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COMBINING JUDGEMENTS WITH SAMPLE AND MODEL INFORMATION**

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December 2008

*The analyses, opinions and findings of these papers represent the views of the authors,
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On the uncertainty and risks of macroeconomic forecasts: Combining judgements with sample and model information

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Abstract

Institutions which publish macroeconomic forecasts usually do not rely on a single econometric model to mechanically generate their forecasts. The combination of judgements with information from different models complicates the problem of characterizing the predictive densities. This paper proposes a flexible (yet parametric) approach to estimate the joint and marginal densities of macroeconomic forecasting errors, combining judgements with sample and model information. We assume that the relevant variables have a multivariate normal skewed distribution, belonging to a class of distributions recently suggested by Ferreira and Steel.

Keywords: Macroeconomic forecasts, density forecasts, fan charts, multivariate skewed normal distribution.

JEL classification: C53, E37.

1 Introduction

An increasing number of institutions which publish macroeconomic forecasts on a regular basis complement the release of point forecasts with additional information on the dispersion and skewness of the forecasting errors probability distributions. The estimation of these distributions, on which indicators of uncertainty and risk of forecasts are based, is complex because usually institutions do not rely on a single econometric model to mechanically generate the forecasts. Even if more weight is attached to one preferred model by the institution's forecasters, expert judgements and, whenever possible, other existing models are also taken into account when producing the forecasts.

In this paper, we propose a flexible (yet parametric) probabilistic approach to estimate the densities of the forecasting errors. It takes into account both the "objective" information on past forecasting errors and the subjective assessments on dispersion and skewness made by the forecasters. In spirit, our approach is similar to the procedures suggested by Blix and Sellin (1998, 1999) and Novo and Pinheiro (2003), while overcoming the informality of the former and the practical limitations of the latter.

We will admit that the paths of the conditioning variables (commodity prices, external demand, exchange rates, etc.) as well as the corresponding baseline point forecasts of the variables of interest (consumer inflation, GDP growth, etc.) are pre-determined relative to the exercise of obtaining the distributions of the forecasting errors. In practise, there is some interaction between the generation of the baseline point forecasts and the assessment of the uncertainty and risks surrounding those point forecasts, but typically this interaction is limited and the exercise is conducted into two almost sequential phases. First, for given trajectories of the conditioning variables, point forecasts are produced for the variables of interest, representing the most likely outcome of these variables taking into account the econometric tools and the expertise available. Then, having agreed on the point forecasts, the forecasters turn to the assessment of the marginal densities of the forecasting errors¹.

The forecasting errors of the variables of interest can be decomposed in two parts: (i) the component that results from the deviations of the conditioning variables from the paths considered when producing the (point) baseline forecasts; (ii) the error that would remain even if no such deviations existed over the forecasting horizon. This second component, which we will refer to as the *pure forecasting error*, is mainly associated with model uncertainty, resulting from estimation and specification errors in the model(s) used to generate the baseline forecasts.

¹In section 3, we will present a set of conditions which validates this separation of the forecasting process in two sequential stages.

For the ease of exposition, we will refer to both components as *input variables*. The *output variables* will be the deviations of the forecasted variables from their baseline figures. Using the available models, the forecaster may obtain impulse responses to unit shocks in each input variable and express each output variable as a linear combination of the input variables, the coefficients being the estimated (or calibrated) interim multipliers. These linear combinations should be interpreted as local linear approximations to the underlying complex model that generated the baseline point forecasts (typically non-linear and often not fully explicit, because the baseline forecasts may have incorporated judgemental information).

We assume that the deviations from the baseline paths for the conditioning variables and the pure forecasting errors are independent, but we will admit non-null correlations between different conditioning variables as well as between different pure forecasting errors. Moreover, we assume that the input variables have a joint multivariate skewed normal distribution belonging to the class of multivariate skewed distributions suggested by Ferreira and Steel (2007a, 2007b). This class is generated by non-singular linear affine transformations of vectors of independent variables, each having an univariate skewed distribution constructed from a symmetric distribution (in our case the standard normal) using the inverse scaling factor method introduced by Fernandez and Steel (1998). If no skewness is considered, the multivariate skewed normal distribution *à la* Ferreira and Steel (henceforth *FS-MSN* distribution) simply collapses to the multivariate normal distribution. Like the latter, the *FS-MSN* distributions are closed under non-singular affine linear transformations, which is a convenient property. However, unlike the multivariate normal, the *FS-MSN* distributions are not closed under marginalization, implying that the marginal distributions have to be obtained by (standard) simulation. This is the (small) price to pay for being possible to accommodate any degree (no matter how large) of skewness. Other well known alternative classes of multivariate skewed distributions available in the literature, like the multivariate skew-normal discussed by Azzalini and Dalla Valle (1996) and Azzalini and Capitanio (1999), as well as the multivariate skew-t distribution developed by Azzalini and Capitanio (2003)², also reduce to the multivariate normal in special cases and are closed under linear transformations and under marginalization. However, they can only accommodate limited degrees of skewness, making them unsuitable for our purposes.

The paper is organized as follows. In section 2, the literature more closely related to our

²The multivariate skew-t distributions constitute an interesting subclass of the *skew-elliptical* distributions (Azzalini and Capitanio (1999) and Branco and Dey (2001), among others). For an extensive bibliography on the (univariate and multivariate) skew-normal, skew-t and related distributions, see <http://azzalini.stat.unipd.it/SN/list-publ.pdf>.

approach is briefly reviewed. In section 3, the notation and the analytical framework are presented. In section 4, the subclass of *FS-MSN* distributions is introduced and its main properties are summarized. The proposed procedure is discussed in section 5. An empirical illustration is presented in section 6. Finally, the last section concludes.

2 Related research

Since the mid 90's, graphical representations of the estimated probability distributions of future inflation and GDP growth outcomes have been published by the *Bank of England* in its quarterly *Inflation Reports*. These density forecasts, known as *fan charts*, are the graphical display of nested prediction intervals. The publication of fan charts by the *Bank of England* was followed soon by other central banks, which developed probabilistic methods adapted to their specific needs and institutional settings. Blix and Sellin (1998, 1999) and Novo and Pinheiro (2003) were the main sources of inspiration for the analytical framework used in this paper.

Regarding the distributional assumption, Blix and Sellin assume that the input variables have marginal *two-piece normal* (*tpn*) distributions³. The corresponding three parameter probability density function (pdf) is formed by taking two halves of normal pdfs with the same mean and possibly different standard deviations. When the "right-side standard deviation" and "left-side standard deviation" are different the distribution is skewed, while when they are identical the *tpn* collapses to the normal distribution. Instead, the procedure proposed by Novo and Pinheiro makes use of (also three parameter) *skewed generalized normal* (*sgn*) distributions⁴, which are generated by linear combinations of two independent variables: a standard normal and a variable with distribution transformed from an exponential distribution by applying a location shift intended to reduce its mean to zero. When the coefficient of the exponential variable in that linear combination is positive (negative) the distribution is right-skewed (left-skewed), while when it is null the distribution collapses to the normal. The parameters defining the marginal distributions of the input variables, either *tpn* or *sgn*, are determined resorting to sample information (namely the errors observed in previous similar forecasting exercises) as well as to judgemental information. The latter is used to set the degree of skewness and to adjust the degree of dispersion over the forecasting horizon.

³This distribution is discussed in detail by John (1982). See also Johnson *et al.* (1994).

⁴Not to be confounded with *skew-normal* variables, which were suggested by Azzalini (1985).

The procedure suggested by Blix and Sellin to derive the output variables marginal distributions from the distributions of the input variables is somewhat informal. Although the linear combination of *tpn* variables is not *tpn*, and in general the mode of the distribution of a linear combination of skewed variables is not the linear combination (with the same coefficients) of the modes in the marginal distributions, both approximations are used for simplicity. Moreover, possible correlations between the input variables are overlooked. Blix and Sellin (1998), consider a non-diagonal covariance matrix, allowing for non-null correlations between the input variables. However, in their analysis these correlations are ignored when determining the skewness of output variables marginal distributions⁵. The *sgn* distribution considered by Novo and Pinheiro, unlike the *tpn*, is closed under linear combinations, i.e. linear combinations of *sgn* variables are *sgn* distributed under certain conditions on the degree of skewness and on the correlations between the variables. This feature allowed Novo and Pinheiro to suggest a more formalized aggregation procedure. However, the conditions under which the closure of *sgn* distributions to linear combinations is verified are quite narrow, implying that in most cases, when using the Novo and Pinheiro procedure, one must resort to a *sgn* approximation of the true distribution.

Cogley *et al.* (2005) suggested an alternative method to estimate the forecasting errors densities, still allowing for judgemental information to be taken into consideration. In a first stage, their method consists of generating the densities of the forecasting errors from a Bayesian vector autoregression (with drifting conditional mean parameters and stochastic volatility of the innovations) and, in a second stage, modify those densities by incorporating additional information (from other available models or judgements). They use the relative entropy method of Roberston *et al.* (2005) to "twist" their first stage densities generated from the *BVAR*, constraining the twisted densities to satisfy a vector of moment conditions on the output variables that contain the relevant additional information. This method does not suffer from the aforementioned shortcomings of the Blix and Sellin and Novo and Pinheiro methods, but raises two different practical difficulties. For most institutions that publish forecasts, the first-stage densities derived from the vector autoregression may represent a rather poor approximation to the generating process of the point baseline forecasts. Moreover, converting the relevant additional information (obtained from econometric models and judgements) into a vector of moment constraints is also problematic. If the judgements refer to the behavior of the input variables over the forecasting horizon, as normally they do, it is not straightforward to convert them into restrictions on the moments of the output variables.

⁵Blix and Sellin (2000) consider a bivariate case which overcomes this simplification but cannot be easily extended to higher dimensions.

More recently, Knuppel and Todter (2007) proposed a non-parametric approach to generate the densities, based on an asymmetric bootstrapping procedure applied to estimated innovations of an augmented version of the main macroeconomic model used by the *Bundesbank*. This approach, like the Cogley *et al.* approach, does not require the specification of a parametric class of skewed multivariate distributions for the input variables⁶. It is based on the past estimated innovations of an augmented version of the main macroeconomic model used to generate the point forecasts. The model is augmented by adding two sets of equations. First, the exogenous variables of the model are "endogeneized" by specifying them as autoregressions, possibly augmented with lags of other exogenous variables. Second, the residuals of the equations of the original endogenous variables are also modelled as autoregressions⁷. The resulting estimated innovations of both types of additional equations are then used in the bootstrapping procedure. The main problem with this approach is that the estimated innovations do not characterize adequately the uncertainty associated with the point baseline forecasts. In particular, the estimated innovations obtained from the autoregressions specified for the exogenous variables may have a rather weak correlation with the corresponding deviations of the exogenous variables from the conditioning paths admitted to generate the baseline point forecasts published in the past. In addition, the estimated innovations obtained from the autoregressive processes considered for the residuals of the model structural equations will also tend to underestimate the model uncertainty.

3 Notation and analytical framework

Let us consider an institution that regularly produces macroeconomic forecasts of N variables for horizons up to H periods. Let $h = 0$ be the last period for which the relevant variables are observed and $h = 1, 2, \dots, H$ be the forecasting periods. We denote

$$y_h = [y_{h,1} \ \cdots \ y_{h,n} \ \cdots \ y_{h,N}]'$$

the N -dimensional vector of output variables at horizon h , i.e. the vector of deviations of the endogenous variables from the point baseline forecasts at horizon h . The baseline point

⁶In their paper, Knuppel and Todter also mention an alternative parametric approach, but with insufficient detail on its practical implementation, in particular when the input variables are correlated. Moreover, the empirical section of the paper focus exclusively on the non-parametric approach, which clearly appears as their preferred approach.

⁷Knuppel and Todter motivate the need to consider the second set of autoregressions with different sample periods considered to estimate the model and to implement the bootstrapping procedure.

forecasts are conditional on given paths for M conditioning variables. The deviations from the paths assumed for conditioning variables at horizon h are

$$z_h = [z_{h,1} \cdots z_{h,m} \cdots z_{h,M}]'$$

Also let

$$u_h = [u_{h,1} \cdots u_{h,n} \cdots u_{h,N}]'$$

be the N -dimensional vector of pure forecasting errors at horizon h , defined as the component of the output variables which would exist even if (z_1, z_2, \dots, z_h) were all null vectors. To better understand the concept, if the point baseline forecasts were generated by a single macroeconomic model with N equations (ordered as in y_h), without including any judgements, u_h would simply be the vector of residuals at period h of the model structural equations.

The vectors y , z and u are defined by stacking y_h , z_h and u_h for $h = 1, \dots, H$:

$$y = [y'_1 \cdots y'_h \cdots y'_H]'$$

$$z = [z'_1 \cdots z'_h \cdots z'_H]'$$

and

$$u = [u'_1 \cdots u'_h \cdots u'_H]'$$

Using the available macroeconometric models and some expert judgements, we admit that forecasters can produce H matrices C_τ ($N \times M$) of estimated dynamic responses of the endogenous variables to unitary shocks in the conditioning variables (interim multipliers). The element (n, m) of C_τ represents the response of the n -th endogenous variable at period $t + \tau$ to a unit shock in the m -th conditioning variable occurred in period t ($\tau = 0, \dots, H - 1$). Similarly, we admit that forecasters can come up with a set of matrices B_τ ($N \times N$) of estimated dynamic responses of endogenous variables to unitary shocks in their structural innovations. The element (n, k) of B_τ represents the response at period $t + \tau$ of the n -th endogenous variable to a unit shock occurred at period t in the structural innovation of the k -th endogenous variable.

Matrices C_τ and B_τ can be envisaged as the coefficients of a linear approximation (around the baseline) of the underlying data generating process of the output variables:

$$y_h = \sum_{t=1}^h (C_{h-t} z_t + B_{h-t} u_t) \quad (h = 1, \dots, H) \quad (1)$$

In a more compact notation,

$$y = Cz + Bu \quad (2)$$

where

$$C = \begin{bmatrix} C_0 & 0 & \cdots & 0 \\ C_1 & C_0 & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ C_{H-1} & C_{H-2} & \cdots & C_0 \end{bmatrix} \quad (HN \times HM)$$

and

$$B = \begin{bmatrix} B_0 & 0 & \cdots & 0 \\ B_1 & B_0 & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ B_{H-1} & B_{H-2} & \cdots & B_0 \end{bmatrix} \quad (HN \times HN)$$

We assume that B_0 is non-singular. This implies that B is also non-singular and that, given data from previous similar forecasting exercises, the dynamic equations (1) can be used to compute estimates of the past pure forecasting errors (and, therefore, to evaluate their historical dispersions and correlations):

$$\tilde{u}_1 = B_0^{-1} (\tilde{y}_1 - C_0 \tilde{z}_1) \quad (3)$$

and

$$\tilde{u}_h = B_0^{-1} \left[(\tilde{y}_h - C_0 \tilde{z}_h) - \sum_{t=1}^{h-1} (C_{h-t} \tilde{z}_t + B_{h-t} \tilde{u}_t) \right] \quad (h = 2, \dots, H) \quad (4)$$

where \tilde{z}_h denotes the observed deviations at horizon h ($h = 1, 2, \dots, H$) of the conditioning variables from the corresponding paths in past similar exercises and \tilde{y}_h are the associated observed output variables in those exercises. Note that using (3)-(4) to estimate the past pure forecasting errors ensures that the model estimation and misspecification errors are both taken into account.

We assume that z and u are independent and that the "block-marginal" pdfs $f_z(z)$ and $f_u(u)$ are unimodal. Using (2), it is straightforward to obtain the joint pdf of (z, y) :

$$f_{z,y}(z, y) = |\det(B_0)|^{-H} f_z(z) f_u(B^{-1}(y - Cz))$$

and the conditional density of y given $z = 0$:

$$f_{y|z=0}(y) = \frac{f_{z,y}(0, y)}{f_z(0)} = |\det(B_0)|^{-H} f_u(B^{-1}y)$$

Both densities $f_{z,y}(z, y)$ and $f_{y|z=0}(y)$ are also unimodal. Moreover, if we interpret the baseline point forecasts as the most likely outcomes conditional on the admitted paths of the conditioning variables, the conditional mode of y given $z = 0$ is the null vector ($\mu(y|z = 0) =$

0)⁸. But from the expression of $f_{y|z=0}(y)$, it becomes obvious that $\mu(y|z=0) = 0$ if and only if the most likely outcome for the vector of pure forecasting errors over the forecasting horizon is also the null vector (i.e. $\mu(u) = 0$). Otherwise, it would be possible to improve the baseline point forecasts, in order to keep them as the most likely outcomes. Thus, the conditions:

- (i) B_0 is non-singular,
- (ii) the densities of z and u are unimodal,
- (iii) z and u are independent, and
- (iv) $\mu(u) = 0$,

are sufficient to ensure that the baseline point forecasts remain unchanged ($\mu(y|z=0) = 0$), irrespective of the dispersion, correlation and shape of the distributions considered for the input variables. In other words, the above stated conditions are sufficient to legitimate the separation of the forecasting exercise in two sequential phases: (1) the production of a set of baseline point forecasts; (2) the assessment of the uncertainty and risks of the forecasts, without impacting on the previously set baseline.

Notice that conditions (i) to (iv) do not restrain $\mu(z)$, the mode of the deviations of the conditioning variables from the paths considered when generating the baseline point forecasts. For some conditioning variables, the assumed paths over the forecasting horizon naturally correspond to the most likely scenario for those variables. However, for other conditioning variables, the chosen paths may represent some "technical assumptions". For instance, it may happen that some policy measures will likely be implemented over the forecasting horizon, but the forecasters are bound to produce a baseline point forecast that only takes into account the measures already approved and in place. Although not incorporated in the baseline point forecast, the most likely paths for these variables have to be taken into account when assessing the uncertainty of the forecasts. In our framework, while continuing to interpret $z = 0$ as associated with the paths for the conditioning variables conditional on which the baseline point forecasts were produced, one may be forced to consider a non-null mode of z (i. e. $\mu(z) \neq 0$).

4 The *FS-MSN* distributions

Our goal is to determine the joint density $f_{z,y}(z, y)$ and the corresponding marginal densities of $z_{h,m}$ and $y_{h,n}$ ($m = 1, \dots, M; n = 1, \dots, N; h = 1, \dots, H$). To simplify the solution, the

⁸ $\mu(v)$ and $\mu(v|w = w_0)$ denote the joint mode of vector v and the mode of v conditional on $w = w_0$, respectively.

distributions of z and u are restricted to belong to the class of multivariate skewed distributions suggested by Ferreira and Steel (2007a, 2007b). In order to ensure that distributions collapse to the multivariate normal when no skewness is considered, we admit that distributions of z and u belong to the subclass of *FS-MSN* distributions (i.e. multivariate skewed normal distributions *à la* Ferreira and Steel). This class is generated by non-singular linear affine transformations of vectors of independent variables, each having a basic univariate skewed normal distribution constructed from the $N(0, 1)$ distribution using the inverse scaling factor method introduced by Fernandez and Steel (1998). The subclass of *FS-MSN* distributions is very flexible, accommodating any degrees of skewness and being unimodal and very parsimonious in the number of parameters. Furthermore, its parameters have a straightforward interpretation.

A scalar random variable ε_j is distributed according to a basic Fernandez and Steel univariate skewed normal distribution (basic univariate *FS-SN* distribution) if its pdf is the following:

$$f_{\varepsilon_j}(\varepsilon_j; \gamma_j) = \begin{cases} 2 \left(\gamma_j + \gamma_j^{-1} \right)^{-1} \varphi(\gamma_j \varepsilon_j) & (\varepsilon_j \leq 0) \\ 2 \left(\gamma_j + \gamma_j^{-1} \right)^{-1} \varphi(\gamma_j^{-1} \varepsilon_j) & (\varepsilon_j > 0) \end{cases} \quad (5)$$

where $\varphi()$ denotes the $N(0; 1)$ pdf and $\gamma_j (> 0)$ is the shape parameter. When $\gamma_j = 1$, the density collapses to $\varphi(\varepsilon_j)$. Values of γ_j above (below) unity correspond to densities skewed to the right (left). From the expression above, one can easily confirm that

$$f_{\varepsilon_j}(\varepsilon_j; \gamma_j) = f_{\varepsilon_j}(-\varepsilon_j; \gamma_j^{-1})$$

so that inverting γ_j produces the mirror image of the density around zero. Therefore, irrespective of γ_j , the basic univariate *FS-SN* distribution is unimodal with mode zero.

The cumulative distribution function can easily be derived from (5):

$$F_{\varepsilon_j}(\varepsilon_j; \gamma_j) = \begin{cases} 2 \left(1 + \gamma_j^2 \right)^{-1} \Phi(\gamma_j \varepsilon_j) & (\varepsilon_j \leq 0) \\ 2 \gamma_j^2 \left(1 + \gamma_j^2 \right)^{-1} \Phi(\gamma_j^{-1} \varepsilon_j) + \left(1 - \gamma_j^2 \right) \left(1 + \gamma_j^2 \right)^{-1} & (\varepsilon_j > 0) \end{cases} \quad (6)$$

where $\Phi()$ is the $N(0; 1)$ cumulative distribution function.

For any positive integer r , the moment of order r of ε_j exists finite:

$$E(\varepsilon_j^r) = \Theta_r \frac{\gamma_j^{r+1} + (-1)^r \gamma_j^{-(r+1)}}{\gamma_j + \gamma_j^{-1}} \quad \text{with} \quad \Theta_r = 2 \int_0^\infty \eta^r \varphi(\eta) d\eta$$

In particular,

$$E(\varepsilon_j) = \sqrt{\frac{2}{\pi}} \left(\gamma_j - \gamma_j^{-1} \right) \quad \text{and} \quad V(\varepsilon_j) = \left(1 - \frac{2}{\pi} \right) \left(\gamma_j^2 + \gamma_j^{-2} \right) + \frac{4 - \pi}{\pi}$$

Skewness, measured as one minus twice the probability mass to the left of the mode (as proposed by Arnold and Groeneveld (1995)), is simply

$$Sk(\varepsilon_j) = 1 - \frac{2}{1 + \gamma_j^2}$$

This measure takes values in the range $[-1, 1]$ and is equal to zero when the distribution is symmetric. It is positive (negative) when the distribution is right (left) skewed. In the case of the basic univariate *FS-SN*, the whole range of possible values can be covered. When γ_j tends to zero (infinity) the distribution approaches the left (right) half-normal distribution.

The joint density of a J -dimensional random vector $\varepsilon = [\varepsilon_1 \cdots \varepsilon_j \cdots \varepsilon_J]'$ of independent components such that each ε_j has a basic univariate *FS-SN* distribution with parameter γ_j is given by

$$f_\varepsilon(\varepsilon; \gamma) = \prod_{j=1}^J f_{\varepsilon_j}(\varepsilon_j; \gamma_j) \quad (7)$$

where each $f_{\varepsilon_j}(\varepsilon_j; \gamma_j)$ is as in (5) and $\gamma = [\gamma_1 \cdots \gamma_j \cdots \gamma_J]'$. The random vector $x = [x_1 \cdots x_j \cdots x_J]'$ is said to follow a *FS-MSN* distribution if there are two $(J \times 1)$ vectors of constants μ and γ , and a non-singular $(J \times J)$ matrix A such that

$$x = \mu + A\varepsilon$$

where ε has density (7). Vector x has a *FS-MSN* distribution with parameters μ , γ and A , and its joint pdf is simply

$$f_x(x; \mu, A, \gamma) = |\det(A)|^{-1} f_\varepsilon(A^{-1}(x - \mu); \gamma)$$

Notice that, by construction, if A is block diagonal

$$A = \begin{bmatrix} A_I & 0 \\ 0 & A_{II} \end{bmatrix}$$

with A_I and A_{II} non-singular matrices $(J_I \times J_I)$ and $(J_{II} \times J_{II})$, respectively, and if x is partitioned accordingly, then x_I and x_{II} are independent. It is also straightforward to confirm that the *FS-MSN* distributions are unimodal with joint mode $\mu(x) = \mu$, irrespective of A or γ .

The class of *FS-MSN* is closed under linear affine non-singular transformations. Indeed, if

$$s = \beta + Rx = \beta + R\mu + RA\varepsilon$$

where β is a $(J \times 1)$ vector of constants and R is a $(J \times J)$ non-singular matrix of constants, then

$$s \sim FS - MSN(\beta + R\mu; RA; \gamma)$$

If $x \sim FS-MSN(\mu; A; \gamma)$, then $E\left(\prod_{j=1}^J x_j^{r_j}\right)$ exists finite for any vector of non-negative integers $[r_1, \dots, r_j, \dots, r_J]'$ (see Ferreira and Steel (2007b)). In particular,

$$E(x) = \mu + \sqrt{\frac{2}{\pi}} A \left(\gamma - \begin{bmatrix} \gamma_1^{-1} & \dots & \gamma_j^{-1} & \dots & \gamma_J^{-1} \end{bmatrix}' \right)$$

The variance-covariance matrix of x is

$$V(x) = AD(\gamma)A' \quad (8)$$

where $D(\gamma)$ is the (positive definite) diagonal variance-covariance matrix of ε :

$$D(\gamma) = \text{diag}_j \left(\left(1 - \frac{2}{\pi} \right) \left(\gamma_j^2 + \gamma_j^{-2} \right) + \frac{4 - \pi}{\pi} \right)$$

The ability to cover all possible mean, non-degenerate covariance and skewness structures, together with the closure under non-singular linear transformations and the unimodality (with joint mode equal to one of the vectors of parameters), are the main attractive features of the *FS-MSN* class of distributions. However, there are also two drawbacks. First, this class is not closed under marginalization, i.e. if $x \sim FS-MSN(\mu; A; \gamma)$, in general the marginal densities of x_j ($j = 1, \dots, J$) are not of the same type and their expressions are rather cumbersome to derive analytically, even in the simplest case of $J = 2$. This problem to obtain the marginal distributions may be easily solved by resorting to standard simulation.

The second drawback is that the *FS-MSN* class of distributions, although parsimonious in the number of parameters, is not uniquely parameterized when setting the joint mode μ , the variance-covariance matrix $V(x)$ and the vector of shape parameters γ .⁹ Indeed, if $x \sim FS-MSN(\mu; A; \gamma)$, by fixing $\mu = \bar{\mu}$, $\gamma = \bar{\gamma}$ and the symmetric matrix $V(x) = \bar{V}$, the latter $J(J+1)/2$ restrictions are not enough to uniquely determine A , which has J^2 elements (and is only restricted to be non-singular). Additional $J(J-1)/2$ restrictions are required in order to obtain a unique parameterization of the *FS-MSN* distribution. Note that

$$Cov(x, \varepsilon) = E\{[x - E(x)][\varepsilon - E(\varepsilon)]'\} = AV(\varepsilon) = AD(\gamma)$$

Thus

$$A = Cov(x, \varepsilon) D^{-1}(\gamma) \quad (9)$$

If the variables x are observable and if ε is interpreted as a vector of independent structural sources of dispersion and skewness, the extra $J(J-1)/2$ restrictions may be specified by imposing *a priori* reasonable nullity restrictions on the covariances between the x_j and the ε_i

⁹This issue is related to the non-sphericity of this class of distributions. For a discussion, see Ferreira and Steel (2007a).

$(i, j = 1, \dots, J)$, which by (9) translates into imposing nullity restrictions on the corresponding elements of A . Equivalently, one can define an ordering of the variables in x and restrict A to be lower triangular¹⁰. The ordering of the variables associated with A being lower triangular implies a recursive propagation of the structural sources of dispersion and skewness that greatly simplifies the determination of the joint distribution of the input variables.

5 The aggregation procedure

From (2), (z, y) may be expressed as a non-singular linear transformation of the input variables (z, u) :

$$\begin{bmatrix} z \\ y \end{bmatrix} = \begin{bmatrix} I & 0 \\ C & B \end{bmatrix} \begin{bmatrix} z \\ u \end{bmatrix}$$

The matrices of interim multipliers C and B can be estimated/calibrated using the macro-econometric models available and some expert judgements, and B is admitted non-singular, thus rendering the above transformation non-singular. Assuming additionally that both z and u follow *FS-MSN* distributions, then from the properties stated in the previous section, the joint distribution (z, y) will also belong to the same class.

Let

$$z \sim FS - MSN(\mu(z); A(z); \gamma(z))$$

and (taking into account that, by construction, the mode of u is the null vector)

$$u \sim FS - MSN(0; A(u); \gamma(u))$$

By assumption z and u are independent. Thus

$$\begin{bmatrix} z \\ u \end{bmatrix} \sim FS - MSN \left(\begin{bmatrix} \mu(z) \\ 0 \end{bmatrix}; \begin{bmatrix} A(z) & 0 \\ 0 & A(u) \end{bmatrix}; \begin{bmatrix} \gamma(z) \\ \gamma(u) \end{bmatrix} \right)$$

and, hence

$$\begin{bmatrix} z \\ y \end{bmatrix} \sim FS - MSN \left(\begin{bmatrix} \mu(z) \\ C\mu(z) \end{bmatrix}; \begin{bmatrix} A(z) & 0 \\ CA(z) & BA(u) \end{bmatrix}; \begin{bmatrix} \gamma(z) \\ \gamma(u) \end{bmatrix} \right)$$

Therefore, in this framework, obtaining the joint distribution of (z, y) requires determining (besides the matrices of multipliers C and B):

¹⁰Ferreira and Steel (2007a, 2007b) discuss the additional restrictions needed to render the parametrization unique. In general, one may specify A as the product of non-singular lower triangular matrix and any given orthogonal matrix.

- the (non-singular) matrices $A(z)$ and $A(u)$;
- the vector of joint modes of z , $\mu(z)$; and
- the shape vectors $\gamma(z)$ and $\gamma(u)$.

Regarding the matrices $A(z)$ and $A(u)$, recall that the forecaster needs to specify recursive covariance structures between z and u , on the one hand, and the latent structural sources of dispersion and skewness ε , on the other hand. In other words, as remarked at the end of the previous section, the components of vector z , as well as the components of vector u , should be ordered and *a priori* restrictions should be imposed such that $A(z)$ and $A(u)$ become lower triangular matrices (reflecting the judgement of the forecaster on the propagation of the structural shocks affecting the economy). For different orderings, the resulting joint density of (z, y) will not be exactly the same and, consequently, the marginal densities will also change. However, as we will illustrate in the next section, in general the empirical results are not very sensitive to different orderings, meaning that the assessment on the dispersion and skewness of the forecasting errors is quite robust to the choice of the identification of the structural shocks affecting the economy.

After having specified $A(z)$ and $A(u)$ as lower triangular matrices, in order to determine these matrices, the next step consists of computing the Cholesky matrices associated with the variance-covariances matrices $V(z)$ and $V(u)$, i.e. the (non-singular) lower triangular matrices $K(z)$ and $K(u)$ such that

$$V(z) = K(z)K(z)' \quad \text{and} \quad V(u) = K(u)K(u)'$$

From (8),

$$A(z) = K(z)D^{-1/2}(\gamma(z)) \quad \text{and} \quad A(u) = K(u)D^{-1/2}(\gamma(u)) \quad (10)$$

which shows that, given $V(z)$ and $V(u)$ (and thus $K(z)$ and $K(u)$), the matrices $A(z)$ and $A(u)$ can be expressed as functions of the vectors of shape parameters $\gamma(z)$ and $\gamma(u)$, respectively. Note that because $K(z)$ is lower triangular and $D(\gamma(z))$ is diagonal, the element $(1, 1)$ of $A(z)$ depends only on the first element of $\gamma(z)$, the non-null elements of the second row of $A(z)$ depend on the two first elements of $\gamma(z)$, and so on. A similar relationship exists between the elements of $A(u)$ and $\gamma(u)$.

The variances-covariances $V(z)$ and $V(u)$ may be obtained from a two-steps procedure. Firstly, the "historical" covariance matrices are computed from the input variables observed in past similar forecasting exercises, $\tilde{V}(\tilde{z})$ and $\tilde{V}(\tilde{u})$. If the number of past similar exercises is not enough to generate non-singular matrices, some form of censoring is required to ensure that the matrices become non-singular¹¹. In a second step, the forecaster may introduce some

¹¹One way to achieve non-singularity is to impose a (small) lower bound on the eigenvalues of the variance-

judgement on how different will be the dispersions of the input variables over the forecasting horizon as compared with the observed past dispersions. This can be achieved by pre- and post-multiplying the (possibly censored) historical variance-covariance matrices by diagonal adjustment matrices:

$$V(z) = S(z)\tilde{V}(\tilde{z})S(z) \quad \text{and} \quad V(u) = S(u)\tilde{V}(\tilde{u})S(u)$$

where

$$S(z) = \underset{m,h}{diag}(s_{h,m}(z)) \quad \text{and} \quad S(u) = \underset{n,h}{diag}(s_{h,n}(u))$$

$s_{h,m}(z)$ and $s_{h,n}(u)$ being the judgmental adjustment factors to the historical standard deviations of $z_{h,m}$ and $u_{h,n}$, respectively. When $S(z) = I$ and $S(u) = I$, no judgemental based adjustments to the historical variances-covariances are admitted. But when there is the perception by the forecaster that the uncertainty on the future developments of some input variable differs significantly from the past behavior, an adjustment factor higher or lower than one should be considered.

As regards $\mu(z)$, the joint mode of z , it must be null when all the paths for the conditioning variables take their (joint) most likely values. As already mentioned, non-zero values may be required when the baseline paths of the conditioning variables are chosen according to some technical assumptions. Whenever there are conditioning variables subject to technical assumptions that do not correspond approximately to their most likely outcomes, the best procedure is to run an alternative scenario using some policy reaction functions for those variables (therefore endogeneizing them), while maintaining the same paths for the other conditioning variables. The difference between the (endogenous) path so obtained for the problematic variables and the path implied by technical assumptions should be reflected in the mode.

Having chosen $\mu(z)$ and expressed $A(z)$ and $A(u)$ as recursive functions of the elements of $\gamma(z)$ and $\gamma(u)$, it remains to determine the latter vectors of shape parameters. Following the traditional approach, the forecaster should consider vectors $p(z)$ ($HM \times 1$) and $p(u)$ ($HN \times 1$) of "joint mode quantiles", with generic elements defined as $p_{h,m}(z) = P(z_{h,m} < \mu_{h,m})$ and $p_{h,n}(u) = P(u_{h,n} < 0)$ ($m = 1, \dots, M$; $n = 1, \dots, N$; $h = 1, \dots, H$). The vectors $p(z)$ and $p(u)$ can be exclusively based on judgements about the risks on the future developments of the input variables.

We will focus on the determination of $\gamma(z)$, as the procedure is perfectly similar for $\gamma(u)$. To simplify the notation, we will suppress the explicit reference to z in brackets. Let $\varepsilon \sim$

covariance matrices.

$FS - MSN(0; I; \gamma)$ be the latent vector of structural shocks behind z , such that

$$z = \mu + A\varepsilon$$

Thus (because A is lower triangular)

$$\left\{ \begin{array}{l} P(a_{1,1}\varepsilon_{1,1} < 0) = p_{1,1} \\ P(a_{2,1}\varepsilon_{1,1} + a_{2,2}\varepsilon_{1,2} < 0) = p_{1,2} \\ \vdots \\ P(a_{hm,1}\varepsilon_{1,1} + a_{hm,2}\varepsilon_{1,2} + \cdots + a_{hm,hm}\varepsilon_{h,m} < 0) = p_{h,m} \\ \vdots \\ P(a_{HM,1}\varepsilon_{1,1} + a_{HM,2}\varepsilon_{1,2} + \cdots + a_{HM,HM}\varepsilon_{H,M} < 0) = p_{H,M} \end{array} \right.$$

where $a_{i,m}$ refers to the element (i, m) of matrix A and for the vectors μ , ε and p we continue to follow the same notation convention introduced at the beginning of section 3 (for example, $p_{h,m}$ is the joint mode quantile associated with the m -th conditioning variable at horizon h). This system can be solved recursively. In the first equation, $p_{1,1}$ is set by the forecaster, $a_{1,1}$ depends only on $\gamma_{1,1}$ (given the variance-covariance matrix V) and, by construction, $\varepsilon_{1,1}$ follows a basic univariate $FS-SN$ distribution with parameter $\gamma_{1,1}$. Using (6), the first equation can be solved to determine $\gamma_{1,1}$. Given $\gamma_{1,1}$ and $p_{1,2}$, the only unknown of the second equation that remains to be determined is $\gamma_{1,2}$. By simulation, it is rather straightforward to solve the second equation and obtain $\gamma_{1,2}$. The same procedure may be applied recursively to the remaining equations, until the whole vector γ is determined.

6 Empirical illustration

In this section we illustrate the implementation of the suggested methodology using data for Portugal. The required matrices of multipliers B and C were obtained from one of the macro-econometric models in use at the *Banco de Portugal*. As regards the past forecasting errors, we considered those associated with the annual forecasts produced by *Banco de Portugal* in the context of the Spring Eurosystem forecasting exercises (from 1999 onwards). The forecasting horizon includes the current year (for which only very limited information is available in March/April when the Spring Eurosystem exercise is carried out) and one year ahead.

We do not pretend to present a realistic forecasting uncertainty and risk analysis exercise for the Portuguese economy, but simply to illustrate how the suggested methodology may be implemented, emphasizing the relevance of judgmental information in the determination of

the forecasting errors distribution. In order to keep the exercise as simple as possible, only four conditioning variables are considered (annual growth rate of oil price, annual growth rate of foreign demand, first difference of the short-term interest rate and annual growth rate of the USD/euro exchange rate¹²), while the rate of growth of GDP and the consumer inflation are the only two endogenous variables. Thus, in our example, vector z has dimension 8 (the four conditioning variables referred to the first period and same conditioning variables referred to the second period) and u has dimension 4 (the two endogenous variables referred to each period):

$$z = \begin{bmatrix} \text{oil price}_1 \\ \text{foreign demand}_1 \\ \text{interest rate}_1 \\ \text{exchange rate}_1 \\ \text{oil price}_2 \\ \text{foreign demand}_2 \\ \text{interest rate}_2 \\ \text{exchange rate}_2 \end{bmatrix} \quad \text{and} \quad u = \begin{bmatrix} \text{GDP error}_1 \\ \text{inflation error}_1 \\ \text{GDP error}_2 \\ \text{inflation error}_2 \end{bmatrix} \quad (11)$$

Before the Spring 2006 forecasting exercise, the *Eurosystem* conditioned the forecasts on the technical assumption of constant short term interest rates over the horizon. Thereafter, the assumption was modified and the conditioning path for the interest rate is derived from future markets information. In our illustration, the historical pure forecasting errors \tilde{u} were computed using (3)-(4) and the deviations observed for the conditioning variables from their assumed paths. Hence, for the interest rate, before 2006 the observed interest rates were compared with the assumed constant conditioning levels. However, it would not be realistic to base the covariance matrix $V(z)$ on past deviations of interest rates computed using an outdated technical assumption. Instead, for the purpose of obtaining $\tilde{V}(\tilde{z})$, artificial deviations were constructed for the period before 2006 in order to emulate the current technical assumption based on future markets information.

Given that the number of available past forecasting errors is too low to ensure the non-singularity of the historical covariance matrices $\tilde{V}(\tilde{z})$ and $\tilde{V}(\tilde{u})$, a lower bound of 10^{-5} was imposed on the eigenvalues of these matrices¹³. In order to illustrate that the forecasters may modify the historical variances and covariances of the forecasting errors when judgemental

¹²The exchange rate is defined here as the price of 1 dollar, such that an appreciation of the euro translates into a decrease of the exchange rate.

¹³After imposing the lower bound on the eigenvalues, the transformed matrices were rescaled (through pre and post multiplication by appropriate diagonal matrices) with the objective of keeping unchanged the diagonal elements of $\tilde{V}(\tilde{z})$ and $\tilde{V}(\tilde{u})$.

information is available on the dispersion of the errors over the horizon, we considered

$$S(z) = \text{diag}(0.35, 1, 1, 1, 0.9, 1, 1, 1)$$

and

$$S(u) = \text{diag}(1, 1.1, 1, 1.1)$$

In the case of $S(z)$, a possible rationale for the adjustment is the correction of the influence of outliers in the earlier part of the sample, because the deviations of the observed oil price from the conditioning paths admitted in the first two years were particularly high (and most likely will not be repeated in the near future). We also admitted an increase of inflation error dispersion, explained by the recent instability of food prices.

As regards the mode of z , $\mu(z)$, we opted to consider a situation in which the path for the conditioning variables does not coincide with the most likely outcomes for all the variables. Namely, we admit that the interest rate path derived from the futures market (the technical assumption currently adopted in the *Eurosystem* forecasting exercises, as mentioned above) has implicit an outlook of the world economic activity clearly more favorable than the one implicit in the conditioning path for the foreign demand of the Portuguese economy. Using a standard Taylor rule, with the short-term interest rate expressed as a function of the GDP growth and inflation forecasts, we computed the most likely evolution of the short-term interest rate conditional on the outlook for the Euro Area economy implicit in the path admitted for the foreign demand (our most likely scenario for the Euro Area economy)¹⁴. This procedure implied a short-term interest rate path lower than the one generated by the technical assumption by 25 and 100 basis points (annual averages), respectively in the first and the second years of the horizon. Thus, taking into account that interest rates are expressed as first differences, we have

$$\mu(z) = \begin{bmatrix} 0 & 0 & -0.0025 & 0 & 0 & 0 & -0.0075 & 0 \end{bmatrix}'$$

The vectors of baseline quantiles $p(z)$ and $p(u)$ are set to express the beliefs of the forecasters (or policy decision makers) regarding the skewness of the input variables marginal distributions. For a given variable, if no judgements are made on the skewness, the corresponding element of $p(z)$ or $p(u)$ is simply set to 0.5. In this illustration, we admit a negative risk on foreign demand and a risk of appreciation of the euro, implying that the corresponding baseline quantiles are above 0.5. Both risks (assumed to be more pronounced in the first period) might be justified by the assessment that the probability of a harder landing of the US

¹⁴ A similar procedure was used by Coimbra and Esteves (2004).

economy is larger than the probability of a softer landing, with the inevitable consequences on the Euro Area activity and on the exchange rate. All in all, in our illustration we consider the following vector of baseline quantiles for the conditioning variables:

$$p(z) = \begin{bmatrix} 0.50 & 0.65 & 0.50 & 0.55 & 0.50 & 0.60 & 0.50 & 0.60 \end{bmatrix}'$$

Concerning the pure forecasting errors, judgements on the skewness of the distribution should reflect the likelihood of phenomena that econometric models are not able to fully capture and may significantly affect the output variables. In this spirit, in our illustration a negative skewness (joint mode quantile of 0.60) was admitted for the pure forecasting error of GDP growth both in the first and in the second periods reflecting abnormally tighter credit conditions and a sharp decline in confidence indicators. In addition, a (less significant) negative skewness was also considered for the pure forecasting errors of consumer inflation (joint mode quantile of 0.55 in both periods), reflecting a the effect of a possible decrease of the VAT rates. Taking together the two judgements on the pure forecasting errors, we have:

$$p(u) = \begin{bmatrix} 0.60 & 0.55 & 0.60 & 0.55 \end{bmatrix}'$$

Table 1 presents some indicators of the marginal distributions of the output variables, obtained using our probabilistic method and the above described inputs¹⁵. For the figures outside brackets, the ordering of the conditioning variables and pure forecasting errors was set as in (11). In square brackets, we present the minimum and maximum values of the indicators for all the possible orderings of input variables ($48 = 4! \times 2$ orderings). A first conclusion emerging from Table 1 is that the ordering of the input variables does not matter much, because the widths of the ranges are quite narrow and the impact on the assessment of the dispersion and skewness of the forecasting errors distributions is negligible.

[Table 1 here]

Not surprisingly, because in this illustration most risks are towards a less buoyant economic activity, the balance of risks for GDP growth is clearly on the downside, with "baseline quantiles" of 0.626 and 0.563 (i.e. probabilities that it will be below the baseline point forecast), respectively in the first and in the second year of the horizon. The risks for the

¹⁵The marginal densities of the output variables were simulated generating 10000 draws from the *FS-MSN* joint distribution of the conditioning and output variables.

inflation forecast are also on the downside, although less than for GDP growth, with baseline quantiles of 0.534 and 0.563.

[Figure 1 here]

Figure 1 shows the estimated marginal distributions of the forecasting errors of GDP growth and inflation. Because the marginal distributions functions were computed by simulation, the lines in Chart 1 are kernel density estimates¹⁶. Besides the clear downside risks for GDP, these densities emphasize the high level of uncertainty surrounding the forecasts, which tend to increase over the horizon. Considering for example 80 per cent prediction intervals, the deviations of GDP growth from the baseline are within the range $[-0.0059, 0.0031]$ in the first period and $[-0.0141, 0.0106]$ in the second year. The corresponding ranges for the deviations of consumer inflation from the baseline are $[-0.0050, 0.0041]$ and $[-0.0114, 0.0101]$.

Finally, Figure 2 presents the fan charts for GDP growth and inflation forecasting errors, here constructed as the nested prediction intervals for 20, 40, 60 and 80 per cent confidence levels¹⁷.

[Figure 2 here]

7 Conclusion

In this paper, we proposed a new method to estimate the marginal probability distributions of institutional macroeconomic forecasts. We assume that the joint distribution of the relevant variables belongs to the subclass of multivariate skewed normal distributions, contained in a broader class of distributions recently suggested by Ferreira and Steel (2007a, 2007b). This class of distributions accommodates any degree of skewness (no matter how large) and is closed

¹⁶They were computed using the function *density* as available in the software *R*, with all specifications at their default values.

¹⁷Note that, unlike in Figure 2, in the typical fan chart the baseline point forecasts are added to the upper and lower bounds of all the prediction intervals of the forecasting errors.

under non-singular linear transformations of the variables. The latter property considerably simplifies the probabilistic procedure, even taking into account that the marginal distributions have to be determined by numerical simulation (because the distributions are not closed under marginalization). The method requires the identification of a propagation structure of the sources of dispersion and skewness of forecasting errors. One possible and simple way to specify the propagation structure is to make it recursive, by explicitly defining an ordering of the conditioning and endogenous variables. However, we are convinced that the issue has little practical relevance and in most cases different orderings of variables will correspond to very similar forecasting marginal distributions.

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Table 1 – Main indicators of uncertainty and risk

	h	GDP growth		Inflation	
Baseline percentile	1	0.626	[0.613 , 0.634]	0.534	[0.527 , 0.543]
	2	0.563	[0.556 , 0.574]	0.529	[0.521 , 0.549]
Mean	1	-0.001	[-0.001 , -0.001]	0.000	[0.000 , 0.000]
	2	-0.002	[-0.002 , -0.002]	-0.001	[-0.001 , -0.001]
Standard deviation	1	0.004	[0.004 , 0.004]	0.004	[0.004 , 0.004]
	2	0.010	[0.010 , 0.010]	0.008	[0.008 , 0.008]
Skewness ^(*)	1	-0.226	[-0.278 , -0.156]	-0.054	[-0.175 , -0.002]
	2	-0.138	[-0.256 , -0.058]	-0.041	[-0.103 , 0.026]
Kurtosis ^(**)	1	0.059	[-0.097 , 0.209]	-0.003	[-0.129 , 0.061]
	2	0.022	[-0.058 , 0.147]	-0.059	[-0.084 , 0.110]

For the generic variable x_j :

$$^{(*)} \frac{E[x_j - E(x_j)]^3}{V(x_j)^{3/2}}; \quad ^{(**)} \frac{E[x_j - E(x_j)]^4}{V(x_j)^2}$$

Figure 1 - Forecasts density distributions

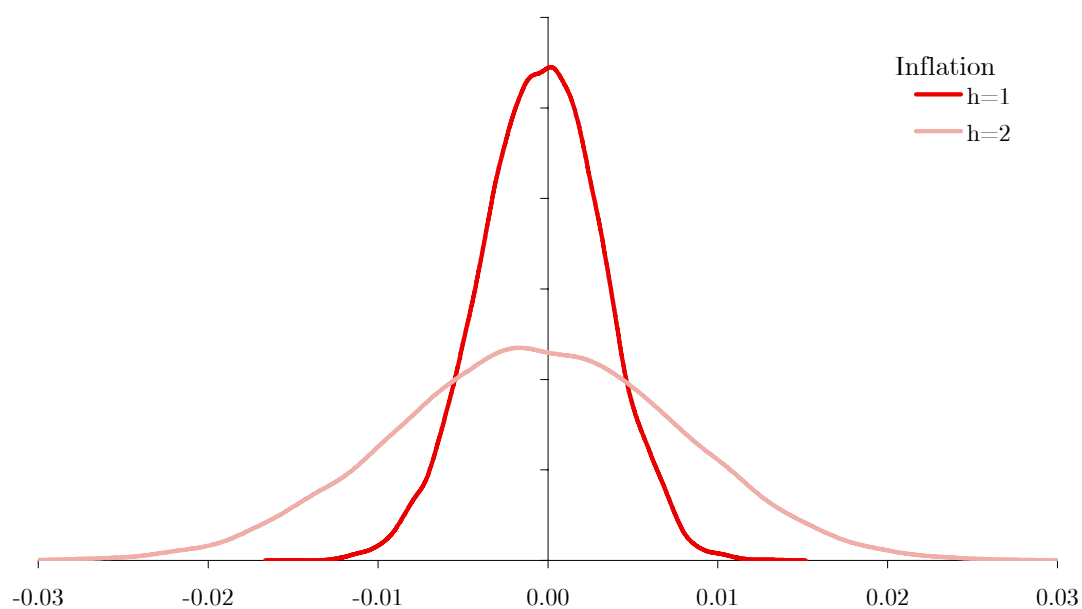
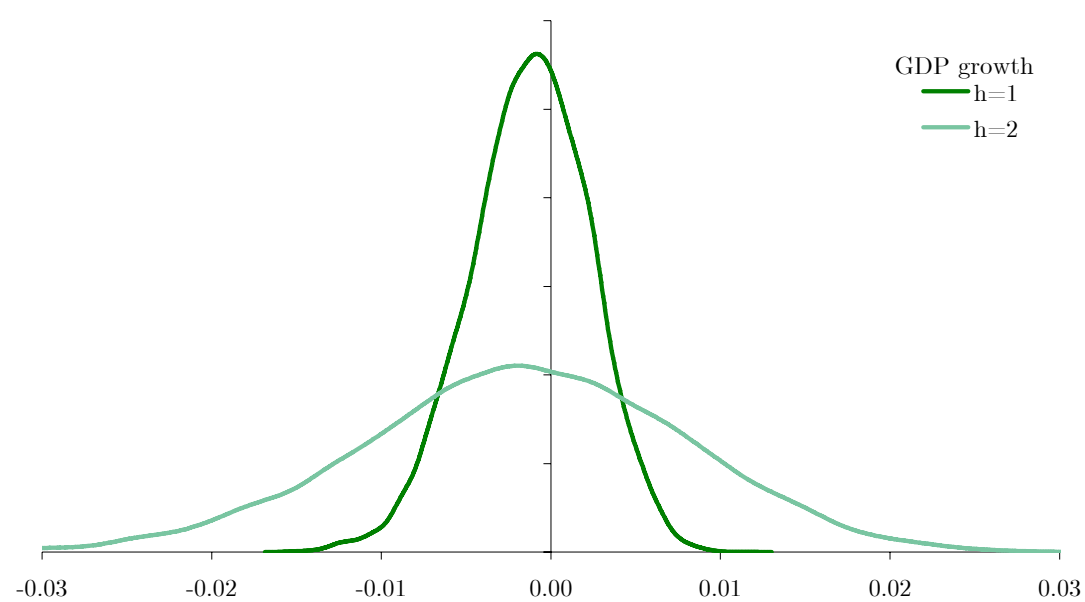


Figure 2 - Fan charts for GDP growth and inflation forecasting errors

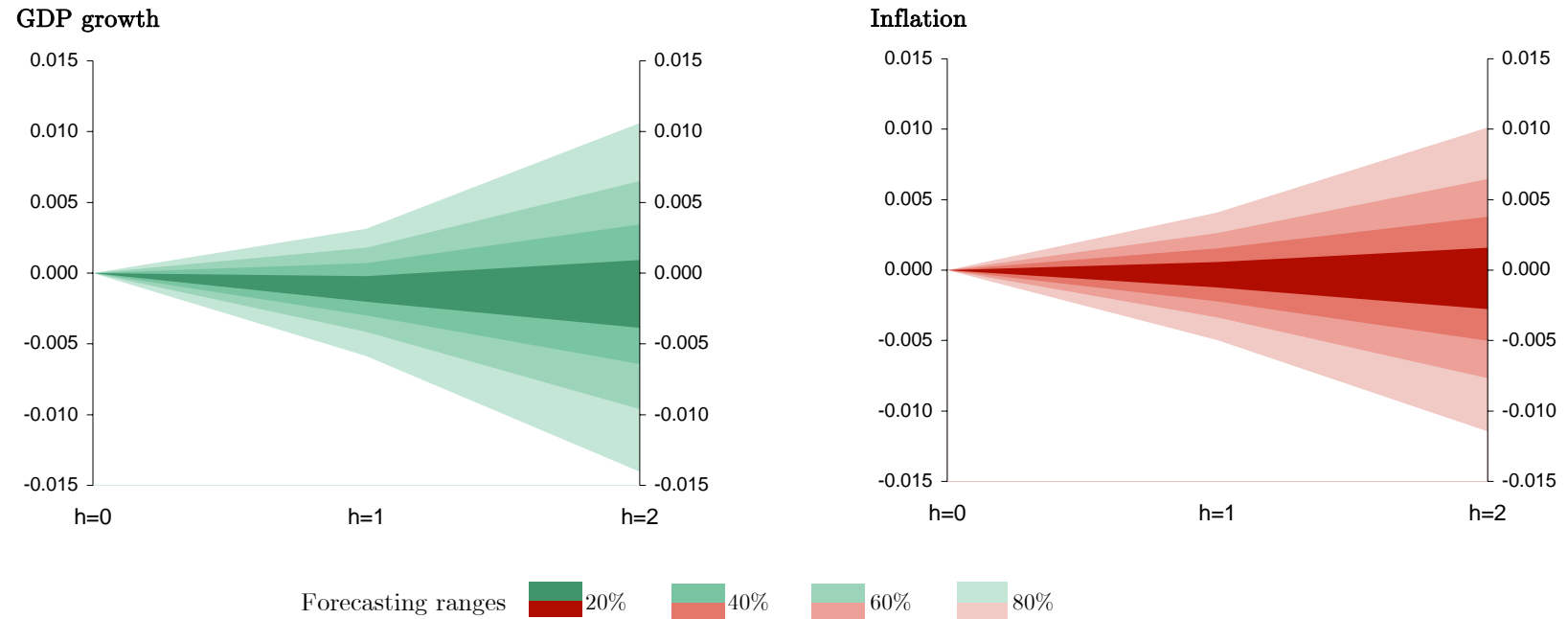
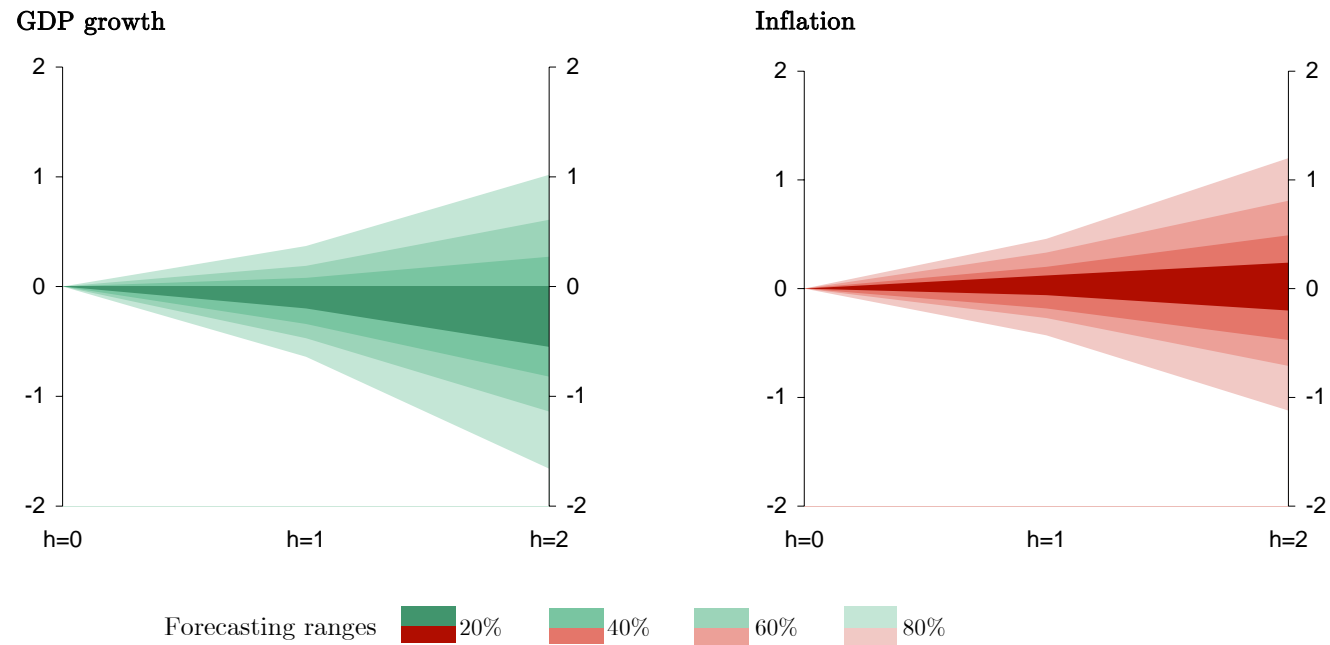


Figure 2 - Fan charts for GDP growth and inflation forecasting errors



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