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Monetary Policy with State Contingent Interest Rates*

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Abstract

What instruments of monetary policy must be used in order to implement a unique equilibrium? This paper revisits the issues addressed by Sargent and Wallace (1975) on the multiplicity of equilibria when policy is conducted with interest rate rules. We show that the appropriate interest rate instruments under uncertainty are state-contingent interest rates, i.e. the nominal returns on state-contingent nominal assets. A policy that pegs state-contingent nominal interest rates, and sets the initial money supply, implements a unique equilibrium. These results hold whether prices are flexible or set in advance.

Key words: Monetary policy; policy instruments; sticky prices; state-contingent interest rates.

JEL classification: E31; E40; E52; E58; E62; E63.

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1. Introduction

This paper revisits the issues addressed by Sargent and Wallace (1975), among many others, on the multiplicity of equilibria when policy is conducted with interest rate rules. We show that while policy on state-noncontingent interest rates is unable to pin down a unique equilibrium, that is not the case when the policy instruments are the returns on state-contingent nominal assets. If monetary policy targets state-contingent interest rates, as well as the initial money supply, in flexible price models and in a class of sticky price models, there is a unique equilibrium.

In a deterministic flexible price model if policy sets exogenously the path of nominal interest rates as well as the initial money supply there is a unique perfect foresight equilibrium. Under uncertainty, setting the initial money supply and the nominal interest rate in every state of the world is not sufficient to pin down a unique equilibrium. The path of the nominal interest rate in a flexible price model pins down the allocations and the average growth rate of the price level, but not the distribution of prices across states. In order to pin down a unique equilibrium, there have to be additional exogenous instruments of policy.

Under sticky prices setting the path for the nominal interest rates not only does not pin down the distribution of prices it also generates multiple equilibria in the allocations. Adao, Correia and Teles (2003), use a model with prices set one period in advance and show that when the nominal interest rate is given, possibly arbitrarily close to the optimal rule which is the Friedman rule of zero nominal interest rates, there is a large set of implementable allocations. In that paper the optimal allocation is implemented by setting exogenously not only the nominal interest rates but also the money supplies in some, but not all, states of the world. Setting the returns on the state-contingent nominal interest rates is an alternative, natural, way to implement that optimal allocation.

There is a vast literature on using interest rate feedback rules to address the issue of multiplicity of equilibria, as in McCallum (1981). This analysis is about local determinacy, not about uniqueness. Appropriately chosen interest rate feedback rules can implement a locally determinate equilibrium, i.e. a single equilibrium in the neighborhood of a steady state. In the deterministic model this means that there is only one initial price level and corresponding money supply that is associated with a path converging to the steady state. In the approximately linear system the other equilibrium paths are divergent. Not necessarily so in the non-linear system, as shown by the examples in Benhabib, Schmitt–Grohe and

Uribe (2001) and Christiano and Rostagno (2002).

We assume throughout the paper that fiscal policy is determined endogenously, in line with an extensive literature on multiplicity of equilibria in monetary models. In contrast, exogenous fiscal policy could be used, as in the fiscal theory of the price level, to determine a unique equilibrium.

This paper is closely related to Adao, Correia and Teles (2004b), as well as Nakajima and Polemarchakis (2003) and Bloise, Dreze and Polemarchakis (2004). Those papers, in contrast with this one, analyze the degrees of freedom in conducting monetary policy when the instruments are the money supply and the nominal interest rate. Nakajima and Polemarchakis (2003) impose restrictions on the structure of the economy and on the policies so that the policy results are the same whether the economy has a finite or an infinite horizon. Instead, Adao, Correia and Teles (2004) show that the policy results in the economy with an infinite horizon can be very different from the ones in the analogous finite horizon economy.

Angeletos (2002) and Buera and Nicolini (2004) have shown that state-contingent debt may be replicated by debt of multiple maturities. A possible implication for the result in this paper is that pegging the prices of the state-contingent nominal assets could be achieved by pegging the prices of debt of different maturities.

The paper proceeds as follows: In Section 2, we consider a simple cash in advance economy with flexible prices. In that economy a policy that sets exogenously the nominal interest rates and the initial money supply implements a unique equilibrium in the deterministic case, but not under uncertainty. We show that a policy that implements a unique equilibrium under uncertainty sets exogenously the state-contingent interest rates, as well as the initial money supply. We show that the set of equilibria can be implemented with zero net supply of nominal state-contingent assets. In Section 3, we show that the results are extended to economies with prices set in advance. We also show that they are not robust to all price setting restrictions. Section 4 contains concluding remarks.

2. A model with flexible prices

We first consider a simple cash in advance economy with flexible prices. The economy consists of a representative household, a representative firm behaving competitively, and a government. The uncertainty in period $t \geq 0$ is described by the random variable $s_t \in S_t$ and the history of its realizations up to period t (state or node at t), $(s_0, s_1, ..., s_t)$, is denoted by $s^t \in S^t$. We assume that s_t has a discrete distribution. The number of states in period $t \geq 0$ is Φ_t.

Production uses labor according to a linear technology. We impose a cash-in-advance constraint on the households' transactions with the timing structure described in Lucas and Stokey (1983). That is, each period is divided into two subperiods, with the assets market operational in the first subperiod and the goods market in the second.

2.1. Competitive equilibria

Households The households have preferences over consumption C_t, and leisure L_t, described by the expected utility function:

$$U = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t u(C_t, L_t) \right\} \tag{2.1}$$

where β is a discount factor. The households start period t with nominal wealth $\mathbb{W}_t$. They decide to hold money, M_t, and to buy B_t nominal bonds that pay $R_t B_t$ one period later. R_t is the gross nominal interest rate at date t. They also buy $B_{t,t+1}$ units of state-contingent nominal securities. Each security pays one unit of money at the beginning of period $t+1$ in a particular state. Let $Q_{t,t+1}$ be the beginning of period t price of these securities normalized by the probability of the occurrence of the state. The households spend $E_t Q_{t,t+1} B_{t,t+1}$ in state-contingent nominal securities. Thus, in the assets market at the beginning of period t they face the constraint

$$M_t + B_t + E_t Q_{t,t+1} B_{t,t+1} \leq \mathbb{W}_t \tag{2.2}$$

where the initial nominal wealth $\mathbb{W}_0$ is given.

Consumption must be purchased with money according to the cash in advance constraint

$$P_t C_t \leq M_t. \tag{2.3}$$

At the end of the period, the households receive the labor income $W_t N_t$, where $N_t = 1 - L_t$ is labor and W_t is the nominal wage rate and pay lump sum taxes, T_t. Thus, the nominal wealth households bring to period $t+1$ is

$$\mathbb{W}_{t+1} = M_t + R_t B_t + B_{t,t+1} - P_t C_t + W_t N_t - T_t \tag{2.4}$$

The households' problem is to maximize expected utility (2.1) subject to the restrictions (2.2), (2.4), (2.3), together with a no-Ponzi games condition on the holdings of assets.

The following are first order conditions of the households' problem:

$$\frac{u_L(t)}{u_C(t)} = \frac{W_t}{P_t}\frac{1}{R_t} \tag{2.5}$$

$$\frac{u_C(t)}{P_t} = R_t E_t\left[\frac{\beta u_C(t+1)}{P_{t+1}}\right] \tag{2.6}$$

$$Q_{t,t+1} = \beta\frac{u_C(t+1)}{u_C(t)}\frac{P_t}{P_{t+1}},\ t \geq 0 \tag{2.7}$$

From these conditions we get

$$E_t Q_{t,t+1} = \frac{1}{R_t} \tag{2.8}$$

Condition (2.5) sets the intratemporal marginal rate of substitution between leisure and consumption equal to the real wage adjusted for the cost of using money, R_t. Condition (2.6) is an intertemporal marginal condition necessary for the optimal choice of risk-free nominal bonds. Condition (2.7) determines the price of one unit of money at time $t+1$, for each state of nature s^{t+1}, normalized by the conditional probability of occurrence of state s^{t+1}, in units of money at time t.

Firms The firms are competitive and prices are flexible. The production function of the representative firm is linear

$$Y_t = A_t N_t$$

The equilibrium real wage is

$$\frac{W_t}{P_t} = A_t. \tag{2.9}$$

Government The policy variables are taxes T_t, nominal interest rates R_t, state-contingent nominal prices $Q_{t,t+1}$, money supplies M_t, state-noncontingent public debt B_t and state-contingent debt $B_{t,t+1}$. The government expenditures, G_t, are exogenous.

The government budget constraints are

$$M_t + B_t + E_t Q_{t,t+1} Z_{t+1} = M_{t-1} + R_{t-1} B_{t-1} + B_{t-1,t} + P_{t-1} G_{t-1} - P_{t-1} T_{t-1},\ t \geq 0$$

together with a no-Ponzi games condition. Let $Q_{t+1} \equiv Q_{0,t+1}$, with $Q_0 = 1$. If $\lim_{T\to\infty} E_t Q_{T+1} \mathbb{W}_{T+1} = 0$, the budget constraints can be summarized by a single budget constraint

$$E_0 \sum_{t=0}^{\infty} Q_{t+1} M_t \left(R_t - 1\right) = \mathbb{W}_0 + E_0 \sum_{s=0}^{\infty} Q_{t+1} P_t \left[G_t - T_t\right] \tag{2.10}$$

Market clearing

$$C_t + G_t = A_t N_t.$$

$$1 - L_t = N_t.$$

We have already imposed market clearing in the money and asset markets.

Equilibria A competitive equilibrium is a sequence of policy variables, quantities and prices such that the private agents solve their problems given the sequences of policy variables and prices, and the budget constraint of the government is satisfied.

The competitive equilibrium conditions for the variables $\{C_t, L_t\}$, $\{R_t, Q_{t,t+1}, M_t, B_t, B_{t,t+1}, T_t\}$ are the resource constraints

$$C_t + G_t = A_t(1 - L_t), \tag{2.11}$$

the intratemporal conditions

$$\frac{u_C(t)}{u_L(t)} = \frac{R_t}{A_t}, \tag{2.12}$$

that are obtained from the households intratemporal conditions (2.12) and the firms optimal condition (2.9), as well as the cash in advance constraints (2.3), the intertemporal conditions (2.6) and (2.7), and the budget constraint (2.10).

The equations identified above determine a set of equilibrium allocations, prices and policy variables. In order for a particular equilibrium in this set to be implemented need to determine exogenous policy rules for a subset of the policy variables. A policy rule for a particular policy variable can be a function of only the state or of other variables. We will primarily consider the case where policy rules are only functions of the state. An exogenous policy rule is one that is not implied by the other equilibrium conditions.

In the next section we will show that if rates are set exogenously in every date and state, as well as the initial money supply, there is a single perfect foresight equilibrium, but multiplicity under uncertainty. The appropriate policy instruments, that allow to implement a unique equilibrium, are the returns on state-contingent nominal assets.

2.2. Multiplicity of equilibria with interest rate rules

In this section, we show that when policy is conducted with constant functions for the monetary policy instruments, and fiscal policy is endogenous, if policy sets the nominal interest rates and the initial money supply, it is unable, under uncertainty, to implement a unique equilibrium.

Without imposing restrictions on the policy variables, the equilibrium conditions are the resources constraint, (2.11), the intratemporal condition (2.12), the cash in advance constraints (2.3), the intertemporal conditions (2.6) and (2.7), as well as the budget constraints (2.10). These conditions define a set of equilibrium allocations, prices and policy variables. The set of allocations is the set of implementable allocations. In order for the set of equilibrium conditions to have a unique solution a subset of the policy variables must be set exogenously. We will call this set, the set of instruments of policy.

We will start by showing that if the policy instruments are the nominal interest rate, as well as the initial money supply, it is possible to implement a unique equilibrium in the deterministic economy, but not under uncertainty. We first consider the case in which the policy are sequences of constant functions for the interest rates. The other policy variables are not exogenous. From the resources constraints, (2.11), the intratemporal conditions (2.12), and the cash in advance conditions, (2.3), we obtain the functions $C(R_t)$ and $N(R_t)$ and $P_t = \frac{M_t}{C(R_t)}$, $t \geq 0$. The system of equations can then be summarized by the following dynamic equations:

$$\frac{u_C(C(R_t), N(R_t))}{\frac{M_t}{C(R_t)}} = \beta R_t E_t \left[\frac{u_C(C(R_{t+1}), N(R_{t+1}))}{\frac{M_{t+1}}{C(R_{t+1})}} \right], \, t \geq 0 \tag{2.13}$$

together with the budget constraints, (2.10).

Suppose the nominal interest rate is set exogenously in every date and state. The allocation is pinned down uniquely. The issue is how can a unique sequence of prices be pinned down. The proposition follows:

Proposition 2.1. *Suppose policy are sequences of numbers for the nominal interest rates. Let the interest rate be determined exogenously in every date and state, as well as the initial money supply. For this policy the allocation and prices are determined uniquely in the deterministic case. Under uncertainty, there is a single solution for the consumption and labor allocations, but not for the price levels.*

Proof: Given the interest rate for every date and state, the allocation is obtained from the functions $C(R_t)$ and $N(R_t)$.

At any period $t \geq 1$, given M_{t-1}, there are Φ_{t-1} equations to determine Φ_t variables, M_t. More specifically, for each state s^{t-1}, there is one equation to determine $\#S_t$ variables. Except for the deterministic case, there are multiple solutions for the money supply, and consequently for the price level, $P_t = \frac{M_t}{C(R_t)}$.■

A policy that delivers a unique equilibrium sets the interest rate in every date and state and the money supply in every state at some period T and thereafter in $\Phi_t - \Phi_{t-1}$ states, for $t \geq 1$. If we take a sequence of economies indexed by T and make T arbitrarily large, those economies will have a single equilibrium, and therefore the limit of these economies will also have a single equilibrium. The economy for $T = \infty$ is one where only interest rates are given exogenously, and where therefore there are multiple equilibria. There is a discontinuity. This exercise suggests the intuition that under uncertainty in order to pin down a unique sequence for the price level it is necessary to pin down one money supply for each history, in every terminal node. In the particular case of the deterministic economy, money supply must be pinned down in any one period $t \geq 0$.

The initial price indeterminacy in the deterministic economy under an interest rate peg, is replaced by a terminal price indeterminacy, in every terminal state, under uncertainty. As we will see below this explosion in degrees of indeterminacy under uncertainty results from pegging the state-noncontingent nominal interest rates instead of the state-contingent nominal returns. If these were pegged instead, there would be a single degree of indeterminacy as in the deterministic case. A single equilibrium could be implemented by setting exogenously the money supply in the initial period.

2.2.1. Feedback rules

We have assumed that policy were sequences of numbers for the policy variables. However it is commonly assumed that policy is conducted with feedback rules, in particular, interest rate feedback rules. These interest rate rules have been proposed in the literature starting with McCallum (1981), as a means to guarantee local determinacy. In Carlstrom and Fuerst (2001) show that in monetary models with transaction technologies depending on assumptions on timing there is local determinacy when policy is conducted with interest rate feedback rules where the nominal interest rate depends on either contemporaneous or past inflation. It is straightforward to see that the use of interest rate rules that depend on current or past variables clearly preserves the same degrees of freedom in the determination of policy. When fiscal policy is endogenous, there are still multiple equilibria when money supply is not used as an additional instrument.

2.3. Policy with state-contingent interest rates

In this section we show that a policy that pegs the state-contingent nominal returns, as well as the money supply in the initial period, implements a unique equilibrium.

The equilibrium conditions are the resources constraint, (2.11), the intratemporal condition (2.12), the cash in advance constraints (2.3), the intertemporal conditions (2.6) and (2.7), as well as the budget constraints (2.10).

As before, from the resources constraints (2.11) and the intratemporal condition (2.12) we obtain the functions $C(R_t)$ and $N(R_t)$.

The system of equilibrium conditions can be summarized by the following dynamic equations:

$$Q_{t,t+1} = \beta \frac{u_C(C(R_{t+1}), L(R_{t+1}))}{u_C(C(R_t), L(R_t))} \frac{P_t}{P_{t+1}}, \ t \geq 0 \tag{2.14}$$

together with (2.8), the cash in advance condition, (2.3), and the budget constraint that determines, not uniquely, the endogenous taxes and debt levels, (2.10).

Clearly if policy is conducted by setting exogenously the state-contingent nominal interest rates, given the initial price level, the price levels are all determined. In order to have a single equilibrium it would still be necessary to set exogenously the initial money supply. The proposition follows:

Proposition 2.2. *If the state-contingent interest rates are set exogenously for every date and state, there is a unique equilibrium for the allocations and prices if the money supply is set exogenously in the initial period.*

Proof:

Let P_0 be given. Given the values for $\{Q_{t-1,t}, t \geq 1\}$, $\{R_t, t \geq 0\}$ are determined uniquely, and given P_{t-1}, P_t is obtained from the intertemporal condition

$$Q_{t-1,t} = \beta \frac{u_C(C(R_t), L(R_t))}{u_C(C(R_{t-1}), L(R_{t-1}))} \frac{P_{t-1}}{P_t}, \; t \geq 1 \tag{2.15}$$

The condition above only holds for $t \geq 1$. Cannot use the condition at $t = 0$, to determine P_0. M_0 pins down the initial price.■

In these economies there is a unique equilibrium if the policy is to peg the nominal returns on the state-contingent nominal assets and in addition money supply is set exogenously in the initial period. A timeless perspective (see Woodford, 2003) abstracts from this initial period by concentrating on the asymptotic behavior of the economy, as if the initial period had happened at an arbitrarily early date.[1] According to the timeless perspective all the government is required to do is to set exogenously the state-contingent interest rates.

2.3.1. State-contingent debt in zero net supply

Even if the government stands ready to supply and demand any quantity of state-contingent bonds at given state-contingent prices, these assets can be in zero net supply in every equilibrium. To see this notice that, when the supply of state-contingent assets is zero, $B_{t-1,t} = 0$, the budget constraints of the government are

[1] One way to interpret the timeless perspective, is that it is possible to use conditions $Q_{t,t+1} = \beta \frac{u_C(C(R_{t+1}), L(R_{t+1}))}{u_C(C(R_t), L(R_t))} \frac{P_t}{P_{t+1}}$ for $t = -1$, instead of only from $t = 0$ on, as well as the assumption that P_{-1} is exogenous. These are additional conditions that allow to determine the price levels at $t = 0$.

$$\sum_{s=0}^{\infty} E_t Q_{t,t+s+1} M_{t+s} \left(R_{t+s} - 1\right) = \mathbb{W}_t + \sum_{s=0}^{\infty} E_t Q_{t,t+s+1} P_{t+s} \left[G_{t+s} - T_{t+s}\right], \; t \geq 0 \tag{2.16}$$

where $\mathbb{W}_t = M_{t-1} + R_{t-1}B_{t-1} + P_{t-1}G_{t-1} - P_{t-1}T_{t-1}$. This must be satisfied for any allocation and prices, at any period and state. Even with $B_{t-1,t} = 0$, there are still multiple solutions of these equations for the endogenous nominal state-noncontingent debt and the lump sum taxes.

3. Price setting restrictions

In this section we show that the results derived above extend to an environment with prices set in advance. We modify the environment to consider price setting restrictions. There is a continuum of goods, indexed by $i \in [0,1]$. Each good i is produced by a different firm. The firms are monopolistic competitive and set prices in advance with different lags.

The households have preferences described by (2.5) where C_t is now the composite consumption

$$C_t = \left[\int_0^1 c_t(i)^{\frac{\theta-1}{\theta}} di\right]^{\frac{\theta}{\theta-1}}, \theta > 1.$$

Households have a demand function for each good given by

$$c_t\left(i\right) = \left(\frac{p_t\left(i\right)}{P_t}\right)^{-\theta} C_t.$$

where P_t is the price level,

$$P_t = \left[\int p_t(i)^{1-\theta} di\right]^{\frac{1}{1-\theta}}. \tag{3.1}$$

The households' intertemporal and intratemporal conditions are as before, (2.5), (2.6) and (2.7).

The government must finance an exogenous path of government purchases $\{G_t\}_{t=0}^{\infty}$, such that

$$G_t = \left[\int_0^1 g_t(i)^{\frac{\theta-1}{\theta}} di\right]^{\frac{\theta}{\theta-1}}, \theta > 0 \tag{3.2}$$

Given the prices on each good i in units of money, $P_t(i)$, the government minimizes expenditure on government purchases by deciding according to

$$\frac{g_t(i)}{G_t} = \left(\frac{p_t(i)}{P_t}\right)^{-\theta} \tag{3.3}$$

Aggregate resource constraints can be written as

$$(C_t + G_t) \int_0^1 \left(\frac{p_t\,(i)}{P_t}\right)^{-\theta} di = A_t N_t. \tag{3.4}$$

A fraction α_j firms set prices j periods in advance with $j = 0, ..., J-1$. Firms decide the price for period t with the information up to period $t-j$ to maximize:

$$E_{t-j}\left[Q_{t-j,t+1}\left(p_t(i)y_t(i) - W_t n_t(i)\right)\right]$$

subject to the production function

$$y_t(i) \leq A_t n_t(i)$$

and the demand function

$$y_t(i) = \left(\frac{p_t(i)}{P_t}\right)^{-\theta} Y_t \tag{3.5}$$

where $y_t(i) = c_t(i) + g_t(i)$

The optimal price is

$$p_t(i) = p_{t,j} = \frac{\theta}{(\theta - 1)} E_{t-j}\left[\eta_{t,j} \frac{W_t}{A_t}\right] \tag{3.6}$$

where

$$\eta_{t,j} = \frac{Q_{t-j,t+1} P_t^{\theta} Y_t}{E_{t-j}\left[Q_{t-j,t+1} P_t^{\theta} Y_t\right]}.$$

The price level at date t can be written as

$$P_t = \left[\sum_{j=0}^{J-1} \alpha_j \left(p_{t,j}\right)^{1-\theta}\right]^{\frac{1}{1-\theta}} \tag{3.7}$$

Compared with the equilibrium conditions under flexible prices, the set of equilibrium conditions when prices are set in advance includes more variables, the prices of the different firms, but it also includes more restrictions, the price setting restrictions. The number of additional variables and restrictions is the same, and the degrees of freedom are the same as under flexible prices. This argument works in this case, because we can write the new equations as functions of current and past variables. The degree of indeterminacy is the same as under flexible prices and therefore the statement in Proposition 2.3, that a peg of state-contingent interest rates delivers a unique equilibrium, still holds when prices are set in advance.

We can replace the nominal wage from the intratemporal condition (2.5) in the price setting restrictions (3.6). The equilibrium conditions can then be summarized as the intertemporal conditions (2.7) and (2.8), as well as

$$P_t = \left[\sum_{j=0}^{J-1} \alpha_j \left(p_{t,j} \right)^{1-\theta} \right]^{\frac{1}{1-\theta}}, \ t \geq 0$$

$$p_{t,j} = \frac{\theta}{(\theta - 1)} E_{t-j} \left[\eta_{t,j} \frac{u_L \left(C_t, L_t \right) P_t R_t}{u_C \left(C_t, L_t \right) A_t} \right]$$

where $\eta_{t,j} = \frac{\left(Q_{t-j,t+1} P_t^{\theta} \right) Y_t}{E_{t-j}\left[\left(Q_{t-j,t+1} P_t^{\theta} \right) Y_t \right]}$, and the resource constraints

$$\left(C_t + G_t \right) \left[\sum_{j=0}^{J-1} \alpha_j \left(\frac{p_{t,j}}{P_t} \right)^{-\theta} \right] = A_t \left(1 - L_t \right).$$

The proposition follows:

Proposition 3.1. *(Prices are set in advance) If the state-contingent interest rates are set exogenously for every date and state, and the money supply is set exogenously in the initial period, there is a unique equilibrium for the allocations and prices.*

Proof: Let $\{Q_{t,t+1}, \, t \geq 0\}$ be set exogenously. Then $\{R_t, \, t \geq 0\}$ are determined uniquely.

At any $t \geq J$, given P_{t-1}, C_{t-1} and L_{t-1} there are Φ_t intertemporal conditions, Φ_t resource constraints, Φ_t price level conditions, Φ_{t-j} price setting conditions, $j = 0, ..., J-1$. The variables are Φ_t consumptions C_t, Φ_t levels of leisure L_t, Φ_t price levels and Φ_{t-j} prices for the different firms, $j = 0, ..., J-1$.

For $t = 0$, there are Φ_0 price level conditions, Φ_0 resources constraints, Φ_0 price setting conditions. The variables are Φ_0 consumptions C_0, Φ_0 levels of leisure L_0,

Φ_0 price levels and Φ_0 prices for the flexible firms in period 0. The other prices are historical. Can use the cash in advance constraint with exogenous M_0 to determine all the variables in period 0.

For $t = 1$, given P_0, C_0 and L_0, there are Φ_1 price level conditions, Φ_1 resources constraints, $\Phi_1 + \Phi_0$ price setting conditions, Φ_0 intertemporal conditions to determine the same number of variables. The variables are Φ_1 consumptions C_1, Φ_1 levels of leisure L_1, Φ_1 price levels, Φ_1 prices for the flexible firms in period 1 and Φ_0 prices for the firms setting the price in period 0 for period 1. Similarly for any period $1 \leq t \leq J-1$.■

The result in this proposition also applies to other forms of price setting, but not to all. If prices were set in a staggered fashion so that the prices for period t are functions of future variables, the proof does not go through, as we show next.

Staggered prices In this section we show that the results obtained so far do not generalize to all forms of price setting, in particular we give an example with prices set as in Taylor (1983) where the policy in Proposition 2.3 of setting exogenously both state-contingent interest rates and the initial money supply does not guarantee uniqueness. However, this is also an economy where, for the deterministic case, setting the initial money supply and the nominal interest rate in every date does not implement a unique equilibrium. This is the example we provide.

A constant share of firms, $\frac{1}{J}$, set prices every J periods. The price level is

$$P_t = \left[\sum_{j=0}^{J-1} \frac{1}{J} \left(p_{t-j}\right)^{1-\theta}\right]^{\frac{1}{1-\theta}}$$

where p_{t-j} is the price chosen by the type of firm that can change the price at date $t-j$. The firms that choose the price at time t,[2] do it according to

$$\frac{p_t}{P_t} = \frac{\theta}{(\theta-1)} \frac{E_t \sum_{j=0}^{J-1} \beta^j \frac{U_c(t+j)}{R_{t+j}} \left(\frac{P_t}{P_{t+j}}\right)^{-\theta} C_{t+j} \frac{W_{t+j}}{P_{t+j} A_{t+j}}}{E_t \sum_{j=0}^{J-1} \beta^j \frac{U_c(t+j)}{R_{t+j}} \left(\frac{P_t}{P_{t+j}}\right)^{1-\theta} C_{t+j}}.$$

We consider for simplicity preferences that are separable and linear in leisure.

[2] See the Appendix for the derivation of the price setting condition.

Proposition 3.2. *Let preferences be separable and linear in leisure and firms be restricted to set prices as in Taylor (1983), for two periods, i.e. $J = 2$. Let the state-contingent interest rates and the initial money supply be set. Then there are multiple equilibria in this economy.*

Proof: Replacing the cash in advance in the pricing equation for the firm that chooses the price at t for two periods t and $t+1$, we get

$$p_t = \frac{\theta}{(\theta-1)} \frac{\frac{M_t}{A_t} + \beta E_t \left(\frac{P_t}{P_{t+1}}\right)^{1-\theta} \frac{M_{t+1}}{A_{t+1}}}{U_c(\frac{M_t}{P_t})\frac{M_t}{P_t R_t} + \beta E_t \left(\frac{P_t}{P_{t+1}}\right)^{1-\theta} U_c(\frac{M_{t+1}}{P_{t+1}})\frac{M_{t+1}}{P_{t+1}R_{t+1}}}, \; t \geq 0 \tag{3.8}$$

The equation for the price level is

$$P_t^{1-\theta} = \frac{1}{2}(p_t)^{1-\theta} + \frac{1}{2}(p_{t-1})^{1-\theta}, \; t \geq 0 \tag{3.9}$$

Another dynamic equation is the intertemporal condition

$$\frac{U_c\left(\frac{M_t}{P_t}\right)}{P_t} = \beta Q_{t,t+1} \frac{U_c\left(\frac{M_{t+1}}{P_{t+1}}\right)}{P_{t+1}}, \; t \geq 0 \tag{3.10}$$

For $t = 0$, there are Φ_0 equations (3.8), Φ_0 equations (3.9) and Φ_1 equations (3.10) to determine (Φ_0) p_0, (Φ_0) P_0, (Φ_1) M_1, (Φ_1) P_1. There are Φ_1 degrees of indeterminacy, say, the prices in period 1 are not determined. For $t = 1$, we add Φ_1 equations (3.8), Φ_1 equations (3.9) and Φ_2 equations (3.10) to determine (Φ_1) p_1, (Φ_1) P_1, (Φ_2) M_2, (Φ_2) P_2. There are now Φ_2 degrees of indeterminacy. The price levels in period 2 are not determined. Similarly for period 3, and any period thereafter. The equilibrium conditions do not have a single solution.■

4. Concluding Remarks

We have shown that a monetary policy that targets the state-contingent nominal returns, as opposed to the state-noncontingent nominal interest rates, is able to eliminate the indeterminacy associated with uncertainty. In economies with flexible prices and prices set in advance, that policy, for a given initial money supply, implements a unique equilibrium.

While in a finite horizon economy the argument that price setting restrictions add the same number of restrictions as unknowns is valid, that is not the case in the infinite horizon economy. As shown in this paper in a staggered price environment, as in Taylor (1983), the policy analyzed in this paper does not deliver a unique equilibrium. However that is also an economy where, in the deterministic version, a policy that sets the nominal interest rate in every date, as well as the initial money supply, is not sufficient to determine a unique equilibrium.

A policy that pegs the returns on state-contingent nominal assets may seem a difficult task for the monetary authority, even if less so when reminded of the result of Angeletos (2002) and Buera and Nicolini (2004) that state-contingent government debt can be replicated by debt of multiple maturities. However, it is also one that has the advantage of solving the multiplicity of equilibria generated by an interest rate rule. Furthermore, it is also a policy that can be used to implement a unique equilibrium at the, consistently optimal, Friedman rule of zero nominal interest rates

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4.1. Appendix: Taylor (1983) price setting

A constant share of firms, $\frac{1}{J}$, set prices every J periods. Firms choose p_t to maximize

$$E_t \sum_{j=0}^{J-1} \beta^j \frac{U_c(t+1+j)\Pi_{t+j}(p_t)}{P_{t+1+j}}$$

where

$$\Pi_{t+j}(p_t) = \left(p_t - \frac{W_{t+j}}{A_{t+j}}\right) y_{t+j}$$

The profits Π_{t+j} should be discounted with $\frac{U_c(t+1+j)}{P_{t+1+j}}$ since they can only be used in the following period. But, since

$$\begin{aligned} E_t \frac{U_c(t+j)\Pi_{t+j}(p_t)}{\beta R_{t+j} P_{t+j}} &= E_t \left[\Pi_{t+j}(p_t) E_{t+j} \frac{U_c(t+1+j)}{P_{t+1+j}}\right] \\ &= E_t \frac{U_c(t+1+j)\Pi_{t+j}(p_t)}{P_{t+1+j}}, \end{aligned}$$

$$c_{t+j} = \left(\frac{p_t}{P_{t+j}}\right)^{-\theta} C_{t+j},$$

and

$$y_{t+j} = c_{t+j},$$

the objective function can be written as

$$E_t \sum_{j=0}^{J-1} \beta^j \frac{U_c(t+j)}{R_{t+j} P_{t+j}} \left[\left(p_t - \frac{W_{t+j}}{A_{t+j}}\right) c_{t+j}\right]$$

or

$$E_t \sum_{j=0}^{J-1} \beta^j \frac{U_c(t+j)}{R_{t+j}} \left[\left(\frac{p_t}{P_{t+j}}\right)^{1-\theta} - \frac{W_{t+j}}{A_{t+j} P_{t+j}} \left(\frac{p_t}{P_{t+j}}\right)^{-\theta}\right] C_{t+j}$$

A first order condition is

$$E_t \sum_{j=0}^{J-1} \beta^j \frac{U_c(t+j)}{R_{t+j}} \left[\frac{(1-\theta)}{p_t} \left(\frac{p_t}{P_{t+j}}\right)^{1-\theta} + \frac{\theta \frac{W_{t+j}}{A_{t+j} P_{t+j}}}{p_t} \left(\frac{p_t}{P_{t+j}}\right)^{-\theta}\right] C_{t+j} = 0$$

that can be rearranged as

$$\frac{p_t}{P_t} = \frac{\theta}{(\theta-1)} \frac{E_t \sum_{j=0}^{J-1} \beta^j \frac{U_c(t+j)}{R_{t+j}} \left(\frac{P_t}{P_{t+j}}\right)^{-\theta} \frac{W_{t+j}}{A_{t+j} P_{t+j}} C_{t+j}}{E_t \sum_{j=0}^{J-1} \beta^j \frac{U_c(t+j)}{R_{t+j}} \left(\frac{P_t}{P_{t+j}}\right)^{1-\theta} C_{t+j}}.$$