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Dynamic factor models with jagged edge panel data: Taking on board the dynamics of the idiosyncratic components

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Abstract

As macroeconomic data are released with different delays, one has to handle unbalanced panel data sets with missing values at the end of the sample period when estimating dynamic factor models. We propose an EM algorithm which copes with such data sets while accounting for autoregressive common factors and allowing for serial correlation in the idiosyncratic components. Based on Monte Carlo simulations, we find that taking on board the dynamics of the idiosyncratic components improves significantly the accuracy of the estimation of both the missing values and the common factors at the end of the sample period.

JEL classification: C32; C33; C53.

Keywords: Factor model; Maximum likelihood; EM algorithm; Kalman filter; Missing data.

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1 Introduction

The literature on dynamic factor models in economics and finance goes back to Geweke (1977), Sargent and Sims (1977), Geweke and Singleton (1981) and Watson and Engle (1983). In a factor model, the data generating process of each variable is the sum of a common component, driven by a small number of latent common factors, and an idiosyncratic component. In the classical formulation, the idiosyncratic components are cross-sectionally and serially independent and also uncorrelated with the common factors. In addition, the common factors are generated by a finite order vector autoregression. For a fixed cross-sectional dimension, the model can be consistently estimated by Gaussian maximum likelihood. In the early literature, the analysis was limited to panels with a small number of variables and the model was estimated by maximum likelihood using either frequency or time domain approaches.

In the context of growing data availability, the existence of large panel data sets led to the development of a non-parametric estimation approach based on least squares. The resulting principal components estimator avoided the feasibility issues and the increased technical complexity of the maximum likelihood estimator when dealing with large cross-sections. Connor and Korajczyk (1986, 1988, 1993) discussed the consistency of the principal components estimator when the number of variables tends to infinity and the time dimension remains fixed. When both panel dimensions tend to infinity, Stock and Watson (1998, 2002b), Bai and Ng (2002), Bai (2003) and Amengual and Watson (2007) have shown that, under slightly different sets of assumptions regarding the data generating processes of the factors and of the idiosyncratic components, the first principal components span the factor space, even if there is some heteroskedasticity and limited dependence of the idiosyncratic components in both dimensions, as well as moderate correlation between the latter and the factors. Related work includes Forni and Reichlin (1998), Forni and Lippi (2001), Forni *et al.* (2000, 2004, 2005), using frequency domain methods.

Doz *et al.* (2012) reconciled the classical factor model estimated by Gaussian maximum likelihood with the strand of literature on factor models for large cross-sections. In a quasi-maximum likelihood approach (in the sense of White, 1982), they treat the classical model as a possibly misspecified model which is used for estimation purposes, henceforth the "estimation model". By imposing the classical assumptions on the estimation model makes the Gaussian maximum likelihood estimation feasible for large cross-sections. They show that the factor space is estimated consistently when both panel dimensions tend to infinity even if the underlying data set is generated by a model with heteroskedastic and serially correlated idiosyncratic components. More recently, the estimation model has been generalized to allow for serially correlated idiosyncratic components (Jungbacker and Koopman, 2008; Reis and Watson, 2010; Banbura and Modugno, 2010; among others).

In practice, macroeconomic data become available with different delays, i.e. one has to handle unsynchronized data releases for a large number of variables. In fact, if one had to wait until all data were available it would be necessary to wait a few months to estimate the factors for the current period. The staggered release of information results in an unbalanced panel data with missing values located at the end of the sample period. The presence of missing values at the end of the sample is by and large the more practically relevant issue for macroeconomic forecasting, nowcasting and policy analysis. Typically, for data of the same frequency, there are no missing values at the middle of the sample whereas if they are located at the beginning one can always shorten the sample and still have long time series in most cases. In light of this, the jagged edge panel data feature is clearly the most challenging feature that one has to deal with. Giannone *et al.*(2008) address this issue in the framework of a dynamic factor model and a large cross-section. They refer to panels with this specific unbalanced feature as having a jagged edge across the most recent periods of the sample. Other authors refer to this problem as ragged edge data (see, for example, Wallis ,1986, and more recently Schumacher and Breitung, 2008, Marcellino and Schumacher, 2010, and Kuzin *et al.,* 2011).

The estimation model considered by Giannone *et al.* (2008) is a dynamic factor model with idiosyncratic components cross-sectionally orthogonal and white noise.[1] As mentioned above, the misspecification of the idiosyncratic components autocorrelation does not jeopardize the consistent estimation of the factor space, but consistency is not the only issue at stake. A more accurate estimation of factors at the end of the sample is key to produce superior forecasts when the panel presents the jagged edge feature. A precise estimation of the factors in the most recent periods may also be important, for

[1] They do not estimate the model by maximum likelihood. Instead, they use the two-step estimator based on Kalman filtering suggested by Doz *et al.* (2007).

example, in real time disaggregation of time series based on factor models estimated with higher frequency panel data sets (see Angelini *et al.*, 2006).

Assuming serially uncorrelated idiosyncratic components can be a strong assumption. In Figure 1 we present the histogram of the first order autocorrelation coefficients of the idiosyncratic components estimated from the well-known US monthly data set of Stock and Watson (2005), using the principal components estimator and setting the number of factors to seven as found by Stock and Watson.[2] We can see that a large fraction of the variables shows clear signs of autocorrelation in the idiosyncratic component.

Classical dynamic factor model and its extension with serially correlated idiosyncratic components can be written in state-space form. The EM algorithm is a well known approach to maximize the Gaussian log-likelihood function of models in state-space form (Shumway and Stoffer, 1982, and Watson and Engle, 1983). Moreover, the EM algorithm is convenient to deal with missing values in the panel data set. For an arbitrary pattern of missing values, Shumway and Stoffer (1982) provided the modifications required to the algorithm in the case of known loadings. Stock and Watson (2002a) suggest an EM algorithm to estimate several types of missing values in the case of a classical model with unknown loadings, fixed factors and white noise idiosyncratic components.

Banbura and Modugno (2010) try to circumvent the difficulties in the general case of unknown loadings and autoregressive factors and idiosyncratic components by adding the latter to the state-vector. Their solution consists of modelling the idiosyncratic component as a sum of a first order autoregressive process ($AR(1)$), which is included in the state vector, and an independent white noise process. By making the variance of the white noise arbitrarily small, they obtain an approximation to the likelihood estimators for the model with $AR(1)$ idiosyncratic components. However, for large cross-sections, as pointed out by Jungbacker *et al.* (2011), the dimension of the augmented state vector becomes very large, which leads to computational inefficiency. To

[2] The panel covers the period from January 1959 up to December 2003 and comprises 132 time series. The data can be downloaded at http://www.princeton.edu/~mwatson and are transformed as suggested by Stock and Watson (2005). Similar results for the autocorrelation coefficients are obtained if the model is estimated by maximum likelihood either with seven or, alternatively, with four dynamic factors, in the latter case also including their first lags in the measurement equation (in line with the results of Bai and Ng, 2007).

overcome the problem, Jungbacker *et al.* (2011) propose a computationally more efficient state-space representation with time-varying state dimensions (and autoregressive idiosyncratic components), augmenting only moderately the size of the state-vector in each period.

In this paper, while allowing for serially correlated idiosyncratic components, we focus on the special case of jagged edge panel data sets. As regards nowcasting, the existence of missing values at the end of the sample period is by large the more practically important feature of the data sets. Our focus on jagged edge data is similar to that of Giannone, Reichlin and Small (2008), but they do not take into account the idiosyncratic serial correlation, which reduces the realism of their model and leads to a poorer estimation and forecasting performance. Our algorithm deals efficiently with the presence of missing values at the end of the data set and is analytical and computationally simpler in this special case than the algorithm for the general case proposed by Jungbacker *et al.* (2011). Using our algorithm, and through Monte Carlo simulations, we assess the performance of the maximum likelihood estimator for different estimated model specifications and data generating processes. We evaluate the accuracy of the estimation of both the common factors at the end of the sample and the missing data. We find that, when the idiosyncratic components are autocorrelated in the data generating process, admitting $AR(1)$ idiosyncratic components (as compared to white noise ones) in the estimation model substantially improves the accuracy.

The paper is organized as follows. In section 2, we present the dynamic factor model with autoregressive factors and $AR(1)$ cross-sectionally independent idiosyncratic components. An EM algorithm for such model and for jagged edge panel data is proposed in section 3. In section 4, we present the Monte Carlo simulation design and discuss the results. Finally, section 5 concludes.

2 The dynamic factor model

Consider a vector of N stationary time series $\mathring{x}_t = \begin{bmatrix} \mathring{x}_{t,1} & \cdots & \mathring{x}_{t,n} & \cdots & \mathring{x}_{t,N} \end{bmatrix}'$ with data generating process given by the dynamic factor model, for $t = 1, \cdots, T$:

$$\mathring{x}_t = \mu + \Lambda(L) f_t + \mathring{v}_t \tag{1}$$

$$A(L)f_t = u_t \tag{2}$$

$$B(L)\mathring{v}_t = \mathring{e}_t \tag{3}$$

$$\begin{bmatrix} \mathring{e}_t \\ u_t \end{bmatrix} \sim i.i.d.N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}; \begin{bmatrix} \Psi & 0 \\ 0 & \Phi \end{bmatrix}\right) \tag{4}$$

$$\begin{bmatrix} f_1' & f_0' & \cdots & f_{2-R}' \end{bmatrix}' \sim N(\zeta; \Omega) \tag{5}$$

where $f_t = \begin{bmatrix} f_{t,1} & \cdots & f_{t,q} & \cdots & f_{t,Q} \end{bmatrix}'$ is a vector of Q latent common dynamic factors, $\mathring{v}_t = \begin{bmatrix} \mathring{v}_{t,1} & \cdots & \mathring{v}_{t,n} & \cdots & \mathring{v}_{t,N} \end{bmatrix}'$ is a vector of N latent idiosyncratic components, u_t and $\mathring{e}_t$ are Gaussian white noise innovations to the vector autoregressive (VAR) processes of f_t and $\mathring{v}_t$, respectively. The vector of (unknown) constants μ is N-dimensional and $\Lambda(L)$ is the polynomial matrix in the lag operator L

$$\Lambda(L) = \Lambda_0 + \Lambda_1 L + \cdots + \Lambda_S L^S \qquad (N \times Q)$$

Λ_s $(N \times Q)$ being the matrix of (also unknown) factor loadings associated with f_{t-s} $(s = 0, 1, \cdots, S)$. Similarly,

$$A(L) = I - A_1 L - \cdots - A_P L^P \qquad (Q \times Q)$$

and

$$B(L) = I - BL \qquad (N \times N)$$

where A_p $(p = 1, \cdots, P)$ and B are the (unknown) matrices of coefficients in the VAR processes of f_t and $\mathring{v}_t$, respectively.[3] Equation (5) states the initial conditions for the dynamic factors, with $R = \max(P; S+2)$. Vector ζ $(RQ \times 1)$ and the symmetric matrix Ω $(RQ \times RQ)$ are also unknown parameters.

We assume that B and Ψ are diagonal, thereby reducing the number of parameters

[3]Only the case of first order autoregressive idiosyncratic components is pursued in the paper, but the extension to allow for autoregressive processes of order larger than one is straightforward (although more cumbersome in terms of notation). The main difference would be that, for each observable variable, the maximization of the concentrated expected log likelihood in subsection 3.1 would not be univariate anymore, and we would need to resort to some quasi-Newton scheme. We are convinced that, in practice, this extension is not very relevant. The results of the simulations reported in Section 4 show that the specification with $AR(1)$ idiosyncratic components continue to perform well when these components are generated according to $AR(2)$ or $MA(1)$ processes instead of $AR(1)$.

to a manageable size and avoiding to blur the separate identification of the common and idiosyncratic components. The resulting specification still encompasses most of the specifications found in the recent literature on dynamic factor models for large cross-sections. Reis and Watson (2010) specify a model equivalent to (1)-(5) in order to breakdown consumption goods' inflation into three components. Jungbacker and Koopman (2008) suggest a likelihood-based analysis of a general dynamic factor model which allows for dynamic factors generated according to a vector autoregressive moving average ($VARMA$) process and for idiosyncratic components generated by a VAR of order possibly larger than one.[4] However, in their empirical illustration, they simplify the specification to the formulation above using $S = 0$. The "approximating factor model" considered by Doz *et al.* (2007,2012), as well as the model considered by Giannone *et al.* (2008), are also particular cases of our model with $B = 0$.[5] Finally, the case $S = P = 0$ and $B = 0$ was considered by Stock and Watson (2002a, Appendix A) to motivate an EM algorithm for dealing with several types of data irregularities.

Model (1)-(5) can be written in a state-space form

$$\mathring{x}_t = \eta_t + \Pi \mathbf{f}_t^{(R)} + \mathring{e}_t \tag{6}$$

$$\mathbf{f}_t^{(R)} = \Theta \mathbf{f}_{t-1}^{(R)} + G u_t \tag{7}$$

$$\mathbf{f}_1^{(R)} \sim N(\zeta; \Omega) \tag{8}$$

where $\mathbf{f}_t^{(R)} = \begin{bmatrix} f_t' & f_{t-1}' & \cdots & f_{t-R+1}' \end{bmatrix}'$ is the $(RQ \times 1)$ vector of state variables, with R defined as above, $\eta_t = (I - B)\mu + B\mathring{x}_{t-1}$ is a $(N \times 1)$ vector of predetermined variables in the measurement equation,

$$\underset{(N \times RQ)}{\Pi} = \begin{cases} \Upsilon & \text{if } P \leq S+2 \\ \begin{bmatrix} \Upsilon & 0 & \cdots & 0 \end{bmatrix} & \text{otherwise} \end{cases}$$

$$\underset{(N \times (S+2)Q)}{\Upsilon} = \begin{bmatrix} \Lambda_0 & (\Lambda_1 - B\Lambda_0) & \cdots & (\Lambda_S - B\Lambda_{S-1}) & -B\Lambda_S \end{bmatrix}$$

[4] In addition, they admit exogenous explanatory variables in equation (1).

[5] Doz *et al.* (2012) mention in a footnote that the restriction of serially uncorrelated idiosyncratic components is only made for expositional simplicity. Doz *et al.* (2007, 2012) also admit that $S = 0$, while the factor model in Giannone *et al.* (2008) is equivalent to a formulation with $S \geq 0$.

$$\underset{(RQ\times RQ)}{\Theta} = \begin{cases} \begin{bmatrix} A_1 & A_2 & \cdots & A_{P-1} & A_P \\ I_Q & 0 & \cdots & 0 & 0 \\ 0 & I_Q & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & I_Q & 0 \end{bmatrix} & \text{if } P \geq S+2 \\ \\ \begin{bmatrix} A_1 & A_2 & \cdots & A_{P-1} & A_P & 0 & \cdots & 0 & 0 \\ I_Q & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & I_Q & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & & & \ddots & & & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & I_Q & 0 \end{bmatrix} & \text{otherwise} \end{cases}$$

$$\underset{(RQ\times Q)}{G} = \begin{bmatrix} I_Q \\ 0 \end{bmatrix}$$

where I_Q stands for the identity matrix of order Q.

In order to allow for a jagged edge feature of the data, we admit that the n-th variable of the panel is observed for any t from 1 to $\mathcal{T}_n$ $(1 < \mathcal{T}_n \leq T)$. Let the $\mathcal{N}_t$-dimensional column vector x_t (with $\mathcal{N}_t \leq N$) denote the sub-vector of $\mathring{x}_t$ comprising the variables with non-missing realizations. One may write $x_t = C_t\mathring{x}_t$, with C_t the $(\mathcal{N}_t \times N)$ selection matrix of zeros and ones such that its (m, n) element is 1 if both the realization of $\mathring{x}_{t,n}$ is not missing and if $x_{t,m} = \mathring{x}_{t,n}$.[6] Note that if $\mathcal{T}_n = T$ for all n (or, equivalently, if $\mathcal{N}_t = N$ for all t), the panel data set is balanced. Also note that the only missing values that we are admitting are associated with the latest time periods of the sample: if $\mathring{x}_{t_0,n}$ is missing for n and t_0 than $\mathring{x}_{t,n}$ is also missing for all $t > t_0$.

3 An EM algorithm for the case of panel data sets with jagged edge

The EM algorithm for maximizing the log-likelihood consists of iterating an "expectation-step" (or "E-step") and a "maximization-step" (or "M-step") until convergence, i.e. un-

[6]If all variables are observed for period t, C_t is the identity matrix of order N.

til the improvement in the value of the log-likelihood function is smaller than some tolerance level. Given a set of values for the model parameters, the E-step corresponds to computing the first and second order moments of the dynamic factors conditional on the realizations of $\{x_1, \cdots, x_t, \cdots, x_T\}$. The Kalman smoother is used to perform this computation. Having obtained the estimated factor moments, the "M-step" corresponds to maximizing the expected value of the joint likelihood of "observables and factors" $\{(\mathring{x}_t, f_t)\}_{t=1,\cdots,T}$ with respect to the model parameters and conditional on $\{x_t\}_{t=1,\cdots,T}$.

In this section, we describe an EM algorithm to estimate our model in the case of a panel data set with jagged edge.[7] Owing to the diagonality of B and Ψ, in order to determine the solution for the diagonal elements of B, the suggested M-step only requires N univariate estimations in the range $]-1;1[$. Given B, the solutions for the remaining parameters are computed resorting to analytical expressions. As usual, the EM algorithm may be initialized with parameter estimates based on the principal component estimator and linear regression methods.

The presence of missing values in the panel data set creates difficulties to the implementation of the EM algorithm. In particular, the expected value of the joint likelihood of observables and factors, conditional on a realization of the observables becomes more complex if there are missing values in the sample. The procedure suggested by Jungbacker *et al.* (2011) consists of developing a state space model with time-varying state dimensions. However, that comes at an analytical and a computational cost.

In their paper, Jungbacker *et al.* (2011) report an assessment of the computational cost incurred by the presence of randomly chosen missing entries, for different dimensions of the panel and different "intensities" of missing observations. For the case of $N = 100$ and 1% and 10% of missing observations, the computation time increases about 20% and 300% relative to the case of a balanced panel, respectively. The algorithm that we suggest in the following subsections, besides being more simple analitically, also deals more efficiently with the missing data. Indeed, for $N = T = 100$ and 1000 panels with 1% and 10% of missing values generated according to the procedure described in subsection 4.1, the average computation time increased only by around 10% and 20% relative to the case of a balanced panel, respectively.

[7] Obviously, the suggested procedure is also valid for the particular case of a balanced panel data set.

3.1 The M-step

Let $g(\cdot)$ denote the joint probability density function of the complete data (observables and factors) and define

$$\ell(\mu, \Lambda, B, \Psi, A, \Phi, \zeta, \Omega | x_1, \cdots, x_T) =$$

$$= E\left[\ln g\left(\mathring{x}_1, \cdots, \mathring{x}_T, \mathbf{f}_1^{(R)}, f_2, \cdots, f_T; \mu, \Lambda, B, \Psi, A, \Phi, \zeta, \Omega\right) | x_1, \cdots, x_T\right]$$

where $\Lambda = \begin{bmatrix} \Lambda_0 & \Lambda_1 & \cdots & \Lambda_S \end{bmatrix}$ $(N \times Q(S+1))$ and $A = \begin{bmatrix} A_1 & \cdots & A_P \end{bmatrix}$ $(Q \times PQ)$. After somewhat lengthy but straightforward calculations (see Appendix 1), we get:

$$\ell(\mu, \Lambda, B, \Psi, A, \Phi, \zeta, \Omega | x_1, \cdots, x_T) \mathrel{\hat{=}} -\frac{1}{2}\ln\left[\det(\Omega)\right] +$$

$$-\frac{1}{2} tr\left\{\left[\mathbf{P}_{1,1|T}^{(R)} + \left(\mathbf{f}_{1|T}^{(R)} - \zeta\right)\left(\mathbf{f}_{1|T}^{(R)} - \zeta\right)'\right]\Omega^{-1}\right\} - \frac{T-1}{2}\ln\left[\det(\Phi)\right] +$$

$$-\frac{1}{2} tr\left\{\left[\sum_{t=2}^{T} M_{t,t|T} + A\left(\sum_{t=1}^{T-1}\mathbf{M}_{t,t|T}^{(P)}\right)A' - 2A\sum_{t=2}^{T}\mathbf{H}_{t,t-1|T}^{(P)\prime}\right]\Phi^{-1}\right\} + \frac{1}{2}\sum_{n=1}^{N}\left\{\ln\left(1 - \beta_n^2\right) +\right.$$

$$-T\ln(\psi_n) - \frac{\mathcal{T}_n}{\psi_n}\left[\bar{y}_n(\beta_n, \mathcal{T}_n) - 2\bar{x}_n(\beta_n, \mathcal{T}_n)\mu_n + \bar{T}(\beta_n, \mathcal{T}_n)\mu_n^2 +\right.$$

$$\left.\left. - 2\bar{z}_n(\beta_n, \mathcal{T}_n)'\lambda_n + 2\bar{\mathbf{f}}(\beta_n, \mathcal{T}_n)'\lambda_n\mu_n + \lambda_n'\bar{\mathbf{M}}(\beta_n, \mathcal{T}_n)\lambda_n\right]\right\} \tag{9}$$

where '$\hat{=}$' stands for 'identity up to a term that does not depend on the parameters', λ_n is the transposed n-th row of Λ, the n-th diagonal elements of B and Ψ are denoted by β_n and ψ_n, respectively, the conditional first and second order moments of the factors are represented by (with i, j and τ non-negative integers and W a positive integer):

$$\underset{(Q\times 1)}{f_{t|\tau}} = E(f_t | x_1, \cdots, x_\tau)$$

$$\underset{(WQ\times 1)}{\mathbf{f}_{t|\tau}^{(W)}} = \begin{bmatrix} f_{t|\tau}' & \cdots & f_{t-W+1|\tau}' \end{bmatrix}'$$

$$\underset{(Q\times Q)}{P_{t-i,t-j|\tau}} = E\left[(f_{t-i} - f_{t-i|\tau})(f_{t-j} - f_{t-j|\tau})' | x_1, \cdots, x_\tau\right]$$

$$\underset{(Q\times Q)}{M_{t-i,t-j|\tau}} = E\left(f_{t-i}f_{t-j}' | x_1, \cdots, x_\tau\right) = P_{t-i,t-j|\tau} + f_{t-i|\tau}f_{t-j|\tau}'$$

$$\underset{(WQ\times WQ)}{\mathbf{P}^{(W)}_{t-i,t-j|\tau}} = E\left[\left(\mathbf{f}^{(W)}_{t-i} - \mathbf{f}^{(W)}_{t-i|\tau}\right)\left(\mathbf{f}^{(W)}_{t-j} - \mathbf{f}^{(W)}_{t-j|\tau}\right)' |x_1, \cdots, x_\tau\right] =$$

$$= \begin{bmatrix} P_{t-i,t-j|\tau} & \cdots & P_{t-i,t-j-W+1|\tau} \\ \vdots & & \vdots \\ P_{t-i-W+1,t-j|\tau} & \cdots & P_{t-i-W+1,t-j-W+1|\tau} \end{bmatrix}$$

$$\underset{(WQ\times WQ)}{\mathbf{M}^{(W)}_{t-i,t-j|\tau}} = E\left(\mathbf{f}^{(W)}_{t-i}\mathbf{f}^{(W)\prime}_{t-j}|x_1, \cdots, x_\tau\right) = \mathbf{P}^{(W)}_{t-i,t-j|\tau} + \mathbf{f}^{(W)}_{t-i|\tau}\mathbf{f}^{(W)\prime}_{t-j|\tau} =$$

$$= \begin{bmatrix} M_{t-i,t-j|\tau} & \cdots & M_{t-i,t-j-W+1|\tau} \\ \vdots & & \vdots \\ M_{t-i-W+1,t-j|\tau} & \cdots & M_{t-i-W+1,t-j-W+1|\tau} \end{bmatrix}$$

$$\underset{(Q\times WQ)}{\mathbf{H}^{(W)}_{t-i,t-j|\tau}} = \begin{bmatrix} M_{t-i,t-j|\tau} & \cdots & M_{t-i,t-j-W+1|\tau} \end{bmatrix}$$

and, furthermore, we used the following aditional notation in order to be able to write (6) more compactly:

$$\bar{\mathbf{M}}\left(\beta_n, \mathcal{T}_n\right) = \frac{1}{\mathcal{T}_n}\left[\sum_{t=1}^{\mathcal{T}_n}\mathbf{M}^{(S+1)}_{t,t|T} + \beta_n^2\sum_{t=2}^{\mathcal{T}_n-1}\mathbf{M}^{(S+1)}_{t,t|T} - \beta_n\sum_{t=2}^{\mathcal{T}_n}\left(\mathbf{M}^{(S+1)}_{t,t-1|T} + \mathbf{M}^{(S+1)\prime}_{t,t-1|T}\right)\right]$$

$$\bar{\mathbf{f}}\left(\beta_n, \mathcal{T}_n\right) = \frac{1}{\mathcal{T}_n}\left[\sum_{t=1}^{\mathcal{T}_n}\mathbf{f}^{(S+1)}_{t|T} + \beta_n^2\sum_{t=2}^{\mathcal{T}_n-1}\mathbf{f}^{(S+1)}_{t|T} - \beta_n\sum_{t=2}^{\mathcal{T}_n}\left(\mathbf{f}^{(S+1)}_{t|T} + \mathbf{f}^{(S+1)}_{t-1|T}\right)\right]$$

$$\bar{T}\left(\beta_n, \mathcal{T}_n\right) = \frac{1}{\mathcal{T}_n}\left[\mathcal{T}_n - 2(\mathcal{T}_n - 1)\beta_n + (\mathcal{T}_n - 2)\beta_n^2\right]$$

$$\bar{y}_n\left(\beta_n, \mathcal{T}_n\right) = \frac{1}{\mathcal{T}_n}\left[\sum_{t=1}^{\mathcal{T}_n}x_{t,n}^2 + \beta_n^2\sum_{t=2}^{\mathcal{T}_n-1}x_{t,n}^2 - 2\beta_n\sum_{t=2}^{\mathcal{T}_n}x_{t,n}x_{t-1,n}\right]$$

$$\bar{z}_n\left(\beta_n, \mathcal{T}_n\right) = \frac{1}{\mathcal{T}_n}\left[\sum_{t=1}^{\mathcal{T}_n}x_{t,n}\mathbf{f}^{(S+1)}_{t|T} + \beta_n^2\sum_{t=2}^{\mathcal{T}_n-1}x_{t,n}\mathbf{f}^{(S+1)}_{t|T} - \beta_n\sum_{t=2}^{\mathcal{T}_n}\left(x_{t,n}\mathbf{f}^{(S+1)}_{t-1|T} + x_{t-1,n}\mathbf{f}^{(S+1)}_{t|T}\right)\right]$$

$$\bar{x}_n\left(\beta_n, \mathcal{T}_n\right) = \frac{1}{\mathcal{T}_n}\left[\sum_{t=1}^{\mathcal{T}_n}x_{t,n} + \beta_n^2\sum_{t=2}^{\mathcal{T}_n-1}x_{t,n} - \beta_n\sum_{t=2}^{\mathcal{T}_n}\left(x_{t,n} + x_{t-1,n}\right)\right]$$

From the first order conditions of the problem of maximization of the expected log likelihood with respect to the model parameters, we can derive analytical expressions for the solutions of ζ, Ω, A and Φ as functions of sufficient statistics based on the first and second order moments of the common factors (see Appendix 2):

$$\hat{\zeta} = \mathbf{f}_{1|T}^{(R)} \tag{10}$$

$$\hat{\Omega} = \mathbf{P}_{1,1|T}^{(R)} \tag{11}$$

$$\hat{A} = \left(\sum_{t=2}^{T} \mathbf{H}_{t,t-1|T}^{(P)}\right)\left(\sum_{t=1}^{T-1} \mathbf{M}_{t,t|T}^{(P)}\right)^{-1} \tag{12}$$

$$\hat{\Phi} = \frac{1}{T-1}\left[\sum_{t=2}^{T} M_{t,t|T} - \left(\sum_{t=2}^{T} \mathbf{H}_{t,t-1|T}^{(P)}\right)\left(\sum_{t=1}^{T-1} \mathbf{M}_{t,t|T}^{(P)}\right)^{-1}\left(\sum_{t=2}^{T} \mathbf{H}_{t,t-1|T}^{(P)}\right)'\right] \tag{13}$$

Additionally, from the first order conditions with respect to μ_n, λ_n and ψ_n ($n = 1, \cdots, N$), we get (see also Appendix 2):

$$\begin{aligned}\hat{\mu}_n(\beta_n) = \frac{1}{\bar{T}(\beta_n, \mathcal{T}_n)}\Big\{ & \bar{x}_n(\beta_n, \mathcal{T}_n) - \bar{\mathbf{f}}(\beta_n, \mathcal{T}_n)' . \\ & . \left[\bar{\mathbf{M}}(\beta_n, \mathcal{T}_n) - \frac{1}{\bar{T}(\beta_n, \mathcal{T}_n)}\bar{\mathbf{f}}(\beta_n, \mathcal{T}_n)\bar{\mathbf{f}}(\beta_n, \mathcal{T}_n)'\right]^{-1} . \\ & . \left[\bar{z}_n(\beta_n, \mathcal{T}_n) - \frac{1}{\bar{T}(\beta_n, \mathcal{T}_n)}\bar{x}_n(\beta_n, \mathcal{T}_n)\bar{\mathbf{f}}(\beta_n, \mathcal{T}_n)\right]\Big\}\end{aligned} \tag{14}$$

$$\begin{aligned}\hat{\lambda}_n(\beta_n) = & \left[\bar{\mathbf{M}}(\beta_n, \mathcal{T}_n) - \frac{1}{\bar{T}(\beta_n, \mathcal{T}_n)}\bar{\mathbf{f}}(\beta_n, \mathcal{T}_n)\bar{\mathbf{f}}(\beta_n, \mathcal{T}_n)'\right]^{-1} . \\ & . \left[\bar{z}_n(\beta_n, \mathcal{T}_n) - \frac{1}{\bar{T}(\beta_n, \mathcal{T}_n)}\bar{x}_n(\beta_n, \mathcal{T}_n)\bar{\mathbf{f}}(\beta_n, \mathcal{T}_n)\right]\end{aligned} \tag{15}$$

$$\begin{aligned}\hat{\psi}_n(\beta_n) = \frac{\mathcal{T}_n}{T}\Big\{ & \left[\bar{y}_n(\beta_n, \mathcal{T}_n) - \frac{1}{\bar{T}(\beta_n, \mathcal{T}_n)}\left(\bar{x}_n(\beta_n, \mathcal{T}_n)\right)^2\right] + \\ & - \left[\bar{z}_n(\beta_n, \mathcal{T}_n) - \frac{1}{\bar{T}(\beta_n, \mathcal{T}_n)}\bar{x}_n(\beta_n, \mathcal{T}_n)\bar{\mathbf{f}}(\beta_n, \mathcal{T}_n)\right]' . \\ & . \left[\bar{\mathbf{M}}(\beta_n, \mathcal{T}_n) - \frac{1}{\bar{T}(\beta_n, \mathcal{T}_n)}\bar{\mathbf{f}}(\beta_n, \mathcal{T}_n)\bar{\mathbf{f}}(\beta_n, \mathcal{T}_n)'\right]^{-1} .\end{aligned}$$

$$\cdot \left[\bar{z}_n (\beta_n, \mathcal{T}_n) - \frac{1}{\bar{T} (\beta_n, \mathcal{T}_n)} \bar{x}_n (\beta_n, \mathcal{T}_n) \bar{\mathbf{f}} (\beta_n, \mathcal{T}_n) \right] \Bigg\} \tag{16}$$

Concentrating $\ell(\cdot)$, we obtain:

$$\ell^{(c)} (B | x_1, \cdots, x_T) \triangleq \frac{1}{2} \sum_{n=1}^{N} \ell_n^{(c)} (\beta_n | x_{1,n}, \cdots, x_{\mathcal{T}_n,n})$$

with

$$\ell_n^{(c)} (\beta_n | x_{1,n}, \cdots, x_{\mathcal{T}_n,n}) = -\ln \frac{\hat{\psi}_n (\beta_n)}{\left(1 - \beta_n^2\right)^{1/T}}$$

For each n, the solution $\hat{\beta}_n$ which maximizes $\ell_n^{(c)} (\beta_n | x_{1,n}, \cdots, x_{\mathcal{T}_n,n})$ can be found by grid search in the range $]-1;1[$. Having obtained $\hat{\beta}_n$ $(n = 1, \cdots, N)$, the corresponding solutions for $\hat{\mu}_n$, $\hat{\lambda}_n$ and $\hat{\psi}_n$ follow from (14)-(16). Note that the computation time of the estimates does not depend significantly on the number of missing values. Indeed, the single difference relative to the case of a balanced panel is that $\mathcal{T}_n$ replaces T in some of the expressions.

3.2 The E-step

The sufficient statistics based on the first and second order moments of the dynamic factors can be computed applying the Kalman smoother to the state-space representation of the model, for given estimates of the model parameters. Expressions for the Kalman filter and smoother for a model such as (6)-(8) and balanced panel data sets can be found e.g. in Harvey (1989), Durbin and Koopman (2001) and Shumway and Stoffer (1982, 2006). In the case of missing values, if the idiosyncratic components of observed and unobserved variables are uncorrelated (as in our model), as noted by Shumway and Stoffer (1982, 2006), the filtered and smoothed state vectors can be calculated from the usual equations by plugging zeros in the observation vector where data is missing and by zeroing out the corresponding rows of the design matrix.

Using our notation and the selection matrices C_t $(t = 1, \cdots, T)$, the Kalman filter and smoother recursions for state-space representation (6)-(8) with missing data can be written as follows:

(i) Filter forward recursions (for $t = 2, \cdots, T$)[8]

$$\mathbf{f}^{(R)}_{t|t-1} = \hat{\Theta}\mathbf{f}^{(R)}_{t-1|t-1}$$

$$\mathbf{P}^{(R)}_{t,t|t-1} = \hat{\Theta}\mathbf{P}^{(R)}_{t-1,t-1|t-1}\hat{\Theta}' + G\hat{\Phi}G'$$

$$\mathbf{P}^{(R)}_{t,t-1|t-1} = \hat{\Theta}\mathbf{P}^{(R)}_{t-1,t-1|t-1}$$

$$\mathbf{K}_t = \mathbf{P}^{(R)}_{t,t|t-1}\hat{\Pi}'C_t'\left\{\left(C_t\hat{\Psi}C_t'\right)^{-1} + \right.$$

$$\left. - \left(C_t\hat{\Psi}C_t'\right)^{-1} C_t\hat{\Pi}\left[I + \mathbf{P}^{(R)}_{t,t|t-1}\hat{\Pi}'C_t'\left(C_t\hat{\Psi}C_t'\right)^{-1} C_t\hat{\Pi}\right]^{-1} \mathbf{P}^{(R)}_{t,t|t-1}\hat{\Pi}'C_t'\left(C_t\hat{\Psi}C_t'\right)^{-1}\right\}$$

$$\mathbf{f}^{(R)}_{t|t} = \mathbf{f}^{(R)}_{t|t-1} + \mathbf{K}_t\left(x_t - C_t(I_N - \hat{B})\hat{\mu} - C_t\hat{B}C_{t-1}'x_{t-1} - C_t\hat{\Pi}\mathbf{f}^{(R)}_{t|t-1}\right)$$

$$\mathbf{P}^{(R)}_{t,t|t} = \mathbf{P}^{(R)}_{t,t|t-1} - \mathbf{K}_tC_t\hat{\Pi}\mathbf{P}^{(R)}_{t,t|t-1}$$

(ii) Smoother backward recursions (for $t = T-1, T-2, \cdots, 1$)

$$\mathbf{J}_t = \mathbf{P}^{(R)}_{t,t|t}\hat{\Theta}'\left(\mathbf{P}^{(R)}_{t+1,t+1|t}\right)^{-1}$$

$$\mathbf{f}^{(R)}_{t|T} = \mathbf{f}^{(R)}_{t|t} + \mathbf{J}_t\left(\mathbf{f}^{(R)}_{t+1|T} - \mathbf{f}^{(R)}_{t+1|t}\right)$$

$$\mathbf{P}^{(R)}_{t,t|T} = \mathbf{P}^{(R)}_{t,t|t} + \mathbf{J}_t\left(\mathbf{P}^{(R)}_{t+1,t+1|T} - \mathbf{P}^{(R)}_{t+1,t+1|t}\right)\mathbf{J}_t'$$

$$\mathbf{P}^{(R)}_{t,t-1|T} = \mathbf{P}^{(R)}_{t,t|t}\mathbf{J}_{t-1}' + \mathbf{J}_t\left(\mathbf{P}^{(R)}_{t+1,t|T} - \mathbf{P}^{(R)}_{t+1,t|t}\right)\mathbf{J}_{t-1}'$$

with

$$\mathbf{P}^{(R)}_{T,T-1|T} = \left(I_{RQ} - \mathbf{K}_TC_T\hat{\Pi}\right)\mathbf{P}^{(R)}_{T,T-1|T-1}$$

We adopt the normalization restriction

$$\frac{1}{T}\sum_{t=1}^{T} f_{t|T}f_{t|T}' = I \tag{17}$$

Even with these $Q(Q+1)/2$ identifying restrictions, any rotation of the dynamic factors (with the offsetting transformation of the associated parameters) will generate an

[8]Note that $C_t\hat{\Psi}C_t'$ is diagonal.

observationally equivalent model. However, it should be mentioned that, conditional on the moments of the common factors, if $\sum_{t=1}^{T-1}\mathbf{M}^{(P)}_{t,t|T}$ is non-singular and $\mathcal{T}_n > 2$ for all n the solution of the M-step is unique.

After running the smoother backward recursions of the E-step, and before moving to a new iteration of the M-step, the factor moments are normalized so as to comply with condition (17). Let $\mathbf{L}$ be the lower triangular $(Q \times Q)$ matrix resulting from the Cholesky decomposition

$$\mathbf{L}\left(\frac{1}{T}\sum_{t=1}^{T} f_{t|T} f'_{t|T}\right)\mathbf{L}' = I_R$$

where $f_{t|T}$ $(t = 1, \cdots, T)$ are calculated from the recursions, before normalization. Also let

$$\mathbb{L} = (I_R \otimes \mathbf{L})$$

where $\otimes$ denotes the Kronecker product. For any t, the normalization corresponds to:

(i) premultiplying by $\mathbb{L}$ all the first order moments $\mathbf{f}^{(R)}_{t|t-1}$, $\mathbf{f}^{(R)}_{t|t-1}$ and $\mathbf{f}^{(R)}_{t|T}$;

(ii) premultiplying by $\mathbb{L}$ and post-multiplying by $\mathbb{L}'$ all the second order moments $\mathbf{P}^{(R)}_{t,t|t-1}$, $\mathbf{P}^{(R)}_{t,t-1|t-1}$, $\mathbf{P}^{(R)}_{t,t|t}$, $\mathbf{P}^{(R)}_{t,t|T}$ and $\mathbf{P}^{(R)}_{t,t-1|T}$.

3.3 Log-likelihood evaluation

For a set of realizations $\{x_1, \cdots, x_t, \cdots, x_T\}$ and for a given set of estimates of the model parameters $\hat{\Xi} = \left(\hat{\mu}, \hat{\Lambda}, \hat{B}, \hat{\Psi}, \hat{A}, \hat{\Phi}, \hat{\zeta}, \hat{\Omega}\right)$, we may use the prediction error decomposition to evaluate the log-likelihood function:[9]

$$\mathcal{L}\left(x_1, \cdots, x_t, \cdots, x_T; \hat{\Xi}\right) = \mathcal{L}\left(x_1; \hat{\Xi}\right) + \sum_{t=2}^{T}\mathcal{L}\left(x_t | x_1, \cdots, x_{t-1}; \hat{\Xi}\right) =$$

$$= -\frac{1}{2}\left\{N\ln(2\pi) + \ln\left[\det\left(\hat{V}\left(x_1\right)\right)\right] + \left[x_1 - \hat{E}\left(x_1\right)\right]'\left[\hat{V}\left(x_1\right)\right]^{-1}\left[x_1 - \hat{E}\left(x_1\right)\right]\right\} +$$

$$-\frac{1}{2}\sum_{t=2}^{T}\left\{\mathcal{N}_t\ln(2\pi) + \ln\left[\det\left(\hat{V}\left(\mathring{x}_t | x_1, \cdots, x_{t-1}\right)\right)\right] +\right.$$

[9]Note that, by construction, $\mathcal{N}_1 = N$ and $C_1 = I$.

$$+\left[x_t - \hat{E}\left(x_t|x_1,\cdots,x_{t-1}\right)\right]'\left[\hat{V}\left(x_t|x_1,\cdots,x_{t-1}\right)\right]^{-1}\left[x_t - \hat{E}\left(x_t|x_1,\cdots,x_{t-1}\right)\right]\Bigg\}$$

where $\hat{E}(\cdot)$ and $\hat{V}(\cdot)$ denote, respectively, the estimated expected values and variances:

$$\hat{E}\left(x_1\right) = \hat{\mu} + \hat{\Lambda}\hat{\zeta}$$

$$\hat{V}\left(x_1\right) = \left(I_N - \hat{B}^2\right)^{-1}\hat{\Psi} + \hat{\Lambda}\hat{\Omega}\hat{\Lambda}'$$

and, for $t = 2,\cdots,T$:

$$\hat{E}\left(x_t|x_1,\cdots,x_{t-1}\right) = C_t\left[\left(I_N - \hat{B}\right)\hat{\mu} + \hat{B}C'_{t-1}x_{t-1} + \hat{\Lambda}\mathbf{f}^{(S+1)}_{t|t-1} - \hat{B}\hat{\Lambda}\mathbf{f}^{(S+1)}_{t-1|t-1}\right]$$

$$\hat{V}\left(x_t|x_1,\cdots,x_{t-1}\right) =$$

$$= C_t\left[\hat{\Psi} + \hat{\Lambda}\mathbf{P}^{(S+1)}_{t,t|t-1}\hat{\Lambda}' + \hat{B}\hat{\Lambda}\mathbf{P}^{(S+1)}_{t-1,t-1|t-1}\hat{\Lambda}'\hat{B} - \hat{\Lambda}\mathbf{P}^{(S+1)}_{t,t-1|t-1}\hat{\Lambda}'\hat{B} - \hat{B}\hat{\Lambda}\mathbf{P}^{(S+1)\prime}_{t,t-1|t-1}\hat{\Lambda}'\right]C'_t$$

4 A Monte Carlo analysis

In this section, a Monte Carlo study is conducted to evaluate the performance of alternative model specifications in the presence of panel data with jagged edge. First, we define the data generating process which will be our base case and discuss the results. Then, we perform a sensitivity analysis to assess the robustness of the findings to different simulation settings.

4.1 The base case

Take the model (1) to (5). We consider a data generating process similar to the one of Stock and Watson (2002b) and Doz *et al.* (2012) and admit the following assumptions:[10]

$\Lambda_{s,nq} \sim i.i.d.\ N(0,1) \qquad (s = 0,\cdots,S; n = 1,\cdots,N; q = 1,\cdots,Q)$

Λ_s independent of Λ_r for any $s \neq r$

[10] For simplicity, we set μ to zero although the model is estimated allowing for $\widehat{\mu}$ different from zero.

$A(L) = I - A_1 L$

$$A_{1,qm} = \begin{cases} \sim i.i.d.\ U([0.3, 0.7]) & \text{if } q = m \\ 0 & \text{otherwise} \end{cases}$$

$B = diag_n(\beta_n)$ with $\beta_n \sim i.i.d.\ U([-0.9, 0.9])$

$(1 - \beta_n L)\, v_{t,n} = \sqrt{\alpha_n \left(1 - \beta_n^2\right)} \varepsilon_{t,n}$ $(n = 1, \cdots, N; t = 1, \cdots, T)$ with $\varepsilon_t \sim i.i.d.\ N(0, H(\delta))$

$$\alpha_n = \frac{1-\gamma_n}{\gamma_n} \left[\frac{1}{T} \sum_{t=1}^{T} \left(\sum_{s=0}^{S} \sum_{q=1}^{Q} \Lambda_{s,nq} f_{t,q} \right)^2 \right]^{-1} \text{ with } \gamma_n \sim i.i.d.\ U([0.1, 0.9]) \quad (n = 1, \cdots, N)$$

$H(\delta) = [h_{n,m}(\delta)]_{n,m}$ with $h_{nm}(\delta) = \delta^{|n-m|}$ $(n, m = 1, \cdots, N)$

In the base case, as regards the dynamics of the factors when generating the data, we set $P = 1$ and allow the autoregressive coefficient of the common factors to be drawn from a uniform distribution on $[0.3, 0.7]$. Concerning the number of dynamic factors, we consider four common factors, $Q = 4$. The number of static factors is set to be equal to the number of dynamic factors, i.e. $S = 0$. The autoregressive coefficients of the idiosyncratic components are drawn from a uniform distribution on $[-0.9, 0.9]$. Another parameter of interest is γ_n, which can be interpreted as the ratio between the variance of the common component and the total variance of variable n. The variance of $v_{t,n}$, denoted α_n, depends on γ_n. We allow γ_n to be drawn from a uniform distribution on $[0.1, 0.9]$. It is worth mentioning that from the results of Stock and Watson (2005), with the model estimated by principal components and the number of common factors set to seven, we roughly get a uniform pattern for the empirical distribution of the ratios between the estimated variances of the common components and the total variances. The parameter δ controls for the amount of contemporaneous cross-correlation between the idiosyncratic components. When $\delta = 0$, $H(\delta)$ reduces to the identity matrix, which corresponds to the base case.

Regarding the size of the panel data, we consider one hundred series and twenty years of monthly data, i.e. $N = 100$ and $T = 240$, which can be seen as the size of a typical large data set. Monthly indicators are usually available at most with a lag of two months (see, for example, Giannone *et al.,* 2008, for the US), so in the base case we assume, as in a stylized calendar, that half of the series have no release lag, one fourth of the series have a lag of one month and the remaining fourth have a lag of two months.

Concerning the estimation model, we consider three alternative specifications. The first is $(B = 0, P = 0)$, which assumes white noise factors and idiosyncratic components (as in Stock and Watson, 2002a). The second specification, $(B = 0, P = 1)$, takes into account only the dynamics of the common factors. Finally, the third specification is $(B \neq 0, P = 1)$, which takes into account the dynamics of both the common factors and the idiosyncratic components.

Several measures are computed for the comparison of the different estimation models and the results are based on $1,000$ sample draws. To evaluate the accuracy in the estimation of the missing values and the factors at the end of the sample in the presence of unbalanced data we resort to the Mean Squared Error (MSE) for the last observation of the sample (observation T) and for the second last observation (observation $T-1$). For ease of comparison, we present the relative MSE (RMSE) for each specification vis-à-vis the specification $(B = 0, P = 0)$. Following Stock and Watson (2002b) and Doz *et al.* (2012), we also compute the trace R^2

$$R^2_{F,\widehat{F}} = \frac{\widehat{E}\left[tr\left(F'\widehat{F}\left(\widehat{F}'\widehat{F}\right)^{-1}\widehat{F}'F\right)\right]}{\widehat{E}\left[tr\left(F'F\right)\right]}$$

where

$$\underset{(T\times Q)}{F} = \begin{bmatrix} f_1' \\ \vdots \\ f_t' \\ \vdots \\ f_T' \end{bmatrix} \qquad \underset{(T\times Q)}{\widehat{F}} = \begin{bmatrix} f_{1|T}' \\ \vdots \\ f_{t|T}' \\ \vdots \\ f_{T|T}' \end{bmatrix}$$

where $\widehat{E}[.]$ denotes the expectation estimated by averaging the relevant statistic over the $1,000$ draws. This statistic is a measure of fit of the multivariate regression of the true factors on the estimated factors, and is commonly used because the common factors are identified only up to a rotation. A value close to one denotes a good approximation of the space spanned by the true common factors.

The simulation results for the base case are presented in Table 1. One can see that the specification $(B = 0, P = 1)$ leads to quite similar results to those obtained with $(B = 0, P = 0)$. However, for the specification $(B \neq 0, P = 1)$, besides the slight increase in $R^2_{F,\widehat{F}}$, there is a substantial improvement in the accuracy of the estimation of both the factors and the missing values. In particular, the gain in the estimation of the factors for observation T is around 26 percentage points (pp) and for observation $T-1$ the improvement is almost 30 pp. For the missing values, the gain is more than 20 pp for observation T and around 27 pp for observation $T-1$. Hence, taking into account the dynamics of the factors seems to have only a limited gain in the estimation of the factors and missing values, while taking on board the dynamics of the idiosyncratic components proves to be quite valuable.

In Table 1 we also report the average running time (in seconds) for our algorithm and for the EM version of the algorithm proposed by Jungbacker *et al.* (2011). For the specification with autoregressive idiosyncratic components our algorithm takes on average about four seconds whereas the general purpose algorithm takes almost seven seconds (*i.e.* a computational gain of 70%). For the other cases, the running time is reduced from more than four seconds to around one second.[11]

4.2 Sensitivity analysis

Due to the high dimensionality of the problem and the existence of infinite possible combinations, the sensitivity analysis was carried out by changing one setting of the simulation design at a time while maintaining all the others constant vis-à-vis the base case. In this way, it is possible to identify the settings of the base case that are more critical for the results (see Table 2).

First, to assess the importance of the amount of serial correlation of the idiosyncratic components, several fixed values for β_n were considered instead of $\beta_n \sim i.i.d.\ U([-0.9, 0.9])$ as in the base case. In particular, we fixed β_n at -0.9, at -0.8, and so on up to 0.9. A noteworthy finding is the fact that when $\beta_n = 0$, that is, when the idiosyncratic components are serially uncorrelated in the data generating process, allowing for the dynamics of idiosyncratic components in the estimation model does not

[11] We only report the average running time for the Junbacker *et al.* (2011) algorithm because the other results are virtually identical to those obtained with our algorithm, as expected. The Matlab codes are available from the authors upon request.

involve any cost in terms of relative performance. Another finding is that the larger (in absolute value) is the serial correlation, the larger are the gains in considering $AR(1)$ idiosyncratic components when estimating both the factors and the missing values. Indeed, specification $(B \neq 0, P = 1)$ ranks always first, with gains that can be quite large in the presence of moderate to strong serial correlation of the idiosyncratic components.

Simulations were also carried out considering different numbers of dynamic factors (both in the data generating process and in the estimated specifications). In particular, we set $Q = 2$ and $Q = 6$. Increasing or decreasing the number of common factors does not seem to make much difference in terms of the relative performance. In fact, the gains are similar to the ones observed in the base case.

Different dimensions of the panel data set were also addressed. Increasing the number of variables to $N = 200$ has a limited effect on the results. Regarding the number of observations, decreasing the sample size to half, that is setting $T = 120$, deteriorates slightly the relative performance of specification $(B \neq 0, P = 1)$ in estimating the common factors at the end of the sample. Nevertheless, the gain in the estimation of the factors for observation T is 21 pp and for observation $T - 1$ the improvement is more than 24 pp. Increasing the number of observations to $T = 480$ leads to larger gains than in the base case. In particular, the estimation of the factors for observation T is improved almost 30 pp whereas for observation $T - 1$ the gain is around 38 pp. Concerning the results for the missing values, whatever the size of the panel the results are almost unchanged vis-à-vis the base case.

Furthermore, we assessed the sensitivity of the results to the value of γ_n, the ratio of the variance of the common component to the total variance of the n-th variable. A low value for γ_n means that the idiosyncratic component is relatively more important and therefore allowing for the dynamics of such component may prove to be crucial. In fact, setting $\gamma_n = 0.1$ results in a noteworthy improvement in the relative performance of specification $(B \neq 0, P = 1)$ both in terms of $R^2_{F,\widehat{F}}$ and in terms of the estimation of the factors at the end of the sample period. The gain in the estimation of the factors for observation T is around 36 pp whereas for the observation $T - 1$ the gain is more than 47 pp. Naturally, as γ_n increases, the gain reduces. Nevertheless, for $\gamma_n = 0.5$, that is, when the common component contributes as much as the idiosyncratic component to the total variance of the series, the gains in the estimation of the factors at the end of the sample are around 35 pp. For $\gamma_n = 0.9$, that is when the idiosyncratic component plays a minor role in the total variance, the gains still turn out to be about 15 pp. Regarding the estimation of the missing values, the results are similar to those of the base case.

We also assessed different values for the autoregressive coefficients of the common factors. In particular, we considered $A_{1,qq} = 0.0$, 0.3, 0.5, 0.7, 0.9 ($q = 1, \cdots, Q$). The results suggest that, as the serial correlation of the common factors increases, the gains in the estimation of the factors by taking into account the dynamics of the idiosyncratic components decrease. When the factors have no serial correlation, the gain is more than 30 pp for observation T and is close to 37 pp for observation $T-1$, whereas in the most unfavorable case, $A_{1,qq} = 0.9$, the improvement is around 14 pp for observation T and more than 10 pp for observation $T-1$. As regards the estimation of the missing values, the results are not influenced by the amount of serial correlation of the common factors.

The jagged edge issue depends on the variables included in the data set as well as on the release calendar which may differ from country to country. In the base case, we assumed that 50% of the series have no release lag, 25% of the series have a lag of one month and the other 25% of the series present a lag of two months. For the sensitivity analysis, we considered two alternatives. In the first, 80% of the series have no release lag, 10% have a lag of one month and 10% have a lag of two months, while in the second case 30% of the series have no release lag, 35% have a lag of one month and 35% have a lag of two months. One can see that the results are quite similar to those in Table 1.

In the base case, we set $S = 0$, which implies that there is no distinction between the dynamic and the static factors. If we consider $S = 1$ both in the data generating process and in the estimated specifications, the results are again similar to the base case.

So far, it has been assumed that there is a match between the specification ($B \neq 0, P = 1$) and the model underlying the data generating process. To assess the performance under misspecification, several exercises were conducted.

First, we assumed a mismatch between the true number of dynamic factors and the number of estimated dynamic factors. When the true number of dynamic factors is two but the model is estimated assuming that there are four dynamic factors, the results remain virtually unchanged. In contrast, when the true number of dynamic factors is six but the model is estimated again assuming that there are only four dynamic factors, the $R^2_{F,\widehat{F}}$ worsens significantly and the gains in the estimation of the common factors at the end of sample period almost vanish. Hence, the underspecification of the model in terms of the number of common factors limits substantially the gains of taking into account the dynamics of the idiosyncratic components. Nevertheless, the specification $(B \neq 0, P = 1)$ continues to present a significant improvement in the estimation of the missing values.

Another robustness exercise relates to the dynamics of the idiosyncratic components. Two variants were assessed. First, $AR(2)$ instead of $AR(1)$ idiosyncratic components were considered in the data generating process while holding the three estimation model specifications unchanged. The two roots for each autoregressive polynomial were generated as the inverse of independent draws from a uniform distribution on $[-0.9, 0.9]$. For this variant, the improvement in the relative performance of specification $(B \neq 0, P = 1)$ is even larger than in the base case. In particular, the gain in the estimation of the factors for observation T exceeds 42 pp and for observation $T-1$ the improvement is around 46 pp. For the missing values, the gain is about 31 pp for observation T and almost 39 pp for observation $T-1$. The second variant corresponds to admit $MA(1)$ idiosyncratic components in the data generating process, the coefficients also being drawn from a uniform distribution on $[-0.9, 0.9]$. The specification $(B \neq 0, P = 1)$ continues to perform better than the other specifications, but the gains are lower than in the base case. However, there is still a gain of more than 10 pp for observation T and 14 pp for observation $T-1$ in the estimation of the common factors.

Two additional exercises allowed for contemporaneous cross-correlation amongst the idiosyncratic components in the data generating process and not taken into account in the estimated specifications. In this respect, we first set $\delta = 0.5$, which corresponds to a moderate contemporaneous cross-correlation between the generated innovations of idiosyncratic components. One can see that the relative performance remain almost unchanged vis-à-vis the base case. Note that in the latter simulation exercise the matrix of coefficients of the lagged idiosyncratic components is kept diagonal and the cross-correlation is generated only through the contemporaneous covariance matrix of their innovations.[12] A more general scheme for generating cross-correlation would consider the non-diagonality of both the contemporaneous covariance matrix of the innovations and the matrix of coefficients of the lagged idiosyncratic components (i.e. non-zero

[12] This way of generating cross-correlation between the idiosincratic components closely follows Doz *et al.* (2012).

off-diagonal entries in matrix B). With this in mind, in a second exercise to assess the effects of cross-correlation on the properties of the estimators, we considered the alternative specification for generating the idiosyncratic components:

$$(1-\beta_n L)\,\tilde{v}_{t,n} = \xi_{n,t} \;\; (n=1,\cdots,N; t=1,\cdots,T) \;\text{ with } \xi_t \sim i.i.d.N(0,I)$$

$$v_t = D(\rho)\,\tilde{v}_t \text{ with } D(\rho) = \underset{n}{diag}\left(\sqrt{\alpha_n}\right) K(\rho)' \left(I-B^2\right)^{1/2}$$

where $K(\rho)$ is such that $K(\rho)' K(\rho) = H(\rho)$.[13] As in the first specification, $V(v_{t,n}) = \alpha_n$ and the generating process reduces to the base case when ρ is set to zero. The results with $\rho = 0.5$, which roughly mimics the cross-sectional correlations of the idiosyncratic components in the US data set, are also presented in Table 2. One can conclude that in this more demanding setup there are still noteworthy gains although smaller than in the base case.

Turning now to the number of factor lags in the measurement equation of the model, we set $S = 1$ in the data generating process while continuing to estimate the model assuming $S = 0$. Similarly to what happens when the number of dynamic factors is underspecified, the $R^2_{F,\widehat{F}}$ is significantly lower for all estimated specifications and the gains in the estimation of the common factors at the end of sample period using specification $(B \neq 0, P = 1)$ vanish. Nevertheless, the specification $(B \neq 0, P = 1)$ continues to be the best in terms of the estimation of the missing values, with a gain of 12 pp for observation T and around 18 pp for observation $T-1$.

In a final exercise, we investigated simultaneously the impact of the underestimation of the number of factors and variations in the ratio of the variance of the common component to the total variance of the n-th variable. In particular, we considered the case where the true number of factors is four but the number of estimated factors is two and the case where the true number of factors is six but the number of estimated factors is four. For both cases, we considered several values for the ratio of the variance of the

[13] More specifically, we set

$$K(\rho) = \left(1-\rho^2\right)^{1/2} \begin{bmatrix} 1 & 0 & \cdots & 0 & 0 \\ 0 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & 0 \\ 0 & 0 & \cdots & 0 & \left(1-\rho^2\right)^{-1/2} \end{bmatrix} \begin{bmatrix} 1 & 0 & \cdots & 0 & 0 \\ \rho & 1 & \cdots & 0 & 0 \\ \vdots & \ddots & & \vdots & \vdots \\ \rho^{N-2} & \rho^{N-3} & \cdots & 1 & 0 \\ \rho^{N-1} & \rho^{N-2} & \cdots & \rho & 1 \end{bmatrix}$$

common component to the total variance of the n-th variable, namely $\gamma_n = 0.1,0.5,0.9$. The $R^2_{F,\widehat{F}}$ is the highest for specification $(B \neq 0, P = 1)$ when $\gamma_n = 0.1,0.5$. For the case where $\gamma_n = 0.9$, differences are negligible. In terms of the estimation of the factors, the specification $(B \neq 0, P = 1)$ presents gains vis-à-vis the other specifications when $\gamma_n = 0.1,0.5$ whereas when $\gamma_n = 0.9$ differences are again marginal. Regarding the estimation of the missing values, the specification $(B \neq 0, P = 1)$ continues to present noteworthy gains in all cases.

5 Conclusions

The staggered release of macroeconomic data results in unbalanced panel data sets with missing values at the end of the sample (the so-called jagged or ragged edge) which raises difficulties to the estimation of dynamic factor models in a real-time environment.

We propose an EM algorithm which copes with panel data sets with jagged edge without significantly increasing the computation time relative to the balanced panel case, while accounting for autoregressive common factors and allowing for serial correlation of the idiosyncratic components. When compared with the general purpose algorithm proposed by Jungbacker *et al.* (2011), our algorithm is much simpler analytically and also significantly faster.

Being able to exploit the dynamics of both the common factors and the idiosyncratic components proves to be quite useful for the estimation of the factors in a context of limited data availability at the end of the sample. Based on a Monte Carlo analysis, we found that taking into account only the dynamics of the factors leads to results similar to those obtained when assuming serially independent factors, as in Stock and Watson (2002a). However, when one also takes into account the dynamics of the idiosyncratic components, besides some increase in the overall fit of the factors, there is a substantial improvement in the relative MSE of the estimation of both the common factors at the end of the sample and the missing values. In particular, the gain in the estimation of the common factors for the last observation is around 26 percentage points and for the second last observation the improvement is almost 30 percentage points, while for the missing values the gain exceeds 20 percentage points for the last observation and is around 27 percentage points for the second last observation.

To assess the robustness of such findings, a thorough sensitivity analysis was performed. The results reinforce the importance of taking into account the dynamics of the idiosyncratic components when dealing with jagged edge panel data sets. The results also show that, in the particular case of underspecification of the number of factors in the estimation model, the overall fit worsens significantly for all specifications and the gains in the relative performance of taking into account the dynamics of the idiosyncratic components decrease.

Appendix 1 - M-step: The expected log likelihood function

For any vectors of random variables w and s, let $g\left(w; parameters\right)$ and $g\left(w|s; parameters\right)$ denote the probability density functions of w and of w conditional on s, respectively. To shorten the length of the expressions, also let any quadratic form $\delta'\Delta\delta$ be written as $\delta'\Delta\left(\cdots\right)$. The log joint density of both the variables and factors generated according to the dynamic factor model (1)-(5) is:

$$\ln g\left(\mathring{x}_1, \cdots, \mathring{x}_T, \mathbf{f}_1^{(R)}, f_2, \cdots, f_T; \mu, \Lambda, B, \Psi, A, \Phi, \zeta, \Omega\right) =$$

$$= \ln g\left(\mathbf{f}_1^{(R)}|\zeta, \Omega\right) + \sum\nolimits_{t=2}^{T} \ln g\left(f_t|\mathbf{f}_1^{(R)}, f_2, \cdots, f_{t-1}; A, \Phi\right) + \ln g\left(\mathring{x}_1|\mathbf{f}_1^{(R)}; \mu, \Lambda, B, \Psi\right) +$$

$$+ \sum\nolimits_{t=2}^{T} \ln g\left(\mathring{x}_t|\mathbf{f}_1^{(R)}, f_2, \cdots, f_t, \mathring{x}_1, \cdots, \mathring{x}_{t-1}; \mu, \Lambda, B, \Psi\right) \mathrel{\mathring{=}}$$

$$\mathrel{\mathring{=}} -\frac{1}{2}\ln\left[\det\left(\Omega\right)\right] - \frac{1}{2}\left(\mathbf{f}_1^{(R)} - \zeta\right)'\Omega^{-1}\left(\cdots\right) - \frac{T-1}{2}\ln\left[\det\left(\Phi\right)\right] +$$

$$-\frac{1}{2}\sum\nolimits_{t=2}^{T}\left(f_t - A\mathbf{f}_{t-1}^{(P)}\right)'\Phi^{-1}\left(\cdots\right) - \frac{1}{2}\ln\left[\det\left(\left(I - B^2\right)^{-1}\Psi\right)\right] +$$

$$-\frac{1}{2}\left(\mathring{x}_1 - \mu - \Lambda\mathbf{f}_1^{(S+1)}\right)'\left(I - B^2\right)\Psi^{-1}\left(\cdots\right) - \frac{T-1}{2}\ln\left[\det\left(\Psi\right)\right] +$$

$$-\frac{1}{2}\sum\nolimits_{t=2}^{T}\left(\mathring{x}_t - B\mathring{x}_{t-1} - \left(I - B\right)\mu - \Upsilon\mathbf{f}_t^{(S+2)}\right)'\Psi^{-1}\left(\cdots\right)$$

For any t (> 1), $\mathring{C}_t = \left[\begin{array}{cc} C_t' & \ddot{C}_t' \end{array}\right]'$ $(N \times N)$ is an orthogonal matrix (i.e. $\mathring{C}_t\mathring{C}_t' = \mathring{C}_t'\mathring{C}_t = I_N$), where $\ddot{C}_t$ $((N - \mathcal{N}_t) \times N)$ is a matrix of zeros and ones such that $\ddot{C}_t\ddot{C}_t' = I_{N-\mathcal{N}_t}$ and $C_t\ddot{C}_t' = 0$. We have $\mathring{C}_t\mathring{x}_t = \left[\begin{array}{cc} x_t' & \ddot{x}_t' \end{array}\right]'$, where $\ddot{x}_t$ is the $(N - \mathcal{N}_t)$-dimensional vector of variables of period t for which the realizations are missing:

$$\mathring{C}_t\mathring{x}_t = \mathring{C}_t\left(B\mathring{x}_{t-1} + (I - B)\mu + \Upsilon\mathbf{f}_t^{(S+2)} + \mathring{e}_t\right) =$$

$$= \left[\begin{array}{c} \left(C_tBC_t'\right)x_{t-1} + \left(I - C_tBC_t'\right)C_t\mu + \left(C_t\Upsilon\right)\mathbf{f}_t^{(S+2)} + e_t \\ \left(\ddot{C}_tB\ddot{C}_t'\right)\ddot{x}_{t-1} + \left(I - \ddot{C}_tB\ddot{C}_t'\right)\ddot{C}_t\mu + \left(\ddot{C}_t\Upsilon\right)\mathbf{f}_t^{(S+2)} + \ddot{e}_t \end{array}\right]$$

with $e_t = C_t\mathring{e}_t$ and $\ddot{e}_t = \ddot{C}_t\mathring{e}_t$. Because the idiosyncratic components are serially and cross-sectionally independent,

$$E_x\left(\ddot{e}_t\right)=E_x\left[\ddot{x}_t-\left(\ddot{C}_tB\ddot{C}_t'\right)\ddot{x}_{t-1}-\left(I-\ddot{C}_tB\ddot{C}_t'\right)\ddot{C}_t\mu-\left(\ddot{C}_t\Upsilon\right)\mathbf{f}_t^{(S+2)}\right]=0$$

where we adopted the simplified notation $E_x\left(w\right)$ instead of $E\left(w|x_1,x_2,\cdots,x_T\right)$. Moreover,

$$E_x\left(\ddot{e}_t\ddot{e}_t'\right)=\ddot{C}_tE_x\left(\mathring{e}_t\mathring{e}_t'\right)\ddot{C}_t'=\ddot{C}_t\Psi\ddot{C}_t'$$

The expected values (conditional on $x_1,x_2,\cdots,x_T$) of the quadratic forms in the log density presented above may be expressed as follows:

i) $$E_x\left[\left(\mathbf{f}_1^{(R)}-\zeta\right)'\Omega^{-1}\left(\cdots\right)\right]=tr\left\{E_x\left[\left(\left(\mathbf{f}_1^{(R)}-\mathbf{f}_{1|T}^{(R)}\right)+\left(\mathbf{f}_{1|T}^{(R)}-\zeta\right)\right)\left(\cdots\right)'\right]\Omega^{-1}\right\}=$$
$$=tr\left\{\left[\mathbf{P}_{1,1|T}^{(R)}+\left(\mathbf{f}_{1|T}^{(R)}-\zeta\right)\left(\mathbf{f}_{1|T}^{(R)}-\zeta\right)'\right]\Omega^{-1}\right\}$$

ii) $$E_x\left[\left(f_t-A\mathbf{f}_{t-1}^{(P)}\right)'\Phi^{-1}\left(\cdots\right)\right]=$$
$$=tr\left\{E_x\left[\left(\left(f_t-f_{t|T}\right)-A\left(\mathbf{f}_{t-1}^{(P)}-\mathbf{f}_{t-1|T}^{(P)}\right)+\left(f_{t|T}-A\mathbf{f}_{t-1|T}^{(P)}\right)\right)\left(\cdots\right)'\right]\Phi^{-1}\right\}=$$
$$=tr\left\{\left[P_{t,t|T}+A\mathbf{P}_{t-1,t-1|T}^{(P)}A'-2\left[P_{t,t-1|T}\quad\cdots\quad P_{t,t-P|T}\right]A'+\left(f_{t|T}-A\mathbf{f}_{t-1|T}^{(P)}\right)\left(\cdots\right)'\right]\right.$$
$$\left..\Phi^{-1}\right\}=tr\left\{\left[M_{t,t|T}+A\mathbf{M}_{t-1,t-1|T}^{(P)}A'-2A\mathbf{H}_{t,t-1|T}^{(P)\prime}\right]\Phi^{-1}\right\}$$

iii) $$E_x\left[\left(\mathring{x}_1-\mu-\Lambda\mathbf{f}_1^{(S+1)}\right)'\left(I-B^2\right)\Psi^{-1}\left(\cdots\right)\right]=\text{[because }\mathcal{T}_n>1\text{ for all }n]$$
$$=E_x\left[\left(x_1-\mu-\Lambda\mathbf{f}_1^{(S+1)}\right)'\left(I-B^2\right)\Psi^{-1}\left(\cdots\right)\right]=$$
$$=tr\left\{E_x\left[\left(x_1-\mu-\Lambda\mathbf{f}_1^{(S+1)}\right)\left(\cdots\right)'\right]\left(I-B^2\right)\Psi^{-1}\right\}=$$
$$=tr\left\{E_x\left[\left(\left(x_1-\mu\right)-\Lambda\left(\mathbf{f}_1^{(S+1)}-\mathbf{f}_{1|T}^{(S+1)}\right)-\Lambda\mathbf{f}_{1|T}^{(S+1)}\right)\left(\cdots\right)'\right]\left(I-B^2\right)\Psi^{-1}\right\}=$$
$$=tr\left\{\left[\left(x_1-\mu-\Lambda\mathbf{f}_{1|T}^{(S+1)}\right)\left(\cdots\right)'+\Lambda\mathbf{P}_{1,1|T}^{(S+1)}\Lambda'\right]\left(I-B^2\right)\Psi^{-1}\right\}=$$
$$=tr\left\{\left[\left(x_1-\mu\right)\left(\cdots\right)'-2\left(x_1-\mu\right)\mathbf{f}_{1|T}^{(S+1)\prime}\Lambda'+\Lambda\mathbf{M}_{1,1|T}^{(S+1)}\Lambda'\right]\left(I-B^2\right)\Psi^{-1}\right\}$$

iv) $$E_x\left[\left(\mathring{x}_t-B\mathring{x}_{t-1}-\left(I-B\right)\mu-\Upsilon\mathbf{f}_t^{(S+2)}\right)'\Psi^{-1}\left(\cdots\right)\right]=$$
$$=E_x\left[\left(\mathring{x}_t-B\mathring{x}_{t-1}-\left(I-B\right)\mu-\Upsilon\mathbf{f}_t^{(S+2)}\right)'\mathring{C}_t'\mathring{C}_t\Psi^{-1}\mathring{C}_t'\mathring{C}_t\left(\cdots\right)\right]=$$

$$= E_x \left[\left(x_t - (C_t B C_t') x_{t-1} - (I - C_t B C_t') C_t \mu - C_t \Upsilon \mathbf{f}_t^{(S+2)} \right)' (C_t \Psi C_t')^{-1} (\cdots) \right] +$$

$$+ E_x \left(\ddot{e}_t' \left(\ddot{C}_t \Psi \ddot{C}_t' \right)^{-1} \ddot{e}_t \right) = tr \left\{ \left[(x_t - (C_t B C_t') x_{t-1} - (I - C_t B C_t') C_t \mu) (\cdots)' + \right. \right.$$

$$\left. -2 (x_t - (C_t B C_t') x_{t-1} - (I - C_t B C_t') C_t \mu) \mathbf{f}_{t|T}^{(S+2)\prime} \Upsilon' C_t' + C_t \Upsilon \mathbf{M}_{t,t|T}^{(S+2)} \Upsilon' C_t' \right] .$$

$$\left. . (C_t \Psi C_t')^{-1} \right\} + (N - \mathcal{N}_t) \mathrel{\hat{=}} tr \left\{ \left[(x_t - (C_t B C_t') x_{t-1} - (I - C_t B C_t') C_t \mu) (\cdots)' + \right. \right.$$

$$-2 (x_t - (C_t B C_t') x_{t-1} - (I - C_t B C_t') C_t \mu) \left(\mathbf{f}_{t|T}^{(S+1)\prime} \Lambda' - \mathbf{f}_{t-1|T}^{(S+1)\prime} \Lambda' B \right) C_t' +$$

$$\left. + C_t \left(\Lambda \mathbf{M}_{t,t|T}^{(S+1)} \Lambda' + B \Lambda \mathbf{M}_{t-1,t-1|T}^{(S+1)} \Lambda' B - \Lambda \left(\mathbf{M}_{t,t-1|T}^{(S+1)} + \mathbf{M}_{t,t-1|T}^{(S+1)\prime} \right) \Lambda' B \right) C_t' \right] .$$

$$\left. . (C_t \Psi C_t')^{-1} \right\}$$

Thus, taking into account that B and Ψ are diagonal matrices, the expected log likelihood may be written as:

$$\ell (\mu, \Lambda, B, \Psi, A, \Phi, \zeta, \Omega | x_1, \cdots, x_T) =$$

$$= E_x \left[\ln g \left(\mathring{x}_1, \cdots, \mathring{x}_T, \mathbf{f}_1^{(R)}, f_2, \cdots, f_T; \mu, \Lambda, B, \Psi, A, \Phi, \zeta, \Omega \right) \right] \mathrel{\hat{=}}$$

$$\mathrel{\hat{=}} -\frac{1}{2} \ln [\det (\Omega)] - \frac{1}{2} tr \left\{ \left[\mathbf{P}_{1,1|T}^{(R)} + \left(\mathbf{f}_{1|T}^{(R)} - \zeta \right) \left(\mathbf{f}_{1|T}^{(R)} - \zeta \right)' \right] \Omega^{-1} \right\} - \frac{T-1}{2} \ln [\det (\Phi)] +$$

$$-\frac{1}{2} tr \left\{ \left[\sum_{t=2}^{T} M_{t,t|T} + A \left(\sum_{t=1}^{T-1} \mathbf{M}_{t,t|T}^{(P)} \right) A' - 2A \sum_{t=2}^{T} \mathbf{H}_{t,t-1|T}^{(P)\prime} \right] \Phi^{-1} \right\} +$$

$$+\frac{1}{2} \ln \left[\det \left(I - B^2 \right) \right] - \frac{T}{2} \ln [\det (\Psi)] +$$

$$-\frac{1}{2} tr \left\{ \left[(x_1 - \mu)(x_1 - \mu)' - 2 (x_1 - \mu) \mathbf{f}_{1|T}^{(S+1)\prime} \Lambda' + \Lambda \mathbf{M}_{1,1|T}^{(S+1)} \Lambda' \right] \left(I - B^2 \right) \Psi^{-1} \right\} +$$

$$-\frac{1}{2} \sum_{t=2}^{T} tr \left\{ \left[((x_t - C_t \mu) - (C_t B C_t') (x_{t-1} - C_t \mu)) (\cdots)' + \right. \right.$$

$$-2 ((x_t - C_t \mu) - (C_t B C_t') (x_{t-1} - C_t \mu)) \left(\mathbf{f}_{t|T}^{(S+1)\prime} \Lambda' - \mathbf{f}_{t-1|T}^{(S+1)\prime} \Lambda' B \right) C_t' +$$

$$\left. \left. + C_t \left(\Lambda \mathbf{M}_{t,t|T}^{(S+1)} \Lambda' + B \Lambda \mathbf{M}_{t-1,t-1|T}^{(S+1)} \Lambda' B - \Lambda \left(\mathbf{M}_{t,t-1|T}^{(S+1)} + \mathbf{M}_{t,t-1|T}^{(S+1)\prime} \right) \Lambda' B \right) C_t' \right] (C_t \Psi C_t')^{-1} \right\} =$$

$$= -\frac{1}{2} \ln [\det (\Omega)] - \frac{1}{2} tr \left\{ \left[\mathbf{P}_{1,1|T}^{(R)} + \left(\mathbf{f}_{1|T}^{(R)} - \zeta \right) \left(\mathbf{f}_{1|T}^{(R)} - \zeta \right)' \right] \Omega^{-1} \right\} - \frac{T-1}{2} \ln [\det (\Phi)] +$$

$$-\frac{1}{2}tr\left\{\left[\sum_{t=2}^{T}M_{t,t|T}+A\left(\sum_{t=1}^{T-1}\mathbf{M}_{t,t|T}^{(P)}\right)A'-2A\sum_{t=2}^{T}\mathbf{H}_{t,t-1|T}^{(P)\prime}\right]\Phi^{-1}\right\}+\frac{1}{2}\sum_{n=1}^{N}\ln\left(1-\beta_n^2\right)+$$

$$-\frac{T}{2}\sum_{n=1}^{N}\ln\left(\psi_n\right)-\frac{1}{2}\sum_{n=1}^{N}\frac{1-\beta_n^2}{\psi_n}\left[\left(x_{1,n}-\mu_n\right)^2-2\left(x_{1,n}-\mu_n\right)\mathbf{f}_{1|T}^{(S+1)\prime}\lambda_n+\lambda_n'\mathbf{M}_{1,1|T}^{(S+1)}\lambda_n\right]+$$

$$-\frac{1}{2}\sum_{n=1}^{N}\psi_n^{-1}\left\{\sum_{t=2}^{\mathcal{T}_n}\left(x_{t,n}-\mu_n\right)^2+\beta_n^2\sum_{t=2}^{\mathcal{T}_n}\left(x_{t-1,n}-\mu_n\right)^2-2\beta_n\sum_{t=2}^{\mathcal{T}_n}\left(x_{t,n}-\mu_n\right)\left(x_{t-1,n}-\mu_n\right)+\right.$$

$$-2\sum_{t=2}^{\mathcal{T}_n}\left(x_{t,n}-\mu_n\right)\mathbf{f}_{t|T}^{(S+1)\prime}\lambda_n+2\beta_n\sum_{t=2}^{\mathcal{T}_n}\left(x_{t,n}-\mu_n\right)\mathbf{f}_{t-1|T}^{(S+1)\prime}\lambda_n+$$

$$+2\beta_n\sum_{t=2}^{\mathcal{T}_n}\left(x_{t-1,n}-\mu_n\right)\mathbf{f}_{t|T}^{(S+1)\prime}\lambda_n-2\beta_n^2\sum_{t=2}^{\mathcal{T}_n}\left(x_{t-1,n}-\mu_n\right)\mathbf{f}_{t-1|T}^{(S+1)\prime}\lambda_n+$$

$$\left.+\lambda_n'\sum_{t=2}^{\mathcal{T}_n}\left[\mathbf{M}_{t,t|T}^{(S+1)}+\beta_n^2\mathbf{M}_{t-1,t-1|T}^{(S+1)}-\beta_n\left(\mathbf{M}_{t,t-1|T}^{(S+1)}+\mathbf{M}_{t,t-1|T}^{(S+1)\prime}\right)\right]\lambda_n\right\}=$$

$$=-\frac{1}{2}\ln\left[\det\left(\Omega\right)\right]-\frac{1}{2}tr\left\{\left[\mathbf{P}_{1,1|T}^{(R)}+\left(\mathbf{f}_{1|T}^{(R)}-\zeta\right)\left(\mathbf{f}_{1|T}^{(R)}-\zeta\right)'\right]\Omega^{-1}\right\}-\frac{T-1}{2}\ln\left[\det\left(\Phi\right)\right]+$$

$$-\frac{1}{2}tr\left\{\left[\sum_{t=2}^{T}M_{t,t|T}+A\left(\sum_{t=1}^{T-1}\mathbf{M}_{t,t|T}^{(P)}\right)A'-2A\sum_{t=2}^{T}\mathbf{H}_{t,t-1|T}^{(P)\prime}\right]\Phi^{-1}\right\}+\frac{1}{2}\sum_{n=1}^{N}\left\{\ln\left(1-\beta_n^2\right)+\right.$$

$$-T\ln\left(\psi_n\right)-\psi_n^{-1}\left[\sum_{t=1}^{\mathcal{T}_n}\left(x_{t,n}-\mu_n\right)^2+\beta_n^2\sum_{t=2}^{\mathcal{T}_n-1}\left(x_{t,n}-\mu_n\right)^2-2\beta_n\sum_{t=2}^{\mathcal{T}_n}\left(x_{t,n}-\mu_n\right)\left(x_{t-1,n}-\mu_n\right)+\right.$$

$$-2\left(\sum_{t=1}^{\mathcal{T}_n}\left(x_{t,n}-\mu_n\right)\mathbf{f}_{t|T}^{(S+1)\prime}+\beta_n^2\sum_{t=2}^{\mathcal{T}_n-1}\left(x_{t,n}-\mu_n\right)\mathbf{f}_{t|T}^{(S+1)\prime}-\beta_n\sum_{t=2}^{\mathcal{T}_n}\left(x_{t,n}-\mu_n\right)\mathbf{f}_{t-1|T}^{(S+1)\prime}+\right.$$

$$\left.\left.-\beta_n\sum_{t=2}^{\mathcal{T}_n}\left(x_{t-1,n}-\mu_n\right)\mathbf{f}_{t|T}^{(S+1)\prime}\right)\lambda_n+\mathcal{T}_n\lambda_n'\bar{\mathbf{M}}\left(\beta_n,\mathcal{T}_n\right)\lambda_n\right\}=$$

$$=-\frac{1}{2}\ln\left[\det\left(\Omega\right)\right]-\frac{1}{2}tr\left\{\left[\mathbf{P}_{1,1|T}^{(R)}+\left(\mathbf{f}_{1|T}^{(R)}-\zeta\right)\left(\mathbf{f}_{1|T}^{(R)}-\zeta\right)'\right]\Omega^{-1}\right\}-\frac{T-1}{2}\ln\left[\det\left(\Phi\right)\right]+$$

$$-\frac{1}{2}tr\left\{\left[\sum_{t=2}^{T}M_{t,t|T}+A\left(\sum_{t=1}^{T-1}\mathbf{M}_{t,t|T}^{(P)}\right)A'-2A\sum_{t=2}^{T}\mathbf{H}_{t,t-1|T}^{(P)\prime}\right]\Phi^{-1}\right\}+\frac{1}{2}\sum_{n=1}^{N}\left\{\ln\left(1-\beta_n^2\right)+\right.$$

$$-T\ln\left(\psi_n\right)-\psi_n^{-1}\left[\left(\sum_{t=1}^{\mathcal{T}_n}x_{t,n}^2+\beta_n^2\sum_{t=2}^{\mathcal{T}_n-1}x_{t,n}^2-2\beta_n\sum_{t=2}^{\mathcal{T}_n}x_{t,n}x_{t-1,n}\right)-2\mu_n\left(\sum_{t=1}^{\mathcal{T}_n}x_{t,n}+\right.\right.$$

$$\left.+\beta_n^2\sum_{t=2}^{\mathcal{T}_n-1}x_{t,n}-\beta_n\sum_{t=2}^{\mathcal{T}_n}\left(x_{t,n}+x_{t-1,n}\right)\right)+\mu_n^2\left(\mathcal{T}_n+\beta_n^2\left(\mathcal{T}_n-2\right)-2\beta_n\left(\mathcal{T}_n-1\right)\right)+$$

$$-2\left(\sum_{t=1}^{\mathcal{T}_n}x_{t,n}\mathbf{f}_{t|T}^{(S+1)\prime}+\beta_n^2\sum_{t=2}^{\mathcal{T}_n-1}x_{t,n}\mathbf{f}_{t|T}^{(S+1)\prime}-\beta_n\sum_{t=2}^{\mathcal{T}_n}\left(x_{t,n}\mathbf{f}_{t-1|T}^{(S+1)\prime}+x_{t-1,n}\mathbf{f}_{t|T}^{(S+1)\prime}\right)\right)\lambda_n+$$

$$\left.\left.+2\mu_n\left(\sum_{t=1}^{\mathcal{T}_n}\mathbf{f}_{t|T}^{(S+1)}+\beta_n^2\sum_{t=2}^{\mathcal{T}_n-1}\mathbf{f}_{t|T}^{(S+1)}-\beta_n\sum_{t=2}^{\mathcal{T}_n}\left(\mathbf{f}_{t|T}^{(S+1)}+\mathbf{f}_{t-1|T}^{(S+1)}\right)\right)^{\prime}\lambda_n+\mathcal{T}_n\lambda_n^{\prime}\bar{\mathbf{M}}\left(\beta_n,\mathcal{T}_n\right)\lambda_n\right]\right\}=$$

$$=(9)$$

Appendix 2 - M-step: The first order conditions

The partial derivatives of the expected log likelihood with respect to all parameters but B are the following:

$$\frac{\partial \ell}{\partial \zeta}\left(\mu, \Lambda, B, \Psi, A, \Phi, \zeta, \Omega | x_1, \cdots, x_T\right) = \Omega^{-1}\left(\mathbf{f}_{1|T}^{(R)} - \zeta\right)$$

$$\frac{\partial \ell}{\partial \Omega}\left(\mu, \Lambda, B, \Psi, A, \Phi, \zeta, \Omega | x_1, \cdots, x_T\right) = \frac{1}{2}\Omega^{-1}\left[\mathbf{P}_{1,1|T}^{(R)} + \left(\mathbf{f}_{1|T}^{(R)} - \zeta\right)\left(\mathbf{f}_{1|T}^{(R)} - \zeta\right)' - \Omega\right]\Omega^{-1}$$

$$\frac{\partial \ell}{\partial A}\left(\mu, \Lambda, B, \Psi, A, \Phi, \zeta, \Omega | x_1, \cdots, x_T\right) = \Phi^{-1}\left[\sum_{t=2}^{T}\mathbf{H}_{t,t-1|T}^{(P)} - A\sum_{t=1}^{T-1}\mathbf{M}_{t,t|T}^{(P)}\right]$$

$$\frac{\partial \ell}{\partial \Phi}\left(\mu, \Lambda, B, \Psi, A, \Phi, \zeta, \Omega | x_1, \cdots, x_T\right) = \frac{1}{2}\Phi^{-1}\left[\sum_{t=2}^{T} M_{t,t|T} + A\left(\sum_{t=1}^{T-1}\mathbf{M}_{t,t|T}^{(P)}\right)A' + \right.$$

$$\left. -2A\sum_{t=2}^{T}\mathbf{H}_{t,t-1|T}^{(P)\prime} - (T-1)\,\Phi\right]\Phi^{-1}$$

and, for $n = 1, \ldots, N$:

$$\frac{\partial \ell}{\partial \mu_n}\left(\mu, \Lambda, B, \Psi, A, \Phi, \zeta, \Omega | x_1, \cdots, x_T\right) = -\frac{\mathcal{T}_n}{\psi_n}\left[\bar{T}\left(\beta_n, \mathcal{T}_n\right)\mu_n - \bar{x}_n\left(\beta_n, \mathcal{T}_n\right) + \bar{\mathbf{f}}\left(\beta_n, \mathcal{T}_n\right)'\lambda_n\right]$$

$$\frac{\partial \ell}{\partial \lambda_n}\left(\mu, \Lambda, B, \Psi, A, \Phi, \zeta, \Omega | x_1, \cdots, x_T\right) = -\frac{\mathcal{T}_n}{\psi_n}\left[\bar{\mathbf{M}}\left(\beta_n, \mathcal{T}_n\right) - \bar{z}_n\left(\beta_n, \mathcal{T}_n\right) + \mu_n\bar{\mathbf{f}}\left(\beta_n, \mathcal{T}_n\right)\right]$$

$$\frac{\partial \ell}{\partial \psi_n}\left(\mu, \Lambda, B, \Psi, A, \Phi, \zeta, \Omega | x_1, \cdots, x_T\right) = -\frac{1}{2}\left\{\frac{T}{\psi_n} - \frac{\mathcal{T}_n}{\psi_n^2}\left[\bar{y}_n\left(\beta_n, \mathcal{T}_n\right) - 2\bar{x}_n\left(\beta_n, \mathcal{T}_n\right)\mu_n + \right.\right.$$

$$\left.\left. +\bar{T}\left(\beta_n, \mathcal{T}_n\right)\mu_n^2 - 2\bar{z}_n\left(\beta_n, \mathcal{T}_n\right)'\lambda_n + 2\bar{\mathbf{f}}\left(\beta_n, \mathcal{T}_n\right)'\lambda_n\mu_n + \lambda_n'\bar{\mathbf{M}}\left(\beta_n, \mathcal{T}_n\right)\lambda_n\right]\right\}$$

Equating these partial derivatives to zero and solving the system of first conditions we obtain (10)-(13) as well as

$$\hat{\mu}_n = \frac{1}{\bar{T}\left(\beta_n, \mathcal{T}_n\right)}\left[\bar{x}_n\left(\beta_n, \mathcal{T}_n\right) - \bar{\mathbf{f}}\left(\beta_n, \mathcal{T}_n\right)'\hat{\lambda}_n\right] \tag{18}$$

$$\bar{\mathbf{M}}\left(\beta_n, \mathcal{T}_n\right)\hat{\lambda}_n = \bar{z}_n\left(\beta_n, \mathcal{T}_n\right) - \hat{\mu}_n\bar{\mathbf{f}}\left(\beta_n, \mathcal{T}_n\right) \tag{19}$$

$$\hat{\psi}_n = \frac{\mathcal{T}_n}{T}\left[\bar{y}_n\left(\beta_n, \mathcal{T}_n\right) - 2\bar{x}_n\left(\beta_n, \mathcal{T}_n\right)\hat{\mu}_n + \bar{T}\left(\beta_n, \mathcal{T}_n\right)\hat{\mu}_n^2 + \right.$$

$$-2\bar{z}_n(\beta_n, \mathcal{T}_n)' \hat{\lambda}_n + 2\bar{\mathbf{f}}(\beta_n, \mathcal{T}_n)' \hat{\lambda}_n \hat{\mu}_n + \hat{\lambda}_n' \bar{\mathbf{M}}(\beta_n, \mathcal{T}_n) \hat{\lambda}_n \Big] \quad (20)$$

From (18)-(19) we get (14) and (15). Finally, replacing $\hat{\mu}_n$ and $\hat{\lambda}_n$ in (20) by (14)-(15) we obtain (16).

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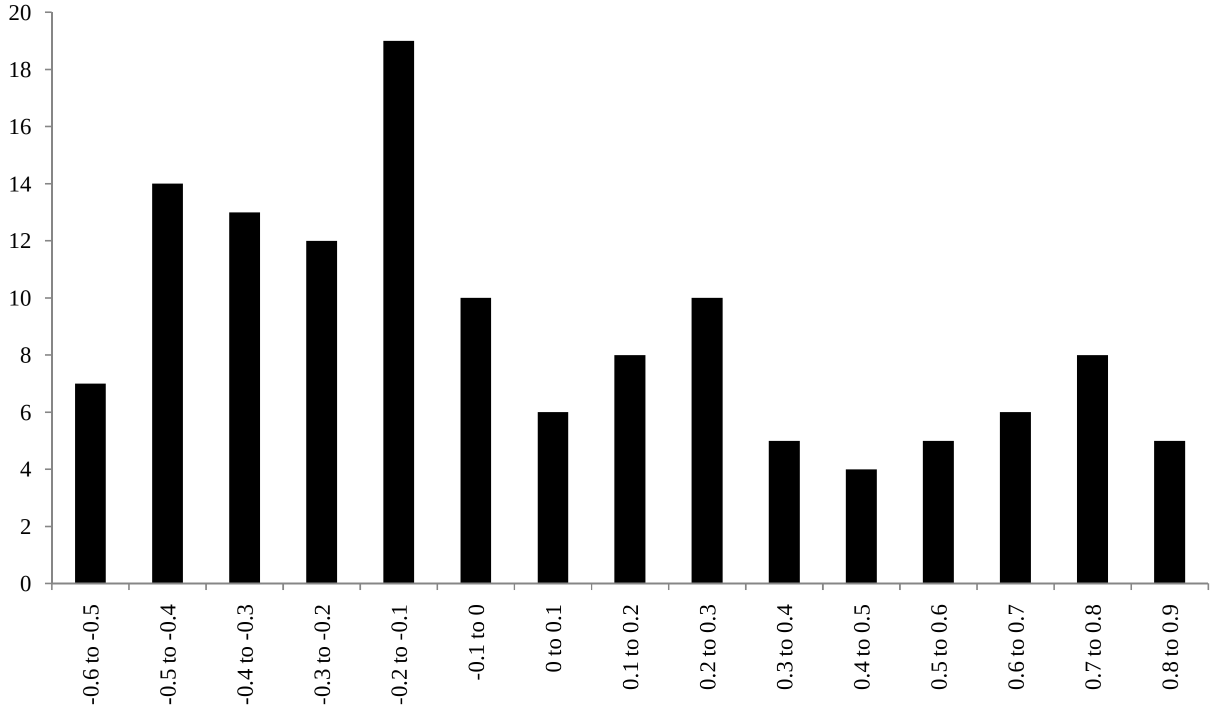
FIGURE 1

Histogram - 1st order autocorrelation coefficients of the estimated idiosyncratic components



Note: The results are for the US monthly data set of Stock and Watson (2005) considering seven factors estimated by principal components.

TABLE 1

Monte Carlo simulation results for the base case

	(B = 0, P = 0)	(B = 0, P = 1)	(B ≠ 0, P = 1)
R^2	0.959	0.960	0.975
RMSE for F_{T-1}	1.000	0.999	0.705
RMSE for F_T	1.000	0.996	0.739
RMSE for x_{T-1}	1.000	1.000	0.731
RMSE for x_T	1.000	1.001	0.793
Running time (seconds)			
Average	1.010 [4.513]	1.033 [4.552]	4.055 [6.883]

Note: The relative Mean Squared Error (RMSE) is computed vis-à-vis the specification (B = 0, P = 0). The running times were obtained using a computer with Intel Core Duo 2.93 Ghz, 64 Bit, 32 Gb RAM. Figures in square brackets refer to the EM version of the algorithm proposed by Jungbacker, Koopman and van der Wel (2011). The codes were developed in Matlab.

TABLE 2

Sensitivity analysis

		R^2			RMSE for F_{T-1}		RMSE for F_T		RMSE for x_{T-1}		RMSE for x_T	
		(B = 0, P = 0)	(B = 0, P = 1)	(B ≠ 0, P = 1)	(B = 0, P = 1)	(B ≠ 0, P = 1)	(B = 0, P = 1)	(B ≠ 0, P = 1)	(B = 0, P = 1)	(B ≠ 0, P = 1)	(B = 0, P = 1)	(B ≠ 0, P = 1)
β	-0.9	0.950	0.953	0.969	0.945	0.683	0.952	0.582	0.998	0.195	0.998	0.273
	-0.8	0.955	0.958	0.966	0.951	0.793	0.955	0.699	0.998	0.366	0.998	0.480
	-0.7	0.957	0.959	0.964	0.956	0.849	0.962	0.775	0.998	0.517	0.999	0.638
	-0.6	0.957	0.959	0.963	0.962	0.889	0.967	0.833	0.998	0.647	0.999	0.757
	-0.5	0.958	0.960	0.962	0.968	0.922	0.972	0.880	0.999	0.757	0.999	0.845
	-0.4	0.958	0.960	0.961	0.973	0.948	0.976	0.917	0.999	0.846	0.999	0.909
	-0.3	0.958	0.960	0.961	0.979	0.968	0.980	0.946	0.999	0.916	1.000	0.953
	-0.2	0.958	0.960	0.960	0.984	0.983	0.984	0.969	0.999	0.966	1.000	0.982
	-0.1	0.958	0.960	0.959	0.990	0.994	0.988	0.985	0.999	0.995	1.000	0.998
	0	0.959	0.959	0.959	0.996	1.000	0.993	0.995	0.999	1.005	1.000	1.003
	0.1	0.959	0.959	0.958	1.002	1.002	0.997	0.998	0.999	0.995	1.001	0.997
	0.2	0.959	0.959	0.958	1.008	0.999	1.002	0.994	1.000	0.964	1.001	0.981
	0.3	0.959	0.959	0.957	1.015	0.991	1.007	0.982	1.000	0.914	1.001	0.951
	0.4	0.959	0.959	0.957	1.022	0.980	1.011	0.961	1.000	0.844	1.001	0.906
	0.5	0.959	0.958	0.956	1.030	0.966	1.016	0.932	1.000	0.754	1.001	0.842
	0.6	0.959	0.958	0.955	1.039	0.950	1.020	0.893	1.001	0.645	1.001	0.754
	0.7	0.958	0.957	0.953	1.048	0.932	1.025	0.846	1.001	0.516	1.001	0.635
	0.8	0.957	0.956	0.951	1.059	0.919	1.029	0.792	1.001	0.367	1.001	0.478
	0.9	0.953	0.951	0.949	1.073	0.913	1.037	0.740	1.002	0.198	1.001	0.274
Q	2	0.972	0.973	0.982	0.995	0.714	0.995	0.774	1.000	0.732	1.000	0.803
	6	0.946	0.947	0.968	0.979	0.652	0.989	0.731	0.999	0.727	0.999	0.793
N	200	0.971	0.972	0.981	1.000	0.737	1.003	0.771	1.000	0.729	1.000	0.801
T	120	0.943	0.944	0.962	1.007	0.759	0.993	0.790	1.000	0.747	1.000	0.809
	480	0.968	0.968	0.982	0.978	0.623	0.988	0.704	1.000	0.738	0.999	0.799
γ_n	0.1	0.653	0.577	0.841	1.136	0.526	1.099	0.641	1.003	0.724	1.000	0.790
	0.5	0.943	0.945	0.968	0.991	0.644	0.991	0.685	0.998	0.726	1.000	0.788
	0.9	0.978	0.979	0.984	1.023	0.864	1.014	0.846	1.000	0.724	1.001	0.785

Note: The relative Mean Squared Error (RMSE) is computed vis-à-vis the specification (B = 0, P = 0).

TABLE 2 (continued)

Sensitivity analysis

		R^2			RMSE for F_{T-1}		RMSE for F_T		RMSE for x_{T-1}		RMSE for x_T	
		(B = 0, P = 0)	(B = 0, P = 1)	(B ≠ 0, P = 1)	(B = 0, P = 1)	(B ≠ 0, P = 1)	(B = 0, P = 1)	(B ≠ 0, P = 1)	(B = 0, P = 1)	(B ≠ 0, P = 1)	(B = 0, P = 1)	(B ≠ 0, P = 1)
A_1	0.0	0.968	0.967	0.984	1.001	0.629	1.002	0.697	1.000	0.730	1.000	0.793
	0.3	0.964	0.964	0.980	1.004	0.664	1.004	0.717	1.000	0.731	1.001	0.793
	0.5	0.960	0.961	0.976	1.001	0.701	0.998	0.736	1.000	0.731	1.001	0.793
	0.7	0.950	0.952	0.965	0.990	0.763	0.980	0.770	0.999	0.731	1.000	0.793
	0.9	0.901	0.906	0.916	0.980	0.896	0.950	0.859	0.995	0.730	0.994	0.791
Missing	10 & 10	0.959	0.960	0.975	1.000	0.695	0.997	0.779	0.999	0.741	1.001	0.793
	35 & 35	0.959	0.959	0.974	0.995	0.722	0.986	0.732	0.999	0.732	1.000	0.815
S	1	0.958	0.958	0.972	1.012	0.756	0.980	0.703	1.000	0.740	0.998	0.802
Q	2	0.971	0.968	0.982	1.077	0.710	1.052	0.777	0.991	0.736	0.990	0.804
	6	0.671	0.671	0.678	1.004	0.996	1.000	0.972	1.002	0.790	1.001	0.845
v_t	AR(2)	0.959	0.960	0.981	1.003	0.534	0.989	0.574	1.000	0.612	0.999	0.692
	MA(1)	0.959	0.959	0.968	0.996	0.857	0.995	0.896	1.000	0.876	1.001	0.943
δ	0.5	0.958	0.959	0.974	0.998	0.696	0.989	0.735	0.999	0.730	1.000	0.796
ρ	0.5	0.957	0.958	0.967	0.996	0.818	0.993	0.852	0.999	0.838	1.000	0.873
S	1	0.674	0.680	0.679	0.993	1.003	1.089	1.086	0.996	0.824	1.000	0.880
Q = 4	$\gamma_n = 0.1$	0.379	0.369	0.461	1.007	0.878	1.006	0.882	1.000	0.748	0.999	0.809
	$\gamma_n = 0.5$	0.514	0.514	0.517	1.001	0.986	0.992	0.984	1.001	0.826	0.998	0.877
	$\gamma_n = 0.9$	0.527	0.526	0.521	0.994	0.993	0.994	0.995	0.999	0.792	0.997	0.875
Q = 6	$\gamma_n = 0.1$	0.399	0.289	0.565	1.152	0.739	1.103	0.812	1.000	0.740	0.997	0.803
	$\gamma_n = 0.5$	0.663	0.664	0.677	0.990	0.966	0.996	0.963	1.000	0.799	1.000	0.855
	$\gamma_n = 0.9$	0.691	0.691	0.688	0.994	1.022	1.005	1.008	0.995	0.816	1.002	0.882

Note: The relative Mean Squared Error (RMSE) is computed vis-à-vis the specification (B = 0, P = 0).

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