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Monetary Policy with Single Instrument Feedback Rules.*

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Abstract

We revisit the issue of multiplicity of equilibria when monetary policy is conducted with either the interest rate or the money supply as the sole instrument of policy. We show that in standard monetary models there are interest rate feedback rules, and also money supply rules, that implement a unique global equilibrium. This is a contribution to a literature that either concentrates on conditions for local determinacy, or criticizes that approach showing that local determinacy might be associated with global indeterminacy. The interest rate rules we propose are price targeting rules that respond to the forecasts of future economic activity and the future price level.

Key words: Monetary policy; interest rate rules; unique equilibrium.

JEL classification: E31; E40; E52; E58; E62; E63.

1. Introduction

In this paper we revisit the issue of multiplicity of equilibria when monetary policy is conducted with either the interest rate or the money supply as the instrument of policy. There has been an extensive literature on this topic starting with Sargent and Wallace (1975), including a recent literature on local and global determinacy in models with nominal rigidities. Most of this literature finds conditions on policy under which there is a single equilibrium locally, in the neighborhood of a steady state. Some of this literature points out that the conditions for local uniqueness are not robust to changes in the environment. Another branch of this literature criticizes the local approach by pointing out that in general there are other equilibria, and arguing that the analysis should be global.

Our analysis is global. We show that it is possible to implement a unique equilibrium globally with an appropriately chosen interest rate feedback rule, and similarly with a money supply feedback rule of the same type.

The interest rate feedback rules that implement unique equilibria are price targeting rules where the nominal interest rate is a function of expectations of the future level of economic activity and the future price level. To the extent that the interest rate reacts to the forecast of an economic aggregate it resembles the rules that central banks appear to follow. In the response to the price level it is in the class of price level targeting rules, that are further apart from the policy debate.

We show the results in the simplest possible model, a cash-in-advance economy with flexible prices. The results are robust to alternative assumptions on the use of money and, as we show in the paper, to the consideration of nominal rigidities. An important assumption, and one that is also standard in this literature, is that fiscal policy is endogenous, meaning that taxes can be adjusted residually to satisfy the budget constraint of the government.

The assumption of an infinite horizon is crucial. In finite horizon economies, the equilibrium is described by a finite dimensional system of equations where the unknowns are the quantities, prices and policy variables. The number of degrees of freedom in conducting policy can be counted exactly. It is easy to see that single instrument policies are not sufficient restrictions on policy. They always generate multiple equilibria. This multiplicity does not depend on the way policy is conducted, whether interest rates are set as sequences of numbers, or as backward, current or forward functions of endogenous variables. This is no longer the case in the infinite horizon economy, as we show in this paper.

As is standard in the literature we do not impose the zero bound on the nominal

interest rate as a restriction on the actions of the government. The interest rate must be nonnegative in equilibrium but is unrestricted out of equilibrium, as in, for example, Bassetto (2004) and Schmitt-Grohe and Uribe, 2001. Benhabib, Schmitt-Grohe and Uribe (2001b) assume that the zero bound restriction applies not only in equilibrium but also to the government actions out of equilibrium. Under this alternative approach there would also be multiple equilibria in our set up. It is not clear in the context of these simple models which assumption is more reasonable, whether the zero bound restriction holds in equilibrium or also out of equilibrium. The alternative assumptions cannot be assessed empirically. In a more deeply founded model, Bassetto (2004) shows that the zero bound restriction should only be satisfied in equilibrium. There is a resemblance between this issue and the heated controversy on Ricardian versus non-Ricardian policies in the fiscal theory of the price level[1].

As mentioned above, after Sargent and Wallace (1975), and McCallum (1981), there has been an extensive literature on multiplicity of equilibria when the government follows either an interest rate rule or a money supply rule. This includes the literature on local determinacy. Contributors to this literature are Woodford (1994, 2003), Clarida, Gali and Gertler (1999, 2000), Carlstrom and Fuerst (2001, 2002), Benhabib, Schmitt-Grohe and Uribe (2001a), Dupor (2001), among many others. In this literature the analysis uses linear approximations of the models, in the neighborhood of a steady state, and identifies the conditions on preferences, technology, timing of markets, and policy rules, under which there is a unique local equilibrium. There is a unique local equilibrium, in the neighborhood of a steady state, when there is also a continuum of divergent solution paths originating close to that steady state. In the linear approximation of the model, the divergent solutions are explosive, and are disregarded using arguments such as the ones in Obstfeld and Rogoff (1983) that are typically outside the model. In the nonlinear model the alternative equilibria may converge to other steady states, or exhibit all kinds of cyclical behavior. It is on the basis of these results that the literature on local determinacy has been criticized by the recent work on global stability showing that the conditions for local determinacy may in fact be conditions for global indeterminacy (see Benhabib, Schmitt-Grohe and Uribe (2001b), Schmitt-Grohe and Uribe (2001) and Christiano and Rostagno (2002)).

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[1]Ricardian policies are policies such that the budget constraint of the government holds also for prices, that are not necessarily equilibrium prices, while non-Ricardian policies satisfy the budget constraint only for the equilibrium prices. In Bassetto (2004) while the budget constraint must hold also out of equilibrium, the zero bound restriction only holds in equilibrium.

Independent work by Loisel (2006) takes a generic linear model and shows that with policy rules analogous to the ones we use in this paper it is possible to exclude explosive paths that originate in the neighborhood of a steady state. He applies this method to the standard new keynesian linear model as in Woodford (2003). Because his analysis is local he cannot establish global uniqueness, as we do. Other related work is by Bloise, Dreze and Polemarchakis (2004) and Nakajima and Polemarchakis (2005). They take a finite horizon monetary model, or an infinite horizon one under particular assumptions, and show that it is not possible to implement unique equilibria with interest rate targeting.

This paper was motivated by previous work by the authors on optimal monetary policy in an economy under sticky prices. In Adao, Correia and Teles (2003), it is shown that after choosing the sequence of nominal interest rates there is still a large set of implementable allocations, each supported by a particular sequence of money supplies. Implicitly it is assumed that policy can set exogenous sequences for both interest rates and money supplies, subject to certain restrictions. Alternatively, as we show in this paper, there are single instrument feedback rules that implement the optimal allocation. Finally, the paper is also related to Adao, Correia and Teles (2004) where we show that it is possible to implement unique equilibria in environments with flexible prices and prices set in advance by pegging state contingent interest rates as well as the initial money supply.

The paper proceeds as follows: In Section 2, we describe the model, a simple cash-in-advance economy with flexible prices. In Section 3, we show that there are single instrument feedback rules that implement a unique equilibrium. We also discuss how a particular equilibrium can be implemented and compare the rules we propose to alternative rules in the literature, that can only guarantee locally determinate equilibria. In Section 4, we interpret the results by showing that the assumption of an infinite horizon is a necessary assumption for the results. In Section 5 we extend the results to the case where prices are set in advance. Section 6 contains concluding remarks.

2. A model with flexible prices

We first consider a simple cash-in-advance economy with flexible prices. The economy consists of a representative household, a representative firm behaving competitively, and a government. The uncertainty in period $t \geq 0$ is described by the random variable $s_t \in S_t$ and the history of its realizations up to period t (state or node at t), $(s_0, s_1, ..., s_t)$, is denoted by $s^t \in S^t$. The initial realization s_0

is given. We assume that the history of shocks has a discrete distribution.

Production uses labor according to a linear technology. We impose a cash-in-advance constraint on the households' transactions with the timing structure described in Lucas and Stokey (1983). Each period is divided into two subperiods, with the assets market operational in the first subperiod and the goods market in the second.

2.1. Competitive equilibria

Households The households have preferences over consumption C_t, and leisure L_t, described by the expected utility function:

$$U = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t u\left(C_t, L_t\right) \right\} \tag{2.1}$$

where β is a discount factor. The households start period t with nominal wealth $\mathbb{W}_t$. They decide to hold money, M_t, and to buy B_t nominal bonds that pay $R_t B_t$ one period later. R_t is the gross nominal interest rate at date t. They also buy $B_{t,t+1}$ units of state contingent nominal securities. Each security pays one unit of money at the beginning of period $t+1$ in a particular state. Let $Q_{t,t+1}$ be the beginning of period t price of these securities normalized by the probability of the occurrence of the state. Therefore, households spend $E_t Q_{t,t+1} B_{t,t+1}$ in state contingent nominal securities. Thus, in the assets market at the beginning of period t they face the constraint

$$M_t + B_t + E_t Q_{t,t+1} B_{t,t+1} \leq \mathbb{W}_t. \tag{2.2}$$

Consumption must be purchased with money according to the cash-in-advance constraint

$$P_t C_t \leq M_t, \tag{2.3}$$

where P_t is the price of the consumption good in units of money.

At the end of the period, the households receive the labor income $W_t N_t$, where $N_t = 1 - L_t$ is labor and W_t is the nominal wage rate and pay lump sum taxes, T_t. Thus, the nominal wealth households bring to period $t+1$ is

$$\mathbb{W}_{t+1} = M_t + R_t B_t + B_{t,t+1} - P_t C_t + W_t N_t - T_t \tag{2.4}$$

The households' problem is to maximize expected utility (2.1) subject to the restrictions (2.2), (2.4), (2.3), together with a no-Ponzi games condition on the holdings of assets.

The following are first order conditions of the households problem:

$$\frac{u_L(t)}{u_C(t)} = \frac{W_t}{P_t} \frac{1}{R_t} \tag{2.5}$$

$$\frac{u_C(t)}{P_t} = R_t E_t \left[\frac{\beta u_C(t+1)}{P_{t+1}} \right] \tag{2.6}$$

$$Q_{t,t+1} = \beta \frac{u_C(t+1)}{u_C(t)} \frac{P_t}{P_{t+1}}, \ t \geq 0 \tag{2.7}$$

From these conditions we get $E_t Q_{t,t+1} = \frac{1}{R_t}$. Condition (2.5) sets the intratemporal marginal rate of substitution between leisure and consumption equal to the real wage adjusted for the cost of using money, R_t. Condition (2.6) is an intertemporal marginal condition necessary for the optimal choice of risk-free nominal bonds. Condition (2.7) determines the price of one unit of money at time $t+1$, for each state of nature s^{t+1}, normalized by the conditional probability of occurrence of state s^{t+1}, in units of money at time t.

Firms   The firms are competitive and prices are flexible. The production function of the representative firm is

$$Y_t \leq A_t N_t.$$

The firms maximize profits $P_t Y_t - W_t N_t$. The equilibrium real wage is

$$\frac{W_t}{P_t} = A_t. \tag{2.8}$$

Government   The policy variables are lump sum taxes, T_t, interest rates, R_t, money supplies, M_t, state noncontingent public debt, B_t. State-contingent debt is in zero net supply, $B_{t,t+1} = 0$. We can define a policy as a mapping for the policy variables $\{T_t, R_t, M_t, B_t, \ t \geq 0, \text{ all } s^t\}$, that maps sequences of quantities, prices and policy variables into sets of sequences of the policy variables.

The period by period government budget constraints are

$$M_t + B_t = M_{t-1} + R_{t-1} B_{t-1} + P_{t-1} G_{t-1} - P_{t-1} T_{t-1}, \ t \geq 0$$

Let $Q_{t,s} \equiv Q_{t,t+1}Q_{t+1,t+2}...Q_{s-1,s}$, with $Q_{t,t} = 1$. If $\lim_{T\to\infty} E_t Q_{t,T+1}\mathbb{W}_{T+1} = 0$, the sequence of budget constraints are

$$\sum_{s=t}^{\infty} E_t Q_{t,s+1}M_s\left(R_s - 1\right) = \mathbb{W}_t + \sum_{s=t}^{\infty} E_t Q_{t,s+1}P_s\left[G_s - T_s\right] \qquad (2.9)$$

that can be written as

$$E_t \sum_{s=0}^{\infty} \beta^s u_C(t+s)C_{t+s}\left(\frac{R_{t+s}-1}{R_{t+s}}\right) = u_C(t)\frac{\mathbb{W}_t}{P_t} + E_t \sum_{s=0}^{\infty} \beta^s u_C(t+s)\frac{[G_{t+s}-T_{t+s}]}{R_{t+s}}$$
$$(2.10)$$

using (2.7).

Market clearing Market clearing in the goods and labor market requires

$$C_t + G_t = A_t N_t,$$

and

$$N_t = 1 - L_t.$$

We have already imposed market clearing in the money and debt markets.

Equilibrium An equilibrium is a sequence of policy variables, quantities and prices such that the private agents maximize given the sequences of policy variables and prices, the budget constraint of the government is satisfied and the policy sequence is in the set defined by the policy.

The equilibrium conditions for the variables $\{C_t, L_t, R_t, M_t, B_t, T_t, Q_{t,t+1}\}$ are the resources constraints

$$C_t + G_t = A_t(1 - L_t), \; t \geq 0, \qquad (2.11)$$

the intratemporal condition that is obtained from the households intratemporal condition (2.12) and the firms optimal condition (2.8)

$$\frac{u_C(t)}{u_L(t)} = \frac{R_t}{A_t}, \; t \geq 0, \qquad (2.12)$$

the cash-in-advance constraints (2.3), the intertemporal conditions (2.6) and (2.7), and the budget constraints (2.9), as well as the government policy rules, to be specified below.

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3. Single instrument feedback rules.

3.1. Rules that implement unique equilibria.

Here we assume that policy is conducted with either interest rate or money supply feedback rules. We show the main result of the paper, that there are single instrument feedback rules that implement a unique equilibrium, globally, for the allocation and prices. The proposition for an interest rate feedback rule follows:

Proposition 3.1. *When policy is conducted with the interest rate feedback rule*

$$R_t = \frac{\xi_t}{E_t \frac{\beta u_C(t+1)}{P_{t+1}}}, \tag{3.1}$$

where ξ_t is an exogenous variable, there is a unique global equilibrium.

Proof: When policy is conducted with the rule (3.1), the intertemporal condition (2.6) can be written as

$$\frac{u_C(t)}{P_t} = \xi_t, \ t \geq 0, \tag{3.2}$$

so that

$$R_t = \frac{\xi_t}{\beta E_t \xi_{t+1}}. \tag{3.3}$$

It follows that the intratemporal conditions (2.12) can be written as

$$\frac{u_C(t)}{u_L(t)} = \frac{\frac{\xi_t}{\beta E_t \xi_{t+1}}}{A_t}, \ t \geq 0 \tag{3.4}$$

These conditions together the resource constraints, (2.11), determine uniquely the variables C_t, L_t, P_t, R_t. The money balances, M_t, is determined uniquely using the cash-in-advance conditions (2.3), with equality.[2]

The budget constraints (2.10) are satisfied for multiple paths of the taxes and state noncontingent debt levels.∎

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[2]Notice that when the nominal interest rate is zero the cash-in-advance constraint does not have to hold with equality. This multiplicity of the money stock has no implications for the uniqueness of the price level or allocation.

The forward looking interest rate feedback rules that implement unique global equilibria resemble to some extent the rules that appear to be followed by central banks. The nominal interest rate reacts positively to the forecast of future consumption. It also reacts positively to the forecast of the future price level. While the reaction to future economic activity is standard in the policy debate, the reaction to the price level is not. Central banks appear to respond to forecasts of future inflation, rather than the price level, when deciding on nominal interest rates.

Depending on the exogenous process for ξ_t, with the feedback rule we consider, it is possible to decentralize any feasible allocation distorted by the nominal interest rate. The first best allocation, at the Friedman rule of a zero nominal interest rate, can also be implemented. We discuss this in the next section.

An analogous proposition to the one above is obtained when policy is conducted with a particular money supply feedback rule.

Proposition 3.2. *Suppose the cash-in-advance constraint holds exactly.[3] When policy is conducted with the money supply feedback rule,*

$$M_t = \frac{C_t u_C(t)}{\xi_t}, \tag{3.5}$$

where ξ_t is an exogenous variable, there is a unique global equilibrium.

Proof: Suppose policy is conducted according to (3.5). Then, using the cash-in-advance conditions (2.3) with equality, we obtain

$$\frac{u_C(t)}{P_t} = \xi_t \tag{3.6}$$

Using the intertemporal conditions (2.6), we have

$$R_t = \frac{\xi_t}{\beta E_t \xi_{t+1}}. \tag{3.7}$$

The two conditions above, (3.6) and (3.7), together with the intratemporal conditions (2.12) and the resource constraints, (2.11) determine uniquely the four variables, C_t, L_t, P_t, R_t in each period $t \geq 0$ and state s^t.

The taxes and debt levels satisfy the budget constraint (2.10).∎

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[3]This is always the case if the interest rate is strictly positive.

Also for this money supply rule, for a particular choice of the process of ξ_t is it possible to implement a particular, desirable, equilibrium. The same process ξ_t implements the same equilibrium whether the rule is the interest rate rule (3.1) or the money supply rule (3.5), with one qualification. The implementation of a unique equilibrium with a money supply rule relies on the cash-in-advance constraint holding exactly. That is not necessarily the case when the interest rate is zero. Instead, with an interest rate rule there is always a unique equilibrium for the allocations and price level. The money stock is not unique when the cash-in-advance constraint does not hold with equality.

3.2. Implementing equilibria with interest rate feedback rules.

3.2.1. The first best allocations and equilibria with constant inflation.

Depending on the particular stochastic process for ξ_t, it is possible to use the interest rate feedback rules in Proposition 3.1, (3.1), to select the unique equilibrium from a large set of possible equilibria, some more desirable than others. The welfare maximizing equilibrium, which in this simple environment is the first best, will have the nominal interest rate equal to zero. Another example of an equilibrium that can be implemented has zero, or constant, inflation. We will now describe the processes for ξ_t that implement either the first best or equilibria with constant inflation.

There is only one first best allocation but there are many possible equilibrium processes for the price level associated with that allocation. Varying the process for ξ_t it is possible to implement uniquely each of those equilibria.

With

$$\xi_t = \frac{1}{k\beta^t}, t \geq 0,$$

where k is a positive constant, from (3.3), $R_t = 1$. Condition (3.4) becomes

$$\frac{u_C(t)}{u_L(t)} = \frac{1}{A_t}, \ t \geq 0$$

which, together with the resource constraint (2.11) gives the first best allocation described by the functions $C_t = C^*(A_t, G_t)$, $L_t = L^*(A_t, G_t)$. The price level $P_t = P(A_t, G_t; .)$ can be obtained as the solution for P_t of (3.2), i.e.

$$\frac{u_C(C^*(A_t, G_t), L^*(A_t, G_t))}{P_t} = \frac{1}{k\beta^t}, \ t \geq 0,$$

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For each k, which is a policy parameter, there is a unique equilibrium process for the price level. The equilibrium money stock is obtained using the cash-in-advance constraint, $M_t = P(A_t, G_t; .)C^*(A_t, G_t)$, if it holds with equality. If it did not hold exactly, there would be multiple equilibrium paths for the money stock that would have no implications for the determination of the prices and allocations.

Notice that there are still other possible equilibrium processes for the path of the price level, or realized inflation, that are associated with the Friedman rule. The interest rate feedback rule with

$$\xi_t = \frac{\mu_t}{k \left(\rho\beta\right)^t},$$

where $\mu_t = \rho\mu_{t-1} + \varepsilon_t$, and ε_t is a white noise, will also imply $R_t = 1$, and achieve the first best allocation with a process for the price level given by

$$\frac{u_C(C^*(A_t, G_t), L^*(A_t, G_t))}{P_t} = \frac{\mu_t}{k \left(\rho\beta\right)^t}, \, t \geq 0. \tag{3.8}$$

The choice of k affects only the level of the price, while the process of μ_t affects realized inflation. The conditional average inflation is the symmetric of the real interest rate.

Allocations where inflation is always zero, $\frac{P_{t+1}}{P_t} = 1$, can also be implemented, as long as the real interest rate is non-negative in every state, $\frac{u_C(t)}{E_t[\beta u_C(t+1)]} \geq 1$. It is a feature of these models that there are multiple equilibrium allocations such that inflation is zero. There are also many price levels consistent with zero inflation. Again, for a particular ξ_t we are able to implement a unique sequence for the allocations and price level with zero inflation.

Let $C_t = C(R_t; .)$ and $L_t = L(R_t; .)$ be the functions that solve the system of equations given by the intratemporal condition, (2.12), and the resource constraint, (2.11), for C_t and L_t as functions of R_t, as well as A_t and G_t. Let

$$R_t = \frac{u_C(C(R_t; .), L(R_t; .))}{\beta E_t u_C(C(R_{t+1}; .), L(R_{t+1}; .))}, \, t \geq 0. \tag{3.9}$$

Any sequence of nominal interest rates $\{R_t\}$ satisfying this difference equation, and corresponding allocations $C(R_t; .)$ and $L(R_t; .)$, is such that inflation is zero. Let's take a particular sequence of interest rates $\{\overline{R}_t\}$ that solves the difference equation (3.9) and let $\overline{C}_t = C(\overline{R}_t; .)$ and $\overline{L}_t = L(\overline{R}_t; .)$. $\{\overline{C}_t\}$ and $\{\overline{L}_t\}$ are

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sequences of numbers so that $u_C(\overline{C}_t, \overline{L}_t)$ is an exogenous stochastic process. A particular sequence of interest rates and allocations, and associated price level, $\overline{P}$, can be implemented with the interest rate feedback rule of Proposition 3.1, with

$$\xi_t = \frac{u_C(\overline{C}_t, \overline{L}_t)}{\overline{P}}, \ t \geq 0,$$

where $\overline{P}$ is any positive constant. To see this notice that, using the interest rate rule

$$R_t = \frac{\frac{u_C(\overline{C}_t, \overline{L}_t)}{\overline{P}}}{\beta E_t \frac{u_C(C_{t+1}, L_{t+1})}{P_{t+1}}},$$

we obtain

$$\frac{u_C(C_t, L_t)}{P_t} = \frac{u_C(\overline{C}_t, \overline{L}_t)}{\overline{P}}, \tag{3.10}$$

which together with

$$\frac{u_C(t)}{u_L(t)} = \frac{\frac{u_C(\overline{C}_t, \overline{L}_t)}{\beta E_t u_C(\overline{C}_{t+1}, \overline{L}_{t+1})}}{A_t}, \ t \geq 0 \tag{3.11}$$

and the resource constraint (2.11) determines a unique equilibrium $C_t = \overline{C}_t$, $L_t = \overline{L}_t$ and $P_t = \overline{P}$. Again, the rule implements a unique equilibrium, but an alternative process $\left\{ \overline{\overline{C}}_t \right\}$ and $\left\{ \overline{\overline{L}}_t \right\}$ induced by another solution $\left\{ \overline{\overline{R}}_t \right\}$ of the difference equation (3.9) would implement an alternative equilibrium with zero inflation. For each exogenous process, M_t is determined uniquely using the cash-in-advance conditions (2.3) with equality.

3.2.2. Cashless economies.

In the economies we have analyzed, the nominal interest rate affects the allocations because it distorts the decision between consumption and leisure. It is common in the recent literature with sticky prices (see Woodford, 2003) to consider economies that in the limit do not have this distortion. In those cashless economies under flexible prices there is a single allocation independent of inflation. All policy does under flexible prices is to determine the price level. A variation of the feedback rule we consider is able to determine a unique equilibrium path for the price level.

The way to interpret in our set up a cashless economy is by considering a cash-in-advance constraint with velocity, and take velocity to the limit where it is

infinite. Let the cash-in-advance condition be

$$\frac{P_t C_t}{v_t} \leq M_t, \tag{3.12}$$

where $v_t \to \infty$.

In the limit case, the intratemporal condition is

$$\frac{u_C(t)}{u_L(t)} = \frac{1}{A_t}, \, t \geq 0, \tag{3.13}$$

because the nominal interest rate does not distort choices, and the intertemporal condition is

$$\frac{u_C(t)}{P_t} = E_t \left[R_{t+1} \frac{\beta u_C(t+1)}{P_{t+1}} \right], \tag{3.14}$$

where the nominal interest rate is now R_{t+1}, rather than R_t, because with infinite velocity the consumption good is a credit good and can be paid at the assets market in the following period according to the timing of transactions that we have considered.

Because the interest rate does not affect the allocations, the allocation is the first best allocation, described by the functions $C^*(A_t, G_t)$ and $L^*(A_t, G_t)$. In contrast to the economy with a monetary distortion, in these economies there is a unique nominal interest rate path consistent with zero inflation that is given by

$$R_{t+1} = \frac{u_C(C^*(A_t, G_t), L^*(A_t, G_t))}{\beta u_C(C^*(A_{t+1}, G_{t+1}), L^*(A_{t+1}, G_{t+1})))}, \, t \geq 0.$$

In this case of a cashless economy the policy rule (3.1), in Proposition 3.1, would have to be modified to

$$R_{t+1} = \frac{\xi_t}{\frac{\beta u_C(t+1)}{P_{t+1}}}, \tag{3.15}$$

and would implement a unique equilibrium for the price level described by

$$\frac{u_C(C^*(A_t, G_t), L^*(A_t, G_t))}{P_t} = \xi_t$$

A particular equilibrium that can be implemented will have zero inflation. As before, if the interest rate rule is followed with

$$\xi_t = \frac{u_C(C^*(A_t, G_t), L^*(A_t, G_t))}{\bar{P}}, \, t \geq 0, \tag{3.16}$$

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it is possible to implement a zero inflation target for the price level, $P_t = \overline{P}$.

In this case of a cashless economy, an alternative rule that would determine a unique equilibrium, with the exogenous target for the price $\{\overline{P}_t\}$, is the modified feedback rule (3.15) with

$$\xi_t = \frac{u_C(C_t, L_t)}{\overline{P}_t}, \, t \geq 0.[4] \tag{3.17}$$

This rule would not work in the more general case where the allocations are distorted by the nominal interest rate. We show this now.

Suppose that the analogous rule

$$R_t = \frac{\frac{u_C(C_t, L_t)}{\overline{P}_t}}{E_t \frac{\beta u_C(t+1)}{P_{t+1}}}, \tag{3.18}$$

with R_t rather than R_{t+1}, was followed for the case analyzed before with unit velocity, $v_t = 1$. In this case the rule would still be able to pin down uniquely (and globally) the price level, but the allocation would not be uniquely pinned down. To see this, notice that we can then write the intertemporal condition as

$$\frac{u_C(C(R_t), L(R_t))}{\overline{P}_t} = R_t E_t \left[\frac{\beta u_C(C(R_{t+1}), L(R_{t+1}))}{\overline{P}_{t+1}} \right] \tag{3.19}$$

This is a first order difference equation in R_t with multiple solutions. Each of those solutions is associated with an equilibrium in the allocations. There are therefore multiple equilibria. The alternative rule is not able to pin down a single equilibrium.

3.3. Alternative interest rate rules

The interest rate rules that we consider in this paper are able to implement unique equilibria globally. This is a contribution to a literature that has concentrated on conditions for local determinacy or on examples of global indeterminacy. In this section we relate the interest rate rule we propose to the ones in the literature. We consider a linear approximation to the model in the neighborhood of a steady state. We consider alternative rules, where the interest rate reacts to inflation or to the price level. The latter rules are less standard, but they are still considered

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[4]We thank an anonymous referee for this suggestion.

in the literature, such as the Wicksellian rules in Woodford (2003), and are closer to the ones we propose. We show that while all the other rules guarantee a determinate equilibrium, meaning that there is a unique local equilibrium but multiple solutions that diverge from the neighborhood of the steady state, and that suggest the existence of alternative equilibria in the nonlinear model[5], the analog to the rule in Proposition 3.1 eliminates all other solutions other than the one in the neighborhood of the steady state.

We consider a cashless economy, with infinite velocity, where the allocations are uniquely determined independently of policy. The cash-in-advance constraint is (3.12), with $v_t \to \infty$. The intratemporal conditions of households and firms imply the marginal conditions (3.13). The allocations are determined uniquely by these marginal conditions together with the resource constraints, (2.11), and are not affected by policy.

We now proceed as is standard in the literature and linearize the model around a deterministic steady state, with constant consumption and leisure and constant inflation. For simplicity, let $G_t = 0$.

The linearized equilibrium conditions (3.13), (3.14) and (2.11), which are the relevant equilibrium conditions together with the policy function, can be summarized by

$$E_t \left(\widehat{R}_{t+1} - \widehat{\pi}_{t+1} \right) = -c_n E_t \left(\widehat{A}_{t+1} - \widehat{A}_t \right) \tag{3.20}$$

where $\pi_{t+1} = \frac{P_{t+1}}{P_t}$, $c_n = \frac{u_{cc}C}{u_c} + \frac{a_n}{b_n} \left(\phi_c + 1 \right)$; $b_n = \phi_c - \kappa\phi_L > 0$; $a_n = \frac{u_{cc}C}{u_c} - \frac{\kappa u_{cl}L}{u_c}$, $\kappa = \frac{N}{L}$ and $\phi_x = \frac{\partial \frac{u_L(t)}{u_C(t)}}{\partial x} \frac{u_C(t)}{u_L(t)} x$, $x = C, L$.

Notice that the equilibrium condition (3.20) has $\widehat{R}_{t+1}$ instead of $\widehat{R}_t$. In the alternative timing used by Woodford (2003) among others the interest rate would be indexed by t. This has implications for whether rules should be forward, current or backward in order to guarantee local determinacy. A current rule in Woodford (2003) with $\widehat{R}_t$ reacting to $\widehat{\pi}_t$ is equivalent to a backward rule in this environment with $\widehat{R}_{t+1}$ reacting to $\widehat{\pi}_t$ (see Carlstrom and Fuerst (2001) for this discussion).

Suppose now that policy was conducted by setting the nominal interest rate path, exogenously, equal to a sequence of numbers. This would allow to determine a unique path for the conditional expectation of inflation $E_t\widehat{\pi}_{t+1}$, but would not determine the initial price level, nor the distribution of realized inflation across

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[5]See Benhabib, Schmit-Grohe and Uribe (2001b) and Schmitt-Grohe and Uribe (2001). Those alternative equilibria cannot be characterized in the linear model that is only a valid approximation in the neighborhood of the steady state.

states. The multiplicity of equilibria in this simple model without nominal rigidities does not affect allocations. Instead, in the model with nominal rigidities, that we will consider in Section 5, the multiplicity is extended to the real allocations. We analyze the simple model to make the points more clearly, but the conclusions go through, more forcefully, in a model with both a monetary distortion and nominal rigidities.

We consider now alternative interest rate rules that we compare to the ones in Proposition 3.1. These alternative rules are able to determine locally a unique equilibrium in the neighborhood of a steady state, but do so at the expense of multiple other solutions of the linear system that diverge from that neighborhood.

Suppose policy was conducted with an interest rate rule where the nominal interest rate $\widehat{R}_{t+1}$ reacts to inflation $\widehat{\pi}_t$,

$$\widehat{R}_{t+1} = \tau \widehat{\pi}_t - c_n \left(\widehat{A}_{t+1} - \widehat{A}_t \right).^6$$

Then

$$\tau \widehat{\pi}_t - E_t \left(\widehat{\pi}_{t+1} \right) = 0.$$

With $\tau > 1$, the solution is locally determinate and given by $\widehat{\pi}_t = 0$. This is the standard case discussed in the literature where the Taylor principle of an active rule, with $\tau > 1$, is necessary to guarantee a determinate equilibrium.

With an active interest rate rule reacting to inflation, there is indeed, in the linear model, a single local equilibrium but multiple explosive solutions. If inflation in period zero was $\widehat{\pi}_t = \varepsilon > 0$, the solution would diverge. These divergent solutions may in the nonlinear model converge to another steady state or cycle around this steady state, and be equilibrium paths[7].

Wicksellian interest rate rules as in Woodford (2003) have the interest rate respond to the price level rather than inflation. Again here the equivalent rule to the one in Woodford will have the interest rate in period $t+1$ respond to the price level in t. That rule is

$$\widehat{R}_{t+1} = \phi \widehat{P}_t - c_n \left(\widehat{A}_{t+1} - \widehat{A}_t \right),^8$$

where $\phi > 0$. Substituting the rule in the equilibrium condition (3.20) with $\widehat{\pi}_{t+1} = \widehat{P}_{t+1} - \widehat{P}_t$, we get

$$(1 + \phi) \widehat{P}_t - E_t \widehat{P}_{t+1} = 0 \qquad (3.21)$$

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[7] See Benhabib, Schmitt-Grohe and Uribe (2001b).

[8] Again, here, the term $-c_n \left(\widehat{A}_{t+1} - \widehat{A}_t \right)$ is irrelevant for the issue of determinacy.

With $\phi > 0$, there is a determinate equilibrium, locally, in the neighborhood of the steady state. The price level will be growing at the constant inflation, possibly zero. There are however, also in this case, other solutions of the linear model, that diverge from the neighborhood of the steady state. Also with these rules it is not possible to exclude those other solutions as possible candidates to equilibria that cannot be analyzed in the linear model. In the linear model they are explosive and do not satisfy certain bounds that may be imposed arbitrarily. In the nonlinear model even those bounds may be satisfied and there will be in general alternative equilibria.

An alternative rule with

$$\widehat{R}_{t+1} = \phi \widehat{P}_t$$

and an arbitrarily high ϕ will have the determinate solution be $\widehat{P}_t = 0$[9]. Again here the solution is unique only in a local sense. There are other divergent solutions.

The rule (3.1) in Proposition 3.1. is also a price targeting rule in the sense that the interest rate reacts to the price level rather than inflation. With our timing it is a current rule (in Woodford (2003) would be a forward rule) and the coefficient on the price level is one. In the linear, cashless, model, the rule would be

$$\widehat{R}_{t+1} = \widehat{\xi}_t + \widehat{P}_{t+1} - c_n \widehat{A}_{t+1}. \tag{3.22}$$

This and (3.20) implies

$$\widehat{P}_t = -\widehat{\xi}_t - c_n \widehat{A}_t$$

which determines a unique equilibrium originating in the neighborhood of the steady state. All the other solutions of the linear system are excluded. The equilibrium is unique. If $\widehat{\xi}_t = -c_n \widehat{A}_t$, then the solution will be

$$\widehat{P}_t = 0$$

as before.

We have shown that the policy rule (3.22) in the linear model is able to generate a unique equilibrium, eliminating the divergent solutions that are present when the alternative rules considered above are followed, whether inflation or price level targeting rules. The result in Proposition 3.1. is stronger because uniqueness is shown in the actual model of the economy, and not in the linear approximation. The interest rate rule (3.1) allows to implement a unique equilibrium globally. With the interest rate feedback rules used above in the nonlinear system, the

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[9]We thank an anonynous referee for this suggestion.

multiplicity of equilibria originating locally is eliminated and so is the multiplicity of equilibria originating anywhere else, except for the single equilibrium that the rule implements.

4. Interpreting the results. The importance of an infinite horizon.

The result that there are single instrument feedback rules that implement unique equilibria is a surprising one. In fact, it is well known that interest rate rules may implement a determinate equilibrium, but not a unique global equilibrium. We have illustrated this in the previous section in a linear approximation to the model. In that model the interest rate rules considered in the literature generate multiple solutions one of which possibly in the neighborhood of a steady state. The analog to the rule we propose eliminates all but one solution.

In this section we show more generally, in the nonlinear model, that the multiplicity of equilibria with interest rate rules is a general result. On this, whether the economy has an infinite horizon or a finite horizon, possibly arbitrarily large, is an important assumption. If the economy had a finite horizon, an equilibrium would be characterized by a finite number of equations and unknowns. In that case the number of degrees of freedom in conducting policy is a finite number that does not depend on whether policy is conducted with sequences of numbers or with feedback rules, functions of future, current or past variables, as long as these functions are truly exogenous, i.e. independent from the remaining equilibrium conditions. Single instrument feedback rules are not sufficient restrictions. They are always unable to pin down unique equilibria. Instead in an infinite horizon, as we show in this paper, the way policy is conducted matters, and there are single instrument feedback rules that guarantee unique global equilibria.

We first consider the case where monetary policy is conducted with sequences of numbers for the policy variables. We will show that in that case an interest rate policy generates multiple equilibria. That result is directly extended to the case where the interest rate is a function of contemporaneous or past variables.

The equilibrium conditions are the resources constraints, (2.11), the intratemporal conditions (2.12), the cash-in-advance constraints (2.3), the intertemporal conditions (2.6) and the budget constraints (2.10).

These conditions define a set of equilibrium allocations, prices and policy variables. There are many equilibria. To see this, notice that, from the resources constraints, (2.11), the intratemporal conditions (2.12), and the cash-in-advance

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constraints, (2.3), we obtain the functions $C_t = C(R_t)$ and $L_t = L(R_t)$ and $P_t = \frac{M_t}{C(R_t)}$, $t \geq 0$. As long as $u_C(C_t, L_t)C_t$ depends on C_t or L_t, excluding therefore preferences that are additively separable and logarithmic in consumption, the system of equations can be summarized by the following dynamic equations:

$$\frac{u_C(C(R_t), L(R_t))}{\frac{M_t}{C(R_t)}} = \beta R_t E_t \left[\frac{u_C(C(R_{t+1}), L(R_{t+1}))}{\frac{M_{t+1}}{C(R_{t+1})}} \right], t \geq 0 \qquad (4.1)$$

together with the budget constraints, (2.10). The budget constraints are satisfied by the choice of lump-sum taxes T_s.

Suppose the path of interest rates is set exogenously in every date and state. To make the point that an interest rate target is unable to pin down a unique equilibrium notice that in order for the difference equation to have a unique solution we would have to add other restrictions on the other policy variable, money supply. Suppose the money supply was set exogenously in period zero, M_0, and that it would also be set exogenously for each $t \geq 1$, and each state s^{t-1}, in $\#S_t - 1$ states, where $\#S_t$ is the number of elements of S_t. In that case there would be a single solution for the allocations and prices. Similarly, there would also be a unique equilibrium if the money supply was set exogenously in every date and state, and so would be the interest rate in period 0, R_0, as well as, for each $t \geq 1$, and for state s^{t-1}, the interest rate in $\#S_t - 1$ states. The budget constraints restrict, not uniquely, the taxes and debt levels.

If policy was conducted with sequence of numbers for either the interest rate or the money supply, it would not be possible, in general, to pin down unique equilibria. The same result holds when policy is conducted with interest rate rules that depend on current or past variables. Those rules clearly preserve the same degrees of freedom in the determination of policy. When fiscal policy is residual, it would still be necessary to add as additional restrictions the exogenous levels of the money supply in some but not all states.

The result that either interest rates or money supplies are not sufficient instruments to pin down unique equilibria, when policy is conducted as a sequence of numbers, is a general result, with an exception that is useful to understand the workings of the feedback rules in Proposition 3.1. In the particular case where the preferences are additively separable and logarithmic in consumption, the difference equation (4.1) becomes

$$\frac{1}{M_t} = \beta R_t E_t \left[\frac{1}{M_{t+1}} \right], t \geq 0. \qquad (4.2)$$

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When the money supply is set exogenously in every state, there is a unique equilibrium for the path of the nominal interest rates $\{R_t\}$. The allocations are therefore uniquely determined from (2.12) and (2.11). With the allocations uniquely determined and the money supply set exogenously, the price level is also determined uniquely from the cash-in-advance constraint (2.3) with equality. In order to have a unique equilibrium, there is no need of additional restrictions on interest rates.

These particular preferences is another exception to the general principle that single instrument policy is not able to pin down unique equilibria. The mechanism is the same as the one that allows the rules in propositions 3.1 and 3.2 to guarantee unique global equilibria. Notice that the right hand side of the difference equation (2.3) is exogenous as in the case of the rules in those propositions.

In the infinite horizon the way policy is conducted matters for global multiplicity. Characteristics of the environment such as preferences also matter. This is to some extent an odd result since there are as many money supplies per state as interest rates, and preferences should not matter when counting equations and unknowns. The reason why this happens is because the horizon is infinite and counting equations and unknowns when these are infinite is not necessarily useful.

In a finite horizon, instead, an equilibrium is described by a finite number of equations and unknowns. In this case the number of necessary policy restrictions needed to have a unique equilibrium can be counted exactly. As we will show, in the analog finite horizon economy, single instrument feedback rules are never able to pin down unique equilibria.

4.1. Finite horizon economies

In a finite horizon economy[10], determining the degrees of freedom in conducting policy amounts to simply counting the number of equations and unknowns. We proceed to considering the case where the economy lasts for a finite number of periods $T + 1$, from period 0 to period T. After T, there is a subperiod for the clearing of debts, where money can be used to pay debts, so that

$$\mathbb{W}_{T+1} = M_T + R_T B_T + P_T G_T - P_T T_T = 0$$

The first order conditions in the finite horizon economy are the intratemporal conditions, (2.12) for $t = 0, ..., T$, the cash-in-advance constraints, (2.3) also for

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[10]See Bloise, Dreze and Polemarchakis (2004) and Nakajima and Polemarchakis (2005).

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$t = 0, ..., T$, the intertemporal conditions

$$\frac{u_C(t)}{P_t} = R_t E_t \left[\frac{\beta u_C(t+1)}{P_{t+1}} \right], \ t = 0, ..., T - 1$$

$$Q_{t,t+1} = \beta \frac{u_C(t+1)}{u_C(t)} \frac{P_t}{P_{t+1}}, \ t = 0, ..., T - 1 \tag{4.3}$$

and, for any $0 \le t \le T$, and state s^t, the budget constraints

$$\sum_{s=0}^{T-t} E_t Q_{t,t+s+1} M_{t+s} (R_{t+s} - 1) = \mathbb{W}_t + \sum_{s=0}^{T-t} E_t Q_{t,t+s+1} P_{t+s} [G_{t+s} - T_{t+s}]$$

where $E_0 Q_{T+1} \equiv \frac{E_0 Q_T}{R_T}$.

The budget constraints restrict, not uniquely, the levels of state noncontingent debts and taxes. Assuming these policy variables are not set exogenously we can ignore this restriction. The equilibrium can then be summarized by

$$\frac{u_C(C(R_t), L(R_t))}{\frac{M_t}{C(R_t)}} = \beta R_t E_t \left[\frac{u_C(C(R_{t+1}), L(R_{t+1}))}{\frac{M_{t+1}}{C(R_{t+1})}} \right], \ t = 0, ..., T - 1 \tag{4.4}$$

Note that the total number of money supplies and interest rates is the same. Let the number of states in period t, $\#S^t$, be denoted by Φ_t. There are $\Phi_0 + \Phi_1 + ... + \Phi_T$ of each monetary policy variable. The number of equations is $\Phi_0 + \Phi_1 + ... + \Phi_{T-1}$. In order for there to be a unique equilibrium need to add to the system $\Phi_0 + \Phi_1 + ... + 2\Phi_T$ independent restrictions. If the interest rates are set exogenously in every state, the degrees of freedom are the number of terminal nodes. Thus, if the money supply could also be determined exogenously it should be so in every terminal node. Similarly there would be a unique equilibrium if the money supply was set exogenously in every state and the interest rates were set in every terminal node. In this sense, the two monetary instruments are equivalent in this economy.

In the finite horizon economy, the number of degrees of freedom in conducting policy does not depend on how policy is conducted, whether with sequences of numbers or with functions of endogenous variables, whether current, backward or forward.

When policy is conducted with the forward looking feedback rule in Proposition 3.1, the policy for the interest rate in the terminal period R_T, cannot be a function of variables in period $T+1$. It still remains to determine the money supply in every state at T. While the rule we propose guarantees a unique equilibria in the infinite horizon, it would not do so in a finite horizon economy, even if an arbitrarily large horizon.

There is an analogous intuition to the one in this model in models with overlapping generations. In those models, while the first welfare theorem always holds in a finite horizon, it does not in the infinite horizon. In the infinite horizon it may be possible to improve welfare of the initial old generation by transferring resources from the successive generations. In a finite horizon those transfers would break down, when the last generation would be unable to obtain its compensation[11].

The multiplicity of equilibria in the finite horizon economy also does not depend on price setting restrictions. The price setting restrictions introduce as many variables as number of restrictions. Instead, in an infinite horizon, the irrelevance of price setting restrictions is not granted. We do this analysis in the following section.

5. Sticky prices

We have shown the results in the simplest model with flexible prices. Under flexible prices, an interest rate target, in the sense of a policy that sets the path of nominal interest rates equal to a sequence of numbers, is able to pin down a unique equilibrium for the real allocations, but not the price level. Instead if prices are sticky, the same policy will generate multiplicity of real allocations. For this reason the interest of policy rules that may guarantee uniqueness is higher when nominal rigidities are considered.

In this section we show that the results derived above extend to an environment with prices set in advance. We modify the environment to consider price setting restrictions. There is a continuum of firms, indexed by $i \in [0,1]$, each producing a differentiated good also indexed by i. The firms are monopolistic competitive and set prices in advance with different lags.

The households have preferences described by (2.5) where C_t is now the com-

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[11]Loosely speaking, in our set up, the infinite horizon allows to bring in from the future additional restrictions, rather than resources, that help determining unique equilibria.

posite consumption

$$C_t = \left[\int_0^1 c_t(i)^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}}, \theta > 1, \tag{5.1}$$

and $c_t(i)$ is consumption of good i. Households minimize expenditure $\int_0^1 p_t(i)c_t(i)di$, where $p_t(i)$ is the price of good i in units of money, to obtain a given level of the composite good C_t, (5.1). The resulting demand function for each good i is given by

$$c_t(i) = \left(\frac{p_t(i)}{P_t} \right)^{-\theta} C_t, \tag{5.2}$$

where P_t is the price level,

$$P_t = \left[\int p_t(i)^{1-\theta} di \right]^{\frac{1}{1-\theta}}. \tag{5.3}$$

The households' intertemporal and intratemporal conditions are as before, (2.5), (2.6) and (2.7).

The government must finance an exogenous path of government purchases $\{G_t\}_{t=0}^{\infty}$, such that

$$G_t = \left[\int_0^1 g_t(i)^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}}, \theta > 0. \tag{5.4}$$

Given the prices on each good i, $p_t(i)$, the government minimizes expenditure on government purchases by deciding according to

$$\frac{g_t(i)}{G_t} = \left(\frac{p_t(i)}{P_t} \right)^{-\theta}. \tag{5.5}$$

Market clearing for each good implies

$$c_t(i) + g_t(i) = A_t n_t(i), \tag{5.6}$$

while in the labor market it must be that, in equilibrium,

$$\int_0^1 n_t(i)di = N_t. \tag{5.7}$$

Using (5.6), (5.7), (5.2), and (5.5), we can write the resource constraints as

$$(C_t + G_t) \int_0^1 \left(\frac{p_t(i)}{P_t} \right)^{-\theta} di = A_t N_t. \tag{5.8}$$

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We consider now that firms set prices in advance. A fraction α_j firms set prices j periods in advance with $j = 0, ...J$. Firms decide the price for period t with the information up to period $t - j$ to maximize profits[12]:

$$E_{t-j} \left[Q_{t-j,t+1} \left(p_t(i)y_t(i) - W_t n_t(i) \right) \right],$$

subject to the production function

$$y_t(i) \leq A_t n_t(i)$$

and the demand function

$$y_t(i) = \left(\frac{p_t(i)}{P_t} \right)^{-\theta} Y_t, \tag{5.9}$$

where $y_t(i) = c_t(i) + g_t(i)$ and $Y_t = C_t + G_t$.

The optimal price for a firm that is setting the price for period t, j periods in advance, is

$$p_t(i) = p_{t,j} = \frac{\theta}{(\theta - 1)} E_{t-j} \left[\eta_{t,j} \frac{W_t}{A_t} \right], \tag{5.10}$$

where

$$\eta_{t,j} = \frac{Q_{t-j,t+1} P_t^\theta Y_t}{E_{t-j} \left[Q_{t-j,t+1} P_t^\theta Y_t \right]}.$$

The price level at date t can be written as

$$P_t = \left[\sum_{j=0}^{J} \alpha_j \left(p_{t,j} \right)^{1-\theta} \right]^{\frac{1}{1-\theta}}. \tag{5.11}$$

When we compare the two sets of equilibrium conditions, under flexible and prices set in advance, here we are adding more variables, the prices of the differently restricted firms, but we also add the same number of equations. To show that the same arguments in the previous section also work here, it is useful to rewrite the equilibrium conditions.

Substituting the state contingent prices $Q_{t-j,t+1}$ in the price setting conditions (5.10), and using the intertemporal condition (2.6) as well as the households' intratemporal condition (2.5), we obtain the intratemporal conditions

$$E_{t-j} \left[\frac{u_C(t)}{R_t} P_t^{\theta-1} A_t \left(1 - L_t \right) - \frac{\theta}{(\theta - 1)} u_L(t) P_t^{\theta-1} \left(1 - L_t \right) \frac{P_t}{p_{t,j}} \right] = 0, j = 0, ...J. \tag{5.12}$$

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[12]Profits at t are priced by $Q_{t-j,t+1}$ because of the timing of transactions where profits are received at the end of the period to be use for consumption the period after.

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Notice that for $j = 0$, the condition becomes

$$\frac{u_C(t)}{u_L(t)} = \frac{\theta R_t}{(\theta - 1) A_t} \frac{P_t}{p_{t,0}}. \tag{5.13}$$

If $J = 0$, meaning that there are only flexible price firms, $p_{t,0} = P_t$ and we would get the intratemporal condition obtained under flexible prices,

$$\frac{u_C(t)}{u_L(t)} = \frac{\theta R_t}{(\theta - 1) A_t}, \tag{5.14}$$

corresponding to (2.12), for the case where $\theta \to \infty$.

The resource constraints can be written as

$$(C_t + G_t) \sum_{j=0}^{J} \alpha_j \left(\frac{p_{t,j}}{P_t} \right)^{-\theta} = A_t N_t. \tag{5.15}$$

The proposition follows:

Proposition 5.1. *When prices are set in advance, if policy is conducted with the interest rate feedback rule*

$$R_t = \frac{\xi_t}{E_t \frac{\beta u_C(t+1)}{P_{t+1}}},$$

where ξ_t is an exogenous variable, there is a unique equilibrium. Similarly, if policy is according to the money supply feedback rule,

$$M_t = \frac{C_t u_C(t)}{\xi_t}$$

and the cash-in-advance constraints holds exactly, there is also a unique equilibrium.

    Proof: When policy is conducted with the interest rate feedback rule $R_t = \frac{\xi_t}{E_t \frac{\beta u_C(t+1)}{P_{t+1}}}$, then the intertemporal condition (2.6) implies

$$\frac{u_C(t)}{P_t} = \xi_t, \ t \geq 0 \tag{5.16}$$

and

$$R_t = \frac{\xi_t}{\beta E_t \xi_{t+1}}, \ t \geq 0 \tag{5.17}$$

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These conditions together with the resource constraints (5.15), the intratemporal conditions (5.12), the conditions on the price level, (5.11), and the cash-in-advance constraints, (2.3), with equality, determine uniquely all the variables C_t, L_t, P_t, $p_{t,j}$, $j = 0, ...J$, and M_t. $p_{0,j}$, $j = 1, ...J$ are exogenous.

The budget constraints (2.10) are satisfied for multiple paths of the taxes and state noncontingent debt levels.

Clearly the same arguments in the proof of Proposition 3.2, for the money supply rule under flexible prices, apply here.∎

We have shown that the results extend to environments with sticky prices, in particular when prices are set in advance in a staggered fashion. In the following section we illustrate the results by describing how the interest rate rule works in a simpler economy where all firms set prices one period in advance.

5.1. An example: All firms set prices one period in advance.

We now consider an economy where all firms set prices one period in advance. This is a simple example that illustrates how the interest rate rule is able to determine unique equilibria also when prices are sticky.

In a model where there is only one type of firms that set prices one period in advance, the equilibrium conditions can be summarized by the conditions that follow. When the nominal interest rate policy is conducted according to $R_t = \frac{\xi_t}{E_t \frac{\beta u_C(t+1)}{P_{t+1}}}$, then, as before we have

$$\frac{u_C(t)}{P_t} = \xi_t, \ t \geq 0 \tag{5.18}$$

and

$$R_t = \frac{\xi_t}{\beta E_t \xi_{t+1}}, \ t \geq 0 \tag{5.19}$$

The other equilibrium conditions are

$$E_{t-1}\left[\frac{u_C(t)}{R_t} A_t (1 - L_t) - \frac{\theta}{(\theta - 1)} u_L(t) (1 - L_t)\right] = 0, \ t \geq 1$$

$$C_t + G_t = A_t (1 - L_t). \tag{5.20}$$

which determine the variables C_t, L_t and the predetermined prices P_t, with P_0 exogenous. Money supply is determined from the cash-in-advance constraint

$$P_t C_t = M_t. \tag{5.21}$$

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6. Concluding Remarks

The problem of multiplicity of equilibria under an interest rate policy has been addressed, after Sargent and Wallace (1975) and McCallum (1981), by an extensive literature on determinacy under interest rate rules. Interest rate feedback rules on endogenous variables such as the inflation rate, or the price level, can, with appropriately chosen coefficients, deliver determinate equilibria, i.e. unique local equilibria in the neighborhood of a steady state. There are still multiple solutions to the system of difference equations that approximates linearly the model. Those additional solutions suggest other equilibria that can be analyzed in the nonlinear model. Indeed, it is a consensual result that there are multiple equilibria when policy is conducted with single instrument rules.

In this paper we show that there are interest rate feedback rules, and also money supply feedback rules, that implement unique global equilibria. This result does not depend on preferences or other similar characteristics of the environment. It is also robust to the consideration of nominal rigidities such as prices set in advance. The way this rule works in pinning down unique equilibria in our simple set up is by eliminating expectations of future variables from the dynamic equations. In alternative more complex environments, the policy rule may need to be modified so that the same result may be achieved.

The feedback rules that we propose can be used to pin down the welfare maximizing equilibria, but the policy maker can also implement uniquely other, less desirable, equilibria.

An important assumption for our results, which is the standard assumption in the literature, is that the time horizon is infinite. Otherwise, single instrument feedback rules would never be able to pin down unique equilibria.

Anther important assumption is that fiscal policy is Ricardian, in the sense that taxes can be used as a residual variable to satisfy the budget constraint of the government. Likewise, it is important that the zero bound condition be imposed in equilibrium and not as a restriction to the out of equilibrium actions of the government. In this we follow most of the literature, in particular Bassetto (2004).

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