-1}{\sigma}}}\right]^{\frac{1}{1-\sigma}}$$
$$\left(\left[\theta+(1-\theta)\frac{\widetilde{c}_{f,t+1}(i)^{\frac{\sigma-1}{\sigma}}}{\widetilde{c}_{h,t+1}(j)^{\frac{\sigma-1}{\sigma}}}\right]^{\frac{\sigma}{\sigma-1}}\widetilde{c}^*_{h,t+1}(j)-\epsilon\frac{\left(N^*_{t+1}\right)^{1+\chi}}{1+\chi}\right),\ t=0,1,...$$

If we add up the home constraints, after multiplying them by θ, and the foreign constraints, after multiplying them by $1-\theta$, and use the resource constraints we get

$$\left(\left[\theta+(1-\theta)\frac{\widetilde{c}_{f,t}(i)^{\frac{\sigma-1}{\sigma}}}{\widetilde{c}_{h,t}(j)^{\frac{\sigma-1}{\sigma}}}\right]^{\frac{\sigma}{\sigma-1}}(A_tN_t-\overline{c}_h)-\epsilon\frac{\theta(N_t)^{1+\chi}+(1-\theta)(N^*_t)^{1+\chi}}{1+\chi}\right)\left[\theta+(1-\theta)\frac{\widetilde{c}_{f,t}(i)^{\frac{\sigma-1}{\sigma}}}{\widetilde{c}_{h,t}(j)^{\frac{\sigma-1}{\sigma}}}\right]^{\frac{1}{1-\sigma}}$$
$$=\left(\beta R_t\frac{p_{h,t}}{p_{h,t+1}}\right)^{-\frac{1}{\phi}}\left[\theta+(1-\theta)\frac{\widetilde{c}_{f,t+1}(i)^{\frac{\sigma-1}{\sigma}}}{\widetilde{c}_{h,t+1}(j)^{\frac{\sigma-1}{\sigma}}}\right]^{\frac{1}{1-\sigma}}$$
$$\left(\left[\theta+(1-\theta)\frac{\widetilde{c}_{f,t+1}(i)^{\frac{\sigma-1}{\sigma}}}{\widetilde{c}_{h,t+1}(j)^{\frac{\sigma-1}{\sigma}}}\right]^{\frac{\sigma}{\sigma-1}}(A_{t+1}N_{t+1}-\overline{c}_h)-\epsilon\frac{\theta(N_{t+1})^{1+\chi}+(1-\theta)\left(N^*_{t+1}\right)^{1+\chi}}{1+\chi}\right),\ t=0,1,... \qquad (40)$$

Equations (40) determine $\left\{\frac{p_{h,t}}{p_{h,t+1}}\right\}_{t=0}^{\infty}$ as all the other variables are already known.

Given $\left\{\frac{p_{h,t}}{p_{h,t+1}}\right\}_{t=0}^{\infty}$ from the intertemporal conditions (39) we obtain $\{\widetilde{c}_{h,t}\}_{t=1}^{\infty}$ as a function of $\widetilde{c}_{h,0}$. Given $\left\{\frac{\widetilde{c}_{h,t}}{\widetilde{c}_{f,t}}\right\}_{t=0}^{\infty}$, we get $\{\widetilde{c}_{f,t}\}_{t=0}^{\infty}$ as a function of $\widetilde{c}_{h,0}$, as well. The intertemporal budget constraint of the representative consumer in the home country is

$$\sum_{t=0}^{\infty} Q_t \left(p_{h,t} \left(\widetilde{c}_{h,t}(j) + \overline{c}_h \right) + p_{f,t} \left(\widetilde{c}_{f,t} + \overline{c}_f \right) - P_{h,t} Y_t \right) = \mathbb{W}_0 + \sum_{t=0}^{\infty} Q_t X_t \quad (41)$$

where $Q_t = \frac{\beta^{t+1} u_{c_{h,t}(j)} p_{h,0}}{u_{c_{h,0}(j)} p_{h,t}}$, is the value at 0 of a monetary unit at $t+1$, $Q_0 = 1$, and $\mathbb{W}_0$ is the initial nominal wealth of the representative household of the home country. Once we rewrite $\{\widetilde{c}_{h,t}(j)\}_{t=0}^{\infty}$ and $\{\widetilde{c}_{f,t}(i)\}_{t=0}^{\infty}$ as functions of $\widetilde{c}_{h,0}(j)$, condition (41) determines the value of $\widetilde{c}_{h,0}(j)$. Given the value of $\widetilde{c}_{h,0}(j)$ we can compute the whole path $\{\widetilde{c}_{h,t}(j), \widetilde{c}_{f,t}(i)\}_{t=0}^{\infty}$. Using equation (2) we obtain the equilibrium path of the home country aggregate consumption. The foreign consumptions $\left\{c_{h,t}^*, c_{f,t}^*, c_t^*\right\}_{t=0}^{\infty}$ can be obtained in a similar manner, or instead by using the resource constraints.

Appendix 2

Complete markets and labor mobility

As claimed in the text when labor is mobile the two country economy is similar to a typical closed economy.

Proposition 5: Independently of the households' preferences if countries have zero initial wealth and labor is mobile across countries then the per capita consumption of every good and the supply of labor are equal across households. Thus, state contingent markets are redundant.

Proof: If labor is mobile implies equal nominal wages across countries, $W_t = W_t^*$, the price-setting behavior of firms, (13) and (14), implies, $p_{h,t}(j) = p_{f,t}(i) = p_t$, for all t, $j \in [0, \theta]$ and $i \in (\theta, 1]$. Therefore the terms of trade are one, i.e. $\mathfrak{p}_t = 1$, $N_t = N_t^*$, $\widetilde{c}_{h,t} = \widetilde{c}_{f,t} = \widetilde{c}_t$ and $\widetilde{c}_{h,t}^* = \widetilde{c}_{f,t}^* = \widetilde{c}_t^*$. In this case the period t intertemporal budget constraints for the representative households are

$$\sum\nolimits_{s=t}^{\infty} E_t Q_{t,s+1} \left[p_s \left(\widetilde{c}_s + \theta \overline{c}_h + (1-\theta) \overline{c}_f \right) - W_s N_s \right] = \mathbb{W}_t + \sum_{t=0}^{\infty} Q_t X_t, \text{ for all dates and states,}$$

and

$$\sum\nolimits_{s=t}^{\infty} E_t Q_{t,s+1}^* \left[p_s \left(\widetilde{c}_s^* + \theta \overline{c}_h + (1-\theta) \overline{c}_f \right) - W_s N_s^* \right] = \mathbb{W}_t^* + \sum_{t=0}^{\infty} Q_t X_t, \text{ for all dates and states,}$$

where $\mathbb{W}_t$ is the nominal wealth of the home representative household in period t, and $Q_{t,t+1}$ is the price, in the home state-contingent market, at date t of one monetary unit at a particular state at date $t+1$. Thus $Q_{t-1,t}\frac{u_{c_{h,t-1}}}{p_{t-1}} = \frac{\beta u_{c_{h,t}}}{p_t}$, for all dates and states. $Q_{t,s} = Q_{t,t+1}...Q_{s-1,s}$, $t \geq 0$, $s \geq t+1$, and $Q_{t,t} = 1$. $\mathbb{W}_t^*$ and $Q_{t,s+1}^*$ are defined similarly. Clearly if $\mathbb{W}_0 = \mathbb{W}_0^* = 0$ then $\widetilde{c}_t = \widetilde{c}_t^*$, for all dates and all states, satisfies all intertemporal budget constraints.■

Thus, in equilibrium per capita aggregate consumption and the supply of labor is independent of the country of residence and the existence of a market for a nominal state contingent bond across countries is redundant.

However, if labor is immobile a single nominal state contingent market for the union is not enough to avoid changes in the terms of trade, and asymmetric responses of output and consumption across countries as a result of common shocks in the union.

Proposition 6: A monetary shock in a monetary union environment with labor immobility across countries and a nominal state global contingent bond market has asymmetric effects across similar countries.

In this environment the terms of trade, hours per capita, productions and real wages across countries, $\left\{\mathfrak{p}_t, N_t, N_t^*, y_{h,t}, y_{f,t}, \frac{W_t}{p_{h,t}}, \frac{W_t^*}{p_{f,t}}\right\}$, continue to be determined in every state and date, by the same equations, (30), (22), (23), (24) and (25). Thus, these variables behave in the same way in the two different environments. The terms of trade and the differences in hours across countries change with a monetary shock and the country with higher trend output will experience lower volatility of output. It remains to see, in this context, how each country's consumption reacts to the aggregate shock. The existence of a global state contingent asset implies that the ratios of the marginal consumptions of dates $t+1$ and t must be equal across countries,

$$\frac{u_{C_{t+1}}}{u_{C_t}} = \frac{u_{C_{t+1}^*}}{u_{C_t^*}}, \text{ for all dates and states.} \tag{42}$$

Condition (42) entails that there is a constant $\alpha > 0$, such that

$$u_{C_t} = \alpha u_{C_t^*}, \text{ for all dates and states.} \tag{43}$$

For the particular instantaneous utility function used (42) implies:

$$\left[C_t - \epsilon\frac{(N_t)^{1+\chi}}{1+\chi}\right] = \alpha'\left[C_t^* - \epsilon\frac{(N_t^*)^{1+\chi}}{1+\chi}\right], \text{ with } \alpha' = \alpha^{-\frac{1}{\phi}}. \tag{44}$$

It is clear, from (44), that the differences in hours across countries, for all dates and states, will be reflected in differences in the aggregate consumption across countries.■

We have shown that even if there is a state contingent asset, tradable across countries, a common shock will have idiosyncratic effects across similar countries. There will be transactions of the state contingent asset to smooth out marginal utilities of consumption and leisure across countries, but nevertheless a common shock will affect the terms of trade and lead to differences across countries in state contingent hours, output and aggregate consumption.

Appendix 3 (not for publication)

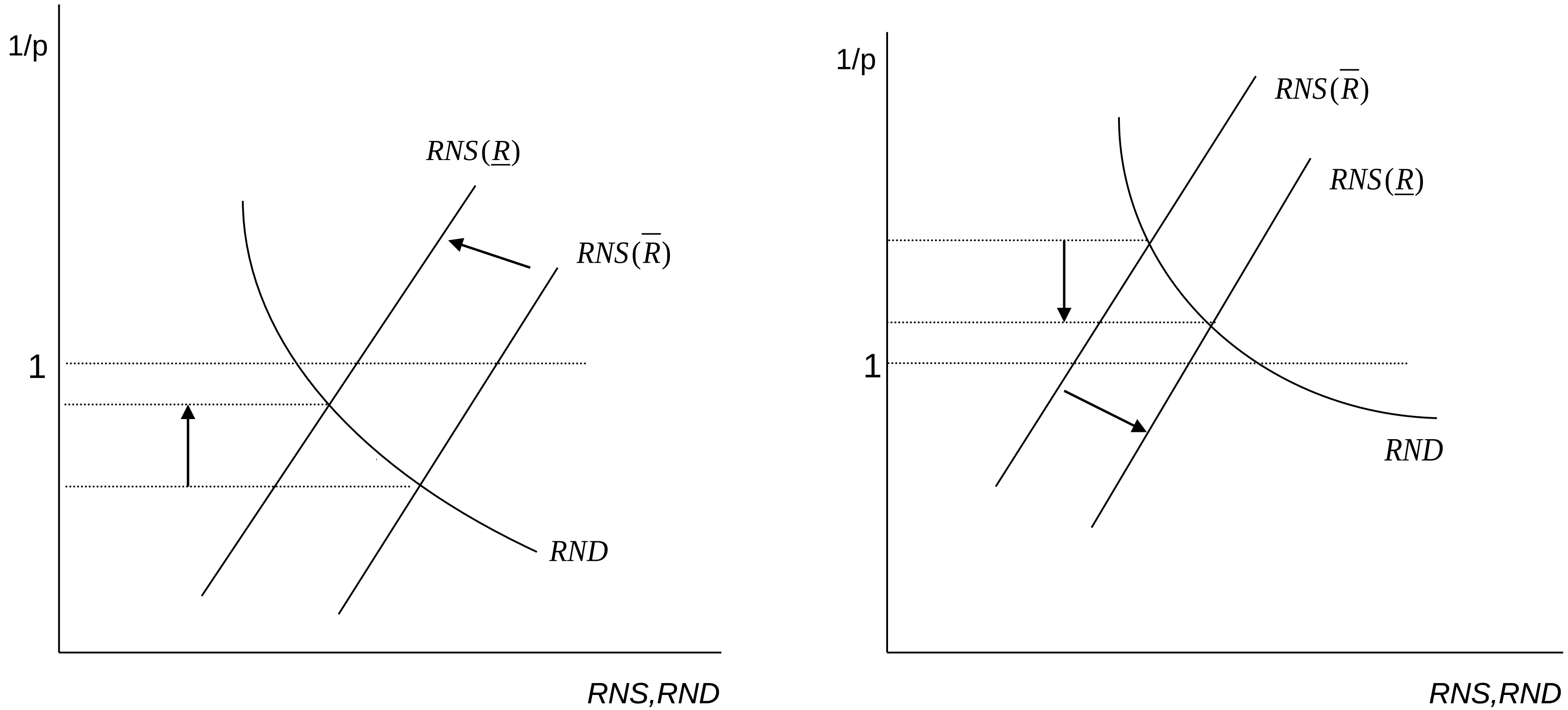
Determination of Labor supply

The labor supply of the home country is implied by the following set of equalities

$$
\begin{aligned}
\frac{\epsilon\left(N_t\right)^{\chi}}{\theta^{\frac{1}{\sigma}}}\left(\frac{\widetilde{c}_{h,t}(j)}{C_t}\right)^{\frac{1}{\sigma}} &= \frac{\epsilon\left(N_t\right)^{\chi}}{\theta^{\frac{1}{\sigma}}\left[\theta^{\frac{1}{\sigma}}+(1-\theta)^{\frac{1}{\sigma}}\left(\frac{\widetilde{c}_{f,t}(i)}{\widetilde{c}_{h,t}(j)}\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{1}{\sigma-1}}} \\
&= \frac{\epsilon\left(N_t\right)^{\chi}}{\left[\theta^{\frac{1}{\sigma}+\frac{\sigma-1}{\sigma}}+(1-\theta)^{\frac{1}{\sigma}+\frac{\sigma-1}{\sigma}}\left(\frac{\theta}{1-\theta}\right)^{\frac{\sigma-1}{\sigma}}\left(\frac{\widetilde{c}_{f,t}(i)}{\widetilde{c}_{h,t}(j)}\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{1}{\sigma-1}}} \\
&= \frac{\epsilon\left(N_t\right)^{\chi}}{\left[\theta+(1-\theta)\left(\frac{\theta}{1-\theta}\frac{\widetilde{c}_{f,t}(i)}{\widetilde{c}_{h,t}(j)}\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{1}{\sigma-1}}} \\
&= \frac{\epsilon\left(N_t\right)^{\chi}}{\left[\theta+(1-\theta)\left(\frac{1}{\mathfrak{p}_t}\right)^{\sigma-1}\right]^{\frac{1}{\sigma-1}}} \\
&= \frac{W_t}{R_t P_{h,t}}.
\end{aligned}
$$

Similarly for the foreign country.

Figure 1



$\overline{c}_h < \overline{c}_f$

$\overline{R} > \underline{R}$

$\overline{c}_h > \overline{c}_f$

$\overline{R} > \underline{R}$

Figure 2: Impulse Responses of Aggregate Variables
Deviations from the Steady-State (percentage points)

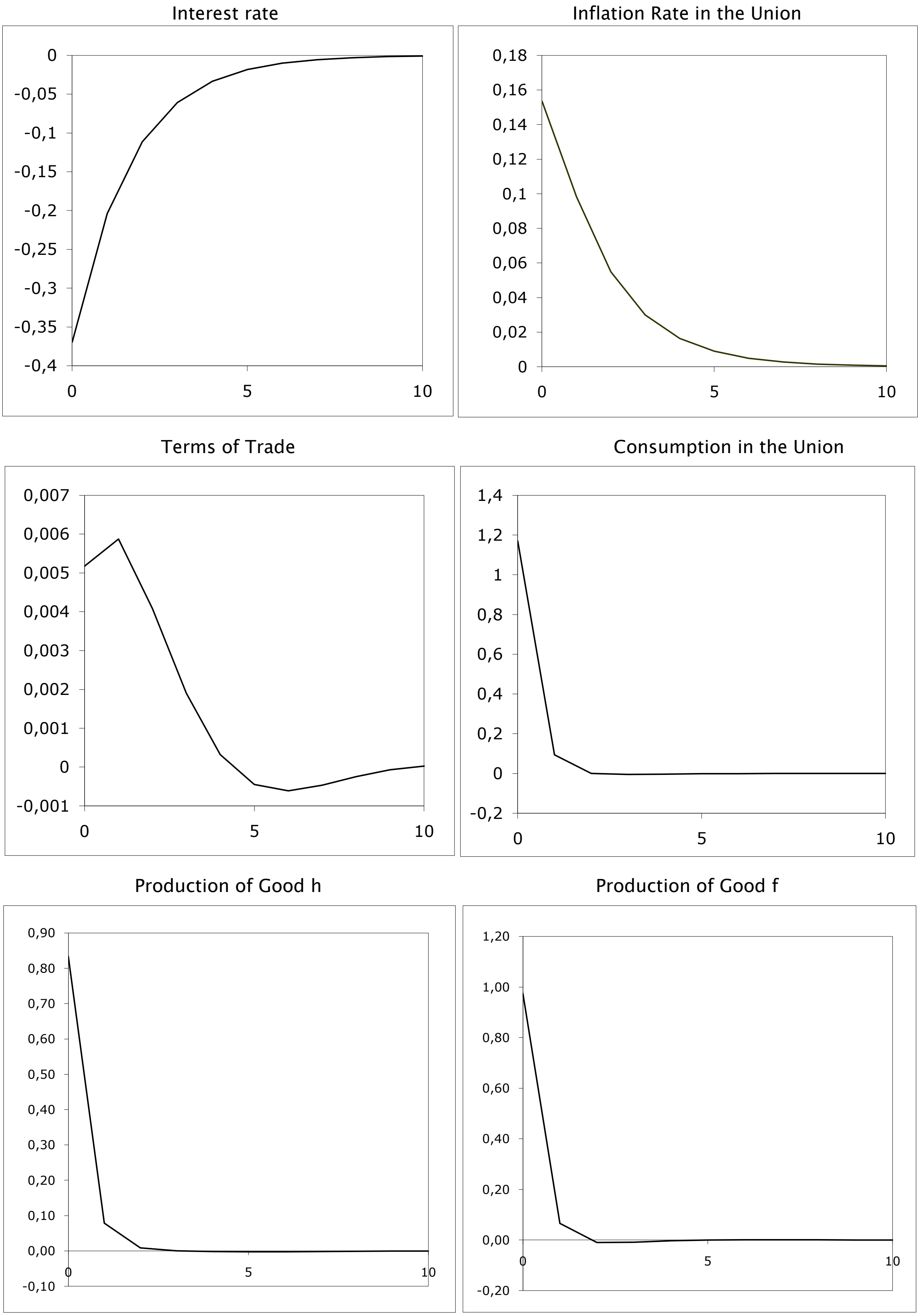


Figure 3: Impulse Responses of Country Variables
Variables in Levels

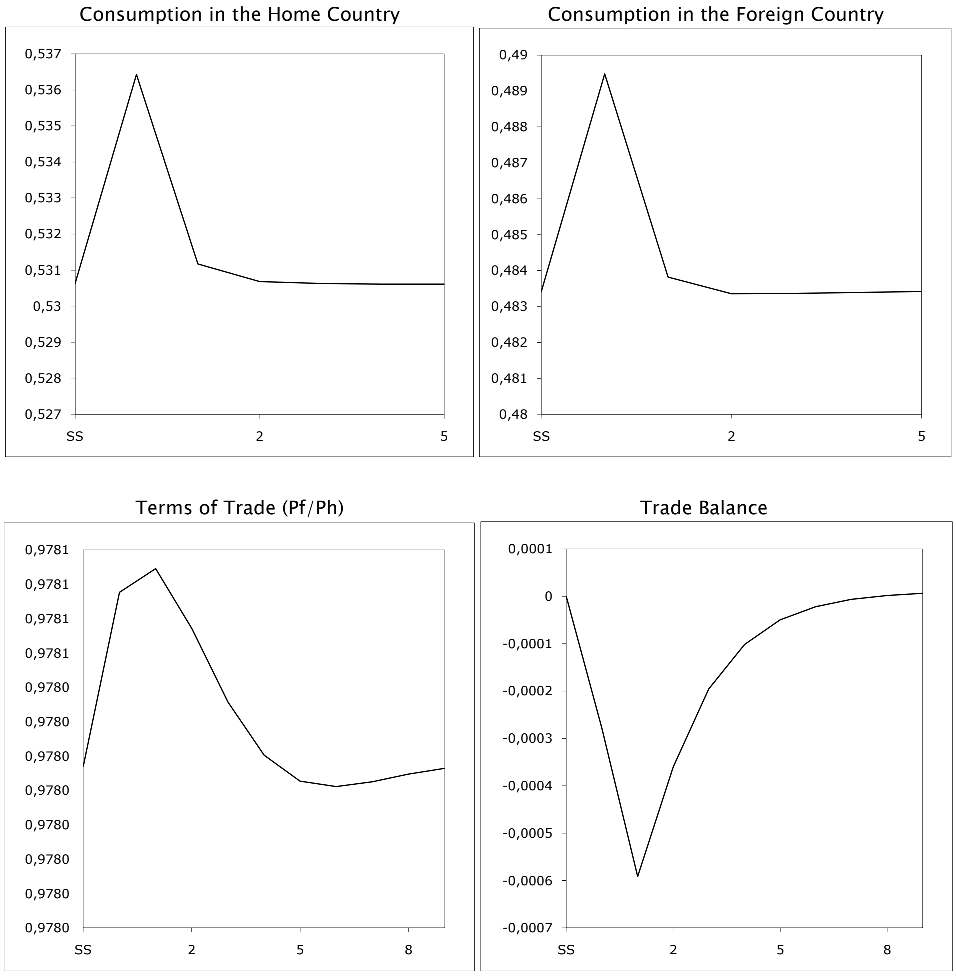
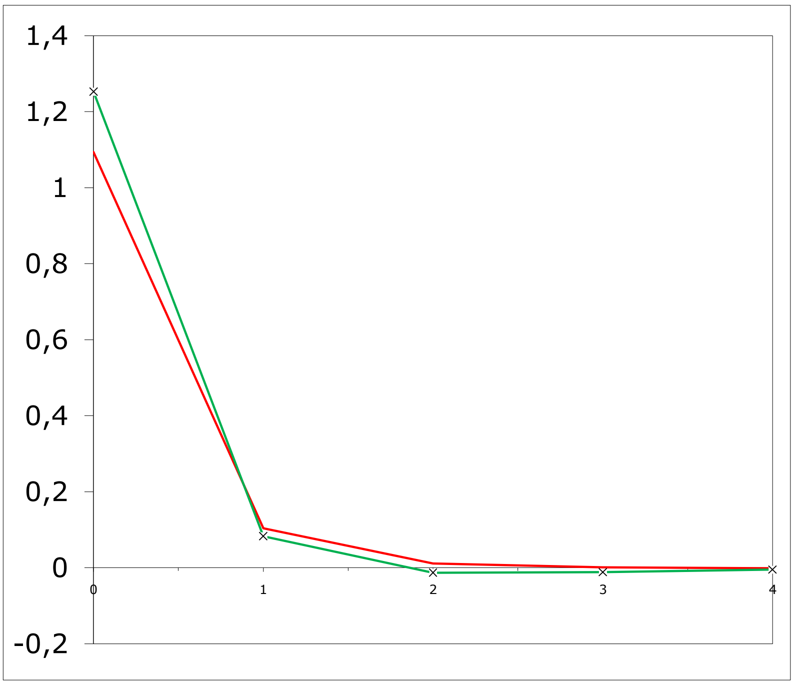


Figure 4: Impulse Responses of Consumption
Percentage Deviations from the Steady-State



Bold lines: Home Country (Red)
Lines with crosses: Foreign Country (Green)

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