# Nominal Debt as a Burden on Monetary Policy*

Javier Díaz-Giménez   Giorgia Giovannetti   Ramon Marimon
Pedro Teles†

November, 2007‡

**Abstract**

We characterize the optimal sequential choice of monetary policy in economies with either nominal or indexed debt. In a model where nominal debt is the only source of time inconsistency, the Markov-perfect equilibrium policy implies the progressive depletion of the outstanding stock of debt, until the time inconsistency disappears. There is a resulting welfare loss if debt is nominal rather than indexed. We also analyze the case where monetary policy is time inconsistent even when debt is indexed. In this case, with nominal debt, the sequential optimal policy converges to a time-consistent steady state with positive – or negative – debt, depending on the value of the intertemporal elasticity of substitution. Welfare can be higher if debt is nominal rather than indexed and the level of debt is not too high.
Keywords: nominal debt; indexed debt; optimal monetary policy; time consistency; Markov-perfect equilibrium
JEL Classification Numbers: E40, E50, E58, and E60

*We thank Narayana Kocherlakota, and two anonymous referees for very useful comments and suggestions. We also thank Isabel Correia and Juan Pablo Nicolini for their comments, as well as the participants in seminars and conferences where this work has been presented. Financial support of the Spanish Ministerio de Ciencia y Tecnología (Grants: SEC2002-004318), FIRB (Grant RBNE03YT7Z-006) and the Fundação de Ciência e Tecnologia of Portugal is also acknowledged. Corresponding author: Ramon Marimon; European University Institute, Florence, Italy, Ramon.Marimon@EUI.eu.

†J. Díaz-Giménez: Universidad Carlos III and CAERP; G. Giovannetti: Università di Firenze; R. Marimon: European University Institute, UPF-CREi, CREA, CEPR and NBER, and P. Teles: Banco de Portugal, U. Catolica Portuguesa and CEPR.

‡This is a revised version of the working paper dated February 2006.

## 1 Introduction

Fiscal discipline has often been seen as a precondition for price stability. Such is, for example, the rationale behind the Growth and Stability Pact in Europe. The underlying policy debate shows the concern regarding a time-inconsistency problem associated with high levels of nominal debt that could be monetized. In this paper we analyze the implications for the optimal sequential design of monetary policy when public debt is nominal and when it is indexed. We characterize the optimal sequential policy choices with both nominal and indexed debt and assess the relative performance of the two in terms of welfare.

The model is a cash-in-advance production economy where agents start the period with predetermined money balances, which are used for transactions during the period, as in Svensson (1985). The government's problem is to finance exogenous government expenditures in the least distortionary manner. In this economy, an increase in the price level decreases the real value of outstanding money and nominal debt and therefore reduces the need for distortionary taxation. However, this also induces a fall in present consumption because of the cash-in-advance constraint. As shown by Nicolini (1998), who analyzes the same class of economies, the incentives to inflate, or deflate, depend on preferences and on whether debt is nominal or real.

If debt is indexed, the decision on whether to use the inflation tax, to tax today or tomorrow, hinges on the intertemporal elasticity of substitution. If the elasticity is one then it is equal to the implicit elasticity of the cash-in-advance constraint and the optimal plan is time consistent. However, with nominal debt, there is a reason to monetize the debt, and the optimal policy plan is no longer time consistent. We show that in a Markov-perfect equilibrium path the debt is asymptotically depleted, and therefore the path for the nominal interest rate is decreasing. In this case of unitary elasticity, the fact that debt is nominal rather than indexed introduces a dynamic distortion that lowers welfare unambiguously.

For the general case of non unitary elasticity, the optimal policy plan is time inconsistent even with indexed debt. Optimal taxation principles dictate whether current or future consumption should be taxed more. In particular, if the intertemporal elasticity of substitution is higher than one – that is, higher than the implicit elasticity of the cash-in-advance constraint – it is efficient to tax more current consumption; along a sequentially optimal path, indexed debt is depleted all the way to the first best, where it is negative and large enough in absolute value to finance all expenditures without the need to collect distortionary taxes. If the intertemporal elasticity is, instead, lower than one, future consumption is taxed more and debt increases asymptotically.

With nominal debt, the incentives to inflate when debt is positive can compensate the incentives to deflate when the intertemporal elasticity is lower than one. Similarly, the incentives to deflate when debt is negative can compensate the incentives to inflate when the intertemporal elasticity is higher than one. At the debt level where these conflicting incentives cancel out there is a steady state. This stationary level of debt is negative for elasticity higher than one, and positive for elasticity lower than one. For different levels of initial debt, optimal sequential paths of nominal debt converge to this steady state.

When the elasticity is different from one, in contrast with the unitary elasticity case, nominal debt solves – in the long-run – a time-inconsistency problem present in the indexed-debt case; in particular, if the elasticity is higher than one, there is no need to accumulate so many assets in order to achieve the first best, as in the indexed-debt case; if the elasticity is lower than one, debt does not increase asymptotically.

A central contribution of this paper is the welfare comparison of the two regimes, nominal or indexed debt. If the intertemporal elasticity of substitution is one, indexed debt unambiguously dominates nominal debt in terms of welfare. In contrast, if the elasticity is non-unitary, the fact that the incentive to monetize the debt can compensate the distortions present with indexed debt can result in nominal debt dominating indexed debt. In particular, as our computations show – when debt is relatively low – nominal debt can be a blessing, rather than a burden, to monetary policy.

Related work includes Calvo (1988), Obstfeld (1997), Nicolini (1998), Ellison and Rankin (2007), Martin (2006), Persson, Persson and Svensson (2006), and Reis (2006). Calvo (1988) addressed the question of the relative performance of nominal versus indexed debt, considering a reduced form model with two periods, where nominal debt creates a time inconsistency. There is an ad-hoc cost of taxation and an ad-hoc cost of repudiation that depends on the volume of debt. The focus of Calvo (1998) is on multiple equilibria, which result from his assumption on repudiation costs. With such a model, it is not possible to understand how debt, either nominal or indexed, can be used as a state variable affecting future monetary policy; how optimal equilibrium paths should evolve, or why different welfare rankings of indexed versus nominal debts are possible.

Obstfeld (1997) and Ellison and Rankin (2007) assume that debt is real, and focus on monetary policy. They compute Markov-perfect equilibria when the source of the time inconsistency of monetary policy is related to the depletion of the real value of money balances. Obstfeld (1997) uses a model where money balances are not predetermined and therefore must consider an ad-hoc cost of a surprise inflation. Ellison and Rankin (2007) use the model in Nicolini (1998) with a class of preferences for which the level of real debt matters for the direction of the time inconsistency problem.

Martin (2006) studies a version of the same model we analyze, in which the government only issues nominal debt, not indexed. He provides an analytical characterization of the long-run behavior of Markov-perfect equilibria in the case of nominal debt, and shows that the long-run behavior depends on the intertemporal elasticity of substitution. Our paper analyzes and contrasts both types of debt regimes, providing a numerical comparison of Markov-perfect equilibrium outcomes, characterizing the equilibria and comparing the indexed and nominal debt regimes in terms of welfare.

A different strand of related literature studies how optimal policies under commitment can be made time consistent by properly managing the portfolio of government assets and liabilities. The closest paper to ours in this literature is Persson, Persson and Svensson (2006)[1]. They use a structure similar to Nicolini (1998) and assume that the government can use both nominal and real debt as well as that there are no restrictions on debt being positive or negative.

Although we use as benchmark economies with full commitment, our main focus is on Markov-perfect equilibria. In fact, the full characterization and computation of the optimal policy in such equilibria – with debt as a state variable – is an additional contribution of our work.[2]

Finally, there is a recent related literature on the characterization of the best sustainable equilibrium in similar optimal taxation problems, which also reaches the conclusion that optimal policies should, asymptotically, eliminate time inconsistency distortions (see, for example, Reis, 2006).

The paper proceeds as follows: in Section 2, we describe the model economy and define competitive equilibria with nominal and indexed debt. In Section 3, we characterize the optimal allocations and policies under commitment, for the purpose of understanding the sources of time inconsistency. In Section 4, we analyze and compute the Markov-perfect equilibria with indexed and nominal debt. Section 5 contains the main results of the paper: we compare the different regimes in terms of welfare. Finally in Section 6, we show that considering alternative taxes does not change the analysis, as long as taxes are set one period in advance.

## 2 The model economy

In this section we describe the model economy *with nominal debt*. We follow very closely the structure in Nicolini (1998). The economy is a production economy with linear technology,

$$c_t + g \leq n_t \tag{1}$$

for every $t \geq 0$, where $c_t$ and $g$ are private and public consumption, respectively and $n_t$ is labor. There is a representative household and a government. The preferences of the household are assumed to be linear in leisure and isoelastic in consumption,

$$\sum_{t=0}^{\infty} \beta^t [u(c_t) - \alpha n_t], \tag{2}$$

[1]See also Alvarez, Kehoe and Neumeyer (2004) and Lucas and Stokey (1983).

[2]In this respect, our work is closely related to Krusell, Martín and Ríos-Rull (2003), who characterize the recursive equilibria that obtain in an optimal labor taxation problem, and other more recent work on Markov-perfect equilibria.

where $u(c) = \frac{(c)^{1-\sigma}-1}{1-\sigma}$. $0 < \beta < 1$ is the time discount factor.

We assume that consumption in period $t$ must be purchased using currency carried over from period $t-1$ as in Svensson (1985). This timing of transactions implies that the representative household takes initial money balances $M_0$ and nominal public debt holdings $B_0(1+i_0)$ as given. A price increase is costly since it reduces consumption. The specific form of the cash-in-advance constraint faced by the representative household is:

$$P_t c_t \leq M_t \tag{3}$$

for every $t \geq 0$, where $P_t$ is the price of one unit of the date $t$ consumption good in units of money and $M_t$ are money balances acquired in period $t-1$ and used for consumption in period $t$.

In each period the representative household faces the following budget constraint:

$$M_{t+1} + B_{t+1} \leq M_t - P_t c_t + B_t(1+i_t) + P_t n_t \tag{4}$$

where $M_{t+1}$ and $B_{t+1}$ denote, respectively, money and nominal government debt that the household carries over from period $t$ to period $t+1$, and $i_t$ is the nominal interest rate on government debt held from period $t-1$ to $t$ . The representative household faces a no-Ponzi games condition:

$$\lim_{T \longrightarrow \infty} \beta^T \frac{B_{T+1}}{P_T} \geq 0 \tag{5}$$

In each period $t \geq 0$, the government issues currency $M^g_{t+1}$ and nominal debt $B^g_{t+1}$, to finance an exogenous and constant level of public consumption $g$.[3] Initially, we abstract from any other source of public revenues. The sequence of government budget constraints is

$$M^g_{t+1} + B^g_{t+1} \geq M^g_t + B^g_t(1+i_t) + P_t g, \ t \geq 0, \tag{6}$$

together with the no-Ponzi games condition $\lim_{T \longrightarrow \infty} \beta^T \frac{B^g_{T+1}}{P_T} \leq 0$. The initial stock of currency, $M^g_0$, and initial debt liabilities, $B^g_0(1+i_0)$, are given. A government policy is, therefore, a specification of $\{M^g_{t+1}, B^g_{t+1}\}$ for $t \geq 0$.

## 2.1 A competitive equilibrium with nominal debt

**Definition 1** *A competitive equilibrium for an economy with nominal debt is a government policy, $\{M^g_{t+1},\ B^g_{t+1}\}^\infty_{t=0}$, an allocation $\{M_{t+1}, B_{t+1}, c_t, n_t\}^\infty_{t=0}$, and a price vector, $\{P_t, i_{t+1}\}^\infty_{t=0}$, such that:*

[3] We assume that government expenditures, $g$, are given, although our analysis can easily be extended to the case of endogenous government expenditures.

*(i) given $M_0^g$ and $B_0^g(1+i_0)$, and $g$, the government policy and the price vector satisfy the government budget constraint described in expression (??) together with a no-Ponzi games condition;*

*(ii) when households take $M_0$, $\mathrm{B}_0(1+\mathrm{i}_0)$ and the price vector as given, the allocation maximizes utility (??), subject to the cash-in-advance constraint (??), the household budget constraint (??), and the no-Ponzi games condition (??); and*

*(iii) all markets clear, that is: $M_{t+1}^g = M_{t+1}$, $B_{t+1}^g = B_{t+1}$, and $g$ and $\{c_t, n_t\}_{t=0}^{\infty}$ satisfy the economy's resource constraint (??), for every $t \geq 0$.*

Given our assumptions on the utility of consumption $u$, it is straightforward to show that the competitive equilibrium allocation of this economy satisfies both the economy's resource constraint (??) and the household's budget constraint (??) with equality, and that the first order conditions of the Lagrangian of the household's problem are both necessary and sufficient to characterize the solution to the household's problem. The cash-in-advance constraint (??) is binding for $t \geq 0$ if $\frac{u_c(c_t)}{\alpha} > 1$. This condition is satisfied for $t \geq 1$ whenever $i_t > 0$, since $\frac{u_c(c_t)}{\alpha} = 1 + i_t$ for $t \geq 1$. For the first-best consumption level, where $\frac{u_c(c_t)}{\alpha} = 1$, the cash-in-advance constraint does not have to hold with equality.

The competitive equilibrium of an economy with nominal debt can be characterized by the following conditions that must hold for every $t \geq 0$:

$$\frac{u_c(c_{t+1})}{\alpha} = 1 + i_{t+1},\ t \geq 0, \tag{7}$$

$$1 + i_{t+1} = \beta^{-1}\frac{P_{t+1}}{P_t},\ t \geq 0, \tag{8}$$

and the cash-in-advance constraint, which, if $\frac{u_c(c_t)}{\alpha} > 1,\ t \geq 0$, must hold with equality[4]

$$c_t = \frac{M_t}{P_t},\ t \geq 0. \tag{9}$$

Furthermore, the following equilibrium conditions must also be satisfied: the government budget constraint (??), the resource constraint (??) with equality, and the transversality condition

$$\lim_{T \longrightarrow \infty} \beta^T \left(\frac{M_{T+1} + B_{T+1}}{P_T}\right) = 0 \tag{10}$$

implied by optimality given the no-Ponzi games condition (??).

[4]Let $\mu_t$ be the multiplier of the cash-in-advance constraint (??). At the optimum, $\beta^t\left(u_c\left(c_t\right) - \alpha\right) = \mu_t P_t$, $t \geq 0$. Therefore, $\mu_t > 0$ as long as $u_c\left(c_t\right)/\alpha > 1$.

## 2.2 An economy with indexed debt

An economy *with indexed debt* is an economy in all identical to the economy with nominal debt except far government assets. The nominal interest rate adjusts with the price level so that $\frac{B_t(1+i_t)}{P_t} \equiv b_t$ is now predetermined for every period $t \geq 0$. The intertemporal budget constraint of the household can then be written as

$$M_{t+1} + \frac{b_{t+1}}{1+i_{t+1}} P_{t+1} \leq M_t - P_t c_t + b_t P_t + P_t n_t.$$

The first order conditions (**??**)-(**??**) are also first order conditions of the optimal problem with indexed debt.

A competitive equilibrium for an economy with indexed debt is defined as a government policy, $\{M^g_{t+1}, b^g_{t+1}\}_{t=0}^{\infty}$, an allocation $\{M_{t+1}, b_{t+1}, c_t, n_t\}_{t=0}^{\infty}$, and a price vector, $\{P_t, i_{t+1}\}_{t=0}^{\infty}$, such that the conditions (i), (ii) and (iii) of Definition 1 are satisfied when nominal liabilities are replaced by real liabilities, according to $\frac{B_t(1+i_t)}{P_t} = b_t$, where $b_t$ is predetermined.

## 2.3 Implementability with nominal debt

When choosing its policy the government takes into account the above equilibrium conditions. These conditions can be summarized with implementability conditions in terms of the allocations. In particular, as long as the cash-in-advance constraint is binding, the government budget constraint (**??**), which is satisfied with equality, can be written as the implementability condition

$$c_{t+1} u_c(c_{t+1}) \frac{\beta}{\alpha} + \beta z_{t+1} c_{t+1} = c_t + z_t c_t + g, \;\; t \geq 0 \tag{11}$$

where

$$z_t \equiv \frac{B^g_t(1+i_t)}{M^g_t}.$$

To see this, notice that the budget constraint (**??**) with equality can be written in real terms as

$$\frac{M^g_{t+1}}{P_t} + \frac{B^g_{t+1}}{P_t} = \frac{M^g_t}{P_t} + \frac{B^g_t(1+i_t)}{P_t} + g \tag{12}$$

and, using the first order conditions of the household problem, (**??**), (**??**) and (**??**), as well as the cash-in-advance constraint with equality, $\frac{M^g_t}{P_t} = c_t$, one obtains the following identities: $\frac{M^g_{t+1}}{P_t} = \frac{M^g_{t+1}}{P_{t+1}} \frac{P_{t+1}}{P_t} = c_{t+1}\beta(1+i_{t+1}) = c_{t+1} u_c(c_{t+1}) \frac{\beta}{\alpha}$; $\frac{B^g_t(1+i_t)}{P_t} = \frac{B^g_t(1+i_t)}{M^g_t} \frac{M^g_t}{P_t} = z_t c_t$, and $\frac{B^g_{t+1}}{P_t} = \frac{B^g_{t+1}(1+i_{t+1})}{M^g_{t+1}} \frac{M^g_{t+1}/P_{t+1}}{P_t(1+i_{t+1})/P_{t+1}} = \beta z_{t+1} c_{t+1}$.

The intertemporal implementability condition (**??**) together with the terminal condition $\lim_{T \longrightarrow \infty} \beta^T (c_{T+1} u_c(c_{T+1}) \frac{\beta}{\alpha} + \beta z_{T+1} c_{T+1}) = 0$, obtained from the transversality condition (**??**), summarize the equilibrium conditions if the cash-in-advance constraint is always binding. Notice that the remaining equilibrium conditions are satisfied since equilibrium interest rates, prices, nominal liabilities and labor supplies can be derived from the competitive equilibrium restrictions; that is $\{i_{t+1}\}_{t=0}^{\infty}$ satisfying (**??**), $\{P_{t+1}\}_{t=0}^{\infty}$ satisfying (**??**), $\{M_{t+1}\}_{t=0}^{\infty}$ and $P_0$ that satisfy (**??**), $\{n_t\}_{t=0}^{\infty}$ satisfying (**??**), and $\{B_{t+1}\}_{t=0}^{\infty}$ so that $z_{t+1} = \frac{B_{t+1}(1+i_{t+1})}{M_{t+1}}$.

Using the terminal condition, the present value government budget constraint takes the form

$$\sum_{t=0}^{\infty} \beta^t \left( c_{t+1} u_c(c_{t+1}) \frac{\beta}{\alpha} - (c_t + g) \right) = z_0 c_0 \tag{13}$$

## 2.4 Implementability with indexed debt

With indexed debt, the government budget constraint (**??** ) with equality can be written as the implementability condition

$$c_{t+1} u_c(c_{t+1}) \frac{\beta}{\alpha} + \beta b_{t+1} = c_t + b_t + g, \;\; t \geq 0, \tag{14}$$

provided the cash-in-advance constraint binds. The transversality condition (**??**) is written as $\lim_{T \longrightarrow \infty} \beta^T (c_{T+1} u_c(c_{T+1}) \frac{\beta}{\alpha} + \beta b_{T+1}) = 0$, which implies that the present value government budget constraint takes the form

$$\sum_{t=0}^{\infty} \beta^t \left( c_{t+1} u_c(c_{t+1}) \frac{\beta}{\alpha} - (c_t + g) \right) = b_0. \tag{15}$$

This condition summarizes, when debt is indexed, the competitive equilibrium restrictions on the sequence of consumption $\{c_t\}_{t=0}^{\infty}$. The only difference between the implementability conditions with nominal and indexed debt is in the right hand side of equations (**??**) and (**??**). With nominal debt, the government can affect the real value of outstanding debt, although this necessarily affects consumption.

Notice also that an economy with nominal debt, and initial nominal liabilities $z_0$, where the government policy results in a choice of $c_0$ has the same period zero, *ex-post,* real liabilities as an indexed economy with initial –but, *predetermined* – real liabilities $b_0 = z_0 c_0$. We use this correspondence in comparing economies with nominal debt with economies with real debt.

## 3 Optimal policy with full commitment

In this section we compare optimal policies under full commitment when debt is indexed and when it is nominal. This is useful because, by observing how the optimal allocations differ in the initial period from subsequent ones, we are able to understand whether policy is time consistent, and when it is not, what is the source of the time inconsistency. In defining a full commitment Ramsey equilibrium with indexed debt, we assume – and, ex-post confirm – that the solution of the problem satisfies $\frac{u'(c_t)}{\alpha} > 1$, $t \geq 0$, so that the cash-in-advance constraint always binds[5].

**Definition 2** *A **full commitment Ramsey equilibrium with indexed debt** is a competitive equilibrium such that* $\{c_t\}$ *solves the following problem:*

$$Max \sum_{t=0}^{\infty} \beta^t \left[u(c_t) - \alpha(c_t + g)\right] \tag{16}$$

*subject to the implementability condition (**??**):*

$$\sum_{t=0}^{\infty} \beta^t \left(c_{t+1} u_c(c_{t+1}) \frac{\beta}{\alpha} - (c_t + g)\right) = b_0.$$

The other competitive equilibrium variables, which are the government policy $\{M^g_{t+1}, B^g_{t+1}\}_{t=0}^{\infty}$, the allocation $\{M_{t+1}, B_{t+1}, n_t\}_{t=0}^{\infty}$, and the price vector $\{P_t, i_{t+1}\}_{t=0}^{\infty}$, are obtained using the competitive equilibrium conditions.

The optimal solution with commitment results in a constant consumption path from period one on, $c_{t+1} = c_1$, $t \geq 0$. The intertemporal condition relating the optimal consumption in the initial period to the optimal stationary consumption in subsequent periods is

$$u_c(c_0) - \alpha = \frac{u_c(c_1) - \alpha}{1 - \frac{u_c(c_1)}{\alpha}(1 - \sigma)}. \tag{17}$$

Clearly, when $\sigma = 1$, the two consumptions are equal and the solution is time consistent. When $\sigma < 1$, (i.e., the intertemporal elasticity of substitution is $1/\sigma > 1$) the government prefers to tax more current consumption than future consumption, and therefore $c_0 < c_1$. When $\sigma > 1$, the government prefers to delay taxation and $c_0 > c_1$. In summary, when $\sigma \neq 1$, the full commitment solution is time-inconsistent due to 'intertemporal elasticity effects.'

In the definition of a full-commitment Ramsey equilibrium, we have imposed a binding cash-in-advance constraint in all periods. When $\sigma \geq 1$ (i.e., $u_c(c_1) \geq u_c(c_0)$), the

[5] See Appendix 1 for a discussion of equilibria with first-best outcomes and with non-binding cash-in-advance constraints.

cash-in-advance constraint binds as long as there is a need to raise distortionary taxes. When $\sigma < 1$ (i.e., $u_c(c_1) < u_c(c_0)$), the cash-in-advance constraint binds as long as it is not possible to attain the first best from period $t = 1$ on[6].

**Definition 3** *A* ***full commitment Ramsey equilibrium with nominal debt*** *is a competitive equilibrium such that $\{c_t\}$ solves the following problem:*

$$Max \sum_{t=0}^{\infty} \beta^t \left[u(c_t) - \alpha(c_t + g)\right] \tag{18}$$

*subject to the implementability condition (**??**):*

$$\sum_{t=0}^{\infty} \beta^t \left( c_{t+1} u_c(c_{t+1}) \frac{\beta}{\alpha} - (c_t + g) \right) = z_0 c_0. \tag{19}$$

As in economies with indexed debt, it is optimal for the government to commit to a constant path of consumption (and nominal interest rates) from period one on, but consumption in period zero may differ. With nominal debt, the intertemporal condition relating consumption in period zero and period one is given by

$$\frac{u_c(c_0) - \alpha}{1 + z_0} = \frac{u_c(c_1) - \alpha}{1 - \frac{u_c(c_1)}{\alpha}(1 - \sigma)}. \tag{20}$$

This condition makes explicit the additional motive for consumption in the two periods to diverge when debt is nominal. Comparing (**??**) with the intertemporal condition with indexed debt (**??**), it can be seen that, as long as $z_0 \equiv \frac{B_0(1+i_0)}{M_0} > 0$, $c_0$ is relatively smaller – with respect to $c_1$ – than the corresponding $c_0$ of the economy with indexed debt. In other words, the incentive to monetize debt always results in relatively lower period zero consumption; whether this results in lower consumption in the initial period with respect to future consumption depends on how this effect interacts with the 'intertemporal elasticity' effect already present in the indexed economy. As in economies with indexed debt, the government commits to a constant path of consumption (i.e. of nominal interest rates) from period one on, but consumption in period zero may be different, due to the 'intertemporal elasticity effect' (as in the indexed debt case) or to the 'nominal effect' of monetizing nominal debts and revaluing nominal assets.

A closer inspection of (**??**) also shows that for every $\sigma$ there is a $\overline{z}_0$ resulting in a constant optimal consumption path from period zero on, and therefore policy is time consistent. Such $\overline{z}_0$ is obtained by solving for $c$, the following steady state implementability condition: $\overline{c}u_c(\overline{c})\frac{\beta}{\alpha} + \beta\overline{z}_0\overline{c} = (1 + \overline{z}_0)\,\overline{c} + g$. That is, substituting $\overline{z}_0 = -\frac{u_c(\overline{c})}{\alpha}(1 - \sigma)$, the steady state equation reduces to $\frac{\overline{c}u_c(\overline{c})}{\alpha}\left[1 - (1 - \beta)\,\sigma\right] = \overline{c} + g$. Therefore, as long as

[6] See Appendix 1.

$(1-\beta)\,\sigma < 1$, there is a solution for $\bar{c}$ and, correspondingly, for $\bar{z}_0$. Notice that $\bar{z}_0$ is negative, zero or positive, depending on whether $\sigma < 1$, $\sigma = 1$ or $\sigma > 1$[7]

In summary, in our economies, with full commitment, it is optimal to set the same inflation tax rate from period one on, resulting in a stationary consumption path starting from that period. With a unitary intertemporal elasticity of substitution (log utility), the optimal policy with indexed debt is time consistent (Nicolini, 1998). The time-consistency is lost for those same preferences if debt is nominal, since there is an additional reason to inflate in the initial period: to reduce the real value of outstanding nominal liabilities. Under more general preferences, there is a time inconsistency even when debt is indexed. Nominal debt in that case exacerbates the time inconsistency problem when the elasticity of substitution is greater than one, since it reinforces the incentive to inflate, and alleviates it when the elasticity is lower than one. If public debt is negative, the incentive for the government is to revalue this asset, so that the 'nominal effect' works in the opposite direction.

## 4 Markov-perfect monetary equilibria

In this section we assume that the government cannot commit to a future path of policy actions, and therefore chooses its monetary policy sequentially. We restrict the analysis to the case where such sequential choices do not depend on the whole history up to period $t$ but can depend on the pay-off relevant state variables – as in Markov-perfect equilibria – and therefore sequential optimal choices are recursively given by stationary optimal policies. In particular, in the case with nominal debt, government policy is recursively defined by $c_t = C(z_t)$ and the corresponding state transition $z_{t+1} = \boldsymbol{z}'(z_t)$. Similarly, with indexed debt, government policy is recursively defined by $c_t = C(b_t)$ and the corresponding state transition $b_{t+1} = \boldsymbol{b}'(b_t)$. Agents have rational expectations and therefore their consumption plans are consistent with government policy choices and the corresponding state transitions. Our definitions of *Markov-perfect monetary equilibria* take these elements into account.

### 4.1 Indexed debt

**Definition 4** *A Markov-perfect monetary equilibrium*[8] *with indexed debt is a value function $V(b)$ and policy functions $C(b)$and $\boldsymbol{b}'(b)$ such that $c = C(b)$ and $b' = \boldsymbol{b}'(b)$ solve*

$$V(b) = \max_{c,b'} \left\{ u(c) - \alpha(c+g) + \beta V(b') \right\}$$

[7]Ellison and Rankin (2007) show that this possibility of having time consistent optimal policies for specific initial real liabilities can also occur with indexed debt for some forms of non-CRRA preferences.

[8]As in economies with full commitment, we assume – and ex-post verify – that the cash-in-advance constraint is always binding.

*subject to the implementability constraint*

$$C(b')u_c(C(b'))\frac{\beta}{\alpha} + \beta b' = c + g + b, \quad t \geq 0.$$

In order to characterize the Markov-perfect equilibrium, notice that the first order condition for $c$ is:

$$u_c(c) - \alpha = \lambda,$$

and for $b'$,

$$V_b(b') + \lambda \left( \frac{C_b(b')u_c(C(b'))}{\alpha} (1-\sigma) + 1 \right) = 0.$$

while the envelope condition is given by

$$V_b(b) = -\lambda.$$

These equations imply the following intertemporal condition along an equilibrium path,

$$\frac{u_c(c')}{\alpha} - 1 = \left( \frac{u_c(c)}{\alpha} - 1 \right) \left[ 1 + \frac{u_c(C(b'))}{\alpha} C_b(b') (1-\sigma) \right]. \tag{21}$$

It follows that in the log case ($\sigma = 1$) the optimal policy is stationary; given any sustainable level of initial debt $b_0$, the level is maintained and, correspondingly, the consumption path is constant.

It is also the case that the first best, where $\frac{u_c(c)}{\alpha} = 1$, is a steady state, independently of the value of $\sigma$. Furthermore, at the first best, where the stationary level of debt is negative and large enough in absolute value to cover expenditures, an increase in the level of assets does not affect consumption $C_b(b') = 0$, for $b' \leq b^*$, where $b^* < 0$ is the level of assets corresponding to the first best[9]

Comparing the equilibrium intertemporal condition of the economy without commitment (**??**) with the corresponding condition of the economy with full commitment (**??**), we see that the 'intertemporal elasticity effect,' when $\sigma \neq 1$, is weighted by $-C_b(b')$, which is the marginal decrease in consumption in response to an increase in debt, as a function of the level of debt. In the numerical computation of Markov-perfect equilibria, $C_b(b')$ is always negative. With $C_b(b')$ negative, the intertemporal condition (**??**) implies that when $\sigma < 1$ the consumption path is increasing, while when $\sigma > 1$ the consumption path is decreasing.

We now turn to the numerical results. In Appendix 2 we discuss the choice of parameters and describe the computational algorithm. Figure 1 shows the optimal debt and consumption policies, $\boldsymbol{b}'(b)$ or rather $\boldsymbol{b}'(b) - b$, and $C(b)$, for $\sigma = 0.6$, $\sigma = 1.0$, and $\sigma = 1.4$, and for the corresponding relevant ranges of $b$. As we have already mentioned, when $\sigma = 1.0$ the initial level of debt is maintained forever, i.e. $\boldsymbol{b}'(b) - b = 0$.

[9]Technically, there is a need to consider positive transfers, meaning that there is free disposal of extra revenues by the government (see Appendix 1).

When $\sigma < 1$ the policy function $\boldsymbol{b}'(b)$ is decreasing and $\boldsymbol{b}'(b) < b$, except at the value of debt that supports the first-best level of consumption where there is no distortionary taxation [10]. The inflation tax is higher initially so that debt may be depleted and assets accumulated, to the point where there are enough assets to finance all expenditures, and the first best is attained.

When $\sigma > 1$, the policy function $\boldsymbol{b}'(b)$ is increasing and $\boldsymbol{b}'(b) > b$, except at the first-best level of debt $b^* < 0$. Furthermore, $\frac{\boldsymbol{b}'(b)}{b} < \beta^{-1}$, so that debt is accumulated at a rate lower than $\beta^{-1} - 1$. The first best is also a steady state when $\sigma > 1$, but it is not the asymptotic state and therefore the 'intertemporal elasticity effect' never disappears. Panel F also shows that when $\sigma = 1.4$, $C_b(b')$ is very close to zero. In general, as our computations have also shown, whenever $\sigma$ is close to one, $C_b(b')$ is very close to zero and, correspondingly, $\boldsymbol{b}'(b) \approx b$. This means that convergence to the first best when $\sigma < 1$, or the accumulation of debt when $\sigma > 1$, is very slow.

### 4.2 Nominal debt

As we have seen, in an economy with full commitment and nominal debt, there is a 'nominal effect' since the government has an incentive to partially monetize the debt in the initial period. In an economy without commitment, this distorting effect is present in every period and therefore is anticipated by households. As in an economy with full commitment, the 'nominal effect' interacts with the possible 'intertemporal elasticity effect'.

**Definition 5** *A Markov-perfect monetary equilibrium with nominal debt is a value function $V(z)$ and policy functions $C(z)$ and $\boldsymbol{z}'(z)$ such that $c = C(z)$ and $z' = \boldsymbol{z}'(z)$ solve*

$$V(z) = \max_{\{c,z'\}} \{u(c) - \alpha(c+g) + \beta V(z')\} \tag{22}$$

*subject to the implementability constraint*

$$C(z')u_c(C(z'))\frac{\beta}{\alpha} + \beta z' C(z') = zc + c + g. \tag{23}$$

To characterize the Markov-perfect monetary equilibrium, notice that the first order conditions of the problem described above are

$$u_c(c) - \alpha = \lambda(1+z)$$

and

$$V_z(z') + \lambda\left(\frac{C_z(z')u_c(C(z'))}{\alpha}(1-\sigma) + C(z')\left[1 + \varepsilon_c(z')\right]\right) = 0,$$

[10] Notice that policies show small high-frequency fluctuations due to our discrete grid algorithm. Such fluctuations do not impinge on our results. Ffor clarity we fit a fourth-order polynomial in Figure 1. Also see Appendix 2, which provides the values of debt supporting the first best.

where $\varepsilon_c(z) = \frac{zC_z(z)}{C(z)}$ is the elasticity of $C(z)$, and the envelope condition is

$$V_z(z) = -\lambda c.$$

This implies

$$\frac{\frac{u_c(c')}{\alpha} - 1}{1+z'} = \frac{\frac{u_c(c)}{\alpha} - 1}{1+z} \left(1 + \varepsilon_c(z') + \frac{u_c(C(z'))C_z(z')}{\alpha C(z')}(1-\sigma)\right). \tag{24}$$

For $z' \neq 0$, this can be written as

$$\frac{\frac{u_c(c')}{\alpha} - 1}{1+z'} = \frac{\frac{u_c(c)}{\alpha} - 1}{1+z} \left(1 + \varepsilon_c(z')\left[1 + \frac{u_c(C(z'))}{z'\alpha}(1-\sigma)\right]\right). \tag{25}$$

As in the case with indexed debt, there is a first-best steady state, with $\frac{u_c(c)}{\alpha} = 1$, where government assets are enough to finance expenditures. If $z \equiv z^* = -1 - \frac{g}{(1-\beta)c^*}$ with $u_c(c^*) = \alpha$, the cash-in-advance constraint holds with equality and the solution is the first best. This is an isolated steady state[11].

As we have seen in the previous section, with nominal debt there is another steady state, where $\overline{z} = -\frac{u_c(\overline{c})}{\alpha}(1-\sigma)$, and $\overline{c}$ solves $\frac{\overline{c}u_c(\overline{c})}{\alpha}[1-(1-\beta)\sigma] = \overline{c} + g$.

To better understand the distortions present in the economy with nominal debt without commitment, it is useful to consider the log case, where the intertemporal condition (**??**) can be rewritten as

$$\frac{\frac{1}{c} - \alpha}{\left[1 + \frac{(1+i)B}{M}\right]} = \frac{\frac{1}{c'} - \alpha}{\left[1 + \frac{(1+i')B'}{M'}\right]}\left[1 + \varepsilon_c\left(\frac{(1+i')B'}{M'}\right)\right]^{-1} \tag{26}$$

where the elasticity $\varepsilon_c(z')$ is negative and less than one in absolute value (as our computations show).

This intertemporal equation reflects the different distortions present as a result of debt being nominal and policy decisions being sequential. The term $\left[1 + \frac{(1+i)B}{M}\right]$ results from the discretionary incentive to reduce the real value of nominal debt. It is present in the problem with commitment only in the initial period (equation (**??**)). The marginal benefit of increasing current consumption is discounted by the current liabilities, $\frac{(1+i)B}{M}$, reflecting the fact that higher consumption in the current period implies a higher real value of outstanding nominal debt and therefore higher future distortionary taxes. Hence, the benefits of higher consumption today for the benevolent government must be discounted to take into account these future costs. The term $\left[1 + \varepsilon_c\left(\frac{(1+i')B'}{M'}\right)\right]$

[11]See Appendix 1 for a more detailed discussion of the case $z \leq -1$.

results from the dynamic nature of this problem and reflects the cost of the time inconsistency problem being exacerbated in the future, due to an increase in outstanding liabilities at the end of the current period [12]. For $\sigma \neq 1$, these two effects are compounded by the 'intertemporal elasticity effects' and, as we have seen, the interaction of all these effects may result in stationary solutions not present in the economy with indexed debt.

In Figure 2 we report our findings for the same three elasticity values, $\sigma = 0.6$, $\sigma = 1.0$, and $\sigma = 1.4$, when debt is nominal. We find that in all three cases, $\boldsymbol{z}'(z) - z$ is decreasing and, as we have shown before, there is a steady state at $\overline{z} = 0$ when $\sigma = 1$, $\overline{z} < 0$ when $\sigma < 1$, and $\overline{z} > 0$ when $\sigma > 1$[13]. Since $\boldsymbol{z}'(z) - z$ is decreasing, these steady states correspond to the asymptotic behavior of nominal debt paths.

In contrast with the indexed debt case illustrated in Figure 1, when $\sigma = 1$ the stock of debt is no longer constant, but is progressively depleted until the 'nominal effect' disappears at $\overline{z} = 0$. When $\sigma < 1$, debt is also progressively depleted and assets are accumulated but, in contrast with the indexed-debt case, this process leads to a level of assets $\overline{z}$ which is lower than the first-best steady-state level, since $-1 < \overline{z} < 0$ and the first best would require $z < -1$.

When $\sigma > 1$, debt is not accumulated without bound – as was the case with indexed debt – but is accumulated or progressively depleted until it reaches the distorted steady state $\overline{z} > 0$ in which the 'nominal' and the 'intertemporal elasticity' effects cancel out. As in the indexed-debt case, for $\sigma = 1.4$, $\boldsymbol{z}'(z) - z$ is fairly flat and, as a result, long-run convergence (divergence in the indexed-debt case) is very slow (see Panel E in Figure 2).

## 5 Welfare comparisons

In the last two sections we have seen how optimal monetary policies may result in different time paths for debt and inflation, corresponding to different distortions, depending on whether monetary authorities can or cannot commit, and whether debt is indexed or nominal. In this section we address the central question of how these different monetary regimes compare in terms of welfare.

There is an immediate and unambiguous comparison between economies with and without commitment with the same type of liabilities (i.e. either indexed or nominal debt in both economies). As one would expect, *Markov-perfect equilibria are less efficient than full-commitment Ramsey equilibria*, unless the initial conditions are such that Ramsey equilibria are stationary and, therefore, time-consistent. This result follows from the fact that the full-commitment Ramsey solution is the choice of a sequence of consumption $\{c_t\}_{t=0}^{\infty}$ which maximizes utility (**??**) in the set of competitive equilibrium sequences

[12] Myopic governments that do not take into account the effect of their choices on the state variables of future government decisions would be solving the problem with only the first term present.

[13] See Appendix 2 for the corresponding values of $\overline{z}$.

defined either by (??) with indexed debt, or (??) with nominal debt, while the Markov-perfect equilibrium imposes additional restrictions to this maximization problem: the optimality of future government decisions. In other words, the Ramsey solution is the solution of a maximization problem of a committed government in period zero, while the Markov-perfect equilibrium can be better thought of the equilibrium of a game between successive governments, in which the optimal plan of the period-zero government has to take into account the sequential decisions of future governments (or its own future revised policies).

It is less straightforward to compare economies with different types of liabilities. Nevertheless, we can compare the welfare of an economy with nominal debt with an – otherwise identical – economy with indexed debt, provided that both have the same initial (equilibrium) levels of either real or nominal liabilities.

We compare economies with the same real value of initial liabilities. In economies with full commitment, if in the indexed-debt economy initial real liabilities are $b_0$, then in the nominal-debt economy nominal liabilities $z_0$ have to be such that $b_0 = z_0 C_0(z_0)$, where $C_0(z_0)$ is the full-commitment optimal choice of initial consumption in the economy with nominal debt[14]. It is then trivial to compare the two economies in terms of welfare under full commitment. Given that both economies have the same initial real liabilities $b_0$, a benevolent government accounting for such real liabilities achieves higher welfare than one which does not, i.e. welfare in the indexed debt economy is higher than in the corresponding nominal debt economy. We state this formally in the following proposition.

**Proposition 1** *Consider two economies with full commitment with an initial money stock $M_0$. One of them has initial nominal debt $B_0(1+i_0) > 0$, and the other has initial indexed debt $b_0$. Suppose $b_0 = \frac{B_0(1+i_0)}{P_0}$, where $P_0$ is the period-zero price in the economy with nominal debt. Then, the economy with nominal debt always gives lower welfare, independently of the value of $\sigma$.*

**Proof:** The nominal debt economy has initial condition $z_0 = \frac{B_0(1+i_0)}{M_0}$, while the indexed economy has initial condition $b_0 = z_0 C_0(z_0)$, where $C_0(z_0)$ is the full-commitment optimal choice of initial consumption in the economy with nominal debt. The optimal solutions in the two economies have to satisfy the same implementability condition. Taking into account that optimal consumption paths are constant after period zero, this implementability condition is:

$$\beta c_1 \left[ \frac{u_c(c_1)}{\alpha} - 1 \right] - (1-\beta)\, c_0 - g = (1-\beta)\, b_0.$$

Given this condition, the solution with the highest welfare is the solution with indexed

[14]If the mapping from $b_0$ to $z_0$ is not unique, we select the lowest $z_0$; however, in our computations it is unique.

debt, satisfying

$$u_c(c_0) - \alpha = \frac{u_c(c_1) - \alpha}{\left(1 - \frac{u_c(c_1)}{\alpha}(1-\sigma)\right)},$$

since the solution with nominal debt is distorted by $z_0$ (even if there is no 'unexpected inflation'; i.e. realized real liabilities are $b_0$) according to

$$\frac{u_c(c_0) - \alpha}{1 + z_0} = \frac{u_c(c_1) - \alpha}{\left(1 - \frac{u_c(c_1)}{\alpha}(1-\sigma)\right)}.$$

If the solution with nominal debt is different, as when $z_0 \neq 0$, then the solution with nominal debt gives strictly lower welfare■

The choice of comparing economies with the same real value of initial liabilities under full commitment is consistent with how the Markov-perfect equilibria of the two economies without full commitment should be compared. A Markov-perfect equilibrium, like any competitive equilibrium, imposes the 'rational expectations' condition that agents have the right expectations regarding future liabilities. In particular, the indexed economy is characterized by having nominal interest rates adjusting to price changes so as to guarantee the predetermined value of real liabilities. With nominal debt, real liabilities are not predetermined, but with rational expectations (and no uncertainty), *ex-post* real returns correspond to agents' *ex-ante* expected values. In a Markov-perfect equilibrium, strategies are policies that only depend on the state variable and therefore such policies do not treat period zero differently. It follows that in a Markov-perfect equilibrium of an economy with nominal debt, the government has no 'free lunch' from unexpected inflation, even in period zero.

As we have just seen, with full commitment indexed debt is unambiguously more efficient than nominal debt, when the pure monetization effect is not considered. Without commitment, along a Markov-perfect equilibrium path, there are no 'free lunches'. Does this mean that indexed debt is better than nominal debt? The following proposition provides an interesting answer to this question.

**Proposition 2** *Consider two economies without commitment and initial money stock $M_0$. One of them has initial nominal debt $B_0(1+i_0) > 0$, and the other has initial indexed debt $b_0$. Suppose $b_0 = \frac{B_0(1+i_0)}{P_0}$, where $P_0$ is the period-zero price in the economy with nominal debt.*

*(i) If $\sigma = 1$, the welfare in the economy with indexed debt is higher than in the economy with nominal debt.*

*(ii) If $\sigma \neq 1$, the welfare in the economy with nominal debt can be higher or lower than in the economy with indexed debt, depending on $b_0$.*

Part (*i*) of Proposition 2 follows from previous results. First, in the log case with indexed debt, policy is time consistent; hence, the full-commitment and the Markov-perfect equilibria coincide. Second, from Proposition 1, the full-commitment equilibrium with nominal debt provides lower welfare than the equilibrium with indexed debt. Third, as we have seen, the Markov-perfect equilibrium introduces additional constraints to the full-commitment maximization problem, resulting in lower welfare in the economy with nominal debt. Therefore, when $\sigma = 1$, indexed debt dominates nominal debt in terms of welfare. Part (*ii*) of Proposition 2 follows from our numerical simulations, showing that welfare reversals may occur.

Figure 3 shows that – in spite of the roughness of our discrete choice algorithm – the value functions are very well behaved; e.g. decreasing and concave in $b$, and fairly smooth. This allows for (robust) welfare comparisons, once indexed and nominal debt value functions take the same domain. Figure 4 illustrates Proposition 2. It compares the value functions with indexed debt $V_i$ and with nominal debt $V_n$ as functions of nominal liabilities $z$; that is, $V_i(\boldsymbol{b}(z))$, where $\boldsymbol{b}(z) \equiv zC(z)$.

Panels C and D of Figure 4 show the unambiguous result in part (*i*) of Proposition 2, that, with unitary elasticity, indexed debt dominates nominal debt in terms of welfare. The remaining panels illustrate the result in the second part of Proposition 2. In particular, when $\sigma \neq 1$, there is a range of assets and debts for which nominal debt Pareto-dominates indexed debt.

There are two effects at play. First, because we compare economies with the same initial real liabilities, indexed debt tends to give higher welfare than nominal debt. This is the case under full commitment as stated in Proposition 1, but the effect is also present in Markov-perfect equilibria. The second effect, which could either compensate or reinforce the first one, is the magnitude of the dynamic distortions induced by the time inconsistency.

With $\sigma = 1$, Ramsey policy with indexed debt is time consistent, while it is time inconsistent with nominal debt. In a Markov-perfect equilibrium, nominal debt is depleted to the point where debt is zero and policy is time consistent. Such departure from stationarity is costly, and as a result indexed debt dominates nominal debt in terms of welfare, when $\sigma = 1$.

If $\sigma \neq 1$, Ramsey policy is time inconsistent when debt is indexed, while when debt is nominal there is a level of nominal debt (or assets) such that policy is time consistent. In particular, as we have seen, when $\sigma < 1$ and $z < 0$ (or alternatively when $\sigma > 1$ and $z > 0$), the 'intertemporal elasticity effect' and the 'nominal effect' tend to mutually offset; in fact both effects fully cancel out at the distorted steady state $\overline{z} < 0$ (alternatively, $\overline{z} > 0$). At the distorted steady state $\overline{z}$, the elimination of the time inconsistency distortions more than compensates for the potential dominance of indexed debt. As a result, welfare is higher with nominal debt. This dominance of nominal debt is still present in a range of initial debt (or asset) levels close to the distorted steady states. This follows from the fact that Markov-equilibrium paths converge to the (nearby) distorted steady state, and therefore the cost of anticipated distortions – due to 'time inconsistencies' – is relatively small. In fact, the dominance of nominal debt can still be present at initial values of debt for which the 'intertemporal elasticity effect' and the 'nominal effect' actually reinforce each other; that is for relatively low debt values, $z > 0$, when $\sigma < 1$, or for relatively low asset values, $z < 0$, when $\sigma > 1$, as can be seen Figure 4.

As the intertemporal equilibrium condition (**??**) shows, the time-inconsistency 'nominal effect' is exacerbated by the size of $z$ (in absolute value); i.e. by the debt (or asset) to money ratio. Therefore, for large values of $z$ (in absolute value) the dynamic distortions due to time inconsistency are very costly. It follows that, for high levels of debt (or assets) relative to money, indexed debt dominates nominal debt in terms of welfare.

When $\sigma < 1$, debt depletion is a characteristic of Markov-perfect equilibria with both indexed and nominal debt. If the initial value of debt is high, the relative advantage of converging to $\overline{z} < 0$, as opposed to the first best, is properly discounted into the distant future and offset by the more immediate cost of having the 'nominal effect' reinforcing the 'intertemporal elasticity effect'. Similarly, when $\sigma > 1$ and the initial level of debt is much higher than $\overline{z} > 0$, the relative advantage of converging to $\overline{z} > 0$ is properly discounted and offset by the cost of depleting the debt, given that the 'intertemporal elasticity effect' calls for postponing taxation and accumulating debt: a recommendation that sequential optimal policy follows with indexed debt, but not with nominal debt. It should also be noticed that a similar effect can happen when $\sigma < 1$ and $-1 < z < \overline{z} < 0$. Then the 'intertemporal elasticity effect' calls for anticipating taxation and accumulating assets, while the 'nominal effect' calls for a relatively costly process of asset depletion; as a result, in this case, welfare with indexed debt is higher when assets are high (see Panel B.)

To summarize we emphasize two points. The first is that we provide an interesting example of the principle that adding a distortion to a second-best problem can actually improve welfare. Nominal debt adds a dynamic distortion to the Markov-perfect equilibrium. In the case where policy is time consistent with indexed debt, adding this distortion reduces welfare. When policy is time inconsistent with indexed debt, there are two dynamic distortions, due to the differing elasticities and to nominal debt. In this case, adding the distortion from nominal debt can actually raise welfare.

The second point has important policy implications. In the calibrated economy with $\sigma$ different from one, when debt is relatively high (relative to output) it is better to have indexed debt, but for moderate levels of debt, it is preferable to have nominal debt, i.e. it is better to converge to the level of debt associated with the nominal-debt distorted steady state (positive or negative), rather than have an indefinite accumulation of debt or its depletion and subsequent accumulation of assets all the way to the first best.

## 6 Additional taxes

In most advanced economies, seigniorage is a minor source of revenue, and government liabilities are financed mostly through consumption and income taxes. This raises the question of whether our previous analysis is still valid when taxes are introduced so as to make seigniorage only a marginal component of government revenues. An additional motivation to introduce taxes in our model is to inquire whether a fully committed fiscal authority can overrun the commitment problems of a monetary authority. In this section we address these questions by introducing both consumption and labor income taxes.

We show first that the introduction of taxes, when the fiscal authority sets taxes one period in advance, reduces the need to raise revenues through seigniorage but does not change the characterization of equilibria with respect to the economies without taxes, analyzed in the previous sections. Our timing assumption is reasonable, taking into account the different frequencies in which monetary and fiscal decisions are typically made and implemented. Second, we show that, if the intertemporal elasticity of substitution is not lower than one and there is full commitment on the part of the fiscal authority (who makes policy choices before the monetary authority does), the fiscal authority can implement the full-commitment Ramsey solution independently of the degree of commitment of the monetary authority who is constrained to follow the Friedman rule of zero nominal interest rated. This result applies to both economies with nominal and indexed-debt.

When the government levies consumption and labor income taxes, the household problem becomes:

$$\max \sum_{t=0}^{\infty} \beta^t \left[ u(c_t) - \alpha n_t \right] \tag{27}$$

subject to:

$$P_t(1+\tau_t^c)c_t \leq M_t \tag{28}$$

and

$$M_{t+1} + B_{t+1} \leq M_t - P_t(1+\tau_t^c)c_t + B_t(1+i_t) + P_t(1-\tau_t^n)n_t, \tag{29}$$

where $\tau_t^c, \tau_t^n$ are consumption and labor income taxes respectively, and subject to the terminal condition: $\lim_{T \longrightarrow \infty} \beta^T \frac{B_{T+1}}{P_T} \geq 0$.

Now, the marginal conditions (**??**), (**??**) and (**??**) characterizing the households's optimal choice become:

$$\frac{u_c(c_{t+1})}{\alpha} = (1+i_{t+1}) \frac{1+\tau_{t+1}^c}{1-\tau_{t+1}^n} \tag{30}$$

$$1 + i_{t+1} = \beta^{-1} \frac{P_{t+1}\left(1-\tau_{t+1}^n\right)}{P_t\left(1-\tau_t^n\right)} \tag{31}$$

and

$$c_t = \frac{M_t}{P_t(1+\tau_t^c)}. \tag{32}$$

These conditions must hold for every $t \geq 0$. As in the economies without taxes, the cash-in-advance constraint (**??**) is binding as long as $u_c(c_t)/\alpha > 1$. However, in contrast with economies without taxes, $u_c(c_t)/\alpha > 1$ does not imply that the nominal interest rate is positive. The non-negativity of nominal interest rates is an additional restriction to the implementation.

The sequence of government budget constraints in this economy is now given by

$$P_t g + M_t^g + B_t^g(1+i_t) \leq P_t \tau_t^c c_t + P_t \tau_t^n n_t + M_{t+1}^g + B_{t+1}^g \tag{33}$$

while the feasibility conditions (**??**) do not change.

Define the *effective labor income tax* as:

$$\tau_t = \frac{\tau_t^c + \tau_t^n}{1+\tau_t^c}, \text{ i.e. } (1-\tau_t) = \frac{1-\tau_t^n}{1+\tau_t^c}$$

Then, using the equilibrium conditions (**?? - ??**), as well as the resource constraint (**??**), the intertemporal government budget constraint can be written as the following implementability condition[15]

$$c_{t+1} u_c(c_{t+1}) \frac{\beta}{\alpha} + \beta z_{t+1}(1-\tau_{t+1})^{-1} c_{t+1} = c_t + z_t(1-\tau_t)^{-1} c_t + g, \tag{34}$$

together with the terminal condition

$\lim_{T \longrightarrow \infty} \beta^T c_{T+1} u_c(c_{T+1}) \frac{\beta}{\alpha} + \beta z_{T+1}(1-\tau_{T+1})^{-1} c_{T+1}) = 0.$

These implementability conditions compare with (**??**). The equations are the same, except that the variable $z_t$ in (**??**) is the variable $z_t(1-\tau_t)^{-1}$ in (**??**).

## 6.1 Optimal monetary policy when the fiscal authority moves one period in advance

We now consider the case where tax decisions for some period $t$ must be made one period in advance, and may depend only on the state at $t-1$. In this case we can define the

[15] Notice that $\frac{M_t^g}{P_t(1-\tau_t^n)} = c_t(1-\tau_t)^{-1}$; $\frac{M_{t+1}^g}{P_t(1-\tau_t^n)} = \frac{M_{t+1}^g}{P_{t+1}(1-\tau_{t+1}^n)} \frac{P_{t+1}(1-\tau_{t+1}^n)}{P_t(1-\tau_t^n)} = c_{t+1}(1-\tau_{t+1})^{-1}\beta(1+i_{t+1}) = c_{t+1} u_c(c_{t+1}) \frac{\beta}{\alpha}$; $\frac{B_t^g(1+i_t)}{P_t(1-\tau_t^n)} = \frac{B_t^g(1+i_t)}{M_t^g} \frac{M_t^g}{P_t(1-\tau_t^n)} = z_t c_t (1-\tau_t)^{-1}$, and $\frac{B_{t+1}^g}{P_t(1-\tau_t^n)} = \frac{B_{t+1}^g(1+i_{t+1})}{M_{t+1}^g} \frac{M_{t+1}^g}{P_t(1-\tau_t^n)(1+i_{t+1})} = \beta z_{t+1} c_{t+1} (1-\tau_{t+1})^{-1}$.

new state variable $\widehat{z}_t \equiv z_t(1-\tau_t)^{-1}$, and the problems are isomorphic to the problems in the previous sections, since the implementability condition (??) reduces to

$$c_{t+1}u_c(c_{t+1})\frac{\beta}{\alpha} + \beta\widehat{z}_{t+1}c_{t+1} = c_t + \widehat{z}_t c_t + g, \tag{35}$$

which is formally identical to (??).

Notice that financing liabilities through taxes reduces the need to use seigniorage, however this does not mean that distortions are reduced, since $u_c(c_{t+1})/\alpha = (1+i_{t+1})(1-\tau_{t+1})^{-1}$.

Consistent with our framework, we assume that the fiscal authority either sets a sequence of taxes $\{\tau_t\}_{t=0}^{\infty}$ – and commits to it – or defines a recursive strategy for taxes – from period one on – as a function of the state; i.e. sets $\tau_0$ and $\tau_{t+1} = \tau(z_t)$. As long as the nominal interest rates are away from the lower bound of zero nominal interest rates, the problem for the monetary authority has the same structure as before, and therefore the same results obtain.

In summary, the monetary authority faces the same problem with pre-determined taxes on consumption and income as with only seigniorage, for any degree of monetary commitment. The recursive strategies of the monetary authority $C(\widehat{z})$ and $\boldsymbol{z}'(\widehat{z})$are the same as in the economy without taxes, as long as nominal interest rates are positive.

**Additional taxes with indexed debt.** In the economy with indexed debt let

$$\widehat{b}_t = b_t(1-\tau_t^n)^{-1} = \frac{B_t(1+i_t)}{P_t(1-\tau_t^n)}.$$

That is, $\widehat{b}_t = z_t c_t(1-\tau_t)^{-1} = \widehat{z}_t c_t$. Then the implementability condition with indexed debt can be written as

$$c_{t+1}u_c(c_{t+1})\frac{\beta}{\alpha} + \beta\widehat{b}_{t+1} = c_t + \widehat{b}_t c_t + g. \tag{36}$$

With labor taxes set one period in advance, as long as nominal interest rates are positive, the policy choices $C(\widehat{b})$ and $\boldsymbol{b}'(\widehat{b})$ are the same as in the economy without taxes. Furthermore, if $\tau_t^n = 0$ then the implementability condition is equivalent to (??) with $\widehat{b}_t = b_t$. This means that when there are only consumption taxes, whether these are decided one period in advance or not, the policy problem and therefore the optimal path of indexed debt do not change.

## 6.2 When the fiscal authority can fully commit

Whenever the discretionary incentive is to have higher current tax (which is the case with $\sigma \leq 1$), it is possible to set taxes in a way that the resulting monetary policy follows the full-commitment Ramsey solution with zero nominal interest rates[16].

Full-commitment solutions, both with indexed and nominal debt, have the characteristic that consumption is constant from period one on. Let $c_0$, $c_t = \bar{c}$, $t \geq 1$, be the optimal path of consumption with full-commitment in a given economy. Let taxes be $\tau_t = \bar{\tau}$, $t \geq 1$,where

$$\frac{u_c(\bar{c})}{\alpha} = (1 + \bar{\tau})^{-1}.$$

In period zero, the monetary authority has no incentive to deviate from such consumption plan. Consider the choice of the monetary authority at any $t \geq 1$. If the incentive is to delay consumption then, given that the government present value budget constraint must be satisfied, there must be some future consumption $c_{t+1+s}, s \geq 0$, such that $c_t < \bar{c} < c_{t+1+s}$, as long as raising the tax rates raises revenue. However, since $u_c(c_{t+1+s})/\alpha = (1 + i_{t+1+s})(1 + \bar{\tau})^{-1}$ it must be that $i_{t+1+s} < 0$. Negative interest rates cannot be an equilibrium in this economy since households would borrow without limit.

It follows that there is no gain in partially monetizing the stock of nominal debt in any period, and monetary policy can only passively implement the full-commitment solution of the corresponding nominal or indexed-debt economy[17].

**Proposition 3** *In an economy with either nominal or indexed debt (with $\widehat{z}_0$ or $\widehat{b}_0$), assume fiscal authorities maximize the welfare of the representative household and fully commit to their policies. If $\sigma \leq 1$ then the fiscal authorities can induce the implementation of the corresponding full-commitment Ramsey equilibrium regardless of the degree of commitment of the monetary authority.*

As we have seen, when $\sigma > 1$ the monetary authority has an incentive to reduce the current price level (i.e. the 'intertemporal elasticity' effect may dominate the 'nominal debt' effect) and so in this case the previous argument cannot be applied.

# 7 Concluding remarks

This paper has discussed the different ways in which nominal and indexed debt affect the sequential choice of optimal monetary and debt policies. To this purpose, we have studied a general equilibrium monetary model where the costs of unanticipated inflation

[16]The Friedman rule is optimal in an economy with cash and credit goods if preferences are separable in leisure and homothetic in the two consumption goods.

[17]Marimon, Nicolini, and Teles (2003) make a similar argument in a different context.

arise from a cash-in-advance constraint with the timing of Svensson (1985), and where government expenditures are exogenous.

In our environment, as in Nicolini (1998), when the utility function is logarithmic in consumption and linear in leisure and debt is indexed, there is no time-inconsistency problem. In this case, the optimal monetary policy is to maintain the initial level of indexed debt, independently of the level of commitment of a benevolent government.

In contrast, for the same specification of preferences, when the initial stock of government debt is nominal, a time-inconsistency problem arises. In this case, the government is tempted to inflate away its nominal debt liabilities. When the government cannot commit to its planned policies, to progressively deplete the outstanding stock is part of an optimal sequential policy. Optimal nominal interest rates in this case are also decreasing and converge asymptotically.

In the rational expectations equilibria of our economies there are no surprise inflations. Still, for a given initial real value of outstanding debt, the sequential optimal equilibrium with indexed debt provides higher welfare. In this sense nominal debt can be a burden on optimal monetary policy.

When we consider CRRA preferences with the intertemporal elasticity of substitution different from one, it is still true that in a Markov-perfect equilibrium the path of nominal debt converges to a stationary level of debt. However, it is not zero, but negative or positive depending on the intertemporal elasticity being greater or lower than one. With such general preferences, optimal sequential policy is time inconsistent even when debt is indexed. The interaction of the two sources of dynamic distortions, resulting from the differing elasticities and from nominal debt, can overturn the above efficiency result and it may actually be the case that nominal debt provides higher welfare than indexed debt. In fact, our computations show that, for relatively low values of debt, welfare is higher when debt is nominal. This is one more illustration of the principle that in a second best, adding a distortion may actually increase welfare. However, our computations also show that, for large levels of debt, indexed debt dominates in terms of welfare and therefore nominal debt is a burden to monetary policy.

The introduction of additional forms of taxation further clarifies the interplay between the various forms of debt and commitment possibilities. Under the natural assumption that fiscal policy choices are predetermined, we have shown that the optimal policy problem has the same characterization, provided that the revenues levied through seigniorage are enough to allow for an optimal monetary policy with non-negative interest rates.

If there is full commitment to an optimal fiscal policy, the fiscal authorities, anticipating monetary policy distortions, may choose to fully finance government liabilities and – provided the elasticity of substitution is greater or equal to one – the resulting monetary policy follows the Friedman rule of zero nominal interest rates. Moreover, this policy results in the equilibrium that obtains in the economy with full commitment with indexed debt, even if debt is nominal and the monetary authority can not commit.

Ours is a normative (second-best) analysis that takes into account the commitment problems which are at the root of institutional design in many developed economies (such as Central Bank independence, constraints on public indebtness, etc.) As such, it throws new light on the ways in which the possibility of monetizing nominal debts can affect monetary policy (a central concern in policy design), and on how optimal debt and monetary policies should be designed. We do not claim that our results on optimal-equilibrium debt paths match – or should match – observed data. Still, it is the case that the prescriptions of our model could be used to provide a more detailed positive analysis of existing monetary policies and some insights on how monetary and debt policies should be redesigned if necessary.

## References


Alvarez, F., Kehoe P. J., and Neumeyer P. A., 2004, "The Time Consistency of Optimal Fiscal and Monetary Policies," *Econometrica* 72, 2, 541-567.

Calvo, G., 1988, "Servicing the Public Debt: the Role of Expectations," *The American Economic Review* 78, 4, 647-661.

Díaz-Giménez, J., Giovannetti G., Marimon R., and Teles P., 2004. "Nominal Debt as a Burden on Monetary Policy, "Working Paper Series WP-04-10, Federal Reserve Bank of Chicago.

Ellison, M., and Rankin N., 2007,"Optimal Monetary policy when Lump-Sum Taxes are Unavailable: A Reconsideration of the Outcomes under Commitment and Discretion, "*Journal of Economics, Dynamics and Control* 31, 1, 219-243.

Krusell, P., Martin F., and Rios-Rull J.-V., 2003, "On the Determination of Government Debt," mimeo, U. Pennsylvania.

Lucas, R. E., JR., and Stokey N. L., 1983, "Optimal Fiscal and Monetary Theory in an Economy without Capital," *Journal of Monetary Economics* 12, 55-93.

Marimon, R., Nicolini J. P., and Teles P., 2003, "Inside-Outside Money Competition," *Journal of Monetary Economics* 50, 1701-1718.

Martin F., 2006, "A Positive Theory of Government Debt,"mimeo, Simon Fraser University.

Nicolini, J. P., 1998, "More on the Time Inconsistency of Optimal Monetary Policy," *Journal of Monetary Economics* 41, 333-350.

Obstfeld M., 1997, "Dynamic Seigniorage Theory: An Explanation," *Macroeconomic Dynamics* 1, 3, 588-614.

Persson, M., Persson T., and Svensson L.E.O., 2006, "Time Consistency of Fiscal and Monetary Policy: A Solution," *Econometrica* 74, 1, 193-212.

Reis, C., 2006, "Taxation without Commitment,"mimeo, MIT.

Svensson, L.E.O., 1985, “Money and Asset Prices in a Cash-in-Advance Economy,” *Journal of Political Economy* 93, 919–944.

## Appendices

### Appendix 1. Equilibria with first-best outcomes

In this Appendix we discuss in more detail equilibria with first-best outcomes; in particular, equilibria where the cash-in-advance constraints may not be binding. To understand these cases it is enough to consider optimal policies with full commitment. In these economies, the implementability constraint, assuming that the cash-in-advance constraint holds with equality and given that optimal consumption is constant from period $t = 1$ on, reduces to

$$\beta \left[ \frac{u_c(c_1)}{\alpha} - 1 \right] c_1 - (1-\beta)c_0 - g = (1-\beta)x_0.$$

where $x_0 = b_0$ if debt is indexed, and $x_0 = z_0 c_0$ if debt is nominal.

Clearly, if initial government assets are large enough for there to be no need to raise distortionary taxes, the first best is achieved, i.e. $c_1 = c_0 = c^*$, where $\frac{u_c(c^*)}{\alpha} = 1$.

Both in economies with nominal and indexed debt, there is a first-best solution where the cash-in-advance constraint holds with equality in every period, even if it is not binding. In the indexed-debt economy that will be the case if initial debt is $b_0 = b^* \equiv -c^* - \frac{g}{(1-\beta)}$, and in the nominal debt economy, if initial debt is $z_0 \equiv z^* = -1 - \frac{g}{(1-\beta)c^*}$, which results in real initial assets $b^* = z^* c^*$. When initial assets are larger $\left(b_0 < b^*\right.$ or $\left.z_0 < z^*\right)$, the government transfers to the consumers the redundant assets lump sum in order to implement the first-best allocation.

With nominal debt, it is possible to implement the first-best allocations for any $z_0 < -1$, i.e. $M_0^g + B_0^g(1+i_0) < 0$, although then the cash-in-advance constraints do not hold with equality and therefore the implementability conditions cannot be written as above. By imposing the transversality condition (**??**), the budget constraint of the government (**??** ) can then be written as

$$\sum_{t=0}^{\infty} \beta^{t+1} \frac{i_{t+1} M_{t+1}^g}{P_{t+1}} = \frac{M_0^g + B_0^g(1+i_0)}{P_0} + \frac{g}{1-\beta}.$$

But if $M_0^g + B_0^g(1+i_0) < 0$, there is a price $P_0$, such that the first-best $c_t = c^*$, for all $t \geq 0$, is achieved. At the first-best, $i_{t+1} = 0$, $t \geq 0$, and therefore

$$\frac{M_0^g}{P_0}(1+z_0) = -\frac{g}{1-\beta}.$$

If $z_0 \in (z^*, -1)$ then $\frac{M_0^g}{P_0} > c^*$, i.e. the cash-in-advance constraint is not satisfied with equality. In particular, if $1 + z_0 = -\varepsilon < 0$, with $\varepsilon \to 0$, in order to achieve the first best, $\frac{M_0^g}{P_0} \to \infty$ (which means that initial real debt is $\frac{B_0^g(1+i_0)}{P_0} = -\frac{M_0^g}{P_0} - \frac{g}{1-\beta} \to -\infty$). But this solution with total assets that are positive but arbitrarily low is not an equilibrium if anticipated. Indeed from the Fisher equation (**??**), the nominal interest rate would have to be negative and approaching $-1$, as the price level approaches zero. Private agents would be able to make infinite profits by borrowing at the negative nominal rate and holding money. As a result, when $z_0 < -1$, but close to $-1$, there is a first-best *full commitment Ramsey equilibrium,* but there is no *Markov-perfect equilibrium* for $z < -1$ and close to $-1$, since the latter imposes that the government policy be anticipated, and the above incentive to deflate is equally present when there is no commitment.

Finally, the case $z_0 = -1$ also deserves to be discussed. The corresponding *full-commitment Ramsey equilibrium* results in a time-inconsistent optimal policy given by $c_0 = c^*$ and $c_1$ being the solution to $\beta \left[\frac{u_c(c_1)}{\alpha} - 1\right] c_1 = g$, while $z_t = -1$, for $t \geq 1$. A *Markov-perfect equilibrium* with $z_0 = -1$ must have $c_0 = c^*$ but $z_1 > -1$, since it is not possible to have $z_1 \leq -1$.

## Appendix 2. Numerical Exercises

We carry out numerical exercises for three different values of the elasticity of substitution: $\sigma_1 = 0.6$, $\sigma_2 = 1.0$, and $\sigma_3 = 1.4$. To solve our model economies we must choose numerical values for $\alpha$, $\beta$, and $g$. These are the same for the different economies. We assume that the value of $\beta$ is such that the real interest rate is approximately two percent. Consequently, $\beta = 0.98$.

We take as reference values a constant government expenditure to output ratio of $g/y = 0.01$ and a constant debt to output ratio of $b/y = 0.8$. The reason for the low expenditure to output ratio is that these are the expenditures to be financed with seigniorage which is a relatively low share of tax revenues. Since in our model economies $g + c = y$, these choices imply that $g/c = 0.01$ and $b/c = 0.81$. Next, we normalize units so that $c = 1$ and, therefore $g = 0.01$ and $y = 1.01$. To obtain the value of $\alpha$ we use the implementability condition, (**??**), with stationary values of consumption and debt:

$$\frac{c^{-\sigma}}{\alpha} = \frac{1}{\beta}\left[1 + \frac{g}{c} + (1-\beta)\frac{b}{c}\right]. \tag{37}$$

Notice that, given our normalization $c = 1$, different values of $\sigma$ result in the same choice of $\alpha$; which, given the rest of the parameters, is $\alpha = 0.95$.

These choices imply that the values of $b$ that support the first best in the economies with indexed debt are $b_1^* = -1.59$, $b_2^* = -1.55$, $b_3^* = -1.54$. These are obtained by computing $c_i^*$ such that $\frac{(c_i^*)^{-\sigma_i}}{\alpha} = 1$, and the corresponding value of $b_i^*$ satisfies (**??**). The choices of parameters also imply that the distorted steady-state values of $z$ for the economies with nominal debt are $\overline{z}_1 = -0.429$ (corresponding to $\overline{c}_1 = 1.05$ and $\overline{b}_1 = -0.451$), $\overline{z}_2 = 0$ (corresponding to $\overline{c}_2 = 1.02$ and $\overline{b}_2 = 0$), and $\overline{z}_3 = 0.419$ (corresponding to $\overline{c}_3 = 1.01$ and $\overline{b}_3 = 0.423$). These are obtained using (**??**) for stationary consumption and the implementability condition (**??**) in the steady state, so that

$$\overline{z} = -\frac{u_c(\overline{c})}{\alpha}(1-\sigma)$$

and

$$\frac{\overline{c}u_c(\overline{c})}{\alpha}\left[1-(1-\beta)\,\sigma\right] = \overline{c} + g.$$

Therefore, since $(1-\beta)\,\sigma < 1$, there is a solution for $\overline{c}$ and, correspondingly, for $\overline{z}$.

**Algorithm**

Let $u(c) = (c^{1-\sigma}-1)/(1-\sigma)$. Then to compute the monetary equilibria numerically, we solve the following dynamic program:

$$V(x) = \max\left\{u(c) - \alpha\,(c+g) + \beta V(x')\right\} \tag{38}$$

subject to

$$\frac{\beta}{\alpha}C(b')^{1-\sigma} + \beta b' = c + g + b \tag{39}$$

when $x = b$ and the debt is indexed, or subject to

$$\frac{\beta}{\alpha}C(z')^{1-\sigma} + \beta z' C(z') = (1+z)c + g \tag{40}$$

when $x = z$ and the debt is nominal.

The Bellman operators associated with these problems are:

$$V_{n+1}(x) = T[V_n(x)] = \max\left\{u(c) - \alpha\,(c+g) + \beta V_n(x')\right\} \tag{41}$$

subject to expression (**??**) when $x = b$ and the debt is indexed, or subject to expression (**??**) when $x = z$ and the debt is nominal.

To solve these problems, we use the following algorithm:

- Step 1: Choose numerical values for parameters $\alpha$, $\beta$, $\sigma$, and $g$.

- Step 2: Define a discrete grid on $x$ (with the first-best level of debt as a lower bound for the grid with indexed debt, $-1$ as a lower bound for the grid with nominal debt as well as for the grid of welfare comparisons, and a large upper bound, so as to capture an unambiguous welfare ranking across regimes for high values of debt. This may require a robustness test on the upper bound; see below how we treat the case $\sigma > 1$).

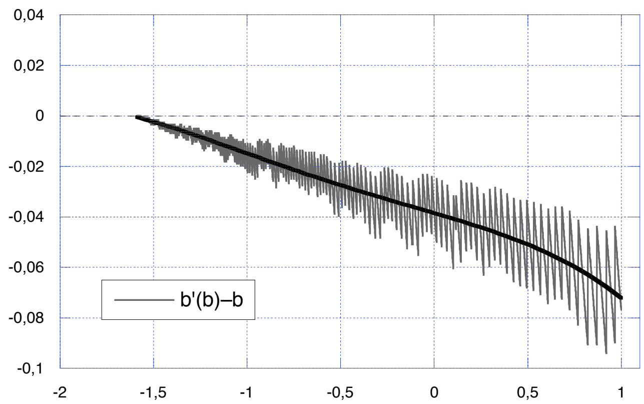
- Step 3: Define a decreasing discrete function $C_n(x)$.

- Step 4: Define an initial discrete function $V_n(x)$ and iterate on the Bellman operator defined above until we find the converged $V^*(x)$, $x'^*(x)$, $C^*(x)$.

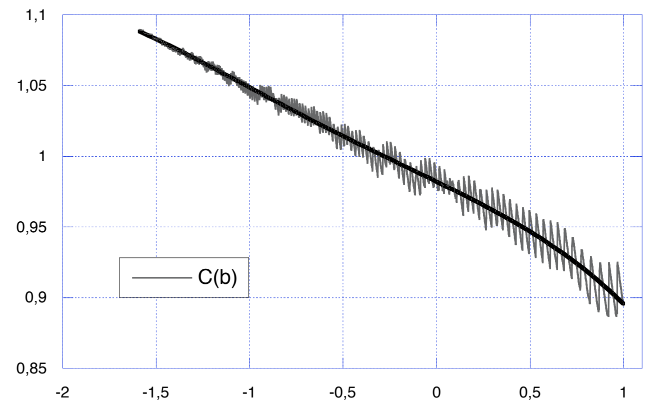
- Step 5: If $C^*(x) = C_n(x)$, we are done. Otherwise, update $C_n(x)$ and go to Step 3.

The above algorithm must be modified to compute indexed economies with $\sigma > 1$ where the level of debt grows (at a rate lower than $\beta^{-1}$). In this case we iterate the above procedure by expanding the upper bound (in Step 2) until successive iterations do not (significantly) change optimal policies in the relevant range.

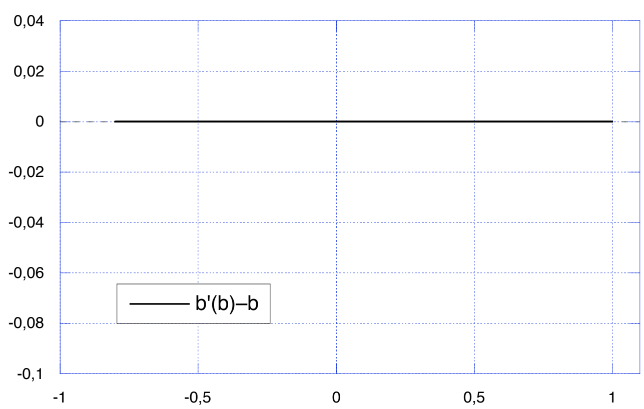
Figure 1: Indexed Debt for various values of $\sigma$



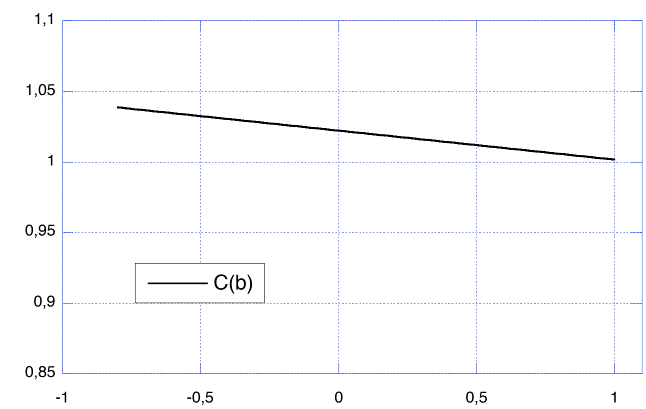

A. Indexed Debt: $b'(b) - b$ for $\sigma = 0.6$



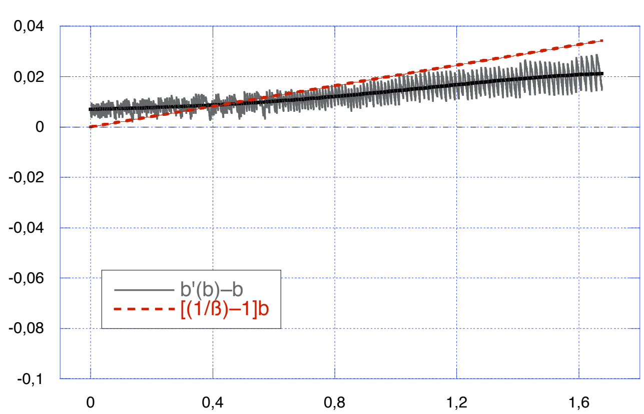

B. Indexed Debt: $C(b)$ for $\sigma = 0.6$



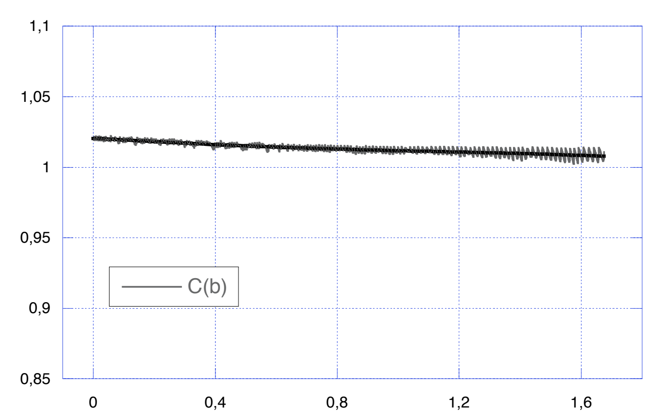

C. Indexed Debt: $b'(b) - b$ for $\sigma = 1.0$




D. Indexed Debt: $C(b)$ for $\sigma = 1.0$

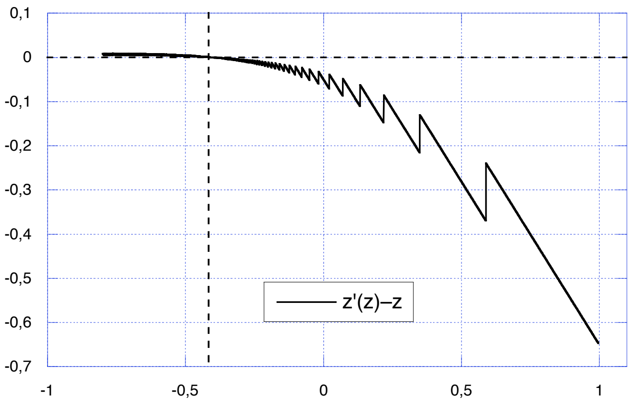



E. Indexed Debt: $b'(b) - b$ for $\sigma = 1.4$

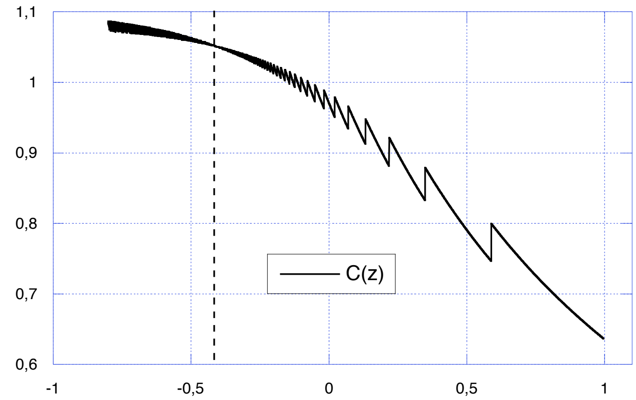



F. Indexed Debt: $C(b)$ for $\sigma = 1.4$

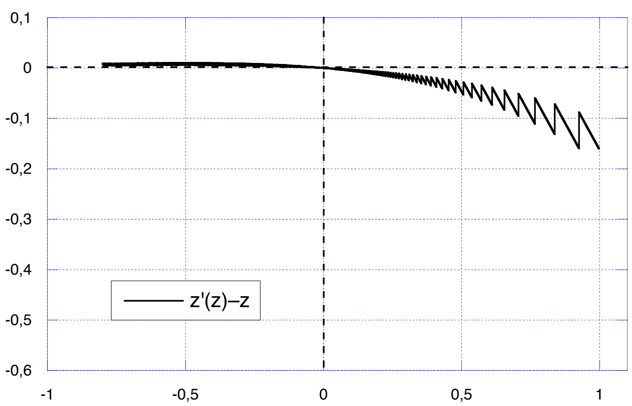
Figure 2: Nominal Debt for various values of $\sigma$



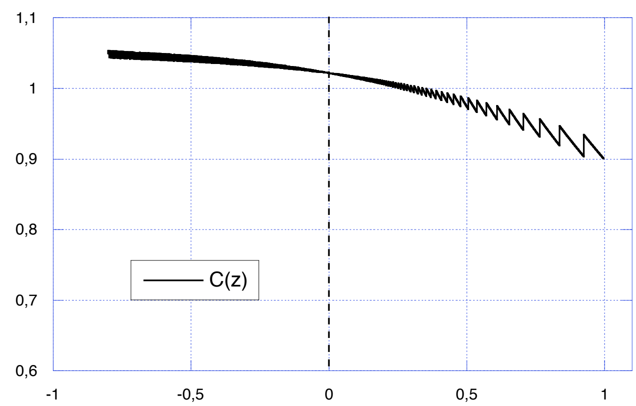

A. Nominal Debt: $z'(z) - z$ for $\sigma = 0.6$



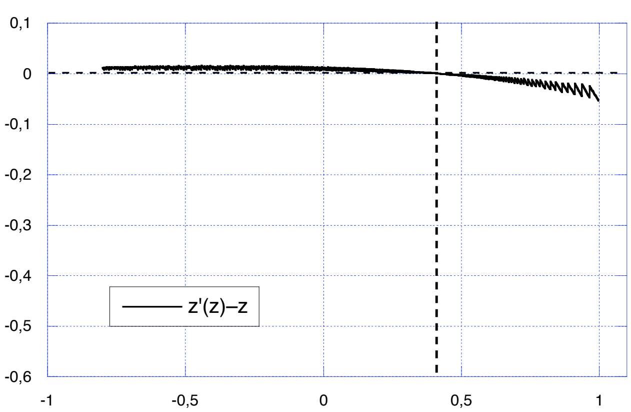

B. Nominal Debt: $C(z)$ for $\sigma = 0.6$



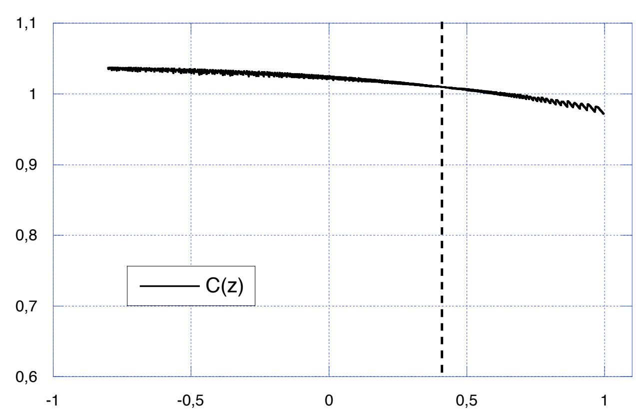

C. Nominal Debt: $z'(z) - z$ for $\sigma = 1.0$




D. Nominal Debt: $C(z)$ for $\sigma = 1.0$

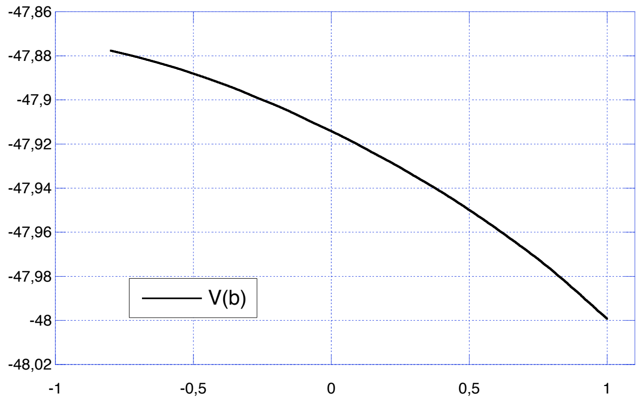



E. Nominal Debt: $z'(z) - z$ for $\sigma = 1.4$

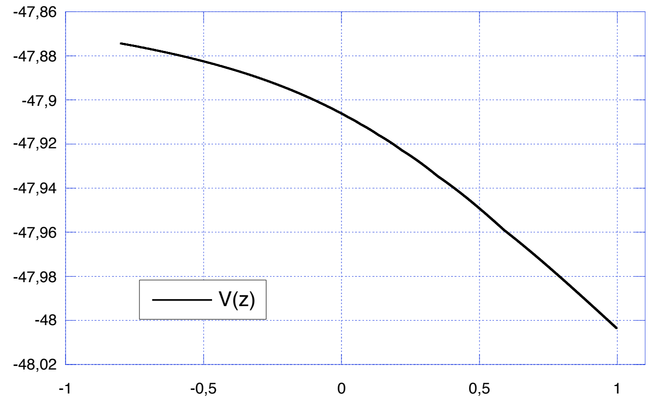



F. Nominal Debt: $C(z)$ for $\sigma = 1.4$

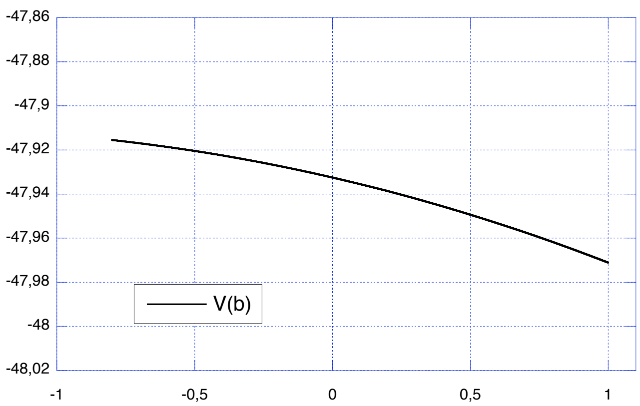
Figure 3: Value Functions for Indexed and Nominal Debt and various values of $\sigma$



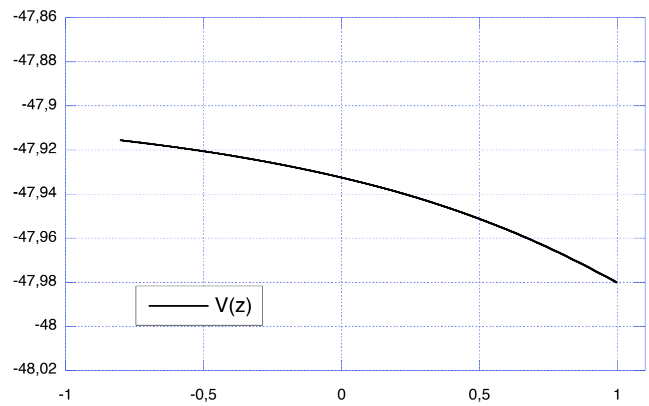

A. Indexed Debt: $V(b)$ for $\sigma = 0.6$



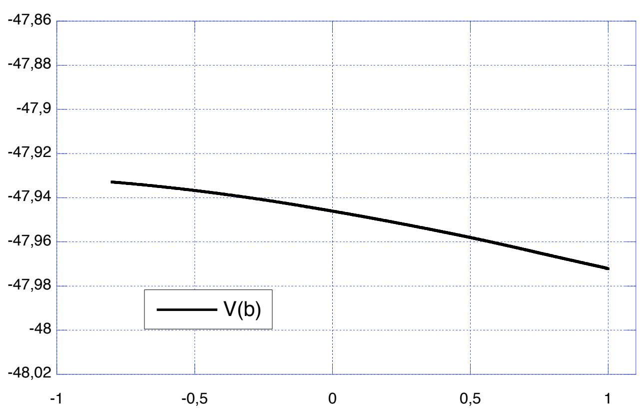

B. Nominal Debt: $V(z)$ for $\sigma = 0.6$



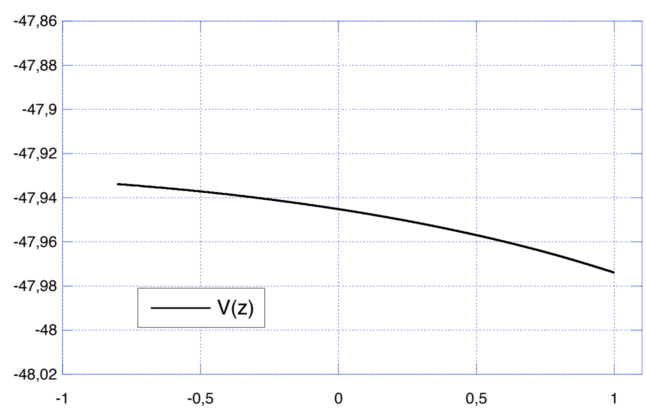

C. Indexed Debt: $V(b)$ for $\sigma = 1.0$




D. Nominal Debt: $V(z)$ for $\sigma = 1.0$

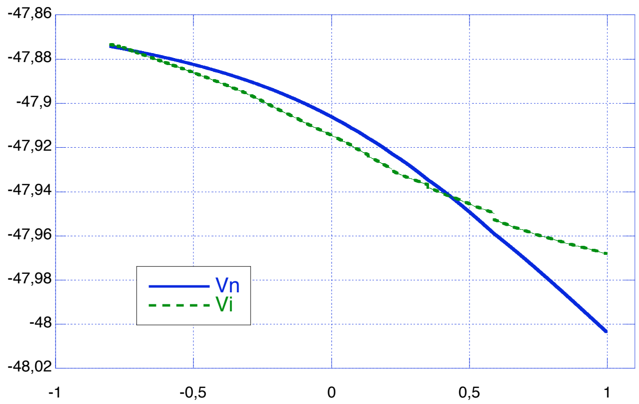



E. Indexed Debt: $V(b)$ $\sigma = 1.4$

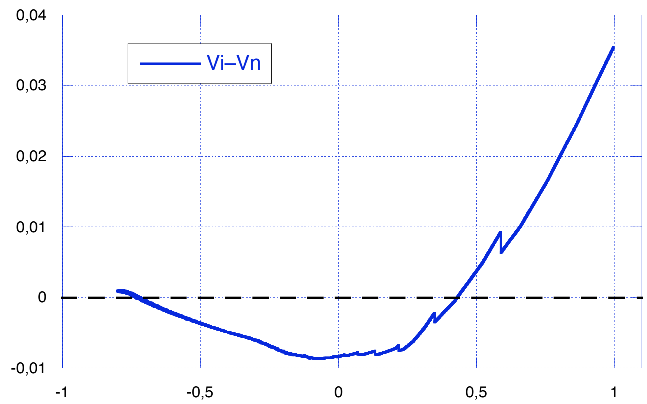



F. Nominal Debt: $V(z)$ for $\sigma = 1.4$

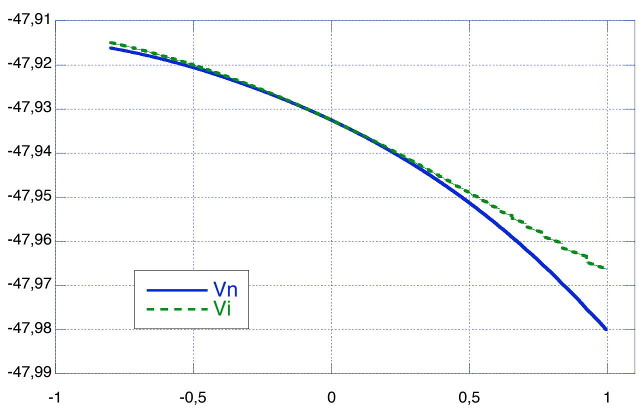
Figure 4: Welfare Comparisons for various values of $\sigma$



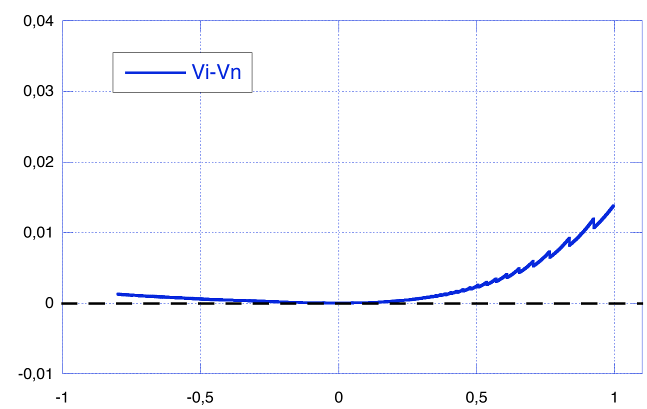

A. $V_n(z)$ and $V_i[b(z)]$ for $\sigma = 0.6$



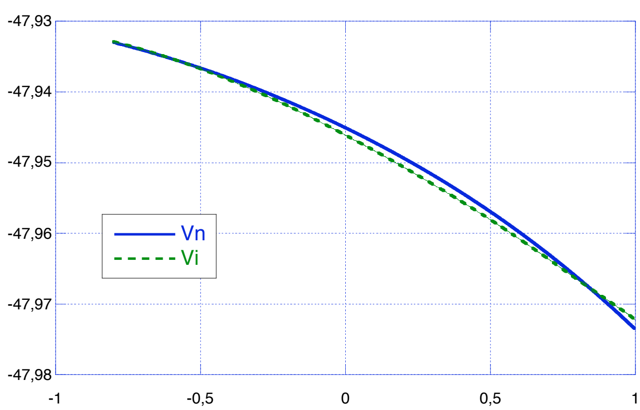

B. $V_i[b(z)] - V_n(z)$ for $\sigma = 0.6$



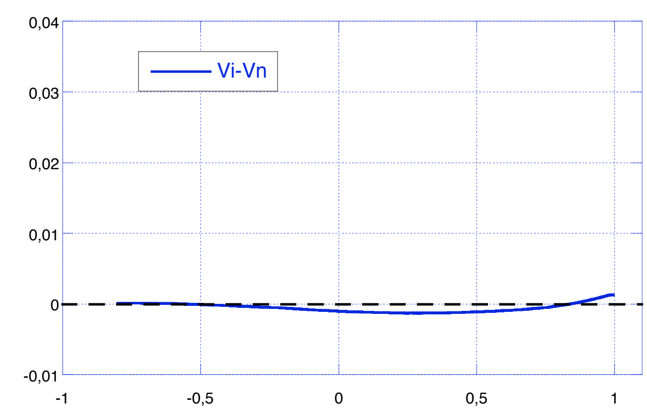

C. $V_n(z)$ and $V_i[b(z)]$ for $\sigma = 1.0$




D. $V_i[b(z)] - V_n(z)$ for $\sigma = 1.0$




E. $V_n(z)$ and $V_i[b(z)]$ for $\sigma = 1.4$




F. $V_i[b(z)] - V_n(z)$ for $\sigma = 1.4$