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Accounting for the hidden economy: barriers to legality and legal failures*

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Abstract

This paper examines how much of the difference in the size of the informal sector and in per capita income across countries can be accounted by regulation costs (*barriers to legality*) and contractual imperfections in financial markets (*legal failures*). It constructs and solves numerically a general equilibrium model with credit constrained heterogenous agents, occupational choices over formal and informal businesses, contractual imperfections and a government sector which imposes taxes and regulations on formal firms. The premium from formalization is better access to outside finance. Differences in regulation costs and the degree of enforcement in financial contracts endogenously generate differences in the size of the informal sector and in total factor productivity (TFP). The numerical exercises suggest that: (i) regulation costs and not financial market imperfections account for the difference in the size of the informal sector between United States and Mediterranean Europe; (ii) this is not the case for countries with very weak enforcement systems, such as Peru, as both contractual imperfections and regulation costs account for the observed difference in the size of the informal sector. Regarding output per capita, regulation costs and the strength of enforcement explain roughly 60% of the difference in observed international incomes.

JEL Classification: E6; O11; O17

Keywords: inequality; credit constraints; corruption; informal sector

1 Introduction

A fundamental issue in economic development is to study the determinants of the informal (unregulated) sector. Why does the size of the informal sector vary so much across countries? Informal production accounts for roughly 10 percent of total production in the United States

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and this statistic is about 25 and 60 percent in Italy and Peru, respectively.[1] De Soto (1989) emphasizes that informal production in developing countries is mainly driven by *entry barriers*, under the form of regulation and corruption, that entrepreneurs face to acquire legal status. Empirical studies have corroborated this hypothesis.[2] In a cross-section of 85 countries, Djankov, La Porta, Lopez-de-Silanes and Shleifer (2002) show that stricter regulation of entry[3] is associated with sharply higher levels of corruption, and a greater relative size of the unofficial economy.

Recently, De Soto (2000) suggests that these *barriers to legality* are not only important to explain differences in the size of the informal sector but also differences in per capita income across countries. His idea is that, without legal status, entrepreneurs cannot exercise full property rights over their assets, and, as a consequence, cannot use their wealth as collateral for a loan and cannot generate capital from their savings. Due to these *legal failures*, they, in general, under-invest and are locked into running low productivity technologies.

This paper investigates the role of corruption, regulation (*barriers to legality*) and credit market imperfections (*legal failures*) in accounting for differences in the size of the unofficial economy across countries. We put forth a general equilibrium model to address De Soto's (2000) hypothesis analytically and quantitatively. Agents in our framework can choose to be either a worker or an (informal or formal) entrepreneur. In this respect, this paper is related to Lucas' (1978) "span of control" model, which was later extended by Rauch (1991) in his study of informal production. Unlike these models, ours is built upon a dynamic framework and uses credit constraints in the analysis of occupational choice. Agents are differentiated by entrepreneurial ability and initial wealth. They care about their own consumption and the initial wealth of their offspring. In order to open a formal/informal business, agents must buy capital in advance to finance their project. Capital markets are imperfect and therefore the best project will not necessarily be undertaken. This interaction between wealth distribution and capital market imperfection is based on Banerjee and Newman (1993).[4]

The theoretical environment, therefore, considers three occupational choices (worker, and formal or informal entrepreneur), inequality in wealth and in entrepreneurial ideas, corruption and regulation, and limited enforcement. Occupational choices and the size of each project are determined endogenously. They depend on the agent's "type" (wealth and project), start up costs, and credit market imperfections. Different levels of bureaucracy and limited enforcement generate not only differences in the occupational choice (size of the informal sector), but also differences in total factor productivity (TFP). Our model therefore provides a theory of differences in TFP, as required by Prescott (1998), that maps differences in corruption and limited enforcement into differences in observed TFP.[5]

Our quantitative experiments based on the empirical evidence on the size of the informal sector, regulation costs and the degree of enforcement across countries suggest that: (i) regulation costs rather than financial market imperfections, account for the difference in the size of the informal sector between United States and Mediterranean Europe; however, (ii) this is not the case for countries with very weak enforcement systems (property rights), such as Peru:

[1] See table 1 (column 1) in section 4. See Johnson, Kaufmann and Zoido-Lobaton (1998), Schneider and Enste (2000), and Friedman, Johnson, Kaufman and Zoido-Lobaton (2000) for additional data, and for an extensive discussion of the underground economy.

[2] See Friedman et al. (2000) and Djankov, La Porta, Lopez-de-Silanes and Shleifer (2002).

[3] They evaluate all formal procedures that an entrepreneur needs to carry out to begin legally operating a firm.

[4] See also Lloyd-Ellis and Bernhardt (2000) for a close framework which studies the macroeconomic and distributional dynamics associated with the process of economic development. They develop important tools that we use to characterize the long-run dynamics of our model economy.

[5] Antunes and Cavalcanti (2003) and Erosa and Cabrillana (2004) also develop model economies where capital market imperfections and regulation costs endogenously generate differences in TFP.

both contractual imperfections and regulation costs account for the observed difference in the size of the informal sector. Regarding differences in output per capita across countries, limited enforcement and corruption costs explain roughly 60% of the difference in observed international incomes. Therefore, the model accounts for the difference in informal sector sizes across countries, but just part of the difference in output per capita.

Another important empirical fact is also consistent with our exercises: the proportion of small scale activities is negatively related with per capita income levels. Tybout (2000), for instance, shows that government regulations and taxes are enforced only among large firms. In order to avoid the costs associated with formal production, entrepreneurs scale down the size of their firms and operate outside the realm of government regulation. This result arises endogenously in our economy.

There is a long tradition in economics to study the hidden economy. One branch of the literature studies the effects of the informal sector on growth and government policies.[6] Another branch, which is more related to this study, investigates the determinants of the hidden economy. Among the empirical studies it is important to highlight the work of Djankov, La Porta, Lopez-de-Silanes and Shleifer (2002), and of Friedman, Johnson, Kaufman and Zoido-Lobaton (2000). Djankov et al. (2002) emphasize the role of start up costs to generate large informal sectors, while Friedman et al. (2000) suggest that it is not tax rates[7] per se that induce entrepreneurs to go underground, but bureaucracy and a weak legal system. Their findings are consistent with our quantitative results. We contribute to this literature by specifically identifying cases in which corruption or start up costs are the main determinant of the hidden economy.

With respect to the theoretical studies,[8] Rauch's (1991) model suggests that entrepreneurs go underground to avoid minimum wages. Consequently, in their model the formal equilibrium wage rate is higher than that in the informal sector.[9] In Dessy and Pallage (2003) entrepreneurs become legal because they can use productive infrastructure (which enters in the production function), while in our model the benefit from formalization is better access to outside finance. Quintin (2002) has a similar premium from formalization. Our model is different from his because we add bureaucracy costs and bequest transfers to the analysis. Regulation costs are empirically and quantitatively important to determine the size of the informal sector and the interaction between credit constraints and bequest inequality is key for occupational choice.

Finally, a literature on two sector growth models and economic development is also related to our quantitative theoretical analysis. Parente, Rogerson and Wright (2000), for instance, introduce home production, while Restuccia (2004) considers a traditional and a modern sector in a Neoclassical growth model. Similar to our model, these economies can generate larger differences in "official" output levels across countries than standard models for a given policy differential. In our model technologies will be the same in each sector but differences in productivity will arise endogenously due to policies that affect differently each sector.[10]

[6] Easterly (1993) and Loayza (1996) show that growth is negatively related to informal production. Cavalcanti and Villamil (2003) show how the optimal monetary policy and the welfare costs of inflation might be affected by the size of the informal sector through tax evasion.

[7] They argue that the burden of tax rates has two effects: (i) it drives agents into the informal sector to avoid official taxes; but (ii) it can also raise revenue to improve government institutions and enforcement, which leads to a lower informal sector. In general, these two effects offset each other, and the relationship between tax rates and the size of the informal sector across countries is not statistically significant.

[8] An interesting article is Azuma and Grossman (2002). They study a related but different question: why do governments impose or tolerate burdensome taxes, bribes and bureaucracy that lead many producers to operate in the informal sector?

[9] See Amaral and Quintin (2003) for a model where differences in worker's characteristics and in the wage rate arise endogenously due to better investment opportunities in the formal sector. The question that they address is different from ours. They explain why informal production emphasizes low-skill work.

[10] Related to this result is Guner, Ventura and Yi (2004), who consider a model with occupational choice and investigate the effects of restrictions on size on productivity.

This paper is divided as follows: the next section describes the model economy. Section 3 describes the agents' optimal behavior, defines the competitive equilibrium allocations, and presents some analytical results. Section 4 solves the model numerically and conducts policy experiments. The last section provides some concluding remarks and policy implications.

2 The model

2.1 Preferences

In each time period ($t = 0, 1, 2, ...$), the economy consists of a continuum of individuals in the unit interval. Each agent lives and is productive for one period, then reproduces another individual so that the population is constant. Agents care about their own consumption and leave bequests to their offspring. Let c_t^i and b_{t+1}^i denote consumption and bequests, respectively, by agent i in period t. Preferences are represented by

$$U^i = (c_t^i)^\gamma (b_{t+1}^i)^{1-\gamma}, \quad \gamma \in (0,1). \tag{1}$$

This utility function implies that agents are risk-neutral with respect to income as the indirect utility function is linear in wealth. This implies that any additive punishment or reward in utility may be measured in terms of income. Notice that, for tractability, we assume that preferences are for the bequest and not the offspring's utility (Banerjee and Newman, 1993, Lloyd-Ellis and Bernhardt, 2000).

2.2 Endowments

Each individual can be either a worker or an entrepreneur. Entrepreneurs create jobs and manage their labor force, n. As in Lucas (1978*a*) each individual is endowed with a talent for managing, x^i, drawn from a continuous cumulative probability distribution function $\Gamma(x)$ with finite support $[\underline{x}, \overline{x}]$, where $\underline{x} \geq 0$. Therefore, in each period agents are distinguished by their initial wealth and ability as entrepreneurs, (b_t^i, x_t^i). We assume that the agent's talent for managing is not hereditary. For notational convenience, in the remainder of the paper we drop agent superscript i.

2.3 Production technologies

Managers operate a technology that uses labor, n, and capital, k to produce a single consumption good, y, that is represented by

$$y = xk^\alpha n^\beta, \quad \alpha,\ \beta\ > 0, \quad \text{and} \quad \alpha + \beta < 1. \tag{2}$$

Capital fully depreciates each period. Managers can operate only one project. Entrepreneurs can choose to declare their establishments (formal sector) or to work in the shadow economy (informal sector). In order to operate in the formal sector, entrepreneurs must pay a start up cost, ς, in the form of complying with regulations and corruption. This cost is assumed to be independent of firm output since it is an *ex-ante* payment to the government. This is a barrier to become formal. De Soto (1989, 2000) has shown that this cost varies across countries and is especially high in developing countries. Firms that are legally declared also pay a uniform payroll tax, τ. Informal firms do not pay any start up costs or taxes. We might assume that informal firms pay fines in case they are detected by the tax authority and assume that the expected punishment rate in the informal sector is a fraction of output, ηy, where $\eta \in [0,1]$.[11]

[11] ηy can also be seen as the costs associated with hiding information for operating in the shadow economy.

This parameter, however, does not add any insights about the effects of corruption costs and limited enforcement on the size of the informal sector. We, therefore, do not use it in the analysis.

2.4 The capital market

Agents can borrow capital from a financial intermediary with access to perfect outside capital markets, in which a risk-free bond earns a gross return of $r \geq 1$. Let l be the amount of funds that an agent borrows from the financial intermediary. In order to finance their projects, constrained agents must put up their initial wealth, b, as collateral. Borrowers cannot commit *ex-ante* to their individual promises and can avoid the repayment obligation, rl, by defaulting on their debt and loosing rb. Those that renege on their debt loose the collateral and incur a cost proportional to what was produced, ϕy. This is equivalent to an additive utility punishment. This cost reflects the strength of contract enforcement in the economy.[12] Higher ϕ means a better quality of the legal system. The point here is that, in contrast to Banerjee and Newman (1993), not only the quality of the enforcement system will be an important determinant of external debt, but also the quality of the project.[13] Since contracts are easily monitored in the formal sector, we assume for simplicity that ϕ is zero in the informal sector.[14] This is consistent with De Soto (2000), who points out that projects and assets in the informal sector are not adequately documented and therefore *"cannot be turned into capital or cannot be used as a collateral for a loan."* Loans will be limited by the agents' inheritance and the degree that contracts are enforced.

Notice that resource allocation involves the division of individuals among formal and informal managers and workers, and then the allocation of factors of production among managers. Occupational choices will be driven by the agent's type, (b, x), the efficiency of the capital market, ϕ, and government tax and regulation, τ and ς.

3 Optimal behavior and equilibrium

3.1 Entrepreneurs

Those who have enough resources and managerial ability to become entrepreneurs choose the level of capital and the number of employees to maximize profit subject to the technological constraint. Since capital markets are imperfect, let us describe the problem of an entrepreneur for a given level of capital k. Let $\mathcal{I}_F$ be an indicator function that takes value one when $j = F$, and zero otherwise. The problem of an entrepreneur with capital k is

$$\pi_j(k, x; w) = \max_{n_j} x k^\alpha n_j^\beta - (1 + \tau \mathcal{I}_F) w n_j, \tag{3}$$

where $j \in \{F, I\}$. Equation (3) gives the labor demand of each entrepreneur in both sectors:

$$n_j(k, x; w) = \left(\frac{\beta x k^\alpha}{w(1 + \tau \mathcal{I}_F)} \right)^{\frac{1}{1-\beta}}. \tag{4}$$

[12]We chose a proportional punishment for convenience. This follows the literature. See Krasa and Villamil (2000) and Krasa, Sharma and Villamil (2004) for extended analysis of enforcement and debt contracts.

[13] Several studies have shown (see, in particular, Cohn and Coleman, 2000) that profitability of the firm is an important predictor of external debt, suggesting that lenders may use individual and business characteristics to evaluate projects.

[14]Strictly speaking, in a simple debt contract the creditor gets everything. Of course debtors can hide assets. Firms in the informal sector are better able to hide information and therefore ϕ is lower for them. In the limit $\phi = 0$ in the informal sector.

Substituting (4) into (3) yields the entrepreneur's profit function for a given level of capital,

$$\pi_j(k,x;w) = [(1-\beta)xk^{\alpha}]^{\frac{1}{1-\beta}} \left(\frac{\beta}{w(1+\tau\mathcal{I}_F)}\right)^{\frac{\beta}{1-\beta}}. \tag{5}$$

In an environment where enforcement is perfect and the entire production is given back to the financier in case of default, $\phi = 1$, managers will solve the following problem

$$\max_{k_j \geq 0} \pi_j(k_j, x; w) - r(k_j + \varsigma\mathcal{I}_F). \tag{6}$$

This gives the optimal physical capital level:

$$k_j^*(x;w) = \left[\left(\frac{\beta}{w(1+\tau\mathcal{I}_F)}\right)^{\beta} \left(\frac{\alpha}{r}\right)^{1-\beta} x\right]^{\frac{1}{1-\alpha-\beta}}. \tag{7}$$

Since agents cannot commit to their promises, debt contracts must be self-enforcing. Let a be the amount of capital that is self-financed (or used as collateral) and l be the amount of funds raised in the outside capital market. The income from running a project is

$$V_j(b,x;w) = \max_{0\leq a_j \leq b,\ l_j \geq 0} \pi_j(a_j + l_j, x; w) - r(a_j + l_j + \varsigma\mathcal{I}_F) \tag{8}$$

subject to

$$\pi_j(a_j + l_j, x; w) - r(a_j + l_j + \varsigma\mathcal{I}_F) \geq (1 - \phi\mathcal{I}_F)\pi_j(a_j + l_j, x; w) - ra_j.$$

This problem yields optimal policy functions $a_j(b,x;w)$ and $l_j(b,x;w)$, and we define the optimal policy function for capital as $k_j(b,x;w) = a_j(b,x;w) + l_j(b,x;w)$. The restriction is an incentive compatibility constraint, which guarantees that individual promises will be fulfilled (Kehoe and Levine, 1993). We can rewrite this constraint as

$$l_j(b,x) \leq \left(\frac{\phi}{r}\pi_j(a_j(b,x;w) + l_j(b,x;w), x; w) - \varsigma\right)\mathcal{I}_F.$$

It can be shown that constrained entrepreneurs put their entire wealth in the project as long as $b \leq k_j^*(x;w)$.[15] This implies that the size of a project of an entrepreneur (b,x) is such that

$$k_j(b,x;w) \leq b + \left(\frac{\phi}{r}\pi_F(b + l(b,x;w), x; w) - \varsigma\right)\mathcal{I}_F. \tag{9}$$

Therefore, projects are limited by the agents' inheritance and the incompleteness of the capital market.

The following lemma summarizes the value of undertaking each project:

Lemma 1 *For any $x \in [\underline{x}, \overline{x}]$, and $w > 0$, the value function $V_j(b,x;w)$, and the associated policy function $l_j(b,x;w)$ for $j \in \{F, I\}$ have the following properties:*

1. *$V_j(b,x;w)$ is continuous and differentiable in x and w. If $x > 0$, it is also strictly increasing in x and strictly decreasing in w.*

2. *For $b < k_j^*(x;w)$, $V_j(b,x;w)$ is continuous, differentiable and strictly increasing in b. For $b > k_j^*(x;w)$, $V_j(b,x;w)$ is constant in b. Moreover, $V_j(b,x;w)$ is continuous at $b = k_j^*(x;w)$.*

[15] See appendix A.

3. *For all b and x,* $l_I(b,x;w) = 0$*.* $l_F(b,x;w)$ *is strictly increasing for* $b < k_F^*(x;w)$ *and* $l_F(b,x;w) = 0$ *for* $b > k_F^*(x;w)$*.*

Proof. See appendix B. ■

It is important to highlight the trade-offs governing the decision to operate in each sector. In the informal sector entrepreneurs do not pay the payroll tax and the start up cost, but projects are limited by the agents' initial wealth. In the formal sector, managers have access to the financial market, but have to pay taxes and costs associated with regulation and corruption.

3.2 Occupational choice

The occupational choice of each agent defines his lifetime income. For any $w > 0$, an agent (b,x) will become an entrepreneur if $(b,x) \in E(w)$, where

$$E(w) = \{(b,x) \in [0,\infty) \times [\underline{x}, \overline{x}] : \max\{V_F(b,x;w), V_I(b,x;w)\} \geq w\}. \quad (10)$$

Let $E^c(w)$ denote the complement set of $E(w)$ in $[0,\infty) \times [\underline{x}, \overline{x}]$. Obviously, if $(b,x) \in E^c(w)$, then agents are workers. Among those who are able to operate a business, they will become a formal entrepreneur if $(b,x) \in E_F(w) \subseteq E(w)$, where

$$E_F(w) = \{(b,x) \in E(w) : V_F(b,x;w) \geq V_I(b,x;w)\}. \quad (11)$$

They operate in the informal sector if $(b,x) \in E_I(w) \subseteq E(w)$, where

$$E_I(w) = \{(b,x) \in E(w) : V_I(b,x;w) > V_F(b,x;w)\}. \quad (12)$$

The inequality is strict by convention. The following lemma characterizes the occupational choice for a given bequest and entrepreneurial ability.

Lemma 2 *Define* $b_e(x;w)$ *as the curve in the* (b,x) *plane such that* $(b,x) \in [0,\infty) \times [\underline{x}, \overline{x}]$ *and* $\max\{V_F(b,x;w), V_I(b,x;w)\} = w$*. Then there exists* $x^*(w)$ *such that* $\frac{\partial b_e(x;w)}{\partial x} < 0$ *for* $x > x^*(w)$ *and* $\frac{\partial b_e(x;w)}{\partial x} = -\infty$ *for* $x = x^*(w)$*.*

1. *For all x, if* $b < b_e(x;w)$*, then* $(b,x) \in E^c(w)$*.*

2. *For all x, if* $b \geq b_e(x;w)$*, then* $(b,x) \in E(w)$*.*

In addition, define $b_s(x;w)$ *as the curve in the* (b,x) *plane such that* $(b,x) \in [0,\infty) \times [\underline{x}, \overline{x}]$ *and* $V_F(b,x;w) = V_I(b,x;w)$*.*

3. *For all x, if* $b \geq b_e(x;w)$ *and* $b > b_s(x;w)$*, then* $(b,x) \in E_I(w)$*.*

4. *For all x, if* $b \geq b_e(x;w)$ *and* $b \leq b_s(x;w)$*, then* $(b,x) \in E_F(w)$*.*

Proof. See appendix C. ■

Figure 1 illustrates this lemma. It shows the occupational choice in the (b,x) space for the baseline economy (see parameters in section 4). Lemma 2 and figure 1 suggest that agents are workers when the quality of their project is low, i.e., $x < x^*(w)$ (the lightest shaded area). For $x \geq x^*(w)$, then agents might become entrepreneurs depending if they are credit constrained or not (notice that for very low bequests agents are workers even though their entrepreneurial ability is higher than $x^*(w)$). The negative association between $b_e(x;w)$ and x suggests that managers with better projects need a lower level of initial wealth to run a project. This is rather intuitive since profits are increasing in the quality of the project. Given the low operational costs

Figure 1: Firm size distribution in the formal and informal sectors.

in the informal sector, unconstrained entrepreneurs will stay illegal. Constrained entrepreneurs will operate in the informal sector only if the premium from formalization (access to outside finance) is not high enough. Since this premium increases with the quality of the project, only high-productivity projects will operate in the formal sector (darkest shaded area). The area in between the darkest and lightest shaded areas corresponds to managers in the informal sector.

The size of the informal sector depends on the institutional and policy parameters τ, ς and ϕ, as well as on distribution Γ.

3.3 Consumers

In period t, the lifetime wealth of an agent characterized by (b_t, x_t) is given by

$$Y_t = Y(b_t, x_t; w_t) = \max\{w_t, V_F(b_t, x_t; w_t), V_I(b_t, x_t; w_t)\} + rb_t. \tag{13}$$

Lifetime wealth is thus a function of agent-specific b_t and x_t, and economy-wide w_t. Given lifetime wealth, (13), agents choose consumption and bequests to maximize preferences (1). This problem defines the optimal consumption, $c_t = c(Y_t)$, and bequest, $b_{t+1} = b(Y_t)$, policies. The functional form of (1) implies that agents leave a proportion $1 - \gamma$ of their lifetime wealth as a bequest. Notice that bequests cannot be negative because every agent is allowed to become a worker. Define $z_t = (b_t, x_t)$ and let W_t be the bequest distribution at period t.[16]

3.4 Competitive equilibrium

Definition 3 *Given* (τ, ϕ, ς), Γ *and* W_t, *equilibrium at date* t *is a list* w_t, $\{n_j(x; w_t)\}_{j\in\{F,I\}}$, $\{l_j(b, x; w_t)\}_{j\in\{F,I\}}$, $\{a_j(b, x; w_t)\}_{j\in\{F,I\}}$, $\{V_j(b, x; w_t)\}_{j\in\{F,I\}}$, $c_t = c(\cdot)$, $b_{t+1} = b(\cdot)$, *such that:*

A. *Given the wage rate and government policies, an agent of type* (b, x) *chooses his occupation to maximize his lifetime wealth, (13).*

B. $l_j(b, x; w_t)$ *and* $a_j(b, x; w_t)$ *solve (8) for* $j \in \{F, I\}$.

C. *Given the lifetime wealth, (13), each agent maximizes utility, (1).*

D. *Given the wage rate, technology constraint, credit markets, and government policies, formal and informal entrepreneurs select their labor force to maximize profits, (3).*

E. *The Labor Market clears:*

$$\iint_{z\in E_F(w_t)} n_F(x; w_t) W_t(db_t)\Gamma(dx_t) + \iint_{z\in E_I(w_t)} n_I(x; w_t) W_t(db_t)\Gamma(dx_t) = \iint_{z\in E^c(w_t)} W_t(db_t)\Gamma(dx_t). \tag{14}$$

In the quantitative exercises it is important to evaluate policy experiments in "stable" economies, where, for instance, the real wage and income distribution are not changing significantly over time. Indeed, it is possible to show that when policies and institutions are stationary a unique steady-state equilibrium exists (i.e., an equilibrium with a constant real wage, w, and invariant distribution, $H = W\Gamma$) and from any initial condition the economy converges to this equilibrium.

Proposition 4 *There exists a unique stationary equilibrium with* $0 < w < \infty$ *and invariant distribution* W. *In addition, for any initial bequest distribution* W_0 *and stationary government policies and institutions, the bequest distribution converges to* W.

Proof. See appendix D. ■

In the calibration and quantitative experiments we will study the economy in this particular equilibrium and therefore we will consider the long-run impact of changes in policies and institutions.

4 Quantitative results

4.1 Parameterization

In order to solve the model numerically we have to choose a functional form for the ability distribution and assign values to the parameters of the model. We parameterized the model such that, in the stationary equilibrium, we could match some key empirical observations of the United States economy.

[16]See the definition of W_t in appendix D.

Table 1: Selected Statistics. Sources: Informal sector size is from Schneider and Enste (2000, tables 2, 3 and 7), and Friedman et al. (2000, table 1, first column), and is total production in the informal sector as a share of GDP. Official GDP per capita is the Gross Domestic Product per capita in U.S. dollars in 1999 (World Bank, 2001). Unofficial GDP per capita is calculated using the first and the second columns above. For countries with a range of informal sector sizes, the upper limit was used. Regulation costs are from Djankov et al. (2002, table 3, column 8). They are direct costs as a fraction of GDP per capita that entrepreneurs face to meet government regulations. Efficiency of the judicial system, protection against expropriation, rule of law, and risk of contract repudiation are from La Porta et al. (1998, table 5, columns 1, 2, 4, and 5). The last column is the average from columns 5 to 8.

Country	Informal sector size, % (1)	Official GDP per capita (2)	Unofficial GDP per capita (3)	Regulation costs (4)	Efficiency of judicial sytem (5)	Protection against expropriat. (6)	Rule of law (7)	Risk of contract repudiat. (8)	average enforc. (9)
Denmark	9.4	32,030	35,041	0.1000	10.00	9.67	10.00	9.31	9.74
Canada	14.8	19,320	22,179	0.0145	9.25	9.67	10.00	8.96	9.47
Germany	13.2	25,350	28,696	0.1569	9.00	9.90	9.23	9.77	9.47
France	13.8	23,480	26,720	0.1430	8.00	9.65	9.05	9.19	8.97
United States	10	30,600	33,660	0.0049	10.00	9.98	10.00	9.00	9.74
Belgium	15.3	24,510	28,260	0.0998	9.50	9.63	10.00	9.48	9.65
Portugal	22.1	10,600	12,942	0.1844	5.50	8.90	8.68	8.57	7.91
Spain	22.4	14,000	17,136	0.1730	6.25	9.52	7.80	8.40	7.99
Italy	26	19,710	24,834	0.2002	6.75	9.35	8.33	9.17	8.40
Argentina	21.8	7,600	9,257	0.1019	6.00	5.91	5.35	4.91	5.54
Brazil	35	4,420	5,967	0.2014	5.75	7.62	6.32	6.30	6.50
Peru	60	2,390	3,824	0.1986	6.75	5.54	2.50	4.68	4.87
Nigeria	76	310	546	2.5700	7.25	5.33	2.73	4.36	4.92

Table 2: Parameter values, baseline economy.

γ	0.8	τ	0.33
β	0.55	ς	0.0004
α	0.35	ϕ	0.25
r	2	ϵ	6

Table 3: Basic statistics, US and baseline economy. Sources: Schneider and Enste (2000), World Bank (2001) and Cagetti and De Nardi (2002); all figures in percentage.

	US economy	Baseline economy
Informal sector size	10	10
Regulation and corruption costs	0.5	0.5
Income Gini	40–44	34
% of entrepreneurs	9	9

Table 2 summarizes the parameter values, which were determined as follows.[17] We interpret the model period to be 35 years and we let $r = 2$, which implies an yearly real interest rate of roughly 2 percent. We set α and β such that about 55 percent of income is paid to labor, 35 percent is paid to the remuneration of capital, and 10 percent are profits.[18] We chose a payroll tax of $\tau = 0.33$, which is consistent with the literature (Jones, Manuelli and Rossi, 1993). The share of bequests in the instantaneous utility function, $1 - \gamma$, was taken to be 0.2, which is consistent with those estimated by Laitner and Juster (1996). Finally, we assumed that the entrepreneurial cumulative distribution function is $\Gamma(x) = Ax^{\frac{1}{\epsilon}}$ and we normalized the support of this distribution to the $[0, 1]$ interval, so that $A = 1$.[19] We chose parameters ϵ, ϕ and ς such that the size of the informal sector was 10 percent, the percentage of entrepreneurs in the steady-state equilibrium was 9 percent, and regulation costs were around 0.5 percent[20] of GDP per capita.

The baseline economy reproduces statistics consistent with those of the US economy, except for the income Gini coefficient (table 3). Since every worker receives the same wage in the model economy, the model income Gini coefficient should underestimate its real world counterpart. The income Gini coefficient when we consider only entrepreneurs' income is higher in the data. For instance, data in Quadrini (1999) imply a value around 45 percent for the US economy. In our model the Gini index for entrepreneurs' income is roughly 49 percent, which is close to the data. We therefore shall focus less on inequality and more on the other key statistics.

Figure 2 shows the distribution of capital allocated in the two sectors. It illustrates well the premium from formalization. The horizontal area (when ability is low) corresponds to those

[17]Appendix E contains some quantitative results using different parameterizations.

[18]Gollin (2002) argues that it is important to adjust factor income shares by the entrepreneurial income, which is often treated incorrectly as capital income share. If we input entrepreneurial profits as labor income as suggested by Gollin, then the effective labor and capital income shares will be 0.65 and 0.35, respectively. These income shares are those that map our model to those observed in national accounts. If we use another adjustment also suggested by Gollin, which assume that entrepreneurial income is a mix of labor and capital income as the rest of the economy, then the effective labor and capital income shares will roughly be 0.61 and 0.39, respectively. In any case, the effective labor income share will be in the range estimated by Gollin, which goes from 0.60 to 0.80.

[19]Chaterjee, Corbae, Nakajima and Ríos-Rull (2002) use a similar parametric form to generate earnings distribution in the United States.

[20]See table 1, column (4).

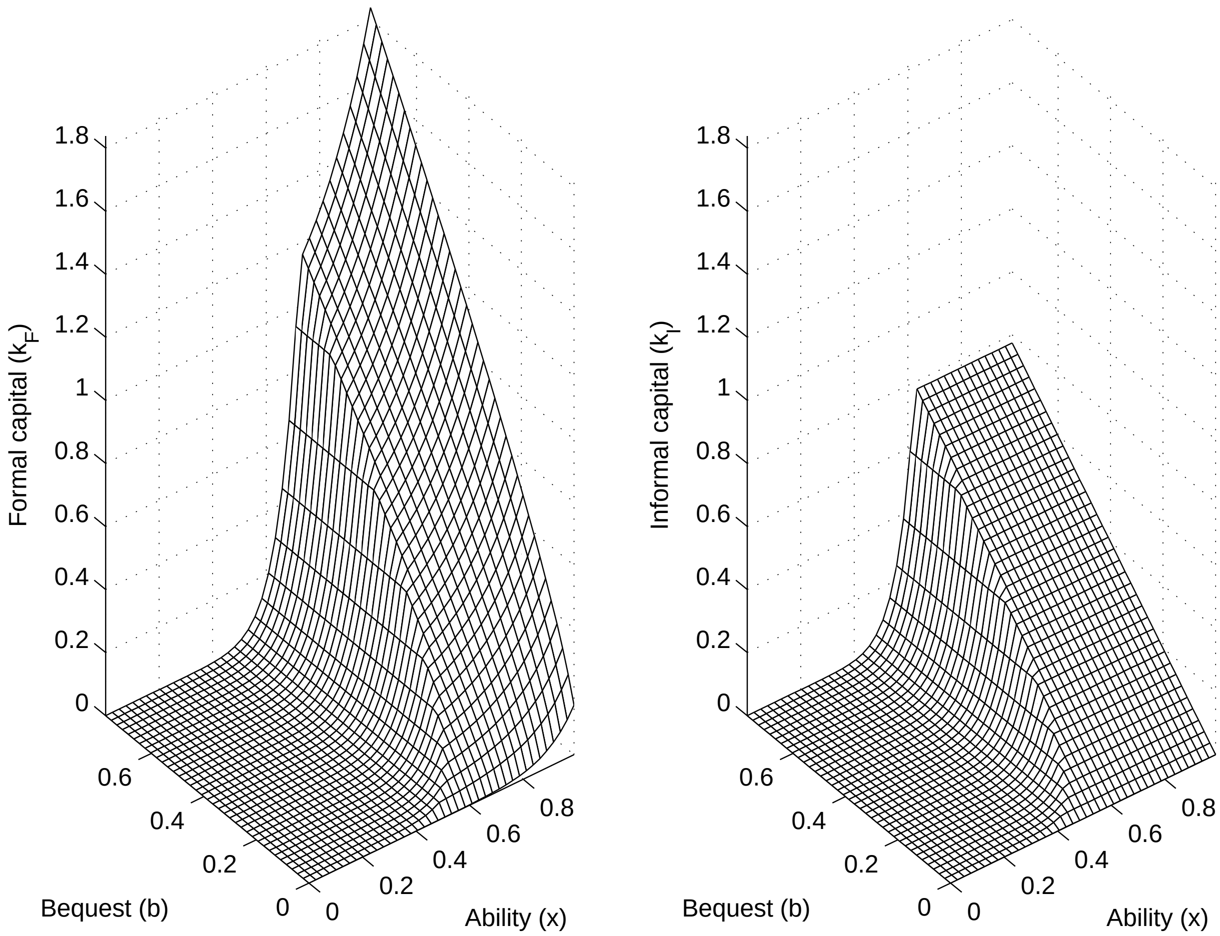

Figure 2: Firm size in the formal and informal sectors.

agents that are workers. As the entrepreneurial ability and bequests increase, the size of the project increases monotonically. The capital in the informal sector, however, is constrained by initial wealth, which implies that agents with different abilities end up having the same level of capital. On the other hand, in the formal sector the total debt increases with the quality of the project. This implies that, for a given level of bequest, the amount of capital employed in this sector is higher for those with better projects. As a consequence, formal entrepreneurs operate more productive technologies.

Next, we will run some quantitative experiments. We will change corruption costs, ς, and the level of enforcement, ϕ, separately to provide their long-run impact on the share of production generated outside the realm of government regulation, productivity, and percentage of entrepreneurs. We will also run experiments by changing these two parameters simultaneously. The values of these parameters will be chosen according to the empirical observations on regulation costs (column (4) of 1) and enforcement (column (9) of 1).

4.2 Contract enforcement versus regulation costs

Contract enforcement

In order to assess how changes in the degree of contract enforcement impact the model, we map linearly the last column of table 1 to ϕ, assuming that the US case corresponds to $\phi = 0.25$

Table 4: Policy Experiments.

	Informal sector size, % output	Total output per capita, % of baseline	Official output per capita, % of baseline	% of entrepreneurs	Income Gini
Baseline	10	100	100	9	34
$\phi_{\text{Med}} = 0.209;\ \frac{\varsigma}{y} = 0.005$	13.4	94	91	9.4	34
$\phi_{\text{Peru}} = 0.13;\ \frac{\varsigma}{y} = 0.005$	28	81	68	10.6	33
$\phi_{\text{US}} = 0.25;\ \frac{\varsigma}{y} = 0.18$	25	94	82	9.1	32
$\phi_{\text{US}} = 0.25;\ \frac{\varsigma}{y} = 0.20$	27	93	79	9.16	32
$\phi_{\text{Med}} = 0.209;\ \frac{\varsigma}{y} = 0.18$	31	87	72	10	32
$\phi_{\text{Peru}} = 0.13;\ \frac{\varsigma}{y} = 0.20$	63	74	41	11	31

and a zero level of enforcement corresponds to $\phi = 0$. Mediterranean Europe (Italy, Portugal, Spain) has a value of ϕ of 0.209, while for Peru (which epitomizes a developing country with a large informal sector) this figure is 0.13.

Table 4 shows that when enforcement decreases from the United States level to the Mediterranean Europe level, the size of the informal sector increases from 10% to 13.4%, total output falls by 6 percent, and measured output decreases by 9 percent. A further decrease of ϕ to 0.13 (Peru level) yields an increase in the size of the informal sector of 18 percentage points relative to the baseline economy, while total output falls in 20%.[21]

From table 4 we can also observe that, as ϕ decreases, the fraction of entrepreneurs in the total population increases. This is a stylized fact in developing economics: the percentage of entrepreneurs over the total working population decreases with output per capita. Less developed countries tend to have more entrepreneurs but less productive entrepreneurs (Lucas, 1978*b*, Tybout, 2000). When, for instance, enforcement improves, the number of entrepreneurs decreases but the quality (size and productivity) of each project increases (see also Kumar, Rajan and Zingales, 2004).

Corruption and regulation costs

We now verify the impacts of corruption costs, ς, on the size of the informal sector and on productivity. We increased ς from the baseline economy (United States) value ($\varsigma = 0.0004$), which corresponds to 0.5% of the output per capita, to 18% of this output ($\varsigma = 0.011$). This new value is the average regulation cost as a fraction of output per capita found in Mediterranean Europe.[22] Notice that the informal sector size increases from 10% to 25%, while observed output per capita decreases by 18%. We further increase corruption costs to the level observed

[21] Notice that in these experiments we kept $\frac{\varsigma}{y} = 0.005$. Since output decreases, ς should also decrease to keep a constant ratio.

[22] See column (4) of table 1.

in Peru, which is roughly 20% of the output per capita ($\varsigma = 0.013$). We observe that the informal sector size increases to 27% and the output per capita decreases to 79% of the United States level. Table 4 also contains the results when we change ϕ and ς simultaneously. As we can see, these parameters can have a sizeable impact on the size of the informal sector and on productivity.

Informal sector size: Model simulation and the data

In order to better evaluate the numerical experiments we construct table 5, which contains the main results from our model simulations (values from table 4) as well as the key statistics observed in the data from table 1. Let us first look at Mediterranean Europe. Table 5 shows that the strength of financial contract enforcement alone cannot account for the differences in the size of the informal sector between Mediterranean Europe and the United States. The model simulation when we only change the enforcement level yields an informal sector size of 13.4%, while in the data it is 24% of output. Corruption and regulation costs, on the other hand, account for all the difference in the size of the informal sector between Mediterranean Europe and the United States. The model simulation when we only change the value of $\frac{\varsigma}{y}$ to the Mediterranean Europe value yields an informal sector size of 25%. Therefore, it is corruption costs rather than limited enforcement in financial contracts that explains the difference in the informal sector size between Mediterranean Europe and the United States.[23]

For Peru, the picture is somewhat different. Model simulations when we change the value of enforcement and corruption costs separately from the US to the Peru level yield an informal sector size of 28% and 27%, respectively. These values are roughly a half of the one observed in the data (60%). The model simulation when we change the two parameters simultaneously generate an informal sector size of roughly the same magnitude as the one observed in the data. Therefore, the numerical exercises suggest that contract enforcement accounts for roughly 50% of the difference in the size of the informal sector between Peru and the United States, and corruption accounts for the other half.

TFP: Model simulation and the data

We now focus on the question of whether differences in contract enforcement and regulation costs between countries account for the differences in output per capita and productivity across countries.

Parente and Prescott (2000) show that the gap in output per capita among rich and poor countries is explained by differences in TFP and not by factor accumulation. Their theory, which is based on technology adoption, shows that inside groups with vested interests block the adoption of more advanced technologies and explains the use of inferior production processes.[24] In our model, differences in productivity across countries arise endogenously due to differences in regulation costs (barriers to legality) and the enforcement system (legal failures). Looking at the last two columns of table 5 we can assess the relevance of those two factors in explaining output differences across countries. Output per capita in Mediterranean Europe is about 55% of the US output per capita. The model simulation when change both corruption and enforcement from the baseline to the US level generates an economy that is about 72% as rich as the US. This leaves another 17% to be explained by other factors that affect TFP, such as vested interests. The simulation for Peru generates an economy that is about 41% as rich as the US,

[23]Similar results are found for Western Europe (average values for Belgium, Denmark, France, and Germany).

[24]This clearly could be extended to explain the presence of institutions that inhibit economic development. Improvements in the judicial system might be blocked by inside groups which benefit from a poor judicial system. Therefore, granted monopoly rights that generate economic rents to specific groups can be mapped into values of ϕ and ς.

Table 5: Empirical data and simulation results for reference economies. Mediterranean Europe comprises Italy, Portugal and Spain.

	ϕ	$\frac{\varsigma}{y}$	Informal sector size, % of official output	Output per capita, % of US level	
				Total	Official
Baseline case	**0.25**	**0.005**	**10**	**100**	**100**
Mediterranean Europe (data)	**0.209**	**0.18**	**24**	**62**	**55**
Simulation					
1) Enforcement	0.209	0.005	13.4	94	91
2) Corruption	0.25	0.18	25	94	82
3) Enforcement and corruption	0.209	0.18	31	87	72
Peru (data)	**0.13**	**0.20**	**60**	**11**	**8**
Simulation					
1) Enforcement	0.13	0.005	28	81	68
2) Corruption	0.25	0.2	27	93	79
3) Enforcement and Corruption	0.13	0.20	63	74	41

while Peru's output per capita is roughly 8% of the US level. Finally, for Peru 60 of the gap in output per capita is explained by corruption costs and contractual imperfections, while the remaining 32 pp are explained by other factors affecting TFP. In sum, the model corruption and credit market imperfections explain roughly 60% of the output gap between the US and other reference economies. These results are still valid if we consider other sets of relatively homogeneous countries.

4.3 Capital income share

In this section we study the robustness of our results to a higher capital income share. As Parente and Prescott (2000) argue, measuring the capital stock is usually problematic and the National Income Statistics usually underestimate the role of capital in the economy. Intangible capital, such as organization and human capital, are not part of capital income in the National Income Statistics. We therefore increased parameter α from 0.35 to 0.45 and parameterized a new economy with a higher capital share. If we use Gollin's (2002) adjustment and assume that for entrepreneurial income the mix of labor and capital is equal to that of the rest of the economy, then the effective capital income share will roughly be 0.5.[25]

Table 6 contains the model simulations with a higher capital share. As before the model does a good job in accounting for the size of the informal sector across countries. Corruption is the main factor explaining the difference in the size of the informal sector between Mediterranean Europe and the United States. With respect to Peru, the results suggest a stronger role of financial market imperfections compared to the findings in table 5. Corruption, however, is still important quantitatively to explain the difference in the informal sector size between Peru and the United States

Regarding differences in output per capita the model explains a higher fraction of the differences in international income.[26] For Mediterranean Europe, for instance, the model with a higher capital share suggests that corruption and credit market imperfections accounts for all the difference in observed output per capita between this region and the United States. This is not the same for Peru. In this case there is still an important part of the observed difference in income that is not accounted by corruption and enforcement. We also simulated the model with a higher capital income share $\alpha = 0.55$, which implies an effective capital share of roughly $\alpha = 0.6$ using Gollin's (2002) adjustment. The same pattern arises: (i) differences in the informal sector can be accounted by corruption and credit market imperfections; (ii) the model is able to explain a higher difference in observed international incomes.[27]

5 Concluding remarks and policy implications

This paper contributes to the literature by characterizing how government policies and institutions interact with the distribution of wealth and entrepreneurial ability in a general equilibrium model with formal and informal sectors, corruption, and contractual imperfections. Formal entrepreneurs have better access to outside finance. The quantitative exercises suggest that:

[25] In this case, $\varsigma = 0.00017$, $\alpha = 0.45$, $\beta = 0.45$, $\phi = 0.25$, and $\epsilon = 5.33$. γ, τ, and r have the same values as before.

[26] Notice that we have to subtract output by investment in intangible capital to make the model compatible with the observed GDP, since national accounts do not measure this investment (Parente and Prescott, 2000). We do this by using the total amount of capital in the economy, and assuming that the interest rate is the same for the physical and intangible capital. Both forms of capital fully depreciate between two periods. The last assumption implies that $\frac{K_P}{K_I} = \frac{\alpha_P}{\alpha_I}$, where α_i is the factor income share of capital $i = P, I$. In table 6 we assumed that $\alpha_P = 0.20$ and $\alpha = 0.25$.

[27] In this case the economy with corruption and enforcement levels as those observed in Peru will be roughly 25% as rich as the United States.

Table 6: Higher capital share ($\alpha = 0.45$). Empirical data and simulation results for reference economies. Mediterranean Europe comprises Italy, Portugal and Spain.

	ϕ	$\frac{\varsigma}{y}$	Informal sector size, % of official output	Output per capita, % of US level	
				Total	Official
Baseline case	**0.25**	**0.005**	**10**	**100**	**100**
Mediterranean Europe (data)	**0.209**	**0.18**	**24**	**62**	**55**
Simulation					
1) Enforcement	0.209	0.005	12.9	84	83
2) Corruption	0.25	0.18	20.4	87	80
3) Enforcement and corruption	0.209	0.18	25.5	75	62
Peru (data)	**0.13**	**0.20**	**60**	**11**	**8**
Simulation					
1) Enforcement	0.13	0.005	27	65	55
2) Corruption	0.25	0.2	23	79	71
3) Enforcement and Corruption	0.13	0.20	54	54	32

i. Regulation costs rather than limited enforcement in financial contracts account for a larger part of the difference in the size of the shadow economy between Mediterranean Europe and the United States.

ii. In countries with very weak enforcement, such as Peru, credit market imperfection explains roughly 50% of the difference in the informal sector size, while corruption costs explain the other half.

iii. Productivity gains from improving the enforcement system and decreasing start up costs are sizeable. The model suggests that contractual imperfections and corruption costs explain roughly 60% of the difference in international incomes. When we increase the capital share to include intangible capital, then corruption and credit market imperfections account for about 75% of the differences in per capita income across countries.

In order to investigate the policy implications of the model it is important to understand the real counterpart of parameters ς and ϕ. Parameter ς measures the *barriers to legality*. It corresponds to costs and procedures to comply with government requirements to start a business. These costs deter entrepreneurs from becoming formal and are most of them sunk. Djankov et al. (2002), for instance, show that in Italy entrepreneurs on average need to follow 16 procedures, pay US$ 3946 in fees, and wait roughly 62 business days to acquire legal status. In Canada, however, the process needs only 2 procedures, takes two days and costs around US$280 in fees. Parameter ϕ, on the other hand, is a proxy for *legal failures* in financial markets. When financial markets operate poorly, capital is misallocated and lucrative investment opportunities are forgone. According to De Soto (2000), capital markets fail in developing countries for two main reasons. First, the majority of residents in developing countries do not have legal tender of their property and therefore cannot use their assets as collateral for a loan to convert it into capital – notice that this is related to parameter ς. Second, even when their assets are formally registered their properties and financial contracts are not well enforced (idem, parameter ϕ).

Policies should therefore simplify the entry process, decrease bureaucracy and provide legal tender to informal assets. Moreover, a set of complementary institutional reforms to secure properties and debt contracts are needed, such as reforms of bankruptcy laws, banking regulation, and judiciary and enforcement systems. Our quantitative experiments suggest that policies that decrease regulation and improve the functioning of credit markets might have important effects on the size of the informal sector and productivity, especially in developing countries. These policies, however, are not easy to implement since they will change the *status quo* of politicians and bureaucrats who are the main beneficiaries from high regulation costs (Djankov et al., 2002) and a weak legal system. Path dependence in economic institutions is an important issue, but it is not the objective of this paper.

Our model could also be used to investigate related questions. One of them is the long-run effect of tax reforms on tax evasion (size of the informal sector) and productivity. Intuitively, for instance, a tax on financial intermediation might not only decrease capital per capita, but also have an effect on the occupational choice of the agents, preventing agents from becoming formal entrepreneurs. In addition, it will be interesting to consider an enforcement mechanism with not only the degree of creditor protection (ϕ) – variable part, but also with a fixed cost to use the enforcement system (court), which is independent of the debtor's output (Krasa and Villamil, 2000, Krasa et al., 2004). In fact, the nature of the judicial enforcement system in most economies embodies characteristics of both assumptions. Although both issues are interesting topics we see them as beyond the objective of this paper. We leave them and other questions for future research.

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A Kuhn-Tucker conditions for problem (8)

The Lagrangean associated with problem (8) is

$$\begin{aligned}L_j &= \pi_j(a_j + l_j, x; w) - r(a_j + l_j + \varsigma\mathcal{I}_{j=F}) \\ &\quad + \lambda_j[\mathcal{I}_{j=F}\phi\pi_j(a_j + l_j, x; w) - r(l_j + \varsigma\mathcal{I}_{j=F})] + \chi_j[b - a_j].\end{aligned}$$

The Kuhn-Tucker conditions are:

$$\frac{\partial L_j}{\partial l_j} = \pi_{j1}(a_j + l_j, x; w) - r + \lambda_j[\mathcal{I}_{j=F}\phi\pi_{j1}(a_j + l_j, x; w) - r] \leq 0, \tag{15}$$

$$\frac{\partial L_j}{\partial a_j} = \pi_{j1}(a_j + l_j, x; w) - r + \lambda_j[\mathcal{I}_{j=F}\phi\pi_{j1}(a_j + l_j, x; w)] - \chi_j \leq 0, \tag{16}$$

$$\lambda_j[\mathcal{I}_{j=F}\phi\pi_j(a_j + l_j, x; w) - r(l_j + \mathcal{I}_{j=F}\varsigma)] = 0, \tag{17}$$

$$\chi_j[b - a_j] = 0, \tag{18}$$

$$l_j \geq 0, \quad \frac{\partial L_j}{\partial l_j} l_j = 0, \quad a_j \geq 0, \quad \frac{\partial L_j}{\partial a_j} a_j = 0, \quad \lambda_j \geq 0, \quad \chi_j \geq 0,$$

along with the incentive compatible constraint and the upper limit on a_j. If the entrepreneur is credit constrained, $\lambda_j > 0$, that is, he would be better off if the credit constraint were eased. Notice first that, from (17), $l_I = 0$. We know that $\pi_{F1}(k_F^*(x; w), x; w) = r$ and $\pi_{F1}(a_F + l_F, x; w)$ is decreasing with l_F. Notice that $a_F + l_F \leq k_F^*(x; w)$, since $k_F^*(x; w)$ is the unconstrained optimal level of capital. Then, equation (16) implies $\chi_F > 0$, which implies by (18) that $a_F = b$.

B Proof of Lemma 1

Continuity of $V_j(b, x; w)$ follows from the *Maximum Theorem* and differentiability from Theorem 4.11 of Stokey and Lucas (1989). From the envelope theorem it is easily seen that, provided $x > 0$,

$$V_{j2}(b, x; w) = \pi_{j2}(b + l_j(b, x; w), x; w)(1 + \lambda_j\phi\mathcal{I}_{j=F}) > 0,$$

$$V_{j3}(b, x; w) = \pi_{j3}(b + l_j(b, x; w), x; w)(1 + \lambda_j\phi\mathcal{I}_{j=F}) < 0,$$

If $b \leq k^*(x; w)$, then

$$V_{j1}(b, x; w) = \pi_{j1}(b + l_j(b, x; w), x; w)(1 + \lambda_j\phi\mathcal{I}_{j=F}) > 0.$$

When $b > k_j^*(x; w)$, then by definition of $k_j^*(x; w)$ the net income from entrepreneurship cannot increase and $V_{j1}(b, x; w) = 0$. $l_I(b, x; w) = 0$ since there is no borrowing in the informal sector. For $b > k_F^*(x; w)$ it is also obvious that $l_F(b, x; w) + a_F(b, x; w) = k_F^*(x; w)$. When agents are credit constrained, the incentive compatible constraint holds with equality and

$$\phi\pi_F(b + l_F(b, x; w), x; w) = r(l_F(b, x; w) + \varsigma).$$

Thus,

$$\frac{\partial l_F(b,x;w)}{\partial b} = \frac{\phi\pi_{F1}(k_F,x;w)}{r-\phi\pi_{F1}(k_F,x;w)}.$$

By condition (15), we have that $r-\phi\pi_{F1}(k_F,x;w) = \frac{\pi_{F1}(k_F,x;w)-r}{\lambda_F}$. Since this is for constrained agents, $\lambda_F > 0$ and, as we have seen previously, $\pi_{F1}(k_F,x;w)$ is greater than r. Therefore,

$$\frac{\partial l_F(b,x;w)}{\partial b} = \lambda_F \frac{\phi\pi_{F1}(k_F,x;w)}{\pi_{F1}(k_F,x;w)-r} > 0.$$

C Proof of Lemma 2

If agents have sufficiently high b and

$$\max_{j\in\{I,F\}}\{V_j(b,x;w)\} \geq w,$$

there is $x^*(w)$ such that for $x < x^*(w)$ agents prefer to be workers rather than managers:

$$x^*(w) = \min_{j\in\{I,F\}}\left\{\left(\frac{r}{\alpha}\right)^{\alpha}\left(\frac{w(1+\tau\mathcal{I}_{j=F})}{\beta}\right)^{\beta}\left(\frac{w+r\varsigma\mathcal{I}_{j=F}}{1-\alpha-\beta}\right)^{1-\alpha-\beta}\right\}.$$

$x^*(w)$ is independent of b. For constrained agents with $x \geq x^*(w)$, we have that

$$\max_{j\in\{I,F\}}\{V_j(b,x;w)\} = w$$

defines $b_e(x;w)$ such that

$$\frac{\partial b_e(x;w)}{\partial x} = -\frac{V_{j2}(b,x;w)}{V_{j1}(b,x;w)},$$

where $j = \arg\max_{j\in\{I,F\}}\{V_j(b,x;w)\}$, in all points where $b_e(x;w)$ is differentiable. This is negative from Lemma 1.

Define

$$G(b,x;w) = V_F(b,x;w) - V_I(b,x;w).$$

Provided $G_1(b,x;w) \neq 0$, by the implicit function theorem $G(b,x;w) = 0$ defines $b_s(x;w)$, where

$$\frac{\partial b_s(x;w)}{\partial x} = -\frac{V_{F2}(b,x;w) - V_{I2}(b,x;w)}{V_{F1}(b,x;w) - V_{I1}(b,x;w)}.$$

We have

$$\begin{aligned}
V_{F2}(b,x;w) - V_{I2}(b,x;w) &= x^{\frac{1}{1-\beta}}\left(\frac{\beta}{w}\right)^{\frac{\beta}{1-\beta}} \times \\
&\left[\left(\frac{(b+l)^{\alpha}}{(1+\tau)^{\beta}}\right)^{\frac{1}{1-\beta}}(1+\lambda_F\phi) - (b^{\alpha})^{\frac{1}{1-\beta}}\right] \\
V_{F1}(b,x;w) - V_{I1}(b,x;w) &= \alpha x^{\frac{1}{1-\beta}}\left(\frac{\beta}{w}\right)^{\frac{\beta}{1-\beta}} \times \\
&\left[\left(\frac{(b+l)^{\alpha}}{(1+\tau)^{\beta}}\right)^{\frac{1}{1-\beta}}\frac{(1+\lambda_F\phi)}{b+l} - \frac{(b^{\alpha})^{\frac{1}{1-\beta}}}{b}\right].
\end{aligned}$$

Notice that $V_F(b, x; w) \geq V_I(b, x; w)$ implies that

$$x^{\frac{1}{1-\beta}} \left(\frac{\beta}{w}\right)^{\frac{\beta}{1-\beta}} \left[\left(\frac{(b+l)^\alpha}{(1+\tau)^\beta}\right)^{\frac{1}{1-\beta}} - (b^\alpha)^{\frac{1}{1-\beta}}\right] \geq \frac{r(l+\varsigma)}{1-\beta} > 0.$$

Since $\lambda_F \geq 0$, this implies that the numerator of $\frac{\partial b_s(x;w)}{\partial x}$ is always positive for $V_F(b, x; w) \geq V_I(b, x; w)$. For a given b, l increases and λ decreases with x, which implies that for sufficiently high x the denominator is negative and $\frac{\partial b_s(x;w)}{\partial x}$ is positive (see figure 1).

D Proof of Proposition 4

First we need to show that, for every bequest distribution, there exists a finite equilibrium wage rate that clears the labor market. Given the bequest and ability distributions, W and Γ, define the excess demand function $ED(w)$ by

$$ED(w) = \iint_{z \in E_F(w)} n_F(x; w) W(db)\Gamma(dx) + \iint_{z \in E_I(w)} n_I(x; w) W(db)\Gamma(dx) - \iint_{z \in E^c(w)} W(db)\Gamma(dx). \tag{19}$$

The excess demand $ED(w)$ is continuous since both $n_j(x; w)$ and $V_j(b, x; w)$ are continuous in w (see equation (4) and Lemma 1). In addition, $n_j(x; w)$ and $V_j(b, x; w)$ are also strictly decreasing in w. Notice that as w goes to zero, no agent wants to become a worker, $V_j(b, x; w)$ is unbounded and $ED(w) > 0$. Analogously, when w increases, then $ED(w) < 0$. Therefore, by continuity of $ED(w)$ there must be some w^* such that $ED(w^*) = 0$.

It remains to show that $w^* \in [\underline{w}, \overline{w}]$, where $\underline{w} > 0$ and $\overline{w} < \infty$. Let us consider an initial bequest distribution that assigns zero bequest to all agents. Set $E_F(w)$ is the measure of all agents for sufficiently small w as long as $\phi > 0$. In this case, the equilibrium wage rate, $\underline{w}$, is positive and finite, as stated in the previous paragraph. Since the wage rate is positive, next periods' bequests will all be positive. Therefore, the set of possible occupational choices cannot shrink, and might even expand. This implies that for the previous wage rate $\underline{w}$, the excess demand is nonnegative, $ED(w) \geq 0$, which in turn means that for this new bequest distribution the wage rate that clears the labor market is $w' \geq \underline{w}$. Consequently, $\underline{w} > 0$ is the lowest equilibrium wage rate for any initial distribution.

Now suppose an initial bequest distribution that assigns $\bar{b}_0$ to all agents such that $\bar{b}_0 \geq k_j^*(\overline{x}; \underline{w})$. By the first argument in this proof, there exists a positive and finite equilibrium wage rate, $\overline{w} < \infty$. In this case, no agent is credit constrained. Either the smallest bequest, $(1-\gamma)(\overline{w} + r\bar{b}_0)$, is higher than $\bar{b}_0$, in which case the next periods' equilibrium wage rate will be the same; or it is smaller than $\bar{b}_0$ and the set of occupation choices might shrink. Therefore, for this new wealth distribution $ED(\overline{w}) \leq 0$. In this case the new equilibrium wage rate is $w' \leq \overline{w}$.

We can thus conclude that $w_t \in [\underline{w}, \overline{w}]$ for all t. The maximum possible bequest is thus $\bar{b}$ such that

$$\bar{b} = (1-\gamma)(\max_{j=F,I}\{\pi_j(k_j^*(\overline{x}; \underline{w}), \overline{x}; \underline{w})\} + r\bar{b}), \tag{20}$$

where we assume that $(1-\gamma)r < 1$. On the other hand the minimum bequest is

$$\underline{b} = (1-\gamma)(\underline{w} + r\underline{b}) \tag{21}$$

Define $Z = [\underline{b}, \bar{b}]$ and $z_t = (b_t, x_t)$. Z is compact. Define the measurable space $(Z, \mathcal{B})$, where $\mathcal{B}$ is the Borel algebra for the set. Define $\Lambda(Z, \mathcal{B})$ as the set of all possible probability measures defined on the measurable space $(Z, \mathcal{B})$. For instance, W_t, which specifies the probability of each event in $\mathcal{B}$ at time t, belongs to $\Lambda(Z, \mathcal{B})$. Measure W_t defines a non-stationary transition probability function,

$$P_t(b_t, A) = \Pr\{b_{t+1} \in A | b_t\},$$

for any (b_t, A) in $(Z, \mathcal{B})$. Function P_t assigns a probability to event A for the descendant of an agent that has bequest b_t but does not know yet x_t. We want to show that the operator $T^* : \Lambda(Z, \mathcal{B}) \to \Lambda(Z, \mathcal{B})$ defined as

$$(T^* W_t)(A) = \int P_t(b_t, A) W_t(db_t), \tag{22}$$

where P_t is the transition function defined above, has a unique fixed point $T^* W = W$ for any Borel subset $A \in \mathcal{B}$, given the initial bequest distribution W_0. $(T^* W_t)(A)$ can be interpreted as the probability that the next period's state lies in A according to the present period's distribution. Of course, $T^* W_t = W_{t+1}$. Notice first that w_t is well defined for every distribution W_t, as we argued previously. Second, we know that $b_{t+1} = h(z_t; w_t)$, where $h(z_t; w_t) = (1 - \gamma) Y(z_t; w_t)$ (see equation 13), is increasing in z_t for any w_t, and Z is compact. Operator $(Tf)(b_t) = \int f(b_{t+1}) P_t(b_t, db_{t+1})$, defined for any bounded function $f : B(Z) \to B(Z)$, where $B(Z)$ is the set of real-valued bounded functions defined on Z, is the conditional expectation of function f at $t+1$ given that the state at t is b_t. Since, for any wage rate $w_t \in [\underline{w}, \overline{w}]$, $h(z_t, u_t)$ is bounded and increasing in b_t, and x_{t+1} is independent of b_t, the conditional expectation of $f(b_{t+1})$ on b_t is also increasing and bounded provided that f is increasing. Intuitively, this means that, given the equilibrium wage rate w_t, an agent's descendant would never be worse off in terms of the expected value of b_{t+1} if, for any $\varepsilon > 0$, the agent's state were $b_t + \varepsilon$ instead of b_t. As function Tf is increasing, T^* is increasing and P_t is a monotonic transition function.[28] By Corollary 2 of Hopenhayn and Prescott (1992), there is a fixed point for map T^*.

It remains to show that P_t satisfies the Monotone Mixing Condition (MMC). First, define $P_{t+n}(b_t, A) = \Pr\{(b_{t+n}) \in A | b_t\}$. This is the n-step transition function beginning at t. We must show that the transition function P_{t+n} satisfies, for all t,

$$P_{t+N}(\underline{b}, [b_a, \overline{b}]) > \epsilon \quad \text{and} \quad P_{t+N}(\overline{b}, [\underline{b}, b_a]) > \epsilon$$

for some $b_a \in Z$, $\epsilon > 0$, and $N \in \mathbb{N}$. Let us, for simplicity and without loss of generality, omit subscript t. Let w be the wage rate associated with the fixed point of map T^*, W. Define the minimum stationary bequest b_l such that $b_l = (1-\gamma)(w + rb_l)$. Let $b_a = (1-\gamma)(w + rb_l) + \varrho$ for some small $\varrho > 0$. We now show that there is a positive probability that the N^{th} descendent of an agent with $b = \underline{b}$ receives a bequest above b_a. Notice first that the agent's descendants will have bequest in the vicinity of b_l in finite time because they will all be workers. Since the measure of sets $E(w)$ and $E^c(w)$ is non-zero and constant (as the labor market clears with wage in $[\underline{w}, \overline{w}]$), and ability is independent across generations, there is a positive probability that a worker becomes entrepreneur and vice-versa. Suppose that agents with ability in the vicinity of $\overline{x}$ and bequest in the vicinity of b_l cannot have descendants that become entrepreneurs. Since all agents' descendants face a positive probability of having bequest in the vicinity of b_l in finite time (as they can have successive low x's), this implies that the measure of agents (workers) in the vicinity of b_l is 1, a contradiction to the fact that $E(w)$ has non-zero measure. Therefore, agents with ability in the vicinity of $\overline{x}$ and bequest in the vicinity of b_l have descendants that become entrepreneurs. Moreover, they can become so in the following generation. This implies that they can also have bequest higher than $b_a > b_l$ as long as they have a sufficiently high x, in which case they have high credit limits. Starting from $b = \overline{b}$ is easier: a succession of low x's leaves the agent's descendants with bequest lower than b_a, as they will become

[28] See Stokey and Lucas (1989, pages 220 and 379).

Table 7: Basic statistics, changes in parameters relative to the baseline.

	Informal sector size, % output	Total output per capita, % of baseline	Official output per capita, % of baseline	% of entrep.	Income Gini
Baseline	10	100	100	9	34
Lifespan = 45	21	91	84	9.1	31
$\gamma = 1$	0	97	107	6.0	40
$\gamma = 0.7$	46	106	80	8.8	27
$\phi = 0$	-	65	0	12	28
$\phi = 0.3$	7.4	106	109	8.1	33

workers and remain so until one of them gets a sufficiently high x. Therefore, by Theorem 2 of Hopenhayn and Prescott (1992), there exists a unique time invariant distribution W and associated equilibrium wage w, such that from any initial distribution W_0, the operator T^*W_t converges to W.

E Sensitivity analysis

This model does not display wage inequality. This could be introduced by means of a "working ability" that would differentiate among workers. This inequality, however, would simply reflect the randomness of working productivity, which is not important to our purposes. We could have increased inequality by increasing the curvature of the ability distribution (i.e., parameter ϵ). The quantitative exercises are roughly the same for a parameterization with higher inequality (income Gini in range 0.40–0.44).[29]

Table 7 shows some quantitative results when we change other parameters of the model. The second row shows the impact of increasing the lifespan to 45 years. The number of entrepreneurs does not change, whereas, as expected, output falls 9 percent.

The third row displays the case where agents are not altruistic. The effect is large on all variables except output. The informal sector is null because there is no bequest in this economy and every entrepreneur needs outside resources to undertake a project. Notice, however, that with $\gamma = 1$ and $\phi = 0$ the economy would collapse because everybody would be credit-constrained. In this case, financial constraints could explain any difference in the size of the informal sector and on output across countries. But this is a rather extreme case. The fourth row shows the results for a higher propensity to leave bequest. Output is higher because agents are less credit constrained and as a consequence productivity increases. Notice that the existence of equilibrium requires that $\gamma > 1 - 1/r$. The model therefore displays some sensitivity to parameter γ, but there is no reason to assume that the altruism degree varies across countries.

The last two rows of table 7 consider two extreme cases: one with an enforcement such that the entrepreneur keeps only 70 percent of the assets in case of default (against 75 percent in the baseline), and another with no enforcement of debt contracts. The informal sector size varies from 7.4 percent of measured output to full informalization, while output per capita varies from 106 to 65 percent of the baseline economy. This confirms our previous findings that credit market policies can account for the differences in the size of the informal sector, especially for countries with low enforcement, but just part of the differences in output per capita.[30]

[29] For the sake of space, we omit these results but we can provide them upon request.

[30] When ϕ goes from 0 to 0.13, total output per capita increases 79 pp, while when it goes from 0.13 to 0.25 total output increases by only 21 pp.