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# THE CROSS SECTIONAL DYNAMICS OF HETEROGENOUS TRADE MODELS

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# The Cross Sectional Dynamics of Heterogenous Trade Models*

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**Abstract**

In this paper we propose a framework for studying export dynamics and market specific flows in a multicountry model of trade with heterogenous firms. Countries are asymmetric in terms of their size, the size distribution of potential entrants, properties of firms idiosyncratic shocks, and trade barriers. The model has predictions in terms of cross-sectional moments and exporters dynamics. We show that persistent productivity shocks are enough to account for, qualitatively, many features of the data. In particular, the model is consistent with observed patterns of entry and exit across markets, export sales distribution, and the life cycle of new exporters.

*JEL Classification*: F10

*Keywords*: Trade, Firm Dynamics, Asymmetric Countries, Cross-Market Entry and Exit

*The analysis, opinions and findings represent the views of the authors, they are not necessarily those of Banco de Portugal. Contact information: Alfonso Irarrazabal, alfonso.irarrazabal@econ.uio.no; Luca David Opromolla, ldopromolla@bportugal.pt.

# 1 Introduction

Research in international trade is increasingly focused on the role of firm heterogeneity in shaping trade flows. A recent strand of the literature has shown empirically that exporters are different from nonexporters in many respects: they are larger, more productive, more skill- and capital-intensive, and pay higher wages (e.g. Bernard et al., 2008b). As a consequence, new models of trade have been developed to explain why some producers export while other do not and to analyze the role of market entry cost in shaping this differential behavior (e.g. Melitz, 2003, Chaney, 2008, Bernard et al., 2003).

An even more recent strand of the literature, triggered by the availability of more detailed datasets, has started to provide evidence on patterns of entry and exit across destinations and products (e.g. Eaton et al., 2007, Bernard et al., 2008a, Amador and Opromolla, 2008, Iacovone and Jarvocik, 2008). Eaton et al. (2008) analyze a cross section of French manufacturing firms selling to 113 destinations. They document, among other facts, that exporters that access less popular markets are larger also on the domestic market and that sales distributions are similar across markets of different size. Eaton et al. (2008) augment the standard Chaney-Melitz model with firm and market specific sales and fixed cost shocks. More importantly, they conclude that nearly half the variation across firms both in market entry and sales can be attributed to a single dimension of underlying firm heterogeneity, namely efficiency.

In a recent paper Eaton et al. (2007), using transaction-level customs data from Colombia, characterize the export dynamics and market specific trade flows for manufacturing firms. In particular, they find that new exporters are very small and begin exporting in a single foreign market, and most of them stop exporting after just one year. Those that survive expand their foreign sales rapidly, gradually expand into additional destinations, and within a decade are responsible for half of total export expansion.

In this paper we introduce persistent productivity into the Chaney-Melitz framework to account for some of the stylized facts outlined in Eaton et al. (2007). We solve for the equilibrium in a model of trade with heterogeneous firms that allows for multiple asymmetric countries. Countries differ in the size of the market, the size distribution of potential entrants, and trade barriers. Firm dynamics are driven by idiosyncratic persistent shocks that can be interpreted as productivity shocks, quality shocks, or preference shocks. The model makes predictions in terms of cross-sectional moments and firm life cycle characteristics that are not possible in static trade models. Nonetheless, the model is parsimonious and can be solved analytically, and is therefore useful for applied work.

To illustrate the properties of the model we analyze a simple scenario where countries are identical except for their variable trade costs. We show that heterogeneity in efficiency together with persistent productivity shocks are enough to explain many features of the data at the qualitative level. Our model is closely related to Luttmer (2007). We extend his industry dynamics framework to allow firms to compete in international markets and we retain the prediction that the firm size distribution is Pareto in the upper tail. In addition, our model implies, consistent with the evidence, that the distribution of sales into a foreign destination is Pareto in the upper tail and possibly increasing in the lower tail. The stationary equilibrium of the economy can also be described in terms of the age distribution of firms and in terms of the set of destinations supplied.

We find that idiosyncratic productivity shocks are able to explain several features of the Eaton et al. facts. First, in our model exporters are not only larger than nonexporters but are also on average older. This implication is consistent with the industrial organization literature which shows that younger firms are usually smaller than older incumbents. Moreover, we provide new predictions that can be tested in the data: Eaton et al. (2008) show that French exporters that are able to penetrate the least popular destinations sell on average more on the domestic market. While we retain this prediction, we also show that these firms are on average older than the ones that sell only in more popular destinations. Second, the model implies that the frequency of firms selling to multiple foreign markets is declining in the number of markets and when firms add or subtract markets, they are likely to do so gradually. Both these features have been documented empirically (e.g. Eaton et al., 2008 and Amador and Opromolla, 2008). Finally, we caracterize the life cycle of a cohort of new exporters. Exporters start out small both in terms of value and number of destinations supplied. Many of them stop exporting soon since they receive a negative shock that pulls them below the export cutoff. The survival rate of exporters jumps after this first period and then gradually increases as exporters grow in sales and in the number of destinations served. Our model is therefore consistent with the behavior of entering exporters. This is important since entrants are considered crucial in the understanding of the long-run trend in total export.

Other theoretical models with heterogeneous firms and firm dynamics include the model of Arkolakis (2008b) and Alessandria and Choi (2007). Arkolakis (2008) also introduces trade dynamics into a multi-country trade model with heterogeneous firms where productivity is subject to a Brownian motion. The author interprets firms as ideas that materialize into production when it is profitable to do so. In his model there is an unbounded stationary productivity distribution of ideas and a bounded (from below) stationary productivity distribution of firms. Depending on the parameterization, firms face constant or increasing cost of reaching new consumers. In the case where the marginal cost of an additional consumer is constant our model becomes similar to his with the difference that in our case the domestic productivity cutoff is an absorbing barrier. In the case where the marginal cost of an additional consumer is increasing his model implies that the growth rate as well as the variance of the growth rate of sales is higher for small firms. In both cases, the author is able to provide an exact dynamic solution of the model along the balanced growth path. Alessandria and Choi develop a two-country model in order to study if firms' export participation decisions alter the co-movement of net exports with the real exchange rate.

The remainder of this paper is organized as follows. In section 2 we develop our model of trade with heterogeneous firms and idiosyncratic productivity shocks. In section 3 we discuss the equilibrium, focusing on the stationary productivity and age distributions, the cross-market dynamic properties, entry and exit into export markets, and cohort analysis. Finally, section 4 concludes.

# 2 Model

In this section we develop a dynamic multi-country model of trade with heterogeneous firms. Firms are potentially heterogeneous from birth, then they hit by persistent idiosyncratic productivity shocks that drive their life cycle. Countries can be asymmetric in size, size distribution of potential entrants, idiosyncratic shocks and trade barriers.

## 2.1 Set-up

Time is continuous, starts at $t = 0$ and goes on forever. There are $N$ potentially asymmetric countries that produce goods using only labor. Country $i$ is populated by $L_i$ infinitely-lived agents that maximize utility derived from the consumption of goods belonging to two sectors. One sector provides a homogeneous good and the other a continuum of differentiated goods.

*Demand.* We assume that an exogenous fraction $\mu$ of income is spent on differentiated products and the remaining fraction $1 - \mu$ on the homogeneous good. Preferences across varieties of the differentiated product have the standard CES form with an elasticity of substitution $\sigma > 1$. Each variety enters symmetrically in the utility functions so that differences in demand across varieties depend only on differences in their prices $p$. These preferences generate a demand function $A_i p_i^{1-\sigma}$ in country $i$ for every brand of the product. The demand level $A_i = \mu R_i P_i^{\sigma-1}$ is exogenous from the point of view of the individual supplier and depends on total expenditure $R_i$ and the consumption-based price index $P_i$.

Each consumer is endowed, at every point in time, with one unit of labor which is supplied inelastically to firms. The homogeneous good is produced, in every country, with one unit of labor per unit output. The common wage rate $w$ is normalized to one and used as *numéraire*. The representative consumer faces a standard present-value budget constraint. Her wealth consists of labor income plus dividends. Each worker owns a single share of a perfectly diversified national portfolio of all the firms and profits earned by firms are distributed as dividends in terms of the *numéraire*.

*Trade barriers and technology.* There are two types of trade barriers: a variable cost $\tau_{ji}$ and a per period cost $\lambda_{ji}^X$. The variable cost takes the form of an iceberg cost: $\tau_{ji} > 1$ units of the good must be shipped from country $i$ in order for one unit of the good to arrive in the country $j$. The cost $\lambda_{ji}^X$, expressed in units of the *numéraire,* has to be paid every period by a country $i$ firm exporting to country $j$.

All countries have access to the same technology. Goods are produced using only labor and firms must sustain a domestic fixed cost of production $\lambda_i^D$. At every moment in time there is a measure of potential entrants proportional to $L_i$, so that larger and wealthier countries have more entrants. This assumption, as in Arkolakis (2008), Chaney (2008) and Irarrazabal et al. (2008), greatly simplifies the analysis and it is similar to Eaton and Kortum (2002) where the set of goods is exogenously given. Arkolakis (2008) was the first paper, to the best of our knowledge, to provide an exact dynamic version of the (asymmetric) multi-country Chaney model. Each potential entrant draws an initial random unit log labor productivity $\bar{z}$ from a distribution

$g(z)$.[1] The cost of producing and selling $c$ units of the good in the domestic market for a firm with productivity $z$ is $ce^{-z} + \lambda_i^D$ while the cost of producing $c$ units of the good in country $i$ and selling them in country $j$ is $\tau_{ji}ce^{-z} + \lambda_{ji}^X$. Firms are price setters. The optimal price in the foreign market, defined as a markup on variable cost of production is $\tau_{ji}e^{-z}\sigma/(\sigma-1)$ where $\sigma/(\sigma-1)$ is the Dixit-Stiglitz markup.

*Productivity shocks and value of the firm.* Following Luttmer (2007), we assume that firm labor productivity evolves identically and independently according to a Brownian motion with drift $\alpha_i$,

$$dz = \alpha_i da + \xi_i dB \tag{1}$$

where $dB$ is the increment of a Wiener process and $\xi_i > 0$ is the diffusion parameter. These permanent idiosyncratic shocks can be interpreted as shocks to technology (producing the same variety at a lower cost), shocks to quality (producing a better variety at the same cost) or as taste shocks to the demand for the firm differentiated good. The value of the firm is a function of current labor productivity $z$. For expositional simplicity, we divide the value of a firm into a domestic and a foreign component. Like in Melitz (2003) we assume that the least productive firms do not export. Moreover, since the presence of a fixed cost implies a minimum size, firms with low productivity choose to exit since they face only a small probability of ever recovering the fixed cost $\lambda_i^D$ required to continue operation. For simplicity, we assume that the exit decision depends only on the domestic value of the firm.[2] The domestic component of the value of a firm is described by the Bellman equation

$$V_i^D(z) = \max\left\{0, \pi_i^D(z) + e^{-(\rho+\delta)dt}E\left[V_i^D(z+dz)\right]\right\}$$

[1] In the remainder of the paper, for simplicity, we refer to $z$, and not to $\exp(z)$, as the firm labor productivity.

[2] Even if the firm is not exporting, the total value of a firm includes also the value of the option to start exporting in the future. This value is small when the lower barrier $z_{ii}$ is distant from the lowest export cutoff and converges to zero, asymptotically, when firm productivity converges to $-\infty$.

where $\delta$ is an exogenous, per unit of time, probability of exiting and $\pi_i^D(z)$ are profits from sales on the domestic market. Domestic profits are a function of the firm labor productivity $z$ and of the endogenously determined price index

$$\pi_i^D(z) = R_i \left(P_i e^z\right)^{\sigma-1} (\sigma-1)^{\sigma-1}\sigma^{-\sigma} - \lambda_i^D.$$

The termination payoff is set to zero. There exists a single cutoff $z_{ii}$ such that for $z > z_{ii}$ the firm decides to stay in operation and for $z < z_{ii}$ it decides to shut down.[3] Notice that in this model there are two reasons why firms can exit: because of a negative productivity shock or because of the exogenous killing rate $\delta$. Bigger firms are less likely to exit because of a negative shock and more likely to exit because of the killing rate $\delta$. In order to solve the Bellman equation we need to apply Ito's Lemma and solve a second-order differential equation subject to a value-matching and a smooth-pasting condition. The domestic value function, valid on the range $(z_{ii}, \infty)$, turns out to be

$$V_i^D(z) = \kappa_{2i} e^{\beta_{2i} z} + \frac{\pi_{ii}(z)/\sigma}{(r+\delta) - \alpha_i(\sigma-1) + \xi_i^2(\sigma-1)^2/2} - \frac{\lambda_i^D}{(r+\delta)},$$

which represents the sum of the value of the option to exit (first term) and the expected discounted value of domestic profits (second and third terms).[4] Variable domestic profits (in the numerator of the second term) evolve as a geometric Brownian motion with drift $\alpha_i(\sigma-1) + \xi_i^2(\sigma-1)^2/2$ and are discounted at the rate $r+\delta$. The present value of revenues, and therefore of variable profits, is finite if the combined discount factor, given by the sum of the interest rate and the exogenous probability of exit $\delta$, is larger than the drift of variable profits. The following assumption guarantees

[3] For this to be true we need: $\pi(z) + e^{-(\rho+\delta)dt} E\left[V(z+dz)\right]$ to be increasing in $z$ and first-order stochastic dominance (which is satisfied by the Brownian motion).

[4] Formal derivations of the value function and cutoffs, as well as definitions of the coefficients $\beta_2$ and $\kappa_{2i}$ are relegated to the Appendix.

that this is the case and therefore guarantees that the value of a firm is finite:

$$r + \delta > \alpha_i(\sigma - 1) + \xi_i^2(\sigma - 1)^2/2.$$

Since $V_i^D(z)$ is increasing in $z$, firms with higher productivity, for a given lower barrier $z_{ii}$, are more valuable. The minimum size $z_{ii}$,

$$e^{z_{ii}} = \frac{\sigma}{\sigma - 1}\left(\frac{\sigma\lambda_i^D}{R_i}\right)^{\frac{1}{\sigma-1}}\frac{\gamma_{di}}{P_i} \tag{2}$$

is endogenously determined, being a function of the price index $P_i$ and absorption $R_i$. The parameter $\gamma_{di}$ is a function of parameters governing the stochastic process for productivity shocks and preferences.[5] Economies with a lower price index $P_i$ and lower total expenditure $R_i$ are economies where firms have to be more productive in order to survive.

All non-exporters have a chance of becoming exporters (the likelihood of this event being increasing in the distance from the lower barrier $z_{ii}$) and some firms do actually export. A firm starts to export to destination $j$ when it becomes profitable to do so, that is, when firm productivity surpasses the cutoff associated with zero profit from exporting to destination $j$,[6]

$$\pi_{ji}(z) = 0 \Leftrightarrow e^{z_{ji}} = \frac{\sigma}{\sigma - 1}\frac{\tau_{ji}}{P_j}\left(\frac{\sigma\lambda_{ji}^X}{R_j}\right)^{\frac{1}{\sigma-1}}. \tag{3}$$

[5] Specifically $\gamma_d$ is: $\left[\frac{\beta_{2i}}{r+\delta}\left(\frac{r+\delta-\alpha_i(\sigma-1)-1/2\xi_i^2(\sigma-1)^2}{\beta_{2i}-\sigma+1}\right)\right]^{1/(\sigma-1)}$.

[6] In a previous paper (Irarrazabal and Opromolla, 2008) we analyze a two country symmetric equilibrium where new or returning exporters have to pay a sunk cost of entry into the foreign market. The presence of a sunk cost creates a wedge between the productivity levels at which firms decide to start exporting and the one at which firms decide to stop exporting. When a firm is between the two cutoffs, the optimal policy is to continue with the status quo, whether it being exporting or non-exporting. As the value of the sunk cost goes to zero the two cutoffs collapse to $z_{ji}$.

## 2.2 Stationary Distributions

We now characterize the stationary probability density over productivity and age. The crucial component is the transition density generated by the Brownian motion (1) subject to the productivity cutoff $z_{ii}$.

*Stationary probability densities.* In steady state there is a time-invariant cross-sectional distribution of firms current productivity $z$ and age $a$,

$$f_i(a,z) = \int_{z_{ii}}^{\infty} e^{-\delta a} h_i(a,z,\bar{z}) g_i(\bar{z}) \frac{L_i}{M_i} d\bar{z}$$

Firms attempt to enter at a constant rate $L_i/M_i$ where $M_i$ is the equilibrium measure of active firms. After strting out with their initial productivity level $\bar{z}$, they receive a sequence of permanent shocks. The transition density $h_i(a,z,\bar{z})$ describes the likelihood that a surviving firm entering at $\bar{z}$ has productivity $z$ at age $a$. This takes into account the possibility that the firm hits the lower barrier $z_{ii}$ and exits forever but not the exogenous killing rate $\delta$ which is controlled by the term $e^{-\delta a}$. In steady state there is a constant ratio between the measure of exporters $M_{Xi}$ and non-exporters $M_{Di}$.[7]

*Transition densities and the entry rate.* The transition density is the core of the stationary probability density $f_i(a,z)$. The transition density $h_i(a,z,\bar{z})$ is the solution of the following Kolmogorov forward equation,

$$\begin{cases} \frac{\partial h_i(a,z,\bar{z})}{\partial a} = -\alpha_i \frac{\partial h_i(a,z,\bar{z})}{\partial z} + \frac{1}{2}\xi_i^2 \frac{\partial^2 h_i(a,z,\bar{z})}{\partial z^2} & z > z_{ii} \\ h_i(a,z,\bar{z}) = 0 \text{ for } z \leq z_{ii}, a \geq 0, \bar{z} \leq z_{ii} \; ; \; h_i(0,z,\bar{z}) = \delta(z - \bar{z}_i) & \end{cases}$$

where $\delta(.)$ is the Dirac-Delta function. Notice that the density $h_i(a,z,\bar{z})$ is a function of the current productivity level $z$ and current age of the firm $a$. We are assuming that this density is the same for different cohorts. Let $J_i(a,z,\bar{z})$ be the net rate of

[7] Recall that $M = M_{Xi} + M_{Di}$.

passage or flux (as $a$ increases infinitesimally) at $z$ (in the $z$ direction) when age is equal to $a$ so that $\partial h_i(a,z,\bar{z})/\partial a = -\partial J_i(a,z,\bar{z})/\partial z$. When the derivative of the flux is positive $h_i(a,z,\bar{z})$ is decreasing in $a$ because, as $a$ increases infinitesimally, the probability of leaving $z$ is higher than the probability of reaching it.[8] The transition density $h_i(a,z,\bar{z})$ turns out to be

$$h_i(a,z,\bar{z}) = \frac{1}{\xi_i\sqrt{a}}\left[\phi\left(\frac{z-\bar{z}_i-\alpha_i a}{\xi_i\sqrt{a}}\right) - e^{-\frac{\alpha_i(\bar{z}_i - z_{ii})}{\xi_i^2/2}}\phi\left(\frac{z+\bar{z}_i-2z_{ii}-\alpha_i a}{\xi_i\sqrt{a}}\right)\right]$$

where $\phi(.)$ is the standard normal density.[9]

The potential entry rate $L_i/M$,

$$\frac{L_i}{M} = \left\{\int_{z_D}^{\infty}\left[\frac{1-e^{-\theta_*(\bar{z}-z_D)}}{\delta}\right]g(\bar{z})d\bar{z}\right\}^{-1} \tag{4}$$

is consistent with $f_i(a,z)$ being a probability density. The higher the killing rate $\delta$ and the lower the initial productivity $\bar{z}$, the higher is the exit rate and also the entry rate.

*Stationary productivity densities and volatility.* The stationary probability density $f_i(a,z)$ describes the equilibrium mass of firms in terms of current productivity and age. If heterogeneity among entrants is small relative to heterogeneity in the overall population, then the density $f_i(a,z)$ looks much like the one obtained by conditioning on a typical $\bar{z}_i > z_{ii}$. In this case, the marginal productivity and age densities, $f_i(z|\bar{z}_i)$ and $f_i(a|\bar{z}_i)$, have a simple representation. Like in Luttmer (2007), $f_i(z|\bar{z}_i)$ is

[8] See Irrarazabal and Opromolla (2008) for a discussion of the flux terms and the intuition behind them.

[9] See Luttmer (2007) for a derivation.

increasing for $z < \bar{z}_i$ and of the Pareto form for $z > \bar{z}_i$,

$$f_i(z|\bar{z}_i) = \begin{cases} \nu_1 \left[e^{\theta_{i*}(z-z_{ii})} - e^{-\theta_i(z-z_{ii})}\right] & \text{for } z < \bar{z}_i \\ \nu_2 \; e^{-\theta_i(z-z_{ii})} & \text{for } z > \bar{z}_i \end{cases}$$

with $\nu_1 = (\theta_i\theta_{i*}) \,/\, \left[(\theta_i + \theta_{i*}) \left(e^{\theta_{i*}(\bar{z}_i - z_{ii})} - 1\right)\right]$ and $\nu_2 = \nu_1 \left[e^{(\theta_i+\theta_{i*})(\bar{z}_i - z_{ii})} - 1\right]$. In order to have a stationary distribution with a finite mean we need to impose the following assumption:

$$\delta > \alpha + \xi^2/2.$$

The coefficient of the Pareto distribution,

$$\theta_i = \frac{1}{\xi_i^2}\left(-\alpha_i + \sqrt{\alpha_i^2 + 2\xi_i^2\delta}\right) > 0$$

shows that the density has a thicker right-tail the higher the volatility coefficient $\xi_i$ of the Brownian motion is. Ceteris paribus, economies with more volatile firm shocks are characterized by a more heterogeneous firm size distribution. The productivity density of exporters to destination $j$, $f_{ji}(z|\bar{z}_i)$, can be easily found by conditioning $f_i(z|\bar{z}_i)$ on being higher than the corresponding export cutoff $z_{ji}$. The exporters' productivity density $f_{ji}(z|\bar{z}_i)$ inherits the shape of the overall productivity density $f_i(z|\bar{z}_i)$. In the case where entrants start selling only on the domestic market before becoming exporters, $\bar{z}_i > z_{ji}$, the productivity density of exporters to country $j$ is

$$f_{ji}(z|\bar{z}_i) = \theta_i e^{-\theta_i(z-z_{ji})}, \quad z > z_{ji} \; .$$

Note that $f_{ji}(z|\bar{z}_i)$ reaches a peak at $z_{ji}$, the productivity cutoff at which firms start exporting to destination $j$. The productivity distribution for exporters $f_{ji}(z|\bar{z}_i)$ is consistent with evidence that the distribution of sales of exporters in the destination market is Pareto in the upper tail (e.g. Eaton et al., 2007, Irarrazabal et al., 2008 and Amador and Opromolla, 2008).

Like in Luttmer (2007), the age distribution $f_i(a|\bar{z}_i)$ is

$$f_i(a|\bar{z}_i) = \left[\frac{1 - e^{-\theta_{i*}(\bar{z}_i - z_{ii})}}{\delta}\right]^{-1} e^{-\delta a}\Lambda_i(a|\bar{z}_i) \tag{5}$$

where

$$\Lambda_i(a|\bar{z}_i) = \Phi\left(\frac{\bar{z}_i - z_{ii} + \alpha_i a}{\xi_i\sqrt{a}}\right) - e^{-\frac{\alpha_i(\bar{z}_i - z_{ii})}{\xi_i^2/2}}\Phi\left(\frac{\alpha_i a - \bar{z}_i + z_{ii}}{\xi_i\sqrt{a}}\right).$$

is the survivor function of a cohort of firms with the same initial productivity $\bar{z}_i$. The age distribution of exporters $f_i^X(a|\bar{z}_i)$ is

$$f_i^X(a|\bar{z}_i) = \left[\nu_2\frac{e^{-\theta_i(z_{ji} - z_{ii})}}{\theta_i}\right]^{-1} f_i(a|\bar{z}_i).$$

The difference between the unconditional age distribution for all firms $f_i(a)$ and the one for exporters $f_i^X(a)$ depends then on differences in the set of initial productivities $\bar{z}_i$ between the two groups. If older firms are, on average, characterized by a higher initial productivity and higher initial productivity firms self-select into export markets, then exporters are on average older than nonexporters.

### 2.3 Trade equilibrium

We close the model by imposing labor market clearing and we then derive the price index. Labor market clearing, together with the entry rate, determines the equilibrium measure of active firms and the exit cutoffs in all countries. Labor supply is fixed. Labor is demanded for sustaining the fixed costs ($L_{fi}$) and sustaining the variable production costs $(L_{pi})$ .

$$L_{Fi} = M_i \left[ \lambda_i^D + \lambda_{ji}^X \int_{z_{ji}}^{\infty} f_i(z)dz \right]$$

$$L_{pi} = M_i \left[ \begin{array}{c} (\sigma - 1)\,\lambda_i^D \gamma_{ii}^{(\sigma-1)} e^{-(\sigma-1)z_{ii}} \int_{z_{ii}}^{\infty} e^{(\sigma-1)z} f_i(z)dz \\ +(\sigma - 1)\,\lambda_j^D \gamma_{jj}^{(\sigma-1)} e^{-(\sigma-1)z_{jj}} \tau_{ji}^{-\sigma} \int_{z_{ji}}^{\infty} e^{(\sigma-1)z} f_i(z)dz \end{array} \right]$$

A firm that exports has to pay both the domestic fixed cost $\lambda_i^D$ and the export fixed cost $\lambda_{ji}^X$ for each destination $j$ served. Labor is then used to produce goods sold on the domestic market (first term of $L_{pi}$) and goods that are shipped abroad (second term of $L_{pi}$). Note that the exit cutoffs enter $L_{Pi}$ since the amount of labor demanded for production depends on firm's sales and these, in turn, depend on the relative productivity of the firm with respect to all the other firms serving the market (as proxied by the distance of $z$ from the lower cutoffs $z_{ii}$ and $z_{jj}$). The set of labor market clearing conditions $\{L_i = L_{fi} + L_{pi}\}_{i=1,...,N}$ determines the set of exit cutoffs $\{z_{ii}\}_{i=1,...,N}$ and, using the entry rate (4), the equilibrium number of firms in any country $\{M_i\}_{i=1,...,N}$. Using (2) and (3) we can then determine all the export cutoff of any country to any other country,

$$\begin{bmatrix} z_{11} & ... & z_{1N} \\ \vdots & z_{ij} & \vdots \\ z_{N1} & ... & z_{NN} \end{bmatrix}. \tag{6}$$

Since the cutoffs are not firm-specific (6) implies that destinations obey a strict hierarchy in the sense that any firm selling to the $k+1$st most popular destination necessarily sold to the $k$th most popular as well. Eaton et al. (2008) show evidence that calls for a model that recognize both a tendency for firms to export according to a hierarchy while allowing them significant latitude to depart from it. In order to account for departure from a hierarchy they include firm- and destination-specific entry and sales shocks into a static heterogeneous trade model.[10] Since our focus is to explore the effects of persistency in productivity on trade dynamics we abstract from introducing another dimension of heterogeneity in our model.

Using the information in (6) we can determine the equilibrium price index in every country as follows

$$P_i^{1-\sigma} = M_i \int_{z_{ii}}^{\infty} \left[ \frac{\sigma}{(\sigma-1)e^z} \right]^{1-\sigma} f_i(z)dz + \sum_{j \neq i} M_j \int_{z_{ij}}^{\infty} \left[ \frac{\tau_{ij}\sigma}{(\sigma-1)e^z} \right]^{1-\sigma} f_j(z)dz.$$

The price index is the minimum expenditure required to purchase one unit of the composite good. As such, it depends on the measure of available varieties in the economy ($M_i$ domestic plus $\sum_{j \neq i} M_j$ imported) and on their average price.

The level of total expenditure $R_i$ is found replacing $P_i$ in the expression for the lower threshold $z_{ii}$. The equilibrium level of the composite good $C_i$ and the equilibrium level of profits $\Pi_i$ are then determined as $C_i = R_i/P_i$ and $\Pi_i = R_i/(1-\mu) - L_i$.

# 3 Analysis of the Equilibrium

In this section, we explore the properties of the equilibrium with asymmetric countries. In order to facilitate the interpretation of the results we keep country heterogeneity at a minimum and consider a simple scenario with five countries that are identical except for the tariff rate. Tariffs are destination-specific and countries are sorted from the least protected (country 1) to the most protected (country 5).[11] We choose country 1 as the reference country.[12] Table 1 reports the main variables as a fraction of country 1's and the exit and export cutoffs in all countries as a fraction of the exit cutoff

[10] Irarrazabal et al. (2008) shows similar evidence for firms' decisions to engage in foreign direct investment. Their model also include export and FDI firm- and destination-specific shocks.

[11] Specifically, we choose the following parameters: $L = 100$, $\lambda_i^D = \lambda_{ji}^X = 10\ \forall i, j$, $\delta = .043$, $r = .05$, $\alpha = .00274$, $\xi = .1$, $\sigma = 2.45$, $g(\bar{z})\tilde{\ }N(0, 1.77)$, $\mu = 0$. Tariffs are set as $\tau_{ii} = 1\ \forall i$ and $\tau_{ji} = 1 + j/10$ with $j = 1, ..., 5$ and $i \neq j$.

[12] Since we focus on one source country from now on we drop the subscript $i$ in order to simplify the notation.

in country 1. We describe the equilibrium in terms of the stationary distribution of productivity and age by export status and sales and age distribution by destination market. Then we focus on the extensive and intensive margins of export, analyzing the expansion path of a cohort of new exporters. Throughout the analysis we study the relationship between the equilibrium outcomes and some key parameter of the model. In particular, we focus on the connection between the cross country entry and exit patterns and the volatility and drift of the shocks.

## 3.1 Stationary distributions

*The distribution of productivity.* Figure 1 depicts the overall productivity distribution and the distributions for exporters and non-exporters. The distribution for exporters is drawn using the lowest exporting cutoff.

The overall distribution is consistent with the observed size distribution of firms (Luttmer, 2007). The size distribution is well approximated over much of its range by a Pareto distribution, but there are fewer small firms than would be the case if the size distribution was Pareto. The shape of the size distribution can be shown to always be Pareto on the right tail, independent of the distribution of entrants. The shape of the distribution is determined by the distribution of potential entrants and characteristics of the economy (in particular parameters that affect the location of the lower barrier). For example if the productivity distribution of potential entrants is degenerate at the lower barrier then the equilibrium distribution is Pareto over all its range.

The figure also shows that exporters are more productive than nonexporters but the productivity distributions do not overlap. Two possible modifications of this model can account for this feature of the data: either shocks to the fixed cost of entry into export markets (as in Eaton et al., 2008) or sunk costs of exporting (as in Irarrazabal and Opromolla, 2008). The thickness of the right tail of the distribution of exporters depends on technology parameters $\alpha$, $\xi$ and on the killing rate $\delta$. Economies with more heterogeneous firms are economies where firms' shocks are more volatile.

*Export sales distributions.* Figure 2 depicts the distributions of export sales in two destinations. In the graph we plot the logarithm of sales against the logarithm of the probability that sales are larger than some particular value. First, we observe that the right tail of both distributions is linear, which is the result of the Pareto shape of the productivity distribution. Eaton et al. (2008) reports that sales distributions across markets of very different size and extent of French participation behave similarly to a Pareto distribution in the upper tail and more like a lognormal distribution in the lower tail. More interestingly, the distribution of exporters has a non-Pareto region for small firms. Eaton et al. (2008) introduce shock to sales to account for this feature based on the work of Arkolakis (2008a). In our model this deviation from Pareto is explained mainly by the distribution of entrants and the effect of selection for small firms. Another implication of figure 2 is that firms selling in the least popular destinations are more likely to be larger as measured by sales.

*Age distribution.* Figure 3 shows the age distribution by export status. We compute the age distribution by integrating the density (5) over the distribution of entrants $g(\bar{z})$. Exporters are on average older than non-exporters. Given that the stochastic process is continuous it takes a longer period for a non-exporter firm with productivity $z$ to reach the lowest cutoff of exporting. The right panel shows the age distribution for exporters for the most and least popular markets. The figure shows that firms that are able to penetrate the least popular destinations are on average older.

## 3.2 Cross market dynamics

We now focus on firms' dynamic behavior and explore the cross-market patterns of entry and exit of our model. In table 2 we categorize steady state firms according to the number of destination markets they supply: none, one, two, three or four, and then document period-to-period transition frequencies between the categories. For expositional simplicity we relabel the cutoffs of country $i$ by sorting them from the smallest ($z_1^*$, that is, country 1 exit cutoff) to the biggest (export cutoff of the least accessible country) as $z_1^*$,...,$z_N^*$. Country 1 firms can access country $j$ market only if their productivity level $z$ is higher than $z_j^*$. In the bottom row of table 2 we report the fraction of firms in each cell in the steady state,

$$M_x^j = M_1 \int_{z_j^*}^{\infty} p_{u1}(z)dz \ , \ \ j = 1, ..., N.$$

The modal number of destination markets is one and, like in Eaton et al. (2007), the frequency of firms selling to multiple foreign markets is declining in the number of markets. While this is true for the set of parameters used, the model is flexible enough to accommodate other sources of heterogeneity. The transition probabilities depends on the relative distance between the cutoffs $\left\{ z_j^* \right\}_{j=1,...,N}$, as implied by the characteristics of the set of countries under analysis, and in particular between country 1 lower barrier and the export cutoff of the most accessible foreign partner.[13] Period-to-period transition frequencies between the categories of table 2 are computed as

$$T_{ab} = \int_{z_b^*}^{z_{b+1}^*} \left[ \Phi\left( \frac{z_{a+1}^* - z - \alpha dt}{\xi\sqrt{dt}} \right) - \Phi\left( \frac{z_a^* - z - \alpha dt}{\xi\sqrt{dt}} \right) \right] p_{ui}(z) \left( M_x^b \right)^{-1} dz,$$
$$a, b = 1, ..., N$$

where $T_{ab}$ is the probability of switching from supplying $b$ destinations to supplying $a$ destinations.[14] The first column of the table shows that, like in Eaton et al. (2007),

[13] The shape of the productivity distribution also affects the results. The ranking is however less likely to be affected if the distribution is monotone in the range $(z_2^*, z_N^*)$. This occurs if new firms are smaller than incumbents, which is usually the case.

[14] $\Phi(.)$ is the standard normal cumulative distribution function.

nonexporters (firms supplying one destination, country 1) almost always enter a single market when they initiate foreign sales. Moreover, when firms add or subtract markets, they are more likely to do so gradually than in large clumps. This feature stems from the fact that shocks to productivity are serially correlated and that the Brownian motion process is characterized by the normally distributed Wiener process. By increasing the diffusion parameter $\xi$ of the Brownian motion, the main diagonal of the transition matrix tends to become less important as it becomes more likely for firms to get bigger productivity shocks (in absolute value) and therefore change the number of destinations reached. A change in the drift parameter $\alpha$ of the Brownian motion instead can introduce some asymmetry around the main diagonal, with a positive drift making upward movements (increasing the number of destinations supplied) more likely than downward movements (decreasing the number of destinations supplied).

### 3.3 Entry and Exit into Export Markets

Patterns of entry and exit into export markets have been shown to affect the behavior of total export in the medium-long run (Eaton et al., 2007). We analyze these patterns in three steps. First, at the aggregate level, we consider the relationship between the volume of export of a country and the number of exporters across destination markets. Second, we compare the relative frequency and size of entering, exiting, continuing and single-period exporters. Third, we characterize the selection and growth processes of a cohort of new exporters.

*Extensive and intensive margins.* We decompose aggregate export into an extensive margin (number of exporters to destination $j$) and an intensive margin (average exporter sales in destination $j$). Eaton et al. (2007 and 2008) find a positive relationship between the volume of export and the number of exporters. The elasticity of the extensive margin is about half in their Colombian data and two-thirds in the

French data. Figure 4 shows that our choice of parameters also delivers a positive relationship with a slope of about 0.45.

*Single-year, exiting, entering and continuing exporters.* Recent evidence (Eaton et al., 2007, Amador and Opromolla, 2008 and Iacovone and Javorcik, 2008) shows that new exporters are usually small, in terms of export value, number of destinations supplied, and number of products exported. In the initial year of export firms are subject to a strict selection so that a large fraction of exporters exit from foreign markets almost right away, usually after one year. Surviving exporters grow quite rapidly and quickly reduce the size gap with incumbent exporters. Exiting exporters are usually smaller than continuing exporters.

We now show that our model is consistent with some of these facts while it is too parsimonious to replicate others. Following Eaton et al. (2007) we consider four categories of firms and compare their frequency and size in term of average export per firm. We define as entrants in period $t$ firms that: (i) did not export in $t-1$, (ii) exported in $t$; and (iii) must export in $t+1$ as well. Exiters in $t$ (i) export in $t-1$; (ii) export in $t$, and (iii) do not export in $t+1$. Continuers (i) export in $t-1$ and (ii) export in $t$ and (iii) export in $t+1$ as well. The remaining firms, those that exported in $t$ but not in $t-1$ or $t+1$ are "single period"exporters. Note that all firms are active (i.e. they sell their good on the domestic market) in the three periods.

Consider the productivity density $f^N(z)$ of new exporters. To compute it we start from the productivity density of nonexporters $f^D(v)$ and then, for each productivity level $v$, we apply the transition density $h(dt, z|v)$ which represents the productivity density after $dt$ time, conditional on survival for a firm with current productivity $v$.

$$f^N(z) = \int_{z_1^*}^{z_2^*} \frac{h(dt, z|v)}{\int_{z_2^*}^{\infty} h(dt, w|v)dw} f^D(v)dv \quad \text{for} \quad z_2^* < z < \infty,$$

The number of newly exporting firms, $M^N$ is found by multiplying the number of nonexporters $M_d$ with the fraction of them that start exporting after $dt$ time, that is, the fraction that receives a shock that is positive enough to make it profitable to export to the most accessible foreign market (i.e. pass the smallest export cutoff $z_2^*$),

$$M^N = M_d e^{-\delta dt} \int_{z_1^*}^{z_2^*} \left[ 1 - \Phi\left( \frac{z_2^* - z - \alpha dt}{\xi\sqrt{dt}} \right) \right] f^D(z) dz.$$

The number of entrants $M^E$, as defined earlier, is found by multiplying the number of newly exporting firms $M^N$ by the fraction that keeps exporting after $dt$ time, that is, the fraction that receives a shock that is large enough to not stop exporting.

$$M^E = M^N e^{-\delta dt} \int_{z_2^*}^{\infty} \left[ 1 - \Phi\left( \frac{z_2^* - z - \alpha dt}{\xi\sqrt{dt}} \right) \right] f^N(z) dz$$

Finally, the productivity density of the entrants is found by weighting $f^N(z)$ by the probability of continuing to export $dt$ time from now for a firm with productivity $z$ (weight equals one when the probability of continuing to export is one), and then normalizing

$$f^E(z) = \frac{f^N(z) \left[ 1 - \Phi\left( \frac{z_2^* - z - \alpha dt}{\xi\sqrt{dt}} \right) \right]}{\int_{z_2^*}^{\infty} f^N(z) \left[ 1 - \Phi\left( \frac{z_2^* - z - \alpha dt}{\xi\sqrt{dt}} \right) \right] dz} \quad \text{for} \quad z_2^* < z < \infty \ .$$

The average export of entrants is

$$\sum_{j=2}^{N} \int_{z_j^*}^{\infty} r_j(z) f^E(z) dz$$

where $r_j(z) = R_j([\tau_j \sigma/(\sigma - 1)P_j e^z]^{1-\sigma}$ is export to country $j$ of a firm with productivity $z$. Finally, the measure and average export of single-period, continuing and exiting exporters are computed similarly.[15]

Table 3 shows the number of firms and the value of average export (as a fraction

[15] Notes are available from the authors upon request.

of the total) in each category for our parameter specification. Figure 5 shows the productivity distributions of the four categories of firms. We reproduce two main features of the Colombian data analyzed by Eaton et al. (2007): (i) continuing exporters are more numerous than entering and exiting exporters and (ii) continuing exporters sell more than all the other firms and single-period exporters sell relatively little. The fact that single-year exporters on average sell less than entrants is consistent with the structure of the model: single-year exporters and entrants initially belong to the same group of firms, those that do not export in $t-1$ and export in $t$. Within this group of firms, single-year exporters are those that stop exporting in $t+1$ while entrants are those that survive to $t+1$. It is then clear that single-year exporters must be more sensitive to a negative productivity shock, be closer to the export cutoff, and therefore export less than entrants. By the same token, continuing exporters are bigger than exiters and entrants.[16] The presence of many continuing exporters depends instead on the location of the export cutoff and in particular on the degree of heterogeneity of the firm size distribution. Economies where firms are hit by shocks that are more dispersed (higher $\xi$) tend to show a higher relative frequency of continuing exporters.

*Cohort analysis.* As mentioned earlier, in the data new exporters usually supply only one foreign destination and undergo a very tough selection in the first year of export. Selection continues after the first year but at a lower rate. Those exporters that survive expand in terms of sales value (Eaton et al., 2007) and in terms of destinations and products supplied (Amador and Opromolla, 2008). Here we characterize the "life cycle"of an exporting episode: we consider a typical cohort of new exporters and follow it through time.[17] The top panel of table 4 shows the survival rate and

[16] Note that, in this exercise we are considering only surviving firms. Since each firm is subject to the killing rate $\delta$, it is possible that some exiters are bigger than continuing exporters.

[17] While different cohorts can potentially follow completely different paths, according to the realizations of the firms idiosyncratic shocks, we describe the characteristics of their (common) expected path. Eaton et al. (2007) find that older cohorts are not always larger than younger ones (in terms of total exports) and cohorts differ in their performance over the years, with leapfrogging in size occurring.

the average exporter size (for the survivors) during the five periods after they start exporting. Figure 6 shows the evolution of the productivity distribution of the surviving exporters. Our choice of parameters imply a strong selection of first-period exporters: almost one third of them stop exporting and return to sell their goods on the domestic market only. The survival rate jumps from 69 percent to 82 percent in the second period and then increases gradually up to 91 percent in the fifth period. As firms gain experience in foreign market their average sales increase. Average export jumps by 33 percent from the first to the second period, up to 116 percent in the fifth period. Part of the increase in export is due to the addition of new markets. The bottom panel of table 4 shows the distribution of exporters in terms of the number of foreign destinations supplied over time. While in the first period about 90 percent of the firms supply only one foreign destination, in the fifth period this decreases to 28 percent, with 41 percent of the firms supplying three or more destinations. The model therefore captures well the main features of the expansion path of newly exporting firms. While entrants usually make only a very small contribution to export growth in their year of entry, they are considered crucial when it comes to understanding long-run trend in total export. Eaton et al. (2007) for example find that gross entry explains almost half of the growth over the full eight year period of their Colombian data sample.

# 4 Conclusions

In this paper we have developed a framework for studying export dynamics and market specific flows in a multicountry model of trade with heterogenous firms. Countries can differ in terms of market size, size distribution of potential entrants, firm dynamics properties and trade barriers. Firm dynamics are driven by idiosyncratic persistent shocks that can be interpreted as productivity shocks, quality shocks or preference shocks. The model makes predictions in terms of cross-sectional moments and firm life cycle characteristics that were not possible in previous models. The model is parsimonious and can be solved analytically and is therefore useful for applied work.

To illustrate the properties of the model we analyze a simple scenario where countries are identical except for the tariff rate. We show that heterogeneity in efficiency together with persistent productivity shocks are enough to account for, qualitatively, for many features of the data. The model retains many of the properties of cross-sectional trade models without dynamics as Chaney (2008) or Eaton et al. (2008). In particular, it is consistent with the observed size distribution of firms, export sales distribution and relative size of exporters and nonexporters. More importantly, it adds new insights into the cross sectional dynamics of these patterns. Our model predicts that the frequency of firms selling to multiple foreign destinations is declining in the number of markets as observed in the data. We also find that nonexporters are more likely to enter only one market when they initiate export activities and that they enter and exit export markets gradually rather than in large clumps. We also study the expansion or life cycle properties of newly exporting firms. We show that while entrants usually make only a very small contribution to export growth in the year of entry, their share becomes more important in the long run as they grow in size.

Future research will need to contrast the properties of the model with the data. In particular, we conjecture that more sources of heterogeneity will be needed to account fully for the entry and exit patterns of small firms.

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# 5 Appendix

## 5.1 Value of the firm

The value function $V_i^D(z)$ and the cutoff $z_{ii}$ are the joint solution of the ordinary differential equation

$$(r+\delta)V_D(z) = \frac{1}{2}\xi^2 V_D''(z) + \alpha V_D'(z) + \pi_D(z)$$

subject to the value-matching and smooth-pasting conditions $V_D(z_D) = 0$ and $V_D'(z_D) = 0$. The general solution of the ordinary differential equation is the sum of the general solution of the corresponding homogenous equation ($V_{Di}^h(z)$) and a particular solution of the non-homogeneous equation ($V_{Di}^p(z)$). The former is

$$V_{Di}^h(z) = \kappa_1 e^{\beta_{1i} z} + \kappa_2 e^{\beta_{2i} z}$$

where$\beta_{1i} = -\alpha_i/\xi_i^2 + \sqrt{\alpha_i^2/\xi_i^4 + 2(r+\delta)/\xi_i^2}$ and $\beta_{2i} = -\alpha_i/\xi_i^2 - \sqrt{\alpha_i^2/\xi_i^4 + 2(r+\delta)/\xi_i^2}$ are the roots of the associated characteristic equation. The general solution of the homogeneous equation represents the value of the option to exit.[18] Since the likelihood of abandonment in the not-too-distant future becomes extremely small as $z$ goes to $\infty$, the value of the abandonment option should go to zero as $z$ becomes very large. Hence the coefficient $\kappa_1$ corresponding to the positive root $\beta_{1i}$ should be zero. This leaves

$$V_{Di}^h(z) = \kappa_2 e^{\beta_{2i} z}$$

The particular solution of the non-homogeneous equation can be found using the "undetermined coefficients" method. When the forcing term has the form $A_{di}e^{(\sigma-1)z} + B_{di}$ the solution assumes the form $C_i e^{(\sigma-1)z} + D_i$. In our case $\pi_{ii}(z) = A_{di}e^{(\sigma-1)z} + B_{di}$ with $A_{di} = R_i P_i^{\sigma-1}(\sigma-1)^{\sigma-1}\sigma^{-\sigma}$ and $B_{di} = -\lambda_i^D$. This delivers

[18] This is discussed in the next section of the Appendix.

$$V^p_{Di}(z) = -\frac{A_{di}}{\frac{1}{2}\xi_i^2(\sigma-1)^2+\alpha_i(\sigma-1)-(r+\delta)}e^{(\sigma-1)z}+\frac{B_{di}}{(r+\delta)}.$$

Overall, the general solution of the non-homogeneous equation is

$$V_i^D(z) = \kappa_{2i}e^{\beta_2 z} - \frac{A_{di}}{\frac{1}{2}\xi_i^2(\sigma-1)^2+\alpha_i(\sigma-1)-(r+\delta)}e^{(\sigma-1)z}+\frac{B_{di}}{(r+\delta)}.$$

The value-matching condition can be used to determine $\kappa_2$,

$$V_i^D(z_{ii}) = 0 \quad \Leftrightarrow \quad \kappa_{2i}e^{\beta_{2i}z_{ii}} - \frac{A_{di}}{\frac{1}{2}\xi_i^2(\sigma-1)^2+\alpha_i(\sigma-1)-(r+\delta)}e^{(\sigma-1)z_{ii}}+\frac{B_{di}}{(r+\delta)} = 0$$

$$\kappa_{2i} = \left(\frac{A_{di}}{\frac{1}{2}\xi_i^2(\sigma-1)^2+\alpha_i(\sigma-1)-(r+\delta)}e^{(\sigma-1)z_{ii}} - \frac{B_{di}}{(r+\delta)}\right)e^{-\beta_{2i}z_{ii}},$$

while the smooth-pasting condition can be used to determine $z_{ii}$,

$$e^{z_{ii}} = \frac{\sigma}{\sigma-1}\left(\frac{\lambda_i^D\sigma}{R_i}\right)^{\frac{1}{\sigma-1}}\frac{\gamma_{di}}{P_i}.$$

Finally we can write $V_i^D(z_D)$ as

$$V_i^D(z) = \frac{\lambda_i^D}{(r+\delta)}\frac{\beta_{2i}}{\beta_{2i}-(\sigma-1)}\left[e^{(\sigma-1)(z-z_{ii})} - \frac{\beta_{2i}-(\sigma-1)}{\beta_{2i}} - \frac{\sigma-1}{\beta_{2i}}e^{\beta_{2i}(z-z_{ii})}\right].$$

Table 1: Aggregate Variables, Exit and Export Cutoffs

| country | $L$ | $P$ | $R$ | | $M$ |
|---|---|---|---|---|---|
| 2 | 1 | 1.09 | 0.85 | | 0.94 |
| 3 | 1 | 1.18 | 0.74 | | 0.90 |
| 4 | 1 | 1.26 | 0.65 | | 0.87 |
| 5 | 1 | 1.35 | 0.59 | | 0.85 |
| | | | source country | | |
| destination country | 1 | 2 | 3 | 4 | 5 |
| 1 | 1 | 1.07 | 1.07 | 1.07 | 1.07 |
| 2 | 1.09 | 1.00 | 1.09 | 1.09 | 1.09 |
| 3 | 1.10 | 1.10 | 1.09 | 1.10 | 1.10 |
| 4 | 1.12 | 1.12 | 1.12 | 1.01 | 1.12 |
| 5 | 1.13 | 1.13 | 1.13 | 1.13 | 1.01 |

Notes: the variables in the top panel are expressed as a fraction country 1's variables while all the cutoffs in the bottom panel are expressed as a fraction of country 1 exit cutoff.

Table 2: Cross-Market Dynamics

| | initial number of destinations | | | | |
|---|---|---|---|---|---|
| final number of destinations | 1 | 2 | 3 | 4 | 5 |
| 1 | 0.86 | 0.29 | 0.06 | 0.01 | 0.00 |
| 2 | 0.09 | 0.37 | 0.22 | 0.07 | 0.00 |
| 3 | 0.02 | 0.24 | 0.33 | 0.21 | 0.01 |
| 4 | 0.00 | 0.08 | 0.24 | 0.30 | 0.03 |
| 5 | 0.00 | 0.02 | 0.15 | 0.43 | 0.95 |
| Fraction in steady state | .32 | .35 | .31 | .26 | .23 |

.

Table 3: Single-Year, Entering, Exiting and Continuing Exporters

| | Entering | Continuing | Exiting | Single-Period |
|---|---|---|---|---|
| Fraction | 0.04 | 0.91 | 0.04 | 0.02 |
| Average Export | 0.14 | 1.08 | 0.15 | 0.13 |

Notes: row one reports the fraction of firms in each category while row two reports the average export of firms in each category as a fraction of average export across all categories.

Table 4: Survival Rate and Number of Destinations Supplied for a Cohort of Exporters

| | Periods | | | | |
|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 |
| Survival Rate | 0.69 | 0.82 | 0.87 | 0.89 | 0.91 |
| Average Export (index) | 1 | 1.33 | 1.67 | 1.94 | 2.16 |
| Number of Destinations | | | | | |
| 1 | 0.90 | 0.57 | 0.41 | 0.33 | 0.28 |
| 2 | 0.09 | 0.33 | 0.36 | 0.33 | 0.32 |
| 3 | 0.01 | 0.08 | 0.15 | 0.18 | 0.19 |
| 4 | 0.00 | 0.02 | 0.08 | 0.16 | 0.22 |

Notes: in the top panel we report the fraction of firms that keep exporting after one to five periods for a cohort of new exporters and their average exports; in the bottom panel we report the fraction of exporters supplying one to four destinations in each period.

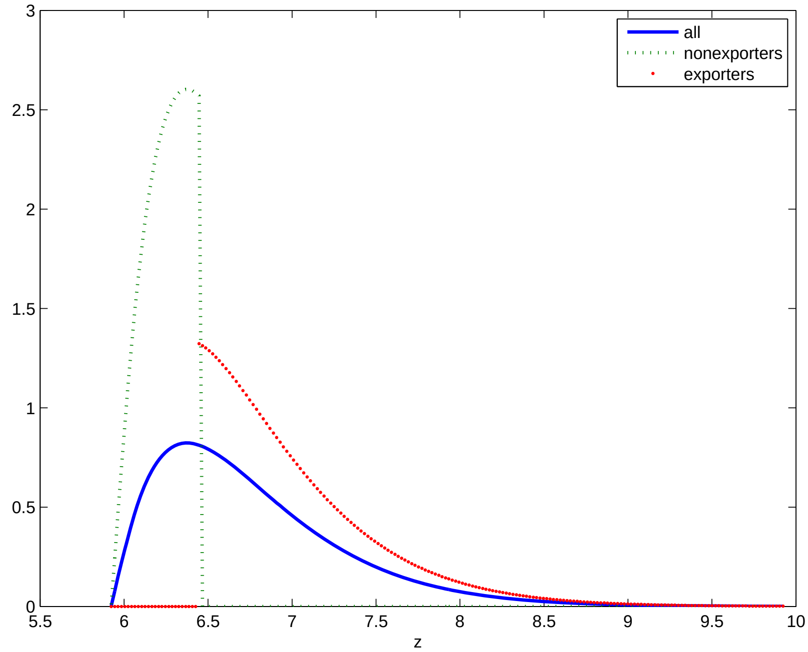


Figure 1: Stationary Productiviy Distributions, by Export Status

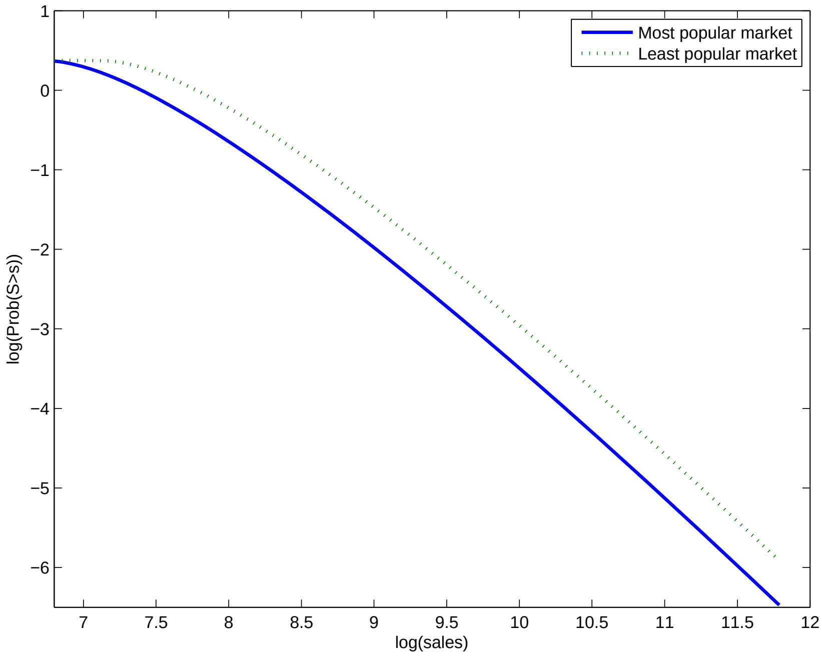


Figure 2: Export Sales Distributions in the Least and Most Popular Markets

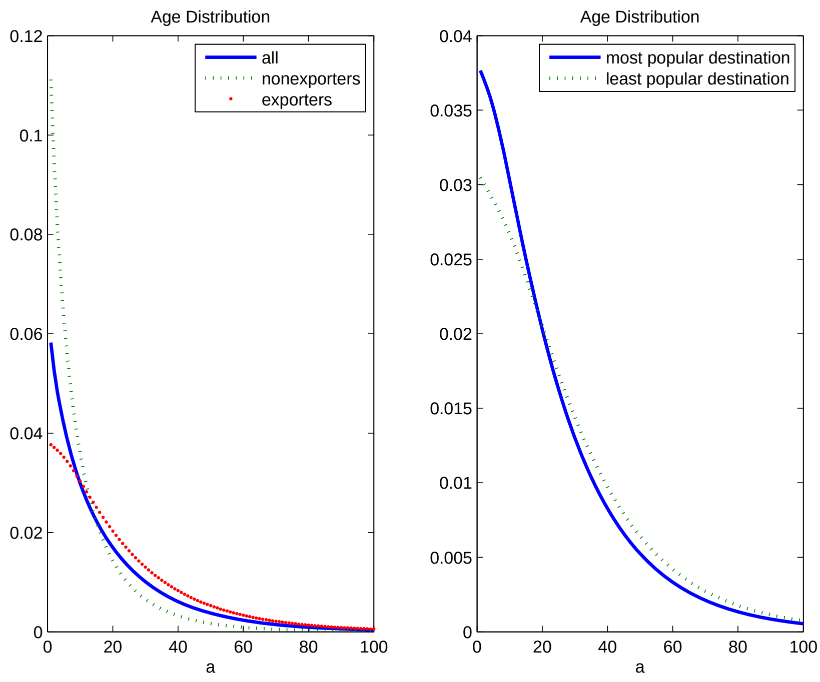


Figure 3: Age Distributions, by Export Status and Destination Market

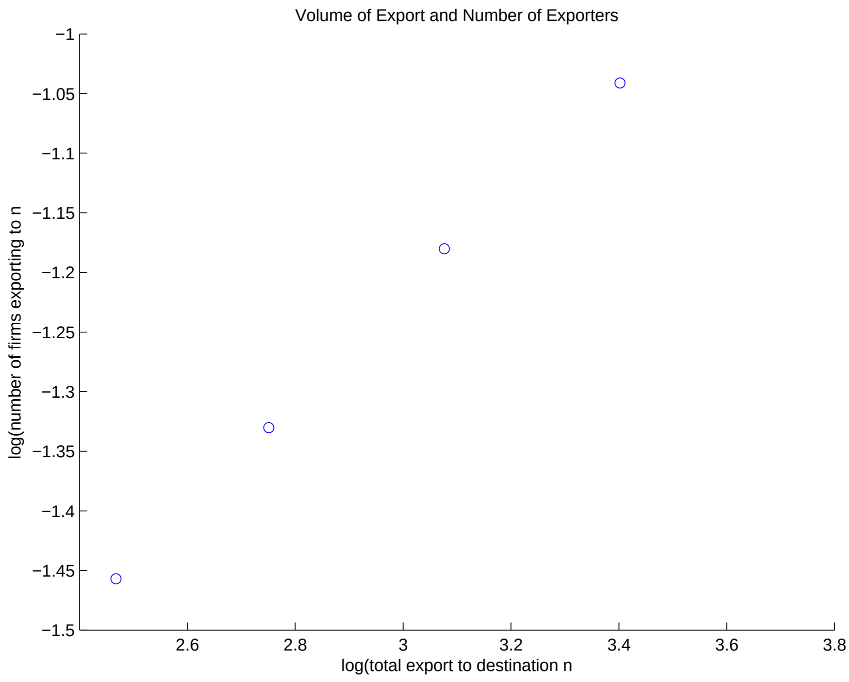


Figure 4: Volume of Exports and Number of Exporters

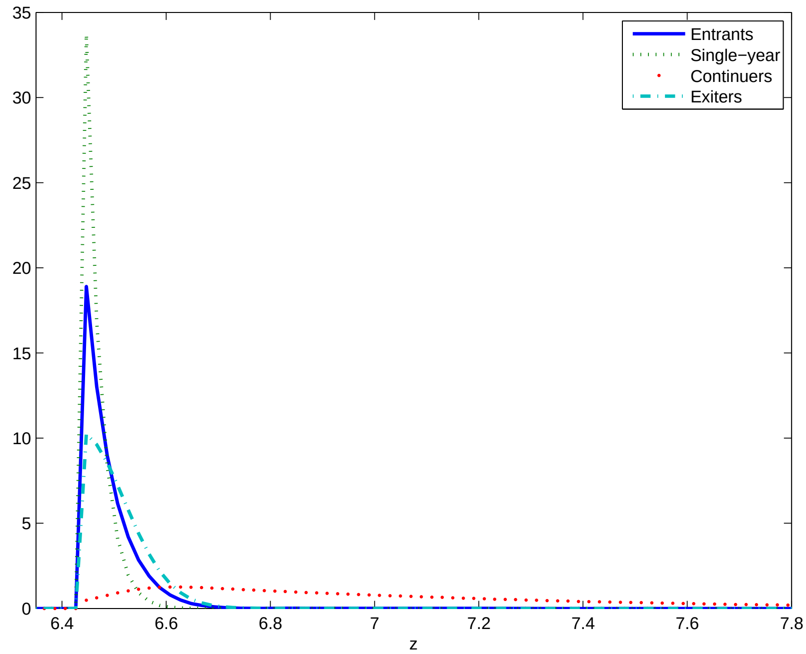


Figure 5: Productivity Density by Type of Exporter

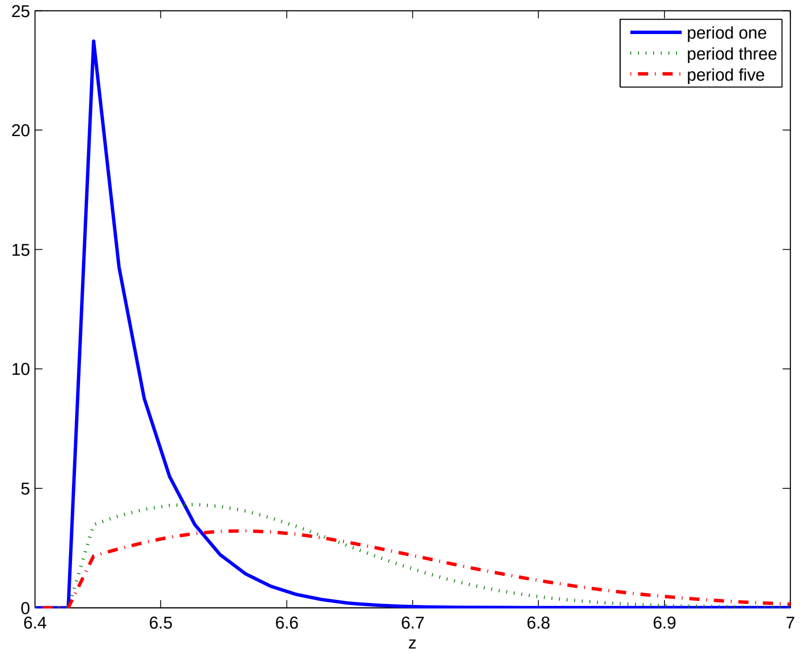


Figure 6: Productivity Density for A Cohort of New Exporters over Time

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