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On LM-Type Tests for Seasonal Unit Roots in the Presence of a Break in Trend*

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Abstract

This paper proposes tests for seasonal unit roots allowing for the presence of a break in the trend slope occurring at an unknown date. In particular, new LM type tests are derived based on the framework introduced by Hylleberg, Engle, Granger and Yoo [HEGY] (1990). Null asymptotic distributions are derived for the no break case as well as when a break is present in the data generating process. A Monte Carlo investigation on the finite sample size and power performance of the new procedures is presented.

Keywords: Seasonal unit roots, structural change, trend breaks, LM type unit root tests.
JEL: C12, C22.

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1 Introduction

Since the seminal work by Nelson and Plosser (1982), there has been great interest in testing whether economic time series are integrated. This has led over the years to the development of numerous unit root tests; see, for instance, Stock (1994), Maddala and Kim (1998) and Phillips and Xiao (1998) for extensive reviews. A similar phenomenon has also been observed in the context of seasonally unadjusted economic time series; see, for instance, Ghysels and Osborn (2001) and Ghysels, Osborn and Rodrigues (2000) for overviews.

Among the procedures proposed, one interesting approach used to test for unit roots is the score principle; Ahn (1993), Oya and Toda (1998), Schmidt and Lee (1991), Schmidt and Phillips (1992) and Solo (1984), *inter alia*, provide Lagrange Multiplier (LM) type test procedures for a zero frequency unit root for nonseasonal data; and Ahn and Cho (1993a, 1993b), Li (1991), Park and Cho (1994) and Rodrigues (2002), *inter alia*, provide LM type tests for zero and seasonal frequencies unit roots.

The underlying main difference between LM and Dickey-Fuller (DF) type tests lies in the nature of detrending. In the former, the parameters used to obtain the residuals are estimated from a model in differences, whereas in the latter these are estimated from a model in levels. However, as argued by Schmidt and Phillips (1992), tests based on the LM principle will have simpler properties than tests based on the DF approach.

With the findings of Perron (1989) unit root test results have been questioned. Perron argues that many macroeconomic time series are well represented by a time trend with a structural break and stationary short run dynamics. This has led over the years to the development of unit root tests which allow for breaks; see for instance, Perron (2006) for an overview. In the seasonal context, developments have been relatively slower, Franses and Vogelsang (1998) introduced one of the earliest procedures to test for seasonal unit roots allowing for the possible presence of breaks in the mean (see also Harvey, Leybourne and Newbold, 2002) and recently Hassler and Rodrigues (2004) investigate the performance of LM tests in this context as well.

In this paper, we contribute to this literature by introducing new seasonal LM type unit root test statistics, based on the approach suggested by Hylleberg, Engle, Granger and Yoo [HEGY] (1990) and extending the work of Rodrigues (2002) and Hassler and Rodrigues (2004) by allowing for a break in the time trend slope occurring at an unknown date. Asymptotic results of the test statistics are presented for the no break case as well as when a break is present in the data generating process (DGP) under the null hypothesis. We also investigate and compare the finite sample performance of alternative seasonal unit root test procedures in this context.

The paper is organized as follows. Section 2 introduces the seasonal LM type unit root tests which allow for a break in the trend slope. The limiting results appear in Section 3. In Section 4 the results of several Monte Carlo experiments comparing the size and power performance of alternative test procedures in the case of quarterly time series are presented. Section 5 concludes the paper. An Appendix collects proofs of the results discussed throughout the text.

2 LM-Type Unit Root Tests with a Break in the Trend Slope

The purpose of this paper is to propose new seasonal LM-type unit root test statistics which allow for a break in the trend. Note that as indicated by, *inter alia,* Breitung and Franses (1998) and Hassler and Rodrigues (2004), breaks in the intercept do not affect the limit distributions of LM type unit root test statistics. However, that is not the case for a break in the trend slope.

Consider the following DGP for data observed S seasons per year (where, for example, $S = 4$ and $S = 12$ correspond to quarterly and monthly periodicities, respectively) with a possible change in the trend slope at time T_B^0, with $S < T_B^0 = [\lambda_0 SN] < SN$ and $0 < \lambda_0 < 1$, such as,

$$y_{Sn+s} - \delta_s - \gamma_1(Sn+s) - \gamma_2 DT^0_{Sn+s} = x_{Sn+s}, \tag{2.1}$$

$$(1-\rho L^S)x_{Sn+s} = \epsilon_{Sn+s} \tag{2.2}$$

where $DT^0_{Sn+s} = (Sn+s-T^0_B)1_{(Sn+s>T^0_B)}$, ϵ_{Sn+s} i.i.d.$(0,\sigma^2)$, $n = 1,\dots,N$, and $s = -S+1,\dots,0$. For the initial conditions we assume that $x_{-S+1},\dots,x_0$ are fixed. The case of no structural change considered for example in Smith and Taylor (1998) and Rodrigues (2002) corresponds in our framework to $\gamma_2 = 0$ in (2.1).

The overall interest lies in testing the null hypothesis of seasonal integration, *i.e.*,

$$H_0 : \rho = 1. \tag{2.3}$$

Under the null hypothesis (2.3) the autoregressive component of (2.2) can be factored as,

$$(1 - L^S) = (1 - L)(1 + L) \prod_{k=1}^{S^*} \left[1 - 2\cos(\omega_k) L + L^2\right] \tag{2.4}$$

where $S^* \equiv (S/2) - 1$ (if S is even) or $[S/2]$ (if S is odd) and $[.]$ represents the integer function. As can be observed from (2.4), this polynomial consists of unit roots at the zero, the Nyquist and the harmonic frequencies.

Assuming normality of the errors,[1] the restricted maximum likelihood estimator of $\gamma = (\gamma_1, \gamma_2)'$, denoted by $\tilde{\gamma} = (\tilde{\gamma}_1, \tilde{\gamma}_2)'$, is obtained by estimating the following regression by OLS,

$$\Delta_S y_{Sn+s} = \gamma_1 S + \gamma_2 \Delta_S DT_{Sn+s} + u_{Sn+s} \tag{2.5}$$

where $\Delta_S = 1 - L^S$ denotes the seasonal difference operator and $DT_{Sn+s} = (Sn + s - T_B)1_{(Sn+s>T_B)}$ is based on some assumed break date denoted by $T_B = [\lambda SN]$ that may differ from the true one, T_B^0, *i.e.*, the assumed break fraction, λ, may be different from the true one, λ_0.

Thus, to set up the seasonal unit root test regression consider the variable,

$$\widetilde{y}_{Sn+s} = y_{Sn+s} - \tilde{\delta}_s^* - \tilde{\gamma}_1 (Sn + s) - \tilde{\gamma}_2 DT_{Sn+s} \tag{2.6}$$

where from (2.1)-(2.2), under the null hypothesis (2.3), it follows that

$$\tilde{\delta}_s^* = y_{S+s} - \tilde{\gamma}_1 (S + s) - \tilde{\gamma}_2 DT_{S+s} = y_{S+s} - \tilde{\gamma}_1 (S + s). \tag{2.7}$$

We observe, following, *inter alia*, HEGY, Rodrigues (2002) and Smith and Taylor (1999), that the regression-based approach to test for seasonal unit roots in the autoregressive component of (2.2), $(1 - \rho L^S)$, consists of the application of the auxiliary regression equation,

$$\begin{aligned}\Delta_S \widetilde{y}_{Sn+s} &= \pi_0 \widetilde{y}_{0,Sn+s-1} + \pi_{S/2} \widetilde{y}_{S/2,Sn+s-1} \\ &\quad + \sum_{k=1}^{S^*} \left(\pi_{\alpha,k} \widetilde{y}^{\alpha}_{k,Sn+s-1} + \pi_{\beta,k} \widetilde{y}^{\beta}_{k,Sn+s-1} \right) + \varepsilon_{Sn+s}\end{aligned} \tag{2.8}$$

where the regressors corresponding to the zero and seasonal frequencies $\omega_j = 2\pi j/S$, $j = 0, ..., [S/2]$, are linear combinations of lags of $\widetilde{y}_{Sn+s}$ given as $\widetilde{y}_{0,Sn+s} = \Delta_0 \widetilde{y}_{Sn+s-j}$, $\widetilde{y}_{S/2,Sn+s} = \Delta_{S/2} \widetilde{y}_{Sn+s}$, $\widetilde{y}^{\alpha}_{k,Sn+s} = \Delta^{\alpha}_k \widetilde{y}_{Sn+s}$, $\widetilde{y}^{\beta}_{k,Sn+s} = \Delta^{\beta}_k \widetilde{y}_{Sn+s}$, where the lag operators are defined as

$$\Delta_0 = \sum_{j=0}^{S-1} L^j, \tag{2.9}$$

$$\Delta_{S/2} = \sum_{j=0}^{S-1} \cos[(j + 1)\pi] L^j, \tag{2.10}$$

$$\Delta^{\alpha}_k = \sum_{j=0}^{S-1} \cos[(j + 1)\omega_k] L^j, \tag{2.11}$$

$$\Delta^{\beta}_k = -\sum_{j=0}^{S-1} \sin[(j + 1)\omega_k] L^j, \tag{2.12}$$

for $k = 1, ..., S^*$, and $\Delta_S \widetilde{y}_{Sn+s} \equiv \widetilde{y}_{Sn+s} - \widetilde{y}_{S(n-1)+s}$. Note that the term $\pi_{S/2} \widetilde{y}_{S/2,Sn+s-1}$ in (2.8) needs to be omitted if S is odd.

[1] The normality assumption is important only for the derivation of the test regression and can therefore, in later applications, be relaxed.

Remark 2.1: For the case of quarterly data, $S = 4$, the relevant transformations are

$$\begin{aligned}\widetilde{y}_{0,Sn+s} &\equiv (1+L+L^2+L^3)\widetilde{y}_{Sn+s}, \;\; \widetilde{y}_{S/2,Sn+s} \equiv -\left(1-L+L^2-L^3\right)\widetilde{y}_{Sn+s}, \\ \widetilde{y}^{\alpha}_{1,Sn+s} &\equiv -L(1-L^2)\widetilde{y}_{Sn+s}, \;\; \widetilde{y}^{\beta}_{1,Sn+s} \equiv -(1-L^2)\widetilde{y}_{Sn+s}.\end{aligned}$$

Remark 2.2: For simplicity of presentation, our discussion is based on a restricted (seasonal) autoregressive (AR) component, however the results are still valid in the context of unrestricted AR of at least order S; see Rodrigues and Taylor (2004) for a discussion.

In order to test H_0 in (2.3) against the alternative of stationarity in at least one of the zero and seasonal frequencies, following HEGY and Smith and Taylor (1998), a left-sided t-statistic, $t_0(T_B)$, for the exclusion of $\widetilde{y}_{0,Sn+s-1}$; a left-sided t-statistic, $t_{S/2}(T_B)$, for the exclusion of $\widetilde{y}_{S/2,Sn+s-1}$ (S even); a left-sided, $t^{\alpha}_k(T_B)$, and a two-sided t-statistic, $t^{\beta}_k(T_B)$, for the exclusion of $\widetilde{y}^{\alpha}_{k,Sn+s-1}$ and $\widetilde{y}^{\beta}_{k,Sn+s-1}$, respectively, and joint tests, $F_k(T_B)$, for the exclusion of *both* $\widetilde{y}^{\alpha}_{k,Sn+s-1}$ and $\widetilde{y}^{\beta}_{k,Sn+s-1}$, $k = 1, ..., S^*$ are considered. Moreover, as in Ghysels, Lee and Noh (1994), Taylor (1998), and Smith and Taylor (1998, 1999), we also consider the joint frequency OLS F-statistics, $F_{1...[S/2]}(T_B)$, for the exclusion of $\widetilde{y}_{S/2,Sn+s-1}$ (S even) and $\{\widetilde{y}^{\alpha}_{k,Sn+s-1}, \widetilde{y}^{\beta}_{k,Sn+s-1}\}^{S^*}_{k=1}$, and $F_{0...[S/2]}(T_B)$ for the exclusion of $\widetilde{y}_{0,Sn+s-1}$, $\widetilde{y}_{S/2,Sn+s-1}$ (S even) and $\{\widetilde{y}^{\alpha}_{k,Sn+s-1}, \widetilde{y}^{\beta}_{k,Sn+s-1}\}^{S^*}_{k=1}$.

Furthermore, since in order to apply this approach and obtain the seasonal unit root test statistics, the unknown break date T_B must be determined first, we consider three possibilities:

i) Assuming a fixed break fraction λ when estimating (2.5).

ii) Using the least squares estimator of the break date, that is, the date that minimizes the sum of squared residuals in (2.5), as in Perron (1997).

iii) Selecting the break date which is least favorable to the null hypothesis H_0 by minimizing or maximizing the value taken by the corresponding seasonal unit root test statistic, as suggested by Zivot and Andrews (1992) for the nonseasonal case.

3 Asymptotic Results

In this section we provide the limiting distributions of the seasonal unit root test statistics under the seasonal unit root ($\rho = 1$) hypothesis and consider two possible scenarios: i) where no break occurs in the DGP even though the test is based on regression (2.5) which allows for a break; ii) a break does occur in the DGP but the assumed or estimated break date may or may not coincide with the true one.

3.1 The case of no break

We first derive the limiting distributions of the LM-type test statistics allowing for a break at a fixed break fraction λ when the DGP satisfies H_0 but without a break, *i.e.*, $\gamma_2 = 0$ in (2.1).

Theorem 3.1 *When the DGP is given by (2.1)–(2.2) with no break, $\gamma_2 = 0$, but the test statistics are computed assuming a break at time $T_B = [\lambda SN]$ with a fixed $\lambda \in (0, 1)$, then under H_0, as $N \to \infty$, the* *following limit results are obtained:*

$$i)\ t_0(T_B) \Rightarrow -\frac{1}{2\left[\int_0^1 [V_0^*(r;\lambda)]^2\,dr\right]^{1/2}} \equiv \tau_0(\lambda); \tag{3.1}$$

$$ii)\ t_{S/2}(T_B) \Rightarrow \frac{\displaystyle\int_0^1 W_{S/2}^*(r)dW_{S/2}^*(r)}{\left(\int_0^1 \left[W_{S/2}^*(r)\right]^2 dr\right)^{1/2}} \equiv \tau_{S/2}; \tag{3.2}$$

$$iii)\ t_k^{\alpha}(T_B) \Rightarrow \frac{\int_0^1 \left[W_k^{\alpha*}(r)dW_k^{\alpha*}(r) + W_k^{\beta*}(r)dW_k^{\beta*}(r)\right]}{\left(\int_0^1 \left\{[W_k^{\alpha*}(r)]^2 + \left[W_k^{\beta*}(r)\right]^2\right\} dr\right)^{1/2}} \equiv \tau_k^{\alpha}; \tag{3.3}$$

$$iv)\ t_k^{\beta}(T_B) \Rightarrow \frac{\int_0^1 \left[W_k^{\alpha*}(r)dW_k^{\beta*}(r) - W_k^{\beta*}(r)dW_k^{\alpha*}(r)\right]}{\left(\int_0^1 \left\{[W_k^{\alpha*}(r)]^2 + \left[W_k^{\beta*}(r)\right]^2\right\} dr\right)^{1/2}} \equiv \tau_k^{\beta}; \tag{3.4}$$

for $k = 1, \ldots S^*$, *where* $V_0^*(r;\lambda) = \frac{1}{\sqrt{S}}\tilde{V}_0(r;\lambda)$,$W_{S/2}^*(r) = \frac{1}{\sqrt{S}}\tilde{W}_{S/2}(r)$, $W_k^{\alpha*}(r) = \sqrt{\frac{2}{S}}\tilde{W}_k^{\alpha}(r)$, $W_k^{\beta*}(r) = \sqrt{\frac{2}{S}}\tilde{W}_k^{\beta}(r)$, *with*

$$\tilde{V}_0(r;\lambda) = \tilde{W}_0(r) - q(r;\lambda)Q(\lambda)^{-1}\int_0^1 g(r;\lambda)'d\tilde{W}_0(r),$$
$$\tilde{W}_0(r) \equiv \sum_{j=0}^{S-1} W_{-j}(r), \qquad \tilde{W}_{S/2}(r) \equiv \sum_{j=0}^{S-1}\left[\cos((j+1)\pi)\right]W_{-j}(r),$$
$$\tilde{W}_k^{\alpha}(r) \equiv \sum_{j=0}^{S-1}\left[\cos((j+1)\omega_k)\right]W_{-j}(r), \qquad \tilde{W}_k^{\beta}(r) \equiv -\sum_{j=0}^{S-1}\left[\sin((j+1)\omega_k)\right]W_{-j}(r),$$

$W_s(r)$, $s = -S+1, ..., 0$ *are independent standard Brownian motions, and*

$$q(r,\lambda) = (r, (r-\lambda)1_{(r>\lambda)}), \qquad Q(\lambda) = \begin{bmatrix} 1 & 1-\lambda \\ 1-\lambda & 1-\lambda \end{bmatrix}, \qquad g(r,\lambda) = (1, 1_{(r>\lambda)}).$$

We observe that the limiting distribution of the zero frequency test statistic $t_0(T_B)$ depends on the value assumed for λ. For the other tests, the limiting distributions coincide with those derived in Rodrigues (2002) for the corresponding seasonal LM type unit root tests allowing for no break.

Corollary 3.1 *Under the same conditions of Theorem 3.1, the following limit results for the joint tests are obtained:*

$$F_k(T_B) \Rightarrow \frac{1}{2}\left\{[\tau_k^{\alpha}]^2 + [\tau_k^{\beta}]^2\right\} \equiv \mathcal{F}_k,\ k = 1, \ldots, S^*, \tag{3.5}$$

$$F_{1\ldots[S/2]}(T_B) \Rightarrow \frac{1}{S-1}\left\{[\tau_{S/2}]^2 + \sum_{k=1}^{S^*}\left([\tau_k^{\alpha}]^2 + [\tau_k^{\beta}]^2\right)\right\} \equiv \mathcal{F}_{1\ldots[S/2]}, \tag{3.6}$$

$$F_{0\ldots[S/2]}(T_B) \Rightarrow \frac{1}{S}\left\{[\tau_0(\lambda)]^2 + [\tau_{S/2}]^2 + \sum_{k=1}^{S^*}\left([\tau_k^{\alpha}]^2 + [\tau_k^{\beta}]^2\right)\right\} \equiv \mathcal{F}_{0\ldots[S/2]}(\lambda). \tag{3.7}$$

Note that the results of this Corollary follow straightforwardly from the asymptotic orthogonality of the regressors of test regression (2.8). As expected, only the limiting distribution of the $F_{0\ldots[S/2]}(T_B)$ test statistic depends on the assumed break fraction λ.

When the break date T_B is unknown, we first consider using the least squares estimator of the break date, *i.e.* the date that minimizes the sum of squared residuals in (2.5). We denote this estimator of T_B

by $\hat{T}_B$. The corresponding estimated break fraction is restricted to lie in a closed subset of $(0,1)$ denoted by Λ. In the following theorem we derive the limiting distributions of the test statistics when evaluated at this estimated break date $\hat{T}_B$.

Theorem 3.2 *When the DGP is given by (2.1)–(2.2) with no break, $\gamma_2 = 0$, but the test statistics are computed assuming a break at the estimated break date $\hat{T}_B$, then under H_0, as $N \to \infty$, the following limit results are obtained:*

$$t_0(\widehat{T}_B) \Rightarrow \tau_0(\widehat{\lambda}); \qquad t_{S/2}(\widehat{T}_B) \Rightarrow \tau_{S/2}; \qquad t_k^{\alpha}(\widehat{T}_B) \Rightarrow \tau_k^{\alpha}; \qquad t_k^{\beta}(\widehat{T}_B) \Rightarrow \tau_k^{\beta}; \tag{3.8}$$

and

$$F_k(\widehat{T}_B) \Rightarrow \mathcal{F}_k; \qquad F_{1\ldots[S/2]}(\widehat{T}_B) \Rightarrow \mathcal{F}_{1\ldots[S/2]}; \qquad F_{0\ldots[S/2]}(\widehat{T}_B) \Rightarrow \mathcal{F}_{0\ldots[S/2]}(\hat{\lambda}); \tag{3.9}$$

for $k = 1, \ldots, S^*$, where $\hat{\lambda} = \underset{\lambda \in \Lambda}{\arg\min} P(\lambda)$ with

$$P(\lambda) = \left(\int_0^1 g(r,\lambda)' d\tilde{W}_0(r) \right)' Q(\lambda)^{-1} \left(\int_0^1 g(r,\lambda)' d\tilde{W}_0(r) \right).$$

As expected, except for the $t_0(\widehat{T}_B)$ and the $F_{0\ldots[S/2]}(\widehat{T}_B)$ statistics, the limiting distributions are the same as in the case where a fixed break fraction is assumed (seeTheorem 3.1 and Corollary 3.1) which also coincide with the corresponding limiting distributions for the no break LM type tests.

Finally, we also consider choosing the break date according to the minimum or maximum value taken by the seasonal unit root test statistics. As above, the corresponding estimated break fractions are restricted to lie in a set Λ. The limiting distributions of these statistics are given in the following theorem.

Theorem 3.3 *When the DGP is given by (2.1)–(2.2) with no break, $\gamma_2 = 0$, but the test statistics are computed as the maximization/minimization of a unit root test statistic over the possible break dates, then under H_0, as $N \to \infty$, the following limit results are obtained:*

$$\min_{T_B} t_0(T_B) \Rightarrow \inf_{\lambda \in \Lambda} \tau_0(\lambda); \qquad \min_{T_B} t_{S/2}(T_B) \Rightarrow \tau_{S/2}; \qquad \min_{T_B} t_k^{\alpha}(T_B) \Rightarrow \tau_k^{\alpha}; \qquad \max_{T_B} |t_k^{\beta}(T_B)| \Rightarrow |\tau_k^{\beta}|;$$

and

$$\max_{T_B} F_k(T_B) \Rightarrow \mathcal{F}_k; \;\; \max_{T_B} F_{1\ldots[S/2]}(T_B) \Rightarrow \mathcal{F}_{1\ldots[S/2]}; \;\; \max_{T_B} F_{0\ldots[S/2]}(T_B) \Rightarrow \sup_{\lambda \in \Lambda} \mathcal{F}_{0\ldots[S/2]}(\lambda),$$

for $k = 1, \ldots, S^*$.

Again, except for the zero frequency test statistic $\min_{T_B} t_0(T_B)$ and the overall test $\max_{T_B} F_{0\ldots[S/2]}(T_B)$, the limiting distributions of the other test statistics coincide with the ones obtained for a fixed break fraction in Theorem 3.1 and Corollary 3.1, and the no break case.

3.2 The case of a break in slope

We now consider the case where a break in the slope is present in the DGP, that is $\gamma_2 \neq 0$ in (2.1). In this context, the following theorems can be provided.

Theorem 3.4 *Suppose the DGP is given by (2.1)–(2.2) with a break of magnitude $\gamma_2 \neq 0$ occurring at time $T_B^0 = [\lambda_0 SN]$ with $\lambda_0 \in (0,1)$. The tests are computed assuming a break at time $T_B = [\lambda SN]$ with a fixed $\lambda \in (0,1)$. Then under H_0, as $N \to \infty$, the following limit results are obtained. When $\lambda = \lambda_0$ we have that*

$$i) \; t_0(T_B) \Rightarrow \tau_0(\lambda_0),$$

and for the seasonal unit root test statistics the limits are as in Theorem 3.1. When $\lambda \neq \lambda_0$, we have that

$$\begin{aligned} ii) (SN)^{1/2} t_0(T_B) \to^p \quad & -\frac{1}{2S} \left(\gamma' \left(\int_0^1 f(r;\lambda,\lambda_0)' f(r;\lambda,\lambda_0) dr \right) \gamma \right)^{-1/2} \\ & \times (\sigma^2 + \gamma' \mathcal{G}(\lambda,\lambda_0)\gamma)^{1/2}; \end{aligned}$$

$$
\begin{aligned}
iii)\ t_{S/2}(T_B) &\Rightarrow \left(\sigma^2 \int_0^1 \left[W^*_{S/2}(r)\right]^2 dr\right)^{-1/2} \\
&\quad \times \left(\sigma^2 \int_0^1 W^*_{S/2}(r) dW^*_{S/2}(r) - \frac{1}{2}\boldsymbol{\gamma}'\mathcal{G}(\lambda,\lambda_0)\boldsymbol{\gamma}\right) \\
&\quad \times (\sigma^2 + \gamma'\mathcal{G}(\lambda,\lambda_0)\gamma)^{-1/2} \\
&\equiv \tau_{S/2}(\lambda,\lambda_0), \\
iv)\ t_k^{\alpha}(T_B) &\Rightarrow \left(\sigma^2 \int_0^1 \left\{[W_k^{\alpha*}(r)]^2 + \left[W_k^{\beta*}(r)\right]^2\right\} dr\right)^{-1/2} \\
&\quad \times \left(\sigma^2 \int_0^1 \left[W_k^{\alpha*}(r) dW_k^{\alpha*}(r) + W_k^{\beta*}(r) dW_k^{\beta*}(r)\right] - \frac{c_k^{\alpha}}{S}\boldsymbol{\gamma}'\mathcal{G}(\lambda,\lambda_0)\boldsymbol{\gamma}\right) \\
&\quad \times (\sigma^2 + \gamma'\mathcal{G}(\lambda,\lambda_0)\gamma)^{-1/2} \\
&\equiv \tau_k^{\alpha}(\lambda,\lambda_0), \\
v)\ t_k^{\beta}(T_B) &\Rightarrow \left(\sigma^2 \int_0^1 \left\{[W_k^{\alpha*}(r)]^2 + \left[W_k^{\beta*}(r)\right]^2\right\} dr\right)^{-1/2} \\
&\quad \times \left(\sigma^2 \int_0^1 \left[W_k^{\alpha*}(r) dW_k^{\beta*}(r) - W_k^{\beta*}(r) dW_k^{\alpha*}(r)\right] - \frac{c_k^{\beta}}{S}\boldsymbol{\gamma}'\mathcal{G}(\lambda,\lambda_0)\boldsymbol{\gamma}\right) \\
&\quad \times (\sigma^2 + \gamma'\mathcal{G}(\lambda,\lambda_0)\gamma)^{-1/2} \\
&\equiv \tau_k^{\beta}(\lambda,\lambda_0), \\
vi)\ F_k(T_B) &\Rightarrow \frac{1}{2}\left\{[\tau_k^{\alpha}(\lambda,\lambda_0)]^2 + [\tau_k^{\beta}(\lambda,\lambda_0)]^2\right\}, \\
vii)\ F_{1\ldots[S/2]}(T_B) &\Rightarrow \frac{1}{S-1}\left\{[\tau_{S/2}(\lambda,\lambda_0)]^2 + \sum_{k=1}^{S^*}\left([\tau_k^{\alpha}(\lambda,\lambda_0)]^2 + [\tau_k^{\beta}(\lambda,\lambda_0)]^2\right)\right\},
\end{aligned}
$$

for $k = 1, \ldots, S^*$. Finally, regarding the overall seasonal unit root tests, when $\lambda = \lambda_0$, it also follows that

$$
viii)\ F_{0\ldots[S/2]}(T_B) \Rightarrow \mathcal{F}_{0\ldots[S/2]}(\lambda)
$$

and when $\lambda \neq \lambda_0$,

$$
ix)\ F_{0\ldots[S/2]}(T_B) \Rightarrow \frac{1}{S}\left\{[\tau_{S/2}(\lambda,\lambda_0)]^2 + \sum_{k=1}^{S^*}\left([\tau_k^{\alpha}(\lambda,\lambda_0)]^2 + [\tau_k^{\beta}(\lambda,\lambda_0)]^2\right)\right\}
$$

where $f(r;\lambda,\lambda_0) = q(r;\lambda_0) - q(r;\lambda)Q(\lambda)^{-1}R(\lambda,\lambda_0)$, $\mathcal{G}(\lambda,\lambda_0) = S^2\left[Q(\lambda_0) - R(\lambda,\lambda_0)'Q(\lambda)^{-1}R(\lambda,\lambda_0)\right]$, $c_k^{\alpha} = \sum\limits_{j=1}^{S} j\cos(j\omega_k)$, $c_k^{\beta} = -\sum\limits_{j=1}^{S} j\sin(j\omega_k)$, and $\gamma = (\gamma_1, \gamma_2)'$.

Note that, only when the chosen break date coincides with the true break date ($\lambda = \lambda_0$), in which case $\mathcal{G}(\lambda,\lambda_0) = 0$, are the limiting distributions of all the test statistics presented in Theorem 3.4 coincident with those obtained in the previous subsection for the case of no break in the DGP (see Theorem 3.1 and Corollary 3.1).

Remark 3.1: When $\lambda \neq \lambda_0$, the zero-frequency t-statistic, $t_0(T_B)$, will converge to zero. However, when $\lambda = \lambda_0$, it converges to $\tau_0(\lambda_0)$. These different orders of magnitude in the case of the $t_0(T_B)$ statistic suggest that the $\operatorname{argmin}_{T_B} t_0(T_B)$ could be used as an estimator of the true break date. Note that this possibility does not hold for the $F_{0\ldots[S/2]}(T_B)$ test as it also depends on the seasonal frequencies. Furthermore, the unit root tests at the seasonal frequencies in this context do not converge to the same limits as in the no break case. Their limiting distributions involve nuisance parameters. Unless the magnitude of the break $\boldsymbol{\gamma}$ is very large, the deterministic quadratic term $\boldsymbol{\gamma}'\mathcal{G}(\lambda,\lambda_0)\boldsymbol{\gamma}$ will be dominated by the stochastic component making these unit root tests useless to estimate the break date.

Next we present the limiting distribution of the test statistics when the break date is estimated by $\hat{T}_B$.

Theorem 3.5 *Suppose the DGP is given by (2.1)–(2.2) with a break of magnitude $\gamma_2 \neq 0$ occurring at time $T_B^0 = [\lambda_0 SN]$ with $\lambda_0 \in \Lambda$. Then under H_0, as $N \to \infty$, the following limit results are obtained.*

$$\begin{aligned} t_0(\hat{T}_B) &\Rightarrow \tau_0(\lambda_0); \\ t_{S/2}(\hat{T}_B) &\Rightarrow \tau_{S/2}; \quad t_k^\alpha(\hat{T}_B) \Rightarrow \tau_k^\alpha; \quad t_k^\beta(\hat{T}_B) \Rightarrow \tau_k^\beta; \\ F_k(\hat{T}_B) &\Rightarrow \mathcal{F}_k; \\ F_{1\ldots[S/2]}(\hat{T}_B) &\Rightarrow \mathcal{F}_{1\ldots[S/2]}; \\ F_{0\ldots[S/2]}(\hat{T}_B) &\Rightarrow \mathcal{F}_{0\ldots[S/2]}(\lambda_0); \end{aligned}$$

for $k = 1, \ldots, S^*$.

It follows that the critical values of the $t_{S/2}(\hat{T}_B)$, $t_k^\alpha(\hat{T}_B)$, $t_k^\beta(\hat{T}_B)$, $F_k(\hat{T}_B)$, and $F_{1\ldots[S/2]}(\hat{T}_B)$ tests are the same as in the no break case considered in the previous subsection which do not depend on the break fraction. However, for the $t_0(\hat{T}_B)$ and $F_{0\ldots[S/2]}(\hat{T}_B)$ tests some size distortions will arise if critical values for the no break case (Theorem 3.2) are used but the DGP has a break in the slope. Conversely, size distortions will also result if critical values for the case of a fixed break date are used (for instance, using the critical values from Theorem 3.1 or Corollary 3.1 with λ set equal to the estimated break fraction) but there is no break in the DGP. A similar result was obtained by Vogelsang and Perron (1998) in the case of DF-type tests for a unit root allowing for a break in the trend. However, as will be discussed in the simulation experiments presented in the next section, since for a wide range of possible values of λ the fixed break case critical values are very close to the no break critical values, such size distortions will in fact be quite small.

4 Critical Values, and Size and Power Simulations

In this section, critical values as well as finite sample size and power simulations in a quarterly context, $S = 4$, are presented for the seasonal LM type unit root tests discussed above. All simulation experiments are based on 5000 replications and data is generated from the following DGP:

$$y_{Sn+s} = \gamma_2 DT_{Sn+s}^0 + x_{Sn+s}, \tag{4.1}$$

$$(1 - \alpha L)(1 - \rho L^S)x_t = \left(1 + \theta L^4\right) e_t, \tag{4.2}$$

where e_t are i.i.d. $N(0, 1)$ random deviates.

To correct for the presence of autocorrelation, we augment regression (2.8) with lags of the dependent variable, as in Schmidt and Lee (1991) and Rodrigues (2002), such that the following test regression is considered

$$\begin{aligned} \Delta_S \widetilde{y}_{Sn+s} &= \pi_0 \widetilde{y}_{0,Sn+s-1} + \pi_{S/2} \widetilde{y}_{S/2,Sn+s-1} \\ &+ \sum_{k=1}^{S^*} \left(\pi_{\alpha,k} \widetilde{y}_{k,Sn+s-1}^\alpha + \pi_{\beta,k} \widetilde{y}_{k,Sn+s-1}^\beta \right) \\ &+ \sum_{j=1}^{p} \phi_j \Delta_S \widetilde{y}_{Sn+s-j} + \varepsilon_{Sn+s} \end{aligned} \tag{4.3}$$

where p is determined using the lag selection procedure suggested by Ng and Perron (1995) with maximum lag order $p^* = 5$. For tests based on an estimated break date, we considered the set $\Lambda = [0.1, 0.9]$.

4.1 Critical values

Table 4.1A presents critical values for the seasonal unit root test statistics when the null hypothesis, $\rho = 1$, holds with no break $\gamma_2 = 0$. Asymptotic critical values were obtained with $SN = 1000$ observations generated from (4.1)-(4.2) with $\gamma_2 = 0$, $\rho = 1$, $\alpha = \theta = 0$, and without correcting for autocorrelation, i.e., $p = 0$ in regression (4.3). In the cases of $t_0(T_B)$ and $F_{0\ldots[S/2]}(T_B)$ where a fixed break date $T_B = [\lambda SN]$ is assumed (Theorem 3.1 and Corollary 3.1), we present critical values for $\lambda = 0.1, ..., 0.9$. Note that the critical values of the LM type tests for unit roots at the seasonal frequencies are practically identical to the critical values presented in Rodrigues (2002, p.186, Table 1) for the corresponding HEGY$_{LM_0}$ tests, confirming in this way the asymptotic results put forward in Theorems 3.1 - 3.5.

Table 4.1A also presents finite sample critical values for $SN = 100$ and 200. These are obtained using a maximum lag order $p^* = 5$.

[**Insert Table 4.1 about here**]

4.2 Finite sample size and power

We now present the finite sample size and power results for $SN = 100$ and 200. Several values for γ_2, both under the null, $\rho = 1$, and under the alternative $\rho < 1$ are considered. For the cases where a break occurs, $\gamma_2 \neq 0$, the true break date was set to $T_B^0 = [\lambda_0 SN]$ with λ_0 taking different values. We consider 5% nominal size and use the finite sample critical values presented in Tables 4.1A and 4.1B.

4.2.1 Experiment A

In this experiment, we analyze the size and power performance of the test procedures by considering data generated from (4.1) and (4.2) with $\rho \in (1, 0.95, 0.8)$, $\gamma_2 \in (0, 1, 2)$, $\lambda_0 \in (0.25, 0.50, 0.75)$ and $\alpha = \theta = 0$. Results are presented in Tables 4.2 and 4.3.

[**Insert Tables 4.2 and 4.3 about here**]

We start by analyzing the size performance of the standard seasonal LM type unit root tests that do not allow for a break as considered in Rodrigues (2002). When a break is present in the DGP, the t_0 and the $F_{0...[S/2]}$ tests are severely undersized. For the seasonal frequencies unit root tests, $t_{S/2}$, F_k and $F_{1...[S/2]}$, only minor size distortions are observed in the presence of a break. Next we consider the size of the tests assuming a fixed break date at exactly the middle of the sample, $\lambda = 1/2$, and using the no break critical values from Theorem 3.1 and Corolary 3.1 (which coincide with the break case critical values if the true break date equals the assumed break date; see Theorem 3.5). As expected, when the true break date is also located in the middle of the sample, there are no size distortions. For different locations of the break, the results are very similar to the tests assuming no break: the t_0 and $F_{0...[S/2]}$ tests are undersized, and minor size distortions are observed for the $t_{S/2}$, F_k and $F_{0...[S/2]}$ tests.

Next, we consider the LM type tests based on the minimization/maximization of the test statistics using critical values for the no break case (from Theorem 3.3). There are significant size distortions for all tests considered in the presence of a break in the DGP. For the min t_0 and max $F_{0...[S/2]}$ tests we also considered using critical values assuming that the estimated break date equals the true break date. However, size distortions remain for the break cases and also appear when the DGP has no break.

Finally, we consider the LM type tests based on the least squares estimator of the break date $\hat{T}_B$. As in the previous case, we start by using critical values for the no break case (considered in Theorem 3.2). The empirical size is correct in almost all cases. As expected, the only case where some minor undersizing is present is for the $t_0(\hat{T}_B)$ and $F_{0...[S/2]}(\hat{T}_B)$ tests when a break is present in the DGP because the no break critical values are being used. Finally, for these two tests, $t_0(\hat{T}_B)$ and $F_{0...[S/2]}(\hat{T}_B)$, we also considered using critical values assuming that the estimated break date equals the true break date. A slight oversizing occurs when the DGP has no break which is reduced as the magnitude of the break increases.

Regarding the power of the procedures, we observe that for all tests considered, the power of the zero frequency t_0 test is low. And when a break is present in the DGP, power generally approaches zero in all cases except for the $t_0(\hat{T}_B)$ test. For the $F_{0...[S/2]}$ case, the only test that performs well in both the break and no break cases is the $F_{0...[S/2]}(\hat{T}_B)$ test. In fact, although the standard $F_{0...[S/2]}$ test ranks first in terms of power when no break is present, its power decreases dramatically in the presence of a break. Nonetheless, it is important to notice that the power of the $t_0(\hat{T}_B)$ and $F_{0...[S/2]}(\hat{T}_B)$ tests is inferior to the power of the corresponding standard tests when no break is present in the DGP. For the seasonal frequencies unit root tests, power is very similar for all variants of the tests when no break is present. However, when a break is present, the standard no break LM type tests $t_{S/2}$, F_k and $F_{0...[S/2]}$, all have very low power. The tests based on the minimization/maximization of the test statistics seem to perform better in the presence of a break. However, such a result is misleading as it is caused by the oversizing problems of these tests in the presence of a break in the DGP. The tests with correct size that perform better overall are the ones based on $\hat{T}_B$.

We also considered the standard HEGY tests which do not allow for a break. Results appear in the first rows of Tables 4.4 and 4.5. In terms of size performance, its results are very similar to the standard LM type tests. However, there are some differences in terms of power performance. For the HEGY zero frequency test, power is also quite low when no break is present, and close to zero if a break is present. For the $F_{0...[S/2]}$ case, when no break is present, power is similar to the LM variants of the tests. When a break is present, although power does not decrease as much as in the LM variant, it is still inferior to the $F_{0...[S/2]}(\hat{T}_B)$ test. Similar power results hold for the remaining seasonal frequencies HEGY tests.

Finally, we also study a modification of the HEGY test allowing for a break in slope based on the following steps: (i) first, the original series is detrended based on a regression in levels:

$$y_{Sn+s} = \phi_1 + \phi_2 t + \phi_3 DT_{Sn+s} + v_{Sn+s}, \tag{4.4}$$

(ii) second, the usual HEGY tests are applied to the resulting residuals. Therefore, in this HEGY variant, the least-squares estimator of the break date is computed from a regression in levels. Simulated critical values for all the HEGY variants of the tests allowing for a break in trend appear in Table 4.1B. The simulation results, presented in Tables 4.4 and 4.5, show that the LM type approach put forward in this paper is in general more powerful than the HEGY approach.

[**Insert Tables 4.4-4.5 about here**]

4.2.2 Experiment B

In this experiment we evaluate the size performance of the LM and HEGY type seasonal unit root tests allowing for AR or MA short-run dynamics in the errors. We consider data generated from (4.1) and (4.2) with $\rho = 1$, $\gamma_2 \in (0, 1, 2)$, $\lambda_0 = 0.50$, $\alpha \in (0, 0.8)$ and $\theta \in (0, -0.8, 0.8)$.

[**Insert Tables 4.6-4.9 about here**]

Tables 4.6-4.9 provide the empirical size of the test procedures. From these tables we observe that the procedures are generally well behaved if autocorrelation is of the AR type. The number of lags considered for augmentation of the test regression is clearly sufficient to account for the short-run dependence. Unfortunately, this is not the case when autocorrelation is of the MA type and in particular when θ is strongly negative. We observe that in this case the test procedures are severely oversized. Furthermore, comparing the LM type procedures with the HEGY type approach we observe that the oversizing is worse in the latter. In general, these results are in accordance with those obtained in Rodrigues (2002) for the no break tests.

5 Conclusion

In this paper new seasonal LM type unit root tests statistics are derived allowing for a break in the trend slope, generalizing the results in Rodrigues (2002) and Hassler and Rodrigues (2004). A Monte Carlo investigation shows evidence of power advantages of the new LM type statistics allowing for a trend break over the standard LM or HEGY type versions. The LM type tests based on the least-squares break date estimator, $\hat{T}_B$, perform best overall. A possible improvement that could be explored in future work is the use of a pre-test for the presence of a break in order to determine if one should apply a standard seasonal unit root test or a test allowing for a break. This could be specially relevant to increase the power of the zero frequency t_0 and the overall $F_{0...[S/2]}$ tests in the case where no break is present in the DGP.

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Appendix A

Throughout this Appendix we assume that the DGP is given by (2.1)-(2.2) under the null hypothesis (2.3) of seasonal integration. Note that the break date considered, $T_B = [\lambda SN]$, may not necessarily coincide with the true break date, $T_B^0 = [\lambda_0 SN]$. This possibility as well as the case when the two dates coincide are accounted for in the derivations of the limit results. We first present some preliminary notation. The superscript 0 will be used to indicate that the true break date T_B^0 is considered in the construction of a variable.

Seasonally differencing (2.1) we observe that

$$\Delta_S y_{Sn+s} = \gamma_1 S + \gamma_2 \Delta_S DT^0_{Sn+s} + \Delta_S x_{Sn+s} \tag{A.1}$$

which can be written in matrix notation as

$$\Delta_S Y = \Delta_S Z^0 \boldsymbol{\gamma} + \Delta_S X \tag{A.2}$$

where $\Delta_S Y = (\Delta_S y_{S+1}, \ldots, \Delta_S y_{SN})'$, $\Delta_S Z^0 = (\Delta_S Z^0_{S+1}{}', \ldots, \Delta_S Z^0_{SN}{}')'$, $\Delta_S X = (\Delta_S x_{S+1}, \ldots, \Delta_S x_{SN})'$, $Z^0_{Sn+s} = (Sn+s, DT^0_{Sn+s})$, $\Delta_S Z^0_{Sn+s} = (S, \Delta_S DT^0_{Sn+s})$ and $\boldsymbol{\gamma} = (\gamma_1, \gamma_2)'$.

Moreover, considering $Z_{Sn+s} = (Sn+s, DT_{Sn+s})$ and $\Delta_S Z_{Sn+s} = (S, \Delta_S DT_{Sn+s})$, the first step regression given in (2.5) can also be written in matrix notation as

$$\Delta_S Y = \Delta_S Z \boldsymbol{\gamma} + U \tag{A.3}$$

where $\Delta_S Z = (\Delta_S Z_{S+1}{}', \ldots, \Delta_S Z_{SN}{}')'$ and $U = (u_{S+1}, \ldots, u_{SN})'$.

We also define $Z = (Z_{S+1}{}', \ldots, Z_{SN}{}')'$, $Z^0 = (Z^0_{S+1}{}', \ldots, Z^0_{SN}{}')'$, $Z_{-1} = (Z_S{}', \ldots, Z_{SN-1}{}')'$ and $Z^0_{-1} = (Z^0_S{}', \ldots, Z^0_{SN-1}{}')'$.

The least squares estimator of γ in (A.3) is given by

$$\tilde{\boldsymbol{\gamma}} = (\Delta_S Z' \Delta_S Z)^{-1} \Delta_S Z' \Delta_S Y. \tag{A.4}$$

From (A.2) we can rewrite (A.4) as

$$\begin{aligned} \tilde{\boldsymbol{\gamma}} &= (\Delta_S Z' \Delta_S Z)^{-1} \Delta_S Z' \Delta_S Z^0 \boldsymbol{\gamma} + (\Delta_S Z' \Delta_S Z)^{-1} \Delta_S Z' \Delta_S X \\ &= H_N \boldsymbol{\gamma} + (\Delta_S Z' \Delta_S Z)^{-1} \Delta_S Z' \Delta_S X \end{aligned} \tag{A.5}$$

where $H_N \equiv (\Delta_S Z' \Delta_S Z)^{-1} \Delta_S Z' \Delta_S Z^0$.

Denoting the vector of least squares residuals from equation (A.3) as $\Delta_S \widetilde{Y} = (\Delta_S \widetilde{y}_{S+1}, \ldots, \Delta_S \widetilde{y}_{SN})'$ and defining the orthogonal projection matrix $M = I - \Delta_S Z (\Delta_S Z' \Delta_S Z)^{-1} \Delta_S Z'$ we have that

$$\Delta_S \widetilde{Y} = M \Delta_S Y. \tag{A.6}$$

Given the definition of $\widetilde{y}_{Sn+s}$ in (2.6) and using (2.1) we also note that,

$$\widetilde{y}_{Sn+s} = x_{Sn+s} - x_{S+s} + (Z^0_{Sn+s} - Z^0_{S+s}) \boldsymbol{\gamma} - (Z_{Sn+s} - Z_{S+s}) \tilde{\boldsymbol{\gamma}} \tag{A.7}$$

or alternatively,

$$\begin{aligned} \widetilde{y}_{Sn+s} = \;& (x_{Sn+s} - x_{S+s}) + (Z^0_{Sn+s} - Z^0_{S+s}) \boldsymbol{\gamma} \\ & - (Z_{Sn+s} - Z_{S+s})(\tilde{\boldsymbol{\gamma}} - H_N \boldsymbol{\gamma}) - (Z_{Sn+s} - Z_{S+s}) H_N \boldsymbol{\gamma}. \end{aligned} \tag{A.8}$$

Before presenting the proofs of the Theorems in the text, it will be convenient to consider the following preparatory Lemmata concerning limit results as $N \to \infty$.

Lemma A.1 *The following results hold:*

$$i)\ \frac{1}{SN}(Z_{S[Nr]+s} - Z_{S+s}) \to \left[r, (r-\lambda) 1_{(r>\lambda)}\right] \equiv q(r;\lambda), \tag{A.9}$$

$$ii)\ \Delta_S Z_{S[Nr]+s} \to S\left[1, 1_{(r>\lambda)}\right] \equiv S g(r;\lambda), \tag{A.10}$$

$$iii)\ \frac{1}{SN}\Delta_S Z'\Delta_S Z \to S^2\begin{bmatrix} 1 & 1-\lambda \\ 1-\lambda & 1-\lambda \end{bmatrix} \equiv S^2 Q(\lambda), \tag{A.11}$$

$$iv)\ \frac{1}{SN}\Delta_S Z'\Delta_S Z^0 \to S^2\begin{bmatrix} 1 & 1-\lambda_0 \\ 1-\lambda & 1-\max(\lambda,\lambda_0) \end{bmatrix} \equiv S^2 R(\lambda,\lambda_0), \tag{A.12}$$

$$v)\ \Delta_i Z_{S[Nr]+s} \to -c_i g(r;\lambda),\ \ for\ i = (S/2), \binom{\alpha}{k}, \binom{\beta}{k}, \tag{A.13}$$

$$vi)\ \frac{1}{SN}\Delta_S Z'\Delta_i Z_{-1} \to -c_i S Q(\lambda),\ \ for\ i = (S/2), \binom{\alpha}{k}, \binom{\beta}{k}, \tag{A.14}$$

$$vii)\ \frac{1}{SN}\Delta_S Z'\Delta_i Z^0_{-1} \to -c_i S R(\lambda,\lambda_0),\ \ for\ i = (S/2), \binom{\alpha}{k}, \binom{\beta}{k}, \tag{A.15}$$

with $c_{S/2} = \sum_{j=1}^{S} j\cos(j\pi)$, $c_k^\alpha = \sum_{j=1}^{S} j\cos(j\omega_k)$ *and* $c_k^\beta = -\sum_{j=1}^{S} j\sin(j\omega_k)$.

Proof of Lemma A.1:

We begin with the proof of $i)$. Given the definition of Z_{Sn+s} and recalling that $DT_{Sn+s} = (Sn+s-T_B)1_{(Sn+s>T_B)}$, we obtain

$$\begin{aligned}
\frac{1}{SN}(Z_{S[Nr]+s} - Z_{S+s}) &= \frac{1}{SN}\left(S[Nr]+s, DT_{S[Nr]+s}\right) - \frac{1}{SN}\left(S+s, DT_{S+s}\right) \\
&= \left(\frac{S[Nr]+s}{SN}, \frac{DT_{S[Nr]+s}}{SN}\right) + o(1) \\
&\to \left[r, (r-\lambda)1_{(r>\lambda)}\right] \equiv q(r;\lambda).
\end{aligned}$$

Regarding the results in $ii) - iv)$ we have that,

$$\begin{aligned}
\Delta_S Z_{S[Nr]+s} &= \left(S, \Delta_S DT_{S[Nr]+s}\right) \\
&\to S(1, 1_{(r>\lambda)}) \equiv S g(r;\lambda);
\end{aligned}$$

$$\begin{aligned}
\frac{1}{SN}\Delta_S Z'\Delta_S Z &= \frac{1}{SN}\sum_{s=1-S}^{0}\sum_{n=2}^{N}\Delta_S Z_{Sn+s}{}'\Delta_S Z_{Sn+s} \\
&\to S^2\begin{bmatrix} 1 & 1-\lambda \\ 1-\lambda & 1-\lambda \end{bmatrix} \equiv S^2 Q(\lambda);
\end{aligned}$$

and

$$\begin{aligned}
\frac{1}{SN}\Delta_S Z'\Delta_S Z^0 &= \frac{1}{SN}\sum_{s=1-S}^{0}\sum_{n=2}^{N}\Delta_S Z_{Sn+s}{}'\Delta_S Z^0_{Sn+s} \\
&\to S^2\begin{bmatrix} 1 & 1-\lambda_0 \\ 1-\lambda & 1-\max(\lambda,\lambda_0) \end{bmatrix} \equiv S^2 R(\lambda,\lambda_0).
\end{aligned}$$

Regarding $v)$ we start with the case of $i = S/2$, so that, given the definition of $\Delta_{S/2}$ in (2.10), we have

$$\Delta_{S/2} Z_{S[Nr]+s} = \Delta_{S/2}(S[Nr]+s, (S[Nr]+s-T_B)1_{(S[Nr]+s>T_B)}).$$

Note that

$$\begin{aligned}
\Delta_{S/2}(S[Nr]+s) &= \sum_{j=0}^{S-1}\cos[(j+1)\pi](S[Nr]+s-j) \\
&= \sum_{j=0}^{S-1}\cos[(j+1)\pi](S[Nr]+s+1) - \sum_{j=0}^{S-1}\cos[(j+1)\pi](j+1) \\
&= (S[Nr]+s+1)\sum_{j=0}^{S-1}\cos[(j+1)\pi] - \sum_{j=0}^{S-1}\cos[(j+1)\pi](j+1) \\
&= 0 - \sum_{j=1}^{S} j\cos(j\pi) \\
&= -c_{S/2}.
\end{aligned}$$

It is also easy to derive that $\Delta_{S/2}((S[Nr]+s-T_B)1_{(S[Nr]+s>T_B)}) \to -c_{S/2}1_{(r>\lambda)}$. Therefore we have that $\Delta_{S/2}Z_{S[Nr]+s} \to -c_{S/2}(1, 1_{(r>\lambda)}) = -c_{S/2}g(r;\lambda)$. The cases $i = \binom{\alpha}{k}, \binom{\beta}{k}$ are derived similarly. The proofs of *vi*) and *vii*) are very similar to the previous ones and are omitted. ■

Lemma A.2 *The following result holds:*

$$\frac{1}{(SN)^{1/2}} x_{S[Nr]+s} \Rightarrow \frac{\sigma}{\sqrt{S}} W_s(r), \;\; s = -S+1, ..., 0 \tag{A.16}$$

where $\Rightarrow$ denotes weak convergence and $W_s(r)$ are independent standard Brownian motions.

Proof of Lemma A.2: This result follows from the FCLT (see Phillips, 1988, Phillips and Perron, 1988 and Chan and Wei, 1988).

Lemma A.3 *The following results hold:*

$$i)\frac{1}{(SN)^{1/2}} \Delta_{S/2} x_{S[Nr]+s} \Rightarrow \frac{\sigma}{\sqrt{S}}(-1)^s \tilde{W}_{S/2}(r)$$

$$ii)\frac{1}{(SN)^{1/2}} \Delta_k^{\alpha} x_{S[Nr]+s} \Rightarrow \frac{\sigma}{\sqrt{S}}\left[\cos(s\omega_k)\tilde{W}_k^{\alpha}(r) + \sin(s\omega_k)\tilde{W}_k^{\beta}(r)\right],$$

$$iii)\frac{1}{(SN)^{1/2}} \Delta_k^{\beta} x_{S[Nr]+s} \Rightarrow -\frac{\sigma}{\sqrt{S}}\left[\sin(s\omega_k)\tilde{W}_k^{\alpha}(r) - \cos(s\omega_k)\tilde{W}_k^{\beta}(r)\right].$$

$$iv)\;\frac{1}{(SN)^{1/2}} \Delta_0 x_{S[Nr]+s} \Rightarrow \frac{\sigma}{\sqrt{S}}\tilde{W}_0(r)$$

where $\tilde{W}_0, \tilde{W}_{S/2}, \tilde{W}_k^{\alpha}, \tilde{W}_k^{\beta}$ are independent Brownian motions as defined in Theorem 3.1.

Proof of Lemma A.3: Consider first *i*). From the definition of $\Delta_{S/2}$ in (2.10) we have

$$\begin{aligned}
\frac{1}{(SN)^{1/2}} \Delta_{S/2} x_{S[Nr]+s} &= \frac{1}{(SN)^{1/2}} \sum_{j=0}^{S-1} \cos[(j+1)\pi] x_{S[Nr]+s-j} \\
&\Rightarrow \frac{\sigma}{\sqrt{S}} \sum_{j=0}^{S-1} \cos((j+1+s)\pi) W_{-j}(r) \\
&= \frac{\sigma}{\sqrt{S}} \sum_{j=0}^{S-1} (\cos(s\pi)\cos((j+1)\pi) - \sin(s\pi)\sin((j+1)\pi))\, W_{-j}(r) \\
&= \frac{\sigma}{\sqrt{S}} \sum_{j=0}^{S-1} \cos(s\pi)\cos((j+1)\pi) W_{-j}(r) \\
&= \frac{\sigma}{\sqrt{S}} (-1)^s \sum_{j=0}^{S-1} \cos((j+1)\pi) W_{-j}(r) \\
&= \frac{\sigma}{\sqrt{S}} (-1)^s \tilde{W}_{S/2}(r),
\end{aligned}$$

where the second line follows from Lemma **A.2** and the CMT. The proofs of *ii*) and *iii*) follow along similar lines, *i.e.*,

$$\begin{aligned}
\frac{1}{(SN)^{1/2}} \Delta_k^{\alpha} x_{S[Nr]+s} &= \frac{1}{(SN)^{1/2}} \sum_{j=0}^{S-1} \cos[(j+1)\omega_k] x_{S[Nr]+s-j} \\
&\Rightarrow \frac{\sigma}{\sqrt{S}} \sum_{j=0}^{S-1} \cos\left[(j+1+s)\,\omega_k\right] W_{-j}(r;\lambda) \\
&= \frac{\sigma}{\sqrt{S}} \sum_{j=0}^{S-1} \left[\cos(s\omega_k)\cos((j+1)\omega_k) - \sin(s\omega_k)\sin((j+1)\omega_k)\right] W_{-j}(r;\lambda) \\
&= \frac{\sigma}{\sqrt{S}} \left[\cos(s\omega_k)\tilde{W}_k^{\alpha}(r) + \sin(s\omega_k)\tilde{W}_k^{\beta}(r)\right],
\end{aligned}$$

and

$$\frac{1}{(SN)^{1/2}}\Delta_k^{\beta}x_{S[Nr]+s} = -\frac{1}{(SN)^{1/2}}\sum_{j=0}^{S-1}\sin[(j+1)\omega_k]x_{S[Nr]+s-j}$$
$$\Rightarrow -\frac{\sigma}{\sqrt{S}}\sum_{j=0}^{S-1}\sin\left[(j+1+s)\,\omega_k\right]W_{-j}(r)$$
$$= -\frac{\sigma}{\sqrt{S}}\sum_{j=0}^{S-1}\left[\sin(s\omega_k)\cos((j+1)\omega_k)+\cos(s\omega_k)\sin((j+1)\omega_k\right]W_{-j}(r)$$
$$= -\frac{\sigma}{\sqrt{S}}\left[\sin(s\omega_k)\tilde{W}_k^{\alpha}(r)-\cos(s\omega_k)\tilde{W}_k^{\beta}(r)\right].$$

Finally

$$\frac{1}{(SN)^{1/2}}\Delta_0 x_{S[Nr]+s} = \frac{1}{(SN)^{1/2}}\sum_{j=0}^{S-1}x_{S[Nr]+s-j}$$
$$\Rightarrow \frac{\sigma}{\sqrt{S}}\tilde{W}_0(r).$$

■

Lemma A.4 *The following results hold:*

$$i)\ \frac{1}{(SN)^{1/2}}\Delta_S Z'\Delta_S X = \frac{1}{(SN)^{1/2}}\sum_{s=1-S}^{0}\sum_{n=2}^{N}\Delta_S Z_{Sn+s}{}'\Delta_S x_{Sn+s}$$
$$\Rightarrow \frac{\sigma}{\sqrt{S}}\int_0^1 Sg(r;\lambda)'d\tilde{W}_0(r) \tag{A.17}$$

$$ii)\ \frac{1}{(SN)^{1/2}}\Delta_i Z_{-1}{}'\Delta_S X = \frac{1}{(SN)^{1/2}}\sum_{s=1-S}^{0}\sum_{n=2}^{N}\Delta_i Z_{Sn+s-1}{}'\Delta_S x_{Sn+s}$$
$$\Rightarrow -\frac{\sigma}{\sqrt{S}}\int_0^1 c_i g(r;\lambda)'d\tilde{W}_0(r)\ \textit{for}\ i=(S/2),\binom{\alpha}{k},\binom{\beta}{k} \tag{A.18}$$

Proof of Lemma A.4: Result *i)* follows directly from Lemmas A.1 *ii)* and A.2. Result *ii)* follows from Lemmas A.1 *v)* and A.2.

Lemma A.5 *The following results hold for $\widetilde{y}_{Sn+s}$ defined in (2.6). i) When $\gamma_2 = 0$ or $\lambda = \lambda_0$:*

$$\frac{1}{(SN)^{1/2}}\widetilde{y}_{S[Nr]+s} \Rightarrow \frac{\sigma}{\sqrt{S}}V_s(r;\lambda), \tag{A.19}$$

and ii) when $\gamma_2 \neq 0$ and $\lambda \neq \lambda_0$:

$$\frac{1}{SN}\widetilde{y}_{S[Nr]+s} \xrightarrow{p} f(r;\lambda,\lambda_0)\gamma, \tag{A.20}$$

where $V_s(r;\lambda) \equiv W_s(r) - q(r;\lambda)S^{-1}Q(\lambda)^{-1}\int_0^1 g(r;\lambda)'d\tilde{W}_0(r)$ and $f(r;\lambda,\lambda_0) \equiv q(r;\lambda_0) - q(r;\lambda)Q(\lambda)^{-1}$ $R(\lambda,\lambda_0)$.

Proof of Lemma A.5:

Given our assumption on the starting values (see Section 2) we have that,

$$\frac{1}{(SN)^{1/2}}x_{S+s} \xrightarrow{p} 0,\ \ s = 1-S,...,0. \tag{A.21}$$

Using (A.11) and (A.17) in (A.5) we obtain

$$(SN)^{1/2}(\tilde{\boldsymbol{\gamma}} - H_N\boldsymbol{\gamma}) = (\frac{1}{SN}\Delta_S Z'\Delta_S Z)^{-1}\frac{1}{(SN)^{1/2}}\Delta_S Z'\Delta_S X$$
$$\Rightarrow \frac{\sigma}{\sqrt{S}} S^{-2}Q(\lambda)^{-1}\int_0^1 Sg(r;\lambda)' d\tilde{W}_0(r). \quad \text{(A.22)}$$

Consider first the case when $\gamma_2 = 0$ or $\lambda = \lambda_0$, i.e. Lemma A.5 i). It follows, when $\lambda = \lambda_0$, that $H_N = I$ and when $\lambda \neq \lambda_0$ but $\gamma_2 = 0$ that $H_N\boldsymbol{\gamma} = \boldsymbol{\gamma}$. Thus, in both cases, $H_N\boldsymbol{\gamma} = \boldsymbol{\gamma}$, so that from (A.22) we have

$$(SN)^{1/2}(\tilde{\boldsymbol{\gamma}} - \boldsymbol{\gamma}) \Rightarrow \frac{\sigma}{\sqrt{S}} S^{-2}Q(\lambda)^{-1}\int_0^1 Sg(r;\lambda)' d\tilde{W}_0(r). \quad \text{(A.23)}$$

In this context, we also have $Z^0\boldsymbol{\gamma} = Z\boldsymbol{\gamma}$, so that from (A.7) we obtain:

$$\widetilde{y}_{Sn+s} = (x_{Sn+s} - x_{S+s}) - (Z_{Sn+s} - Z_{S+s})(\tilde{\boldsymbol{\gamma}} - \boldsymbol{\gamma}). \quad \text{(A.24)}$$

Using (A.9), (A.16), (A.21) and (A.23) in (A.24), we establish that

$$\begin{aligned} \frac{1}{(SN)^{1/2}}\widetilde{y}_{S[Nr]+s} &\Rightarrow \frac{\sigma}{\sqrt{S}}W_s(r) - \frac{\sigma}{\sqrt{S}}q(r;\lambda)S^{-2}Q(\lambda)^{-1}\int_0^1 Sg(r;\lambda)'d\tilde{W}_0(r) \\ &\equiv \frac{\sigma}{\sqrt{S}}V_s(r;\lambda) \end{aligned}$$

proving (A.19).

Consider now the case where $\gamma_2 \neq 0$ and $\lambda \neq \lambda_0$, i.e. Lemma A.5 ii). From (A.11) and (A.12) we have that

$$\begin{aligned} H_N &\rightarrow S^{-2}Q(\lambda)^{-1}S^2R(\lambda,\lambda_0) \\ &= Q(\lambda)^{-1}R(\lambda,\lambda_0). \end{aligned} \quad \text{(A.25)}$$

Using the results from (A.9), (A.16) (A.21), (A.22) and (A.25) in (A.8), it follows that

$$\frac{1}{SN}\widetilde{y}_{S[Nr]+s} \rightarrow^p \left[q(r;\lambda_0) - q(r;\lambda)Q(\lambda)^{-1}R(\lambda,\lambda_0)\right]\boldsymbol{\gamma} \equiv f(r;\lambda,\lambda_0)\boldsymbol{\gamma}$$

proving (A.20). ■

Lemma A.6 *The following results hold,*

$$\begin{aligned} &i)\ \frac{1}{(SN)^{1/2}}\widetilde{y}_{S/2,S[Nr]+s} \Rightarrow \frac{\sigma}{\sqrt{S}}(-1)^s\tilde{W}_{S/2}(r), \\ &ii)\ \frac{1}{(SN)^{1/2}}\widetilde{y}^{\alpha}_{k,S[Nr]+s} \Rightarrow \frac{\sigma}{\sqrt{S}}\left[\cos(s\omega_k)\tilde{W}^{\alpha}_k(r) + \sin(s\omega_k)\tilde{W}^{\beta}_k(r)\right], \\ &iii)\ \frac{1}{(SN)^{1/2}}\widetilde{y}^{\beta}_{k,S[Nr]+s} \Rightarrow -\frac{\sigma}{\sqrt{S}}\left[\sin(s\omega_k)\tilde{W}^{\alpha}_k(r) - \cos(s\omega_k)\tilde{W}^{\beta}_k(r)\right]. \end{aligned}$$

For the zero frequency note that for $\gamma_2 = 0$ or $\lambda = \lambda_0$,

$$iv)\ \frac{1}{(SN)^{1/2}}\widetilde{y}_{0,S[Nr]+s} \Rightarrow \frac{\sigma}{\sqrt{S}}\tilde{V}_0(r;\lambda)$$

and for $\gamma_2 \neq 0$ and $\lambda \neq \lambda_0$,

$$v)\ \frac{1}{SN}\widetilde{y}_{0,S[Nr]+s} \xrightarrow{p} Sf(r;\lambda,\lambda_0)\gamma$$

with $\tilde{W}_{S/2}(r)$, $\tilde{W}^{\alpha}_k(r)$, $\tilde{W}^{\beta}_k(r)$ and $\tilde{V}_0(r;\lambda)$ as defined in Theorem 3.1.

Proof of Lemma A.6:

We first note that independently of whether $\widetilde{y}_{Sn+s}$ is written as in (A.7) or (A.8), the filtering (2.9)-(2.12) used to construct $\widetilde{y}_{S/2,Sn+s}$, $\widetilde{y}^{\alpha}_{k,Sn+s}$ and $\widetilde{y}^{\beta}_{k,Sn+s}$, transform any deterministic trend components in $\widetilde{y}_{Sn+s}$ into constants. It follows that

$$\begin{aligned}
\frac{1}{(SN)^{1/2}}\widetilde{y}_{S/2,S[Nr]+s} &= \frac{1}{(SN)^{1/2}}\sum_{j=0}^{S-1}\cos[(j+1)\pi]\widetilde{y}_{S[Nr]+s-j} \\
&= \frac{1}{(SN)^{1/2}}\sum_{j=0}^{S-1}\cos[(j+1)\pi]x_{S[Nr]+s-j} + o_p(1) \\
&\Rightarrow \frac{\sigma}{\sqrt{S}}(-1)^s\tilde{W}_{S/2}(r),
\end{aligned}$$

where the third line follows from Lemma A.3. The proofs of *ii*) and *iii*) follow along similar lines, *i.e.*,

$$\begin{aligned}
\frac{1}{(SN)^{1/2}}\widetilde{y}^{\alpha}_{k,S[Nr]+s} &= \frac{1}{(SN)^{1/2}}\sum_{j=0}^{S-1}\cos[(j+1)\omega_k]\widetilde{y}_{S[Nr]+s-j} \\
&= \frac{1}{(SN)^{1/2}}\sum_{j=0}^{S-1}\cos[(j+1)\omega_k]x_{S[Nr]+s-j} + o_p(1) \\
&\Rightarrow \frac{\sigma}{\sqrt{S}}\left[\cos(s\omega_k)\tilde{W}^{\alpha}_k(r) + \sin(s\omega_k)\tilde{W}^{\beta}_k(r)\right],
\end{aligned}$$

and

$$\begin{aligned}
\frac{1}{(SN)^{1/2}}\widetilde{y}^{\beta}_{k,S[Nr]+s} &= -\frac{1}{(SN)^{1/2}}\sum_{j=0}^{S-1}\sin[(j+1)\omega_k]\widetilde{y}_{S[Nr]+s-j} \\
&= -\frac{1}{(SN)^{1/2}}\sum_{j=0}^{S-1}\sin[(j+1)\omega_k]x_{S[Nr]+s-j} + o_p(1) \\
&\Rightarrow -\frac{\sigma}{\sqrt{S}}\left[\sin(s\omega_k)\tilde{W}^{\alpha}_k(r) - \cos(s\omega_k)\tilde{W}^{\beta}_k(r)\right].
\end{aligned}$$

From Lemma A.5, when $\gamma_2 = 0$ or $\lambda = \lambda_0$, it follows that

$$\begin{aligned}
\frac{1}{(SN)^{1/2}}\widetilde{y}_{0,S[Nr]+s} &= \frac{1}{(SN)^{1/2}}\sum_{j=0}^{S-1}\widetilde{y}_{S[Nr]+s-j} \\
&\Rightarrow \frac{\sigma}{\sqrt{S}}\sum_{j=0}^{S-1}V_{-j}(r;\lambda) \\
&= \frac{\sigma}{\sqrt{S}}\left\{\tilde{W}_0(r) - q(r;\lambda)Q(\lambda)^{-1}\int_0^1 g(r;\lambda)'d\tilde{W}_0(r)\right\} \\
&\equiv \frac{\sigma}{\sqrt{S}}\tilde{V}_0(r;\lambda)
\end{aligned}$$

and when $\gamma_2 \neq 0$ and $\lambda \neq \lambda_0$, we observe that

$$\begin{aligned}
\frac{1}{SN}\widetilde{y}_{0,S[Nr]+s} &= \frac{1}{(SN)}\sum_{j=0}^{S-1}\widetilde{y}_{S[Nr]+s-j} \\
&\xrightarrow{p} \sum_{j=0}^{S-1} f(r;\lambda,\lambda_0)\gamma = Sf(r;\lambda,\lambda_0)\gamma.
\end{aligned}$$

■

Lemma A.7 *The following results hold.*
i) When $\gamma_2 = 0$ or $\lambda = \lambda_0$:

$$\frac{1}{(SN)^2}\sum_{Sn+s=S+1}^{SN}\widetilde{y}^2_{0,Sn+s-1} \Rightarrow \frac{\sigma^2}{S}\int_0^1\left[\tilde{V}_0(r;\lambda)\right]^2 dr. \tag{A.26}$$

ii) When $\gamma_2 \neq 0$ and $\lambda \neq \lambda_0$,

$$\frac{1}{(SN)^3}\sum_{Sn+s=S+1}^{SN}\widetilde{y}^2_{0,Sn+s-1} \xrightarrow{p} S^2\gamma'\left(\int_0^1 f(r;\lambda,\lambda_0)'f(r;\lambda,\lambda_0)dr\right)\gamma. \tag{A.27}$$

Proof of Lemma A.7: The result in (A.26) follows from Lemma A.6 *iv*):

$$\begin{aligned}\frac{1}{(SN)^2}\sum_{Sn+s=S+1}^{SN}\widetilde{y}^2_{0,Sn+s-1} &= \frac{1}{SN}\sum_{Sn+s=S+1}^{SN}\left(\frac{1}{(SN)^{1/2}}\sum_{j=0}^{S-1}\widetilde{y}_{Sn+s-j}\right)^2 + o_p(1)\\ &\Rightarrow \frac{\sigma^2}{S}\int_0^1\left[\tilde{V}_0(r;\lambda)\right]^2 dr.\end{aligned}$$

Regarding (A.27), from Lemma A.6 v) we have

$$\begin{aligned}\frac{1}{(SN)^3}\sum_{Sn+s=S+1}^{SN}\widetilde{y}^2_{0,Sn+s-1} &= \frac{1}{SN}\sum_{Sn+s=S+1}^{SN}\left(\frac{1}{SN}\sum_{j=0}^{S-1}\widetilde{y}_{Sn+s-j}\right)^2 + o_p(1)\\ &\Rightarrow S^2\int_0^1\gamma' f(r;\lambda,\lambda_0)'f(r;\lambda,\lambda_0)\gamma dr.\end{aligned}$$

■

Lemma A.8 *The following results hold:*

$$\frac{1}{(SN)^2}\sum_{Sn+s=S+1}^{SN}\widetilde{y}^2_{S/2,Sn+s-1} \Rightarrow \frac{\sigma^2}{S}\int_0^1\left[\tilde{W}_{S/2}(r)\right]^2 dr$$

and

$$\begin{aligned}\frac{1}{(SN)^2}\sum_{Sn+s=S+1}^{SN}\left(\widetilde{y}^{\alpha}_{k,Sn+s-1}\right)^2 &\equiv^a \frac{1}{(SN)^2}\sum_{Sn+s=S+1}^{SN}\left(\widetilde{y}^{\beta}_{k,Sn+s-1}\right)^2\\ &\Rightarrow \frac{\sigma^2}{2S}\int_0^1\left\{\left[\tilde{W}^{\alpha}_k(r)\right]^2+\left[\tilde{W}^{\beta}_k(r)\right]^2\right\}dr,\ k=1,...,S^*.\end{aligned}$$

Proof of Lemma A.8:

Based on Lemma A.6 i), it follows that

$$\begin{aligned}\frac{1}{(SN)^2}\sum_{Sn+s=S+1}^{SN}\widetilde{y}^2_{S/2,Sn+s-1} &= \frac{1}{SN}\sum_{Sn+s=S+1}^{SN}\left(\frac{1}{(SN)^{1/2}}\sum_{j=0}^{S-1}(-1)^{j+1}\widetilde{y}_{Sn+s-j}\right)^2 + o_p(1)\\ &\Rightarrow \frac{\sigma^2}{S}\int_0^1\left[W_{S/2}(r)\right]^2 dr\end{aligned}$$

Similarly from Lemma A.6 v) we get

$$\begin{aligned}\frac{1}{(SN)^2}\sum_{Sn+s=S+1}^{SN}\left(\widetilde{y}_{k,Sn+s-1}^{\alpha}\right)^2 &= \frac{1}{SN}\sum_{Sn+s=S+1}^{SN}\left(\frac{1}{(SN)^{1/2}}\sum_{j=0}^{S-1}\cos\left[(j+1)\omega_k\right]\widetilde{y}_{Sn+s-j}\right)^2 + o_p(1). \\ &\Rightarrow \frac{\sigma^2}{S^2}\int_0^1\sum_{j=1-S}^{0}\left\{\cos(j\omega_k)\tilde{W}_k^{\alpha}(r)+\sin(j\omega_k)\tilde{W}_k^{\beta}(r)\right\}^2 dr \\ &\equiv \frac{\sigma^2}{S^2}\int_0^1\sum_{j=1-S}^{0}\left\{\cos^2(j\omega_k)\left[\tilde{W}_k^{\alpha}(r)\right]^2+\sin^2(j\omega_k)\left[\tilde{W}_k^{\beta}(r)\right]^2\right\} dr\end{aligned}$$

and since $\sum\limits_{j=1-S}^{0}\cos^2(j\omega_k)=\sum\limits_{j=1-S}^{0}\sin^2(j\omega_k)=S/2$ it follows that,

$$\frac{1}{(SN)^2}\sum_{Sn+s=S+1}^{SN}\left(\widetilde{y}_{k,Sn+s-1}^{\alpha}\right)^2 \Rightarrow \frac{\sigma^2}{2S}\int_0^1\left\{\left[\tilde{W}_k^{\alpha}(r)\right]^2+\left[\tilde{W}_k^{\beta}(r)\right]^2\right\} dr.$$

Using a similar approach the same result will be obtained for $\frac{1}{(SN)^2}\sum\limits_{Sn+s=S+1}^{SN}\left(\widetilde{y}_{k,Sn+s-1}^{\beta}\right)^2$. ■

Lemma A.9 *The sums of the cross products:*

$$\sum_{Sn+s=S+1}^{SN}\widetilde{y}_{0,Sn+s-1}\widetilde{y}_{S/2,Sn+s-1},\quad \sum_{Sn+s=S+1}^{SN}\widetilde{y}_{0,Sn+s-1}\widetilde{y}_{k,Sn+s-1}^{\alpha},\quad \sum_{Sn+s=S+1}^{SN}\widetilde{y}_{0,Sn+s-1}\widetilde{y}_{k,Sn+s-1}^{\beta},$$
$$\sum_{Sn+s=S+1}^{SN}\widetilde{y}_{S/2,Sn+s-1}\widetilde{y}_{k,Sn+s-1}^{\alpha},\quad \sum_{Sn+s=S+1}^{SN}\widetilde{y}_{S/2,Sn+s-1}\widetilde{y}_{k,Sn+s-1}^{\beta},\ \textit{and}\ \sum_{Sn+s=S+1}^{SN}\widetilde{y}_{k,Sn+s-1}^{\alpha}\widetilde{y}_{k,Sn+s-1}^{\beta}$$

are $O_p(SN)$.

Proof of Lemma A.9: As in Burridge and Taylor (2001) and Rodrigues and Taylor (2004), it is easy to show that there are only a finite number (of order S) of non-zero terms in all the above cross-sums. It is easy to check that all these non-zero terms are $O_p(SN)$. The result then follows. This is clearly an important result given that it provides us with the asymptotic orthogonality of the regressors. ■

Lemma A.10 *The following holds:*

$$\frac{1}{SN}\sum_{Sn+s=S+1}^{SN}\left(\Delta_S\widetilde{y}_{Sn+s}\right)^2 \xrightarrow{p} \sigma^2+\gamma'\mathcal{G}(\lambda,\lambda_0)\gamma$$

where $\mathcal{G}(\lambda,\lambda_0)=S^2\left[Q(\lambda_0)-R(\lambda,\lambda_0)'Q(\lambda)^{-1}R(\lambda,\lambda_0)\right]$.

Remark: When $\gamma_2=0$ or $\lambda=\lambda_0$ it is easy to show that $\gamma'\mathcal{G}(\lambda,\lambda_0)\gamma=0$ so that the following holds:

$$\frac{1}{SN}\sum_{Sn+s=S+1}^{SN}\left(\Delta_S\widetilde{y}_{Sn+s}\right)^2 \xrightarrow{p} \sigma^2.$$

Proof of Lemma A.10:

From (A.2) and (A.6) we have that

$$\begin{aligned}\sum_{Sn+s=S+1}^{SN}\left(\Delta_S\widetilde{y}_{Sn+s}\right)^2 &= \Delta_S\tilde{Y}'\Delta_S\tilde{Y} \\ &= \Delta_S Y' M\Delta_S Y \\ &= \Delta_S X' M\Delta_S X + 2\Delta_S X' M\Delta_S Z^0\boldsymbol{\gamma} \\ &+ \boldsymbol{\gamma}'\Delta_S Z^{0\prime} M\Delta_S Z^0\boldsymbol{\gamma}.\end{aligned} \tag{A.28}$$

The first term in (A.28) is equal to

$$\Delta_S X' M \Delta_S X = \Delta_S X'\Delta_S X - \Delta_S X'\Delta_S Z(\Delta_S Z'\Delta_S Z)^{-1}\Delta_S Z'\Delta_S X.$$

By the law of large numbers, under H_0, $\frac{1}{SN}\Delta_S X'\Delta_S X \to^p \sigma^2$. By (A.11) and (A.17), $\Delta_S X'\Delta_S Z$ $(\Delta_S Z'\Delta_S Z)^{-1}\Delta_S Z'\Delta_S X$ is $O_p(1)$. The second term in (A.28) is $O_p((SN)^{1/2})$ by (A.11), (A.12) and (A.17). Finally, regarding the third term in (A.28) we have that:

$$\begin{aligned} & \frac{1}{SN}\boldsymbol{\gamma}'\Delta_S Z^{0\prime} M \Delta_S Z^0 \boldsymbol{\gamma} \\ = & \frac{1}{SN}\boldsymbol{\gamma}'\Delta_S Z^{0\prime}(I - \Delta_S Z(\Delta_S Z'\Delta_S Z)^{-1}\Delta_S Z')\Delta_S Z^0\boldsymbol{\gamma} \\ = & \frac{1}{SN}\boldsymbol{\gamma}'\left(\Delta_S Z^{0\prime}\Delta_S Z^0 - \Delta_S Z^{0\prime}\Delta_S Z(\Delta_S Z'\Delta_S Z)^{-1}\Delta_S Z'\Delta_S Z^0\right)\boldsymbol{\gamma} \\ \to & \boldsymbol{\gamma}'\left(S^2 Q(\lambda_0) - S^2 R(\lambda,\lambda_0)' S^{-2} Q(\lambda)^{-1} S^2 R(\lambda,\lambda_0)\right)\boldsymbol{\gamma} \\ \equiv & \boldsymbol{\gamma}'\mathcal{G}(\lambda,\lambda_0)\boldsymbol{\gamma}. \end{aligned}$$

Hence, it follows that

$$\frac{1}{SN}\sum_{Sn+s=S+1}^{SN}(\Delta_S \widetilde{y}_{Sn+s})^2 \to^p \sigma^2 + \boldsymbol{\gamma}'\mathcal{G}(\lambda,\lambda_0)\boldsymbol{\gamma}. \qquad \text{(A.29)}$$

■

Lemma A.11 *The following hold:*

$$i)\ \frac{1}{SN}\sum_{Sn+s=S+1}^{SN}\widetilde{y}_{0,Sn+s-1}\Delta_S\widetilde{y}_{Sn+s} \xrightarrow{p} -\frac{1}{2}(\sigma^2 + \boldsymbol{\gamma}'\mathcal{G}(\lambda,\lambda_0)\boldsymbol{\gamma})$$

$$ii)\frac{1}{SN}\sum_{Sn+s=S+1}^{SN}\widetilde{y}_{S/2,Sn+s-1}\Delta_S\widetilde{y}_{Sn+s} \Rightarrow \frac{\sigma^2}{S}\int_0^1 \tilde{W}_{S/2}(r)d\tilde{W}_{S/2}(r) - \frac{1}{2}\boldsymbol{\gamma}'\mathcal{G}(\lambda,\lambda_0)\boldsymbol{\gamma}$$

$$iii)\ \frac{1}{SN}\sum_{Sn+s=S+1}^{SN}\widetilde{y}^{\alpha}_{k,Sn+s-1}\Delta_S\widetilde{y}_{Sn+s} \Rightarrow \frac{\sigma^2}{S}\int_0^1\left[\tilde{W}^{\alpha}_k(r)d\tilde{W}^{\alpha}_k(r) + \tilde{W}^{\beta}_k(r)d\tilde{W}^{\beta}_k(r)\right] - \frac{c^{\alpha}_k}{S}\boldsymbol{\gamma}'\mathcal{G}(\lambda,\lambda_0)\boldsymbol{\gamma}$$

$$iv)\frac{1}{SN}\sum_{Sn+s=S+1}^{SN}\widetilde{y}^{\beta}_{k,Sn+s-1}\Delta_S\widetilde{y}_{Sn+s} \Rightarrow \frac{\sigma^2}{S}\int_0^1\left[\tilde{W}^{\alpha}_k(r)d\tilde{W}^{\beta}_k(r) - \tilde{W}^{\beta}_k(r)d\tilde{W}^{\alpha}_k(r)\right] - \frac{c^{\beta}_k}{S}\boldsymbol{\gamma}'\mathcal{G}(\lambda,\lambda_0)\boldsymbol{\gamma}.$$

Proof of Lemma A.11:

To prove *i)* we first note that since the first step regression (2.5) contains a constant, the residuals sum to zero, so that $\sum_{t=S+1}^{SN}\Delta_S\widetilde{y}_t = 0$. Therefore, it follows that

$$\begin{aligned} 0 &= \left(\sum_t \Delta_S\widetilde{y}_t\right)^2 = \sum_t(\Delta_S\widetilde{y}_t)^2 + 2\sum_t\sum_{\tau<t}\Delta_S\widetilde{y}_t\Delta_S\widetilde{y}_t \\ &= \sum_t(\Delta_S\widetilde{y}_t)^2 + 2\sum_t\Delta_S\widetilde{y}_t(\widetilde{y}_{t-1}+\cdots+\widetilde{y}_{t-S}) \\ &= \sum_t(\Delta_S\widetilde{y}_t)^2 + 2\sum_t\Delta_S\widetilde{y}_t\widetilde{y}_{0,t-1}. \end{aligned}$$

It follows that $\sum_t\Delta_S\widetilde{y}_t\widetilde{y}_{0,t-1} = -\frac{1}{2}\sum_t(\Delta_S\widetilde{y}_t)^2$. The result in *i)* then follows directly from Lemma A.10.

Consider next *ii)*. From equations (A.2) and (A.6) we have $\Delta_S\widetilde{y} = M\Delta_S Z^0\gamma + M\Delta_S X$. From (A.8) we also have that

$$\widetilde{y}_{S/2,Sn+s} = \Delta_{S/2}Z^0_{Sn+s}\gamma + \Delta_{S/2}x_{Sn+s} - \Delta_{S/2}Z_{Sn+s}(\tilde{\gamma} - H_N\gamma) - \Delta_{S/2}Z_{Sn+s}H_N\gamma.$$

Therefore, we can write

$$\begin{aligned} &\sum_{Sn+s=S+1}^{SN} \widetilde{y}_{S/2,Sn+s-1}\Delta_S \widetilde{y}_{Sn+s} \\ = \;& (M\Delta_S Z^0\gamma + M\Delta_S X)'(\Delta_{S/2}Z^0_{-1}\gamma + \Delta_{S/2}X_{-1} \\ & -\Delta_{S/2}Z_{-1}H_N\gamma - \Delta_{S/2}Z_{-1}(\tilde{\gamma} - H_N\gamma)) + O_p(1) \\ = \;& I_1 + I_2 - I_3 - I_4 + I_5 + I_6 - I_7 - I_8 \end{aligned} \qquad \text{(A.30)}$$

where $I_1 = \gamma'\Delta_S Z^{0\prime} M\Delta_{S/2}Z^0_{-1}\gamma$; $I_2 = \gamma'\Delta_S Z^{0\prime} M\Delta_{S/2}X_{-1}$; $I_3 = \gamma'\Delta_S Z^{0\prime} M\Delta_{S/2}Z_{-1}H_N\gamma$; $I_4 = \gamma'\Delta_S Z^{0\prime} M\Delta_{S/2}Z_{-1}(\tilde{\gamma}-H_N\gamma)$; $I_5 = \Delta_S X' M\Delta_{S/2}Z^0_{-1}\gamma$; $I_6 = \Delta_S X' M\Delta_{S/2}X_{-1}$; $I_7 = \Delta_S X' M\Delta_{S/2}Z_{-1}H_N\gamma$ and $I_8 = \Delta_S X' M\Delta_{S/2}Z_{-1}(\tilde{\gamma} - H_N\gamma)$.

Hence, regarding I_1 in (A.30), making use of the results in Lemma A.1, we have that

$$\begin{aligned} \frac{1}{SN}\gamma'\Delta_S Z^{0\prime} M\Delta_{S/2}Z^0_{-1}\gamma \;=\;& \gamma'(\frac{1}{SN}\Delta_S Z^{0\prime}\Delta_{S/2}Z^0_{-1} \\ & -\frac{1}{SN}\Delta_S Z^{0\prime}\Delta_S Z(\frac{1}{SN}\Delta_S Z'\Delta_S Z)^{-1}\frac{1}{SN}\Delta_S Z'\Delta_{S/2}Z^0_{-1})\gamma \\ \to\;& \gamma'(-c_{S/2}SQ(\lambda_0) - S^2R(\lambda,\lambda_0)'(S^2Q(\lambda))^{-1}(-c_{S/2}SR(\lambda,\lambda_0)))\gamma \\ =\;& -\frac{c_{S/2}}{S}\gamma'\mathcal{G}(\lambda,\lambda_0)\gamma, \end{aligned}$$

with $\mathcal{G}(\lambda,\lambda_0)$ as defined in Lemma A.10. Since $c_{S/2} = \frac{S}{2}$, it follows that

$$\frac{1}{SN}\gamma'\Delta_S Z^{0\prime} M\Delta_{S/2}Z^0_{-1}\gamma \to -\frac{1}{2}\boldsymbol{\gamma}'\mathcal{G}(\lambda,\lambda_0)\boldsymbol{\gamma}.$$

It is easy to see that I_2 in (A.30), $\gamma'\Delta_S Z^{0\prime} M\Delta_{S/2}X_{-1}$, is $O_p((SN)^{1/2})$. For instance

$$\begin{aligned} \sum_{Sn+s=1}^{SN}\Delta_S(Sn+s)\Delta_{S/2}x_{Sn+s-1} \;=\;& S\sum_{Sn+s=1}^{SN}\Delta_{S/2}x_{Sn+s-1} \\ =\;& S\sum_{j=0}^{S-2}\sum_{i=0}^{j}x_{SN-1-j} + O_p(1) \\ =\;& O_p((SN)^{1/2}). \end{aligned}$$

Similarly, $\sum\limits_{Sn+s=1}^{SN}\Delta_S DT^0_{Sn+s}\Delta_{S/2}x_{Sn+s-1}$ can also be shown to be $O_p((SN)^{1/2})$.

Next we show that I_3 is $o_p(SN)$. Making use of the results in Lemma A.1, we have

$$\begin{aligned} & \frac{1}{SN}\Delta_S Z^{0\prime} M\Delta_{S/2}Z_{-1}H_N \\ =\;& \frac{1}{SN}\Delta_S Z^{0\prime}\Delta_{S/2}Z_{-1}(\frac{1}{SN}\Delta_S Z'\Delta_S Z)^{-1}\frac{1}{SN}\Delta_S Z'\Delta_S Z^0 \\ & -\frac{1}{SN}\Delta_S Z^{0\prime}\Delta_S Z(\frac{1}{SN}\Delta_S Z'\Delta_S Z)^{-1}\frac{1}{SN}\Delta_S Z'\Delta_{S/2}Z_{-1}(\frac{1}{SN}\Delta_S Z'\Delta_S Z)^{-1}\frac{1}{SN}\Delta_S Z'\Delta_S Z^0 \\ \to\;& -c_{S/2}SR(\lambda,\lambda_0)'(S^2Q(\lambda))^{-1}S^2R(\lambda,\lambda_0) \\ & -S^2R(\lambda,\lambda_0)'(S^2Q(\lambda))^{-1}(-c_{S/2}SQ(\lambda))(S^2Q(\lambda))^{-1}S^2R(\lambda,\lambda_0) \\ =\;& 0. \end{aligned}$$

Given (A.22) and the result obtained for I_1, it is easy to derive that I_4 is $O_p((SN)^{1/2})$.

Regarding I_5 note that,

$$\Delta_S X' M\Delta_{S/2}Z^0_{-1}\gamma = \Delta_S X'\Delta_{S/2}Z^0_{-1}\gamma - \Delta_S X'\Delta_S Z(\Delta_S Z'\Delta_S Z)^{-1}\Delta_S Z'\Delta_{S/2}Z^0_{-1}\gamma.$$

Given the results in Lemma A.1 and Lemma A.4, it follows that this term is $O_p((SN)^{1/2})$.

Regarding I_6, we have that

$$\Delta_S X' M \Delta_{S/2} X_{-1} = \Delta_S X' \Delta_{S/2} X_{-1} - \Delta_S X' \Delta_S Z (\Delta_S Z' \Delta_S Z)^{-1} \Delta_S Z' \Delta_{S/2} X_{-1}. \tag{A.31}$$

From the results obtained for I_2 in (A.30) and Lemmas A.1 and Lemma A.4 *i*), it follows that the second term in (A.31) is $O_p(1)$. By Lemma A.2 and the continuous mapping theorem we finally have that $\frac{1}{SN}$ $\Delta_S X' M \Delta_{S/2} X_{-1} \Rightarrow \frac{\sigma^2}{S} \int_0^1 \tilde{W}_{S/2}(r) d\tilde{W}_{S/2}(r)$.

Similarly to the case of I_5, I_7 can be shown to be $O_p((SN)^{1/2})$. Finally, it is straightforward to show that I_8 is $O_p(1)$.

Combining all these results together we conclude that

$$\frac{1}{SN} \sum_{Sn+s=1}^{SN} \widetilde{y}_{S/2,Sn+s-1} \Delta_S \widetilde{y}_{Sn+s} \Rightarrow -\frac{1}{2} \boldsymbol{\gamma}' \mathcal{G}(\lambda, \lambda_0) \boldsymbol{\gamma} + \frac{\sigma^2}{S} \int_0^1 \tilde{W}_{S/2}(r) d\tilde{W}_{S/2}(r).$$

The proofs of *iii*) and *iv*) follow along similar lines and are omitted. ■

Lemma A.12 *When $\gamma_2 = 0$ or $\lambda = \lambda_0$, it follows that*

$$\begin{aligned} i)\ SN\widehat{\pi}_0 &\Rightarrow \left(\frac{\sigma^2}{S} \int_0^1 \left[\tilde{V}_0(r;\lambda) \right]^2 dr \right)^{-1} \left(-\frac{1}{2} \sigma^2 \right) \\ &= -\frac{S}{2} \left(\int_0^1 \left[\tilde{V}_0(r;\lambda) \right]^2 dr \right)^{-1} \end{aligned}$$

and when $\gamma_2 \neq 0$ and $\lambda \neq \lambda_0$, it follows that

$$\begin{aligned} ii) (SN)^2 \widehat{\pi}_0 &\rightarrow \left(S^2 \gamma' \left(\int_0^1 f(r;\lambda,\lambda_0)' f(r;\lambda,\lambda_0) dr \right) \gamma \right)^{-1} \left(-\frac{1}{2} (\sigma^2 + \gamma' \mathcal{G}(\lambda,\lambda_0) \gamma) \right) \\ &= -\frac{1}{2S^2} \left(\gamma' \left(\int_0^1 f(r;\lambda,\lambda_0)' f(r;\lambda,\lambda_0) dr \right) \gamma \right)^{-1} (\sigma^2 + \gamma' \mathcal{G}(\lambda,\lambda_0) \gamma), \end{aligned}$$

Moreover, we have that,

$$\begin{aligned} i)\ SN\widehat{\pi}_{S/2} &\Rightarrow \left(\frac{\sigma^2}{S} \int_0^1 \left[\tilde{W}_{S/2}(r) \right]^2 dr \right)^{-1} \left(\frac{\sigma^2}{S} \int_0^1 \tilde{W}_{S/2}(r) d\tilde{W}_{S/2}(r) - \frac{1}{2} \boldsymbol{\gamma}' \mathcal{G}(\lambda,\lambda_0) \boldsymbol{\gamma} \right) \\ ii)\ SN\widehat{\pi}_k^{\alpha} &\Rightarrow \left(\frac{\sigma^2}{2S} \int_0^1 \left\{ \left[\tilde{W}_k^{\alpha}(r) \right]^2 + \left[\tilde{W}_k^{\beta}(r) \right]^2 \right\} dr \right)^{-1} \\ &\quad \times \left(\frac{\sigma^2}{S} \int_0^1 \left[\tilde{W}_k^{\alpha}(r) d\tilde{W}_k^{\alpha}(r) + \tilde{W}_k^{\beta}(r) d\tilde{W}_k^{\beta}(r) \right] - \frac{c_k^{\alpha}}{S} \boldsymbol{\gamma}' \mathcal{G}(\lambda,\lambda_0) \boldsymbol{\gamma} \right) \\ ii)\ SN\widehat{\pi}_k^{\beta} &\Rightarrow \left(\frac{\sigma^2}{2S} \int_0^1 \left\{ \left[\tilde{W}_k^{\alpha}(r) \right]^2 + \left[\tilde{W}_k^{\beta}(r) \right]^2 \right\} dr \right)^{-1} \\ &\quad \times \left(\frac{\sigma^2}{S} \int_0^1 \left[\tilde{W}_k^{\alpha}(r) d\tilde{W}_k^{\beta}(r) - \tilde{W}_k^{\beta}(r) d\tilde{W}_k^{\alpha}(r) \right] - \frac{c_k^{\beta}}{S} \boldsymbol{\gamma}' \mathcal{G}(\lambda,\lambda_0) \boldsymbol{\gamma} \right) \end{aligned}$$

$k = 1, ..., S^*$.

Proof of Lemma A.12: Denote $\nabla \widetilde{y}_{Sn+s} = (\widetilde{y}_{0,Sn+s-1}, \widetilde{y}_{S/2,Sn+s-1}, \{\widetilde{y}^{\alpha}_{k,Sn+s-1}, \widetilde{y}^{\beta}_{k,Sn+s-1}\}_{k=1}^{S^*})'$, and $\nabla \widetilde{Y} = (\nabla \widetilde{y}_{S+1}, \cdots, \nabla \widetilde{y}_{SN})'$. The least squares estimator of $\pi = (\pi_0, \pi_{S/2}, \{\pi_{\alpha,k}, \pi_{\beta,k}\}_{k=1}^{S^*})'$ in (2.8) can be written as:

$$\widehat{\pi} = \left(\nabla \widetilde{Y}' \nabla \widetilde{Y} \right)^{-1} \nabla \widetilde{Y}' \Delta_S \widetilde{Y}.$$

From Lemmas A.7, A.8 and A.9, we may conclude that i) when $\gamma_2 = 0$ or $\lambda = \lambda_0$, all the terms on the diagonal of $\nabla \widetilde{y}' \nabla \widetilde{y}$ are $O_p((SN)^2)$. ii) when $\gamma_2 \neq 0$ and $\lambda \neq \lambda_0$, the first term on the diagonal is $O_p((SN)^3)$ while all other terms in the diagonal are $O_p((SN)^2)$. iii) off-diagonal terms are always $O_p(SN)$.

It follows that, when $\gamma_2 = 0$ or $\lambda = \lambda_0$, all the terms on the diagonal of $\left(\nabla\widetilde{Y}'\nabla\widetilde{Y}\right)^{-1}$ are $O_p((SN)^{-2})$, and all off-diagonal terms are $O_p((SN)^{-4})$. When $\gamma_2 \neq 0$ and $\lambda \neq \lambda_0$, the first term on the diagonal is $O_p((SN)^{-3})$, all the other terms in the diagonal are $O_p((SN)^{-2})$, the off-diagonal terms in the first row and in the first column are $O_p((SN)^{-4})$, and all the other off-diagonal terms are $O_p((SN)^{-3})$.

The results then follows from Lemmas A.7, A.8, A.11 and the CMT. ∎

Lemma A.13 *Let $\hat{\sigma}^2$ denote the usual residual variance estimator for equation (2.8). We have that $\hat{\sigma}^2 \xrightarrow{p} \sigma^2 + \gamma'\mathcal{G}(\lambda, \lambda_0)\gamma$.*

Proof of Lemma A.13:

The proof follows easily from Lemmas A.10 and A.12. ∎

Lemma A.14 *When $\gamma_2 = 0$ or $\lambda = \lambda_0$, it follows that*

$$\begin{aligned} i)\ t_0 &\Rightarrow -\frac{\sqrt{S}}{2}\left(\int_0^1 \left[\tilde{V}_0(r;\lambda)\right]^2 dr\right)^{-1/2} \\ &\equiv \tau_0(\lambda) \end{aligned}$$

and when $\gamma_2 \neq 0$ and $\lambda \neq \lambda_0$, it follows that

$$\begin{aligned} ii)(SN)^{1/2}t_0 \to^p \quad & -\frac{1}{2S}\left(\gamma'\left(\int_0^1 f(r;\lambda,\lambda_0)'f(r;\lambda,\lambda_0)dr\right)\gamma\right)^{-1/2} \\ & (\sigma^2 + \gamma'\mathcal{G}(\lambda,\lambda_0)\gamma)^{1/2}. \end{aligned}$$

Moreover, we have that,

$$\begin{aligned} iii)\ t_{S/2} &\Rightarrow \left(\frac{\sigma^2}{S}\int_0^1 \left[\tilde{W}_{S/2}(r)\right]^2 dr\right)^{-1/2} \\ &\quad \times \left(\frac{\sigma^2}{S}\int_0^1 \tilde{W}_{S/2}(r)d\tilde{W}_{S/2}(r) - \frac{1}{2}\boldsymbol{\gamma}'\mathcal{G}(\lambda,\lambda_0)\boldsymbol{\gamma}\right) \\ &\quad \times(\sigma^2 + \gamma'\mathcal{G}(\lambda,\lambda_0)\gamma)^{-1/2} \\ &\equiv \tau_{S/2}(\lambda,\lambda_0) \\ iv)\ t_k^{\alpha} &\Rightarrow \left(\frac{\sigma^2}{2S}\int_0^1 \left\{\left[\tilde{W}_k^{\alpha}(r)\right]^2 + \left[\tilde{W}_k^{\beta}(r)\right]^2\right\} dr\right)^{-1/2} \\ &\quad \times \left(\frac{\sigma^2}{S}\int_0^1 \left[\tilde{W}_k^{\alpha}(r)d\tilde{W}_k^{\alpha}(r) + \tilde{W}_k^{\beta}(r)d\tilde{W}_k^{\beta}(r)\right] - \frac{c_k^{\alpha}}{S}\boldsymbol{\gamma}'\mathcal{G}(\lambda,\lambda_0)\boldsymbol{\gamma}\right) \\ &\quad \times(\sigma^2 + \gamma'\mathcal{G}(\lambda,\lambda_0)\gamma)^{-1/2} \\ &\equiv \tau_k^{\alpha}(\lambda,\lambda_0) \\ v)\ t_k^{\beta} &\Rightarrow \left(\frac{\sigma^2}{2S}\int_0^1 \left\{\left[\tilde{W}_k^{\alpha}(r)\right]^2 + \left[\tilde{W}_k^{\beta}(r)\right]^2\right\} dr\right)^{-1/2} \\ &\quad \times \left(\frac{\sigma^2}{S}\int_0^1 \left[\tilde{W}_k^{\alpha}(r)d\tilde{W}_k^{\beta}(r) - \tilde{W}_k^{\beta}(r)d\tilde{W}_k^{\alpha}(r)\right] - \frac{c_k^{\beta}}{S}\boldsymbol{\gamma}'\mathcal{G}(\lambda,\lambda_0)\boldsymbol{\gamma}\right) \\ &\quad \times(\sigma^2 + \gamma'\mathcal{G}(\lambda,\lambda_0)\gamma)^{-1/2} \\ &\equiv \tau_k^{\beta}(\lambda,\lambda_0) \end{aligned}$$

$k = 1, ..., S^*$.

Proof of Lemma A.14: The results stated for the t-statistics follow directly from the previous results and the CMT. ∎

Lemma A.15 *The following results hold:*

$$i)\ F_k \Rightarrow \frac{1}{2}\left\{[\tau_k^{\alpha}(\lambda,\lambda_0)]^2 + [\tau_k^{\beta}(\lambda,\lambda_0)]^2\right\},\ k=1,\ldots,S^*, \tag{A.32}$$

$$ii)\ F_{1\ldots[S/2]} \Rightarrow \frac{1}{S-1}\left\{[\tau_{S/2}(\lambda,\lambda_0)]^2 + \sum_{k=1}^{S^*}\left([\tau_k^{\alpha}(\lambda,\lambda_0)]^2 + [\tau_k^{\beta}(\lambda,\lambda_0)]^2\right)\right\}, \tag{A.33}$$

Moreover, when $\gamma_2 = 0$ or $\lambda = \lambda_0$, we have that

$$iii)\ F_{0\ldots[S/2]} \Rightarrow \frac{1}{S}\left\{[\tau_0(\lambda)]^2 + [\tau_{S/2}]^2 + \sum_{k=1}^{S^*}\left([\tau_k^{\alpha}]^2 + [\tau_k^{\beta}]^2\right)\right\}$$

and when $\gamma_2 \neq 0$ and $\lambda \neq \lambda_0$, we have that

$$iv) F_{0\ldots[S/2]} \Rightarrow \frac{1}{S}\left\{[\tau_{S/2}(\lambda,\lambda_0)]^2 + \sum_{k=1}^{S^*}\left([\tau_k^{\alpha}(\lambda,\lambda_0)]^2 + [\tau_k^{\beta}(\lambda,\lambda_0)]^2\right)\right\}.$$

Proof of Lemma A.15: The results follow directly from Lemma A.14 and the aymptotical orthogonality of the regressors.

Proof of Theorem 3.1.

When $\gamma_2 = 0$ we have that $\gamma'\mathcal{G}(\lambda,\lambda_0)\gamma = 0$. Theorem 3.1 then follows directly from Lemma A.14. ■

Proof of Theorem 3.2.

First note that when $\gamma_2 = 0$, we have that $\Delta_S Z^0\gamma = \Delta_S Z\gamma$. Moreover $M\Delta_S Z = 0$. From (A.28) we get that the residual sum of squares for the first step regression (2.5), assuming a break occurring at time T_B, is given by $RSS(T_B) \equiv \Delta_S X' M \Delta_S X$. Following Nunes *et al.* (1995), it follows that asymptotically, the break fraction that minimizes $RSS(T_B)$ is the same that maximizes $P_N(T_B) \equiv \Delta_S X'\Delta_S Z(\Delta_S Z'\Delta_S Z)^{-1}\Delta_S Z'\Delta_S X$. By (A.11), (A.17) and the CMT, we obtain that $P_N([\lambda SN]) \Rightarrow \frac{\sigma^2}{S}P(\lambda)$. The results in (3.8) and (3.9) for $t_0(\widehat{T}_B)$ and $F_{0\ldots[S/2]}(\widehat{T}_B)$, respectively, then follow from Theorem 3.1, Corollary 3.1 and the CMT as in Vogelsang and Perron (1998). The remaining results are straightforward to derive since the limiting distributions in Theorem 3.1 and Corollary 3.1 do not depend on λ. ■

Proof of Theorem 3.3.

The proof of this Theorem follows using the result in Theorem 3.1, Corollary 3.1 and the CMT as in Zivot and Andrews (1992) and Vogelsang and Perron (1998). ■

Proof of Theorem 3.4.

The results follow directly from Lemmas A.14 and A.15. ■

Proof of Theorem 3.5.

Bai (1994) showed that the least squares break fraction estimator, denoted as $\widehat{\lambda}$, for a model with a mean shift and stationary errors, as in our case, will be $O_p((SN)^{-1})$ consistent for the true break fraction λ_0. Following the same approach as in the proof of Propositions 3 and 8 in Kim and Perron (2009), this rate of convergence is fast enough for the terms involving the error process x_{Sn+s} to have a order of magnitude larger than that of the remaining terms involving the deterministic terms, so that the limiting distributions of the test statistics evaluated at $\widehat{\lambda}$ are the same as when the true break date λ_0 is used. In particular, note that in (A.8), the second and fourth terms are the ones that dominate when $\lambda \neq \lambda_0$ as shown in Lemma A.5 *ii*), which causes the order of magnitude of $\sum_{Sn+s=S+1}^{SN} \widetilde{y}_{0,Sn+s-1}^2$ to be $O_p((SN)^3)$ instead of $O_p((SN)^2)$ for the case $\lambda = \lambda_0$ as shown in Lemma A.7. Straightforward algebra can show that these extra terms in $\sum_{Sn+s=S+1}^{SN} \widetilde{y}_{0,Sn+s-1}^2$ are $O_p((SN)^3) \times O(|\lambda-\lambda_0|^3)$. If those terms are evaluated at $\widehat{\lambda}$, given the rate of convergence of $\widehat{\lambda}$, it follows that the extra terms are in fact only $O_p(1)$. It is also easy to derive that $\mathcal{G}(\lambda,\lambda_0) = O(|\lambda-\lambda_0|)$ so that when evaluated at $\widehat{\lambda}$ it is $O_p((SN)^{-1})$. The proof of the theorem then follows immediately given the results in Lemmas A.14 and A.15. ■

Table 4.1.A: Critical values for the LM seasonal unit root tests allowing for a break when the DGP has no break.

		$4N = 100$			$4N = 200$			$4N = \infty$		
	λ	1%	5%	10%	1%	5%	10%	1%	5%	10%
$t_0(\lambda \times 4N)$	0.1	-3.43	-2.76	-2.45	-3.51	-2.90	-2.55	-3.42	-2.86	-2.56
	0.2	-3.66	-3.02	-2.69	-3.66	-3.00	-2.70	-3.52	-2.97	-2.69
	0.3	-3.72	-3.07	-2.76	-3.67	-3.04	-2.75	-3.58	-3.02	-2.73
	0.4	-3.68	-3.07	-2.78	-3.63	-3.07	-2.77	-3.55	-3.03	-2.76
	0.5	-3.70	-3.10	-2.82	-3.69	-3.10	-2.78	-3.62	-3.07	-2.77
	0.6	-3.76	-3.12	-2.80	-3.64	-3.06	-2.78	-3.58	-3.04	-2.75
	0.7	-3.70	-3.08	-2.78	-3.64	-3.04	-2.75	-3.57	-3.04	-2.73
	0.8	-3.58	-3.02	-2.70	-3.67	-3.01	-2.72	-3.54	-2.97	-2.69
	0.9	-3.54	-2.91	-2.57	-3.50	-2.87	-2.54	-3.44	-2.86	-2.56
$F_{012}(\lambda \times 4N)$	0.1	4.82	3.44	2.79	4.58	3.44	2.89	4.53	3.32	2.79
	0.2	5.03	3.75	3.11	4.83	3.62	3.04	4.72	3.48	2.94
	0.3	5.13	3.86	3.25	4.80	3.71	3.14	4.82	3.55	3.01
	0.4	5.11	3.90	3.25	4.80	3.70	3.17	4.73	3.61	3.08
	0.5	5.13	3.86	3.28	4.90	3.69	3.16	4.86	3.59	3.13
	0.6	5.31	3.86	3.28	4.85	3.68	3.13	4.81	3.57	3.08
	0.7	5.33	3.79	3.18	4.94	3.63	3.12	4.80	3.56	3.03
	0.8	5.14	3.67	3.10	4.70	3.60	3.06	4.72	3.48	2.97
	0.9	4.87	3.57	2.93	4.51	3.47	2.89	4.58	3.36	2.79
$t_0(0.5 \times 4N)$		-3.70	-3.10	-2.82	-3.69	-3.10	-2.78	-3.62	-3.07	-2.77
$t_2(0.5 \times 4N)$		-2.67	-1.99	-1.65	-2.56	-1.95	-1.61	-2.50	-1.92	-1.58
$F_1(0.5 \times 4N)$		5.04	3.23	2.46	4.90	3.03	2.39	4.83	3.12	2.42
$F_{012}(0.5 \times 4N)$		5.13	3.86	3.28	4.90	3.69	3.16	4.86	3.59	3.13
$F_{12}(0.5 \times 4N)$		4.37	2.91	2.34	4.09	2.80	2.21	4.06	2.74	2.21
$t_0(\widehat{T}_B)$		-3.79	-3.14	-2.81	-3.66	-3.06	-2.76	-3.54	-2.98	-2.71
$t_2(\widehat{T}_B)$		-2.64	-1.99	-1.63	-2.60	-1.93	-1.61	-2.50	-1.92	-1.58
$F_1(\widehat{T}_B)$		5.02	3.27	2.46	4.99	3.06	2.42	4.85	3.11	2.42
$F_{12}(\widehat{T}_B)$		5.10	3.88	3.25	4.96	3.63	3.10	4.66	3.52	2.98
$F_{012}(\widehat{T}_B)$		4.32	2.92	2.35	4.04	2.80	2.24	4.05	2.74	2.21
$\min_{T_B} t_0(T_B)$		-4.49	-3.87	-3.57	-4.37	-3.79	-3.48	-4.29	-3.71	-3.44
$\min_{T_B} t_2(T_B)$		-2.81	-2.12	-1.76	-2.67	-1.99	-1.67	-2.51	-1.92	-1.59
$\max_{T_B} F_1(T_B)$		5.55	3.62	2.75	5.21	3.20	2.54	4.85	3.13	2.43
$\max_{T_B} F_{012}(T_B)$		6.67	5.13	4.45	6.27	4.86	4.22	5.92	4.71	4.00
$\max_{T_B} F_{12}(T_B)$		4.67	3.20	2.58	4.26	2.91	2.32	4.06	2.75	2.22

Note: When the DGP has a break at a break fraction λ_0, the asymptotical critical values for the following LM tests are still valid: $t_2(\widehat{T}_B)$, $F_1(\widehat{T}_B)$, and $F_{12}(\widehat{T}_B)$ provided $\lambda_0 \in \Lambda = [0.1, 0.9]$; $t_0(\lambda \times 4N)$, $t_2(\lambda \times 4N)$, $F_1(\lambda \times 4N)$, $F_{012}(\lambda \times 4N)$, and $F_{12}(\lambda \times 4N)$ provided $\lambda = \lambda_0$. If a break is present, the asymptotical critical values for the $t_0(\widehat{T}_B)$ and $F_{012}(\widehat{T}_B)$ tests correspond to those of the $t_0(\lambda \times 4N)$ and $F_{012}(\lambda \times 4N)$ tests, respectively, with $\lambda = \lambda_0$.

Table 4.1.B: Critical values for the HEGY seasonal unit root tests allowing for a break when the DGP has no break.

		$4N = 100$			$4N = 200$			$4N = \infty$		
	λ	1%	5%	10%	1%	5%	10%	1%	5%	10%
$t_0(\lambda \times 4N)$	0.1	-4.19	-3.57	-3.25	-4.17	-3.52	-3.17	-4.05	-3.51	-3.19
	0.2	-4.33	-3.68	-3.38	-4.31	-3.69	-3.38	-4.24	-3.69	-3.39
	0.3	-4.46	-3.84	-3.50	-4.42	-3.82	-3.53	-4.34	-3.79	-3.49
	0.4	-4.50	-3.94	-3.62	-4.49	-3.91	-3.61	-4.43	-3.83	-3.53
	0.5	-4.53	-4.00	-3.67	-4.50	-3.94	-3.65	-4.43	-3.87	-3.59
	0.6	-4.57	-3.99	-3.68	-4.56	-3.96	-3.63	-4.41	-3.88	-3.61
	0.7	-4.55	-3.99	-3.66	-4.47	-3.94	-3.63	-4.41	-3.86	-3.58
	0.8	-4.45	-3.89	-3.57	-4.49	-3.87	-3.56	-4.36	-3.77	-3.50
	0.9	-4.33	-3.76	-3.43	-4.37	-3.74	-3.43	-4.28	-3.69	-3.39
$F_{012}(\lambda \times 4N)$	0.1	8.83	6.95	6.08	8.01	6.67	5.79	7.94	6.44	5.67
	0.2	8.97	7.05	6.21	8.42	6.87	5.98	8.19	6.70	5.90
	0.3	9.09	7.38	6.50	8.76	7.08	6.21	8.46	6.83	6.11
	0.4	9.31	7.55	6.70	8.79	7.19	6.40	8.53	7.00	6.28
	0.5	9.48	7.71	6.80	8.87	7.31	6.47	8.50	7.05	6.33
	0.6	9.53	7.77	6.82	8.97	7.38	6.53	8.49	7.06	6.32
	0.7	9.54	7.67	6.77	8.86	7.39	6.51	8.53	7.00	6.31
	0.8	9.33	7.56	6.63	8.78	7.29	6.41	8.47	6.86	6.16
	0.9	9.16	7.34	6.45	8.52	7.12	6.21	8.45	6.71	5.98
$t_0(0.5 \times 4N)$		-4.53	-4.00	-3.67	-4.50	-3.94	-3.65	-4.43	-3.87	-3.59
$t_2(0.5 \times 4N)$		-3.59	-2.86	-2.55	-3.40	-2.84	-2.55	-3.48	-2.87	-2.58
$F_1(0.5 \times 4N)$		9.52	7.04	5.87	9.35	6.79	5.70	9.03	7.02	5.77
$F_{012}(0.5 \times 4N)$		9.48	7.71	6.80	8.87	7.31	6.47	8.50	7.05	6.33
$F_{12}(0.5 \times 4N)$		8.14	6.35	5.42	7.85	6.11	5.22	7.73	6.13	5.27
$t_0(\widehat{T}_B)$		-4.94	-4.37	-4.07	-4.87	-4.32	-4.01	-4.86	-4.28	-3.97
$t_2(\widehat{T}_B)$		-3.60	-2.86	-2.55	-3.41	-2.85	-2.54	-3.47	-2.87	-2.58
$F_1(\widehat{T}_B)$		9.53	7.03	5.90	9.29	6.79	5.69	9.08	7.01	5.78
$F_{12}(\widehat{T}_B)$		10.60	8.54	7.54	9.54	8.07	7.25	9.40	7.76	6.99
$F_{012}(\widehat{T}_B)$		8.21	6.35	5.43	7.92	6.10	5.21	7.74	6.13	5.27
$\min_{T_B} t_0(T_B)$		-5.08	-4.45	-4.19	-4.92	-4.38	-4.07	-4.86	-4.31	-4.00
$\min_{T_B} t_2(T_B)$		-3.71	-2.96	-2.66	-3.50	-2.90	-2.60	-3.48	-2.88	-2.58
$\max_{T_B} F_1(T_B)$		9.87	7.45	6.24	9.70	7.04	5.90	9.17	7.04	5.80
$\max_{T_B} F_{012}(T_B)$		10.79	8.83	7.81	9.65	8.21	7.38	9.42	7.84	7.07
$\max_{T_B} F_{12}(T_B)$		8.67	6.64	5.70	8.02	6.27	5.35	7.75	6.15	5.28

Table 4.2: LM tests: results of simulation experiment A for $4N = 100$ (% of rejections of the null hypothesis)

Panel A. $\lambda_0 = 0.5$

	$\rho = 1$			$\rho = 0.95$			$\rho = 0.90$			$\rho = 0.80$		
	$\gamma_2 = 0$	$\gamma_2 = 1$	$\gamma_2 = 2$	$\gamma_2 = 0$	$\gamma_2 = 1$	$\gamma_2 = 2$	$\gamma_2 = 0$	$\gamma_2 = 1$	$\gamma_2 = 2$	$\gamma_2 = 0$	$\gamma_2 = 1$	$\gamma_2 = 2$
t_0	5.0	0.0	0.0	5.4	0.0	0.0	6.5	0.0	0.0	11.3	0.0	0.0
t_2	5.0	4.4	5.1	8.2	6.7	8.1	12.9	10.8	12.3	29.1	22.4	20.6
F_1	5.0	5.7	5.4	7.9	7.4	5.7	15.0	12.8	7.9	42.7	28.3	10.6
F_{012}	5.0	2.4	3.3	8.1	3.4	4.1	16.9	6.8	6.5	51.2	20.1	11.0
F_{12}	5.0	5.7	5.9	9.4	7.7	7.2	19.1	14.8	11.5	55.0	37.4	19.4
$t_0(0.5 \times 4N)$	5.0	5.0	5.0	4.9	4.9	4.9	5.6	5.6	5.6	7.9	7.9	7.9
$t_2(0.5 \times 4N)$	5.0	5.0	5.0	8.2	8.2	8.2	13.0	13.0	13.0	28.9	28.9	28.9
$F_1(0.5 \times 4N)$	5.0	5.0	5.0	8.3	8.3	8.3	15.6	15.6	15.6	44.3	44.3	44.3
$F_{012}(0.5 \times 4N)$	5.0	5.0	5.0	7.4	7.4	7.4	14.3	14.3	14.3	43.3	43.3	43.3
$F_{12}(0.5 \times 4N)$	5.0	5.0	5.0	9.1	9.1	9.1	19.2	19.2	19.2	55.0	55.0	55.0
$\min_{T_B} t_0(T_B)$	5.0	1.8	1.0	5.5	1.8	1.4	5.8	2.1	1.4	7.1	2.9	2.0
$\min_{T_B} t_2(T_B)$	5.0	8.3	10.1	8.1	13.0	16.2	13.0	20.8	24.8	28.6	41.7	48.3
$\max_{T_B} F_1(T_B)$	5.0	9.3	11.0	7.4	13.9	18.1	15.0	25.7	32.0	42.2	60.8	66.8
$\max_{T_B} F_{012}(T_B)$	5.0	3.0	2.7	7.2	4.5	4.5	12.8	9.5	9.6	33.8	29.2	31.1
$\max_{T_B} F_{12}(T_B)$	5.0	10.0	11.7	8.9	15.7	19.3	18.5	30.0	34.8	53.9	71.5	75.5
$t_0(\widehat{T}_B)$	5.0	4.9	4.5	4.9	4.9	4.4	4.7	5.2	5.0	4.7	7.1	7.1
$t_2(\widehat{T}_B)$	5.0	5.1	5.1	8.4	8.3	8.3	13.1	13.2	13.1	28.1	28.6	28.7
$F_1(\widehat{T}_B)$	5.0	4.9	4.9	7.4	7.9	7.9	15.0	15.1	15.1	41.5	43.1	43.5
$F_{12}(\widehat{T}_B)$	5.0	5.0	4.7	6.9	7.1	6.9	12.7	14.2	13.8	33.1	41.8	42.4
$F_{012}(\widehat{T}_B)$	5.0	5.1	4.9	9.1	9.0	9.0	18.7	19.0	19.2	53.7	54.5	54.8
$t_0(\widehat{T}_B)$ c.v. w/break	6.5	5.3	5.0	6.7	5.4	4.8	6.7	5.9	5.5	6.5	7.7	7.8
$F_{012}(\lambda \times 4N)$ c.v. w/break	6.2	5.2	4.8	8.4	7.4	7.3	15.3	14.5	14.3	37.6	42.5	43.1

Panel B. $\lambda_0 = 0.25$

	$\rho = 1$		$\rho = 0.95$		$\rho = 0.90$		$\rho = 0.80$	
	$\gamma_2 = 1$	$\gamma_2 = 2$	$\gamma_2 = 1$	$\gamma_2 = 2$	$\gamma_2 = 1$	$\gamma_2 = 2$	$\gamma_2 = 1$	$\gamma_2 = 2$
t_0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
t_2	4.3	5.1	6.8	7.8	10.7	10.7	19.3	13.2
F_1	5.7	5.5	7.1	4.7	11.3	4.5	19.1	2.4
F_{012}	2.3	3.4	3.1	3.2	5.7	3.9	13.1	3.3
F_{12}	5.9	6.0	7.5	5.5	13.2	6.9	27.0	6.3
$t_0(0.5*4N)$	0.0	0.1	0.0	0.1	0.0	0.0	0.0	0.1
$t_2(0.5*4N)$	4.8	5.0	7.0	8.4	11.4	12.6	22.9	20.8
$F_1(0.5*4N)$	5.9	5.5	7.9	6.2	13.4	8.5	28.3	10.8
$F_{012}(0.5*4N)$	2.3	3.6	3.0	4.4	6.2	7.1	17.7	12.4
$F_{12}(0.5*4N)$	6.0	6.2	7.8	7.1	15.3	11.5	38.0	19.3
$\min_{T_B} t_0(T_B)$	1.7	1.2	1.6	1.1	1.8	1.3	2.9	1.8
$\min_{T_B} t_2(T_B)$	9.0	11.8	14.5	19.6	22.2	27.9	44.0	49.6
$\max_{T_B} F_1(T_B)$	9.9	12.8	15.5	19.9	27.7	33.2	61.4	67.8
$\max_{T_B} F_{012}(T_B)$	2.9	3.5	4.4	5.5	9.5	11.4	30.2	35.0
$\max_{T_B} F_{12}(T_B)$	10.5	13.9	17.0	21.4	31.9	37.1	72.7	76.6
$t_0(\widehat{T}_B)$	4.5	4.3	4.9	4.7	5.1	4.9	7.0	6.5
$t_2(\widehat{T}_B)$	5.1	5.1	8.4	8.4	13.1	13.1	28.5	28.9
$F_1(\widehat{T}_B)$	5.2	5.2	7.8	7.8	15.2	15.1	42.7	43.0
$F_{12}(\widehat{T}_B)$	4.7	4.5	6.8	6.6	13.3	13.0	39.3	39.2
$F_{012}(\widehat{T}_B)$	5.2	5.2	9.1	9.1	18.9	19.1	54.4	54.7
$t_0(\widehat{T}_B)$ c.v. w/break	5.5	5.3	5.8	5.7	6.2	5.8	8.4	7.7
$F_{012}(\lambda*4N)$ c.v. w/break	4.8	4.6	7.2	6.8	13.8	13.3	40.4	39.9

Panel C. $\lambda_0 = 0.75$

	$\rho = 1$		$\rho = 0.95$		$\rho = 0.90$		$\rho = 0.80$	
	$\gamma_2 = 1$	$\gamma_2 = 2$	$\gamma_2 = 1$	$\gamma_2 = 2$	$\gamma_2 = 1$	$\gamma_2 = 2$	$\gamma_2 = 1$	$\gamma_2 = 2$
t_0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
t_2	4.4	5.3	7.1	8.3	11.5	13.0	23.2	26.7
F_1	5.5	5.4	7.5	6.9	14.0	11.8	35.8	26.1
F_{012}	2.5	3.6	3.8	5.1	7.7	9.9	25.6	28.6
F_{12}	5.7	5.9	8.5	8.4	17.5	16.0	45.8	40.8
$t_0(0.5 \times 4N)$	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
$t_2(0.5 \times 4N)$	4.6	5.9	7.6	9.4	11.8	14.1	25.2	30.0
$F_1(0.5 \times 4N)$	6.1	5.4	7.6	7.4	14.5	12.9	38.3	34.9
$F_{012}(0.5 \times 4N)$	2.0	3.5	3.4	5.4	6.9	10.8	23.3	35.6
$F_{12}(0.5 \times 4N)$	5.8	5.9	8.6	9.4	17.7	17.8	48.8	51.0
$\min_{T_B} t_0(T_B)$	1.7	1.0	2.0	1.2	2.1	1.3	3.3	2.2
$\min_{T_B} t_2(T_B)$	7.6	8.1	12.2	12.8	18.3	19.4	38.9	41.0
$\max_{T_B} F_1(T_B)$	8.7	9.8	12.8	15.6	23.9	28.4	58.5	64.7
$\max_{T_B} F_{012}(T_B)$	2.5	2.7	4.4	4.4	9.0	9.5	30.1	32.8
$\max_{T_B} F_{12}(T_B)$	9.0	9.8	14.3	16.0	28.0	31.2	69.2	72.9
$t_0(\widehat{T}_B)$	4.5	3.9	4.7	4.4	5.5	5.3	8.0	7.5
$t_2(\widehat{T}_B)$	5.0	5.2	8.3	8.3	13.2	13.1	28.8	28.9
$F_1(\widehat{T}_B)$	5.1	5.0	7.8	8.0	15.2	15.2	42.8	42.9
$F_{12}(\widehat{T}_B)$	4.6	4.5	7.0	6.7	14.1	13.8	41.5	41.6
$F_{012}(\widehat{T}_B)$	5.0	5.0	8.9	8.9	18.6	18.7	54.7	54.9
$t_0(\widehat{T}_B)$ c.v. w/break	5.7	5.5	6.2	5.8	7.1	6.9	10.3	10.1
$F_{012}(\lambda \times 4N)$ c.v. w/break	5.8	5.8	8.5	8.4	16.6	16.6	47.1	47.2

Table 4.3: LM tests: results of simulation experiment A for $4N = 200$ (% of rejections of the null hypothesis)

Panel A. $\lambda_0 = 0.5$

	$\rho = 1$			$\rho = 0.95$			$\rho = 0.90$			$\rho = 0.80$		
	$\gamma_2 = 0$	$\gamma_2 = 1$	$\gamma_2 = 2$	$\gamma_2 = 0$	$\gamma_2 = 1$	$\gamma_2 = 2$	$\gamma_2 = 0$	$\gamma_2 = 1$	$\gamma_2 = 2$	$\gamma_2 = 0$	$\gamma_2 = 1$	$\gamma_2 = 2$
t_0	5.0	0.0	0.0	6.2	0.0	0.0	11.8	0.0	0.0	29.8	0.0	0.0
t_2	5.0	4.3	4.8	15.0	11.7	12.5	32.5	24.6	22.7	75.1	53.8	42.8
F_1	5.0	5.1	5.6	18.5	18.7	14.1	48.5	45.3	26.1	95.0	83.5	45.3
F_{012}	5.0	1.5	2.5	17.0	6.7	7.8	54.4	23.5	19.1	97.8	68.6	42.2
F_{12}	5.0	4.8	5.3	20.0	19.3	15.0	58.6	49.7	34.1	98.3	90.7	63.6
$t_0(0.5 * 4N)$	5.0	5.0	5.0	5.6	5.6	5.6	7.7	7.7	7.7	16.9	16.9	16.9
$t_2(0.5 * 4N)$	5.0	5.0	5.0	14.8	14.8	14.8	31.8	31.8	31.8	75.1	75.1	75.1
$F_1(0.5 * 4N)$	5.0	5.0	5.0	18.4	18.4	18.4	48.7	48.7	48.7	94.9	94.9	94.9
$F_{012}(0.5 * 4N)$	5.0	5.0	5.0	15.9	15.9	15.9	46.7	46.7	46.7	95.7	95.7	95.7
$F_{12}(0.5 * 4N)$	5.0	5.0	5.0	20.1	20.1	20.1	58.8	58.8	58.8	98.3	98.3	98.3
$\min_{T_B} t_0(T_B)$	5.0	2.0	1.5	5.7	2.3	1.8	7.0	3.1	2.3	13.9	6.6	4.5
$\min_{T_B} t_2(T_B)$	5.0	7.8	9.4	15.6	21.6	25.1	33.3	43.0	48.0	77.1	85.4	88.3
$\max_{T_B} F_1(T_B)$	5.0	9.2	10.8	18.2	28.3	30.5	48.5	64.4	65.5	95.5	98.4	98.4
$\max_{T_B} F_{012}(T_B)$	5.0	2.7	2.4	13.0	8.5	8.5	35.9	30.3	30.5	91.2	89.6	90.0
$\max_{T_B} F_{12}(T_B)$	5.0	9.0	10.6	20.4	31.4	33.8	59.1	72.9	74.4	98.5	99.5	99.7
$t_0(\widehat{T}_B)$	5.0	5.4	5.7	5.3	6.2	6.0	4.6	8.1	8.0	6.4	17.8	17.6
$t_2(\widehat{T}_B)$	5.0	5.1	5.2	15.3	15.4	15.4	32.5	32.8	32.8	75.4	76.0	76.0
$F_1(\widehat{T}_B)$	5.0	4.9	4.9	18.0	18.0	18.1	47.3	47.9	48.0	94.3	94.9	94.8
$F_{12}(\widehat{T}_B)$	5.0	5.5	5.4	14.9	17.4	17.0	39.0	48.8	49.0	92.0	96.0	95.9
$F_{012}(\widehat{T}_B)$	5.0	5.0	5.0	20.0	20.2	20.1	58.2	58.6	58.6	98.2	98.3	98.3
$t_0(\widehat{T}_B)$ c.v. w/break	6.0	4.8	5.1	6.2	5.6	5.5	5.8	7.4	7.5	8.0	16.0	16.3
$F_{012}(\lambda * 4N)$ c.v. w/break	5.4	5.1	5.0	15.4	16.2	15.9	40.1	46.9	46.7	92.7	95.5	95.5

Panel B. $\lambda_0 = 0.25$

	$\rho = 1$		$\rho = 0.95$		$\rho = 0.90$		$\rho = 0.80$	
	$\gamma_2 = 1$	$\gamma_2 = 2$	$\gamma_2 = 1$	$\gamma_2 = 2$	$\gamma_2 = 1$	$\gamma_2 = 2$	$\gamma_2 = 1$	$\gamma_2 = 2$
t_0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
t_2	4.7	5.2	11.8	11.5	22.6	17.8	46.5	23.9
F_1	5.3	5.9	17.2	9.7	37.4	11.6	68.6	11.0
F_{012}	1.5	2.3	6.4	5.4	18.8	8.3	50.7	10.5
F_{12}	5.0	5.5	17.7	11.0	42.4	17.1	79.5	22.3
$t_0(0.5 * 4N)$	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
$t_2(0.5 * 4N)$	4.5	5.0	12.1	12.6	25.2	22.5	54.4	43.0
$F_1(0.5 * 4N)$	5.6	6.0	18.1	13.2	43.6	24.2	82.2	42.7
$F_{012}(0.5 * 4N)$	1.1	2.6	5.3	7.6	19.3	17.9	63.4	40.9
$F_{12}(0.5 * 4N)$	4.9	5.3	19.5	14.2	49.8	33.1	90.5	62.7
$\min_{T_B} t_0(T_B)$	2.2	1.7	2.2	1.8	2.9	2.2	6.0	4.6
$\min_{T_B} t_2(T_B)$	8.4	10.8	22.9	28.0	44.6	51.0	85.5	88.9
$\max_{T_B} F_1(T_B)$	9.7	12.3	29.8	32.5	64.9	66.6	98.2	98.3
$\max_{T_B} F_{012}(T_B)$	2.7	3.0	8.7	10.2	30.3	33.1	89.9	91.6
$\max_{T_B} F_{12}(T_B)$	9.5	12.1	32.4	36.1	73.4	74.9	99.5	99.7
$t_0(\widehat{T}_B)$	5.1	4.9	5.7	5.9	7.6	7.7	15.8	16.4
$t_2(\widehat{T}_B)$	5.2	5.1	15.1	15.1	32.9	33.0	75.7	75.9
$F_1(\widehat{T}_B)$	5.0	5.0	18.0	17.9	47.6	47.6	94.7	94.7
$F_{12}(\widehat{T}_B)$	5.3	5.3	15.3	15.6	46.3	46.0	95.2	95.4
$F_{012}(\widehat{T}_B)$	5.1	5.1	20.2	20.3	58.6	58.7	98.4	98.3
$t_0(\widehat{T}_B)$ c.v. w/break	5.3	5.0	5.9	6.0	7.9	8.1	16.5	16.7
$F_{012}(\lambda * 4N)$ c.v. w/break	4.9	4.7	14.1	14.2	44.1	43.7	94.5	94.6

Panel C. $\lambda_0 = 0.75$

	$\rho = 1$		$\rho = 0.95$		$\rho = 0.90$		$\rho = 0.80$	
	$\gamma_2 = 1$	$\gamma_2 = 2$	$\gamma_2 = 1$	$\gamma_2 = 2$	$\gamma_2 = 1$	$\gamma_2 = 2$	$\gamma_2 = 1$	$\gamma_2 = 2$
t_0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
t_2	4.3	4.6	12.1	13.2	25.7	27.0	59.2	59.3
F_1	5.1	5.4	19.5	15.6	49.6	39.8	91.4	80.1
F_{012}	1.5	2.4	7.1	9.8	26.8	30.7	80.8	78.6
F_{12}	5.0	5.1	20.4	18.2	54.1	49.3	95.3	91.0
$t_0(0.5 * 4N)$	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
$t_2(0.5 * 4N)$	4.2	4.3	12.1	13.0	26.5	28.3	62.4	65.3
$F_1(0.5 * 4N)$	5.6	5.4	19.3	16.6	50.2	45.0	94.2	92.3
$F_{012}(0.5 * 4N)$	1.1	2.1	5.2	9.5	23.8	33.5	79.4	90.2
$F_{12}(0.5 * 4N)$	4.9	4.7	20.8	18.6	55.4	53.5	97.0	97.3
$\min_{T_B} t_0(T_B)$	1.9	1.6	2.0	1.6	2.9	2.4	7.3	5.7
$\min_{T_B} t_2(T_B)$	7.2	8.1	20.3	22.6	41.4	44.5	84.2	86.5
$\max_{T_B} F_1(T_B)$	8.5	9.4	26.4	27.0	61.5	62.5	98.0	98.1
$\max_{T_B} F_{012}(T_B)$	2.6	2.4	8.4	8.9	30.6	33.1	90.9	91.9
$\max_{T_B} F_{12}(T_B)$	8.4	9.3	29.1	30.6	70.9	71.6	99.4	99.7
$t_0(\widehat{T}_B)$	4.3	4.1	5.2	5.2	8.1	7.9	19.2	19.2
$t_2(\widehat{T}_B)$	5.0	5.0	15.3	15.4	32.6	32.6	75.3	75.5
$F_1(\widehat{T}_B)$	4.8	4.8	18.2	18.2	47.7	47.8	94.8	94.8
$F_{12}(\widehat{T}_B)$	4.9	5.1	16.5	16.4	47.1	47.1	95.9	96.0
$F_{012}(\widehat{T}_B)$	4.9	5.0	20.1	20.2	58.7	58.7	98.3	98.3
$t_0(\widehat{T}_B)$ c.v. w/break	4.9	5.0	5.8	5.8	9.0	9.0	20.9	21.1
$F_{012}(\lambda * 4N)$ c.v. w/break	5.0	5.3	16.7	16.7	47.8	48.0	95.9	96.2

Table 4.4: HEGY tests: results of simulation experiment A for $4N = 100$ (% of rejections of the null hypothesis)

Panel A. $\lambda_0 = 0.5$

	$\rho = 1$			$\rho = 0.95$			$\rho = 0.90$			$\rho = 0.80$		
	$\gamma_2 = 0$	$\gamma_2 = 1$	$\gamma_2 = 2$	$\gamma_2 = 0$	$\gamma_2 = 1$	$\gamma_2 = 2$	$\gamma_2 = 0$	$\gamma_2 = 1$	$\gamma_2 = 2$	$\gamma_2 = 0$	$\gamma_2 = 1$	$\gamma_2 = 2$
t_0	5.0	0.0	0.0	5.4	0.0	0.0	6.2	0.0	0.0	10.3	0.0	0.0
t_2	5.0	4.4	5.1	7.0	6.2	7.6	9.1	7.8	9.6	15.6	12.3	14.9
F_1	5.0	5.3	4.1	7.8	7.8	5.8	11.2	10.8	7.7	20.7	20.3	14.3
F_{012}	5.0	1.3	1.4	8.3	2.3	2.6	11.9	3.6	3.8	27.0	8.6	9.5
F_{12}	5.0	5.0	4.5	8.1	8.3	7.3	12.4	12.1	10.4	26.5	25.4	22.3
$t_0(0.5 * 4N)$	5.0	5.0	5.0	5.3	5.3	5.3	5.7	5.7	5.7	7.6	7.6	7.6
$t_2(0.5 * 4N)$	5.0	5.0	5.0	7.1	7.1	7.1	9.0	9.0	9.0	15.0	15.0	15.0
$F_1(0.5 * 4N)$	5.0	5.0	5.0	7.6	7.6	7.6	10.7	10.7	10.7	20.3	20.3	20.3
$F_{012}(0.5 * 4N)$	5.0	5.0	5.0	7.7	7.7	7.7	10.1	10.1	10.1	22.7	22.7	22.7
$F_{12}(0.5 * 4N)$	5.0	5.0	5.0	7.9	7.9	7.9	12.2	12.2	12.2	26.3	26.3	26.3
$\min_{T_B} t_0(T_B)$	5.0	5.2	2.5	5.4	5.4	2.8	5.6	6.2	3.2	6.9	7.4	4.1
$\min_{T_B} t_2(T_B)$	5.0	8.6	9.0	7.1	11.9	12.2	9.1	15.1	15.0	14.8	24.3	24.1
$\max_{T_B} F_1(T_B)$	5.0	9.7	14.2	7.5	15.0	21.6	10.6	20.1	28.6	20.3	35.7	46.8
$\max_{T_B} F_{012}(T_B)$	5.0	5.4	4.9	6.9	8.5	8.0	9.1	11.8	11.9	18.6	23.3	24.0
$\max_{T_B} F_{12}(T_B)$	5.0	10.0	12.6	8.2	16.4	20.1	12.2	22.7	27.4	25.9	43.8	49.3
$t_0(\widehat{T}_B)$	5.0	4.1	2.9	5.3	5.0	3.3	5.8	5.5	3.6	7.3	7.0	4.6
$t_2(\widehat{T}_B)$	5.0	5.0	5.0	7.1	7.4	7.2	8.8	9.3	9.0	14.7	15.1	15.2
$F_1(\widehat{T}_B)$	5.0	5.1	5.1	7.6	7.6	7.8	10.8	11.0	11.1	20.8	21.4	21.1
$F_{12}(\widehat{T}_B)$	5.0	4.7	3.6	7.2	7.3	5.7	9.4	9.9	8.1	19.3	21.0	17.9
$F_{012}(\widehat{T}_B)$	5.0	5.1	5.0	8.0	8.5	8.1	12.0	12.6	12.9	26.6	27.1	27.0
$t_0(\widehat{T}_B)$ c.v. w/break	14.6	10.7	8.0	15.3	11.5	8.8	16.0	12.3	9.6	18.7	14.9	11.4
$F_{012}(\lambda * 4N)$ c.v. w/break	10.3	8.7	7.2	14.6	13.2	10.9	18.8	17.0	14.6	34.1	33.0	29.1

Panel B. $\lambda_0 = 0.25$

	$\rho = 1$		$\rho = 0.95$		$\rho = 0.90$		$\rho = 0.80$	
	$\gamma_2 = 1$	$\gamma_2 = 2$	$\gamma_2 = 1$	$\gamma_2 = 2$	$\gamma_2 = 1$	$\gamma_2 = 2$	$\gamma_2 = 1$	$\gamma_2 = 2$
t_0	0.1	0.1	0.1	0.1	0.0	0.2	0.0	0.1
t_2	4.1	4.5	5.7	6.4	7.3	8.2	12.3	12.9
F_1	5.1	3.7	7.4	4.9	10.2	5.9	18.7	10.7
F_{012}	2.6	2.8	4.4	4.1	6.7	5.8	13.8	12.5
F_{12}	4.7	3.5	7.8	5.8	11.5	7.8	24.2	16.8
$t_0(0.5 * 4N)$	0.7	5.0	0.6	5.2	0.6	5.0	0.5	4.7
$t_2(0.5 * 4N)$	4.2	4.5	5.6	6.4	7.1	8.3	12.3	12.6
$F_1(0.5 * 4N)$	5.1	3.6	7.6	5.2	9.9	6.2	18.6	11.6
$F_{012}(0.5 * 4N)$	3.2	6.4	5.2	9.0	7.4	11.7	13.7	21.7
$F_{12}(0.5 * 4N)$	4.7	3.7	8.2	5.8	11.4	8.2	23.9	17.6
$\min_{T_B} t_0(T_B)$	12.0	57.3	11.9	52.9	11.7	49.1	11.6	43.1
$\min_{T_B} t_2(T_B)$	8.7	9.3	12.1	12.5	14.9	15.1	23.6	24.0
$\max_{T_B} F_1(T_B)$	10.0	15.1	15.2	22.6	19.9	29.5	35.6	46.7
$\max_{T_B} F_{012}(T_B)$	10.5	36.9	16.6	49.0	21.5	56.9	36.4	75.4
$\max_{T_B} F_{12}(T_B)$	10.2	12.9	16.4	21.0	22.9	28.3	42.9	50.2
$t_0(\widehat{T}_B)$	1.9	1.5	2.0	1.5	2.6	1.8	3.7	2.8
$t_2(\widehat{T}_B)$	5.1	5.1	7.0	7.3	9.5	9.7	15.3	15.5
$F_1(\widehat{T}_B)$	5.0	4.9	7.8	7.8	10.8	10.8	20.5	20.8
$F_{12}(\widehat{T}_B)$	2.3	2.1	3.7	3.4	5.8	5.4	14.0	12.6
$F_{012}(\widehat{T}_B)$	5.1	5.2	8.3	8.2	12.7	12.7	26.7	26.8
$t_0(\widehat{T}_B)$ c.v. w/break	8.0	6.7	8.8	7.3	9.9	8.2	13.2	10.3
$F_{012}(\lambda * 4N)$ c.v. w/break	6.8	6.2	10.5	9.3	14.0	12.8	29.4	26.9

Panel C. $\lambda_0 = 0.75$

	$\rho = 1$		$\rho = 0.95$		$\rho = 0.90$		$\rho = 0.80$	
	$\gamma_2 = 1$	$\gamma_2 = 2$	$\gamma_2 = 1$	$\gamma_2 = 2$	$\gamma_2 = 1$	$\gamma_2 = 2$	$\gamma_2 = 1$	$\gamma_2 = 2$
t_0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
t_2	4.5	4.7	6.2	6.5	7.7	8.3	12.0	13.1
F_1	5.3	3.5	7.6	4.8	10.2	6.3	18.5	11.2
F_{012}	1.1	1.1	1.8	1.8	2.7	2.6	6.9	6.0
F_{12}	4.9	4.0	8.3	6.0	11.7	8.3	24.1	18.2
$t_0(0.5 * 4N)$	0.2	0.4	0.1	0.5	0.1	0.4	0.1	0.4
$t_2(0.5 * 4N)$	4.5	4.6	6.1	6.3	7.6	7.9	12.0	12.7
$F_1(0.5 * 4N)$	5.0	3.4	7.4	4.7	10.0	6.3	18.0	11.1
$F_{012}(0.5 * 4N)$	1.5	2.2	2.3	3.3	2.8	4.7	6.9	9.4
$F_{12}(0.5 * 4N)$	4.4	3.8	7.8	5.7	11.3	8.2	23.1	18.0
$\min_{T_B} t_0(T_B)$	6.9	20.0	7.0	17.7	7.7	16.3	9.0	13.9
$\min_{T_B} t_2(T_B)$	7.7	8.0	11.2	11.3	14.2	14.6	22.9	23.2
$\max_{T_B} F_1(T_B)$	9.1	12.4	14.2	19.6	19.7	26.6	35.1	44.2
$\max_{T_B} F_{012}(T_B)$	7.8	18.3	12.1	27.7	16.1	36.1	30.5	57.3
$\max_{T_B} F_{12}(T_B)$	9.3	11.0	15.7	18.1	22.0	25.7	42.2	47.7
$t_0(\widehat{T}_B)$	2.7	2.2	3.3	2.7	3.8	3.2	5.8	4.5
$t_2(\widehat{T}_B)$	5.2	5.1	7.1	7.2	9.0	9.0	15.5	15.7
$F_1(\widehat{T}_B)$	5.1	5.2	7.6	7.7	11.2	11.1	20.8	20.7
$F_{12}(\widehat{T}_B)$	3.4	3.3	5.6	4.8	7.8	7.3	18.2	16.3
$F_{012}(\widehat{T}_B)$	4.9	5.2	8.1	8.6	12.7	12.6	27.2	27.3
$t_0(\widehat{T}_B)$ c.v. w/break	8.9	7.8	10.3	8.9	11.5	9.7	15.1	12.6
$F_{012}(\lambda * 4N)$ c.v. w/break	7.5	6.8	11.4	10.6	15.7	14.3	31.1	28.8

Table 4.5: HEGY tests: results of simulation experiment A for $4N = 200$ (% of rejections of the null hypothesis)

Panel A. $\lambda_0 = 0.5$

	$\rho = 1$			$\rho = 0.95$			$\rho = 0.90$			$\rho = 0.80$		
	$\gamma_2 = 0$	$\gamma_2 = 1$	$\gamma_2 = 2$	$\gamma_2 = 0$	$\gamma_2 = 1$	$\gamma_2 = 2$	$\gamma_2 = 0$	$\gamma_2 = 1$	$\gamma_2 = 2$	$\gamma_2 = 0$	$\gamma_2 = 1$	$\gamma_2 = 2$
t_0	5.0	0.0	0.0	6.0	0.0	0.0	8.5	0.0	0.0	20.8	0.0	0.0
t_2	5.0	4.2	4.6	8.8	7.2	8.0	14.2	11.1	12.0	37.6	26.5	30.4
F_1	5.0	5.9	5.0	10.2	12.5	10.0	19.2	22.9	18.3	58.4	60.0	53.8
F_{012}	5.0	0.9	1.2	11.4	2.7	3.1	25.2	6.9	7.7	78.4	32.0	37.2
F_{12}	5.0	5.3	4.6	12.0	13.0	11.9	25.5	26.0	24.3	74.9	68.1	68.5
$t_0(0.5 * 4N)$	5.0	5.0	5.0	6.1	6.1	6.1	7.8	7.8	7.8	15.9	15.9	15.9
$t_2(0.5 * 4N)$	5.0	5.0	5.0	8.6	8.6	8.6	13.9	13.9	13.9	37.1	37.1	37.1
$F_1(0.5 * 4N)$	5.0	5.0	5.0	10.2	10.2	10.2	19.3	19.3	19.3	58.1	58.1	58.1
$F_{012}(0.5 * 4N)$	5.0	5.0	5.0	11.0	11.0	11.0	23.0	23.0	23.0	72.9	72.9	72.9
$F_{12}(0.5 * 4N)$	5.0	5.0	5.0	11.9	11.9	11.9	25.6	25.6	25.6	74.6	74.6	74.6
$\min_{T_B} t_0(T_B)$	5.0	5.2	3.6	5.4	5.5	3.7	6.3	7.1	5.1	11.6	11.9	8.9
$\min_{T_B} t_2(T_B)$	5.0	7.4	8.5	8.7	12.9	14.2	13.9	20.8	22.5	36.8	49.2	51.4
$\max_{T_B} F_1(T_B)$	5.0	8.0	8.7	10.0	16.6	17.8	18.6	29.8	31.0	57.2	72.9	74.6
$\max_{T_B} F_{012}(T_B)$	5.0	5.4	5.0	10.2	11.4	10.7	20.4	23.4	22.3	65.2	69.8	69.1
$\max_{T_B} F_{12}(T_B)$	5.0	8.0	8.8	12.1	19.4	20.3	25.6	37.6	38.3	74.4	86.4	86.5
$t_0(\widehat{T}_B)$	5.0	4.6	3.5	5.7	5.1	4.0	6.6	6.4	5.4	12.1	12.3	9.2
$t_2(\widehat{T}_B)$	5.0	4.9	5.1	8.5	8.8	8.7	13.8	13.8	14.1	36.9	36.6	37.2
$F_1(\widehat{T}_B)$	5.0	5.2	5.2	10.4	10.3	10.7	19.0	19.6	19.9	58.5	58.7	58.8
$F_{12}(\widehat{T}_B)$	5.0	4.7	4.3	10.2	10.1	9.1	20.5	21.4	19.7	65.7	66.7	64.8
$F_{012}(\widehat{T}_B)$	5.0	5.2	5.2	12.0	12.7	12.7	25.9	26.4	26.2	75.0	74.9	75.5
$t_0(\widehat{T}_B)$ c.v. w/break	14.02	11.58	9.96	15.88	13.48	11.22	18.46	16.06	13.16	30.9	26.12	21.92
$F_{012}(\lambda * 4N)$ c.v. w/break	10.64	9.48	8.44	19.3	18.52	17.02	33.94	33.42	30.8	81.6	80.88	79.06

Panel B. $\lambda_0 = 0.25$

	$\rho = 1$		$\rho = 0.95$		$\rho = 0.90$		$\rho = 0.80$	
	$\gamma_2 = 1$	$\gamma_2 = 2$	$\gamma_2 = 1$	$\gamma_2 = 2$	$\gamma_2 = 1$	$\gamma_2 = 2$	$\gamma_2 = 1$	$\gamma_2 = 2$
t_0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
t_2	4.3	4.3	7.0	7.3	10.9	11.3	25.8	28.5
F_1	5.8	4.7	12.4	9.2	22.0	16.1	58.1	48.5
F_{012}	2.3	2.8	6.2	6.4	13.8	14.2	47.6	51.1
F_{12}	5.3	4.4	12.6	10.3	25.2	21.4	66.8	63.7
$t_0(0.5 * 4N)$	0.0	0.2	0.0	0.3	0.0	0.2	0.0	0.2
$t_2(0.5 * 4N)$	4.3	4.1	7.0	7.2	10.8	11.0	26.3	28.3
$F_1(0.5 * 4N)$	5.7	4.6	11.8	8.7	21.3	15.6	57.1	47.4
$F_{012}(0.5 * 4N)$	2.1	4.3	4.9	9.6	11.4	18.3	42.6	57.5
$F_{12}(0.5 * 4N)$	5.1	4.4	12.3	10.0	25.0	20.8	67.0	63.5
$\min_{T_B} t_0(T_B)$	3.6	7.1	4.0	6.8	4.8	6.9	9.5	9.2
$\min_{T_B} t_2(T_B)$	7.1	8.0	12.5	13.9	20.3	22.1	48.1	51.0
$\max_{T_B} F_1(T_B)$	7.8	8.4	16.8	17.5	30.1	30.9	72.4	73.9
$\max_{T_B} F_{012}(T_B)$	6.7	13.5	15.0	27.7	27.7	47.2	74.3	90.1
$\max_{T_B} F_{12}(T_B)$	7.7	8.4	19.3	20.3	37.5	38.5	85.5	86.7
$t_0(\widehat{T}_B)$	2.6	2.1	2.9	2.5	3.9	3.3	8.5	7.4
$t_2(\widehat{T}_B)$	5.0	5.0	8.6	8.6	13.9	14.0	36.9	37.1
$F_1(\widehat{T}_B)$	5.1	5.1	10.4	10.4	19.4	19.4	58.4	58.7
$F_{12}(\widehat{T}_B)$	3.0	2.6	6.8	6.2	15.1	14.5	59.1	57.9
$F_{012}(\widehat{T}_B)$	5.2	5.0	12.3	12.4	26.3	26.4	75.0	75.0
$t_0(\widehat{T}_B)$ c.v. w/break	9.12	7.76	10.76	9.38	13.4	11.84	24.68	21.76
$F_{012}(\lambda * 4N)$ c.v. w/break	7.14	6.4	14.9	13.86	28.88	27.76	79.1	78.04

Panel C: $\lambda_0 = 0.75$

	$\rho = 1$		$\rho = 0.95$		$\rho = 0.90$		$\rho = 0.80$	
	$\gamma_2 = 1$	$\gamma_2 = 2$	$\gamma_2 = 1$	$\gamma_2 = 2$	$\gamma_2 = 1$	$\gamma_2 = 2$	$\gamma_2 = 1$	$\gamma_2 = 2$
t_0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
t_2	4.4	4.3	7.5	7.4	10.9	11.4	26.0	28.2
F_1	5.5	4.7	12.3	8.9	22.1	16.0	59.0	48.7
F_{012}	0.6	0.8	2.0	2.0	5.2	5.0	26.6	28.5
F_{12}	5.1	4.3	12.5	10.5	25.7	21.5	67.5	64.6
$t_0(0.5 \times 4N)$	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
$t_2(0.5 \times 4N)$	4.5	4.1	7.3	7.1	10.9	11.0	26.3	28.0
$F_1(0.5 \times 4N)$	5.4	4.4	11.7	8.5	21.4	15.4	57.7	46.9
$F_{012}(0.5 \times 4N)$	0.5	1.1	1.6	2.6	4.2	6.6	22.9	31.8
$F_{12}(0.5 \times 4N)$	5.0	4.1	12.4	10.1	25.4	21.0	67.2	63.5
$\min_{T_B} t_0(T_B)$	4.7	6.0	5.5	6.4	7.0	7.1	12.5	10.7
$\min_{T_B} t_2(T_B)$	7.0	7.8	12.4	13.8	20.5	22.4	48.0	51.1
$\max_{T_B} F_1(T_B)$	7.9	8.3	16.8	17.2	29.6	30.8	72.9	74.0
$\max_{T_B} F_{012}(T_B)$	7.4	12.9	15.3	26.5	29.3	46.7	76.0	89.7
$\max_{T_B} F_{12}(T_B)$	7.8	8.4	19.4	20.2	37.7	37.8	86.0	87.2
$t_0(\widehat{T}_B)$	3.6	3.0	4.3	3.8	5.6	5.0	11.7	9.6
$t_2(\widehat{T}_B)$	4.9	4.9	8.7	9.0	14.1	14.1	37.3	37.3
$F_1(\widehat{T}_B)$	5.0	5.1	10.7	10.7	19.6	19.9	58.4	58.8
$F_{12}(\widehat{T}_B)$	4.1	3.8	8.9	8.5	19.7	18.5	65.4	63.7
$F_{012}(\widehat{T}_B)$	4.9	5.0	12.7	12.8	26.5	26.9	75.2	75.5
$t_0(\widehat{T}_B)$ c.v. w/break	9.7	9.48	11.7	10.94	15.56	13.62	27.04	24.82
$F_{012}(\lambda \times 4N)$ c.v. w/break	8.04	7.6	15.98	15.12	30.6	29.54	79.52	78.64

Table 4.6: LM tests: results of simulation experiment B for $4N = 100$ (% of rejections of the null hypothesis)

	$\alpha = 0.8$	$\alpha = 0.8$	$\alpha = 0.8$	$\alpha = 0$	$\alpha = 0$	$\alpha = 0$	$\alpha = 0$	$\alpha = 0$	$\alpha = 0$
	$\theta = 0$	$\theta = 0$	$\theta = 0$	$\theta = 0.8$	$\theta = 0.8$	$\theta = 0.8$	$\theta = -0.8$	$\theta = -0.8$	$\theta = -0.8$
	$\gamma_2 = 0$	$\gamma_2 = 1$	$\gamma_2 = 2$	$\gamma_2 = 0$	$\gamma_2 = 1$	$\gamma_2 = 2$	$\gamma_2 = 0$	$\gamma_2 = 1$	$\gamma_2 = 2$
$t_0(0.5 * 4N)$	5.2	5.2	5.2	11.1	11.1	11.1	22.4	22.4	22.4
$t_2(0.5 * 4N)$	4.6	4.6	4.6	9.3	9.3	9.3	40.3	40.3	40.3
$F_1(0.5 * 4N)$	5.2	5.2	5.2	6.1	6.1	6.1	39.9	39.9	39.9
$F_{012}(0.5 * 4N)$	5.7	5.7	5.7	11.0	11.0	11.0	47.4	47.4	47.4
$F_{12}(0.5 * 4N)$	5.2	5.2	5.2	7.7	7.7	7.7	50.8	50.8	50.8
$\min_{T_B} t_0(T_B)$	5.6	4.1	4.5	12.6	8.2	6.3	33.0	13.4	9.9
$\min_{T_B} t_2(T_B)$	6.5	8.2	12.6	8.6	11.1	13.8	44.0	55.5	65.2
$\max_{T_B} F_1(T_B)$	5.5	7.5	10.9	5.5	6.9	9.7	43.0	60.9	66.4
$\max_{T_B} F_{012}(T_B)$	5.7	4.7	6.1	11.5	9.1	8.3	50.0	53.7	61.3
$\max_{T_B} F_{12}(T_B)$	6.3	8.1	12.0	7.0	8.7	11.9	53.6	71.5	76.7
$t_0(\widehat{T}_B)$	4.0	5.1	4.4	7.3	9.1	9.7	8.7	17.0	18.9
$t_2(\widehat{T}_B)$	4.6	4.6	4.8	9.0	9.3	9.2	39.0	40.2	40.2
$F_1(\widehat{T}_B)$	4.8	5.0	4.7	6.0	5.6	5.7	36.4	39.3	39.6
$F_{12}(\widehat{T}_B)$	4.3	5.4	5.1	8.3	9.7	10.4	33.7	43.2	45.0
$F_{012}(\widehat{T}_B)$	5.0	5.1	5.2	7.8	7.6	7.6	48.2	50.9	51.0

Table 4.7: LM tests: results of simulation experiment A for $4N = 200$ (% of rejections of the null hypothesis)

	$\alpha = 0.8$	$\alpha = 0.8$	$\alpha = 0.8$	$\alpha = 0$	$\alpha = 0$	$\alpha = 0$	$\alpha = 0$	$\alpha = 0$	$\alpha = 0$
	$\theta = 0$	$\theta = 0$	$\theta = 0$	$\theta = 0.8$	$\theta = 0.8$	$\theta = 0.8$	$\theta = -0.8$	$\theta = -0.8$	$\theta = -0.8$
	$\gamma_2 = 0$	$\gamma_2 = 1$	$\gamma_2 = 2$	$\gamma_2 = 0$	$\gamma_2 = 1$	$\gamma_2 = 2$	$\gamma_2 = 0$	$\gamma_2 = 1$	$\gamma_2 = 2$
$t_0(0.5 * 4N)$	4.5	4.5	4.5	14.3	14.3	14.3	47.4	47.4	47.4
$t_2(0.5 * 4N)$	4.8	4.8	4.8	10.5	10.5	10.5	56.1	56.1	56.1
$F_1(0.5 * 4N)$	5.4	5.4	5.4	6.4	6.4	6.4	60.6	60.6	60.6
$F_{012}(0.5 * 4N)$	4.9	4.9	4.9	13.7	13.7	13.7	85.2	85.2	85.2
$F_{12}(0.5 * 4N)$	5.2	5.2	5.2	8.0	8.0	8.0	76.5	76.5	76.5
$\min_{T_B} t_0(T_B)$	5.5	3.7	4.2	16.2	11.7	9.9	62.4	24.3	19.6
$\min_{T_B} t_2(T_B)$	6.2	8.1	11.3	10.7	12.9	15.9	57.2	62.0	69.3
$\max_{T_B} F_1(T_B)$	5.8	7.5	9.9	6.1	7.2	8.5	60.3	77.0	78.3
$\max_{T_B} F_{012}(T_B)$	5.3	4.1	4.7	15.3	11.6	10.0	89.7	75.7	75.9
$\max_{T_B} F_{12}(T_B)$	5.7	7.7	10.4	7.9	9.6	12.3	76.8	85.8	87.7
$t_0(\widehat{T}_B)$	5.2	5.1	4.6	12.0	14.6	14.9	16.8	39.7	43.5
$t_2(\widehat{T}_B)$	5.1	4.9	5.0	11.0	10.7	10.7	56.8	56.7	56.6
$F_1(\widehat{T}_B)$	5.2	5.2	5.2	6.3	6.3	6.3	59.0	59.9	60.1
$F_{12}(\widehat{T}_B)$	5.5	5.3	5.0	12.8	14.0	14.3	74.3	83.1	83.6
$F_{012}(\widehat{T}_B)$	5.0	5.1	5.3	8.0	8.0	8.0	75.8	76.4	76.4

Table 4.8: HEGY tests: results of simulation experiment A for $4N = 100$ (% of rejections of the null hypothesis)

	$\alpha = 0.8$	$\alpha = 0.8$	$\alpha = 0.8$	$\alpha = 0$	$\alpha = 0$	$\alpha = 0$	$\alpha = 0$	$\alpha = 0$	$\alpha = 0$
	$\theta = 0$	$\theta = 0$	$\theta = 0$	$\theta = 0.8$	$\theta = 0.8$	$\theta = 0.8$	$\theta = -0.8$	$\theta = -0.8$	$\theta = -0.8$
	$\gamma_2 = 0$	$\gamma_2 = 1$	$\gamma_2 = 2$	$\gamma_2 = 0$	$\gamma_2 = 1$	$\gamma_2 = 2$	$\gamma_2 = 0$	$\gamma_2 = 1$	$\gamma_2 = 2$
$t_0(0.5 * 4N)$	5.5	5.5	5.5	15.5	15.5	15.5	77.2	77.2	77.2
$t_2(0.5 * 4N)$	4.6	4.6	4.6	10.8	10.8	10.8	77.4	77.4	77.4
$F_1(0.5 * 4N)$	4.3	4.3	4.3	10.7	10.7	10.7	90.1	90.1	90.1
$F_{012}(0.5 * 4N)$	5.1	5.1	5.1	17.1	17.1	17.1	95.2	95.2	95.2
$F_{12}(0.5 * 4N)$	3.8	3.8	3.8	12.4	12.4	12.4	94.4	94.4	94.4
$\min_{T_B} t_0(T_B)$	7.4	12.6	16.3	17.5	31.4	18.6	74.2	68.8	61.5
$\min_{T_B} t_2(T_B)$	5.2	8.0	10.4	10.7	14.3	14.4	80.9	92.0	94.5
$\max_{T_B} F_1(T_B)$	5.0	7.8	11.7	10.1	11.1	11.8	91.0	97.4	98.3
$\max_{T_B} F_{012}(T_B)$	6.0	10.1	17.7	17.7	24.0	17.5	95.4	97.6	98.1
$\max_{T_B} F_{12}(T_B)$	4.7	8.0	12.0	12.3	13.9	14.1	95.3	98.2	98.8
$t_0(\widehat{T}_B)$	5.0	8.1	7.7	12.4	17.3	14.3	71.9	68.2	64.4
$t_2(\widehat{T}_B)$	4.6	4.4	4.9	10.9	10.2	9.8	76.2	76.5	77.3
$F_1(\widehat{T}_B)$	4.5	4.3	4.8	10.8	9.4	9.2	89.9	90.2	90.1
$F_{12}(\widehat{T}_B)$	4.7	6.8	8.3	14.9	17.2	14.6	94.8	94.4	93.7
$F_{012}(\widehat{T}_B)$	3.9	3.7	4.1	12.7	11.3	10.9	94.8	94.6	94.4

Table 4.9: HEGY tests: results of simulation experiment A for $4N = 200$ (% of rejections of the null hypothesis)

	$\alpha = 0.8$	$\alpha = 0.8$	$\alpha = 0.8$	$\alpha = 0$	$\alpha = 0$	$\alpha = 0$	$\alpha = 0$	$\alpha = 0$	$\alpha = 0$
	$\theta = 0$	$\theta = 0$	$\theta = 0$	$\theta = 0.8$	$\theta = 0.8$	$\theta = 0.8$	$\theta = -0.8$	$\theta = -0.8$	$\theta = -0.8$
	$\gamma_2 = 0$	$\gamma_2 = 1$	$\gamma_2 = 2$	$\gamma_2 = 0$	$\gamma_2 = 1$	$\gamma_2 = 2$	$\gamma_2 = 0$	$\gamma_2 = 1$	$\gamma_2 = 2$
$t_0(0.5 \times 4N)$	5.3	5.3	5.3	18.2	18.2	18.2	87.9	87.9	87.9
$t_2(0.5 \times 4N)$	4.7	4.7	4.7	12.0	12.0	12.0	83.1	83.1	83.1
$F_1(0.5 \times 4N)$	5.0	5.0	5.0	9.7	9.7	9.7	95.5	95.5	95.5
$F_{012}(0.5 \times 4N)$	5.3	5.3	5.3	20.7	20.7	20.7	99.8	99.8	99.8
$F_{12}(0.5 \times 4N)$	4.8	4.8	4.8	12.7	12.7	12.7	98.7	98.7	98.7
$\min_{T_B} t_0(T_B)$	5.7	9.7	13.9	19.6	33.4	23.4	83.1	79.1	74.1
$\min_{T_B} t_2(T_B)$	5.0	6.9	8.4	11.9	15.3	17.1	83.7	88.9	89.9
$\max_{T_B} F_1(T_B)$	5.2	6.7	8.9	9.4	10.0	10.3	95.3	98.3	98.0
$\max_{T_B} F_{012}(T_B)$	5.5	8.4	13.2	22.3	30.8	23.8	99.7	99.7	99.6
$\max_{T_B} F_{12}(T_B)$	5.0	6.7	8.5	12.5	14.0	14.5	98.6	99.3	99.3
$t_0(\widehat{T}_B)$	4.7	7.1	7.9	18.1	23.2	19.7	83.5	79.7	75.8
$t_2(\widehat{T}_B)$	4.6	4.7	4.9	11.7	11.6	11.8	83.1	83.2	83.1
$F_1(\widehat{T}_B)$	5.0	4.9	5.1	10.1	9.1	8.9	95.7	95.6	95.5
$F_{12}(\widehat{T}_B)$	4.8	6.6	7.5	20.5	23.7	20.6	99.7	99.7	99.5
$F_{012}(\widehat{T}_B)$	4.9	4.9	4.8	13.0	11.7	11.5	98.7	98.7	98.7

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