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On the Fisher-Konieczny Index of Price Changes Synchronization

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Abstract

This note provides a structural interpretation for the index of price changes synchronization proposed by Fisher and Konieczny (2000, *Economics Letters*, 68, 271-277) and shows that it can be used to test the hypothesis of uniform staggering.

Key words: Homogeneity tests, Price changes synchronization, Uniform staggering

JEL classification codes: C12, D40, L16.

1. INTRODUCTION

Fisher and Konieczny (2000) studied the price setting behaviour of Canadian newspapers using an ad hoc measure of price synchronization, and tested the hypothesis of uniform price staggering using a χ^2 goodness-of-fit test.[1] In this note we provide a structural interpretation for the index of price changes synchronization proposed by Fisher and Konieczny (2000), and show that it can be used to test the hypothesis of uniform staggering.

Let us assume that the researcher is interested in studying the price setting behaviour of N firms selling the same product and that a data set with the N prices in $T+1$ equally

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[1]Here we focus on price synchronization and staggering across firms. The situation is similar if interest is focused on the intra-firm price setting behaviour.

spaced periods is available.[2] Let p_t denote the proportion of firms that changed the price of the product between periods $t-1$ and t. The synchronization index proposed by Fisher and Konieczny (2000, p. 274, Table 2) can be computed as

$$FK = \sqrt{\frac{1}{T}\frac{\sum_{t=1}^{T}(p_t - \bar{p})^2}{\bar{p}(1-\bar{p})}} = \frac{\sqrt{s^2_{p_t}}}{\sqrt{\bar{p}(1-\bar{p})}},$$

where $\bar{p} = \frac{1}{T}\sum_{t=1}^{T} p_t$ and $s^2_{p_t}$ are the sample mean and variance of p_t, respectively.

The rational behind this index is as follows. According to Fisher and Konieczny (2000), in case of perfect synchronization, either all firms change their prices, or no price is changed. In this case p_t is a binary variable and $s^2_{p_t} = \bar{p}(1-\bar{p})$, leading to $FK = 1$. On the other hand, when $p_t = \bar{p}$, $\forall t$, the index will be equal to 0. This corresponds to the case of uniform price staggering in which a proportion $\bar{p}$ of all firms change their prices every period. Therefore, given $\bar{p}$, FK measures the proximity of $s^2_{p_t}$ to its upper bound.

Values of FK between 0 and 1 are interpreted as measuring the degree of price synchronization. However, from the work of Fisher and Konieczny (2000) it is not possible to give a structural interpretation to values of FK different from 0 or 1, which makes the index difficult to interpret. For example, apparently, there is no reason to prefer FK to its square, which is a ratio of variances rather than standard deviations.

2. AN INTERPRETATION OF FK

Consider a stylized economy were there are two kinds of firms. Firms of type 1 are characterized by uniform staggering, with a fixed proportion of firms adjusting their prices every period (as in Taylor, 1980, p. 4). Let θ_1 be the proportion of firms of type 1 that adjust their prices in a given period and define α as the proportion of firms of type 1 in the population. Firms of type 2 have perfect price synchronization as defined by Fisher and Konieczny (2000). Suppose that the probability that firms of type 2

[2] Throughout we assume that the same number of firms is observed in all periods. In case some observations are missing in some periods, only the firms with complete records are used to avoid the potential problems caused by non-random missing values.

change their prices in a given period is equal to θ_2. In this case, the proportion of price changes in a given period is

$$p_t = \alpha\theta_1 + s_t (1 - \alpha)$$

where s_t is an indicator which equals 1 if firms of type 2 change their prices in period t, being zero otherwise. Therefore, s_t is a Bernoulli random variable with parameter θ_2. It is easy to see that

$$\begin{aligned} E(p_t) &= \alpha\theta_1 + \theta_2 (1 - \alpha); \\ V(p_t) &= \theta_2 (1 - \theta_2) (1 - \alpha)^2 . \end{aligned}$$

Under suitable conditions, θ_1, θ_2 and α can be estimated by the method of moments. If it is further assumed that $\theta_1 = \theta_2 = \theta$, estimation becomes much simpler. In this case, $E(p_t|\theta_1 = \theta_2) = \theta$, $V(p_t|\theta_1 = \theta_2) = \theta(1-\theta)(1-\alpha)^2$. Using the notation introduced before, $\bar{p}$ is an estimator for θ and α can be estimated from the relation

$$\frac{V(p_t|\theta_1 = \theta_2)}{\theta(1-\theta)} = \frac{\theta(1-\theta)(1-\alpha)^2}{\theta(1-\theta)} = (1-\alpha)^2 .$$

The empirical counterpart of this expression is

$$\frac{s_{p_t}^2}{\bar{p}(1-\bar{p})} = FK^2.$$

Therefore, the FK index can be interpreted as a method of moments estimator of $(1-\alpha)$, the proportion of firms in the economy that have synchronized prices, in a world where there are two types of firms with opposite price setting behaviours and $\theta_1 = \theta_2$. Of course, this estimator is only valid under the restrictive assumptions made to characterize the stylized economy considered here. Consequently, in general, FK will not estimate a structural parameter. Nevertheless, in empirical applications, the FK index may always be interpreted as the proportion of type 2 firms that would be needed in our stylized economy to generate the degree of price changes synchronization found in the real world.

3. TESTING FOR UNIFORM PRICE STAGGERING

Under the assumptions of the previous section, uniform staggering occurs only when $s_{p_t}^2 = FK = 0$. This is because type 1 firms are assumed to have a non-random behaviour. If, on the other hand, it is assumed that price changes of type 1 firms are uniformly staggered as in Calvo (1983), FK cannot be used to estimate α. However, in this context, the FK index can be used to test the uniform price staggering hypothesis $H_0 : E(p_t) = \theta, \forall t \iff \alpha = 0$.

Fisher and Konieczny (2000) used a χ^2 goodness-of-fit test to check the validity of the hypothesis that the $NT\bar{p}$ observed price changes are uniformly distributed over the T time periods. Although the authors do not present the test statistic used, from their description is possible to conclude it was computed as

$$Q_{FK} = \sum_{t=1}^{T} \frac{(Np_t - N\bar{p})^2}{N\bar{p}},$$

which the authors treat as having a $\chi^2_{(T-1)}$ distribution. However, the validity of the test based on Q_{FK} hinges on the independence between the observations, but in the present setup the observed price changes for each firm are not independent. Indeed, in this problem, the maximum number of price changes that can be observed in a given period is N, whereas under independence this upper bound would be $NT\bar{p}$. Therefore, this form of dependence reduces the variance of the data, making the denominator of Q_{FK} too large. Hence, a test based on Q_{FK} and on the assumed reference distribution will under-reject the null, leading to a power loss.

In each period, the N firms decide whether or not they change their prices. The hypothesis of uniform staggering as defined by Calvo (1983) implies that the decisions are independent and that $E(p_t) = \theta, \forall t$. Therefore, under the null, the total number of price changes in each period is a random variable with binomial distribution with parameters N and θ. This hypothesis can be tested using a χ^2 homogeneity test for $H_0 : E(p_t) = \theta, \forall t$. The appropriate test statistic has the form

$$Q = \sum_{t=1}^{T} \left\{ \frac{(Np_t - N\bar{p})^2}{N\bar{p}} + \frac{(N(1-p_t) - N(1-\bar{p}))^2}{N(1-\bar{p})} \right\},$$

and under the null $Q \dot{\sim} \chi^2_{(T-1)}$.[3] It is easy to see that $Q = (NT)\, FK^2$,[4] and therefore the FK index can be used to test the hypothesis of uniform staggering as defined by Calvo (1983). Notice that when the null is not rejected that does not imply the acceptance of Calvo's model since uniform staggering is just one of the model's characteristics.

4. AN EXAMPLE

In order to compute the Portuguese Consumer Price Index, Instituto Nacional de Estatística periodically registers the prices of a large number of products in a representative set of outlets. We illustrate the results presented above using the monthly data on the price of rice, collected by Instituto Nacional de Estatística from January 1998 to January 2001 across the 54 stores with complete records. Since we have data on the prices for 37 months, it is possible to compute p_t, the proportion of price changes, for 36 pairs of consecutive observations. Table 1 displays the main results obtained with these data.

Table 1:
Statistics for the percentage of price changes

Mean ($\bar{p}$)	Std. Dev. (s_{p_t})	FK	Q
0.31790	0.08253	0.17723	61.062

On average, about 32% of the prices change every month. In the context of the stylized economy described in section 2, these results correspond to a situation in which about 18% of the stores synchronize their price changes.[5] The p-value associated with

[3] This statistic can be easily generalized to test whether the proportions of price increases and price decreases are constant over the sampling period. However, in this case the test statistic does not have direct relation with the FK index.

[4] Indeed,

$$Q = \sum_{t=1}^{T} \left\{ \frac{(Np_t - N\bar{p})^2}{N\bar{p}\,(1-\bar{p})} \right\} = N \frac{\sum_{t=1}^{T} (p_t - \bar{p})^2}{\bar{p}\,(1-\bar{p})} = \frac{(NT)\, s^2_{p_t}}{\bar{p}\,(1-\bar{p})} = (NT)\, FK^2.$$

Notice that $Q_{FK} = (1-\bar{p})\, Q$ and since both tests use the same critical value, it is clear that Q leads to tests with higher power.

[5] Without imposing the restriction $\theta_1 = \theta_2$, the estimates obtained with the first three moments are as follows (standard errors in parenthesis): $\tilde{\theta}_1 = 0.305$ (0.016), $\tilde{\theta}_2 = 0.382$ (0.063), $\tilde{\alpha} = 0.830$ (0.019).

the observed value of Q is 0.00413 and, therefore, in this example the hypothesis of uniform price staggering is rejected at any standard significance level. (In this case $Q_{FK} = 41.65048$, to which corresponds a p-value of 0.20381 that would not lead to the rejection of the null.) Taking the usual 5% significance level as a reference, the null of uniform staggering would be rejected for values of FK larger than 0.02635.

5. CONCLUDING REMARKS

The FK index takes uniform staggering as the opposite of perfect synchronization. However, these two situations can coexist: suppose that in a given market all prices are changed every single period. In this case there is uniform staggering with $\bar{p} = 1$ and it can be argued that there is perfect synchronization in the sense that all firms make the same decision in every period. Not surprisingly, in this situation the FK index is indeterminate and Q cannot be computed. Therefore, despite the attractive characteristics of the FK index highlighted here, the quest for appropriate measures of price staggering and synchronization is not over.

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