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*November 2008*



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# Approximating and Forecasting Macroeconomic Signals in Real-Time

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**Abstract**

We optimally incorporate factors estimated from a large panel of macroeconomic time series in the estimation of two relevant signals related to real activity: business cycle fluctuations and the medium to long-run component of output growth. This latter signal conveys information on the growth of real activity but contains no high-frequency oscillations. For forecasting purposes we show that targeting this object can prove more useful than targeting the original (noisy) time series. In fact, with conventional models, high-frequency fluctuations are always approximated despite being (possibly) unpredictable or idiosyncratic. We illustrate the methodology and provide forecasting comparisons for the U.S.

*JEL Classification:* C14, C32, E32

*Keywords:* Dynamic Factor Models, Band-Pass filter, Business Cycle fluctuations, Coincident indicator

# 1 Introduction

We present a method to estimate in real-time relevant macroeconomic signals. We apply the method to approximate two measures of economic activity in the U.S. and in the Euro Area: business cycle fluctuations of aggregate output and the smooth component of output growth. Following Baxter and King (1999), business cycle fluctuations are defined as "fluctuations with a specified range of periodicities" in the spectrum of the time series of interest. We will pick the standard (although arbitrary) $[6, 32]$ quarters band[1]. The smooth component of output growth (hereafter, smooth growth) is defined as output growth cleaned of fluctuations with period less than one year. The signals just defined can be extracted by applying well-known two-sided infinite moving averages (or filters) to the series of interest. Extraction in real-time is therefore restricted by the availability of data and is thus a difficult task. Our method provides reliable estimates of these signals, it is timely and displays a remarkable forecasting ability at short horizons (less than one year).

Our main contribution is to implement a solution that combines information derived from a large panel of time series (reduced by estimation of common factors) to optimally approximate the signals of interest in a multivariate context. Thus, we integrate the recent developments in the analysis of dynamic factor models in the approximation of band-pass filters. We stress that our method can be used in any similar signal extraction problem demanding precise real-time estimates. Specifically, our method can straightforwardly be adapted to produce optimal approximations to any (absolutely summable and stationary) distributed lag of the series of interest. We prove consistency of the estimated signal when factors are used in the approximation and second moments are estimated. On the empirical side, our approach is shown to deliver real activity indicators that have several desirable properties:

i) they are timely, since our approach is flexible enough so as to take into account the release

[1] See, e.g., the definition in Stock and Watson (1999) for the U.S. and Fagan and Mestre (2001) for the Euro area.

delays of all the series used in the exercise; both that of Gross Domestic Product (GDP), which typically arrives in its final version 6 months after the beginning of the quarter to which it refers and also of the indicators included in our panel of predictors.

ii) they display little short-run oscillations, thereby giving a clear picture of current cyclical and growth prospects.

iii) they are based on a large and comprehensive panel of predictors, whose idiosyncratic and short-run components are eliminated through factor analysis.

iv) they have a remarkable forecasting performance at short horizons (less than one year). To be specific, we view forecasts of the smooth growth indicator as being useful to forecast GDP itself. We highlight an important insight: for forecasting purposes, targeting a smooth version of a time series may be more useful than targeting the original series. We offer a possible justification: with conventional models, short-run fluctuations are being approximated despite being (possibly) unpredictable or idiosyncratic.

Our approach can be summarized as follows: we assume the panel of predictors is appropriately described by a dynamic factor model, as originally developed by Geweke (1977) and Sargent and Sims (1977). We estimate the common factors using either principal components as in Stock and Watson (2002a and 2002b) or generalized principal components as in Forni et al. (2005)[2]. We thus compare the two methods of approximating the factor space in this context. The next step is to use the extracted factors as covariates in a (minimum mean squared error) approximation to the relevant distributed lags of the series of interest. To this effect we use the solution in Valle e Azevedo (2007) that provides such approximation in a multivariate context. The implied multivariate filter is an extension of the univariate filter developed by Christiano and Fitzgerald (2003), itself an extension of the two-sided symmetric filter of Baxter

[2]See also Forni et al. (2000 and 2004) and Forni and Lippi (2001) for a detailed analysis of the generalized dynamic factor model and the alternative approach to estimation of the factor space.

and King (1999). We should stress that such solutions had been provided in the case of stationary processes by Geweke (1978) and in a univariate context including unit-roots by Pierce (1980).

Following Stock and Watson (1989), model-based (or parametric) methods assuming a factor structure have also been used to construct business cycle (or growth) indicators. There have also been attempts to extract signals similar to those we target in a parametric setting. Harvey and Trimbur (2003) propose unobserved components models for which the extraction of a cycle component is equivalent to using a band-pass filter. Using the components in Harvey and Trimbur (2003) and incorporating an extension by Rünstler (2004) that allows for phase shifts in the cyclical components of multiple time series, Valle e Azevedo, Koopman and Rua (2006) construct a business cycle indicator which can be seen as a multivariate band-pass filter. Although a common factor structure is assumed to describe a small set of time series, the representation is far from general and the method is not aimed at approximating an ideal filter isolating a pre-defined range of frequencies. Such is the aim of this paper. The approach closest to ours is that of Altissimo et al. (2007) that resulted in the New Eurocoin indicator. The indicator is obtained by projecting estimated factors on the smooth component of output growth, disregarding the information contained in past observations of the series of interest (GDP). We will later contrast in more detail our approach to theirs.

The remainder of the paper is organized as follows. In section 2 we define precisely the objects (signals) that we are targeting and describe the way to optimally approximate them. In section 3 we discuss the estimation of factors used as covariates in the approximation to the signals. Section 4 deals with the consistency of the estimated signals and section 5 provides an assessment of the real-time performance of our methodology, analyzing results for the Euro area and the U.S. Section 6 analyzes the forecasting performance of the filtering procedure and section 7 offers a summary of the main conclusions.

# 2 Signals of Interest and Approximation

## 2.1 Business cycle fluctuations and Smooth growth

Our variable of interest throughout the paper will be (log of) real quarterly GDP, the best available proxy of aggregate economic activity. Define $x_t$ as the log of real GDP and $\Delta x_t = (1-L)x_t$ , where $L$ is the lag operator, as its growth rate. We define business cycle fluctuations as fluctuations with period in the range $[6, 32]$ quarters in the (pseudo-) spectrum of $x_t$. Smooth growth is defined as GDP growth with fluctuations with period less than one year (or 4 quarters) removed. Specifically, take the following decompositions of $x_t$ and $\Delta x_t$:

$$x_t = BC(L)x_t + (1 - BC(L))x_t$$

$$\Delta x_t = SG(L)\Delta x_t + (1 - SG(L))\Delta x_t$$

where $BC(L) = \sum_{j=-\infty}^{\infty} BC_j L^j$ and $SG(L) = \sum_{j=-\infty}^{\infty} SG_j L^j$ with filter weights given by:

$$BC_o = \frac{\omega_h - \omega_l}{\pi}, \qquad BC_j = \frac{\sin[\omega_h j] - \sin[\omega_l j]}{\pi j}, |j| \geq 1 \text{ with } \omega_h = \frac{2\pi}{6} \text{ and } \omega_l = \frac{2\pi}{32} \tag{1}$$

$$SG_o = \frac{\omega_h}{\pi}, \qquad SG_j = \frac{\sin[\omega_h j]}{\pi j}, |j| \geq 1 \text{ with } \omega_h = \frac{2\pi}{4} \tag{2}$$

Business cycle fluctuations are defined as $BC(L)x_t$ and smooth growth is defined as $SG(L)\Delta x_t$. The band-pass filters $BC(L)$ and $SG(L)$ isolate indeed the desired fluctuations in the integrated $x_t$ and in $\Delta x_t$ (see e.g., Baxter and King, 1999 and Christiano and Fitzgerald, 2003 for a discussion). It is clear that in finite samples it is not possible to extract with arbitrary precision these signals. However, accurate approximations can be obtained in the middle of a sample. Figure 1 presents the two (approximate) decompositions for the U.S. and Euro area, using the filter approximation developed by Baxter and King (1999) (BK filter). Additional (past and future) data would lead to negligible differences between these estimates and those obtained with the filters $BC(L)$ or $SG(L)$. Using the BK filter implies loosing information in the beginning and end of the sample, whereas our interest is in obtaining estimates of these signals in real-time (using only available data at each point).

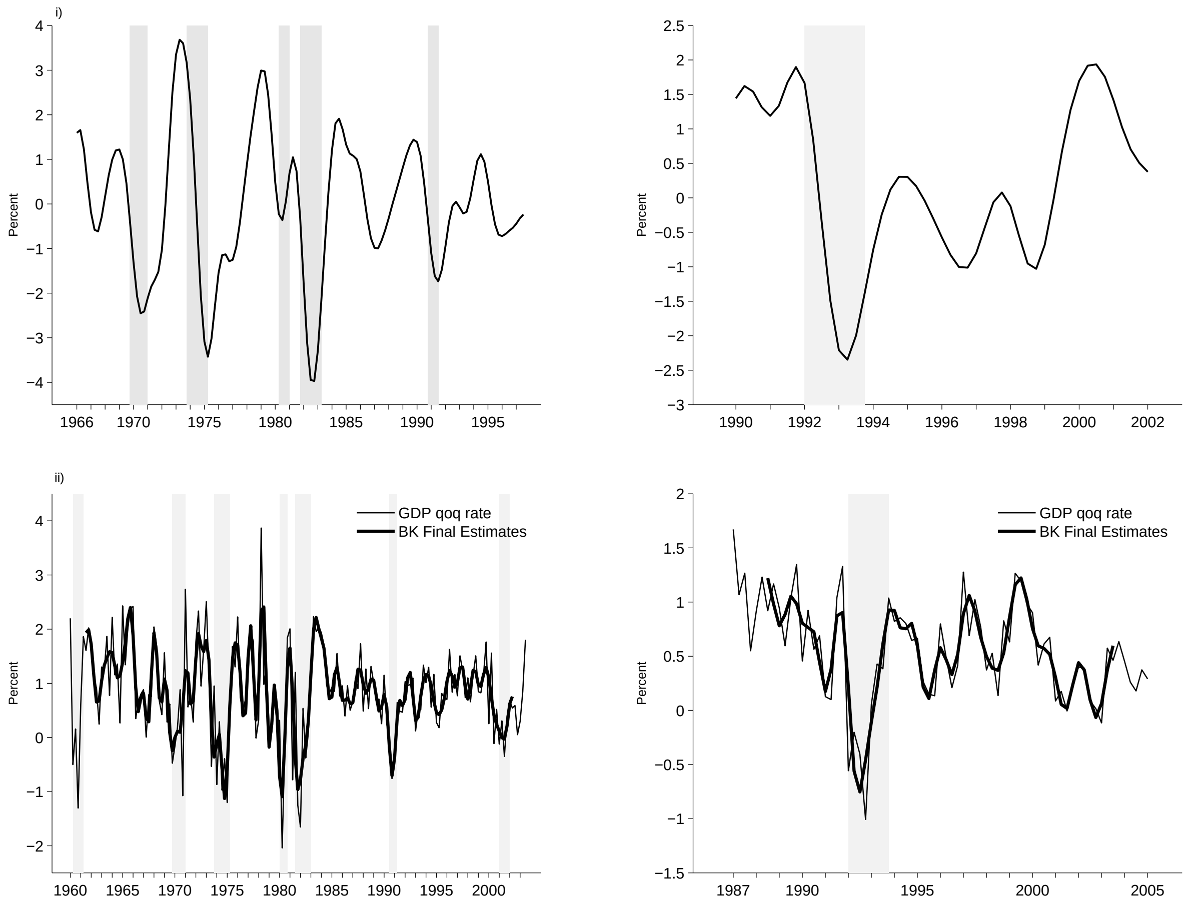


Figure 1

i) Business cycle fluctuations of U.S. (left) and Euro Area (right) GDP (more precisely, approximation to $100 \times BC(L)x_t$, the percentage deviation of output from trend), obtained with the BK filter that truncates the ideal filter at lead and lag $j = 24$ and corrects the weights so as to guarantee removal of one unit-root. ii) Smooth growth rate of U.S. (left) and Euro Area (right) GDP (more precisely, approximation to $100 \times SG(L)\Delta x_t$), obtained with the BK filter that truncates the ideal filter at lead and lag $j = 6$. Grey areas represent the recession dating of the NBER (for the U.S.) and the CEPR (for the Euro Area)

The defined business cycle fluctuations can reasonably be interpreted as those fluctuations in real GDP that are not attributable either to long-run growth or to high-frequency measurement error or to other short-run fluctuations not associated with business cycle phenomena. This definition has been extensively used in the literature (see, e.g., Stock and Watson 1999). Smooth growth is a measure of output growth but it is free of short-run oscillations that make difficult the assessment of current aggregate economic situation. This signal has been targeted by Altissimo et al. (2007) in the construction of a coincident indicator for the Euro area, Eurocoin (denoted there as the medium to long-run component of output growth). Arguably, targeting these objects reduces the need to measure precisely GDP, or similarly, it reduces the need to account for GDP data revisions[3]. Still, the approximations will be computed using observations of the GDP series itself. We should however note that it would be straightforward to adjust the methodology so as to target data not subject to revisions and to neglect the most recent GDP data (that is subject to revisions). The consequences of not taking data revisions into account, extensively analyzed by Croushore and Stark (2001, 2003), are definitely relevant if we want to be serious about the "real-time" performance of our indicators. Bearing this in mind, we have decided to use the latest available vintage of GDP data in the approximation to the desired signals[4].

$BC(L)x_t$ and $SG(L)\Delta x_t$ are our targets, but we will seek in each month an estimate of the signals regarding the current quarter[5]. Thus, within each quarter, new monthly information will be used to update our estimates. It turns out that this additional infra-quarterly information is useful in that it provides more precise estimates of the signals of interest. As a by-product, we will seek in each month an estimate of current year growth based on the most up-to-date approximation to smooth growth and its relevant forecasts.

[3]Obviously, this last statement is only correct if data revisions have most of its power concentrated in the high frequencies that we disregard with both $BC(L)x_t$ and $SG(L)\Delta x_t$, something we have not verified.

[4]Similar data revision problems are present in our panels, but in a much lesser extent since only a few series are revised (e.g., industrial production indices).

[5]So, e.g., in January we will target the signals regarding the first quarter, in April those of the second quarter and in September those of the third quarter. It would be trivial to target in each month any other quarter.

## 2.2 How to approximate the signals of interest

To be general, suppose $x_t$ is integrated of order 1 and we are interested in isolating the signal $y_t = B(L)x_t$ (or alternatively $y_t = B(L)\Delta x_t$), where $B(L)$ is an arbitrary (absolutely summable and stationary) filter that defines a signal on $x_t$ ($\Delta x_t$). Without loss of generality, assume further that $B(1) = 0$ (respectively, $B(1) = 1$), which is verified by $BC(L)$ defined in (1) ($SG(L)$ defined in (2)). We want to isolate the signal in the finite sample $\{x_t\}_{t=1}^{T}$(or $\{\Delta x_t\}_{t=1}^{T}$). Suppose also we have $c$ series of covariates $z_1, ..., z_c$. We make the following assumption:

**Assumption DGP 1.** *The vector $(\Delta x_t, z_{1,t}, ..., z_{c,t})$, where $\Delta = 1-L$, is covariance-stationary with a finite moving average representation (of order $M$, say).*

Now let $x = (x_1, x_2, ..., x_T)'$, $z_1 = (z_{1,1}, z_{1,2}, ..., z_{1,T})'$, ..., $z_c = (z_{c,1}, z_{c,2}, ..., z_{c,T})'$ and $y_t = B(L)x_t$ (or, alternatively, $y_t = B(L)\Delta x_t$) be the signal at time $t$. The estimate $\widehat{y}_t$ of the signal $y_t$ is a weighted sum of elements of $x$ (or $\Delta x$) and elements of $z_1, ..., z_c$:

$$\widehat{y}_t = \sum_{j=-f}^{p} \widehat{B}_j^{p,f} x_{t-j} + \sum_{s=1}^{c}\sum_{j=-f}^{p} \widehat{R}_{s,j}^{p,f} z_{s,t-j} \tag{3}$$

or, in the case we are interested in $y_t = B(L)\Delta x_t$:

$$\widehat{y}_t = \sum_{j=-f}^{p} \widehat{B}_j^{p,f} \Delta x_{t-j} + \sum_{s=1}^{c}\sum_{j=-f}^{p} \widehat{R}_{s,j}^{p,f} z_{s,t-j} \tag{4}$$

$p$ denotes the number of observations in the past that are considered and $f$ the number of observations in the future that are considered. To obtain the minimum mean squared error estimate of $y_t$, we choose the weights $\{\widehat{B}_j^{p,f}, \widehat{R}_{1,j}^{p,f}, ..., \widehat{R}_{c,j}^{p,f}\}_{j=-f,...,p}$ associated with the series of interest and the available covariates that solve the following problem:

$$\underset{\{\widehat{B}_j^{p,f},\widehat{R}_{1,j}^{p,f},\ldots,\widehat{R}_{c,j}^{p,f}\}_{j=-f,\ldots,p}}{Min} E[(y_t - \widehat{y}_t)^2] \tag{5}$$

where the information set is implicitly restricted by $p$ and $f$. The solution to this problem under Assumption DGP 1 is derived and analyzed in Valle e Azevedo (2007) for both the case when the series of interest is integrated (like $x_t$) and also when it is stationary (as $\Delta x_t$). In the first case, a restriction must be imposed on $\widehat{B}(L) = \sum_{j=-f}^{p} \widehat{B}_j^{p,f} L^j$ (namely $\widehat{B}(1) = 0$, since $B(1) = 0$) to ensure the problem is well-defined (ensuring also stationarity of the extracted signal, as desired). We note that dropping the second term in the right hand side of (3) and (4) delivers the univariate approximation of Christiano and Fitzgerald (2003). As we will verify, incorporating information from series other than GDP improves substantially the approximations.

**Remark 1** $f$ is allowed to be negative, which is of particular interest if at time $T$ (say, the current quarter) the series of interest $x_t$ (or $\Delta x_t$) is not available. Thus, it is straightforward to extract the signal $y_{T+k} = B(L)x_{T+k}$ for $k > 0$. One just needs to set $f = -k$ in the solution, so that only the available information (that is, up to period $T$ in this case) is taken into consideration.

**Remark 2** Since we will seek in each month estimates of the (quarterly) signal $y_t$, it is convenient to incorporate time series recorded at mixed frequencies, e.g., quarterly $x_t$ (say GDP) and quarterly $z_{l,t}$'s (say, the preliminary estimates of GDP) as well as monthly $z_{l,t^*}$'s (say Industrial production or a common factor extracted from a large panel). To this effect, we break the monthly series into three quarterly series. To be specific, a monthly indicator $z_{l,t^*}$ will be split into the "quarterly" series $\{z_{l,t}^1\} = (z_{l,t^*}, z_{l,t^*-3}, z_{l,t^*-6}, ...)'$, $\{z_{l,t}^2\} = (z_{l,t^*-1}, z_{l,t^*-4}, z_{l,t^*-7}, ...)'$ and $\{z_{l,t}^3\} = (z_{l,t^*-2}, z_{l,t^*-5}, z_{l,t^*-8}, ...)'$.

**Remark 3** We can deal explicitly with missing observations in the $z_{l,t}$'s in the end of the sample. We just need to trivially relabel the time subscript $t$, shifting the series so that they match the last available observation for $x_t$. Thus, we are able to take into account all the release delays in the data: both the delays of GDP and those of the indicators.

**Remark 4** Assumption DGP 1 implies that the covariates $z_{1,t}, ..., z_{c,t}$ are covariance-stationary. We thus rule out integrated covariates. This is not restrictive, as long as we assume that no cointegration relations exist within the vector $(x_t, z_{1,t}, ..., z_{c,t})$, where now $z_{1,t}, ..., z_{c,t}$ are allowed to be integrated. To see this, suppose we have some indicators $z_{l,t}$ which are integrated of order 1. In this situation, the polynomials $\widehat{R}_l(L) = \sum_{j=-f}^{p} \widehat{R}_{l,j}^{p,f} L^j$ would need to have a unit-root so that the term $\sum_{j=-f}^{p} \widehat{R}_{l,j}^{p,f} z_{l,t-j}$ from the solution to (5) is reduced to stationarity. Otherwise the criterion is infinite. But this is equivalent to initially take first differences to the integrated $z_{l,t}$'s. Thus, the solutions (3) or (4) to (5) depend on the second order moments of $\Delta z_l$, the information from the level of the integrated $z_{l,t}$ is irrelevant. However, in the presence of cointegration relations we would have stationary linear combinations of $x_t, z_{1,t}, ..., z_{c,t}$ (or of $z_{1,t}, ..., z_{c,t}$ only). This would allow us to have a finite variance solution $\widehat{y}_t = \sum_{j=-f}^{p} \widehat{B}_j^{p,f} x_{t-j} + \sum_{s=1}^{c} \sum_{j=-f}^{p} \widehat{R}_{s,j}^{p,f} z_{s,t-j}$ (or $\widehat{y}_t = \sum_{j=-f}^{p} \widehat{B}_j^{p,f} \Delta x_{t-j} + \sum_{s=1}^{c} \sum_{j=-f}^{p} \widehat{R}_{s,j}^{p,f} z_{s,t-j}$ ) even in the presence of integrated $z_{l,t}$'s, without resorting to the unit-root restrictions $\widehat{B}(1) = \widehat{R}_l(1) = 0$ (or only $\widehat{R}_l(1) = 0$). In principle, one could exploit such information and incorporate it in the solution. In practice however, we regard $z_1, ..., z_c$ as available indicators without much of a structural content.

**Remark 5** Vector $(\Delta x_t, z_{1,t}, ..., z_{c,t})$ is not required to have zero mean so long as a constant $z_{1,t} = 1$ is included. In the case of the approximation to $BC(L)x_t$, this is equivalent to linearly detrend $x_t$ and normalize the covariates to have zero mean. In fact, $BC(L)$ has two unit-roots, implying that $BC(L)x_t$ has zero mean when $x_t$ is integrated of order 1. In the case of the approximation to $SG(L)\Delta x_t$ , including $z_{1,t} = 1$ is equivalent to extract the signal with both $\Delta x_t$ and the covariates normalized to have zero mean and then adding the mean of $\Delta x_t$ . This is warranted since $SG(1) = 1$ implies the mean of the signal equals the mean of $\Delta x_t$.

As referred already, we use solution to (5) under Assumption DGP 1 derived and discussed in Valle e Azevedo (2007) to approximate both $BC(L)x_t$ and $SG(L)\Delta x_t$ defined in section 2.1. The weights of the filter are obtained by simply solving a linear system with $(p+f+1)\times(c+1)$ equations and unknowns. The solution depends only on the second moments of $(\Delta x_t, z_{1,t}, ..., z_{c,t})'$ (that need to be estimated, see below) and on the weights of the ideal filter. Define $\widehat{B} = (\widehat{B}_p^{p,f}, \widehat{B}_{p-1}^{p,f}, ..., \widehat{B}_0^{p,f}, ..., \widehat{B}_{-f+1}^{p,f}, \widehat{B}_{-f}^{p,f})'$ and $\widehat{R}_s = (\widehat{R}_{s,p}^{p,f}, \widehat{R}_{s,p-1}^{p,f}, ..., \widehat{R}_{s,0}^{p,f}, ..., \widehat{R}_{s,-f+1}^{p,f}, \widehat{R}_{s,-f}^{p,f})'$, where $s = 1, ..., c$. Stack these vectors in the vector of weights $\widehat{W} = (\widehat{B}', \widehat{R}_1', ..., \widehat{R}_c')'$. The linear system solved to recover the solution $\widehat{W}$ is the following:

$$V = Q\widehat{W} \tag{6}$$

where $Q$ is a $(p+f+1)\times(p+f+1)$ matrix that depends only on the second moments of the vector $(\Delta x_t, z_{1,t}, ..., z_{c,t})$ and $V$ is a vector of dimension $p+f+1$ that depends also on the second moments of the vector $(\Delta x_t, z_{1,t}, ..., z_{c,t})$ as well as on the weights of the infinite sample filter ($BC(L)$ or $SG(L)$). Specific adaptations need to be made in $V$ and $Q$ when we approximate $BC(L)x_t$. This occurs because we impose the restriction $\widehat{BC}(1) = \sum_{j=-f}^{p} \widehat{BC}_j^{p,f} = 0$ that guarantees the removal of one unit-root.

In the remainder of the paper, we will set $p = 50$ (larger values of $p$ lead to negligible differences in the approximations). To approximate the signals of interest in real-time we set either $f = -2$ (in a first or second month of a given quarter) or $f = -1$ (in the third month of a given quarter). This mimics the release delays of the final estimate of GDP both for the U.S. and Euro area (see table 1, that contains additionally the release delays of the earlier estimates of U.S. GDP that can potentially be used as covariates in the approximations). For forecasting purposes we set $f = -$ *Forecasting horizon* (in quarters) + *release delay*, where *release delay* is again 2 in a first or second month of the current quarter and 1 in a third month of the current quarter. In the middle of the sample we set $f$ such that all observations until

| GDP estimate | $1^{st}$ month | $2^{nd}$ month | $3^{rd}$ month |
|---|---|---|---|
| Final | 2 quarters | 2 quarters | 1 quarter |
| Preliminary | 2 quarters | 1 quarter | 1 quarter |
| Advanced | 1 quarter | 1 quarter | 1 quarter |

Table 1
Release delays of U.S. GDP estimates by month of the quarter

the end of the sample are used. We are left with the estimation of the needed second moments of $(\Delta x_t, z_{1,t}, ..., z_{c,t})$, to which we turn in the next sub-section.

## 2.3 Estimation of second moments

We propose two alternatives to the estimation of the needed autocovariance function (or spectrum) of vector $(\Delta x_t, z_{1,t}, ..., z_{c,t})$. The first is based on a standard non-parametric estimator of the spectrum, given by:

$$\widehat{S}_{\Delta x, z_1, ..., z_n}(\omega) = \frac{1}{2\pi}(\widehat{\Gamma}(0) + \sum_{k=1}^{M(T)} \kappa(k)(\widehat{\Gamma}(k)e^{i\omega k} + \widehat{\Gamma}(k)' e^{-i\omega k}))$$

where $\kappa(k, T) = (1 - \frac{k}{M(T)+1})$ denotes the Bartlett lag window, $\widehat{\Gamma}(k)$, $k = 0, 1, ..., M(T)$ is the sample autocovariance of vector $(\Delta x_t, z_{1,t}, ..., z_{c,t})$ at lag $k$ and the truncation point $M(T) < T$ is a function of the sample size $T$. $M(T)$ is typically required to grow slower than $T$ to guarantee consistency of $\widehat{S}_{\Delta x, z_1, ..., z_n}(\omega)$. Given Assumption DGP 1 that includes the finiteness of the moving average ($MA$) representation of $(\Delta x_t, z_{1,t}, ..., z_{c,t})$, we need only $M(T)$ fixed at some order higher than that of the $MA$ representation, together with $\kappa(k, T) = 1$ for $T$ sufficiently large, which is obviously needed to establish consistency of the estimated signals. For all empirical purposes we set in this estimator $M = 30$ in the U.S. case (in the range $20 < M < 40$ results are very similar) and $M = 6$ for the Euro area (in the range $4 < M < 12$ results are also similar). We justify the difference in the choice of $M$ based on the time dimension of Euro area data ($T = 73$ quarters versus $T = 175$ quarters for the U.S.). For the Euro area we have verified that higher values of $M$ resulted in a deteriorated performance of the approximations, most probably due to poor estimation of the high order autocovariances.

Alternatively, following Priestley (1981) together with Den Haan and Levin (1998)[6], we first pre-whiten vector $(\Delta x_t, z_{1,t}, ..., z_{c,t})$ by estimating a SUR-VAR with lag length in each equation determined either by the BIC criterion (U.S. case only, since in the Euro area estimation of the SUR-VAR proved unfeasible and other alternatives for pre-whitening did not deliver encouraging results). Given the estimated VAR polynomial, $\widehat{VAR}(L)$, and the estimated autocovariance function of the resulting residuals ($\widehat{\Gamma}^{\widehat{\varepsilon}}(k)$ with maximum order $k = M$ chosen as above), we derive the autocovariance function of the original process $(\Delta x_t, z_{1,t}, ..., z_{c,t})$ implicit in the estimated autocovariance generating function $\widehat{G}(z) = \widehat{VAR}(z)^{-1} \sum_{k=-M}^{M} \widehat{\Gamma}^{\widehat{\varepsilon}}(k) z^k \widehat{VAR}(z^{-1})^{-1\prime}$. To apply the filter we then truncate the estimated autocovariance function at lead and lag $k = 40$[7]. Increasing the number of covariates used in the approximations complicates the estimation of the VAR but not at all the non-parametric approach, hence the consideration of the two methods.

# 3 Multivariate information

## 3.1 Factor model

In our empirical applications the covariates used in the approximations to the signals will be estimated factors derived from a large panel of monthly time series. Consider the following "monthly" panel of covariance-stationary time series:

$$\mathbf{W} = \{w_{it}\} \quad i = 1, ..., n; \quad t = 1, ..., T^* \quad ,$$

[6] In Den Haan and Levin (1998), the focus is on the estimation of the spectrum only at zero frequency. We need instead the whole autocovariance function.

[7] Higher order autocovariances are very close to zero so that including them makes virtually no difference, as we have verified empirically. Furthermore, in the U.S. case, using the moments estimated from the SUR-VAR or those implicit in $\widehat{G}(z)$ turns out to deliver very similar results.

an $n \times T^*$ triangular array. We use $T^*$ instead of $T$ to make clear that we have a monthly panel of time series. We stress that all the series in the panel are organized in such a way that for month $t$ they are in fact available. E.g., if an Industrial Production index refers to a month but it is only and always released one month after, we use the one-period lagged series as an $w_{it}$. Thus, we effectively take into account the release delays of all the indicators, which is crucial if we want to be serious in the analysis of the real-time performance of our procedure.

We assume each variable can be decomposed into a common component, $\chi_{it}$, driven by a small number of common orthogonal shocks (or so-called dynamic factors) $u_{lt}$, $l = 1, 2, ..., q$ that are loaded differently across $i$, and an idiosyncratic component $\xi_{it}$, orthogonal to the $u_{lt}$ at all leads and lags (and therefore orthogonal to $\chi_{jt}$, $j = 1, 2, ..., n$ at all leads and lags). Specifically:

$$w_{it} = \chi_{it} + \xi_{it} \tag{7}$$

$$\chi_{it} = \beta_{i1}(L)u_{1t} + \beta_{i2}(L)u_{2t} + ... + \beta_{iq}(L)u_{qt} \tag{8}$$

where the $\beta_{ij}(L)$, $j = 1, 2, ..., q$ are distributed lags of finite order. We can thus write:

$$\chi_{it} = \lambda_{i1}F_{1t} + \lambda_{i2}F_{2t} + ... + \lambda_{ir}F_{rt} \tag{9}$$

where the $F_{jt}$ are a function of current and past values of the shocks $u_{lt}$ , $l = 1, 2, ..., q$. Finiteness of the distributed lags $\beta_{ij}(L)$, $j = 1, 2, ..., q$ is crucial here. No stringent restrictions are imposed on the second-moments structure of the $F_{jt}$. We require vector $\mathbf{F}_t = (F_{1t}, F_{2t}, ..., F_{rt})'$ to be covariance-stationary and following a finite moving average representation. More precisely, we make the following assumptions on the factor model:

**Assumption F1** *(Factors and Factor Loading)*

a. $(\Lambda'\Lambda/n) \to I_r$, where $\Lambda = [\lambda_{ij}]_{i=1,...,n\ ;j=1,...,r}$

b. $E[\mathbf{F}_t\mathbf{F}_t'] = \Sigma_{FF}$ where $\Sigma_{FF}$ is a diagonal matrix with elements $\sigma_{ii} > \sigma_{jj} > 0$ for $i < j$

c. $|\lambda_{i,m}| \leq \overline{\lambda} < \infty$

d. $\{\mathbf{F}_t\}$ is a zero mean covariance-stationary process with a finite moving average representation

**Assumption M1** *(Moments of* $\xi_t = (\xi_{1t}, \xi_{2t}, ..., \xi_{nt})'$*)*

a. $E[\xi_t'\xi_{t+u}/n] = \gamma_{N,t}(u)$, and $\lim_{N\to\infty} \sup_t \sum_{u=-\infty}^{\infty} |\gamma_{N,t}(u)| < \infty$

b. $E[\xi_{it}\xi_{jt}] = \tau_{ij,t,}$with $\lim_{N\to\infty} \sup_t n^{-1}\sum_{i=1}^{n}\sum_{j=1}^{n}|\tau_{ij,t}| < \infty$

c. $\lim_{N\to\infty} \sup_{t,s} n^{-1}\sum_{i=1}^{n}\sum_{j=1}^{n}|cov(\xi_{is}\xi_{it}, \xi_{js}\xi_{jt})| < \infty$

Assumption F1 enables identification of the factors (up to a change of sign) and Assumption M1 limits the amount of serial correlation, cross-correlation and size of fourth moments in the idiosyncratic components. These are essentially the assumptions in Stock and Watson (2002a). We restrict however $\mathbf{F}_t$ to be covariance stationary with a finite moving average representation, which we require to apply the multivariate filter and establish consistency of the approximations.

## 3.2 Estimation of factors

The factor space $G(\mathbf{F}, t)$ generated by $\mathbf{F}_t = \{F_{1t}, F_{2t}, ..., F_{rt}\}$, can be approximated in a number of ways. We will take two approaches. The first is based on Stock and Watson (2002b), who follow Chamberlain and Rothschild (1983) and Connor and Korajczyk (1986) in estimating the factor space by principal components. Specifically, the space $G(\mathbf{F}, t)$ spanned by the so-called static factors $F_{jt}$ is estimated by:

$$\widehat{\mathbf{F}}_t^{SW} = (\widehat{F}_{1t}, \widehat{F}_{2t}, ..., \widehat{F}_{rt})' = \widehat{S}\mathbf{w}_t = (\widehat{S}_1\mathbf{w}_t, \widehat{S}_2\mathbf{w}_t, ..., \widehat{S}_r\mathbf{w}_t)'$$

where $\widehat{S}$ is the matrix of row eigenvectors (ordered according to the eigenvalues) of $\widehat{\Gamma}_{w,0} = \sum_{t=1}^{T^*}\mathbf{w}_t\mathbf{w}_t'$ where $\mathbf{w}_t = (w_{1t}, w_{2t}, ..., w_{nt})'$. Thus, $\widehat{\mathbf{F}}_t^{SW}$ contains simply the first $r$ principal components of $\mathbf{w}_t$. The second approach follows Forni et al. (2005) who, based on a slightly different set of assumptions, propose estimating the factor space by generalized principal components:

$$\widehat{\mathbf{F}}_t^{FLHR} = (\widehat{F}_{1t}, \widehat{F}_{2t}, ..., \widehat{F}_{rt})' = \widehat{Z}\mathbf{w}_t = (\widehat{Z}_1\mathbf{w}_t, \widehat{Z}_2\mathbf{w}_t, ..., \widehat{Z}_r\mathbf{w}_t)'$$

where the $\widehat{Z}_j$ are the generalized eigenvectors of the pair of matrices $(\widehat{\Gamma}_0^{\chi}, \widehat{\Gamma}_0^{\xi})$, which denote, respectively, the (estimated) contemporaneous covariance matrices of $\chi_t = (\chi_{1t}, \chi_{2t}, ..., \chi_{nt})'$ and $\xi_t = (\xi_{1t}, \xi_{2t}, ..., \xi_{nt})'$. This requires estimating $q$, the number of dynamic factors. Following Hallin and Liška (2007) we set $\widehat{q} = 4$ for the U.S. panel and $\widehat{q} = 2$ for the Euro area panel. $(\widehat{\Gamma}_0^{\chi}, \widehat{\Gamma}_0^{\xi})$ are then estimated using the same settings as in Altissimo et al. (2007). $\widehat{\mathbf{F}}_t^{FLHR}$ explores the information in $(\widehat{\Gamma}_0^{\chi}, \widehat{\Gamma}_0^{\xi})$ whereas the estimators $\widehat{\mathbf{F}}_t^{SW}$exploit only $\widehat{\Gamma}_{w,0}$. Ultimately, which method to use is an empirical question that we address explicitly. We will thus compare both methods of estimating the factor space in this context.

The number of static factors, $r$, is usually estimated according to the criteria in Bai and Ng (2002). This is reasonable if our purpose is to estimate the factor space $G(\mathbf{F}, t)$ and subsequently a common component $\chi_{it}$. However, for approximation of signals or forecasting purposes we should check whether additional factors are helpful. Stock and Watson (2002 a, b) analyze this issue concluding that in factor augmented regressions the inclusion of $r$ (or an estimate of $r$) static factors does not necessarily result in a superior forecasting performance. Hence, only the first few (estimated) static factors are usually considered, the exact number (and lags) being determined by the BIC or AIC criteria. Altissimo et al. (2007) also use the first generalized principal components (additionally cleaned of short-run oscillations) as regressors, stopping the inclusion if the increase in the $R$-squared of their approximation becomes negligible. We report results for some choices of $k$, the number of static factors included, estimated either by principal components or by generalized principal components. Specifically:

i) we use the first 2 estimated monthly static factors that are split into 6 quarterly series (see Remark 2 in Section 2.2). This delivers very good approximations and is a useful benchmark, as most gains in the comparison with the univariate filter arise from considering only these covariates. Also, estimation of the SUR-VAR is only feasible with a small number of covariates (however, no complications arise in the non-parametric estimation of the needed second-moments when $k$ is increased).

ii) we add split static factors up to the point where the increase in the estimated $R$-squared of our approximation in the third month of each quarter becomes negligible. This $R$-squared is the squared empirical correlation between our final estimates and the real-time estimates[8], both obtained by considering estimated second moments and static factors obtained with the full-sample. This lead to values of $k$ that never exceeded the estimates of $r$ obtained by applying the criteria CP1 in Bai and Ng (2002) ($\widehat{r} = 6$ for the U.S. and $\widehat{r} = 12$ for the Euro area).

### 3.3 Covariates used in the approximations

In month $t^*$ (which is of quarter $t$), the set of covariates to be used in the approximations contains $k$ static factors estimated as above, each split into 3 quarterly series. Also, two earlier estimates of GDP growth, the Advanced and Preliminary estimates referred in Section 2 (denoted, respectively, $\Delta GDP_t^{Adv}$ and $\Delta GDP_t^{Prelim}$) will be considered in some approximations[9]. All the covariates are organized in such a way that their last observation refers to quarter $t$. We have thus the set of covariates updated at most with all the elements of $Z_{t^*}$, where:

[8]Final estimates are obtained by approximating the signals using $f = T - t$ and $p = 50$ in the univariate filter and then disregarding either the last 12 observations (in the case of the approximation to business cycle fluctuations, $BC(L)$) or the last 4 observations (in the case of the approximation to Smooth Growth, $SG(L)$). This is enough to ensure that only negligible revisions occur once more data becomes available (see Valle e Azevedo, 2007 for business cycle fluctuations and Altissimo et al, 2007 for Smooth Growth). Real-time estimates are obtained with the multivariate filter in the third month of each quarter and use $f = -1$ and $p = 50$.

[9]These two candidate covariates are only available with a reasonable time span in the U.S. However, as we will discuss later, incorporating these early estimates did not produce better results.

$$Z_{t^*}{=}(\widehat{F}^1_{1,t}, \widehat{F}^2_{1,t}, \widehat{F}^3_{1,t}..., \widehat{F}^1_{k,t}, \widehat{F}^2_{k,t}, \widehat{F}^3_{k,t}, \Delta GDP^{Adv}_t, \Delta GDP^{Prelim}_t)',$$

and:

$$\begin{aligned} \{\widehat{F}^1_{l,s}\}^t_{s=1} &= (\widehat{F}_{l,t^*}, \widehat{F}_{l,t^*-3}, \widehat{F}_{l,t^*-6}, ...)' \\ \{\widehat{F}^2_{l,s}\}^t_{s=1} &= (\widehat{F}_{l,t^*-1}, \widehat{F}_{l,t^*-4}, \widehat{F}_{l,t^*-7}, ...)' \quad l = 1, ..., k \\ \{\widehat{F}^3_{l,s}\}^t_{s=1} &= (\widehat{F}_{l,t^*-2}, \widehat{F}_{l,t^*-5}, \widehat{F}_{l,t^*-8}, ...)' \end{aligned} \tag{10}$$

Again, we write $t^*$ to make clear that the factors frequency is monthly, whereas $t$ denotes a quarter. As a remainder, our targets are defined on the GDP of the current quarter, but we compute (or update) the approximations every month. The splitting of the monthly factors into quarterly series is valid, in the sense that vector $(F^1_{1,t}, F^2_{1,t}, F^3_{1,t}..., F^1_{k,t}, F^2_{k,t}, F^3_{k,t})$ of true factors still has a moving average representation, given Assumption F1 d). It is trivial to verify this fact. To apply the multivariate filter we require further that $(\Delta x_t (= \Delta GDP_t), F^1_{1,t}, F^2_{1,t}, F^3_{1,t}..., F^1_{k,t}, F^2_{k,t}, F^3_{k,t},$ $\Delta GDP^{Adv}_t, \Delta GDP^{Prelim}_t)'$ verifies assumption DGP1, which implies that $(\Delta x_t, \Delta GDP^{Adv}_t, \Delta GDP^{Prelim}_t)'$ has a finite moving average representation.

# 4 Consistency of the estimated signals

We discuss now the consistency of the extracted signal to the signal obtained by considering the use of true factors and true second moments of the data. Showing consistency turns out to be a bit more involved than the merging of the proof of consistency of the factors to true factors and consistency of sample second moments to true second moments. We focus on the case where the signal is defined as $B(L)x_t$ with $x_t$ integrated of order 1 (e.g., business cycle fluctuations). The case where the signal is defined on a stationary variable, as in $B(L)\Delta x_t$ (e.g., smooth growth) can be handled in a similar, but simpler, way. More importantly, our result only deals with factors estimated by principal components and with the autocovariance function estimated non-parametrically.

**Theorem 1**

*Suppose the "quarterly" process* $\{x_t\}$ *is integrated of order* $1$ *and consider the approximation to* $y_T = B(L)x_T$ *, where* $B(L) = \sum_{j=-\infty}^{\infty} B_j L^j$ *verifies* $\sum_{j=-\infty}^{\infty} |B_j| < \infty$ *and* $B(1) = 0$*, given by:*

$$\widehat{y}_T^* = \sum_{j=-f}^{p} \widehat{B}_j^{p,f*} x_{T-j} + \sum_{s=1}^{l} \sum_{j=-f}^{p} \widehat{R}_{s,j}^{p,f*} \widehat{F}_{s,T-j}^{Q} + \sum_{s=l+1}^{c} \sum_{j=-f}^{p} \widehat{R}_{s,j}^{p,f*} z_{s,T-j}$$

$\widehat{y}_T^*$ is *the estimated solution to (5) that includes as covariates variables that are observed,* $z_s$ $s = l+1, ..., c$*, as well as estimated "quarterly" factors,* $\widehat{z}_s = \widehat{F}_s^Q$ $s = 1, ..., l$*, obtained by splitting as in (10) "monthly" factors* $\widehat{F}_s$ (with $s \leq r$), *estimated by principal components, of the panel that follows the approximate factor model described by (7),(8) and (9). The weights* $\widehat{W}^* = (\widehat{B}^{*\prime}, \widehat{R}_1^{*\prime}, ..., \widehat{R}_c^{*\prime})'$*solve the system (equivalent to that in (6))*

$$V^* = Q^* \widehat{W}^* \tag{11}$$

*where* $V^*$ *and* $Q^*$ *are obtained by substituting in* $V$ *and* $Q$ *true moments by estimated second moments (using a fixed number,* $M_{\max}$*, of sample autocovariances with data from* $t = 1$ *through* $t = T + f$*). Define the signal obtained with true factors (*$z_s = F_s^Q$ *obtained by splitting monthly factors* $F_s$*) and true second moments as:*

$$\widehat{y}_T = \sum_{j=-f}^{p} \widehat{B}_j^{p,f} x_{T-j} + \sum_{s=1}^{l} \sum_{j=-f}^{p} \widehat{R}_{s,j}^{p,f} F_{s,T-j}^{Q} + \sum_{s=l+1}^{c} \sum_{j=-f}^{p} \widehat{R}_{s,j}^{p,f} z_{s,T-j}$$

*where* $\widehat{W} = (\widehat{B}', \widehat{R}_1', ..., \widehat{R}_c')'$ *solves (6). Then, under Assumption DGP 1 on vector* $(\Delta x_t, z_{1,t}, ..., z_{c,t})$ *, if we choose* $M_{\max} > M$ *and Assumptions M1 and F1 on the factors* $F_s$ hold, *we have* $\widehat{y}_T^* \xrightarrow{p} \widehat{y}_T$

*as $n$ and $T \to \infty$*

**Proof**: Appendix A

# 5 Performance of the indicators

## 5.1 Evaluation of the indicators in a Pseudo Real-time Exercise

The performance of the proposed business cycle and growth indicators (the approximations to $BC(L)$ and $SG(L)$) will be assessed by analyzing their real-time performance. Specifically, and in line with Orphanides and Van Norden (2002), we will look at the approximation errors observed by using our method in real-time. These approximation errors are the difference between the real-time approximations and the highly accurate approximations obtained by considering future data. Obviously, once new data is available, the approximations that explore new information vary near the end of the sample. This variation is due to revisions in the data itself, which we do not analyze here, and revisions due to the nature of the one-sided filters used in the end of the sample (and in our case, along with re-estimation of factors and second moments). The magnitude of the revisions if often large, whatever the method used, even in a multivariate context (Orphanides and Van Norden 2002). The fact that the filters' performance deteriorates near the end of the sample is conceptually not different from the fact that forecasts (generally) deteriorate if the forecast horizon is larger. Any approximation to an unobserved signal (or yet unobserved variable) will suffer revisions. Our approach is an attempt to mitigate revisions (or approximation errors) in the estimates of signals that we believe are relevant for the policy-maker.

We now make explicit in what dimensions our exercise can be seen as a real-time exercise. We make all the necessary transformations in the data, estimate factors, estimate the second moments necessary to solve the projection problem and compute the filter weights in real-time. Further, we take into account all the data release delays, so our datasets are organized in such a way that at each filtering moment we use data that would be in fact available. As an example, U.S. and Euro area GDP of a given quarter is only available in the third month of the next quarter. So, in the first two months of that quarter, the latest observation of GDP refers to 2 quarters before. This makes any attempt to approximate the signals of interest a rather demanding task. Also, if we regard smooth growth as a target useful for forecasting GDP growth itself, in real-time we are in fact forecasting two quarters ahead (one in the case of the third month of the quarter). However, and as already referred, we use the latest vintage of GDP data, that can eventually be subject to revisions. The same is true for a few variables in the panel.

Several statistics will be computed to compare the real-time estimates with the estimates obtained in the "middle" of the sample, that we will denote as "final". These "final" estimates are again obtained by approximating the signals using the whole sample (setting $f = T - t$ and $p = 50$ in the univariate filter with moments derived from an AR model) and then disregarding either the last 12 observations (in the case of the approximation to business cycle fluctuations, $BC(L)$) or the last 4 observations (in the case of the approximation to smooth growth, $SG(L)$)[10]. This is enough to ensure that only negligible revisions occur in the "final" estimates once more data becomes available (see Valle e Azevedo, 2007 for business cycle fluctuations and Altissimo et al., 2007 for smooth growth). We now establish the criteria used to evaluate the real-time performance of the approximations. We focus on (or estimates of):

a) $Corr_t[y_t, \widehat{y}_t]$ , where $\widehat{y}_t$ is the optimal approximation to the signal $y_t$. It is easy to show that if $\widehat{y}_t$ minimizes $E[(y_t - \widehat{y}_t)^2]$, then $E[(y_t - \widehat{y}_t)^2] = (1 - Corr_t[y_t, \widehat{y}_t]^2)Var[y_t]$. The dependence on $t$ is eliminated if we fix $p$ and $f$ in the real-time approximations, as we do. $Corr_t[y_t, \widehat{y}_t]$ is therefore a good measure of the variance of the approximation error. We compute the sample counterpart of this statistic, using the estimated signal (say $\widehat{y}_t^*$) as

[10] Using alternatively multivariate filters and/or different methods to estimate second moments delivers "final" estimates that are indistinguishable from these.

$\widehat{y}_t$ and approximating $y_t$ by the "final" estimates, denoted by $y_t^F$.

b) Noise to Signal Ratio, computed as $\sum_t (\widehat{y}_t^* - y_t^F)^2 / \sum_t (y_t^F - \overline{y}^F)^2$

c) The percentage of times $\widehat{y}_t^* - \widehat{y}_{t-1}^{*\,t}$ (where $\widehat{y}_{t-1}^{*\,t}$ is the approximation to the signals at $t-1$ using information up to time $t$ ) correctly signs the change $y_t^F - y_{t-1}^F$ (see Pesaran and Timmerman, 1992).

d) For the approximation to business cycle fluctuations, the percentage of times $\widehat{y}_t^*$ and $y_t^F$ share the same sign (which gives an indication on whether $\widehat{y}_t^*$ indicates correctly if GDP is below or above the long-term trend)

Our benchmark will be the univariate filter of Christiano and Fitzgerald (2003), using either estimated moments derived from an AR model or from a non-parametric estimator. This would be equivalent to extend the GDP series given the estimated moments and then apply the filter of Baxter and King (1999), as in Watson (2007). In the multivariate approximations, we considered some variations in the filters' settings. These have been described earlier in detail[11]:

- estimation of second order moments is either fully non-parametric (simpler) or by means of initially estimating a VAR for pre-whitening, see section 2.3.

- the factor space of the monthly panel is estimated by principal components (PC) or by generalized principal components (GPC), see section 3.2.

[11]We have also considered approximations that included the early released estimates of GDP in the U.S. (Advanced and Preliminary estimates), but the results were not encouraging. Given the high correlation between the final estimate of GDP and these early estimates in the full sample, we expected that adding these covariates would deliver relevant gains in the approximations done in the first and second months of each quarter. In the third month of a quarter no improvements would be expected since the final GDP estimate from the previous quarter becomes available. We performed the analysis with various settings and found no evidence of significant gains. After inspection of the advanced and preliminary estimates of GDP we found that their correlation with GDP had deteriorated in the part of the sample where we evaluated the approximations. Additionally, we also considered approximations to U.S. signals that included estimated factors from the Euro area and vice-versa. It turned out that improvements, if any, were marginal. We thus do not report the results.

- we include only two monthly factors, split into six quarterly series, or split factors up to the point where the increase in the R-squared of the approximation is negligible[12], see section 3.2.

- estimation of second order moments and factors takes into account only information available at each point (*real-time*) or, alternatively, it uses the whole sample (*today onwards* approximation), while still setting in the filter $f = -1$ in the third month of each quarter and $f = -2$ in the first or second month. With this exercise, we hope to understand the revisions stemming from second moments and factor space uncertainty. We expect these to be less severe as the sample size grows whereas the *real-time* approximation includes poorly estimated objects in the beginning of the evaluation period.

## 5.2 Business cycle fluctuations

We start by reporting the real-time evaluation of our approximations to business cycle fluctuations in the U.S. and in the Euro area. Table 2 contains the evaluation statistics just mentioned, for the approximations done in a third month of the quarter and the variations considered. Table 3 contains additionally the evaluation of the approximations in the first and second month of the quarter for a selection of the best performing approximations (in *real-time* and *today onwards*) of table 2. The main conclusions are the following[13]:

- for the U.S., the multivariate filters clearly outperform the univariate filters in the dimensions analyzed. The gains are only modest in *real-time* when 5 monthly static factors (estimated by principal components) are used in the approximations. This may indicate an overffiting behavior and/or poor estimation in the beginning of the evaluation period.

[12] Again, short time span considerations lead us to consider only two monthly factors in the case of the Euro area approximations.

[13] Although not reported, we also evaluated the performance of the indicators in sub-samples with approximately 24 quarters. The results are qualitatively similar to the ones shown for the full evaluation period.

| Performance with respect to Business Cycle fluctuations ($3^{rd}$ month of the quarter) | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | Correlation | | Noise to Signal | | Sign Concord. | | % Correct Change Sign | |
| | Real Time | Today onwards | Real Time | Today onwards | Real Time | Today onwards | Real Time | Today onwards |
| **U.S.** | | | | | | | | |
| *Benchmark Filters* | | | | | | | | |
| BPF AR | 0.74 | 0.75 | 0.61 | 0.60 | 0.71 | 0.73 | 0.70 | 0.70 |
| BPF KERNEL | 0.72 | 0.78 | 0.62 | 0.57 | 0.69 | 0.70 | 0.67 | 0.69 |
| *with factors,* $k = 5 < \widehat{r}, \widehat{q} = 4$ | | | | | | | | |
| MBPF PC KERNEL | 0.75 | 0.96 | 0.59 | 0.26 | 0.69 | 0.92 | 0.76 | 0.87 |
| MBPF GPC KERNEL | 0.77 | 0.95 | 0.57 | 0.29 | 0.68 | 0.89 | 0.73 | 0.88 |
| *with factors,* $k = 2 < \widehat{r}, \widehat{q} = 4$ | | | | | | | | |
| MBPF PC KERNEL | 0.78 | 0.91 | 0.57 | 0.39 | 0.72 | 0.77 | 0.70 | 0.82 |
| MBPF GPC KERNEL | 0.77 | 0.90 | 0.58 | 0.41 | 0.68 | 0.77 | 0.71 | 0.82 |
| MBPF PC VAR | 0.84 | 0.86 | 0.49 | 0.46 | 0.72 | 0.76 | 0.76 | 0.78 |
| MBPF GPC VAR | 0.83 | 0.87 | 0.50 | 0.46 | 0.77 | 0.71 | 0.77 | 0.79 |
| **Euro Area** | | | | | | | | |
| *Benchmark Filters* | | | | | | | | |
| BPF AR | 0.87 | 0.89 | 0.41 | 0.38 | 0.91 | 0.91 | | |
| BPF KERNEL | 0.87 | 0.88 | 0.43 | 0.40 | 0.91 | 0.91 | | |
| *with factors,* $k = 2 < \widehat{r}, \widehat{q} = 2$ | | | | | | | | |
| MBPF PC KERNEL | 0.86 | 0.89 | 0.46 | 0.48 | 0.77 | 0.82 | | |
| MBPF GPC KERNEL | 0.85 | 0.88 | 0.48 | 0.48 | 0.82 | 0.86 | | |

Table 2

Evaluation statistics for the approximations to business cycle fluctuations in the U.S and in the Euro area in the third month of the quarter. BPF AR - univariate filter with second moments estimated by AR model (BIC criterion for lag length); BPF KERNEL - univariate filter with second moments estimated non-parametrically; MBPF - Multivariate band-pass filter; PC - factor space estimated by principal components; GPC - factor space estimated by generalized principal components; KERNEL - Non-parametric estimation of second moments; VAR - estimation of second moments through pre-whitening with a SUR-VAR for the U.S. (BIC criterion for lag length in the various equations). Evaluation period: 1978(3) - 2000(4) for the U.S. and 1999(2) - 2004(3) for the Euro Area.

For the Euro area, both the univariate and multivariate filters perform very well in the (nonetheless short) evaluation period.

- the fit of the *today onwards* approximations is very high for the multivariate filters, especially in the U.S. and when moments are estimated non-parametrically. However, we notice that in this case, using 5 static monthly factors instead of 2 does not improve substantially the approximation whereas it deteriorates the performance in the *real-time* exercise.

- the results obtained with factors estimated by principal components (PC) are very similar to those obtained with generalized principal components (GPC).

| Performance with respect to Business Cycle fluctuations | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | Correlation | | Noise to Signal | | Sign Concord. | | % Correct Change Sign | |
| | Real | Today | Real | Today | Real | Today | Real | Today |
| **U.S.** | Time | onwards | Time | onwards | Time | onwards | Time | onwards |
| BPF AR | | | | | | | | |
| 1st /2nd months | 0.72 | 0.73 | 0.62 | 0.62 | 0.71 | 0.73 | 0.57 | 0.59 |
| 3rd month | 0.74 | 0.75 | 0.61 | 0.60 | 0.71 | 0.73 | 0.70 | 0.70 |
| MBPF PC KERNEL | | | | | | | | |
| ($k = 5 < \widehat{r}$) | | | | | | | | |
| 1st month | 0.74 | 0.94 | 0.61 | 0.32 | 0.73 | 0.88 | 0.60 | 0.82 |
| 2nd month | 0.75 | 0.95 | 0.60 | 0.29 | 0.74 | 0.90 | 0.59 | 0.89 |
| 3rd month | 0.75 | 0.96 | 0.59 | 0.26 | 0.69 | 0.92 | 0.76 | 0.87 |
| MBPF PC VAR | | | | | | | | |
| ($k = 2 < \widehat{r}$) | | | | | | | | |
| 1st month | 0.79 | 0.82 | 0.55 | 0.52 | 0.73 | 0.72 | 0.70 | 0.74 |
| 2nd month | 0.81 | 0.83 | 0.52 | 0.50 | 0.73 | 0.72 | 0.71 | 0.77 |
| 3rd month | 0.84 | 0.86 | 0.49 | 0.46 | 0.72 | 0.76 | 0.76 | 0.78 |
| **Euro Area** | | | | | | | | |
| BPF AR | | | | | | | | |
| 1st /2nd months | 0.85 | 0.88 | 0.46 | 0.38 | 0.77 | 0.91 | | |
| 3rd month | 0.87 | 0.89 | 0.41 | 0.38 | 0.91 | 0.91 | | |
| MBPF PC KERNEL | | | | | | | | |
| ($k = 2 < \widehat{r}$) | | | | | | | | |
| 1st month | 0.88 | 0.89 | 0.41 | 0.42 | 0.86 | 0.86 | | |
| 2nd month | 0.87 | 0.88 | 0.42 | 0.41 | 0.86 | 0.86 | | |
| 3rd month | 0.86 | 0.89 | 0.46 | 0.48 | 0.77 | 0.82 | | |

Table 3

Evaluation statistics in every month of the quarter, for the approximation to business cycle fluctuations in the U.S and in the Euro area. BPF AR - univariate filter with second moments estimated by AR model (BIC criterion for lag length); BPF KERNEL - univariate filter with second moments estimated non-parametrically; MBPF - Multivariate band-pass filter; PC - factor space estimated by principal components; GPC - factor space estimated by generalized principal components; KERNEL - Non-parametric estimation of second moments; VAR - estimation of second moments through pre-whitening with a SUR-VAR for the U.S. (BIC criterion for lag length in the various equations). Evaluation period: 1978(3) - 2000(4) for the U.S. and 1999(2) - 2004(3) for the Euro Area.

- for the U.S., using the VAR to estimate second moments leads to a superior real-time performance, while the *today onwards* approximation does not differ substantially from the *real-time* one in this case.

- for the U.S., and as expected, the approximations tend to be more accurate in the third month of the quarter, followed by the second month and then the first. The clear monotonicity is worth noting. In the case of the Euro area this does not hold, the quality of the approximations is very similar across the months of the quarter.

Figure 2 displays the best performing *real-time* and *today onwards* approximations (respectively, MBPF PC VAR and MBPF PC KERNEL with $k = 5$) in a third month of the quarter as well as the "final" estimates of business cycle fluctuations for the U.S. We visually confirm the quality of the approximations and argue that our method is the first to provide such accurate approximations to business cycle fluctuations in real-time. In Figure 3 we further compare the "final" estimates to the best multivariate approximations in the third month of the quarter when $4, 3, ..., 0$ quarterly observations of GDP as well as the corresponding monthly series of the panel are missing, and also when $1, ..., 5$ additional quarters of information are available. In the horizontal axis, $-1$ represents the *real-time* estimate (recall, in the filter $f = -1$ in a third month of a quarter since the latest available GDP is from the previous quarter, results are already in table 2), 1 represents the estimate obtained when one future data point is available and so forth. All the measures improve as more data becomes available and in all cases the multivariate filter has by far the best performance. The differences across methods tend to disappear after 5 additional quarters of data are considered.

Figures 4 and 5 display the results for the Euro area. We notice again that in this case the performance of the univariate filter is very good and similar to that of the multivariate filters, with the latter performing superiorly when additional data becomes available. We argue that these approximations with additional data are in practice relevant given the fact that one is interested in detecting a signal that has (persistent) fluctuations with period between 6 and 32 quarters.

## 5.3 Smooth Growth

We now report the real-time evaluation of our approximations to smooth growth in the U.S. and in the Euro area. Table 4 contains the evaluation statistics for the approximations done in a third month of the quarter and the variations considered. Table 5 contains additionally the evaluation of the approximations in a first and second month of the quarter for a selection of

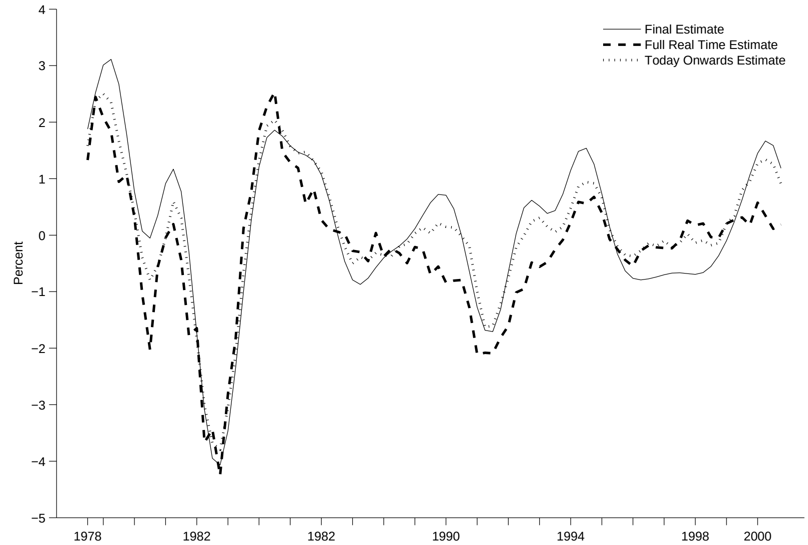


Figure 2

U.S. GDP Business cycle fluctuations: Final estimates, *real-time* (MBPF PC VAR) and *today onwards* (MBPF PC KERNEL, $k = 5$ monthly factors) approximations. Evaluation period: 1978(3)-2000(4).

the best performing approximations (in *real-time* and *today onwards*) of table 4.

The main conclusions are the following:

- the multivariate filters clearly outperform the univariate filters in the dimensions under consideration. Loosing one observation of GDP, as in a first or second month of the quarter, completely deteriorates the performance of the univariate filter but not so much that of the multivariate filters.

- as referred, the multivariate approximations are more accurate in the third month of the quarter, followed by the second month and then the first. In contrast to the case of business cycle fluctuations, there are now considerable gains if factors are used in the Euro area approximations, and very clearly so in the first two months of the quarter. We thus confirm again the usefulness of exploiting infra-quarterly dynamics of the split monthly factors.

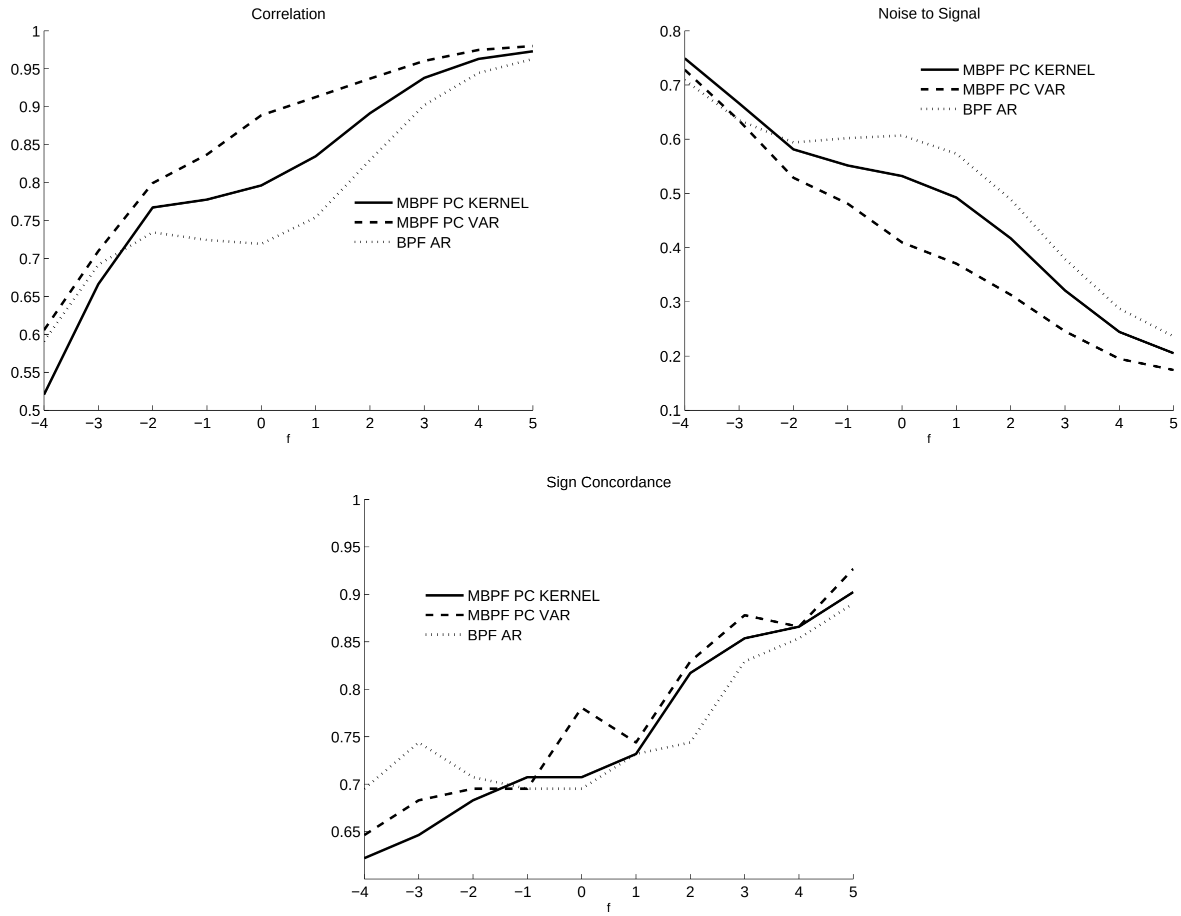


Figure 3

Evaluation of U.S. *real-time* approximations to business cycle fluctuations. Correlation with final estimates, Noise to signal ratio and sign concordance when $f$ future quarters of data are considered (MBPF PC VAR and MBPF PC KERNEL with $k = 5$ monthly factors). Evaluation period: 1978(3)-2000(4).

- again, the fit of the *today onwards* approximations is very high for the multivariate filters, and more so if 4 monthly factors are used and moments are estimated non-parametrically. However, in this case the performance of the *real-time* approximation is relevantly deteriorated in the first and second months of the quarter. This does not occur if only two factors are used and moments are estimated non-parametrically (MBPF PC KERNEL with $k = 2$, not reported in table 5). These findings alert to the fact that more factors do not necessarily produce better out-of -sample results, justifying our claim that the approach of adding covariates until the in-sample fit gains are negligible, followed e.g.

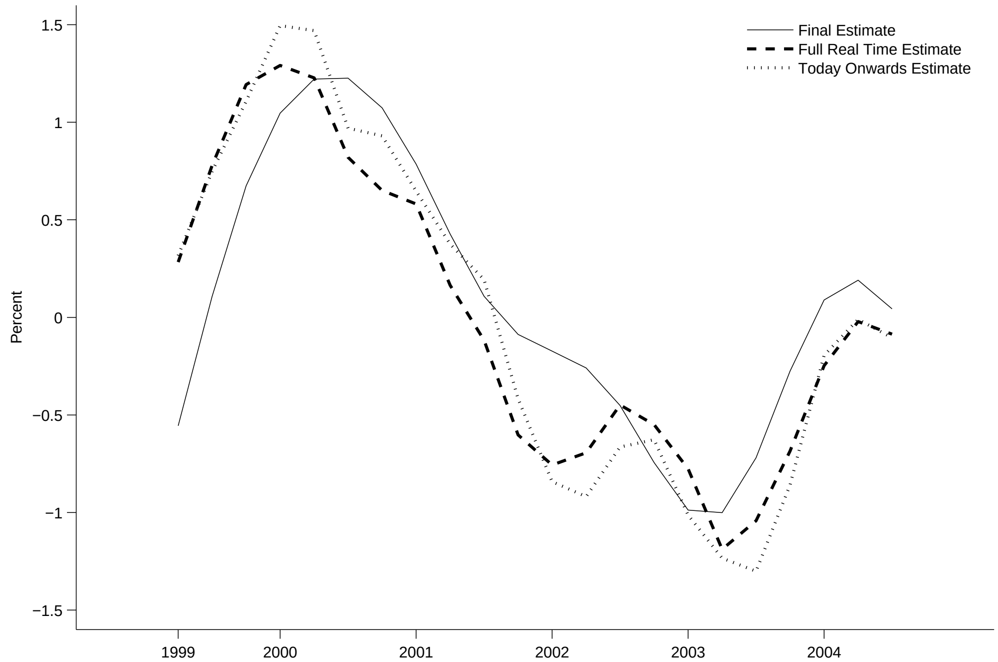


Figure 4

Euro area GDP business cycle fluctuations: Final estimates, *real-time* (MBPF PC KERNEL, $k = 2$ monthly factors)and *today onwards* (MBPF PC KERNEL, $k = 2$ monthly factors) approximations. Evaluation period: 1999(2)-2004(3).

by Altissimo et al. (2007), can be misleading. We found that using $k = 2$ (no more, no less) split monthly factors produced always the best results in *real-time*. Perhaps a larger time dimension would be needed to usefully incorporate a larger number of factors, given the clearly distinct performance of the in-sample (*today onwards*) and out-of-sample (*real-time*) approximations in this case.

- as before, the results obtained with principal components (PC) are very similar to those obtained with generalized principal components (GPC).

- for the U.S., using a VAR to estimate second moments leads to the best performance in *real-time* , while in this case the *today onwards* approximation does not differ substantially from the *real-time* one.

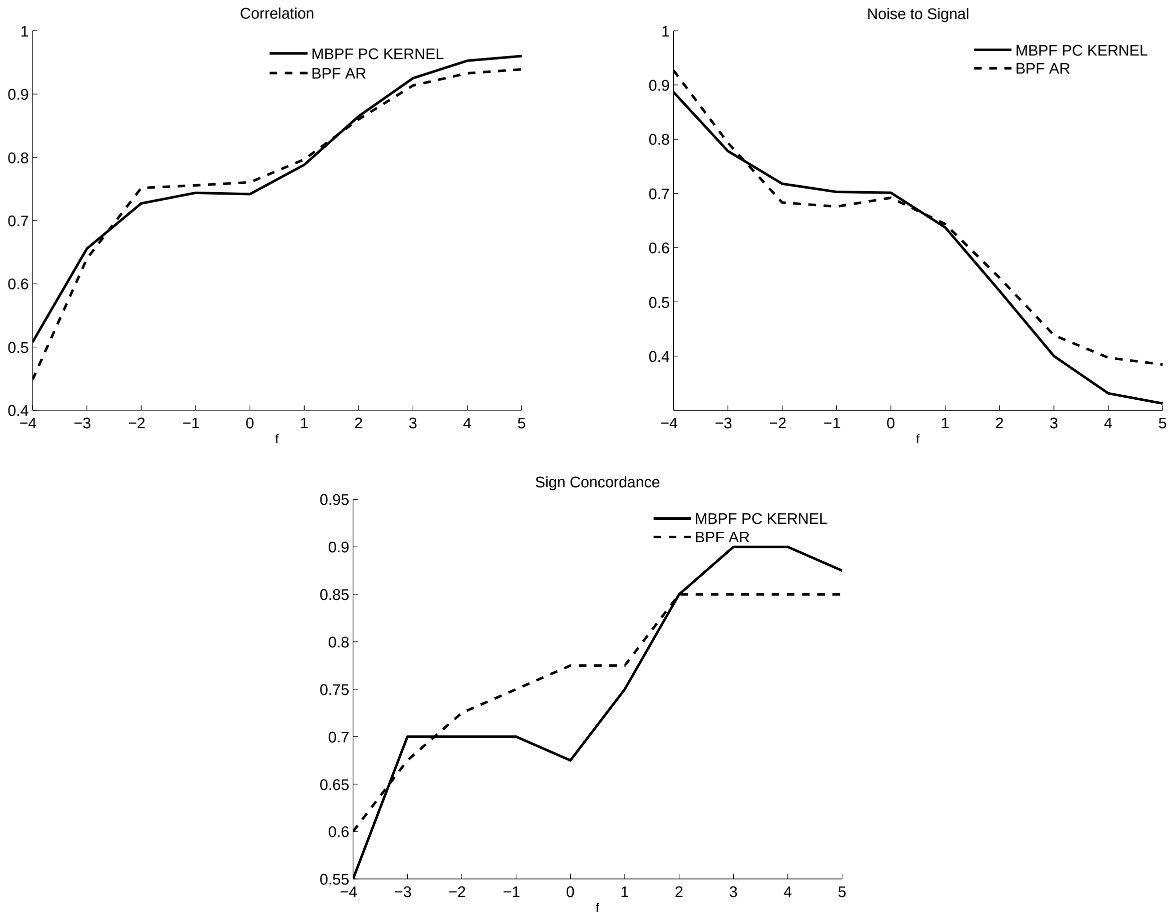


Figure 5

Evaluation of Euro area *real-time* approximations to business cycle fluctuations. Correlation with final estimates, Noise to signal ratio and sign concordance when $f$ future quarters of data are considered (MBPF PC VAR and MBPF PC KERNEL with $k = 2$ monthly factors). Evaluation period: 1999(4)-2003(1).

To further inspect the quality of the approximations, figure 6 displays the "final" estimates of U.S. smooth growth as well as the best performing *real-time* and *today onwards* approximations in a third month of each quarter. As is clear, the multivariate indicators track very accurately the signal. We also analyze in figure 7 the behavior of the best indicators in the third month of the quarter when $4, 3, ..., 0$ quarterly observations of GDP as well as from the covariates are missing (so we are also analyzing the forecasting performance of the approximations to the target), and also when $1, ..., 5$ additional quarters of information are available. Again, in the horizontal axis $-1$ represents the *real-time* estimate ( $f = -1$ in this case, results are

| Performance with respect to Smooth Growth ($3^{rd}$ month of the quarter) | | | | | | |
|---|---|---|---|---|---|---|
| | Correlation | | Noise to Signal | | % Correct Change Sign | |
| **U.S.** | Real Time | Today onwards | Real Time | Today onwards | Real Time | Today onwards |
| *Benchmark Filters* | | | | | | |
| BPF AR | 0.72 | 0.76 | 0.59 | 0.55 | 0.72 | 0.74 |
| BPF KERNEL | 0.70 | 0.81 | 0.60 | 0.50 | 0.76 | 0.79 |
| *with factors,* $k = 4 < \widehat{r}, \widehat{q} = 4$ | | | | | | |
| MBPF PC KERNEL | 0.80 | 0.98 | 0.57 | 0.15 | 0.74 | 0.94 |
| MBPF GPC KERNEL | 0.80 | 0.98 | 0.57 | 0.16 | 0.76 | 0.93 |
| *with factors,* $k = 2 < \widehat{r}, \widehat{q} = 4$ | | | | | | |
| MBPF PC KERNEL | 0.83 | 0.96 | 0.52 | 0.22 | 0.77 | 0.86 |
| MBPF GPC KERNEL | 0.82 | 0.96 | 0.52 | 0.22 | 0.76 | 0.90 |
| MBPF PC VAR | 0.87 | 0.89 | 0.40 | 0.37 | 0.84 | 0.87 |
| MBPF GPC VAR | 0.85 | 0.89 | 0.42 | 0.38 | 0.81 | 0.83 |
| **Euro Area** | | | | | | |
| *Benchmark Filters* | | | | | | |
| BPF AR | 0.77 | 0.78 | 0.52 | 0.51 | | |
| BPF KERNEL | 0.77 | 0.78 | 0.53 | 0.51 | | |
| *with factors,* $k = 2 < \widehat{r}, \widehat{q} = 2$ | | | | | | |
| MBPF PC KERNEL | 0.81 | 0.86 | 0.47 | 0.41 | | |
| MBPF GPC KERNEL | 0.79 | 0.87 | 0.50 | 0.41 | | |

Table 4

Evaluation statistics in a third month of the quarter, for the approximation to Smooth growth in the U.S and in the Euro area. BPF AR - univariate filter with second moments estimated by AR model (BIC criterion for lag length); BPF KERNEL - univariate filter with second moments estimated non-parametrically; MBPF - Multivariate band-pass filter; PC - factor space estimated by principal components; GPC - factor space estimated by generalized principal components; KERNEL - Non-parametric estimation of second moments; VAR - estimation of second moments through pre-whitening with a SUR-VAR for the U.S. (BIC criterion for lag length in the various equations). Evaluation period: 1981(3) - 2002(4) for the U.S. and 1996(4) - 2004(3) for the Euro Area.

already in table 4). The main conclusion is that the multivariate approximations still clearly outperforms the univariate filter while giving an informative (albeit deteriorated) signal even when 3/4 quarters of data are missing. Also, as expected, the differences across methods vanish after 1 additional quarter of information is available.

Figures 8 and 9 repeat the exercise for the Euro area. The conclusions are similar but we notice that in real-time the univariate filter has a performance closer to that of the multivariate filter. Both approximations are less informative than those of the U.S. when a large number of quarters is missing. We believe this is a consequence of the short time span of Euro area data that precludes accurate estimation of the necessary second moments.

| **Performance with respect to Smooth Growth** | | | | | | |
|---|---|---|---|---|---|---|
| | Correlation | | Noise to Signal | | % Correct Change Sign | |
| **U.S.** | Real Time | Today onwards | Real Time | Today onwards | Real Time | Today onwards |
| *BPF AR* | | | | | | |
| 1st month/2nd month | 0.31 | 0.40 | 0.80 | 0.75 | 0.72 | 0.72 |
| 3rd month | 0.72 | 0.76 | 0.59 | 0.55 | 0.72 | 0.74 |
| MBPF PC KERNEL | | | | | | |
| ($k = 4 < \widehat{r}$) | | | | | | |
| 1st month | 0.37 | 0.97 | 0.77 | 0.22 | 0.71 | 0.91 |
| 2nd month | 0.56 | 0.98 | 0.73 | 0.19 | 0.74 | 0.93 |
| 3rd month | 0.80 | 0.98 | 0.57 | 0.15 | 0.74 | 0.93 |
| MBPF PC VAR | | | | | | |
| ($k = 2 < \widehat{r}$) | | | | | | |
| 1st month | 0.74 | 0.77 | 0.56 | 0.52 | 0.71 | 0.73 |
| 2nd month | 0.81 | 0.85 | 0.48 | 0.43 | 0.77 | 0.76 |
| 3rd month | 0.87 | 0.89 | 0.40 | 0.37 | 0.83 | 0.81 |
| **Euro Area** | | | | | | |
| BPF AR | | | | | | |
| 1st month / 2nd month | 0.38 | 0.42 | 0.74 | 0.73 | | |
| 3rd month | 0.77 | 0.78 | 0.53 | 0.51 | | |
| MBPF PC KERNEL | | | | | | |
| ($k = 2 < \widehat{r}$) | | | | | | |
| 1st month | 0.60 | 0.75 | 0.65 | 0.56 | | |
| 2nd month | 0.70 | 0.78 | 0.59 | 0.52 | | |
| 3rd month | 0.81 | 0.86 | 0.47 | 0.41 | | |

Table 5

Evaluation statistics in every month of the quarter, for the approximation to Smooth growth in the U.S and in the Euro area. BPF AR - univariate filter with second moments estimated by AR model (BIC criterion for lag length); BPF KERNEL - univariate filter with second moments estimated non-parametrically; MBPF - Multivariate band-pass filter PC - factor space estimated by principal components; GPC - factor space estimated by generalized principal components; KERNEL - Non-parametric estimation of second moments VAR - estimation of second moments through pre-whitening with a SUR-VAR for the U.S. (BIC criterion for lag length in the various equations) and a VAR(1) for the Euro area. Evaluation period: 1981(3)-2002(4) for the U.S. and 1996(4) - 2004(3) for the Euro Area.

### 5.3.1 Comparison with Eurocoin

The new Eurocoin indicator of Altissimo et al. (2007) targets a monthly measure of quarterly GDP growth free of fluctuations with period less than one year (denoted there as the medium to long run component of output growth, henceforth MLRG). This monthly GDP is obtained through linear interpolation of quarterly figures, which we view as undesirable for statistical and aesthetic reasons. We target instead quarterly GDP growth short of fluctuations with period less than one year, and we do so in the three months of each quarter. The two targets are thus only comparable in the third month of each quarter.

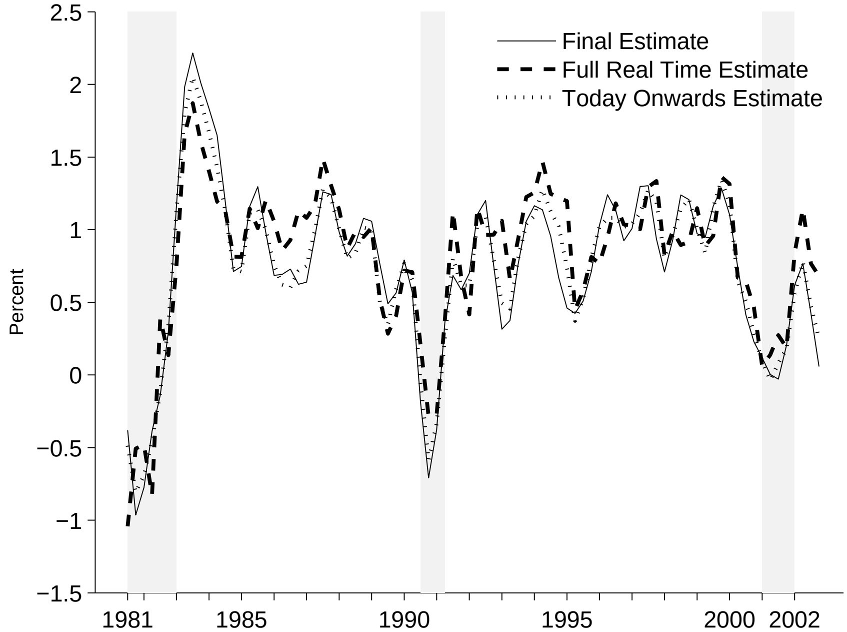


Figure 6

Smooth growth of U.S. GDP: Final estimates, *real-time* (MBPF PC VAR) and *today onwards* (MBPF PC KERNEL, $k = 4$ monthly factors) approximations. Evaluation period: 1981(3)-2002(4).

New Eurocoin is obtained by projecting smooth monthly factors, estimated by generalized principal components, on MLRG. These smooth monthly factors span the subspace of the factor space $G(\mathbf{F}, t)$ that does not contain fluctuations with period less than 12 months. The objective is to have an indicator free of short-run oscillations, just as the target. In our approach this step is not needed and would be undesirable since every vintage of our indicator (that uses all the available information) is by definition smooth, although subject to revisions. The multivariate band-pass filter optimally mitigates these revisions. Furthermore, we can explore infra-quarterly dynamics of the split monthly factors that are instrumental to update efficiently the indicator in the three months of each quarter. Split smooth factors would be almost collinear. More importantly, we are able to incorporate the available observations of GDP in our solution. It turns out that these are the single most important observations in the approximation (as we have verified, and easily seen by analyzing the performance of the univariate filter).

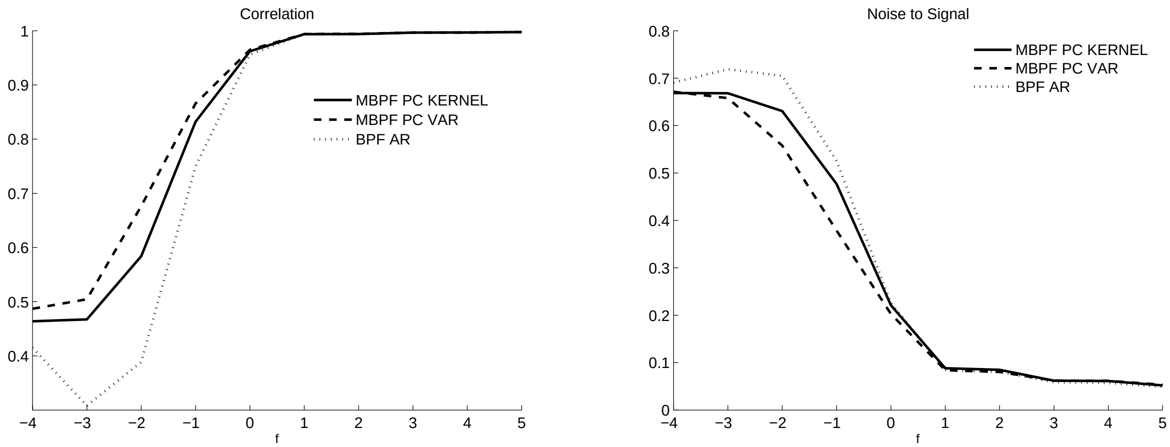


Figure 7

Evaluation of U.S. *real-time* approximations to smooth growth: correlation with final estimates and Noise to signal ratio when $f$ future quarters of data are considered (MBPF PC VAR and MBPF PC KERNEL with $k = 4$ monthly factors). Evaluation period: 1981(3)-2002(4).

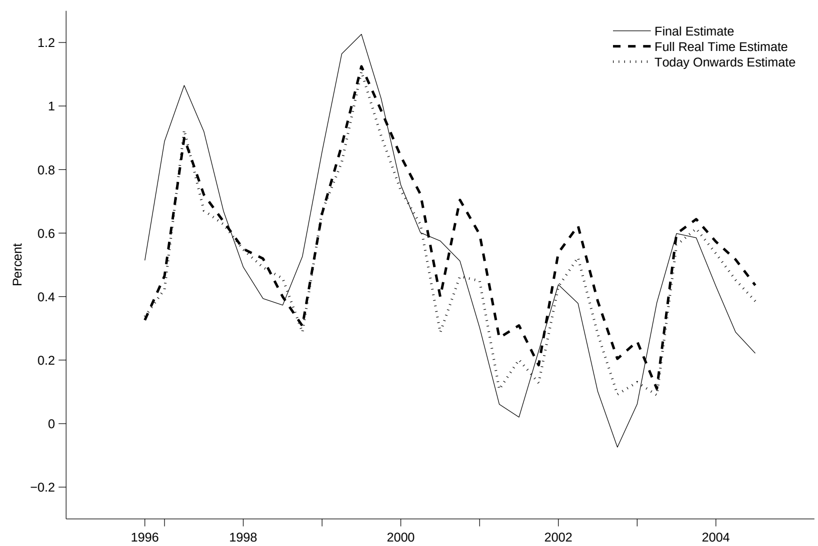


Figure 8

Smooth growth of Euro area GDP: Final estimates, *real-time* (MBPF PC VAR )and *today onwards* (MBPF PC KERNEL, $k = 2$ monthly factors) approximations. Evaluation period: 1996(4)-2004(3).

Finally, we have objections regarding the criterion used to choose the number of static factors that are included in the Eurocoin approximation. Just as we have tried, static factors are added up to the point where the increase in the $R$-squared of the approximation becomes

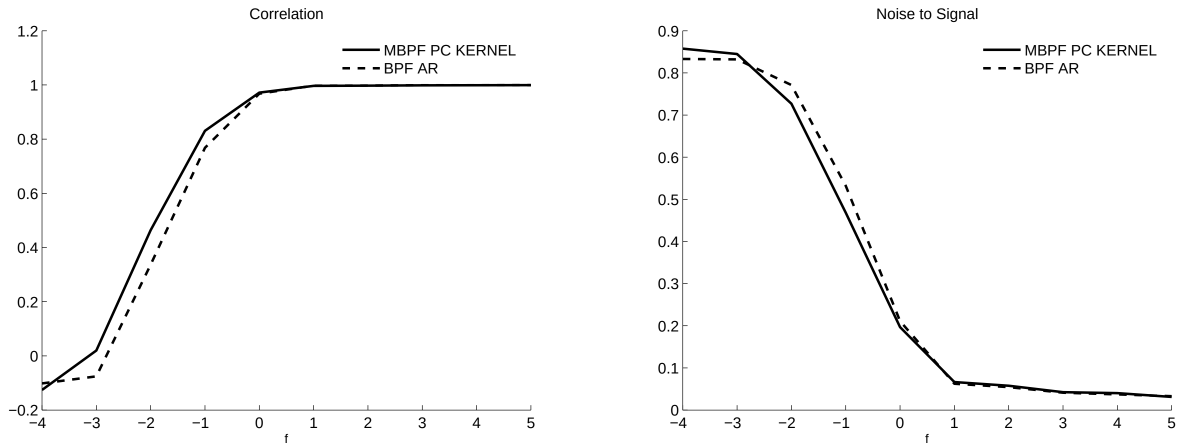


Figure 9

Evaluation of Euro area *real-time* approximations to smooth growth: correlation with final estimates and Noise to signal ratio when $f$ future quarters of data are considered (MBPF PC VAR and MBPF PC KERNEL with $k = 2$ monthly factors). Evaluation period: 1996(4)-2004(3).

negligible. This may lead to overfitting, as seems to be a concern in our indicator that includes split factors using the same criterion. This is true even if we do not increase three split factors in each iteration, but just one. The message here is that a few factors may lead to a superior out-of-sample performance in the context of band-pass filtering, as is clearly the case in our approach.

# 6 Forecasting Performance

## 6.1 Quarterly growth

In this section we report results concerning the forecasting performance of our best multivariate approximations in real-time as well as that of competing methods. To be clear, we will approximate smooth growth at various horizons (quarters ahead) and compare its estimates with the actual observations of quarterly GDP growth. Although our main objective is to approximate a specific signal, we can justify the use of the method for forecasting if we assume from the onset the impossibility of forecasting the high frequencies of GDP growth (those with period less than one year). In theory, if the noisy (or for this matter any other) fluctuations of a time series are unpredictable given a set of covariates, it is not ideal to develop models aimed at approximating them as well. Focusing on approximations to the predictable component of the series may lead to a superior forecasting performance if the assumed restriction (unpredictability at some frequencies) in fact holds. This idea is still far from developed but we believe that important insights are given by our exercise.

Again, we make all the necessary transformations in the data, estimate factors, estimate the second moments necessary to solve the projection problems and compute filter weights in real-time. When lag lengths and number of factors are chosen for some methods, they are so in real-time. We take into account all the data release delays, so that at each forecasting moment we use data that would be in fact available. The exercise focuses on forecasts made in a third month of the quarter, from 1981(3) to 2003(4) in the U.S. and from 1992(1) to 2004(4) in the Euro area. The competing methods are the following:

- univariate approximation to smooth growth with non-parametric estimation of second moments, denoted BPF KERNEL

- univariate approximation to smooth growth, with second moments derived from an AR model (BIC criterion for lag length), denoted BPF AR

- multivariate approximation to smooth growth, with second moments estimated through pre-whitening with a SUR-VAR and $k = 2$ split monthly factors estimated by principal components, denoted MBPF PC VAR (U.S. only)

- multivariate approximation to smooth growth, with second moments estimated non-parametrically and $k = 2$ split monthly factors estimated by principal components, denoted MBPF PC KERNEL

- the linear projection targeting GDP growth with second moments estimated exactly as in MBPF PC VAR above (U.S. only), denoted VAR

- the linear projection obtained with second moments estimated as in MBPF PC KERNEL above (Euro area only), denoted PC KERNEL
- regression of GDP on (a maximum of 2) split factors estimated by principal components and past GDP, with lag length determined by the BIC criterion, exactly as in Stock and Watson (2002a), denoted DI -AR SW. More factors did not lead to a better performance.

The results are presented in tables 6 and 7. The main conclusions follow:

- the multivariate approximations to smooth growth rank very well at one and two steps ahead, dominating overall the "no-smoothing" comparable approximations, VAR for the U.S. and PC KERNEL for the Euro area. That is, using exactly the same estimated second moments while targeting smooth growth instead of GDP growth leads to a superior forecasting performance. This is also generally true for the univariate approximations to smooth growth, but in this case the gains are modest in comparison with the standard autoregression.
- at one step ahead in the U.S., DI AR - SW beats the multivariate approximations to smooth growth, but by a small margin.
- also, in the case of the U.S. the results across methods are very similar for 2 steps ahead, but notice the low RMSE for the benchmark AR in this case (oddly lower than the RMSE of the one step ahead forecast)
- DI AR - SW performs poorly in the Euro area but also in the U.S at 2 and 3 steps ahead.
- In the case of 3 steps ahead forecasts (and higher horizons, results not reported), all methods perform rather poorly, confirming the well-known difficulty of forecasting quarterly GDP growth at long horizons (For recent overviews, see Runstler et al. 2008, and

Angelini et al., 2008). In fact, at 3 steps ahead the root mean squared error of the AR forecast is basically the standard deviation of GDP growth, informing us that we are as better off with the mean growth rate as forecast.

| **Simulated Out-of -Sample Forecasting Results: U.S. GDP growth rate** | | | |
|---|---|---|---|
| | | Relative MSE | |
| Method | One step ahead (current quarter) | 2 steps ahead (1 quarter ahead) | 3 steps ahead (2 quarters ahead) |
| VAR | 0.84 | 1.10 | 1.30 |
| BPF AR | 1.00 | 1.00 | 0.98 |
| BPF KERNEL | 0.95 | 0.99 | 1.00 |
| $k = 2 < \widehat{r}$ | | | |
| **MBPF PC KERNEL** | 0.76 | 0.95 | 0.98 |
| **MBPF PC VAR** | 0.74 | 0.99 | 1.14 |
| DI AR - SW | 0.69 | 1.19 | 1.49 |
| RMSE, AR | 0.00508 | 0.00496 | 0.00524 |

Table 6

Ratio of the Mean squared Error of the forecasts with each method to the Mean squared error of a univariate regression forecast (BIC for lag length). Evaluation period: 1981(3) - 2003(4).

| **Simulated Out-of -Sample Forecasting Results: Euro Area GDP growth rate** | | | |
|---|---|---|---|
| | | Relative MSE | |
| Method | One step ahead (current quarter) | 2 steps ahead (1 quarter ahead) | 3 steps ahead (2 quarters ahead) |
| PC KERNEL | 1.03 | 0.82 | 0.80 |
| BPF AR | 0.93 | 0.99 | 1.00 |
| BPF KERNEL | 0.93 | 1.03 | 1.02 |
| **MBPF PC KERNEL** | 0.78 | 0.84 | 0.81 |
| DI AR - SW | 1.71 | 1.77 | 0.79 |
| RMSE, AR | 0.00297 | 0.0035 | 0.0039 |

Table 7

Ratio of the Mean squared error of the forecasts with each method to the Mean squared error of a univariate regression forecast (BIC for lag length). Evaluation period: 1992(1) - 2004(4).

Overall, the results suggest that our multivariate approximations to smooth growth, while designed for a different purpose, are useful for short-term forecasting of GDP quarterly growth. We argue however that our approximations can have a much more striking role in the forecasting of yearly GDP growth. First, the results in figures 7 and 9 reveal that our approximations to smooth growth at longer horizons (up to 4 quarters) still reveal some of the signal, whereas the forecasts of quarterly growth in tables 6 and 7 and elsewhere reveal the uselessness of all the methods at more than (at most) 2 quarters ahead. Second, the yearly growth rate implied by the quarterly smooth growth rates is indistinguishable from the annual growth implied by the quarter on quarter GDP growth rates (see figure 10). Thus, by approximating accurately smooth growth at various horizons, we will approximate accurately yearly growth. If we use instead useless quarterly growth forecasts to forecast yearly GDP growth, the results are not likely to be promising. The next subsection analyzes this issue in more detail, presenting the evaluation of forecasts of yearly GDP growth made throughout the months of the year for various methods.

### 6.2 Yearly Growth

We forecast, using various methods, yearly GDP growth at various points throughout the year. We do so by forecasting the missing quarterly GDP growth rates and then deriving the implied yearly growth. When using our approximations to smooth growth, we derive yearly growth from the most up-to-date approximations to smooth growth of the relevant quarters. Therefore, in the forecasts that use our approximations, we do not use the already known GDP quarterly rates as an input to derive the yearly growth rate, we use instead the latest vintage of approximations to the relevant quarterly smooth growth rates (recall figure 10 that justifies this approach). For the methods that do not target smooth growth, yearly growth rates are derived from known quarterly rates as well as from the relevant forecasts of quarterly growth. In order to be fair in the comparisons, we substituted the useless (worse than the mean in the evaluation period) quarterly growth forecasts by real-time estimates of the mean growth rate of quarterly GDP. Tables 8 and 9 displays the results for forecasts made in the end of each quarter. In the end of the first quarter (March), no information on quarterly GDP of the current year is available, so

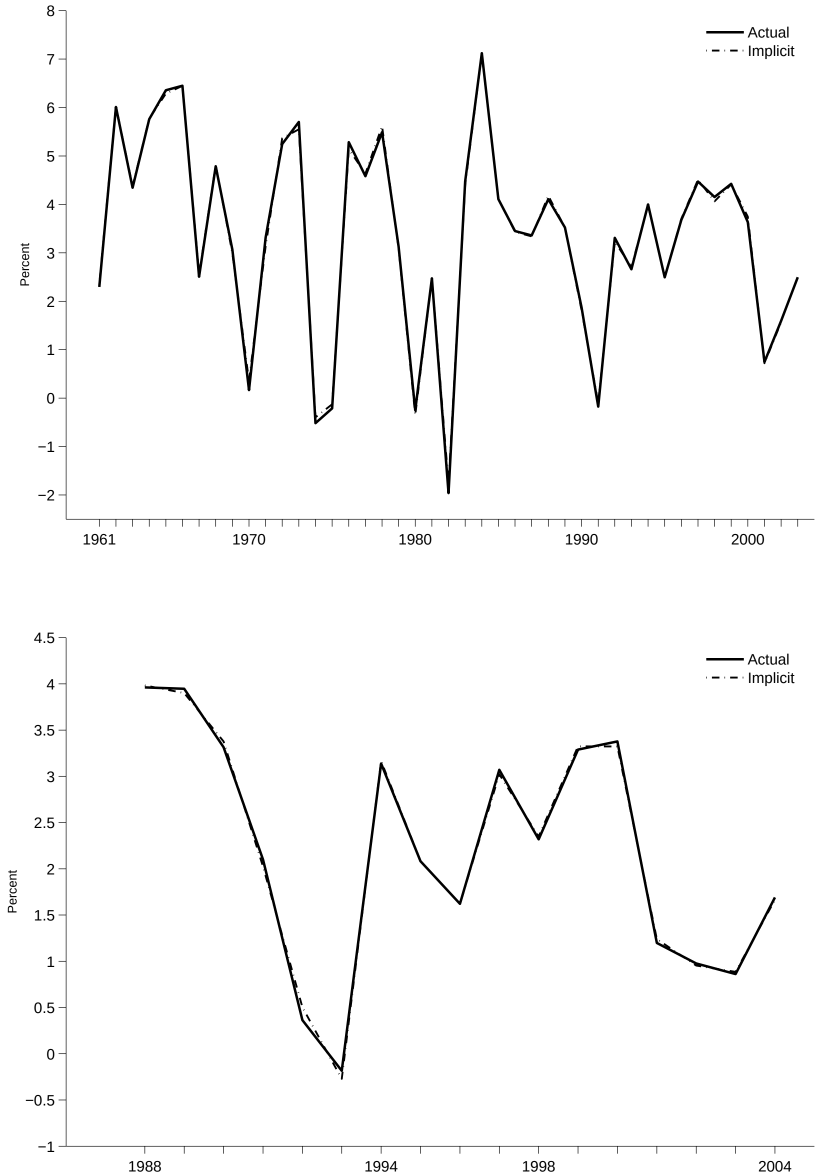


Figure 10

Yearly real GDP growth rate: actual and implicit in smooth growth of quarterly real GDP. Top: U.S., Bottom: Euro area

that forecasts for the four quarters are necessary. In the end of the fourth quarter (December) we have information on GDP growth from the first three quarters.

Table 10 analyzes the forecasting performance of our approximation in the 12 months of the

| Simulated Out-of -Sample Forecasting Results: U.S. GDP yearly growth rate | | | | |
|---|---|---|---|---|
| | Relative MSE of forecasts made at the end of: | | | |
| Methods | 1st quarter | 2nd quarter | 3rd quarter | 4th quarter |
| BPF AR | 1.13 | 0.92 | 1.06 | 0.85 |
| $k = 2 < \widehat{r}$ | | | | |
| **MBPF PC KERNEL** | 1.02 | 0.71 | 0.71 | 0.64 |
| **MBPF PC VAR** | 1.12 | 0.63 | 0.54 | 0.55 |
| DI AR - SW | 0.98 | 0.75 | 0.55 | 0.49 |
| RMSE, AR | 0.0076 | 0.0057 | 0.0035 | 0.0014 |

Table 8

Ratio of the Mean squared error of the forecasts with each method to the Mean squared error of a univariate regression forecast (BIC for lag length). Evaluation period: 1981(3) - 2003(4).

| Simulated Out-of -Sample Forecasting Results: Euro Area GDP yearly growth rate | | | | |
|---|---|---|---|---|
| | Relative MSE of forecasts made at the end of: | | | |
| Methods | 1st quarter | 2nd quarter | 3rd quarter | 4th quarter |
| BPF AR | 1.00 | 0.98 | 1.12 | 0.72 |
| **MBPF PC KERNEL** | 1.02 | 0.83 | 0.99 | 0.81 |
| DI AR - SW | 2.08 | 1.76 | 2.23 | 3.56 |
| RMSE, AR | 0.0069 | 0.0046 | 0.0018 | 0.0007 |

Table 9

Ratio of the Mean squared error of the forecasts with each method to the Mean squared error of a univariate regression forecast (BIC for lag length). Evaluation period: 1993(1) - 2004(4) .

year, and figure 11 shows the forecasts made in March, June, September and December with the actual observations of yearly GDP growth. The results can be summarized as follows:

- at the end of the first quarter all methods perform similarly while afterwards significant gains are achieved by the multivariate methods. For the U.S., MBPF PC VAR performs very well except in March, with DI AR - SW being a tough competitor and MBPF PC KERNEL less so but performing well. For the Euro area, there are also significant (but less striking) gains if MBPF PC KERNEL is used to forecast yearly growth while the performance of DI AR - SW is rather poor.

- table 10 reveals that the quality of the forecasts improves mainly due to the release

| Simulated Out-of -Sample Forecasting Results: GDP yearly growth rate | | |
|---|---|---|
| | RMSE of forecasts | |
| Forecast moment | U.S., *MBPF PC VAR* | Euro, *MBPF PC KERNEL* |
| January | 0.0090 | 0.0108 |
| February | 0.0086 | 0.0108 |
| March | 0.0081 | 0.0070 |
| April | 0.0087 | 0.0062 |
| May | 0.0085 | 0.0058 |
| June | 0.0046 | 0.0042 |
| July | 0.0044 | 0.0035 |
| August | 0.0041 | 0.0029 |
| September | 0.0026 | 0.0018 |
| October | 0.0028 | 0.0020 |
| November | 0.0027 | 0.0018 |
| December | 0.0010 | 0.0006 |

Table 10

Root Mean Squared Error of the forecasts of yearly growth made a the end of each month. Evaluation period: 1981(3) - 2003(4) for the U.S. and 1993(1) - 2004(4) for the Euro area

of GDP quarterly figures, as evidenced by the significant jumps in the RMSE of the approximations in the end of each quarter, exactly when GDP from the previous quarter becomes available. Still, within each quarter new monthly information generally improves the forecasts.

- the performance of the best multivariate filters after the end of the first quarter is remarkable in the U.S. case as evidenced by figure 11. The forecasts correctly predict the clear downturns in 1991 and 2001, and in this case even the March estimate is accurate. For the Euro area the results are not as promising. Only after June can we have a clear and accurate picture of current year growth during the period under consideration. Nonetheless, the 1993 recession is clearly well predicted by the end of March.

# 7 Concluding remarks

We have shown how to usefully integrate the recent developments in the analysis of dynamic factor models in the approximation of band-pass filters (or arbitrary distributed lags of a series of interest). The resulting multivariate band-pass filter, fuelled with factors extracted from a large panel of time series, is reliable and clearly outperforms in various dimensions the optimal univariate approximation. Further, it possesses several advantages over similar (multivariate) attempts to track macroeconomic signals in real-time.

We have analyzed in detail the approximations to two relevant macroeconomic signals related to real activity in the U.S. and in the Euro area: business cycle fluctuations and the smooth growth of real GDP. Our exercise provided real-time approximations that take fully into account the data release delays of all the variables involved. In the analysis of the forecasting performance of smooth growth, we have highlighted an important insight: targeting a smooth version of a time series may be more useful than targeting the sometimes erratic (or unpredictable at high frequencies) original series. Conventional forecasting models fit the variables of interest at every frequency, regardless of the predictive content of the available covariates at each frequency. We plan to further explore this idea in future research.

# Appendix A: Proof of Theorem 1

We begin by collecting some important lemmas used in the proof.

Suppose the approximate factor model described by (7),(8) and (9) satisfies Assumptions F1 and M1. Let $s_i$ denote a variable with value $\pm 1$ and $S$ denote a column with $s_i$'s. Then:

**Lemma 1**: Let $q_t$ denote a sequence of random variables such that: $T^{-1}\sum_{t=1}^{T} q_t^2 \xrightarrow{p} \sigma_q^2$, $T^{-1}\sum_{t=1}^{T-i}\mathbf{F}_{t+i}q_t \xrightarrow{p} \Gamma_{Fq}(i)$ and $T^{-1}\sum_{t=i+1}^{T}\mathbf{F}_{t-i}q_t \xrightarrow{p} \Gamma_{Fq}(-i)$ for any fixed $i$ such that $0 \leq |i| \leq T-1$.Then, the $s_i$ can be chosen so that: $T^{-1}\sum_{t=1}^{T-i} S\widehat{\mathbf{F}}_{t+i}q_t \xrightarrow{p} \Gamma_{Fq}(i)$ and $T^{-1}\sum_{t=i+1}^{T} S\widehat{\mathbf{F}}_{t-i}q_t \xrightarrow{p} \Gamma_{Fq}(-i)$ for any fixed $i$ such that $0 \leq |i| \leq T-1$

This is a direct consequence of **R16** in Stock and Watson (2002b).

**Lemma 2**: $T^{-1}\sum_{t=i+1}^{T} \widehat{\mathbf{F}}_t\widehat{\mathbf{F}}_{t-i} \xrightarrow{p} \Gamma_{FF}(i) = E[\mathbf{F}_t\mathbf{F}_{t-i}]$ for any fixed $i$ such that $0 \leq i \leq T-1$

Slight adaptation of **R18** in Stock and Watson (2002b), obtained by setting $q_t = s_j\widehat{F}_{j,t-i}$

$j = 1, ..., r$ in Lemma 1 and noting that $T^{-1} \sum_{t=i+1}^{T} \widehat{F}_{j,t-i}^2 \xrightarrow{p} \sigma_{F_j}^2$ by **R13** in Stock and Watson (2002b).

**Lemma 3** (Theorem 1 of Stock and Watson, 2002b): Suppose $k$ factors are estimated ($k$ may be $\leq$ or $> r$, the true number of factors). The $s_i$ can be chosen so that:

For $i = 1, 2, ..., r$ , $T^{-1} \sum_{t=1}^{T} (s_i \widehat{F}_{it} - F_{it})^2 \xrightarrow{p} 0$

For $i = 1, 2, ..., r$ , $T^{-1} \sum_{t=1}^{T} s_i \widehat{F}_{it} \xrightarrow{p} F_{it}$

For $i = r + 1, ..., k$ , $T^{-1} \sum_{t=1}^{T} \widehat{F}_{it}^2 \xrightarrow{p} 0$.

Throughout, we assume factors $F_{it}$ are identified (we don't need to sign-adjust the estimators $\widehat{F}_{it}$). Otherwise, trivial changes of sign in true factors, second moments and weights would be needed, but we omit them for tractability. To prove Theorem 1 we first note that assumption F1 d) implies that $\mathbf{F}_t^Q = (F_{1,t}^Q, F_{2,t}^Q, F_{3,t}^Q, ..., F_{l-2,t}^Q, F_{l-1,t}^Q, F_{l,t}^Q)$ , where $l \leq 3r$, has a finite moving average representation. This is easy to check. Suppose for simplicity that $r = 1$, there is only one monthly factor $F_{t^*}$,that can be written by F1 d) as:

$$F_{t^*} = \sum_{j=0}^{M^*} \Psi_j \varepsilon_{t^*-j},$$

where $\{\varepsilon_{t^*}\}$ is a white noise sequence with mean zero and variance $\sum_{\varepsilon}$. Obviously $\gamma^*(j) = E[F_{t^*} F_{t^*-j}]$ does not depend on $t^*$ and equals zero for $|j| > M^*$. The vector $\mathbf{F}_t^Q$ of quarterly factors is $\mathbf{F}_t^Q = (F_{1,t}^Q, F_{2,t}^Q, F_{3,t}^Q)' = (F_{t^*}, F_{t^*-1}, F_{t^*-2})'$ whereas $\mathbf{F}_{t-1}^Q = (F_{1,t-1}^Q, F_{2,t-1}^Q, F_{3,t-1}^Q)' = (F_{t^*-3}, F_{t^*-4}, F_{t^*-5})$ and so on. Now, $E[\mathbf{F}_t^Q] = 0$ and for any integer $k$, $\Gamma^Q(k) = E[\mathbf{F}_t^Q \mathbf{F}_{t-k}^Q] = [\gamma^*(3(k-i) - 3(k-j))]_{i,j=1,2,3}$ , which does not depend on $t$. Also, for order $k = M > M^*/3$ we obtain $\Gamma^Q(k) = 0$ since $\gamma^*(j) = 0$ for $|j| > M^*$. Hence, $\mathbf{F}_t^Q$ has a vector finite moving average representation:

$$\mathbf{F}_t^Q = \sum_{j=0}^{M_F} \Psi_j^Q \eta_{t-j},$$

where $\{\eta_t\}$ is a white noise sequence with mean zero and variance $\sum_\eta$. The argument is similar for $r > 1$. Assumption DGP 1 on the vector $(\Delta x_t, z_{1,t}, ..., z_{c,t})$ is needed to apply the filter and is consistent with assumption F1 d) and the implied finite moving average representation of the quarterly factors $\mathbf{F}_t^Q$. It guarantees additionally covariance stationarity of the quarterly factors but together with the additional covariates , $z_{l+1,t}, ..., z_{c,t}$ as well as $\Delta x_t$. Also, it implies that the additional covariates , $z_{l+1,t}, ..., z_{c,t}$ as well as $\Delta x_t$ have a finite moving average representation.

*Consistency of second moments*

Take the vector $(\Delta x_t, z_{1,t}, ..., z_{c,t})$, normalized for simplicity to have zero mean, and consider its autocovariance function $\Gamma(k)$. The estimate $\widehat{y}_T^*$ of $\widehat{y}_T$ uses as covariates estimated quarterly factors $\widehat{z}_s = \widehat{F}_s^Q$ $s = 1, ..., l$ as well as observed $z_s$ $s = l+1, ..., c$. Take the estimator of the autocovariance function $\Gamma(k)$, with entries $\widehat{\gamma}_{\cdot,\cdot}(k)$, given by:

$$\widehat{\Gamma}(k) = \kappa(k,T)\frac{1}{T}\sum_{t=k+1}^{T+f}((\Delta x_t, \widehat{z}_{1,t}, ..., \widehat{z}_{l,t}, z_{l+1,t}, ..., z_{c,t})')((\Delta x_{t-k}, \widehat{z}_{1,t-k}, ..., \widehat{z}_{l,t-k}, z_{l+1,t-k}, ..., z_{c,t-k})')' \tag{a.0}$$

We set $\kappa(k,T)$=$(1 - \dfrac{k}{M_{\max}+1})$ for $T$ less than some large $T_1$ and $\kappa(k,T)$=1 otherwise. Using this definition of $\kappa(k,T)$ and standard results on the sample autocovariance function (see, e.g., Theorem 11.2.1 of Brockwell and Davis, 1991) we have that:

$\widehat{\gamma}_{\Delta x, z_j}(k) \xrightarrow{p} \gamma_{\Delta x, z_j}(k)$, for any fixed $k$ such that $|k| < T$ and $l < j \leq c$

Also, $\widehat{\gamma}_{z_i, z_j}(k) \xrightarrow{p} \gamma_{z_i, z_j}(k)$ for any fixed $k$ such that $|k| < T$ and $l < i, j \leq c$

However, using Lemma 1 and the definition of $F_{i,t}^Q$ we have further that $\widehat{\gamma}_{\Delta x, \widehat{z}_j}(k) \xrightarrow{p} \gamma_{\Delta x, z_j}(k)$ as $n$ and $T \to \infty$ for any fixed $|k| < T$ and $1 \leq j \leq l$ and $\widehat{\gamma}_{z_i, \widehat{z}_j}(l) \xrightarrow{p} \gamma_{z_i, z_j}$ for any fixed $k$ such that $|k| < T$ with $1 \leq j \leq l$ and $l+1 < i \leq c$

Also, using Lemma 2 and the definition of $F^{Q}_{i,t}$, we have $\widehat{\gamma}_{\widehat{z}_i,\widehat{z}_j}(k) \xrightarrow{p} \gamma_{z_i,z_j}$ as $n$ and $T \to \infty$ for any fixed $k$ such that $|k| < T$ and $1 \leq i,j \leq l$

This implies:

$$\widehat{\Gamma}(k) \xrightarrow{p} \Gamma(k) \text{ as } n \text{ and } T \to \infty \text{ , for any fixed } k \text{ such that } |k| < T \tag{a.1}$$

*Consistency of the weights*

The optimal weights to the solution $\widehat{y}_T$, $\widehat{W} = (\widehat{B}', \widehat{R}'_1, ..., \widehat{R}'_c)'$, solve the linear system (see Valle e Azevedo, 2007):

$$V = Q\widehat{W} \tag{a.2}$$

where:

$$Q = \begin{bmatrix} Q_{-f} \\ \vdots \\ Q_{p-1} \\ \tilde{Q}_p \\ U \end{bmatrix}, U = \begin{bmatrix} \underbrace{1 \;\; 1 \cdots 1}_{p+f+1} & \underbrace{0 \;\; 0 \cdots 0 \;\; 0 \;\; 0 \cdots 0}_{(p+f+1)\times n} \end{bmatrix}$$

Each entry of of $Q$ contains either one autocovariance with finite maximum lag $\overline{L} < M$, or a finite linear combination of $\widehat{\gamma}_{\Delta x}(l)$, $l \leq M$ (if it multiplies in the system an unknown $\widehat{B}^{p,f}_j$). Also,

$$V = \begin{bmatrix} S_{-f} & \cdots & S_{p-1} & \tilde{S}_p & 0 \end{bmatrix}' \text{ with } S_j = S_{j-1} + V_j, j = -f+1, ..., p$$

where

$$V_j = \begin{bmatrix} B_j\gamma_{\Delta x}(0) + \sum_{i=1}^{M}(B_{j+i} + B_{j-i})\gamma_{\Delta x}(i) \\ B_j\gamma_{z_1,\Delta x}(0) + \sum_{i=1}^{M}(B_{j-i}\gamma_{z_1,\Delta x}(i) + B_{j+i}\gamma_{z_1,\Delta x}(-i)) \\ \vdots \\ B\gamma_{z_c,\Delta x}(0) + \sum_{i=1}^{M}(B_{j-i}\gamma_{z_n,\Delta x}(i) + B_{j+i}\gamma_{z_c,\Delta x}(-i)) \end{bmatrix}, \ j = -f+1, ..., p$$

$\tilde{S}_p$ is defined as $S_p$ above but with the first row deleted. Finally,

$$S_{-f} = \int_{-\pi}^{\pi} [e^{i\omega f}\overline{S}_{\Delta x, z_1, ..., z_n}(\omega)\frac{B(e^{i\omega})}{1-e^{i\omega}}]d\omega \tag{a.3}$$

where $\overline{S}_{\Delta x, z_1, ..., z_n}(\omega)$ is the Fourier transform of $\overline{S}_{\Delta x, z_1, ..., z_n}(z)$, a column of the autocovariance generating function. That is,

$$\overline{S}_{\Delta x, z_1, ..., z_n}(z) = \begin{bmatrix} S_{\Delta x}(z) \\ S_{z_1,\Delta x}(z) \\ \vdots \\ S_{z_n,\Delta x}(z) \end{bmatrix} = \frac{1}{2\pi}. \begin{bmatrix} \gamma_{\Delta x}(0) + \gamma_{\Delta x}(1)(z+z^{-1}) + ... + \gamma_{\Delta x}(M)(z^M + z^{-M}) \\ \gamma_{z_1,\Delta x}(-M)z^{-M} + ... + \gamma_{z_1,\Delta x}(0) + ... + \gamma_{z_1,\Delta x}(M)z^M \\ \vdots \\ \gamma_{z_n,\Delta x}(-M)z^{-M} + ... + \gamma_{z_n,\Delta x}(0) + ... + \gamma_{z_n,\Delta x}(M)z^M \end{bmatrix}$$

We derive now a closed form expression for $S_{-f}$ , needed to carry out with the proof. First, note that $B(e^{i\omega}) = (1-e^{i\omega})b(e^{i\omega})$ for some function $b(e^{i\omega}) = \sum_{j=-\infty}^{\infty} b_j e^{i\omega j}$, given the fact that $B(1) = 0$ ($B(e^{i\omega}) = 0$ for $\omega = 0$). Thus, $S_{-f} = \int_{-\pi}^{\pi} [e^{i\omega f}\overline{S}_{\Delta x, z_1, ..., z_n}(\omega)b(e^{i\omega})]d\omega$. Noting that $\int_{-\pi}^{\pi} e^{i\omega j}d\omega = 0$, $j = \pm 1, \pm 2, ...$ and obviously $2\pi$ if $j = 0$, we conclude that $S_{-f}$ is the coefficient on $z^0$ of $b(z)z^{-f}\overline{S}_{\Delta x,z_1,...,z_n}(z)$. Thus,

$$S_{-f} = \begin{bmatrix} b_f\gamma_{\Delta x}(0) + \sum_{i=1}^{M}(b_{f+i} + b_{f-i})\gamma_{\Delta x}(i) \\ b_f\gamma_{z_1,\Delta x}(0) + \sum_{i=1}^{M}(b_{f-i}\gamma_{z_1,\Delta x}(i) + b_{f+i}\gamma_{z_1,\Delta x}(-i)) \\ \vdots \\ b_f\gamma_{z_c,\Delta x}(0) + \sum_{i=1}^{M}(b_{f-i}\gamma_{z_n,\Delta x}(i) + b_{f+i}\gamma_{z_c,\Delta x}(-i)) \end{bmatrix}$$

Instead, $\widehat{y}_T^*$, is obtained with weights $\widehat{W}^* = (\widehat{B}^{*\prime}, \widehat{R}_1^{*\prime}, ..., \widehat{R}_c^{*\prime})'$ that solve the system (equivalent to that in (a.2))

$$V^* = Q^*\widehat{W}^* \tag{a.4}$$

where $V^*$ and $Q^*$ are obtained by substituting in $V$ and $Q$ true moments by estimated second moments (using sample autocovariances as in (a.0) with data from $t = 1$ through $t = T + f$). Given the consistency of estimated autocovariances to true covariances in (a.1), it follows trivially that $Q^* \xrightarrow{p} Q$. Given the finiteness of the sums in $V_i$ and $S_{-f}$ it follows that $V_i^* \xrightarrow{p} V_i$ and $S_{-f}^* \xrightarrow{p} .S_{-f}$. Repeating the argument, given the definition of $\tilde{S}_p$ and the fact that $S_j = , \sum_{i=-f}^{j} V_i + S_{-f}$ , $j = -f, ..., p$ with $V_{-f} = 0$, we conclude that $V^* \xrightarrow{p} V$ .Finally, it follows by Slutsky that:

$$\widehat{W}^* = Q^{*-1}V^* \xrightarrow{p} \widehat{W} = Q^{-1}V \tag{a.5}$$

*Consistency of the approximation*

Now, given that

$$\widehat{y}_T = \sum_{j=-f}^{p} \widehat{B}_j^{p,f} x_{T-j} + \sum_{s=1}^{l}\sum_{j=-f}^{p} \widehat{R}_{s,j}^{p,f} F_{s,T-j}^{Q} + \sum_{s=l+1}^{c}\sum_{j=-f}^{p} \widehat{R}_{s,j}^{p,f} z_{s,T-j}$$

we have:

$$\widehat{y}_T^* - \widehat{y}_T = \sum_{j=-f}^{p}(\widehat{B}_j^{p,f*} - \widehat{B}_j^{p,f})x_{T-j} + \sum_{s=1}^{l}\sum_{j=-f}^{p}(\widehat{R}_{s,j}^{p,f*} - \widehat{R}_{s,j}^{p,f})F_{s,T-j}^{Q} +$$
$$+ \sum_{s=1}^{l}\sum_{j=-f}^{p}\widehat{R}_{s,j}^{p,f}(\widehat{F}_{s,T-j}^{Q} - F_{s,T-j}^{Q}) + \sum_{s=l+1}^{c}\sum_{j=-f}^{p}(\widehat{R}_{s,j}^{p,f*} - \widehat{R}_{s,j}^{p,f})z_{s,T-j}$$

Each of the terms $(\widehat{B}_j^{p,f*} - \widehat{B}_j^{p,f})$ $j = -f, -f+1, ..., p$ is $o_p(1)$ by (a.5), and the same is true of $(\widehat{R}_{s,j}^{p,f*} - \widehat{R}_{s,j}^{p,f})$ $s = 1, ..., c$; $j = -f, -f+1, ..., p$. Further, each of the terms $\widehat{F}_{s,T-j}^{Q} - F_{s,T-j}^{Q}$ $s = 1, ..., c$ is $o_p(1)$ by Lemma 3 and the definition of $F_{i,t}^{Q}$. Finally, $x_{T-j}$, $F_{s,T-j}^{Q}$ and $z_{s,T-j}$ $j = -f, -f+1, ..., p$ are $O_p(1)$ since they have finite second moments by assumption DGP 1. We have a finite number of terms so this implies that $\widehat{y}_T^* - \widehat{y}_T$ is $o_p(1)$.

■

# Appendix B: Data

In the case of the U.S., we use the same vintage panel of U.S. monthly time series studied in Stock and Watson (2005) to estimate the common factors. The panel was downloaded directly at *http://www.princeton.edu/˜mwatson* and transformed as suggested in Stock and Watson (2002a). This dataset covers the period from February of 1960 to December of 2003 ($T^* = 527$), and the $n = 132$ available time series were realigned to account for release delays. In addition, as quarterly data we consider U.S. real GDP from the first quarter of 1959 to the third quarter of 2003 and its advance and preliminary estimates (in growth rates) from the first quarter of 1976 to the third quarter of 2003. These latter series were promptly provided by the Bureau of Economic Analysis.

In the case of the Euro area, we thank Giovanni Veronese for providing us with a transformed and realigned version of the dataset used to compute the New Eurocoin Indicator of Altissimo et al. (2007). The dataset encompasses 144 monthly economic variables of national and Euro area aggregate economies from May of 1987 through August of 2005 ($T^* = 220$). A detailed description of the monthly variables and its transformations is available upon request. The dataset also contains information on Euro area GDP growth rate from the first quarter of 1987 to the first quarter of 2005. Further details on this series can be found in Altissimo et al. (2007).

Both panels include several types of macroeconomic variables that can be organized into groups, specifically price indexes, labor market statistics, survey indicators, production and consumption series, financial variables, interest rates, money aggregates and trade statistics.

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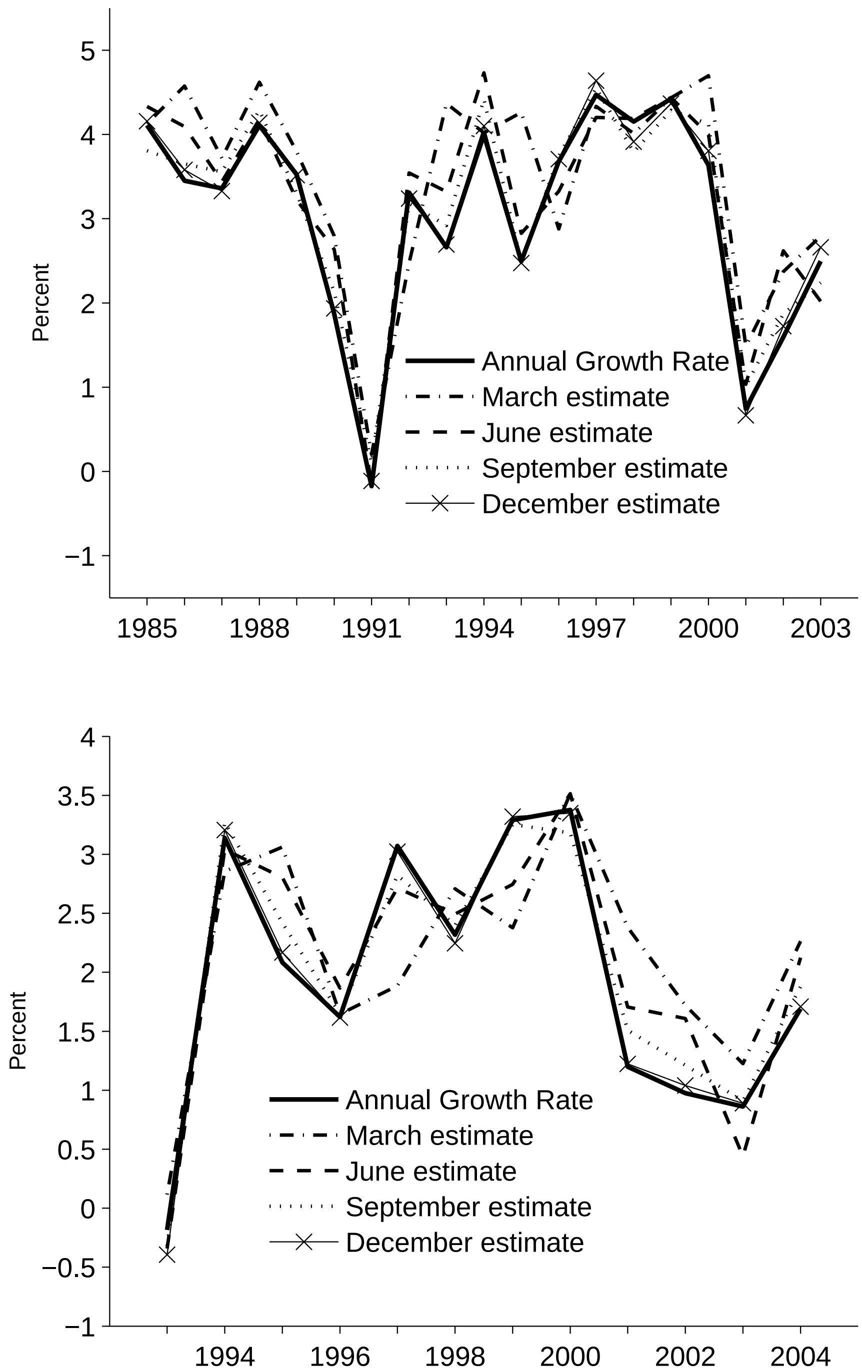


Figure 11

Yearly GDP growth rate and real-time forecasts made at the end of March, June, September and December.

Top: U.S., Bottom: Euro Area.

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