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20 | 2008

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December 2008

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Printed and distributed by
Administrative Services Department
Av. Almirante Reis, 71-2nd floor
1150-012 Lisboa

Number of copies printed
170 issues

Legal Deposit no. 3664/83
ISSN 0870-0117
ISBN 978-989-8061-59-1

A Theory of Entry and Exit into Exports Markets*

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December 3, 2008

Abstract

This paper introduces persistent productivity shocks in a continuous-time mononopolistic competition model of trade with hetererogenous firms similar to Melitz (2003). In our model, the presence of sunk costs and uncertainty have three main consequences: first, firms export decisions become history-dependent. Second, the model generates firm dynamics and allows for substantial heterogeneity in export growth conditional on survival. Policy experiments modify the equilibrium along both the cross-sectional and time dimensions. Third, both the generated equilibrium firm size distribution and sales distribution of exporters into a foreign market are Pareto in the upper tail. All three consequences have been supported by empirical evidence. To solve the model we derive the stationary productivity distributions for exporters and non-exporters in general equilibrium. We point to the presence of a link between intra-industry firm heterogeneity and the degree of persistence in export status. Finally, we perform a numerical exercise to show how per-period fixed cost and up-front entry costs are differently related to persistence in export status for exporters and non-exporters.

JEL Codes: F10, L11

Keywords: Export, Hysteresis, Brownian Motion, Sunk Costs

*Acknowledgements : We would like to express our gratitude to Jonathan Eaton for his advice and constant support. We greatly benefited from discussions with Pol Antràs, Gian Luca Clementi and Costas Arkolakis and we are especially gratefull to Atilla Yilmaz who helped us dealing with stochastic processes. A previous version of this paper circulated under the title "Hysteresis in Export Markets". The analysis, opinions and findings represent the views of the authors, they are not necessarily those of the Banco de Portugal. Contact information: Alfonso Irarrazabal, alfonso.irarrazabal@econ.uio.no; Luca David Opromolla, ldopromolla@bportugal.pt.

1 Introduction

In recent years many studies have highlighted the importance of producer heterogeneity in international trade. Only the most productive firms engage in exporting activities, suggesting substantial hurdles to accessing foreign markets.[1] There are at least two alternative theories of why more productive firms enter exports markets. Bernard et al. (2003) propose a model of trade with heterogenous firms with country-wide Ricardian differences, where firms in more productive countries are better able to compete and therefore access more foreign markets. This theory does not require any fixed cost of entry into export markets. Alternatively, Melitz (2003) put forward a monopolistic competition model of trade with heterogenous firms where exporters need to pay an entry cost to access a foreign market. After these theories were laid out, more detailed firm level and transaction level datasets have been used to show new facts: Eaton et al. (2007) report that sales distributions across markets of very different size and extent of French participation behave similarly to a Pareto in the upper tail and more like a lognormal in the lower tail. Irarrazabal et al. (2008) and Amador and Opromolla (2008) find similar results for Norwegian and Portuguese exporters respectively. Looking at the dynamics of export participation, Eaton et al. (2008) for Colombia and Amador and Opromolla (2008) for Portugal find substantial heterogeneity in export growth conditional on survival: firms are very different in the way they perform after entering export market. Moreover, the industry dynamics literature has shown that firms size distribution tends to behave as a Pareto in the upper tail (Axtell (2001), Luttmer (2007)) and many trade models have subsequently assumed the equilibrium distribution to be Pareto (Chaney (forthcoming), Helman, Melitz and Yeaple (2004)).

This paper provides a new model which is consistent with many of the old and new trade facts. Our framework is similar to Melitz (2003) but we assume that firm productivities evolve following a Brownian motion. This single modification generates

[1] Among many, see for instance Bernard and Jensen (1995) and (1999).

three main consequences: first, unlike in a model without market uncertainty, the way trade entry costs are modelled affects firms decisions. When trade entry costs are sunk upon entry in the export market firms export decisions are history-dependent while when they are paid on a per-period basis they are not.[2] Second, the model generates firm dynamics: firms are created, grow or shrink in size, start or stop exporting and possibly shut down. It is possible to analyze how policy changes modify the equilibrium both along the cross-sectional and the dynamic dimensions. Third, both the equilibrium firm size distribution and the sales distribution of exporters into a foreign market are Pareto in the upper tail and increasing in the lower tail.

In our model, the presence of sunk costs and uncertainty generates new predictions in terms of the decision to stay or leave the export markets. Entrepreneurs make investments to set up firms and draw their initial labor productivity level from a common distribution. Production for the domestic market starts even if profits are negative (as long as they are not too negative) and continues until the expected net present value of current and future profits and the value of the option to exit are high enough. If firm productivity exceeds a threshold it becomes profitable to enter foreign markets by paying a sunk cost. This entry cost has to be paid every time a firm starts to export. If, later on, productivity falls below the level at which the firm had started exporting the entrepreneur prefers to keep exporting, as long as the net present value of exporting profits plus the value of the option to stop exporting is bigger than the value of the option of reentering the export market. In other words, the presence of uncertainty introduces an option value in the decision to enter or leave the export markets. Current exporters wait longer to leave the export market in order to avoid to repay the entry cost later on, even at the expense of periods of negative profits. Similarly, non-exporters wait for higher productivity levels before entering export markets. There is a range of productivity (or size) levels within which nonexporters decide not to enter export markets and exporters decide not to leave them. This is known in the irreversibility literature (Dixit (1989), Dixit and Pyndick (1994)) as the

[2] This point is stressed by Antras (2004).

band of inaction. We adapt the model of Dixit (1989), which consider the decision of entry and exit from an industry. In his model firms need to decide to enter or leave an industry when prices are stochastic. Our model retains the same qualitative characteristics of Dixit, but because we consider entry and exit decision from the export market we need to generalize Dixit's system of value matching and smooth pasting conditions to account for the aggregate changes in the demand. Sunk cost of exporting and uncertainty are present in the work of Tybout and Roberts (1997), who find evidence in favor of history dependence in export participation consistent with our formulation. More recently, Das et al. (2007) develop and estimate a dynamic structural model of export supply with plant-level heterogeneity in export profits, uncertainty and market entry costs. Our model, maintaining the same three main ingredients, embeds these models into a general equilibrium framework.

Our paper is closely related to Luttmer (2007). We extend his industry dynamics framework to allow firms to compete in international markets and we retain the prediction that firm size distribution is Pareto in the upper tail. In addition, our model implies, consistently with recent evidence (Eaton et al. (2007)), that also the distribution of sales into a foreign destination is Pareto in the upper tail and increasing in the lower tail. Arkolakis (2008) explains this fact by introducing per consumer access costs. In our model, instead, there is a measure of exporters that sell little in the destination market because they do not want to abandon it since they still hope for a surge in productivity in the future.

Another contribution of this paper is to propose a methodology to solve for the distribution by types of agents deciding to change status in a continuos time environment with uncertainty and adjustment costs. When the underlying uncertainty follow a standard Brownian motion the distribution of types overlaps along the band of inaction. Closure of the model requires the solution of a complicated system of partial differential equations for the transition probability densities. We use Laplace transform methods to solve for the distribution of exporters and non-exporters for the entire range of productivity levels. We then use this distributions to compute the aggregate price level of the economy. The method is easy to implement and it does not rely particularly on the assumptions of our model.

Other papers have analzyed related issues in a discrete time framework: Ghironi and Melitz (2005) use the Melitz framework as the microeconomic base of a DSGE model and provide a microfounded, endogenous explanation of the Harrod-Balassa-Samuleson effect. Alessandria (2007) looks at the comovement of net exports and the real exchange rate in a equilibrium business cycle model with heterogeneous firms and sunk costs of entry into export markets. Costantini and Melitz (2007) and Atkeson and Burstein (2007) focus on the joint decision of exporting and innovating.

The structure of the paper is as follows. Section 2 presents the setup of the model, solves for the stationary productivity distributions by export status and describes the equilibrium. Section 3 performs a quantitative analysis of the model focusing on persistence in export status. Finally, section 4 concludes.

2 Model

2.1 Set-up

In this section we introduce the basic ingredients of the model. We define preferences, technologies, trade costs and we characterize the value of a firm and the band of inaction.

Demand. Time is continuous, starts at $t = 0$ and goes on forever. There are two symmetric countries, each populated by a measure L of infinitely-lived agents.[3] Consumers in each country maximize utility derived from the consumption of goods from one sector. The sector is made of a continuum of differentiated goods. There is a representative consumer with preferences over sequences of consumption of a

[3] A version of the model with multiple asymmetric countries is available upon request.

composite good C defined by:[4]

$$E\left[\int_0^\infty e^{-\rho t} U(C) dt\right]$$

where $U(.)$ is the period utility function. The discount rate ρ is positive and the representative consumer chooses to consume $c(\omega)$ units of each variety ω, for all varieties in the set Ω (determined in equilibrium), to minimize the cost of acquiring the composite good C,

$$C = \left(\int_\Omega c(\omega)^{\frac{\sigma-1}{\sigma}} d\omega\right)^{\frac{\sigma}{\sigma-1}},$$

where $\sigma > 1$ is the elasticity of substitution between any two varieties.

Each consumer is endowed, at every point in time, with one unit of labor which is supplied inelastically to firms. The wage rate w is normalized to one and used as *numéraire*. The representative consumer faces a standard present-value budget constraint. Her wealth consists in labor income plus dividends. Each worker owns a single share of a perfectly diversified national portfolio of all the firms and profits earned by firms are distributed as dividends in terms of the *numéraire*. Given the prices set by firms, the representative consumer chooses how to allocate her budget across all varieties. Since each variety enters symmetrically in the utility function, differences in demand across varieties depend only on differences in their prices $p(\omega)$,

$$c(\omega) = C\left[\frac{p(\omega)}{P}\right]^{-\sigma} \tag{1}$$

where $c(\omega)$ represents the units demanded of variety ω. The consumption-based price index P, to be defined later, is the minimum expenditure required to purchase one unit of the composite good.

Trade barriers and technology. There are three types of trade barriers: a variable cost τ, a per period cost λ_X and an up-front cost λ_H.[5] The variable cost takes the form of an "iceberg cost": $\tau > 1$ units of the good must be shipped in order for one

[4] Since we focus on the analysis of the steady state and we do not introduce aggregate growth in the model, we drop the time subscript whenever possible in order to simplify the notation.

[5] See Das et al. (2007) for some examples of sunk start-up costs and per-period fixed costs.

unit of the good to arrive in the other country. The cost λ_X has to be paid every period by an exporting firm. The cost λ_H has to be paid up-front every time a firm starts (or restarts) to export. Both the per period cost and the up-front cost are expressed in units of the *numéraire*. These two trade costs play different roles in our model as it will become clear soon.

Both countries have access to the same technology. Goods are produced using only labor and firms must sustain a fixed cost of production. Creating a firm requires sustaining a sunk cost λ_E. Afterwards, the firm draws an initial random unit log labor productivity $\bar{z}$ from a distribution $g(z)$.[6] For expositional purposes, we present the model under the assumption that all new firms enter with the same productivity level $\bar{z}$. This implies that we assume $g(z)$ to be the Dirac-Delta function $\delta(z - \bar{z})$. Before turning to the numerical section of the paper we allow $g(z)$ to be any function and we show how this affects the closure of the model. The cost of producing and selling c units of the good in the domestic market for a firm with productivity z is $ce^{-z} + \lambda_D$ while the cost of producing and selling c units of the good in the foreign market is $\tau ce^{-z} + \lambda_X$. Firms are price setters. The optimal price in the foreign market, set as a markup on variable cost of production is $\tau e^{-z}\sigma/(\sigma - 1)$ where $\sigma/(\sigma - 1)$ is the Dixit-Stiglitz markup.

Productivity shocks and value of the firm Following Luttmer (2007), we assume that firm labor productivity evolves identically and independently according to a Brownian motion with drift α and $\bar{z}$ as initial condition,

$$dz = \alpha da + \xi dB \tag{2}$$

where dB is the increment of a Wiener process and $\xi > 0$ is the diffusion parameter. These permanent idiosyncratic shocks can be interpreted as shocks to technology (producing the same variety at a lower cost), shocks to quality (producing a better variety at the same cost) or as taste shocks to the demand for the firm differentiated

[6] In the remainder of the paper, for simplicity, we refer to z, and not to $\exp(z)$, as the firm labor productivity.

good.

As a result of these shocks, firms prices, labor demand, revenue, profits, export participation evolve over time. The value of the firm is a function both of current labor productivity z and of its export participation status. For expositional simplicity, we divide the value of a firm into a domestic and a foreign component. Like in Melitz (2003) we assume that the least productive firms do not export. Moreover, since the presence of a fixed cost implies a minimum size, firms with low productivity choose to exit since they face only a small probability of ever recovering the fixed cost λ_D required to continue the firm. The domestic component of the value of a firm is described by the Bellman equation

$$V_D(z) = \max\left\{0, \pi_D(z) + e^{-(\rho+\delta)dt} E\left[V_D(z+dz)\right]\right\}$$

where δ is an exogenous, per unit of time, probability of exiting and $\pi_D(z)$ are profits from sales on the domestic market. Domestic profits are a function of the firm labor productivity z and of the endogenously determined price index P,

$$\pi_D(z) = R\left(Pe^z\right)^{\sigma-1}(\sigma-1)^{\sigma-1}\sigma^{-\sigma} - \lambda_D$$

where $R = PC$ is total expenditure. The termination payoff is set to zero. There exists a single cutoff z_D such that for $z > z_D$ continuation (of the firm) is optimal and for $z < z_D$ it is optimal to shut down.[7] Notice that in this model there are two reasons why firms can exit: because of a negative productivity shock or because of the killing rate δ. Bigger firms are less likely to exit because of a negative shock and more likely to exit because of the killing rate δ. In order to solve the Bellman equation, we need to apply Ito's Lemma to find the expected continuation value $E\left[V_D(z+dz)\right]$. It turns out that the domestic value of the firm, in the continuation region $z > z_D$, is the solution of the second-order differential equation

$$(r+\delta)V_D(z) = \frac{1}{2}\xi^2 V_D''(z) + \alpha V_D'(z) + \pi_D(z) \tag{3}$$

[7] For this to be true we need: (1) $\pi(z) + e^{-(\rho+\delta)dt}E\left[V(z+dz)\right]$ to be increasing in z (2) first-order stochastic dominance (which is satisfied by the Brownian motion).

where the right-hand side is the expected total return per unit of time from keeping the firm open. In order to solve jointly for $V_D(z)$ and the cutoff z_D we need the value-matching and smooth-pasting conditions

$$V_D(z_D) = 0 \text{ and } V_D'(z_D) = 0. \tag{4}$$

The domestic value function, valid on the range (z_D, ∞),

$$V_D(z) = \kappa_2 e^{\beta_2 z} + \frac{\pi_D(z)/\sigma}{(r+\delta) - \alpha(\sigma-1) + \xi^2(\sigma-1)^2/2} - \frac{\lambda_D}{(r+\delta)}$$

is the sum of the value of the option to exit (first term) and the expected present discounted value of domestic profits (second and third terms).[8] Variable domestic profits (at the numerator of the second term) evolve as a geometric Brownian motion with drift $\alpha(\sigma-1) + \xi^2(\sigma-1)^2/2$ and are discounted at the rate $r+\delta$. The present value of revenues, and therefore of variable profits, is finite if the combined discount factor, given by the sum of the interest rate and the exogenous probability of exit δ, is bigger than the drift of variable profits. The following assumption guarantees that this is the case and therefore guarantees that the value of a firm is finite:

$$\rho + \delta > \alpha(\sigma-1) + \xi^2(\sigma-1)^2/2.$$

Since $V_D(z)$ is increasing in z, firms with higher productivity, with respect to the lower barrier z_D, are more valuable. The minimum size z_D,

$$e^{z_D} = \frac{\sigma}{\sigma-1}\left(\frac{\lambda_D \sigma}{R}\right)^{\frac{1}{\sigma-1}} \frac{\gamma_d}{P}$$

is endogenously determined, being a function of the price index P and total expenditure R. The parameter γ_d is a function of parameters governing the stochastic process for productivity shocks, time discounting and preferences.[9] Economies with a lower price index P and lower total expenditure R are economies where firms have to be more productive in order to survive.

[8] Formal derivations of all the value functions and cutoffs, as well as definitions of the coefficients β_2 and κ_2 are relegated to the Appendix.

[9] Specifically γ_d is: $\left[\frac{\beta_2}{r+\delta}\left(\frac{r+\delta-\alpha(\sigma-1)-1/2\xi^2(\sigma-1)^2}{\beta_2-\sigma+1}\right)\right]^{1/(\sigma-1)}$.

All non-exporters have a chance of becoming exporters (the likelihood of this event being increasing in the distance from the lower barrier z_D) and some firms do actually export. The presence of a sunk cost of entry and re-entry into export markets creates a wedge between the productivity level at which firms decide to start exporting (z_H) and the one at which firms decide to stop exporting (z_L). In the range of productivity levels between the thresholds z_L and z_H, the optimal policy is to continue with the status quo, whether it be exporting or non-exporting. The interval (z_L, z_H) is therefore a band of inaction that is endogenously determined in the model. The value of the firm has a foreign component which depends on the firm productivity z and its current export participation status. As explained by Dixit and Pindyck (1994) (in a more general context), exporting "is really a composite asset, part of which is an option to abandon. If that option is exercised, the firm (...) acquires another asset, namely the option to invest", that is, start exporting. We denote with $V_L(z)$ the value of the "option to invest" and with $V_H(z)$ the value of being an exporter, including the value of the "option to abandon" export markets. The two foreign components must be determined simultaneously. They are described by two Bellman equations,

$$\begin{aligned} V_L(z) &= \max\left\{V_H(z) - \lambda_H, e^{-(\rho+\delta)dt} E\left[V_L(z+dz)\right]\right\} \text{ and} && (5) \\ V_H(z) &= \max\left\{V_L(z), \pi_X(z) + e^{-(\rho+\delta)dt} E\left[V_H(z+dz)\right]\right\} && (6) \end{aligned}$$

where $\pi_X(z)$ are profits from exporting,

$$\pi_X(z) = \tau^{1-\sigma} R \left(P e^z\right)^{\sigma-1} (\sigma-1)^{\sigma-1} \sigma^{-\sigma} - \lambda_X. \tag{7}$$

Notice that the main difference between λ_H and λ_X is that, under the former, the decision of stopping to export today affects the payoff from exporting tomorrow. Both the entry cost λ_H and uncertainty about future productivity z are necessary for the presence of a band of inaction. Nonexporters are continuously comparing the value of becoming exporters $V_H(z) - \lambda_H$ with the value of choosing the status quo $V_L(z)$. At $z = z_H$, $V_H(z) - \lambda_H = V_L(z)$ and firms are indifferent between exporting and not exporting. Instead, if z is slightly above z_H firms strictly prefers to be exporters. If, as a consequence of negative shocks, z falls below z_H firms do not stop exporting (even if current export profits are negative) because the state is different and the payoffs to be compared are different: the value of stopping to export is still $V_L(z)$ but the value of choosing the status quo is $V_H(z)$. At $z = z_H$, for example, firms are not indifferent anymore since $V_H(z_H) > V_L(z_H)$. When λ_H is replaced by λ_X instead, the value of becoming an exporter, being always $V_H(z)$, does not depend on the firm current export status. In a model without sunk trade entry costs (i.e. $\lambda_H = 0$) and with a positive per period cost λ_X the cutoffs z_L and z_H would coincide and have a closed form solution equal to $\tau \left(\lambda_X/\lambda_D\right)^{1/(\sigma-1)} \gamma_d^{-1} e^{z_D}$. Firms would start and stop exporting at this unique productivity threshold.

Cutoffs and band of inaction

The termination payoff of a non-exporter is the value of an exporter with the same productivity level, $V_H(z)$, minus the sunk cost λ_H. Likewise, the termination payoff of an exporter is the value of a non-exporter with the same productivity level, $V_L(z)$. The value functions $V_L(z)$, $V_H(z)$ and the cutoffs z_L and z_H are the solutions of two, linked, second-order differential equations subject to the following value-matching and a smooth-pasting conditions,

$$\begin{aligned} V_H(z_H) - \lambda_H &= V_L(z_H) \\ V_L(z_L) &= V_H(z_L) \\ V_H'(z_H) &= V_L'(z_H) \\ V_L'(z_L) &= V_H'(z_L). \end{aligned} \tag{8}$$

The value functions $V_L(z)$, valid on the range (z_D, z_H), and given by

$$V_L(z) = \kappa_l e^{\beta_1 z} \tag{9}$$

is the value of the option to start exporting. It is increasing in z since more productive firms gain from entering the export market. The value function $V_H(z)$, valid on the range (z_L, ∞),

$$V_H(z) = \kappa_h e^{\beta_2 z} - \frac{\pi_X(z)/\sigma}{\frac{1}{2}\xi^2(\sigma-1)^2 + \alpha(\sigma-1) - (r+\delta)} - \frac{\lambda_X}{(r+\delta)} \tag{10}$$

is the sum of the value of the option to stop exporting (first term) and the expected present discounted value of export profits (second and third terms).[10] More productive firms gain higher profits on the foreign markets (higher second term) but the value of the option to stop exporting is lower because of the small likelihood of this event (lower first term). The lower and higher exports cutoffs z_L and z_H are endogenously determined (depending on the price index P and total expenditure R through the lower barrier z_D) but do not have a straightforward closed form solution. To summarize, the value of a firm with current productivity z is $V_D(z) + V_L(z)$ if the firm does not export and $V_D(z) + V_H(z)$ if the firms exports.

Free entry and the cutoffs We assume that at every point in time there is an unbounded pool of prospective entrants into the economy. Successful entrants (with initial productivity $\bar{z}$ larger than z_D) can be new exporters or new non-exporters. Assuming the latter, that is $z_D < \bar{z} < z_H$, in equilibrium the expected value of entering has to equal the sunk cost of creating a firm,

$$\lambda_E = V_D(\bar{z}) + V_L(\bar{z}). \tag{11}$$

The free entry condition and the system (8) determine the three cutoffs z_D, z_L and z_H and the two positive constants κ_l and κ_h in the value functions $V_L(z)$ and $V_H(z)$. Up to now it was not necessary to disentangle the price index P from total expenditure R. All the quantities that we have determined were function of P and R through z_D. In order to determine the price index (and to be able to close the model and derive welfare implications) we need to derive the stationary distributions of productivity, the measure of active firms and the measure of exporters.

Due to high non-linearity in the system, it is difficult to obtain an analytical solution for the thresholds. A partial characterization is possible. First, the thresholds

[10] Notice that κ_h is a coefficient to be determined in the hysteresis system.

satisfy $0 < z_L < z_H < \infty$ and the coefficients κ_l and κ_h are positive. Second, suppose that the firm is not an exporter and that it believes that z will persist unchanged forever. The firm will decide to become an exporter if $\pi_X(z) \geq (r+\delta)\lambda_H$. This is the exporting cutoff when there is no uncertainty and z is constant over time. In our case instead, $\pi_X(z_H) > (r+\delta)\lambda_H > 0$ which means that z_H, the exporting cutoff, is larger than the productivity level at which the firm decides to become an exporter when there is no uncertainty and z is constant over time.[11] When domestic producers take into account the uncertainty over future profits, they are more reluctant to become exporters. Similarly, exporters are also more reluctant to abandon foreign markets.

Figure (1) shows some sample paths for firms' productivity and export status: in case 1, a firm starts as a non-exporter but as productivity increases the firm becomes, at age a_h, an exporter. Case 2 portraits the evolution of a firm that exports from the very beginning but is on a decreasing productivity path and eventually stops exporting at age a_l. Finally, in case 3, an initially non-exporting firm starts exporting when productivity overtakes z_H at age a_h but then receives some bad shocks and stop exporting at age a_l.

2.2 Stationary Distributions

In this section, we characterize the stationary probability density over productivity and age and we show how this density can be decomposed into a density for exporters and a density for nonexporters. The crucial ingredients are the transition densities generated by the Brownian motion (2) subject to the productivity cutoffs z_D, z_L and z_H.

Stationary probability densities In steady state there is a time-invariant cross-sectional distribution of firm current productivity z and age a,

$$f(a,z) = e^{-\delta a} h(a,z) \frac{M_A}{M} \tag{12}$$

[11] This is the case in Melitz (2003).

Firms attempt to enter at a constant rate M_A/M where M is the equilibrium measure of active firms and M_A is the equilibrium measure of attempting entrants. After getting their initial productivity $\bar{z}$, they receive a sequence of permanent shocks. The transition density $h(a,z)$ describes the likelihood that a surviving firm has productivity z at age a. This takes into account the possibility that the firm hit the lower barrier z_D and exit forever but not the exogenous killing rate δ which is controlled by the term $e^{-\delta a}$. Expanding the analysis of Luttmer (2007), we can decompose the overall distribution $f(a,z)$ into the weighted sum of a distribution for exporters and a distribution for nonexporters. In steady state there is a constant ratio between the measure of exporters M_X and non-exporters M_D.[12] Let $h_X(a,z)$ describes the likelihood that a surviving firm has productivity z *and is exporting* at age a. Let $h_D(a,z) = h(a,z) - h_X(a,z)$ be the likelihood that a surviving firm has productivity z *and is not exporting* at age a. The probability density $f(a,z)$ can be decomposed as

$$f(a,z) = \frac{M_X}{M} f_X(a,z) + \frac{M_D}{M} f_D(a,z) \tag{13}$$

where

$$\begin{aligned} f_X(a,z) &= e^{-\delta a} h_X(a,z) \frac{M_A}{M_X} \text{ and} \\ f_D(a,z) &= e^{-\delta a} h_D(a,z) \frac{M_A}{M_D} \end{aligned}$$

are time-invariant cross-sectional distribution for exporters and non-exporters respectively.

Transition densities and the entry rate The transition densities are the core of the stationary probability densities $f(a,z)$, $f_X(a,z)$ and $f_D(a,z)$. The transition density $h(a,z)$ is the solution of the following Kolmogorov forward equation,[13]

$$\begin{cases} \frac{\partial h(a,z)}{\partial a} = -\alpha \frac{\partial h(a,z)}{\partial z} + \frac{1}{2}\xi^2 \frac{\partial^2 h(a,z)}{\partial z^2} \quad z > z_D \\ h(a,z) = 0 \text{ for } z \leq z_D, a \geq 0 \ ; \ u(0,z) = \delta(z - \bar{z}) \end{cases} \tag{14}$$

[12] Recall that $M = M_X + M_D$.

[13] Formal solutions of all the Kolmogorov equations are relegated to the Appendix.

where $\delta(.)$ is the Dirac-Delta function. The above Kolmogorov equation describes a stationary (over time) probability density function $h(a,z)$. Notice that the density $h(a,z)$ is a function of the current productivity level z and current age of the firm a. We are assuming that this density is the same for different cohorts. Let $J(a,z)$ be the net rate of passage or flux (as a increases infinitesimally) at z (in the z direction) when age is equal to a so that $\partial h(a,z)/\partial a = -\partial J(a,z)/\partial z$. When the derivative of the flux is positive $h(a,z)$ is decreasing in a because, as a increases infinitesimally, the probability of leaving z is higher than the probability of reaching it.

The transition densities $h_X(a,z)$ and $h_D(a,z)$ are the solutions of two similar but linked Kolmogorov forward equation,

$$\begin{cases} \frac{\partial h_X(a,z)}{\partial a} = -\alpha \frac{\partial h_X(a,z)}{\partial z} + \frac{1}{2}\xi^2 \frac{\partial^2 h_X(a,z)}{\partial z^2} + J_D(a, z_H^-)\delta(z - z_H) & z > z_L \\ h_X(a,z) = 0 \text{ for } z \leq z_L \text{ }, a \geq 0 \text{ ; } v(0,z) = \delta(z - \bar{z}) \end{cases} \tag{15}$$

and

$$\begin{cases} \frac{\partial h_D(a,z)}{\partial a} = -\alpha \frac{\partial h_D(a,z)}{\partial z} + \frac{1}{2}\xi^2 \frac{\partial^2 h_D(a,z)}{\partial z^2} - J_X(a, z_L^+)\delta(z - z_L) & z_D < z < z_H \\ h_D(a,z) = 0 \text{ for } z \geq z_H \vee z \leq z_D \text{ }, a \geq 0 \text{ ; } w(0,z) = \delta(z - \bar{z}). \end{cases}$$

where $J_X(a,z)$ and $J_D(a,z)$ are the flux rates corresponding to the X and D processes. The two equations have to be solved simultaneously because non-exporters become exporters at z_H and exporters become non-exporters at z_L. The derivative of the transition density with respect to a, when calculated at the relevant export cutoff, depends on the flux of the other density as well: it takes into account the change of export participation status when a non-exporter passes z_H from below or when an exporter passes z_L from above.[14]

The potential entry rate M_A/M,

$$\frac{M_A}{M} = \frac{\delta}{1 - e^{-\theta_*(\bar{z} - z_D)}} \tag{16}$$

is consistent with $f(a,z)$ being a probability density. The higher is the killing rate δ and the lower is the initial productivity $\bar{z}$, the higher is the exit rate and therefore the entry rate. The equilibrium ratio between the measure of exporters and the measure

[14] See the appendix for a discussion of the flux terms and the intuition behind it.

of active firms, M_X/M, is constant and consistent with $f_X(a,z)$ being a probability density.

Solving a system of partial differential equations

We use Laplace transform to convert the system of partial differential equations (15) with z and a as independent variables into a system of ordinary differential equations with only z as independent variable. We find the solution of the transformed system of ordinary differential equations, $\hat{h}_X(z)$ and $\hat{h}_D(z)$ using Green's functions and show that $\hat{h}_X(z)$ and $\hat{h}_D(z)$ are functions of the Laplace transformed solution for (14), $\hat{h}(z)$ and its derivatives. We derive analytically $h(a,z)$, the inverse Laplace transform of $\hat{h}(z)$, while we use numerical methods to solve for $h_X(a,z)$ and $h_D(a,z)$, the inverse Laplace transforms of $\hat{h}_X(z)$ and $\hat{h}_D(z)$. In the appendix we show the details of the solution method. After solving for the transition densities, the cutoffs, the entry rate and the ratio between the measure of exporters and active firms we can derive the stationary densities $f(a,z)$, $f_X(a,z)$ and $f_D(a,z)$.

Stationary productivity densities and volatility The stationary probability densities $f(a,z)$, $f_X(a,z)$ and $f_D(a,z)$ describe the equilibrium mass of firms in terms of current productivity, age and export status. The marginal probability densities $f(z)$ and $f_X(z)$ are needed to derive the distributions of prices of domestically produced and imported varieties and therefore to compute the price index P. From (13) we know that $f(z)$ is a weighted average of $f_X(z)$ and $f_D(z)$,

$$f(z) = \frac{M_X}{M} f_X(z) + \frac{M_D}{M} f_D(z).$$

Like in Luttmer (2007), the marginal density $f(z)$ is increasing in the lower tail and of the Pareto form in the upper tail,

$$f(z) = K_1 e^{-\theta(z-z_D)} \tag{17}$$

where K is endogenous but does not depend on z.[15] In order to have a stationary distribution with a finite mean we need to impose the following assumption,

[15] The proof is similar to the one provided in Luttmer (2007) and is available upon request.

$$\delta > \alpha + \xi^2/2.$$

The coefficient of the Pareto,

$$\theta = \frac{1}{\xi^2}\left(-\alpha + \sqrt{\alpha^2 + 2\xi^2\delta}\right) > 0$$

shows that the density has a thicker right-tail the higher is the volatility coefficient ξ of the Brownian motion. For high productivity levels, the exporters productivity density $f_X(z)$ inherits the shape of the overall productivity density $f(z)$. An example of $f(z)$ and $f_X(z)$ is given in Figure (2). Notice that, $f_X(z)$ reaches a peak in correspondence of z_H, the productivity cutoff at which firms become exporters.[16] The productivity distribution for exporters $f_X(z)$ is consistent with evidence (see Eaton et al. (2007), Irarrazabal et al. (2008) and Amador and Opromolla (2008)) that the distribution of sales of exporters in the destination market is Pareto in the upper tail and resembles a lognormal in the lower tail. Arkolakis (2006) explains this fact by introducing an increasing marginal cost to access additional consumers. In our model, instead, there is a measure of exporters that sell little in the destination market (and some of them make negative profits) because they do not want to abandon it since they still hope for a surge in productivity in the future.

For numerical exercises, it is more convenient to express the size distribution of firms in terms of sales or employees. Using the demand equation it turns out that the sales (r) density is still of the Pareto form in the upper tail but it has a different coefficient that includes the elasticity of substitution σ,

$$p_u(r) = K_2 r^{-\theta/(\sigma-1)}. \tag{18}$$

Similarly, the upper tail of the distribution of sales within a foreign destination (r_x) is of the form

$$p_v(r_x) = K_3 r_x^{-\theta/(\sigma-1)}. \tag{19}$$

[16] This occurs as long as new firms enter below z_H.

An important characteristic of our model is that the volatility coefficient ξ of the Brownian motions affects both the shape of the productivity distribution and the width of the band of inaction (z_L, z_H). There is a link between firm level heterogeneity and the degree of persistence into export status: more heterogeneity in the firm size distribution (a higher ξ and therefore a higher θ) is generally associated with a wider band of inaction.

2.3 Trade equilibrium

We close the model through labor market clearing and the derivation of the price index P. Before turning to the quantitative section of the paper, we generalize the model to allow for a generic initial productivity distribution $g(z)$.

Labor market and the price index Labor market equilibrium determines the equilibrium measure of active firms M. Labor supply is fixed. Labor is demanded for creating firms (L_E), sustaining the fixed costs (L_F), sustaining the variable production costs (L_P) and entering into export markets (L_H),

$$L_E = \lambda_E M_A \tag{20}$$

$$L_F = \lambda_D M + \lambda_X M_X$$

$$L_P = RP^{\sigma-1}\left(\frac{\sigma-1}{\sigma}\right)^{\sigma}\left[\int_{z_D}^{\infty} e^{(\sigma-1)z} f(z)dz + \int_{z_D}^{\infty} e^{(\sigma-1)z}\tau^{-\sigma} f_X(z)dz\right]$$

$$L_H = \lambda_H M_D \int_{z_D}^{z_H}\left[1-\Phi_{0,dt}\left(\frac{z_H - z - \alpha dt}{\xi}\right)\right] f_D(z)e^{-\delta dt}dz$$

The last two equations deserve some explanation. For any firm, variable costs of production for domestic sales are a fraction $(\sigma-1)/\sigma$ of the domestic revenues of the firm while variable costs associated to exports are a fraction $(\sigma-1)/\sigma\tau$ of the corresponding revenues. Aggregating, total labor demand for production is a weighted sum of total expenditure on domestic varieties and total expenditure on imported varieties. The sunk cost of entering into export markets are sustained by non-exporting incumbents that start exporting because they pass the z_H threshold.[17] From labor

[17] $\Phi_{0,dt}(.)$ is the distribution function of a Normal with mean zero and standard deviation dt.

market clearing we derive the equilibrium measure of active firms M. In turn, this allow us to determine the equilibrium price index,

$$P^{1-\sigma} = M\int_{z_D}^{\infty}\left[\frac{\sigma}{(\sigma-1)e^z}\right]^{1-\sigma} f(z)dz + M_X\int_{z_L}^{\infty}\left[\frac{\tau\sigma}{(\sigma-1)e^z}\right]^{1-\sigma} f_X(z)dz. \qquad (21)$$

The price index is the minimum expenditure required to purchase one unit of the composite good. As such, it depends on the measure of available varieties in the economy (M domestic plus M_X imported) and on their average price.

The level of total expenditure R is found replacing P in the expression for the lower threshold z_D. The equilibrium level of the composite good C and the equilibrium level of profits Π are then determined as $C = R/P$ and $\Pi = R - L$.

Generalizing the model

In the next section we compute the equilibrium of the model assuming $g(\bar{z})$ to be a Normal density with mean p_1 and standard deviation p_2. Allowing the initial productivity density $g(\bar{z})$ to be a general function does not complicate substantially the model presented above but it is obviously worth when comparing the model with the data. Note that attempts to create new firms can be unsuccessful, with probability $G(z_D)$, while successful entrants start exporting immediately with probability $[1-G(z_H)]\,/\,[1-G(z_D)]$. The free entry condition takes this into account,

$$\lambda_E = \int_{z_D}^{z_H}\left[V_D(\bar{z}) + V_L(\bar{z})\right]dG(\bar{z}) + \int_{z_H}^{+\infty}\left[V_D(\bar{z}) + V_H(\bar{z})\right]dG(\bar{z})$$

and the equilibrium entry rate as well

$$\frac{M_A}{M} = \left[\int_{z_D}^{\infty}\left[\frac{1-e^{-\theta_*(\bar{z}-z_D)}}{\delta}\right]g(\bar{z})d\bar{z}\right]^{-1}.$$

Finally, labor market equilibrium requires to take into account that new firms that enter as exporters pay the sunk cost λ_H as well. This implies an additional term in the labor demand equation

$$L_H = \lambda_H\left[M_A\left(1-G(z_H)\right) + M_D\int_{z_D}^{z_H}\left(1-\Phi_{0,dt}\left(\frac{z_H - z - \alpha dt}{\xi}\right)\right)f_D(z)e^{-\delta dt}dz\right].$$

The stationary distribution $f(z)$ and $f_X(z)$ are still Pareto in the limit. Like in Luttmer (2007), if $g(\bar{z})$ is a distribution with few firms that are much larger than the exit barrier z_d then $f(z)$ will inherit the exponentially declining tail over most of the support (z_d, ∞).

3 Numerical simulations

In this section, we provide a numerical solution to the model to illustrate the behavior of endogenous variables for which closed-form analytical solutions do not exist. We choose plausible parameters that are roughly consistent with stylized facts about firm size distribution and firm dynamics for the US economy. To gain some intuition of the main forces involved in the model we start the analysis by describing how the band of inaction is determined in general equilibrium. We then use the model to examine the determinants of persistence of export status: in particular we analyze the impact of a reduction in trade entry costs under two alternative specifications of the entry cost of exporting: in the first scenario, we consider an economy where the fixed trade cost is paid on a per-period basis while in the second scenario we consider an economy where the entry cost is sunk upon entering the export markets.

Table 1 displays the main parameters used in the simulations. There are large range of values of the elasticity of substitution used in the literature. We set the elasticity of substitution σ equal to 2 as in Ruhl (2008). We set the value of the interest rate to 5% consistent with many calibration exercises for the US economy (for example, Gibson (2007)). Luttmer (2007) finds that the firm size distribution is well approximated over much of its range by a Pareto distribution with a tail index around 1.06. This corresponds to $\theta/(\sigma-1)$ in our model. Taking this into account, the variance of firm growth, pinned down through the entry rate which for the US is 11.6 percent in 2002, is .45 and the drift parameter α is -0.08. [18] The exogenous death

[18] See Luttmer (2007) for more details.

shock is set to 5% as in Constantini and Melitz (2007). We assume the distribution $g(z)$ of entrants to be normal with mean $-.1$ and standard deviation 1.6. We set the standard deviation to match the average domestic sales of entrants relative to incumbents. We choose the variable trade cost to be 1.25. We follow Atkenson and Burstein (2007) and Gibson (2007) in normalizing the fixed cost of setting-up a firm to 1, and choose the per period operation cost λ_D to be .1. Although this is not a calibration exercise, in Table 2 we compare moments from the model with actual moments computed using US data. The model delivers reasonable estimates of the number of firms engaging in export activities, especially in the second scenario. It also captures very well the overall degree of firm size heterogeneity. The model overpredicts exports as a share of total GDP. The reason is that our model generates too large firms as seen in the fraction of employment accounted by exporters.

Figure (3) depicts the stationary distribution of productivity and age by export status. Panel (a) shows the overall productivity distribution and the distributions of exporters and non-exporters. First, notice that within the band of inaction (between z_L and z_H) the distribution of exporters and non-exporters overlap. Some exporters are less productive than some non-exporters. This come to grips with plant level facts (for example Bernard et al. (2003)), an aspect that is missing in the parsimonious Melitz (2003) model. Eaton et al. (2007), introducing firm- (and market-) specific fixed cost and demand shocks into a static framework, also provide a model that is consistent with overlapping exporters and nonexporters distributions. In our model, the overlap is not due to heterogeneous fixed costs but to dynamic factors which imply that each single firm start and stop exporting at two different z levels. Panel (b) displays the distributions of age by export status. Exporters need more time to reach the exporting cutoff and therefore will be on average older.

3.1 Band of inaction

The main mechanism through which changes in parameters affect the response of the model is through the band of inaction. To gain some intuition we describe how the cutoffs are determined in the model. We study in some detail the impact of changes in the sunk cost of exporting λ_H on the steady state equilibrium values of z_D, z_L and z_H. In particular, we consider the response of the model to a 20 percent increase in the sunk cost of exporting λ_H. To analyze how the cutoffs are determined we define the function $S(z) = V_H(z) - V_L(z)$ on the interval (z_L, z_H). Figure 5 shows the change in $S(z)$ before and after the increase in λ_H.Panel (a) shows how the lower cutoffs rises and the higher cutoff falls. Notice that the fall in z_H is larger than the increase in z_L.

Effect on z_H

In order to explain changes in z_H we must consider a nonexporter facing the choice of starting to export. The nonexporter compares the sunk cost λ_H and the lost of an asset whose value is $V_L(z)$ with the benefit of acquiring a different asset whose value is $V_H(z)$. We consider how this trade-off changes for values of z around the old value of z_H: a higher λ_H means that becoming an exporter requires a higher investment but the increase in $V_H(z)-V_L(z)$ is smaller than $\Delta\lambda_H$ and, as a consequence, z_H increases. At the old value of z_H, a nonexporter prefers to keep selling its good only on the domestic market. Notice that $V_H(z) - V_L(z)$ increases both because $V_H(z)$ increases and $V_L(z)$ decreases. Since $V_L(z)$ represents the option of becoming an exporter in the future in order to explain changes in $V_L(z)$ we must consider what happens to $V_H(z)$ (and viceversa). Since we are looking at values of z close to z_H, changes in $V_H(z)$ are explained mainly by changes in the net present value (NPV) component. Equation (7) shows that profits from exporting are higher since an increase in λ_H lowers the barrier z_D (a lower z_D is in this case equivalent to higher price index P and/or higher total expenditure R, that is, profits are higher for any surviving firm). This is why the change in the NPV component of $V_H(z)$, the second and third terms of (10), is positive. Notice that the effect on the NPV component through changes in the price index is a general equilibrium effect absent in a partial equilibrium analysis where the net present value of an exporter would not be affected by changes in the sunk cost. The reduction in $V_L(z)$, the value of the option to become an exporter, is due to the fact that even though profits from exporting are higher their expected value (from the point of view of a nonexporter) is lower since it is less likely that the firm will become an exporter in the future.

Effect on z_L

In order to explain changes in z_L we must consider an exporter facing the choice of stopping to export. The exporter compares the lost of an asset whose value is $V_H(z)$ with the benefit of acquiring a different asset whose value is $V_L(z)$. We consider how this trade-off changes for values of z around z_L. First we look at changes in $V_H(z)$: the change in $V_H(z)$ is mainly driven by the change in the option value component (the NPV component is small to start with since z is low). This is negative since profits from exporting around z_L increase: the option to stop exporting values less when profits from exporting are higher. This contributes to an increase in z_L. Second we look at changes in $V_L(z)$: the change in $V_L(z)$ is negative as explained previously. This contributes to a decrease in z_L. Overall the reduction in $V_L(z)$ more than compensates the reduction in $V_H(z)$ and therefore z_L decreases.

3.2 Persistency of export status

We now use our quantitative model to show how different assumptions about the trade entry costs affect the probability that firms do not change their export status. Recently, several papers have considered quantitative dynamic models of trade with monopolistic competition using annualized cost of access to foreign markets (for example Gibson (2007)). Other papers instead (for example Das et al. (2007)) have estimated models where the cost paid by firms that want to start exporting is sunk upon entry. In our exercise, we consider two alternative scenarios. In the first scenario, we suppose that firms need to pay a per period cost to operate in foreign markets. In the second scenario, we assume instead that the entry cost is paid upfront every time firms want to enter or reenter the export markets. A reduction in the entry cost has different implications for the persistence in export status in the two economies, explained mainly by the differential response of the cutoffs z_D, z_L and z_H. We measure persistence in export status by considering the elements on the diagonal of a one-period transition matrix with three categories: exporters, non-exporters and exiting firms. We consider the probability that an exporter in period t keeps exporting in period $t+1$ (instead of turning into a non-exporter or exiting) and the probability that a non-exporter in period t keeps non-exporting in period $t+1$ (instead of turning into an exporter or exiting).

The probability of remaining an exporter is equal to the ratio between the measure of exporters that remain exporters (M_{XX}) and the original measure of exporters (M_X),

$$M_{XX}/M_X = e^{-\delta dt}\left[1 - \int_{z_L}^{\infty} \Phi_{0,1}\left(\frac{z_L - z - \alpha dt}{\xi\sqrt{dt}}\right) f_X(z)dz\right] \tag{22}$$

where $\Phi_{0,1}(.)$ is the distribution function of a standard Normal. The probability of remaining an exporter is equal to the probability of surviving the killing rate (first term outside the brackets) times the probability of receiving a shock that is not too negative, a shock that would make it unprofitable to keep exporting (terms in the brackets).[19]

Similarly, we compute the probability of remaining a nonexporter as the ratio between the measure of nonexporters that remain nonexporters (M_{DD}) and the original measure of non-exporters (M_D),

$$M_{DD}/M_D = e^{-\delta dt}\int_{z_D}^{z_H}\left[\Phi_{0,1}\left(\frac{z_H - z - \alpha dt}{\xi\sqrt{dt}}\right) - \Phi_{0,1}\left(\frac{z_D - z - \alpha dt}{\xi\sqrt{dt}}\right)\right] f_D(z)dz. \tag{23}$$

The probability of remaining an nonexporter is equal to the probability of surviving the killing rate (first term outside the brackets) times the probability of receiving a shock that is not too negative, in order not to exit, and not too positive, in order not to be profitable to start exporting (terms in the brackets).[20]

Scenario I represents an economy with no sunk cost. We set the value of λ_X to match the ratio export/output as .3. In Scenario II we set the sunk cost λ_H to match

[19] Note that the killing rate is independent from the Brownian motion shocks. Note also that the requirement that the shock is higher than $z_L - z$ both implies that these firms remain exporters and do not exit.

[20] In this case, we require the shock to be between $z_D - z$ and $z_H - z$, so that the firm does not become an exporter and does not exit either.

the same export/output ratio as in Scenario I. In Figure 6 we compare the effect on persistency in export status of a reduction in the entry cost under the two scenarios. First, we reduce λ_X by half from 1 to .5. Panel (a) plots the probability for an exporter to keep exporting in the next period and for a non-exporter to keep selling only on the domestic market in the next period for different levels of per period fixed cost λ_X. A reduction in λ_X increases persistence in export status for exporters but decreases persistence for non-exporters. The intuition for these results is the following. When the per-period trade cost λ_X decreases, exporters, conditional on their current productivity level, enjoy higher profits from their current sales and are more likely to be able to cover the fixed trade cost in the future as well. On the contrary, non-exporters are more likely to receive a positive shock to productivity that is big enough to make it profitable to start exporting. This result, is in line with previous models of trade with heterogeneity like Melitz (2003) or Chaney (forthcoming). Second, we reduce the sunk cost λ_H by half from 5 to 2.5. Panel (b) of Figure 6 shows that a reduction in the sunk costs of exporting decreases persistence in export status for both exporters and non-exporters. When the sunk trade cost decreases, uncertainty about future productivity matters less. An exporter that receives a bad shock is more likely to stop exporting. The risk of having to repay the sunk cost in the future is less important because the magnitude of the sunk cost is lower. Similarly, a non-exporter that receives a positive shock is more likely to start exporting since it is now easier to cover the additional cost with future export revenues. Recall that in our numerical exercise for each scenario we calibrate the sunk/per-period costs to match the same export/output moment. However the implications for the persistence of the export status are quantitatively and qualitatively different. Comparing panels (a) and (b) we observe that an economy with the sunk cost λ_H generates substantially more persistency both for exporters and non-exporters. Therefore, a model with uncertainty that does not take into account the option value to start and resume exporting could underestimate the level of persistence in export status.

We have shown that per-period and sunk trade costs affect differently the persistence in export status of exporters and non-exporters. Another way to characterize changes in persistence is to look at the survivor function or at the average time spent as an exporter or a nonexporter. In panels (c) and (d) of Figure (6) we depict the survival functions (where the event is a change of export status) for low and high values of λ_X and for low and high values of λ_H. The survival function is computed iterating the one-period transition probabilities. Comparing survival rates for both scenarios, we observe that survival rates are larger for both exporters and non/exporters in an economy with sunk cost. Figure (7) plots the average time spent as an exporter and as a nonexporters as a function of the magnitude of the sunk cost (in panel (a)) or the per period cost (in panel (b)).

4 Conclusions

In this paper we introduce persistence productivity shocks in a continuous-time monopolistic model of trade with heterogeneous firms. We show that the presence of sunk cost of entering the export markets and uncertainty give rise to hysteresis in export markets participation. Firms start exporting once they have achieved a certain size, but may remain into export markets even after their size has fallen below that on entry. The model steady state is characterized by a productivity distribution that is Pareto in the upper tail and increasing in the lower tail, consistently with the empirical evidence. We solve the model analytically, and provide a framework to analyze birth, growth, entry and exit into foreign markets. In the steady state firms are created, other firms are shut down and the surviving firms experience different growth dynamics and export participation patterns. However, the sales distribution of exporters in foreign market is Pareto in the upper tail as shown in recent empirical works. We show the presence of a link between intra-industry firm heterogeneity, the width of the band of inaction and persistence in export status.

We solve for the distribution by types of agents deciding to change status in an environment with uncertainty and adjustment costs. When the underlying uncertainty follows a standard Brownian motion the distribution of types overlaps within the band of inaction. This leads to a complicated system of partial differential equations for the transition probability densities. We solve the system using Laplace transform methods. This method may be extended to other setting in which researchers may need to retrieve the probability distributions of types. Finally, we simulate the model using reasonable set of parameters to explore the links between sunk trade costs, uncertainty, per-period fixed costs and persistence in export status.

Appendix

Value of the firm

The value function $V_D(z)$ and the cutoff z_D are the joint solution of the ordinary differential equation (3) subject to the value-matching and smooth-pasting conditions (4). The general solution of the ordinary differential equation is the sum of the general solution of the corresponding homogenous equation ($V_D^h(z)$) and a particular solution of the non-homogeneous equation ($V_D^p(z)$). The former is

$$V_D^h(z) = \kappa_1 e^{\beta_1 z} + \kappa_2 e^{\beta_2 z}$$

where

$$\beta_1 = \frac{-\alpha + \sqrt{\alpha^2 + 2(r+\delta)\xi^2}}{\xi^2} > 0 \text{ and}$$
$$\beta_2 = \frac{-\alpha - \sqrt{\alpha^2 + 2(r+\delta)\xi^2}}{\xi^2} < 0$$

are the roots of the associated characteristic equation. The general solution of the homogeneous equation represents the value of the option to exit.[21] Since the likelihood of abandonment in the not-too-distant future becomes extremely small as z goes to ∞, the value of the abandonment option should go to zero as z becomes very large. Hence the coefficient κ_1 corresponding to the positive root β_1 should be zero. This leaves

$$V_D^h(z) = \kappa_2 e^{\beta_2 z}$$

The particular solution of the non-homogeneous equation can be found using the "undetermined coefficients" method. When the forcing term has the form $A_d e^{(\sigma-1)z} + B_d$ the solution assumes the form $Ce^{(\sigma-1)z} + D$. This delivers

$$V_D^p(z) = -\frac{A_d}{\frac{1}{2}\xi^2(\sigma-1)^2 + \alpha(\sigma-1) - (r+\delta)} e^{(\sigma-1)z} + \frac{B_d}{(r+\delta)}.$$

[21] This is discussed in the next section of the Appendix.

Overall, the general solution of the non-homogeneous equation is

$$V_D(z) = \kappa_2 e^{\beta_2 z} - \frac{A_d}{\frac{1}{2}\xi^2(\sigma-1)^2 + \alpha(\sigma-1) - (r+\delta)} e^{(\sigma-1)z} + \frac{B_d}{(r+\delta)}.$$

The value-matching condition can be used to determine κ_2,

$$\begin{aligned} V_D(z_D) &= 0 \;\Leftrightarrow\; \kappa_2 e^{\beta_2 z_D} - \frac{A_d}{\frac{1}{2}\xi^2(\sigma-1)^2 + \alpha(\sigma-1) - (r+\delta)} e^{(\sigma-1)z_D} + \frac{B_d}{(r+\delta)} = 0 \\ \kappa_2 &= \left(\frac{A_d}{\frac{1}{2}\xi^2(\sigma-1)^2 + \alpha(\sigma-1) - (r+\delta)} e^{(\sigma-1)z_D} - \frac{B_d}{(r+\delta)} \right) e^{-\beta_2 z_D}, \end{aligned}$$

while the smooth-pasting condition can be used to determine z_D,

$$e^{z_D} = \frac{\sigma}{\sigma-1} \left(\frac{\lambda_D \sigma}{R} \right)^{\frac{1}{\sigma-1}} \frac{\gamma_d}{P}.$$

Finally we can write $V_D(z_D)$ as

$$V_D(z) = \frac{\lambda_D}{(r+\delta)} \frac{\beta_2}{\beta_2 - (\sigma-1)} \left[e^{(\sigma-1)(z-z_D)} - \frac{\beta_2 - (\sigma-1)}{\beta_2} - \frac{\sigma-1}{\beta_2} e^{\beta_2(z-z_D)} \right].$$

The value functions $V_L(z)$ and $V_H(z)$ can be derived following similar steps.[22]

Interpretation of $V_D(z)$

The value function $V_D(z)$ has two components: $V_D^h(z)$, the general solution of the homogeneous equation, representing the value of the option to exit and $V_D^p(z)$, a particular solution of the non-homogeneous equation, representing the expected present discounted value of total profits. Ito's Lemma and the stochastic process for z imply that the stochastic process for domestic variable profits $\pi_D^v(z) = R\,(Pe^z)^{\sigma-1}\,(\sigma-1)^{\sigma-1}\sigma^{-\sigma}$ is a geometric BM with drift $\left[\alpha(\sigma-1) + 1/2\xi^2(\sigma-1)^2\right]\pi_D^v(z)$ and diffusion coefficient $\xi(\sigma-1)\pi_D^v(z)$. Denoting today's variable profits by $\pi_D^v(z_a)$, the expected value and variance of variable profits a^* years from now are

$$\begin{aligned} E\left[\pi_{D,a+a^*}^v\right] &= \pi_D^v(z_a) e^{\left[\alpha(\sigma-1)+1/2\xi^2(\sigma-1)^2\right]a^*} \\ V\left[\pi_{D,a+a^*}^v\right] &= \pi_D^v(z_a) e^{2\left[\alpha(\sigma-1)+1/2\xi^2(\sigma-1)^2\right]a^*}\left(e^{\xi^2(\sigma-1)^2 a^*} - 1\right) \end{aligned}$$

[22] Derivation available upon request.

so that the expected present discounted value of variable profits over an infinite period of time is

$$E\left[\int_0^\infty \pi^v_{D,a+a^*} e^{-(r+\delta)a^*} da^*\right] = \frac{\pi^v_D(z_a)}{(r+\delta) - \alpha(\sigma-1) - 1/2\xi^2(\sigma-1)^2}$$

which represents the value of a firm without fixed costs λ_D. Since λ_D is constant over time, the expected present discounted value of total profits over some period of time is

$$\frac{\pi^v_D(z_a)}{(r+\delta) - \alpha(\sigma-1) - 1/2\xi^2(\sigma-1)^2} - \frac{\lambda_D}{(r+\delta)} = V^p_D(z)$$

so that the other component of the general solution of the ordinary differential equation, $V^h_D(z)$, represents the value of the option to exit. Notice that $V^h_D(z) > 0$ since $\kappa_2 > 0$.

Solving a system of partial differential equations

We solve the Kolmogorov equation (14) and the system of linked Kolmogorov equations (15) using Laplace transforms. Letting $\hat{h}(z) = \int_0^\infty e^{-\chi a} h(a,z) da$ be the Laplace transform[23] of $h(a,z)$, (14) can be rewritten as an ordinary differential equation

$$\begin{cases} \frac{1}{2}\xi^2 \hat{h}''(z) - \alpha \hat{h}'(z) - \chi \hat{h}(z) = -\delta(z-\bar{z}) & z > z_D \\ \hat{h}(z_D) = 0. \end{cases} \tag{24}$$

Let $m_1(z) = e^{\lambda_1 z}$ and $m_2(z) = e^{\lambda_2 z}$, with $\lambda_{1,2} = \alpha \pm \sqrt{\alpha^2/\xi^4 + 2\chi/\xi^2}$, be the solutions of the homogeneous equation with, without loss of generality, $\lambda_1 < 0$ and $\lambda_2 > 0$. Let $m_-(.)$ and $m_+(.)$ be two linear combinations of $m_1(.)$ and $m_2(.)$ such that $\lim_{z\to\infty} m_+(z) = 0$ and $m_-(z_D) = 0$. The general solution of the nonhomogeneous equation is

$$\hat{h}(z) = \Psi(z,\bar{z})\Delta(\bar{z})dy = \Upsilon(z,\bar{z})$$

[23] We suppress the χ argument in order to simplify the notation.

where

$$\Delta(y) = \left[\frac{1}{2}\xi^2(m'_-(y)m_+(y) - m_-(y)m'_+(y))\right]^{-1} \text{ and } \Psi(z,y) = \begin{cases} m_+(z)m_-(y) \text{ for } y < z \\ m_-(z)m_+(y) \text{ for } y > z \end{cases}$$

Let $m_+(z) = e^{\lambda_1 z}$ and $m_-(z) = e^{\lambda_1 z} + Be^{\lambda_2 z}$. The boundary condition $m_-(z_D) = 0$ implies $B = -e^{(\lambda_1-\lambda_2)z_D}$ so that $m_-(z) = e^{\lambda_1 z} - e^{\lambda_1 z_D}e^{\lambda_2(z-z_D)}$. Since $m_+(.)$ and $m_-(.)$ are linear combinations of the solutions of the homogeneous equation, $\Delta(y) = \frac{2e^{(\lambda_2-\lambda_1)z_D}e^{-\frac{2\alpha}{\xi^2}y}}{\xi^2(\lambda_1-\lambda_2)}$.The inverse Laplace transform of $\hat{h}(z)$ is[24]

$$u(a,z|\bar{z}) = \frac{1}{\xi\sqrt{a}}\left[\phi\left(\frac{z-\bar{z}-\alpha a}{\xi\sqrt{a}}\right) - e^{-\frac{2\alpha}{\xi^2}(\bar{z}-z_D)}\phi\left(\frac{z+\bar{z}-2z_D-\alpha a}{\xi\sqrt{a}}\right)\right]. \tag{25}$$

We now solve (15). Since $w(a,z_H) = v(a,z_L) = 0$ the flux terms simplify to $J_D(a,z_H^-) = -\frac{1}{2}\xi^2\frac{\partial w(a,z_H^-)}{\partial z}$ and $J_X(a,z_L^+) = -\frac{1}{2}\xi^2\frac{\partial v(a,z_L^+)}{\partial z}$. Let $\hat{h}_X(z) = \int_0^\infty e^{-\chi a}h_X(a,z)da$ be the Laplace transform of $h_X(a,z)$ and rewrite the system for $h_X(a,z)$ as

$$\begin{cases} \frac{1}{2}\xi^2\hat{h}''_X(z) - \alpha\hat{h}'_X(z) - \chi\hat{h}_X(z) = -\delta(z-\bar{z}) + \frac{1}{2}\xi^2\hat{h}'_D(z_H)\delta(z-z_H) \quad\quad z > z_L \\ \hat{h}_X(z_L) = 0 \end{cases} \tag{26}$$

Define $\Upsilon_v(z,y) = \Upsilon(z-(z_L-z_D), y-(z_L-z_D))$. Then the solution to (26) is

$$\begin{aligned} \hat{h}_X(z) &= \int_{z_L}^{\infty}\Upsilon_v(z,y)\left[\delta(y-\bar{z}) - \frac{1}{2}\xi^2\hat{h}'_D(z_H)\delta(z-z_H)\right]dy \\ &= \int_{z_H}^{\infty}\Upsilon_v(z,y)\delta(y-\bar{z})dy - \frac{1}{2}\xi^2\hat{h}'_D(z_H)\Upsilon_v(z,z_H) \\ &= \Upsilon_v(z,\bar{z})1_{\{\bar{z}>z_H\}} - \frac{1}{2}\xi^2\hat{h}'_D(z_H)\Upsilon_v(z,z_H) \end{aligned}$$

Notice that

$$\hat{h}'_X(z_H^-) = \frac{\partial}{\partial z}_{|z=z_H^-}\Upsilon_v(z,\bar{z})1_{\{\bar{z}>z_H\}} - \frac{1}{2}\xi^2\left[\hat{h}'(z_H^-) - \hat{h}'_X(z_H^-)\right]\frac{\partial}{\partial z}_{|z=z_H^-}\Upsilon_v(z,z_H)$$

[24]Luttmer (2007) shows that (25) is the solution to (14). The Laplace transform of (25) coincides with our "Laplace transformed" solution. We use Laplace transform to solve (14) because it makes it much easier to solve the system (15).

so that

$$\hat{h}_X(z) = \Upsilon_v(z,\bar{z})1_{\{\bar{z}>z_H\}} - \frac{1}{2}\xi^2 \frac{\Upsilon'(z_H^-,\bar{z}) - \Upsilon'_v(z_H^-,\bar{z})1_{\{\bar{z}>z_H\}}}{1-\frac{\xi^2}{2}\Upsilon'_v(z_H^-,z_H)}\Upsilon_v(z,z_H)$$

The inverse Laplace transform of $\hat{h}_X(z)$ is $h_X(a,z)$.

The flux term

Figure (8) plots $h(a,z)$ against z for a particular age a and initial condition $\bar{z}$. Consider the change in the probability mass in the shaded area when age changes infinitesimally. The area is approximately equal to $h(a,z)dz$ and its change is $\frac{\partial h(a,z)dz}{\partial a} = J(a,z) - J(a,z+dz)$. Equivalently, $\frac{\partial h(a,z)}{\partial a} = -\frac{\partial J(a,z)}{\partial z}$. The probability mass in the shaded area increases when $\frac{\partial J(a,z)}{\partial z} < 0$ because when the flux is decreasing in z the mass of particles exiting from the shaded area is bigger than the mass of entering particles. The right-hand side of the Kolmogorov equation (14) is equal to $\frac{\partial J(a,z)}{\partial z}$. This clearly shows that $J(a,z) = \alpha h(a,z) - \frac{1}{2}\xi^2\frac{\partial h(a,z)}{\partial z}$. Now we can consider the Kolmogorov equation for $h_X(a,z)$ and conclude that

$$\frac{\partial}{\partial a}\int_z^{z+dz} h_X(a,s)ds = -\left[J_X(a,z+dz) - J_X(a,z)\right] + \int_z^{z+dz} J_D(a,z_H^-)\delta(s-z_H)ds \quad z > z_D$$

so that when $z = z_H$ we have,

$$\frac{\partial}{\partial a}\int_{z_H}^{z_H+dz} h_X(a,s)ds = -\left[J_X(a,z_H+dz) - J_X(a,z_H)\right] + J_D(a,z_H^-)$$

which shows that the change, when age increases infinitesimaly, in the mass of exporting firms with productivity slightly higher than z_H depends on the mass of exporting firms whose productivity becomes slightly higher than z_H and on the mass of newly exporting firms. A similar intuition is behind the presence of the $J_X(z_L^+,a)\delta(z-z_L)$ term in (15).

Distributions in terms of sales

Using (1), total sales of an exporter with productivity z are

$$r(z) = RP^{\sigma-1}\left(e^{-z}\frac{\sigma}{\sigma-1}\right)^{1-\sigma}\left(1+\tau^{1-\sigma}\right)$$

or, rearranging,

$$r^{\theta/(1-\sigma)} R^{\theta/(\sigma-1)} P^{\theta} \left(1+\tau^{1-\sigma}\right)^{-\theta} \left(\frac{\sigma}{\sigma-1}\right)^{-\theta} = e^{-\theta z}$$

which can be plugged in (17) to obtain (18),

$$p_u(r) = K_2 r^{-\theta/(\sigma-1)}$$

where $K_2 = K_1 e^{\theta z_D} R^{\theta/(\sigma-1)} P^{\theta} \left(1+\tau^{1-\sigma}\right)^{-\theta} \left(\frac{\sigma}{\sigma-1}\right)^{-\theta}$. Similar steps lead to (19).

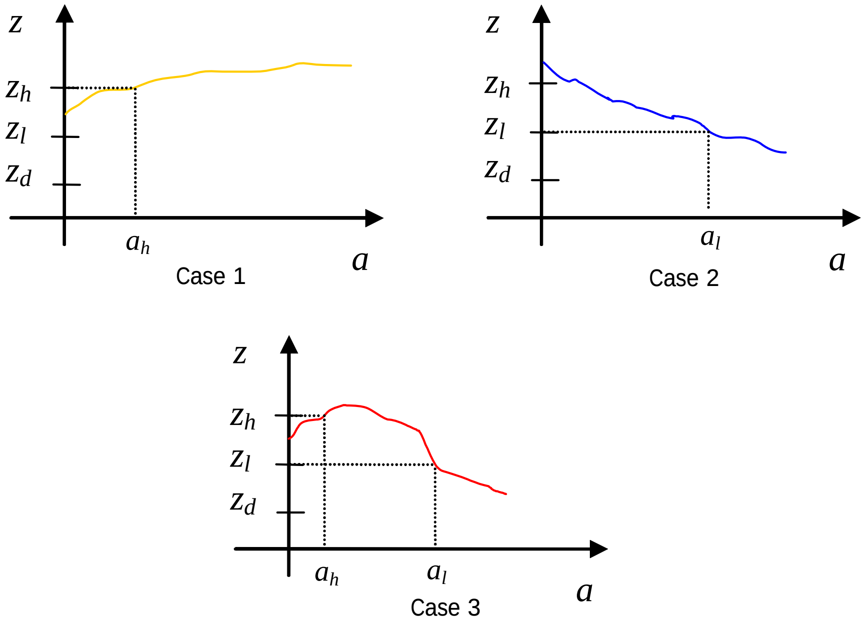


Figure 1: Sample paths for exporters and non-exporters

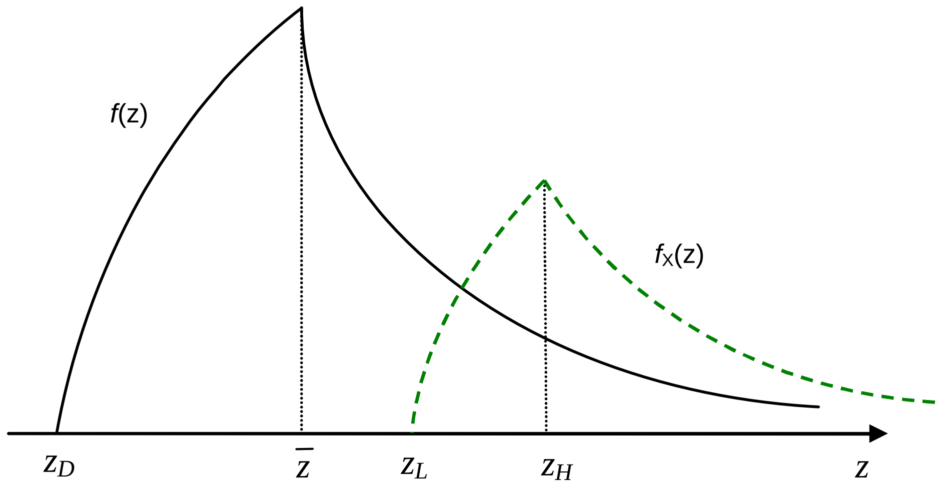


Figure 2: Equilibrium Productivity Distribution, Overall and Exporters

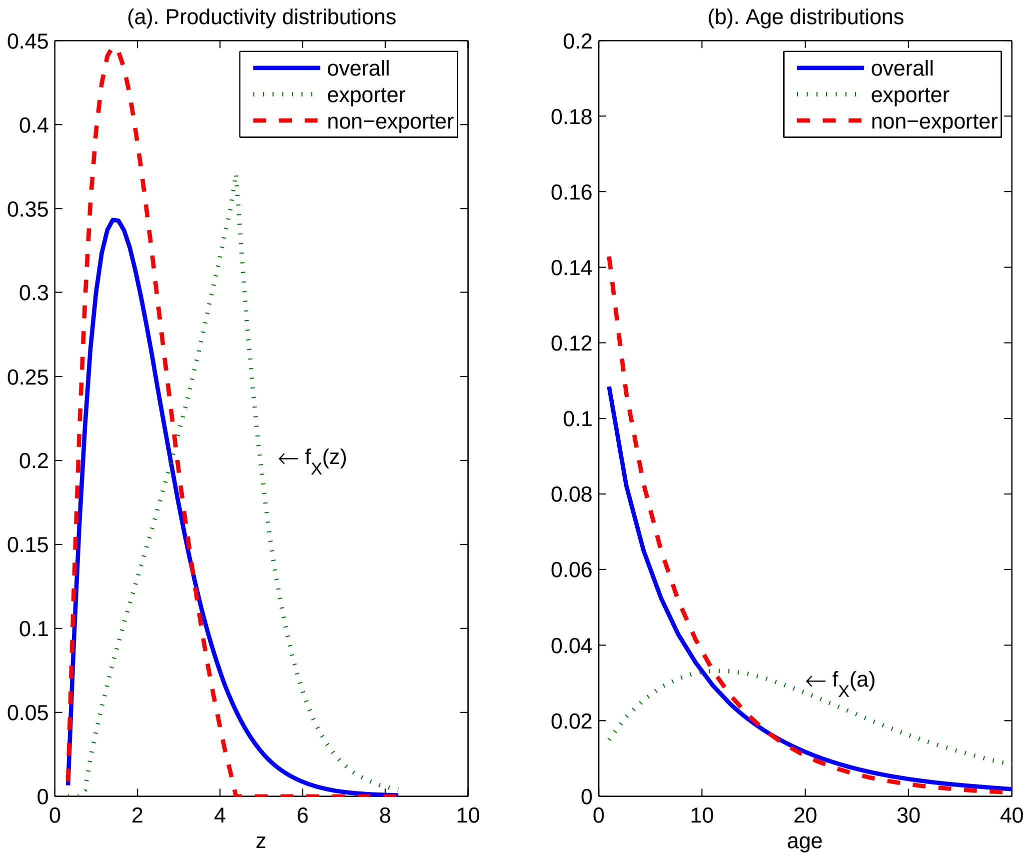
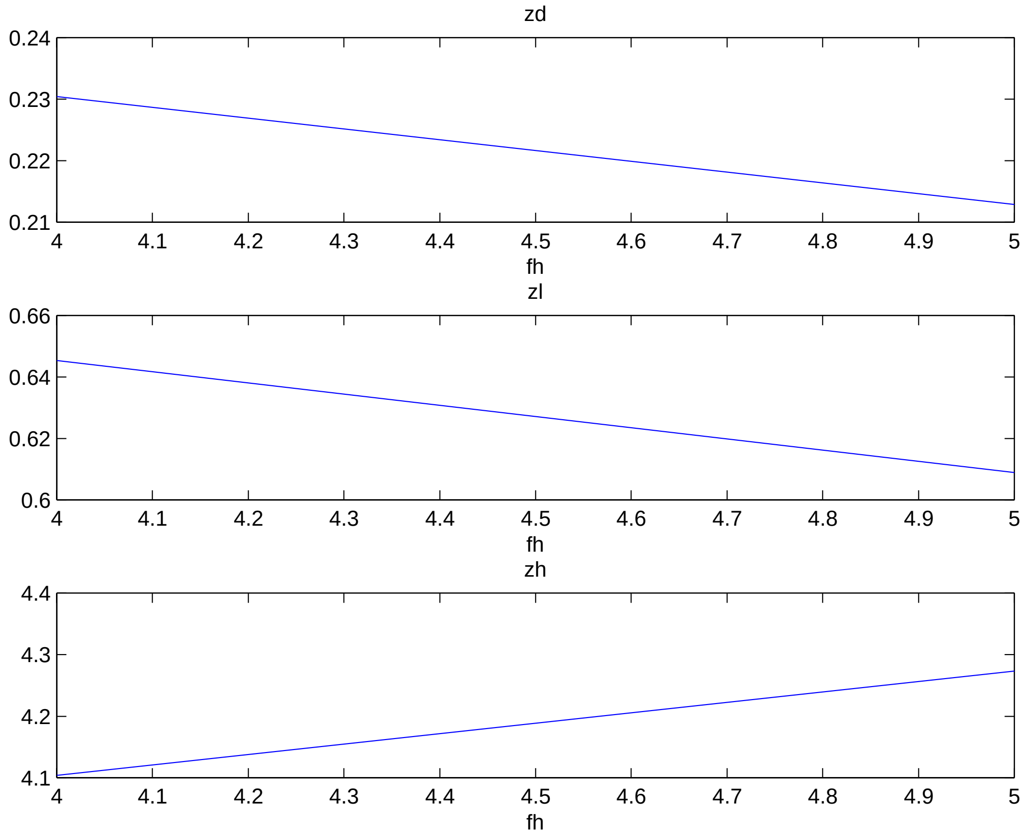


Figure 3: Stationary Distribution over Productivity and Age


zd
0.24
0.23
0.22
0.21
4
4.1
4.2
4.3
4.4
4.5
4.6
4.7
4.8
4.9
5
fh
zl
0.66
0.64
0.62
0.6
4
4.1
4.2
4.3
4.4
4.5
4.6
4.7
4.8
4.9
5
fh
zh
4.4
4.3
4.2
4.1
4
4.1
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4.5
4.6
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4.8
4.9
5
fh

Figure 4:

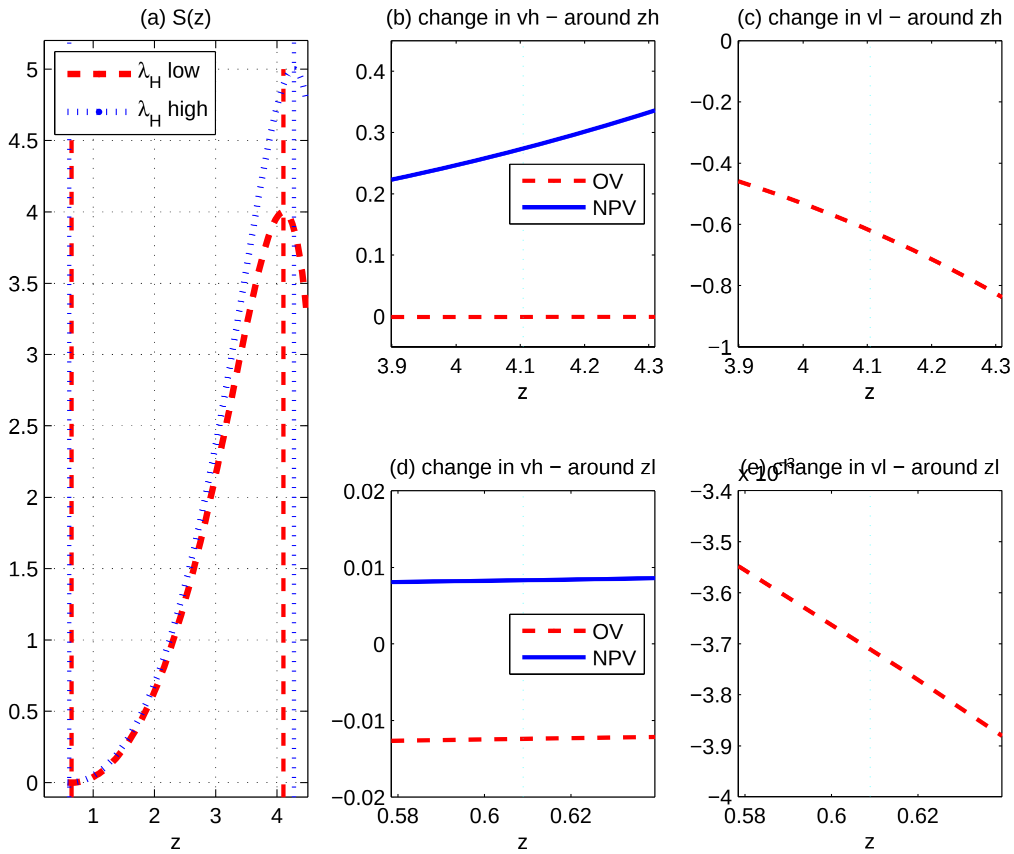

Figure 5: Determinants of the Band of Inaction

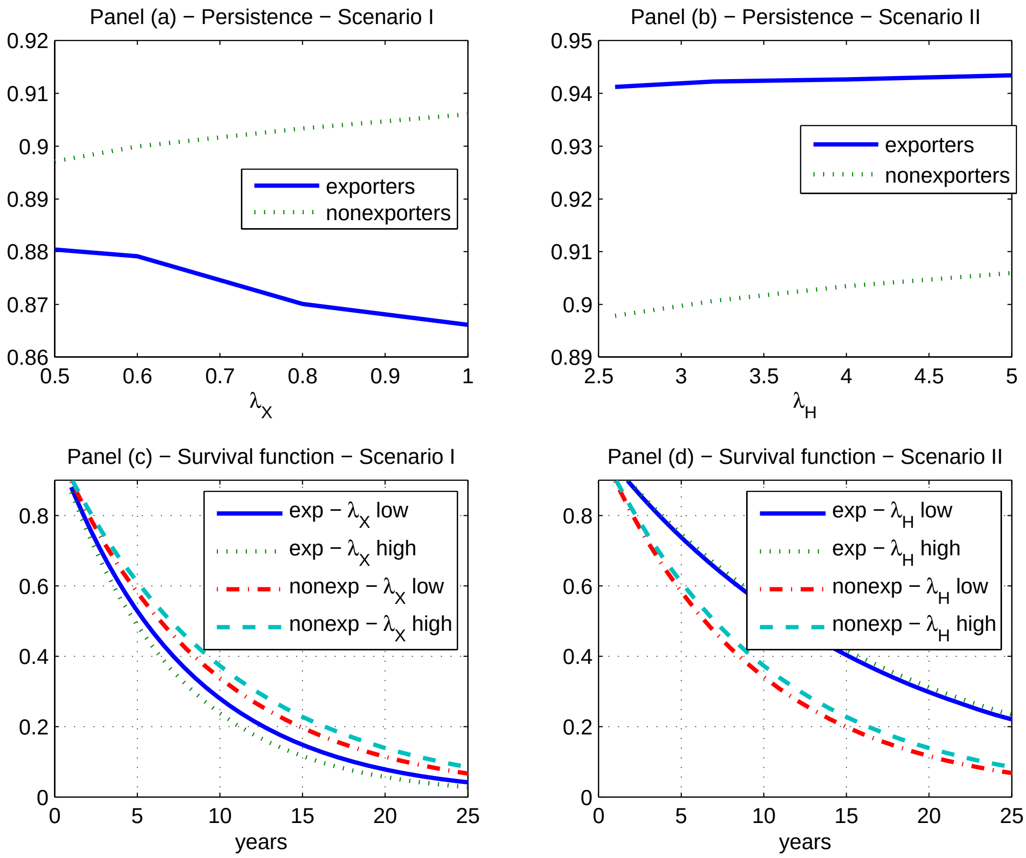

Figure 6: Persistence in Export Status and Survivor Functions, Scenarios I and II

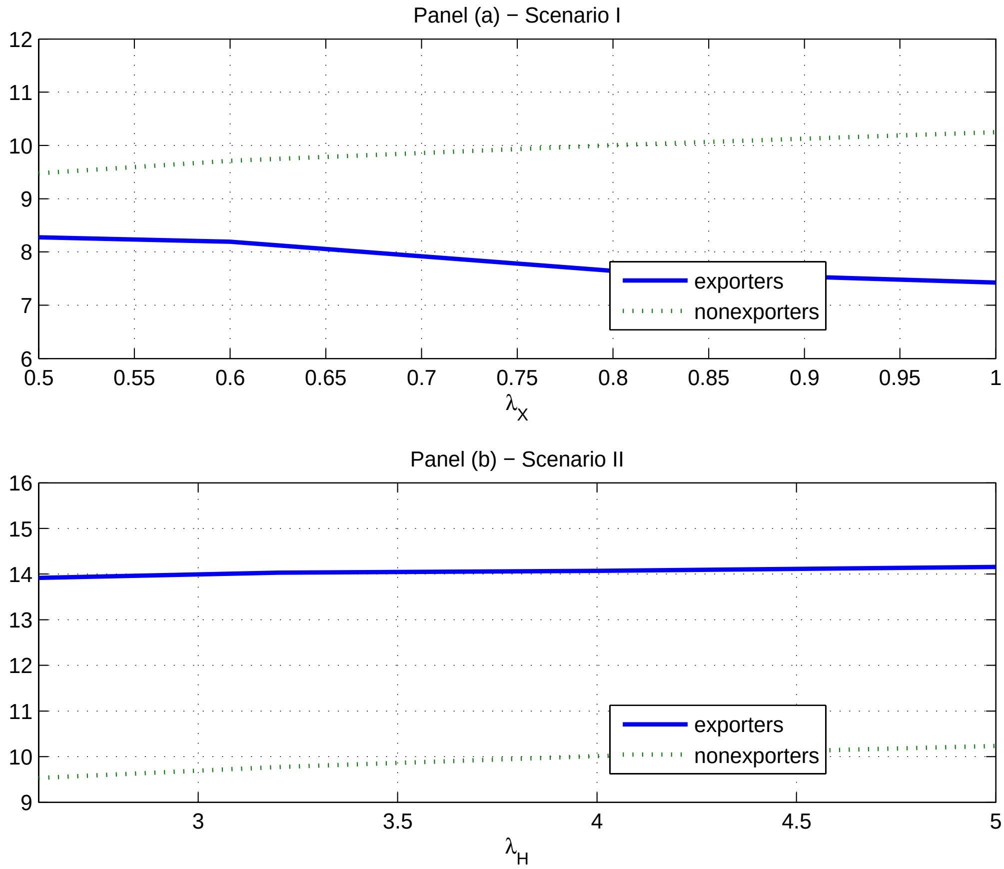

Figure 7: Average Time in Export Status, Scenarios I and II

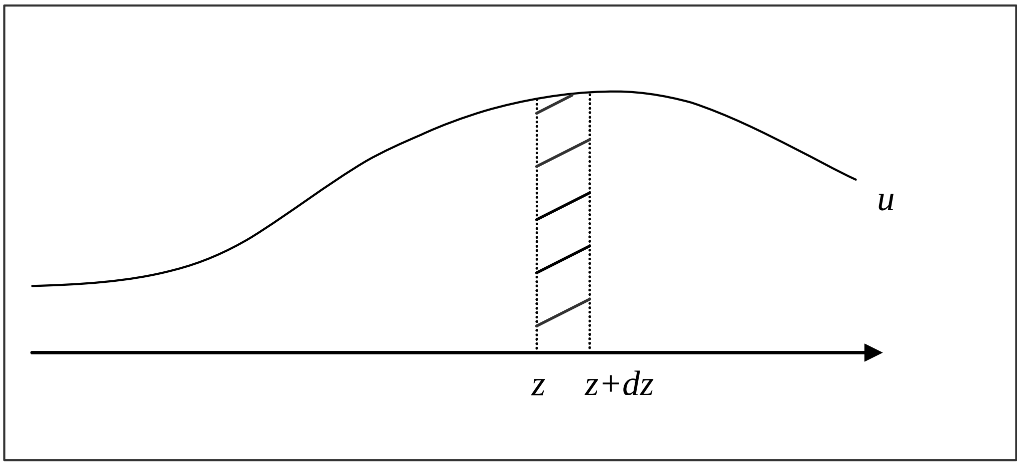

Figure 8: The Flux Term

Table 1: Parameterization

Parameter	*Value*
Consumers	
σ	2
r	0.05
Technology and size distribution	
α	-0.08
χ	0.45
δ	0.05
p_1	-.1
p_2	1.6
Operation and trade costs	
τ	1.25
λ_E	1
λ_D	0.1
λ_X	1 (Scenario I) and .1 (Scenario II)
λ_H	.1 (Scenario I) and 5 (Scenario II)

Table 2: Moments

Moments	Moment USA	Simulated values	
		Scenario I	Scenario II
Proportion of exporters	21% (BEJK)	7%	14%
Std deviation of log of domestic sales	1.7 (BEJK)	1.2	1.2
Avg total sales entrants/incumbents	25% (DRS)	35%	34%
% of employment accounted for by exporters	40% (AB)	68%	71%
% of exports over GDP	7.5% (AB)	33%	35%

References: AB [Atkenson and Burstein, 2007], BEJK [Bernard et al., 2003], DRS [Dunne, Roberts and Samuelson, 1988]

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References

[1] Amador, João and Luca D. Opromolla "Product and Destination Mix in Export Markets", mimeo, 2008.

[2] Antras, Pol "Advanced Topics in International Trade: Firms and International Trade", lecture notes, Harvard University, 2004.

[3] Arkolakis Konstantinos "Market Penetration Costs and the New Consumers Margin in International Trade", mimeo Yale University, 2008.

[4] Atkeson, Andrew and Ariel Burstein "Innovation, Firm Dynamics, and International Trade," mimeo UCLA, August 2007.

[5] Axtell, Robert L. (2001): "Zipf Distribution of U.S. Firm Sizes," Science, 1818-1820.

[6] Bernard, Andrew B. and Jensen, J. Bradford. "Exporters, Jobs, and Wages in U.S. Manufacturing: 1976–1987." *Brookings Papers on Economic Activity: Microeconomics*, 1995, pp. 67–119.

[7] Bernard, Andrew B. and Jensen, J. Bradford. "Exceptional Exporter Performance: Cause, Effect, or Both?" *Journal of International Economics*, February 1999a, *47*(1), pp.1–25.

[8] Bernard, Andrew B.; Eaton, Jonathan; Jensen, J. Bradford and Kortum, Samuel "Plants and Productivity in International Trade" *American Economic Review*, September 2003, *93*(4), pp.1268-1290.

[9] Chaney, Thomas "Distorted Gravity: The Intensive and Extensive Margins of International Trade", *American Economic Review* (forthcoming).

[10] Costantini, James A. and Marc J. Melitz "The Dynamics of Firm-Level Adjustment to Trade Liberalization", mimeo, 2007.

[11] Das, Sanghamitra; Roberts Mark J. and James R. Tybout, "Market Entry Costs, Producer Heterogeneity, and Export Dynamics", *Econometrica*, *75*(3), (2007), 837-873.

[12] Dixit, Avinash "Entry and Exit Decisions under Uncertainty" *Journal of Political Economy*, June 1989, *97*(3), pp. 620-38

[13] Dixit, Avinash and Pindyck Robert S. "Investment under Uncertainty" Princeton University Press, 1994.

[14] Eaton, Jonathan; Eslava, Marcela; Kugler, Maurice and James Tybout, "The Margins of Entry into Export Markets: Evidence from Colombia" in Elhanan Helpman, Dalia Marin and Thieery Verdier, eds., The Organization of Firms in a Global Economy, forthcoming, Harvard University Press.

[15] Gibson, Mark J. "Trade Liberalization, Reallocation and Productvity", mimeo 2006.

[16] Helpman Elhanan; Melitz, Marc J. and Stephen R. Yeaple "Export Versus FDI with Heterogeneous Firms", *American Economic Review*, 2004.

[17] Irarrazabal Alfonso; Moxnes, Andreas and Luca D. Opromolla, "The Spatial Pattern of Exports and Multinational Production", mimeo, 2008.

[18] Luttmer, Erzo G.J. "Selection, Growth and the Size Distribution of Firms", *Quarterly Journal of Economics*, August 2007, *122*(3), 1103-44.

[19] Melitz, Mark "The Impact of Trade on Intra-Industry Reallocations and Aggregate Industry Productivity" *Econometrica,* November 2003, *71*(6), pp. 1695-1725.

[20] Ruhl, Kim J. "The International Elasticity Puzzle in International Economics", mimeo, 2008.

[21] Roberts, Mark J. and Tybout, James R. "The Decision to Export in Colombia: An Empirical Model of Entry with Sunk Costs" *American Economic Review*, September 1997, *87*(4), pp. 545-564.

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