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Statistical Arbitrage with Default and Collateral

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In this paper we study the implications of the absence of statistical arbitrage opportunities (SAO) in a two-period incomplete market economy where default is allowed but there are collateral requirements. We study the existence of state price deflators and the existence of a solution for the individual optimality problem, obtaining modified versions of the fundamental theorems of asset pricing. Then, we address the existence of equilibrium.

Keywords: Equilibrium, Collateral, Statistical Arbitrage

JEL classification: D52, G11, G12

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1. INTRODUCTION

It is very well known that default is a real and important fact in the economy. One way to get protection against default is to require collateral, as happens in the Collateralized Mortgages Obligations market (CMO). In the literature, several models have been suggested in order to price claims which are subject to default, *i.e.*, the possibility that sellers of claims will not honor their obligations. Among these models we have the ones that cover the possibility of default by demanding collateral. The first work in this context was carried out by Dubey, Geanakoplos and Zame (1995), who modeled collateral as a bundle of durable goods. In their model they avoid adverse selection problems since all the lenders know what to expect to receive in each state of nature (the minimum between the value of the claim and the value of the exogenously fixed collateral) and utility penalties are absent.

Recently, Geanakoplos and Zame (2007) studied the impact of collateral on commodity and asset prices, since in many situations collateral can be scarce. Araujo, Páscoa and Torres-Martínez (2002) prove that collateral avoids Ponzi schemes. They prove the existence of equilibrium without using either debt constraints or transversality conditions. Kubler and Schmedders (2003) have shown that the use of collateral facilitates the computation of equilibria.

In none of the above models was there an analysis of arbitrage implications. Araujo, Fajardo and Páscoa (2005) were the first to present the implications of absence of arbitrage in an endogenous collateral model where borrowers are obliged to constitute collateral at the same time they short sales and utility penalties are absent. This collateral is modeled as a bundle of goods, as in in Dubey, Geanakoplos and Zame (1995). In the exogenous collateral context, we have the results of Fajardo (2005) and Orrillo (2005), who use the arbitrage definition introduced by Araujo, Fajardo and Páscoa (2005).

However, in many cases the pricing implications derived by the absence of pure arbitrage strategies are not too relevant from a practical point of view, because the spreads obtained for asset prices are too wide and give no useful information. Hence, in order to have more realistic spreads we need to refine our notion of arbitrage.

There is evidence that traders operate in the market using different ratios or measures to obtain spreads and in the literature many models have been proposed to deal with trades of this kind. In particular, we have the models that use the *statistical arbitrage* concept, as suggested by Bondarenko (2003). These arbitrageurs use strategies that produce positive returns on average in each state of nature. An analogous concept is used in Amaro de Matos and Lacerda (2006) to study the pricing of contingent claims under the presence of liquidity constraints.

In this paper we use three different notions of statistical arbitrage, equivalent under some conditions and very related to the one introduced by Bondarenko (2003). Then, we study the implications of the absence of statistical arbitrage opportunities. We obtain a spread in the context of exogenous collateral requirements and we prove that our arbitrage concept is compatible with the existence of the solution to the individual utility maximization problem.

The paper is organized as follows: in Section 2 we present the model and introduce our notation; in Section 3 we define generalized statistical arbitrage and state our main results; in Section 4 we address the existence of equilibrium; in the last section we state our conclusions.

2. Model

We consider an exchange economy over two periods. In the second period, a finite number of states $s \in S = \{1, 2, ..., S\}$ can occur. There are H agents, J assets and L durable goods. In the first period, there is a market where physical commodities and assets are traded against each other. In the second period asset returns are delivered.

Let $\theta_j \geq 0$ be the number of units of asset j the consumer bought and $\varphi_j \geq 0$ be the number of units he sold. Every sale should be backed by a bundle of goods (collateral). The collateral is the vector of goods that the lender is allowed to hold and the borrower is obliged to hold, and which assures that the lender will receive something if default occurs. Let $C_j \in I\!R^L_+$ denote the collateral that backs up asset j. Hence, an asset j is defined by the promise of goods $R_j \in I\!R^L_+ \backslash \{0\}$ and the collateral backing it (R_j, C_j). In this model the collateral is kept by the borrowers, who will have utility returns from the use of the collateral, as in the case of the Collateralized Mortgage Obligation markets (CMO). We also suppose that agents are not allowed to use their future endowments as collateral, since markets anonymity is required. Moreover, as we are allowing for all goods to be durable with different degrees of depreciation (depending on the state of nature), we assume that durability is not affected by the identity of the user, or by intensity of its use. In this way market anonymity is maintained.

Now we formalize our notation:

- $l \in L = \{1, \ldots, L\}$ set of commodities.
- $s \in S = \{1, 2, \ldots, S\}$ set of states in the second period. Each state s can occur with probability γ_s. We denote the vector $(\gamma_1, \gamma_2, \ldots, \gamma_S)$ by γ.
- $h \in H = \{1, \ldots, H\}$ set of agents.
- $j \in J = \{1, 2, \ldots, J\}$ set of assets.
- $e^h = (e^h_0, (e^h)_{s \in S}) \in \mathbb{R}^L_+ \times \mathbb{R}^{SL}_+$, initial endowments of agent h, such that $e^h_s \neq 0, \ \ \forall s \in S \bigcup \{0\}$.
- $U^h : \mathbb{R}^L_+ \times \mathbb{R}^{SL}_+ \rightarrow \mathbb{R}$ is the utility function of agent h.
- $x = (x_0, (x_s)_{s \in S})$ is the consumption plan, x_0 is the first period consumption and $(x_1, x_2, .., x_S)$ is the vector of second period consumption, denoted by x_{-0}.
- $p = (p_0, (p_s)_{s \in S}) \in \triangle_{L-1} \times \triangle_{SL-1}$ denotes the commodity price system and $\pi \in \triangle_{J-1}$ denotes the asset prices of the economy, where $\triangle_{n-1}$ denotes the n - simplex in $\mathbb{R}^n$.

- $(R_j, C_j)_{j\in J}$ are real assets where $R_j : S \mapsto I\!R^L_+\backslash\{0\}$ is the promise made by the asset j and $C_j \in R^L_+$ is the collateral that backs it.

- $Y_j : S \mapsto I\!R^L_{++}$ are random variables that represent the durability of goods

- $D^h_j : S \mapsto I\!R^L_+$ is the true return on asset j in state s, *i.e.*, $D^h_j(s)$ is the total amount of money to be delivered by agent h on asset j in state s.

Our economy is defined by

$$\mathcal{E} = ((U^h, e^h)_{h\in H}, (R_j, C_j)_{j\in J}, (Y^l)_{l\in L})$$

The budget constraints of each agent are:

$$p_0(x_0 - e^h_0) + \pi(\theta - \varphi) + p_0 \sum_{j\in J} C_j\varphi_j \leq 0, \tag{1}$$

$$p_s(x_s - e^h_s - Y(s)x_0) - \sum_{j\in J}\varphi_j p_s[Y(s)C_j] - \sum_{j\in J} D_j(s)\theta_j + \sum_{j\in J} D_j(s)\varphi_j \leq 0, \quad \forall s \in S, \tag{2}$$

where

$$D_j(s) = \min\{p_s R_j(s), p_s Y(s) C_j\}, \quad \forall s \in S. \tag{3}$$

In this setting each agent $h \in H$ faces the following problem:

$$\max_{(x,\theta,\varphi)\in B^h(p,\pi,C)} U^h(x_0 + C\varphi, x_{-0}) \tag{4}$$

where

$$B^h : \triangle_{L-1} \times \triangle_{SL-1} \times \triangle_{J-1} \times \triangle_{J-1} \mapsto I\!R^{L(S+1)}_+ \times I\!R^J_+ \times I\!R^J_+$$

is defined by:

$B^h(p,\pi,C) := \{(x,\theta,\varphi) \in I\!R_+^{L(S+1)} \times I\!R_+^J \times I\!R_+^J : (1),\ (2) \quad \text{and} \quad (3) \quad \text{hold}\}$,

which is a convex set.

Now, let us use the following matrix form:

$$P \cdot (x - e^h) \leq \begin{bmatrix} -\Pi \\ A \end{bmatrix} \Psi,$$

where $P \cdot (x - e) = (p_0(x_0 - e_0), p_1(x_1 - e_1 - Y_1 x_0), .., p_S(x_S - e_S - Y_S x_0))$, $\Psi = (\theta, \varphi)$,
$\Pi = (\pi, p_0 C - \pi)$ and

$$A_{(C)} = \begin{bmatrix} D(1) & p_1 Y(1)C - D(1) \\ D(2) & p_2 Y(2)C - D(2) \\ . & . \\ . & . \\ D(S) & p_S Y(S)C - D(S) \end{bmatrix}$$

Observe that π is the buy price vector and $\pi - p_0 C$ is the net sale price vector.

3. Statistical Arbitrage

Let us start by defining statistical arbitrage opportunities in a nontrivial context where $p >> 0$, because monotonicity of preferences assures that the commodity arbitrage opportunities derived from zero spot prices have to be ruled out. It should be stressed that Statistical Arbitrage is not riskless, whereas pure arbitrage is. However, it is widely used by traders in the market.

Let us denote by $\gamma \cdot x$ the expected value of the variable x with respect to the historical probabilities γ.

Definition 1 *We say that there exist strong statistical arbitrage opportunities (SSAO) if* $\exists\ \Psi \in I\!R_+^{2J}$ *such that*

$$\Pi\Psi < 0 \quad \textit{and } \gamma \cdot A_{(C)}\Psi \geq 0$$

The following definition follows in spirit Bondarenko (2003), *i.e.*, a zero cost strategy with positive expected return in the next period. Our definition does not coincide with Bondarenko (2003), because Bondarenko's definition considers that the expected payoff as well as the conditional expected payoff in each final state of the nature has to be nonnegative. In our two-period economy, as there is one way to achieve each state of nature, the conditional expected at each final state equals the return at that state. Hence, if we use Bondarenko's definition, requiring that the conditional expected return has to be nonnegative at each state of nature, the definition of an arbitrage opportunity would be obtained.

Definition 2 *We say that there exist pure statistical arbitrage opportunities (PSAO) if* $\exists\ \Psi \in I\!R_+^{2J}$ *such that*

$$\Pi\Psi = 0 \quad \textit{and } \gamma \cdot A_{(C)}\Psi > 0$$

Our last definition (3) is a more general definition of statistical arbitrage.

Definition 3 *We say that* $\Psi \in \mathbb{R}^{2J}_{+}$ *is a generalized statistical arbitrage opportunity (GSAO) if it is either a SSAO or PSAO. i.e.*

$$\Pi\Psi \leq 0 \quad and \quad \gamma \cdot A_{(C)}\Psi \geq 0$$

with at least one strict inequality.

The first theorem of asset pricing with statistical arbitrage and default is presented below:

Theorem 1 *a) There is no SSAO if and only if there exists* $\beta \in \mathbb{R}^{1+2J}_{+}$ *such that the equalities in (5) are satisfied.*

b) There is no GSAO if and only if there exists $\beta \in \mathbb{R}^{1+2J}_{++}$ *such that the equalities in (5) are satisfied.*

$$\sum_{s=1}^{S} \beta_0 \gamma_s D_j(s) + \beta_j = \pi_j = (p_0 - \sum_{s=1}^{S} \gamma_s p_s Y(s)\beta_0) C_j + \sum_{s=1}^{S} \gamma_s \beta_0 D_j(s) - \beta_{j+J}, \quad \forall j \in J. \tag{5}$$

Proof:

Let us denote the expected returns matrix $\gamma \cdot A$ by A^e, which is a $1 \times 2J$ matrix, *i.e.*,

$$A^e = \left[\sum_s \gamma_s D_1(s) \cdots \sum_s \gamma_s D_J(s) \sum_s \gamma_s (p_s Y(s) C_1 - D_1(s)) \cdots \sum_s \gamma_s (p_s Y(s) C_J - D_J(s)) \right].$$

Now, we construct the following $(1 + 2J) \times 2J$ matrix:

$$\hat{A^e}_{(C)} = \begin{bmatrix} & A^e & \\ I & & 0 \\ 0 & & I \end{bmatrix},$$

where I is the $J \times J$ identity matrix and 0 is the $J \times J$ null matrix.

We can observe that for each C:

$$\exists y \in I\!R^{2J} : \hat{A}^e{}_{(C)} y \geq 0 \Leftrightarrow \exists y \in I\!R^{2J} : A^e_{(C)} y \geq 0 \text{ and } y \geq 0.$$

Then, the absence a of SSAO is equivalent to $\nexists y \in I\!R^{2J}$ such that $\hat{A}^e{}_{(C)} y \geq 0$ and $\Pi y < 0$.By the Farkas' Lemma it is equivalent to $\exists\ \beta = (\beta_0, .., \beta_{2J}) \in I\!R^{1+2J}_+$ such that:

$$\hat{A}^{e'}_{(C)} \beta = \Pi.$$

From the previous equality we obtain:

$$\pi_j = \sum_{s=1}^{S} \beta_0 \gamma_s D_j(s) + \beta_j \text{ and } p_0 C_j - \pi_j = \sum_{s=1}^{S} \beta_0 \gamma_s p_s Y(s) C_j - \sum_{s=1}^{S} \beta_0 \gamma_s D_j(s) + \beta_{j+J}.$$

In an analogous way we obtain (b) using another version of Farkas' Lemma (see Luenberger (1969), pag. 167).□

Remark 1 *Under the assumption that there exists* $j^* \in J$ *such that* $C_{j^*} \neq 0$[1]*, the existence of a SSAO implies the existence of a PSAO. In order to check this consider that* Ψ *is a SSAO. Define another strategy* $\hat{\Psi}$ *equal to* Ψ *with the exception on the amount bought of asset* j^**, which is such that* $\pi^{j^*} \hat{\theta}^{j^*} = \pi^{j^*} \theta^{j^*} - \Pi\Psi$*. Then, if* $\pi^{j^*} = 0, \hat{\Psi}$ *will obviously be a PSAO, since we can buy as much as we want of asset* j^* *and we know that* $D_{j^*}(s) > 0$*. If* $\pi^{j^*} > 0$*, let* $\hat{\theta}^{j^*} = \theta^{j^*} - \Pi\Psi/\pi^{j^*}$*. Then,* $\hat{\theta}^{j^*} > 0$*,* $\Pi\hat{\Psi} = 0$ *and* $\gamma \cdot A_{(C)} \hat{\Psi} > 0$*. We conclude that* $\hat{\Psi}$ *is a PSAO.*

Remark 2 *Under the assumption that* $C_j \neq 0, \forall j \in J$*, the existence of a non trivial PSAO*[2] *implies the existence of a SSAO. To verify this, consider*

[1]This guarantees $D_{j^*}(s) > 0$.

[2]By trivial PSAO we mean the strategies derived from $\pi^j = 0$ or $(p_0 C_j - \pi^j) = 0$.

two possible scenarios concerning Ψ*. Firstly, consider the case when* Ψ *is a PSAO with an* $j^* : \theta^{j^*} > 0$*. If this is the case, define another strategy* $\hat{\Psi}$ *equal to* Ψ *with the exception of the amount bought of asset* j^* *which is equal to* $\hat{\theta}^{j^*} = \theta^{j^*} - \epsilon\gamma \cdot A_{(C)}\Psi / \sum_s \gamma_s D_{j^*}(s)$*, with* $\epsilon \leq 1$ *and such that* $\hat{\theta}^{j^*} > 0$*. Hence, as* Ψ *is a non trivial PSAO, we have* $\Pi\hat{\Psi} < 0$ *and* $\gamma \cdot A_{(C)}\hat{\Psi} \geq 0$*, which allows to conclude that* $\hat{\Psi}$ *is a SSAO. Secondly, consider the case that* Ψ *is a PSAO such that* $\nexists j^* : \theta^{j^*} > 0$*. However, as* Ψ *is a PSAO there must exist* $\hat{j} : \varphi^{\hat{j}} > 0$*. By contradiction to what we intend to verify, suppose that* Ψ *is an SSAO. By Theorem 1, we have* $p_0 C - \pi \in I\!R^J_+$ *and, as* Ψ *is a nontrivial PSAO, we have* $p_0 C_j - \pi^j > 0, \forall j$*. Hence, from* $\Pi\Psi = 0$*, we must have* $\Psi = 0$*, obtaining a contradiction.*

Remark 3 *If* $C \in I\!R^J_{++}$ *and* $\Pi \in I\!R^{2J}_{++}$*, then the three statistical arbitrage notions are equivalents.*

Now we present a kind of second part of the fundamental theorem of asset pricing, in a default context with collateral, derived from absence of generalized statistical arbitrage. To this end we assume that $C_j \neq 0$, $\forall j \in J$, which from the economic point of view is a natural assumption.

Theorem 2 *Under the assumption that utility functions are continuous and strongly monotonic, (ii) implies (i):*

i) The individual optimization problem described in (4) for the agent $h \in H$ *has a solution.*

ii) There are no generalized statistical arbitrage opportunities for the agent $h \in H$*.*

Proof: We know that if there is no GSAO then there can not exist pure arbitrage opportunities in the sense introduced by Araujo, Fajardo and Páscoa (2005)[3], since this demands positive returns in each state of nature. Then, by Orrillo (2005), (i) follows. □

In order to establish the inverse implication, some restrictions on the utility function are needed. Hence, we say that the utility function satisfies condition (C) if we have

$$\sum_s \gamma_s p_s x_s > \sum_s \gamma_s p_s y_s \implies u(\ .\ , x) \geq u(\ .\ , y),\ \forall x, y \in I\!R_+^{SL}.$$

Theorem 3 *Under the assumptions that utility functions are continuous, strongly monotonic and satisfy condition (C), (i) implies (ii).*

Proof:

Let $(x^h, \theta^h, \varphi^h)$ be a solution of the individual optimization problem. Hence, it respects

$$p_0(x_0^h - e_0^h) + \pi\theta^h + (p_0 C - \pi)\varphi^h = 0,$$

$$p_s(x_s^h - e_s^h - Y(s)x_0^h) = D(s)\theta^h + [p_s Y(s)C - D^h(s)]\varphi^h,\ \ \forall s \in S. \quad (6)$$

Additionally, let $\tilde{\Psi} = (\tilde{\theta}, \tilde{\varphi})$ be a GSAO. Then, two possibilities: (a) $\Pi\tilde{\Psi} = 0$ and $\gamma \cdot A_{(C)}\tilde{\Psi} > 0$ or (b) $\Pi\tilde{\Psi} < 0$ and $\gamma \cdot A_{(C)}\tilde{\Psi} \geq 0$. Due to Remark 1 is enough to consider (a).

Consider a new allocation $(\hat{\theta}, \hat{\varphi}) := (\theta^h + \tilde{\theta}, \varphi^h + \tilde{\varphi})$, and consumption $\hat{x}$, such that $\hat{x}_0 = x_0^h$ and $\hat{x}_s$ such that

$$p_s(\hat{x}_s - e_s^h - Y(s)\hat{x}_0) = D(s)\hat{\theta} + [p_s Y(s)C - D(s)]\hat{\varphi},\ \ \forall s \in S. \quad (7)$$

[3] Also used in Fajardo (2005) and Orrillo (2005)

Then, multiplying equations (6) and (7) by γ_s and assuming that case (a) holds, we have

$$\sum_s \gamma_s p_s \hat{x}_s > \sum_s \gamma_s p_s x_s^h, \quad \forall s \in S.$$

Applying condition (C) for future consumption and strict monotonicity for present consumption we have an allocation that is feasible and that increases utility. □

4. Equilibrium

In our economy an equilibrium is a price system (p, π) and allocations $(x^h, \theta^h, \varphi^h)_{h \in H}$ such that $(x^h, \theta^h, \varphi^h)$ solves (4) and given (p, π), markets clear:

- $\sum_h x_0^h + \sum_h C\varphi^h = \sum_h x_0^h$
- $\sum_h x_s^h = \sum_h e_s^h + \sum_h Y(s)e_0^h + \sum_h Y(s)C\varphi^h$
- $\sum_h \varphi^h = \sum_h \theta^h$

As we have seen in theorem 2, if there is no GSAO then there exists a solution for the individual optimization problem. Hence, we can state the following result

Theorem 4 *Under the usual assumptions of strictly positive endowments, strictly positive depreciation rates, strictly positive collateral, return matrix different from zero, and utility functions continuous, concave and strictly increasing, the absence of GSAO implies the existence of equilibrium with default and collateral.*

Proof:

As we have not PSAO we can apply theorem 3 in Araujo, Fajardo and Páscoa

(2005), because our exogenous collateral context is a particular case of their endogenous model[4] .

Basically, in the absence of arbitrage, we can obtain endogenous bounds for short sales, Then, as pointed out by Radner (1972), the failure of existence of equilibrium is due to discontinuities in the budget set, that can be avoided by putting bounds on short sales.

More precisely, By Th. (1) we know that absence of GSAO implies eq. (5). Then, $p_0 C^j - \pi^j > 0,\ \forall j \in J$[5]. Using eq. (1) is easy to see that:

$$\varphi^j \leq \frac{e_0}{p_0 C^j - \pi^j},\quad \forall j \in J \tag{8}$$

In other words short sales are bounded. $\square$.

The main idea behind the equilibrium result is that in the presence of default and durable commodities used as collateral, the absence of statistical arbitrage implies existence of equilibrium. We can resume our analysis as follows:

$\nexists GSAO \Rightarrow \nexists PSAO \Rightarrow$ Existence of Individual Optimality $\Rightarrow$ Existence of Equilibrium,

and, reciprocally,

Existence of Equilibrium $\Rightarrow$ Existence of Individual Optimality $\Rightarrow \nexists PSAO \nRightarrow \nexists GSAO$.

[4]See also theorem 2 in Orrillo (2006) or, alternatively, theorem 1 in Geanakoplos and Zame (2007).

[5]See also Fajardo (2005) and Orrillo (2005).

5. Conclusions

This paper has addressed the implications of absence of generalized statistical arbitrage in a two period economy with incomplete markets and default with exogenous collateral, obtaining in this way a spread for asset prices. It is worth noting that, as in Araújo, Fajardo and Páscoa (2005), asset prices are not martingales, resulting from the presence of market frictions. Also, we obtain a type of fundamental theorem of asset pricing derived from the absence of statistical arbitrage, proving the existence of state price deflators and guaranteeing the existence of optimality for an individual agent. Finally, we have addressed the existence of equilibrium.

Interesting extensions of our model can be obtained by considering utility penalties as in Dubey, Geanakoplos and Shubik (2005).

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