

**BANCO DE PORTUGAL**  
**Economic Research Department**

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An Application of the Smooth Transition  
Autoregressive Model to Characterize  
GDP Dynamics for the Euro-area and  
Portugal**

*Francisco Craveiro Dias*

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Please address correspondence to Francisco Craveiro Dias, Economic Research Department, Banco de Portugal, Av. Almirante Reis nº 71, 1150-165 Lisboa, Portugal, Tel.#351-213128327; Fax#351-213107804; email:fadias@bportugal.pt;

# Nonlinearities over the Business Cycle: An application of the Smooth Transition Autoregressive Model to characterize GDP dynamics for the Euro-area and Portugal.

Francisco Craveiro Dias\*

Banco de Portugal

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## Abstract

Many business cycle indicators present asymmetric features that have long been recognized in economics. Basically the contraction periods in an economy are more violent but also more short-lived than the expansion periods, where the dynamics of GDP/GNP growth present asymmetric cyclical developments with upswings which last longer than downturns. Nonlinear models are therefore required to capture the features of the data generating mechanisms of such macroeconomic business cycle series, since linear models are incapable of generating such behaviour. Relying on the smooth transition autoregressive (STAR) model, one of the many non-linear models developed in the nineties, this paper presents empirical evidence in favour of the proposition that the dynamic behaviour of GDP changes over the business cycle. Using quarterly growth rates for seasonally unadjusted GDP data, both, for the Euro-area and Portugal we uncover evidence in favour of asymmetric behaviour for these variables. The nonlinear features are empirically far more evident for the case of Portugal and less sticking so for the GDP for the Euro-area. These nonlinear features show up not only in the form of distinct impulse response functions calculated from different starting points for the shocks, but are also apparent in the completely different dynamics displayed by the two extreme regimes that characterize the "recession" periods and the "high" growing phases of the business cycle.

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\*Correspondence: Francisco Dias, Research Department (DEE), Banco de Portugal, Av. Almirante de Reis, n<sup>o</sup> 71, 1150-012 Lisboa, Portugal; Tel: 351-21328327; E-mail: fadias@bportugal.pt

# 1 Introduction.

Many business cycle indicators present asymmetric features that have long been recognized in economics. Mitchell (1927), for example, discussed this issue and presented statistical evidence in favour of and against the asymmetry of business cycle. Keynes (1936) pointed out that the contraction periods in an economy are more violent but also more short-lived than the expansion periods, so that the dynamics of GDP/GNP growth follows an asymmetric cyclical process with upswings which last longer than downturns. Along these lines and to describe the non-linear features of these macroeconomic variables, some authors have introduced new concepts associated with this issue, such as steepness and deepness, Sichel (1993). The former adjective refers to dynamics in which contractions occur much faster than expansions or the other way round. The latter intends to describe situations in which troughs are situated much below trend than peaks are above or vice-versa. Others have confirmed statistically the asymmetric properties of several business cycles indicators, Kontolemis (1997) or alternatively have built tests for empirical purposes for symmetry in recent years, Andreano and Savio (2002).

As Kontolemis (1997) pointed out, possible economic explanations for this asymmetric dynamics can be found, for example, in the industrial production literature: exit from industry is less costly than entry and as a result overall activity could fall rapidly and expansion could come at a slower pace; furthermore as reduction of production by firms below full capacity, when orders decline, is easier compared with the difficulty of increasing output when capacity constraint are in force, will also generate asymmetric cycles.

In spite of this wide recognition that the relations between economic time series are often of a nonlinear nature, economists have generally relied essentially on linear model to describe those relationships. This is possibly due to the more sound based theoretical developments referring to linear econometric theory as well as probably due to the existence of easy to use standard computational packages available for that purpose. Nonlinear models are therefore needed to capture the features of the data generating mechanisms of inherently asymmetric realizations of some of the macroeconomic business cycle series, since linear models in general and autoregressive or ARMA model are incapable of generating the asymmetric features observed in many business cycle variables.

In the last decade or so statisticians, time-series analysts and econometricians have produced

a great amount of literature suggesting possible nonlinear models and techniques to circumvent this shortcoming. Tong (1990), Granger and Teräsvirta (1993) and Franses and van Dijk (2000) present surveys on some of these nonlinear time-series models. All in all, these models are quite general and highly flexible and so they can approximate quite satisfactorily a wide variety of actual nonlinearities encountered in observed time series. One major difficulty that appears in this context is that the number of alternative nonlinear models is enormous and economic theory is not specific about the dynamics of the relationship among variables.

Among the nonlinear models that have flourished in the applied econometric literature, the ones that rely on state-dependent or regime-switching behaviour have received a great deal of theoretical and empirical attention. One of these regime-switching models that has deserved particular relevance, owing to the fact that it represents a particularly suitable device to capture nonlinear characteristics of business cycle indicators of the type mentioned above, is the smooth transition autoregressive (STAR) model. This modelling framework was introduced in the nonlinear time series literature by Chan and Tong (1986) and later popularised by Granger and Teräsvirta (1993) and Teräsvirta (1994). The general specification of this model nests the linear autoregressive model as a special case, and the introduction of additional parameters in that model provides room for extra flexibility to pick up nonlinear features present in some time series, which may prove helpful for both modelling and forecasting purposes. It is also more general in comparison to other nonlinear model as it nests the threshold autoregressive (TAR) models - initially developed by Tong (1978) and extensively discussed later in Tong (1990) - and in particular the self-exciting TAR (SETAR) model as special cases of the STAR model. Additionally it provides the elegant computational feature of having a well established and tested empirical modelling cycle developed by Teräsvirta (1994) very similar in spirit to one developed for the linear time series models by Box and Jenkins (1970).

The objective of this study is to collect evidence in favour of such nonlinear regularities associated with variables that present asymmetric characteristics over the business cycle. We are going to adopt explicitly, the STAR-modelling device from the very beginning as the alternative nonlinear model to uncover possible nonlinear/asymmetric behaviour in case the linearity hypothesis is rejected. In this paper we address the issue of nonlinear modelling of quarterly GDP for the Euro-area and Portugal.

The plan of the paper is as follows. In section 2 the STAR model - the nonlinear modelling tool we

are going to use to capture the asymmetric dynamics - is presented, as well as the corresponding interpretation of the relevant model parameters. The overall modelling cycle of this nonlinear methodology follows in section 3. The next section presents a brief overview of the issues of calculations of forecasts and impulse response functions in the nonlinear modelling context. The empirical applications, along with the main dynamic features of the estimated models follow in section 5. Conclusions are finally set foreword in the last section.

## 2 Representation of the basic STAR model.

The nonlinear modelling framework that we are going to use belongs to the class of regime-switching models and, as mentioned above, is known as the smooth transition autoregressive (STAR) model. The idea of smooth transition regimes dates back to Bacon and Watts (1971). Granger and Teräsvirta (1993) and Teräsvirta (1994, 1998) amongst others have provided important theoretical and empirical contributions in this area. A recent appealing survey on this specific modelling method can be found in van Dijk, Teräsvirta and Franses (2002)

Putting it simply, this class of models assumes that a variable that follows a stationary and ergodic process moves smoothly between two extreme regimes characterized possibly by completely different specification and dynamics, instead of switching abruptly from one regime to the other as is assumed in the threshold autoregressive (TAR) models developed by Tong in 1978. These extreme regimes may represent, for example, the two different phases of the business cycle mentioned in the introduction – the “expansion and contraction” phases. Possible rationale for this smooth movement between the two extreme regimes can be justified on the following grounds: as the upward or downward adjustment process for each and every economic units does not take place simultaneously, at each moment in time there will be some productive units that are adjusting to the new environment while others continue following their previous dynamics and therefore the overall activity variable presents dynamics that shows up as a smooth changing process between regime and not as an abrupt change from one to the other.

The simplest STAR model for a univariate time series  $y_t$  can be expressed as:

$$y_t = \left( \pi_{10} + \pi'_1 w_t \right) + \left( \pi_{20} + \pi'_2 w_t \right) F(y_{t-d}; \gamma, c) + u_t \quad (1)$$

where  $u_t \cap \text{nid}(0, \sigma_u^2)$ ,  $\pi'_j = (\pi_{j1}, \dots, \pi_{jp})$ ,  $j = 1, 2$ ,  $w_t = (y_{t-1}, \dots, y_{t-p})'$ , vector containing the

first  $p$  lags of  $y_t$  and  $F$ , called the transition function, is by convention a continuous, bounded function between zero and one and at least twice differentiable everywhere in the sample space. The previous equation also can be re-written as:

$$y_t = \left(\pi_{10} + \pi_1' w_t\right) (1 - F(y_{t-d}; \gamma, c)) + \left(\theta_{10} + \theta_1' w_t\right) F(y_{t-d}; \gamma, c) + u_t \quad (2)$$

where  $\theta_{10} = (\pi_{10} + \pi_{20})$  and  $\theta_1' = (\pi_1' + \pi_2')$ .

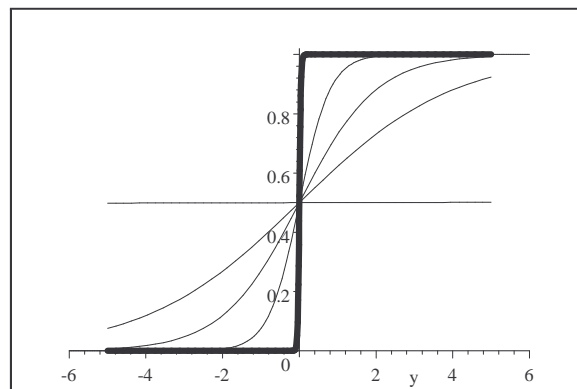
This expression shows that the STAR model can be seen to be a regime-switching model that accommodates two distinct extreme regimes, each associated with the two extreme values of the transition function  $F=0$  and  $F=1$ . The transition between these two extreme regimes is allowed to be smooth and is governed by the transition variable -  $y_{t-d}$ , some lagged value of the variable under analysis. Alternatively, this model can also be read as to accommodate a full spectrum of continuum regimes each associated with a different value of the function  $F(y_{t-d}; \gamma, c)$  in the interval 0 and 1 and which moves more or less smoothly between the two outer regimes characterized by the extreme value of  $F$ . The regime that characterizes the dynamic at each moment in time is completely linear and is dependent on the lagged value of the transition variable -  $y_{t-d}$ .

Basically two different transition functions have received a great deal of emphasizes in the empirical applications although any continuous distribution function could also be used in their place, as they fulfil the conditions mentioned above. One is the logistic function of order one:

$$F^L(y_{t-d}; \gamma, c) = (1 + \exp[-\gamma(y_{t-d} - c)])^{-1}, \gamma > 0 \quad (3)$$

and the resultant model (1) coupled with this transition function has been coined as the logistic STAR model of order one (LSTAR1)

Examples of this logistic function for a fixed parameter  $c = 0$  and for the following set of alternative values of  $\gamma = 50, 2.5, 1.0, 0.5, 0.001$ , is provided in the following chart.



The parameter  $c$  identifies the "midpoint" between the two extreme regimes as  $F^L(c; \gamma, c) = 0.5$  and can be interpreted as the threshold between the two extremes. The parameter  $\gamma$ , known as the smoothness parameter, determines the slope of the logistic function and therefore governs the speed with which the transition takes place between regimes. This function gets steeper as  $\gamma$  increases, and therefore the almost flat line corresponds to the logistic function with  $\gamma = 0.001$  and the bold line to  $\gamma = 50$ . As  $\gamma$  becomes very large the logistic function approaches the indicator function that takes up basically two values and changes abruptly from 0 to 1 or vice versa as  $y_{t-d}$  skips over the threshold value  $c$ . Therefore we can see that the LSTAR1 model nests a two-regime threshold autoregressive (TAR) model as a special case. On the other hand, as  $\gamma \rightarrow 0$  the logistic functions approaches a constant and the LSTAR1 model reduces to the linear AR(p) model when  $\gamma = 0$ .  $y_{t-d}$  is called the transition variable, as at each moment in time, it is the value of this variable that triggers the movement inter-regimes.

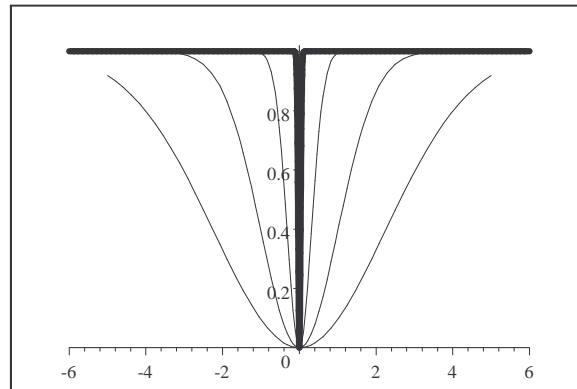
As this logistic is monotonic, it is extremely convenient and recommendable in modelling processes involving variables of the type of business cycle indicators described above. In fact, due to the properties of this function the LSTAR1 model allows one to identify two different regimes: one associated with small values of the transition variable -  $y_{t-d}$ , relative to  $c$  - and the other with large values, thus providing room to identify one regime with the expansion phases of the business cycle and the other with the contraction phases. The processes that describe the two extreme regimes may present very different dynamics and the transition function provides room for smooth changes between the two. For fixed values of the transition function the STAR model is linear, and therefore may, for certain values of  $y_{t-d}$  present non-stationary behaviour although presenting an overall globally stable property.

The other transition function popularised in the literature is the exponential function.

$$F^E(y_{t-d}; \gamma, c) = (1 - \exp[-\gamma(y_{t-d} - c)^2]), \gamma > 0 \quad (4)$$

When equation (1) is combined with this particular transition function the model is called the exponential STAR (ESTAR) model.

Graphs for the exponential function again for  $c = 0$  and for the following alternative set of values of  $\gamma = 500, 5, 0.5, 0.1$ , follows.



This function is characterized as having a bowl shape where the bowl gets wider as  $\gamma$  decreases. Therefore the outer line correspond to  $\gamma = 0.001$  and the bold line to the exponential function  $F^E(y_{t-d}; 500, 0)$ .

It is also symmetric and satisfies the following properties:  $F^E(y_{t-d}; \gamma, c) \rightarrow 1$  both as  $y_{t-d} \rightarrow \pm\infty$  and  $F^E(c; \gamma, c) = 0$ . On the other hand, the exponential function approaches a constant, and thus the ESTAR model collapses to a linear AR(p) model, both when  $\gamma \rightarrow 0$  or  $\gamma \rightarrow \infty$ .

Owing to the features of this function, it is highly suitable for empirical analysis in situations where the dynamic of the variable under consideration depends on its value as in relation to a reference point ( $c$ ) - the threshold. In these situations two different regimes are also unfolded by this particular transition equation. But now the extreme regimes are dependent on the value attained by the transition variable being close or far away from the threshold. This model has been used in empirical applications to analysis the dynamics of real (effective) exchange rate developments. This is motivated by the argument that the behaviour of the real exchange rate depends differently only on the size of the deviation from the purchasing power parity - the threshold, independently of the sign.

This nonlinear modelling instrument has been extended in several different directions: other weakly stationary variables can be included in the set of regressores besides the autoregressive variables. For the transition variable besides some lagged value of the left-hand side variables other weakly stationary variables, or linear combinations of various other variables or of lagged variables have also been considered in the transition function. Multiple regime STAR models have also been unfolded as well as time varying parameter STAR models. On the other hand, the STAR models



have been developed in a multivariate/vector framework recently although empirical applications in this context are still in its infancy. For a very recent and up-to-date survey on these and other issues referring to this modelling method see Van Dijk, Teräsvirta and Franses (2002).

### **3 The STAR modelling cycle.**

Teräsvirta (1994) has developed an appealing and elegant data-based modelling cycle for the STAR model. This cycle, designed very much along the lines of the Box and Jenkins (1970) time-series modelling methodology, is composed of the following sequence of steps:

- 1 - Specification of a linear autoregressive model of order  $p$  for the variable under consideration using an appropriate model selection criterion. This model is to be used for further analysis and to serve as a benchmark against which the final nonlinear model is to be compared with.
- 2 - Test the null of linearity against the well specified nonlinear alternative STAR model. If linearity is rejected determine the transition variable most suitable from the data and simultaneously the transition function that is most appropriate for the case.
- 3 - Estimate the parameters of the chosen STAR model.
- 4 - Test for the final estimated nonlinear specification using appropriate adequacy tests to diagnose for possible inadequate model using data-based misspecification techniques.
- 5 - Adjust the model and reiterate if necessary.
- 6 - Use the model for descriptive or forecasting purposes.

Next we are going to give a simply and brief presentation of some of these modelling steps concentrating essentially on the more innovative and non-standard elements of the procedure. For a more detailed and technical description see Van Dijk, Teräsvirta and Franses (2002) or the references therein mentioned. Step one is more or less obvious, where one can use either AIC or BIC or some other model selection criterion to adjust the linear model, assuring at the same time that the residual are well behaved, else omitted autocorrelation may, at times, cause rejection of the linearity hypothesis.

In step two we proceed to test linearity against the STAR model alternative to see whether this model adequately characterizes the dynamics of the variable under consideration. The reason for this concern is that it could be possible to estimate "successfully" a non-linear specification,

possibly due the existence of some outliers in the series, even though the true data generating process of the variable is almost symmetric. Testing from the very beginning for linearity aims at avoiding this kind of unnecessary complications in the model building process.

One of the ways in which the null hypothesis of linearity can be formulated, although others are equally possible, is by setting the null as  $H_0 : \gamma = 0$ , against  $H_1 : \gamma > 0$ . In this specific modelling environment one can see that under this null there are parameters that are not identified  $(\pi_{20}, \pi_2', c)$ , called nuisance parameters in the literature and that are only identified under the alternative hypothesis. In these cases the classical statistical theory is no longer directly applicable, and Davies (1977,1987) was the first to address and solve this issue. Following his approach, a Lagrange multiplier-type test was derived by Luukkonen, Saikkonen and Teräsvirta (1988) and later by Granger and Teräsvirta (1993) in the STAR modelling context. It is based on the following artificial regression which is obtained by substituting the transition function in (1) by a third order Taylor approximation around  $\gamma = 0$ . Proceeding so and after some reparametrization we get:

$$y_t = \beta_0 + \beta_1' w_t + \beta_2'(w_t y_{t-d}) + \beta_3'(w_t y_{t-d}^2) + \beta_4'(w_t y_{t-d}^3) + e_t \quad (5)$$

where  $\beta_i' = (\beta_{i1}, \beta_{i2}, \dots, \beta_{ip})$  for  $i = 1, 2, 3$  and  $4$ , and  $e_t = u_t + (\pi_{20} + \pi_2' w_t) R_3(y_{t-d}; \gamma, c)$  and  $R_3(y_{t-d}; \gamma, c)$  is the remainder of the Taylor's expansion.

Here the relationships between the original and the new set of parameters in equation 5 translates the initial null hypothesis  $H_0 : \gamma = 0$ , into the corresponding null  $H_0 : \beta_2 = \beta_3 = \beta_4 = 0$ . Under this new equivalent null as  $e_t = u_t$  when  $\gamma = 0$ , the standard Lagrange Multiplier principle can be applied to conduct this test delivering a test statistics that follows an asymptotic  $\chi_{3p}^2$  distribution although the use of the equivalent F distribution version is recommended in small samples to improve the size and power of the test. Simultaneously, one can determine the transition variable -  $y_{t-d}$  - performing the same test for a range of alternative values  $1 \leq d \leq D$ , considered as appropriate and choosing the value for  $d$  for which the above mentioned test rejects the null hypothesis most strongly. The rationale for this selection procedure is based on the fact that the test provides maximum power if  $d$  is chosen correctly, whereas incorrect choices of the transition variable weakens the power of the test, Teräsvirta (1994).

After having determined  $d$ , one has to choose the transition function that determines the LSTAR or ESTAR model as the most adequate for the variable under investigation. This involves a sequence of hypothesis testing undertaken within equation (4). The testing sequence proceeds as follows

(Teräsvirta 1994):

$$H_{04} : \beta_4 = 0 \tag{6}$$

$$H_{03} : \beta_3 = 0 \mid \beta_4 = 0 \tag{7}$$

$$H_{02} : \beta_2 = 0 \mid \beta_3 = \beta_4 = 0 \tag{8}$$

and is based on the relationships between the original set of parameters of the model and the third order linear approximation used in the testing procedure. In our case, however, as where we are going to model business cycle indicators we are going to rely on the logistic transition function for reasons presented earlier, except when the exponential alternative turns out to be very highly evident. All these tests are known, as Lagrange multiplier-type tests as they do not oblige the estimation of the nonlinear alternative model during the testing process.

Once the model and the transition variables have been identified, the estimation sequence of the parameters within the model, in the next stage, follows the general to specific methodology using the AR model obtained in the first step as the starting point. We sequentially drop all the non-significant lags from both the linear and nonlinear part of the equation and simultaneously impose symmetric parameter restrictions on the two extreme regime models when this is empirically sustained by the data. The estimation is performed using standard nonlinear least squares, which under the assumption that the errors are normally distributed is equivalent to conditional maximum likelihood. Under fairly general regularity conditions this procedure guarantees consistent and asymptotically normal estimators. One of the key assumptions in this context is that  $y_t$  follows a weakly stationary and geometrically ergodic process.

Finally, after having estimated the STAR model, we conduct a battery of tests to inquire for its adequacy using the data-based misspecification analysis proposed by Eitrheim and Teräsvirta (1996). When necessary the final model is adjusted in accordance with the information delivered by the test results and the whole cycle is reiterated. The set of tests seek for the following misspecification aspects in the estimated structure: test for no autocorrelation in the residuals, test for no remaining nonlinearities and test of parameter constancy.

## 4 Forecasts and impulse response functions in nonlinear models.

Calculating forecasts and impulse response function with nonlinear models has to be undertaken with a greater deal of care due to the intrinsic nonlinearities of the models. These issues gain a new dimension because the statistical expectational operator used to obtain these results satisfies linear properties and is therefore not directly applicable as in the linear context.

Lundberg and Teräsvirta (2002) provide a detailed review of the forecasting procedure in the context of the STAR modelling framework, although the general idea can also be found elsewhere and is extendable to other nonlinear models. The following brief presentation of this issue is gathered from their article. For example the LSTAR1 nonlinear model can be rewritten as:

$$y_t = \left( \pi_{10} + \pi'_1 w_t \right) + \left( \pi_{20} + \pi'_2 w_t \right) (1 + \exp[-\gamma(y_{t-d} - c)])^{-1} + u_t = g(w_t; \Theta) + u_t \quad (9)$$

From here we can easily derive the one period ahead unbiased forecast which follows the usual procedure as in any linear model. Since the random residual is additive and linear in this case we get:

$$E(y_{t+1}|\psi_t) = y_{t+1}^f = g(w_{t+1}; \Theta) \quad (10)$$

where  $\psi_t$  represents the set of historic information available at the time of forecast, which contains all the necessary information for  $w_{t+1}$ . However forecasting two or more periods ahead is not as straightforward as in the one period case. For instance for the two period ahead forecast we have:

$$E(y_{t+2}|\psi_t) = y_{t+2}^f = E[g(w_{t+2}^f; \Theta) + u_{t+2}|\psi_t] = E[g(w_{t+2}^f; \Theta)|\psi_t] \quad (11)$$

where  $w_{t+2}^f = (1, y_{t+1}^f + u_{t+1}, y_t, y_{t-1}, \dots, y_{t-p+2})$ . This expected value cannot be unfolded as in the linear case where  $u_{t+1}$  is replaced simply by it's expected value since  $g$  is generally a nonlinear function of  $w_{t+2}^f$ . Therefore the value for this expression has to be calculated by numerical integration using the following equivalent equation:

$$y_{t+2}^f = \int_{-\infty}^{+\infty} g(w_{t+2}^f; \Theta) d\Phi(u_{t+1}) \quad (12)$$

where  $\Phi(u_{t+1})$  is the cumulative distribution function of the residual  $u_{t+1}$ . Forecasts for longer time horizons will involve multiple numerical integrations that may get computationally demanding. A simpler equivalent computational device has been proposed by various authors to ease these calculations, which involves using simulation technique to obtain the forecast for the different horizons.

In this case either Monte-Carlo experiments are performed by taking independent draws from the adequate random residuals distribution or alternatively by using the bootstrapping method that obliges that the draws be taken with replacement from the set of estimated residuals. This second procedure is advisable if we do not want to assume a specific distribution for the random error. In either case the set of random draws provides elements to generate a full set of random forecasts for each horizon, which are thereafter averaged to get the required point forecasts. For the case of the two period ahead forecast we get:

$$y_{t+2|t}^f = \frac{1}{N} \sum_{n=1}^N g(w_{t+2,n}^f; \Theta) \quad (13)$$

where each of the N values of  $u_{t+1}$  in  $w_{t+2}^f$  are the draws described above. This procedure assures that the forecasts are asymptotically unbiased. Basically this simulation device averages out the intermediate residuals instead of using numerical integration. Besides allowing one to get the aforementioned multi-period forecasts, additional descriptive forecasting elements can also be obtained. Before averaging we have a whole set of "empirical" forecasts for each horizon - in fact a whole distribution - that contains relevant information that can be used for generating interval forecasts for different levels of confidence, or highest density regions along the lines proposed by Hyndman (1995,1996). This will very likely provide room for the appearance of asymmetric interval forecast due to the nonlinear nature of the model.

Sampling parameter uncertainties in the forecasting process could also be addressed similarly with the same simulation procedure herein described. However, with this generalization, the whole procedure gets highly computer intensive and requires a lot of human resources to carefully scrutinize for each set of parameter outcomes if unstable and explosive specifications appear during the forecasting process.

The impulse response function is an alternative tool to analyse the dynamics of a series provided by a linear or a nonlinear model. This function measures how a shock affects a series over time. It can be calculated as the difference between the conditional expected value of the series with and without the shock. However the conditional expectation in the nonlinear context has to be addressed with similar care as in the case of the forecast exercise.

Koop Pesaran and Potter (1996) and Potter (2000) developed instruments to generalize the impulse response function for the nonlinear modelling context. The point of departure of their analysis is the impulse response function calculated for any linear model. This instrument of analysis has

been extensively used in the empirical literature to trace out how different policy or exogenous shocks feed into the system. It also delivers important elements to judge the overall structural features of the model under analysis providing important identifying elements to pinpoint possible weaknesses of the overall structure. Although the impulse response function can be defined in various ways the basic issue is to find a definition that translates well the following concept: "an impulse response function measures the time profile of the effect of a shock on the behaviour of a series" (Koop, Pesaran and Potter (1996)). Thus, in the linear framework the traditional impulse response function may be defined as:

$$\begin{aligned} TIR_y(h, \delta, \psi_{t-1}) &= E(y_{t+h}|u_t = \delta, u_{t+1} = u_{t+2} = \dots = u_{t+h} = 0, \psi_{t-1}) - \\ &- E(y_{t+h}|u_t = 0, u_{t+1} = u_{t+2} = \dots = u_{t+h} = 0, \psi_{t-1}) \end{aligned} \quad (14)$$

This concept therefore defines the impulse response function as the difference between two profiles of  $y_{t+h}$  that start from identical historic values of the time series up to time  $(t-1)$  - given by  $\psi_{t-1}$ : one is hit by a shock of size  $\delta$  at time  $t$ , while the other - called the benchmark profile - develops with no such shock. Shocks in all intermediate periods are set equal to zero.

It has long been recognized that this function presents the following properties in linear models. Namely, it is *symmetric* - a shock of size  $-\delta$  delivers an exactly opposite effect of one of size  $\delta$ , *linear* - the time profile of the shock is exactly proportional to the intensity of the shock - and *time independent* - does not depend on when the shock occurs i.e. the time profile of the shock does not vary with the position of the economy over the business cycle. These properties can be summarized by saying that the impulse response functions in linear models are history independent and linearly homogeneous.

However these properties no longer stand up in nonlinear models. In the nonlinear framework the response to shocks are dependent not only on the size and sign of the shocks as well as on the historic moment when the shocks hit the "system". Furthermore we can no longer set all the intermediate shocks besides the first to zero to get the whole profile of the impulse response function as this may provide biased results.

Koop, Pesaran and Potter (1996) generalized the concept of the impulse response function to address the measurement of the time profile of shocks for nonlinear models. They called it the generalized impulse response function and defined it as:

$$GIR_y(h, \delta, \psi_{t-1}) = E(y_{t+h}|u_t = \delta, \psi_{t-1}) - E(y_{t+h}|\psi_{t-1}) \quad (15)$$

Now we can see that this impulse response function is dependant only on the shock  $u_t = \delta$  that hits the alternative profile and on the specific history that characterizes the moment at which the shock occurs -  $\psi_{t-1}$ . The whole set of intermediate shocks can no longer be set to zero as in the linear case but have to be averaged out in the expectational expressions as in the forecast exercise. This is as necessary as previously due to the combined nonlinear feature of the model and the aforementioned linear property of the expectational operator. The benchmark profile is also subjected to the same averaging out process not only of all the intermediate shocks but also including the first period shock -  $u_t$ . This new expression is equally applicable for linear models where it reproduces the same set of results as the ones obtained using the traditional definition.

As Koop, Pesaran and Potter (1996) pointed out  $\delta$  and  $\psi_{t-1}$  are both specific realizations of  $u_t$  and  $\Psi_{t-1}$  respectively, where the first is a random variable and the second is a random stationary process. Therefore the previous expression can be interpreted as a special case of a more general one defined as:

$$GIR_y(h, u_t, \Psi_{t-1}) = E(y_{t+h}|u_t, \Psi_{t-1}) - E(y_{t+h}|\Psi_{t-1}) \quad (16)$$

This equation calculates the difference between two conditional expectations, which are themselves random variables. Various particular conditional versions from this expression can therefore be defined: for example conditioning the history to a particular regime or conditioning the shock to the set of positive or negative ones or a combination of these two alternative. For further details see Koop, Pesaran and Potter (1996).

## 5 Modelling GDP with STAR Models.

### 5.1 Empirical application for Portugal.

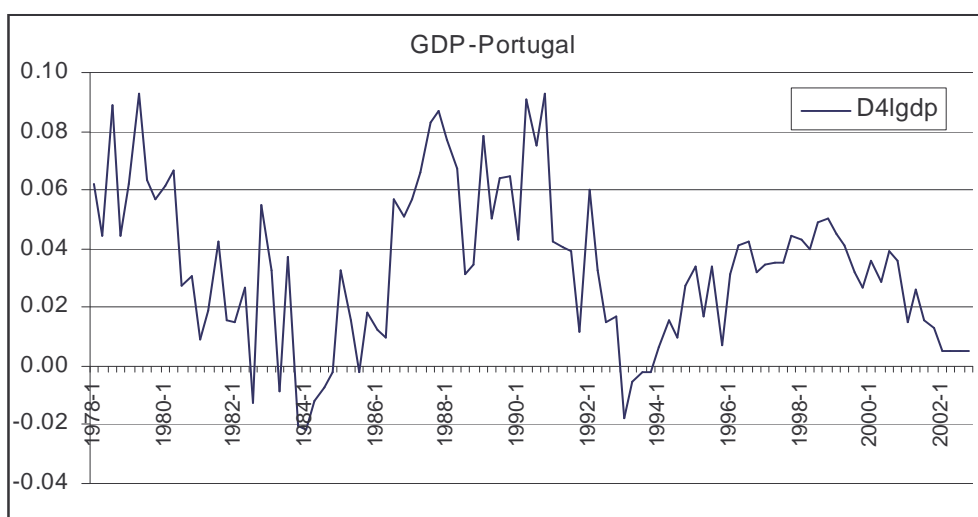
In this section we present the results of the empirical exercise conducted on quarterly GDP data for Portugal within the nonlinear STAR framework.

The quarterly series was constructed at Banco de Portugal (BP) using the quarterly GDP data published by Instituto Nacional de Estatística (INE) to interpolate the annual series for GDP available at BP. This step was undertaken because the original quarterly series published by INE was subjected to various base drifts in its construction as well as methodological changes along the years. These changes originated different levels in comparison to the corresponding annual series

and also implicitly contain annual growth rates divergent from those apparent in the annual series. However they happen to be the only series providing infra-annual information on the developments of GDP growth. For these reasons this interpolating procedure is normally used at BP and is developed to guarantee that the overall annual growth rate of the so constructed quarterly series is consistent with the growth rate of the annual series. The sample period goes from 1977Q1-2002Q4. The quarterly values for 2002 were extended based on very provisional internal forecasts.

As the modelling process requires the series to be stationary, following Teräsvirta and Anderson (1992), we take seasonal differences of the log of the quarterly series to make it approximately stationary. The p-value of the corresponding ADF test for the unit root test hypothesis exceeds marginally the standard 5% level of significance.

The graph for the seasonal difference of GDP - (D4lgdp) - is presented in the following figure.



This annual growth rate series is a reasonable business cycle indicator. Simultaneously it is through this transformation of GDP that the above-mentioned asymmetric business cycle characteristics show up more clearly and that we intend to describe through the STAR modelling procedure. The overall behaviour of GDP dynamics shows that, on average, the "dips" in GDP growth rates come relatively fast and are followed by longer periods of recovery but where growth rates are smaller.

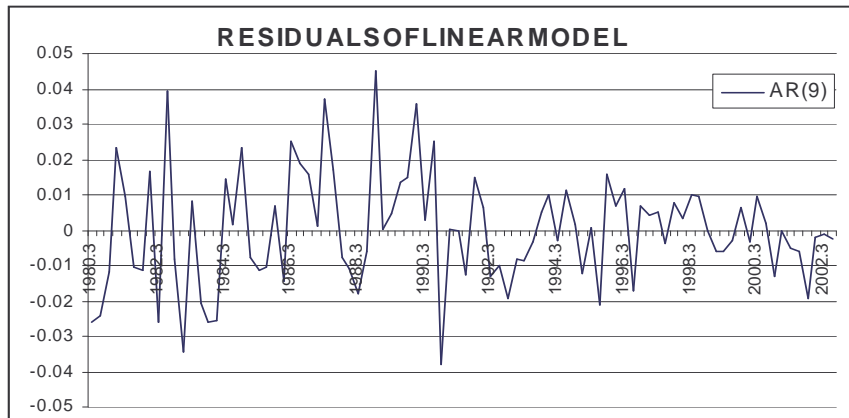
The first step of the modelling cycle requires the specification of a linear auto-regressive model as described above. An AR(9) model is delivered relying both on AIC and BIC criterions. The unrestricted linear model specification we end up with is the following:



$$\begin{aligned}
y_t &= \underset{(2.13)}{0.008} + \underset{(4.72)}{0.512}y_{t-1} + \underset{(2.83)}{0.326}y_{t-2} + \underset{(1.48)}{0.176}y_{t-3} - \underset{(-4.39)}{0.527}y_{t-4} + \underset{(2.86)}{0.359}y_{t-5} + \\
&\quad + \underset{(1.21)}{0.144}y_{t-6} - \underset{(-0.51)}{0.058}y_{t-7} - \underset{(-3.25)}{0.352}y_{t-8} + \underset{(1.47)}{0.153}y_{t-9} + \hat{u}_t \tag{17} \\
SER &= 0.017, AIC = -8.07, BIC = -7.80, SK = 0.30, EK = 0.35, JB = 1.80(0.40), \\
LM_{SI}(1) &= 1.60(0.21), LM_{SI}(3) = 2.86(0.043), LM_{SI}(4) = 2.71(0.036), \\
ARCH(1) &= 0.03(0.853), ARCH(2) = 0.70(0.499), ARCH(4) = 0.79(0.532).
\end{aligned}$$

where  $y_t = \Delta_4 \ln(gdp)$  represents the annual growth rate of GDP, and OLS t-ratio are presented in parenthesis below the parameter estimates. Along with the estimated equation, we also present the following statistics: SER - standard error of regression, the AIC and the BIC criterion values, SK is the skewness, EK the excess kurtosis, JB the Jarque-Bera test of normality of the residuals,  $LM_{SI}(j)$  is the Breusch-Godfrey test for no serial autocorrelation up to and including lag j, and  $ARCH(q)$  is the LM test of no ARCH effects up to order q. *P-values* for the test statistics are also presented in parenthesis following the value of each of them.

The linear model contains too many lags that may, in part, be due to the fact that we are modelling annual growth rates and not chain growth rates - first differences of logged GDP. Seasonal dummies were also included in this specification though they turned out to be highly non-significant. All in all the entire structure seems globally adequate although there still seems to exist some autocorrelation in the residuals, apparent in the Breusch-Godfrey test. Nevertheless the residuals pass the conventional Ljung-Box test of no serial correlation well robustly. In spite of these mixed results referring to autocorrelation we retain this linear specification as the benchmark against which to compare the non-linear version to be developed. All the other statistics comply fairly well with standard criterions. The residuals from this linear structure are the following:



The next set of steps in the modelling cycle involve conducting the linearity tests along with the identification of the transition function and the delay of the transition variable. In the following table we present the results that allow us to decide on these issues following the procedures described in the previous section.

Linearity test based on the AR(9) linear specification

delay	p-values			
	$H_0 : p(F_L)$	$H_{04} : p(F_4)$	$H_{03} : p(F_3)$	$H_{02} : p(F_2)$
1	0.0122	-	-	-
2	0.0055	0.2475	0.0019	0.2085
3	0.0084	0.2738	0.0991	0.0046
4	0.6975	-	-	-
5	0.3057	-	-	-
6	0.2331	-	-	-

These set of results clearly reject linearity in favour of a STAR nonlinear model. Additionally both an ESTAR model with a delay parameter of  $d=2$  and a LSTAR alternative with a delay of  $d=3$  are equally favoured. However for the reasons mentioned in the previous section, we choose the LSTAR version as the recommended alternative to model GDP dynamics.

The next stage involves the estimation of the aforementioned nonlinear alternative<sup>1</sup>. The starting point of the estimation process takes off with the unrestricted version of the AR model that includes nine lags in both the extreme regime, and progressively goes on dropping the lagged variables of GDP with the lowest absolute t-ratios. Simultaneously symmetric restrictions are also imposed on the parameters of equal lags in both regimes, when the corresponding data based empirical tests support those restrictions. The final LSTAR specification we end up with is reproduced in the following equation:

$$\begin{aligned}
 y_t = & \left[ \begin{array}{cccc} 0.372 & -1.224 & 0.832 & -0.314 \\ (5.39) & (-4.26) & (3.48) & (-5.70) \end{array} y_{t-2} - y_{t-4} + y_{t-6} - y_{t-8} \right] + \\
 & + \left[ \begin{array}{cccc} 0.017 & 0.483 & 0.851 & 0.418 \\ (4.38) & (6.76) & (2.71) & (5.30) \end{array} y_{t-1} + y_{t-4} + y_{t-5} - y_{t-6} \right] \times \\
 & \times \left( 1 + \exp \left[ \begin{array}{c} -2.658 \\ (2.70) \end{array} (y_{t-3} + \begin{array}{c} 0.0006 \\ (-0.14) \end{array}) / \hat{\sigma}(y_{t-3}) \right] \right)^{-1} + \hat{u}_t \tag{18}
 \end{aligned}$$

$$SER = 0.015, AIC = -8.23, BIC = -7.96, SK = 0.36, EK = 0.37, JB = 2.54(0.28),$$

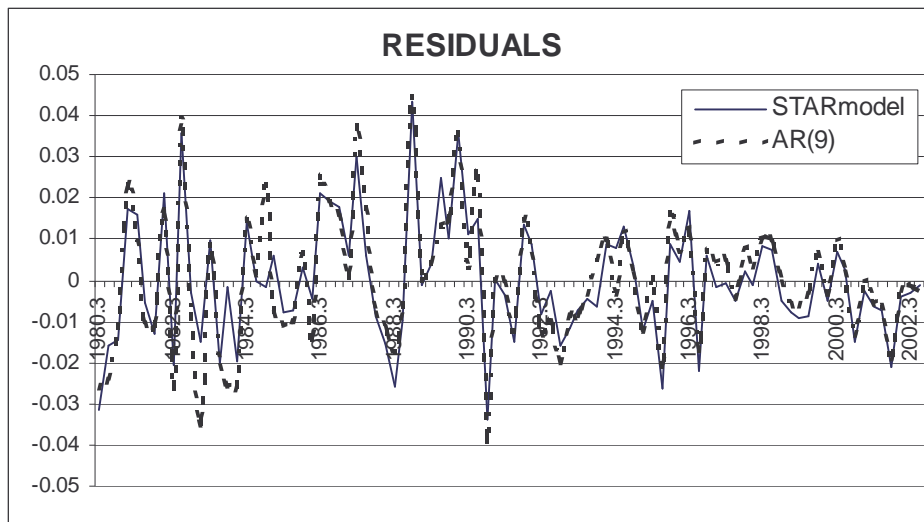
$$ARCH(1) = 0.11(0.74), ARCH(2) = 0.81(0.45), ARCH(4) = 1.51(0.21),$$

$$LB(4) = 4.08(0.395), LB(8) = 5.21(0.735), LB(12) = 9.27(0.680).$$

<sup>1</sup>As mentioned in van Dijk and al.(2002) the codes for the STAR modelling cycle are available at <http://ideas.uqam.ca/ideas/data/SoftwareSeries.html> from where they were downloaded, for which I would like to thank the authors.

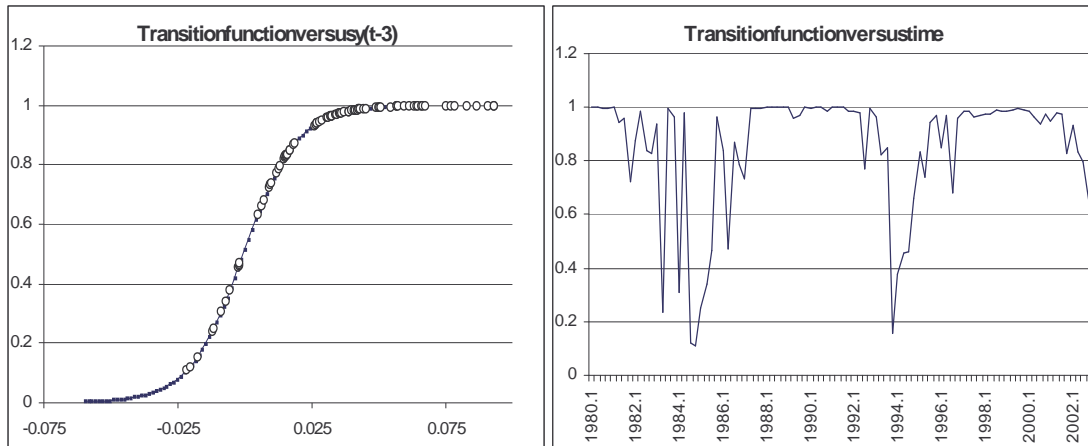
where  $\hat{\sigma}(y_{t-3})$  is the standard deviation of the delay variable in the sample, and LB is the Ljung-Box statistic for no serial correlation in the residual series. The rescaling of the exponent in the transition equation turns out to be necessary for a more accurate estimation of  $\gamma$ . A detailed explanation for this requirement can be found in Teräsvirta (1994).

These results show that this parsimonious structure of the nonlinear model has just the same number of parameters as the linear version. Additionally, here, all the parameters are highly significant with the exception of the threshold parameter. The reduction obtained in the standard deviation is approximately 8 per cent in comparison to the linear version. Overall this model passes all the evaluation tests conducted on the residuals - it follows approximately a white noise behaviour, no further auto-correlation in the residuals is found, also there are no signs of left out non-linearity and finally no symptoms of parameter non-constancy was apparent. The residuals of this model along with the one of the linear version follow next:



From here we can see that this nonlinear version shows a clear improvement in the fit roughly in the following three periods: 1983:2-1985:4, 1992:3-1994:1 and 1997:2-1998:4. The first two of these are clearly periods of low growth rate in GDP developments - lower regime - therefore showing that the additional flexibility of this instrument provides room for capturing dynamics in the slow growing "era" without deteriorating the overall performance in the rest of the sample.

The transition function is characterized as being relatively smooth with the estimated threshold value being estimated as close to zero growth rate of GDP. The transition function has the following shape where each circle corresponds to an observed value of  $y_{t-3}$  - the transition variable:

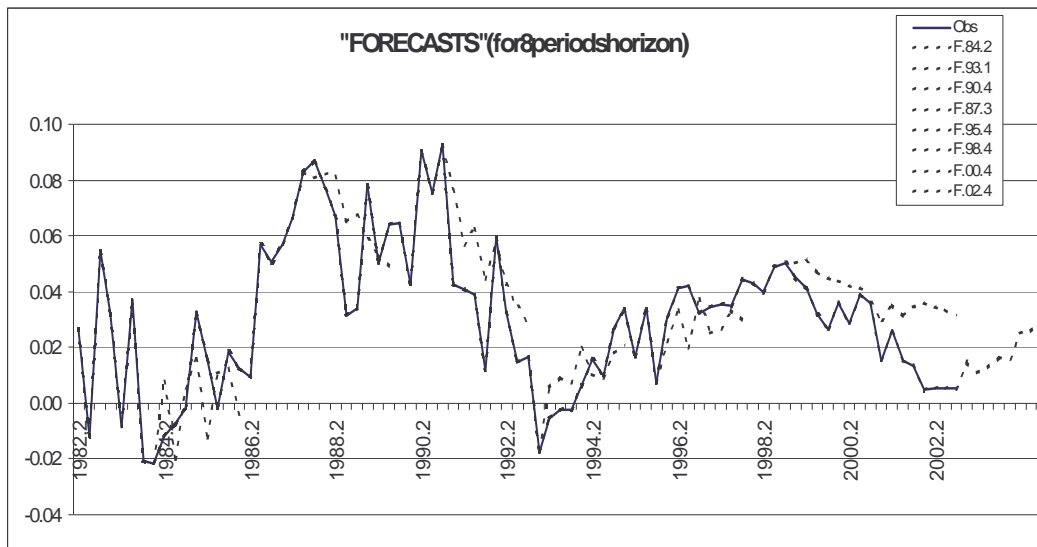


Furthermore these graphs also show that the economy has crossed the threshold towards the lower regime for quite a number of times although never reaching exactly the lower bound regime.

The two outer regimes identified through this structure present completely different dynamics. The "lower" regime, characterized by low growth of GDP ( $F=0$ ), has a characteristic polynomial given by  $h(z) = z^8 - 0.37z^6 + 1.22z^4 - 0.83z^2 + 0.31$  which has complex roots with modulus of 1.06 and 0.71 respectively, thus following an explosive dynamics. The "upper" regime on the contrary is described by the characteristic polynomial  $h(z) = z^8 - 0.48z^7 - 0.37z^6 + 0.37z^4 - 0.42z^3 + 0.31$ , which has dominant roots with the following modulus 0.94, 0.90 and 0.74, which shows that this regime is stationary. This stationary property implies that once the economy is in this upper regime it will stay there and only when the economy is hit again by a large negative exogenous shock it can drive GDP to the lower regime. On the contrary; the local non-stationary dynamics of the lower regime leads GDP growth out of the lower regime to "normal" growth rates as time elapses, even in the absence of positive shocks. This property is apparent through the overall long-run behaviour of the model which shows explicitly that independently of where the economy stands i.e. independently of the recent past history of GDP the model converges to the unique stable stationary point of 0.039 (3.9% growth rate) - which compares with the sample average of 3.4%. This long run behaviour can be obtained by projecting the "skeleton" of the model ignoring the random residual, using different starting point for the projection exercise.

Obtaining forecasts for GDP with an autoregressive model, even if it is a nonlinear one is hardly a reliable exercise. Nevertheless we conduct this exercise just to provide a flavour of how the forecasting process works out in the nonlinear context. With the exception of the last forecast (F.02.4), which is the only genuine out of sample forecast, all the other exercises are in-sample "forecast",

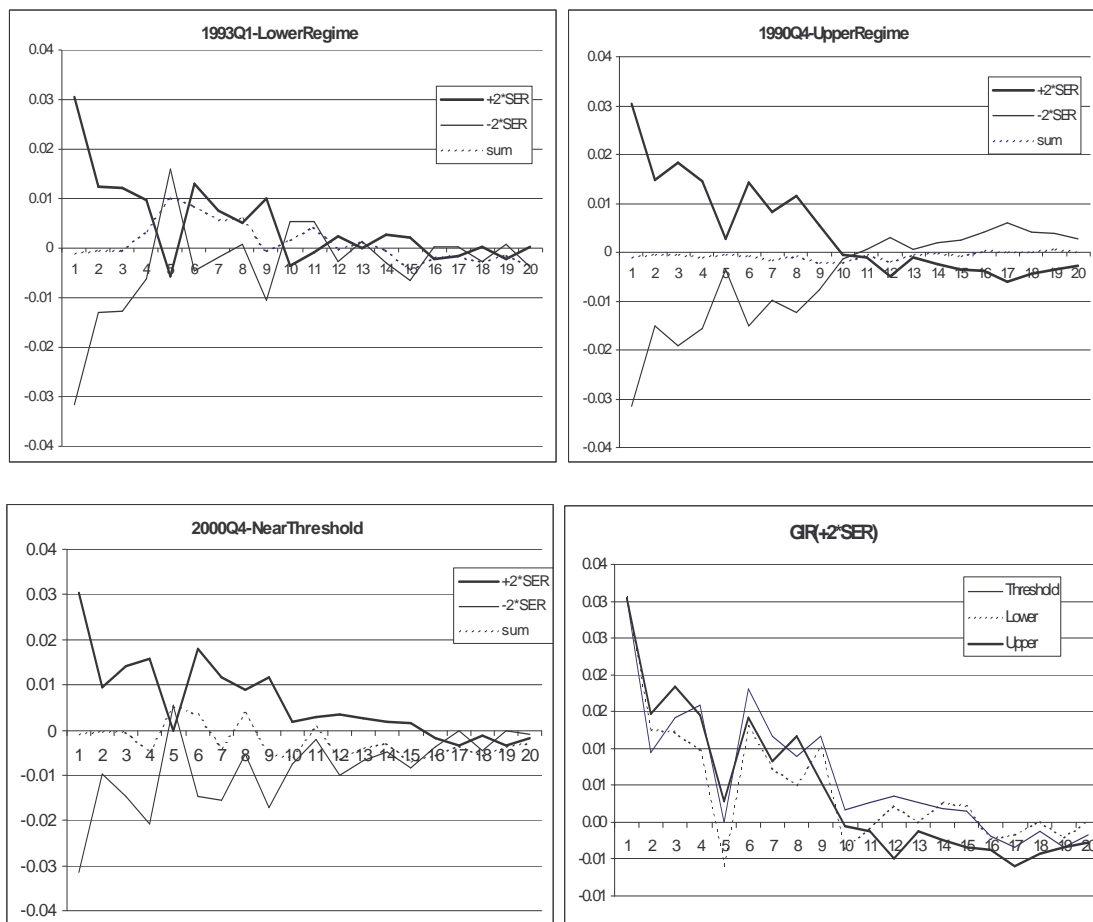
and therefore these results should be read taking this fact into account. For the overall forecasting procedure we relied on the Monte-Carlo simulation, although alternatively bootstrapping method could have equally been applied, where the independent draws for the successive residuals were taken from the following normal distribution -  $N(0; 0.105)$  - where the standard deviation is fixed and equal to the standard error of regression obtained from the STAR structure in equation (18). The forecast horizon, for this exercise, was fixed to eight periods - a two years period horizon - since time-series models are only reliable for performing short-term forecasts, and generally fail over a longer-term horizon.



The choice of the launching points for the different forecasting exercises were picked somewhat subjectively, with the evident exception of the last out-of-sample forecast. The overall criterion was to choose the most extreme values of the growth rates of GDP to provide elements to gain insight on the dynamic features of the collected nonlinear model for GDP.

As mentioned previously the impulse response function is another statistical device that is commonly used to analyse the dynamics of a variable. The impulse response functions were calculated following the description presented in the previous section and we provide the results obtained for a horizon of twenty quarters - a five years period - following the shock. Three historical dates were purposely chosen to shock the baseline profile: 1993Q1 - when observed growth rate of GDP was -1.8%, identified as being close to the lower regime; 1990Q4 (upper regime), when GDP growth rate was 9.3% and finally 2002Q4 when growth rate (+0.5%) was close to the threshold. In each of the three cases we shocked the equation with  $\pm 2SER$  (from the STAR structure). Simultaneously we also graphed the sum of the two corresponding profile to give an idea of possible asymmetries

present in the model.



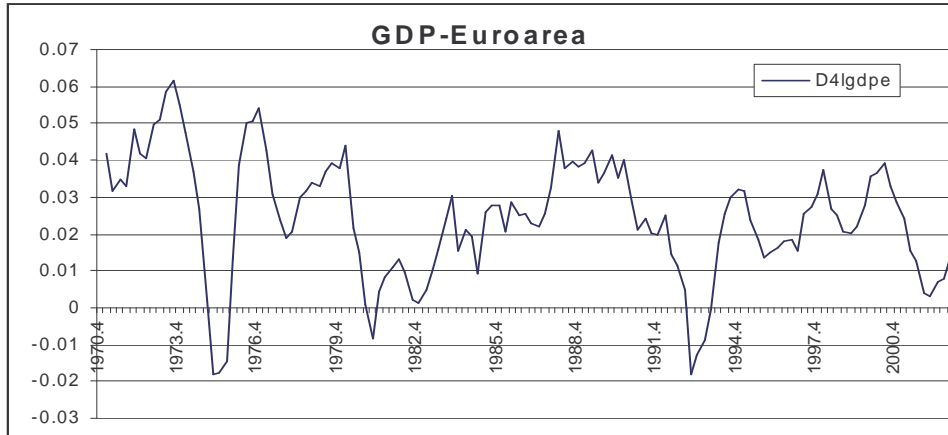
The asymmetries between positive and negative shocks are clearly observable in the lower regime and in the one that characterizes the "near threshold". It does not show up in the upper regime possibly due to the high growth rate (9.3%) observed for GDP in 1990Q4 figure. To further enhance the non symmetric features of this model, we also present the stacked version of the three "positive" impulse response functions in the final panel where the overall difference in the time profile are clearly visible, thus proving that the impulse responses are also *time dependent*.

## 5.2 Empirical application for the Euro-Area.

A similar modelling exercise was also conducted with GDP data for the Euro-area. The elements for this series were obtained from the database that was gathered for the Area-wide model of the European Central Bank. This series was updated for the most recent period with elements collected from one of the forecasting exercises. Therefore the information for 2002 is highly preliminary.

However this fact should not change the major conclusions of the nonlinear modelling exercise.

Following the same steps as in the previous exercise, we hereby present the graph of the seasonal difference of GDP for the Euro-area - (D4lgdpe).



As expected, once again the same business cycle features are present in this series: the economic downturns come up fairly fast and are relatively shorter-lived than the recoveries that, on average, last longer and present smaller growth rates while the recovery lasts.

The linear autoregressive model we collected as the benchmark based on both AIC and BIC criterions is of order five. The specification of the unrestricted model in this case is the following:

$$y_t = \underset{(3.28)}{0.005} + \underset{(12.57)}{1.101}y_{t-1} - \underset{(-0.44)}{0.056}y_{t-2} - \underset{(-0.60)}{0.076}y_{t-3} - \underset{(-4.03)}{0.510}y_{t-4} + \underset{(3.82)}{0.331}y_{t-5} + \hat{u}_t \quad (19)$$

$$SER = 0.007, AIC = -10.00, BIC = -9.86, SK = -0.61, EK = 1.32,$$

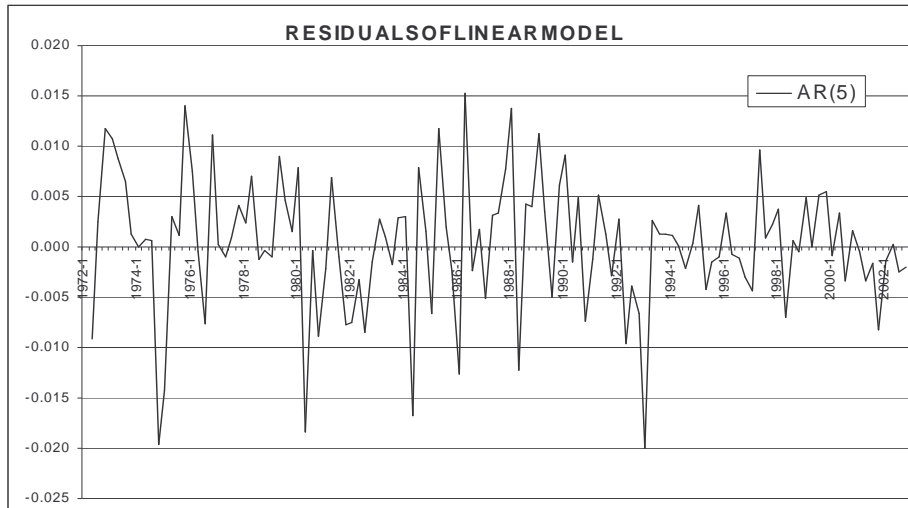
$$JB = 16.0(3.33 * 10^{-4}), LM_{SI}(1) = 0.07(0.79), LM_{SI}(3) = 0.03(0.99),$$

$$LM_{SI}(4) = 1.01(0.40), ARCH(1) = 1.50(0.223), ARCH(2) = 2.15(0.121),$$

$$ARCH(4) = 1.72(0.148).$$

here  $y_t = \Delta_4 \ln(gdpe)$  represents the annual growth rate of GDP for the Euro-area. In this case seasonal dummies were also experimented in the linear specification but they all turned out to be non-significant. The final linear specification that was obtained is globally adequate. The residual errors do not seem to contain any significant serial correlation as well as heteroskedastic errors are ruled out. However the normality of the errors are rejected owing to the existence of some large errors in the tails of the error distribution.

The residuals of the linear structure are presented in the following graph.



The linearity tests along with the identification of the transition variable are performed based on the results presented in the following table:

Linearity test based on the AR(5) linear specification

delay	p-values			
	$H_0 : p(F_L)$	$H_{04} : p(F_4)$	$H_{03} : p(F_3)$	$H_{02} : p(F_2)$
1	0.2636	-	-	-
2	0.3435	-	-	-
3	0.1212	-	-	-
4	0.0394	0.1719	0.0644	0.0706
5	0.2368	-	-	-

The results in favour of the nonlinear model are not as strong as in the previous application. In fact these results reject linearity with a  $p - value$  just marginally above the standard 5% level of significance in favour of a STAR nonlinear model with a delay parameter of  $d = 4$ . Additionally they also favour marginally the ESTAR specification as the adequate one. Nevertheless for the reasons mentioned earlier the LSTAR version is the non-linear model we are going to estimate owing to the features of aforementioned alternative. However one must mention that the ESTAR alternative was equally estimated although the LSTAR version presented superior empirical results.

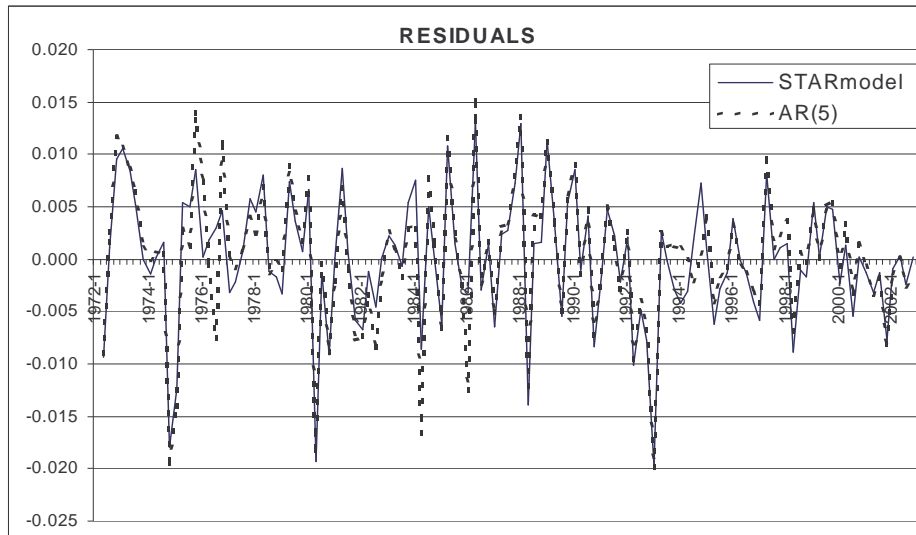
The LSTAR estimated structure that was finally obtained through the estimation procedure previously discussed is the following:



$$\begin{aligned}
y_t &= \left[ \begin{matrix} 0.006 & + & 1.093y_{t-1} & - & 0.446y_{t-2} & - & 0.600y_{t-4} & + & 0.271y_{t-5} \\ (6.45) & & (35.72) & & (-6.53) & & (-10.23) & & (4.87) \end{matrix} \right] + \\
&+ \left[ \begin{matrix} 0.446y_{t-2} \\ (-6.53) \end{matrix} \right] \times \left( 1 + \exp \left[ \begin{matrix} -82.072(y_{t-4} - 0.0122) / \hat{\sigma}(y_{t-4}) \\ (0.800) & (24.32) \end{matrix} \right] \right)^{-1} + \hat{u}_t \quad (20) \\
SER &= 0.006, AIC = -10.11, BIC = -9.95, SK = -0.61, EK = 1.17, \\
JB &= 14.75(6.27 * 10^{-4}), ARCH(1) = 0.71(0.40), ARCH(2) = 0.66(0.52), \\
ARCH(4) &= 0.40(0.81), LB(4) = 3.57(0.47), LB(8) = 9.77(0.28), LB(12) = 14.99(0.24).
\end{aligned}$$

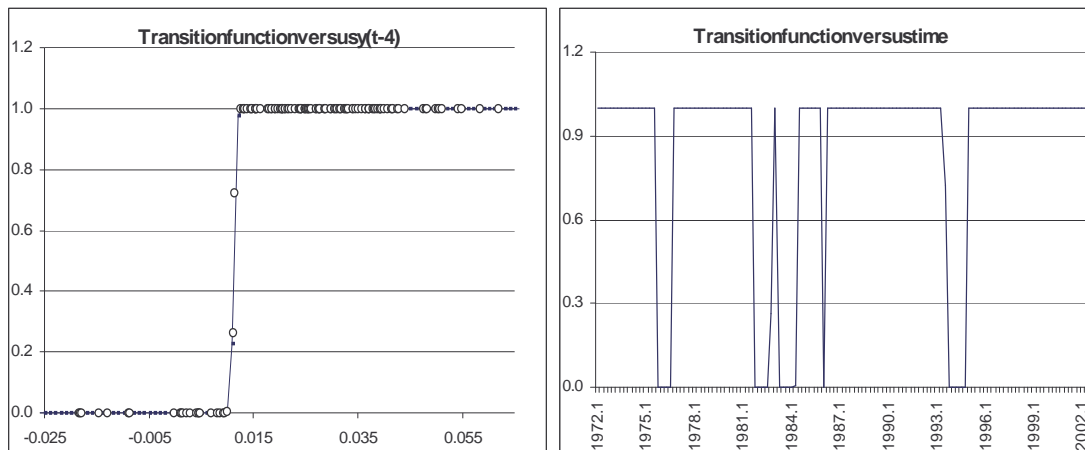
The reduction in the standard deviation in comparison to the linear version amounts to approximately 6 per cent in this application. With the exception of the normality of the residual that is once again not verified due to the existence of large errors both positive and negative ones in the tails of the distribution, all the other evaluation tests conducted on the residuals pass satisfactorily. No further correlation in the residuals is apparent, no further non-linearity seems to be left out and there is no case for parameter non-constancy in the data. The transition parameter of the logistic function shows that the change in regimes in this case occurs fairly fast, situation that will be more evident when we present the graph of the transition function.

The residuals of the linear and the non-linear version are displayed in the next chart:



As was mentioned above the gains in the reduction of the residuals in the non-linear model versus the linear benchmark is somewhat lower than in the previous empirical application. The major gains are basically concentrated in the following periods: 1981:3-1983:3, 1984:2-1986:2 and 1972:2-1981:1. The first of these is characterized as a low growing GDP period while in the second a major "dip" occurred in 1985:1.

The transition functions which follows presents in the left hand figure a steep step-shape figure reflecting the fact that the change inter-regimes occurs very abruptly. This function delivers a specification very close to a two-regime self-exciting threshold autoregressive model.

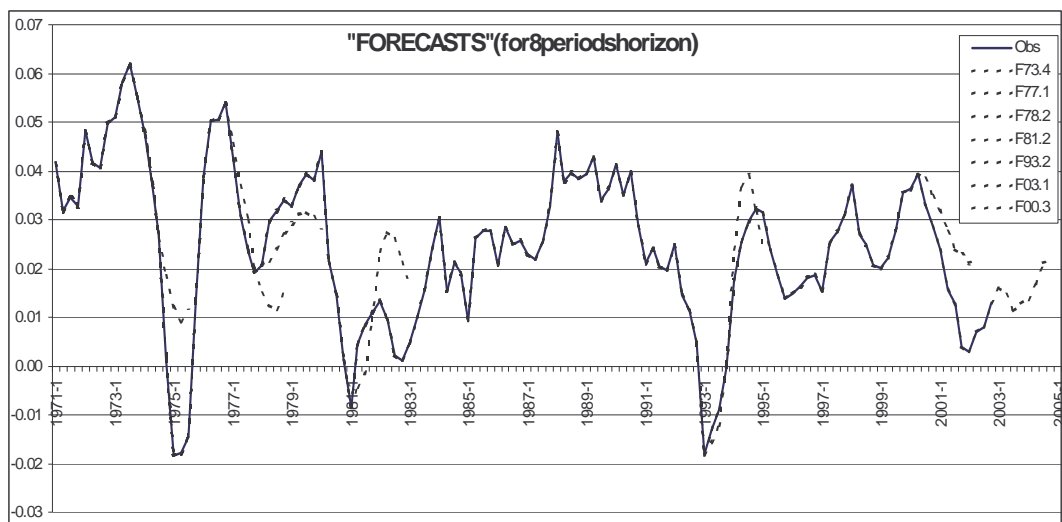


From here we can read that GDP for the Euro-area has been evolving according to the expression given by the upper-regime specification for most of the time, as expected. However the lower regime dynamics has also played a role quite often in the development of GDP and the growth rate of this series passed the estimated threshold -  $c = 0.0122$  - more frequently than in the previous empirical example. In fact the economy of the Euro-area actually "reached" the lower bound regime in this case quite a number of times.

The dynamics of the outer regimes are the following: the "lower" regime which characterizes the dynamics when GDP presents growth rates in the transition period under approximately 1.0% ( $F=0$ ) is given by the autoregressive equation with the following characteristic polynomial  $h(z) = z^5 - 1.09z^4 + 0.45z^3 + 0.60z - 0.27$  which shows having only one of the roots - a complex one - outside the unit circle with modulus of 1.04 describing a non-stationary property; the "upper" regime which is "triggered" whenever the transition variable - lagged GDP growth rate - is just above 1.3% is described by the characteristic polynomial given by  $h(z) = z^5 - 1.09z^4 + 0.60z - 0.27$  - associated with a different autoregressive specification that can be obtained from equation 20 by making ( $F=1$ ) - has the following roots with modulus in descending order equal to 0.91, 0.79, and 0.52 thus portraying a stationary behaviour. In spite of the local non-stationarity property found in the non-linear specification it is globally stationary. This feature can once again be shown through the overall long-run behaviour of the model, which converges to the unique stable stationary point of 0.026 (2.6% growth rate), independently of the past history of GDP i.e. of the regime in which

we are.

Once again the forecasts were undertaken for his empirical exercise. Although an autoregressive model is hardly an advisable instrument for forecasting GDP, even with a non-linear specification we present the results just to show once more how the forecasting procedure works in the non-linear environment. The forecast period is fixed to an eight quarters horizon.



The choice of the forecast origin was chosen subjectively trying however to have in the sample representatives of the extreme growth rates, both large and small as well as one intermediate one too.

## 6 Conclusions.

This paper has found support for the proposition that the dynamic behaviour of GDP changes over the business cycle. Using quarterly growth rates for seasonally unadjusted GDP data for the Euro-area and Portugal we found evidence in favour of asymmetric behaviour for these variables. The nonlinear features that were already apparent in graphs of the series are far more evident empirically for the case of Portugal and less sticking so for the GDP data for the Euro-area.

The existence of these asymmetries over the business cycle have long been uncovered in the empirical literature for various time-series - industrial production index, unemployment rate and even private consumption expenditures - in other countries. The smooth transition autoregressive model supports the evidence of the nonlinear dynamics in our case where the final structure at which we

arrive supports asymmetries found in the data. These nonlinear features show up not only in the form of distinct impulse response functions calculated for different starting points for the shocks but is also apparent in the completely different dynamic displayed by the two extreme regimes that characterize the "recession" period and the "high" growing phases of the business cycle.

The shape of the estimated transition function for the Portuguese GDP series shows that the transition between the two identified extreme regimes is fairly smooth, while in the case of GDP for the Euro-area is very abrupt portraying somewhat the shape of a two-regime self-exciting threshold autoregressive model.

This modelling device can also be used for forecasting purposes, as is done in the paper. However, as in any other time-series model, either linear or nonlinear the final results should be interpreted with greater caution, as these models tend to project into the future simply the historical regularities found in the series though within the nonlinear context the changing environment can affect the future pattern of the forecast. In our case where we are modelling GDP growth within a univariate nonlinear time-series framework this projection exercise should be addressed simply as an exemplifying device of the forecasting procedure in the non-linear modelling framework. Nevertheless this very modelling procedure has been generalized to a multivariate context where other explanatory variables can also be handled in the model giving room to a family of models coined in the literature as smooth transition regression (STR) models. In that context forecast can definitively gain a greater role other than in the autoregressive context of our application.

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