BANCO DE PORTUGAL

Economic Research Department

FORECASTING EURO AREA AGGREGATES WITH BAYESIAN VAR AND VECM MODELS

Ricardo Mourinho Félix
Luís C. Nunes

WP 4-03 February 2003

Please address correspondence to Ricardo Mourinho Félix, Economic Research Department, Banco de Portugal, Av. Almirante Reis nº 71, 1150-165 Lisboa, Portugal, Tel.#351-213130321; Fax#351-213107804; email:rfelix@bportugal.pt; or Luís C. Nunes, Faculdade de Economia, Universidade Nova de Lisboa, Tv Estevão Pinto, P-1099-032 Lisboa, Portugal, email: lcnunes@fe.unl.pt

FORECASTING EURO AREA AGGREGATES WITH BAYESIAN VAR AND VECM MODELS[*]

Ricardo Mourinho Félix[**]

Luís C. Nunes[***]

June 2002

Abstract

This paper focuses on Bayesian Vector Auto-Regressive (BVAR) models for the euro area. A modified hyperparameterization scheme based on the Minnesota prior that takes into account the economic nature of the variables in the model is used. The merits of incorporating long-run relationships are also discussed. Alternative methods to estimate eventual cointegrating relations in the variables are considered, and the problem of choice of appropriate prior distributions for BVAR with Error Correction Mechanism (BECM) models is addressed. Results show that using a flat prior on factor loadings can seriously endanger the forecasting performance of BECM models. Overall, the BVAR model in levels outperforms all other models across variables and forecasting horizons. This is in contrast with other empirical studies where some gains could be obtained when incorporating long-run relationships in the model.

Keywords: *BVAR models; Euro area models; Forecasting; Cointegration*

[**] Banco de Portugal, Research Department. Email: rfelix@bportugal.pt; Tel.: +351 213130000.

[***] Corresponding author. Faculdade de Economia, Universidade Nova de Lisboa. Address: Tv Estevao Pinto, P-1099-032 Lisboa, Portugal. Email: lcnunes@fe.unl.pt; Tel.: +351 213801600. Fax: +351 213886073.

1. Introduction

Multi-country models are frequently used to produce forecasts of the main euro area economic variables. In this approach, forecasts of euro area aggregates are obtained by aggregating the forecasts obtained for each one of the constituent countries. However, with the introduction of the euro in 1999 and the growing economic integration among the countries that have adopted it, one would expect area wide models to become increasingly used. This is the approach followed in this paper, with the euro area modelled as a single country and using aggregate time series for each variable[1].

This paper also focuses on Bayesian Vector Auto-Regressive (BVAR) models. Over the past twenty years, the BVAR approach has gained widespread acceptance as a practical tool to provide reasonably accurate macroeconomic forecasts when compared to conventional macroeconomic models or alternative time series approaches. The Bayesian methodology allows imposing prior restrictions on the model parameters, thereby greatly reducing the dimensionality problem of VAR models, resulting in efficiency gains in the estimation of the parameters and, consequently, in more accurate forecasts.

The majority of the BVAR models proposed in the literature rely on the specification of a prior distribution known as the Minnesota prior as presented in Doan *et al*. (1984). In this paper, a modified hyperparameterization scheme that takes into account the economic nature of the variables in the model is used. In particular, a distinction is made between real variables and price variables, and between endogenous and exogenous variables.

Another aspect discussed in this work concerns the modelling of long-run relationships. In spite of the theoretical attractiveness[2], results presented in some studies using BVAR models not always agree about the nature of the hypothetical gains from incorporating cointegrating relationships. For example, in LeSage (1990), with labor market data for Ohio industries, BVAR with Error Correction Mechanism (BECM) models perform

[1] Bikker (1998), using standard BVAR models in levels for the European Union, provides evidence of the superiority of area wide models in terms of forecasting performance compared to averages of forecasts for individual countries.
[2] See for example Engle and Yoo (1987).

better at increasing forecasting horizons. On the other hand, in the context of electricity demand, Joutz *et al*. (1995) find improvements only at shorter horizons. For the US economy, Shoesmith (1995) finds improvements at all forecasting horizons. Amisano and Serati (1999) find that a BECM model with an informative prior on factor loadings provides the best results at all forecasting horizons for the Italian economy. This paper also considers the merits of incorporating long-run relationships but in the context of BVAR models for the euro area. Alternative methods to estimate eventual cointegration relations among the variables are considered, and the problem of choice of appropriate prior distributions in BECM models is addressed.

The paper is structured as follows. Section 2 summarizes the BVAR framework. The issue of incorporating long-run relationships in a BVAR model is discussed in Section 3. In Section 4, Bayesian and non-Bayesian models for the euro area are compared in terms of forecasting performance. The last section presents some conclusions.

2. BVAR models

Consider a $(n \times 1)$ vector Y of variables to be forecasted. In a VAR model each one of these variables is assumed to be linearly correlated with its past values up to p lags, the past values of the remaining variables included in Y up to the same lag, and a vector D of deterministic components (such as an intercept and seasonal dummy variables), such that,

$$Y_t = CD_t + A_1 Y_{t-1} + \cdots + A_p Y_{t-p} + \varepsilon_t$$

where C, A_1, ... , A_p denote matrices of coefficients to be estimated, and ε_t is a vector of unobserved innovations.

BVAR models present a solution to the excessive number of parameters to estimate in VAR models by imposing some general restrictions through prior probability distribution functions. The posterior distribution function for each parameter is obtained by combining the prior distribution and the sample likelihood using Bayes rule (see for example Lütkepohl, 1993).

The prior specification is an important step in BVAR modelling. An excessively diffuse prior, that is, a prior with a large variance around the prior mean, can be easily modified by accidental sample variability (noise). An informative prior with reasonable values for the variances can only be influenced by systematic sample variability (signal), diminishing the risks of overfitting and of producing unreliable forecasts.

In this work, we use a prior specification inspired by the well-known Minnesota prior. As in Doan *et al*. (1984), it is postulated that most macroeconomic series can be described as pure random walks. Accordingly, it is assumed that the prior distributions for the VAR parameters are independent normal distributions, with their means set equal to the parameters implied by a random walk.

The variances of the prior distributions are defined according to a functional relation linking these to a second set of parameters, smaller than the first one, known as hyperparameters. The way each equation is tightened around the random walk prior mean is determined by a set of overall tightness hyperparameters that can differ from equation to equation. To control the increase of tightness around the random walk prior for lags farther apart in time and to avoid an excessive number of hyperparameters, it is assumed that tightness increases with the lag, that is, the variance for higher lags decays inversely with the lag. Finally, there are hyperparameters controlling cross-variable relationships that can differ from variable to variable and across equations.

In particular, the variance of the prior distribution of the coefficient associated with lag l of variable j in the equation for variable i is set equal to

$$v_{ij,l} = \begin{cases} (\lambda_i / l)^2 & \text{if} \quad i = j \\ (\lambda_i \theta_{ij} \sigma_i / \sigma_j l)^2 & \text{if} \quad i \neq j \end{cases}$$

where λ_i is the overall tightness hyperparameter for equation i, θ_{ij} are the hyperparameters controlling cross-variable relationships, and the term σ_i / σ_j accounts for different units of measurement in the variables. Values for σ_i are set equal to the estimated standard error of a univariate autoregressive model for each variable.

The values assumed by the hyperparameters are crucial in a BVAR model because they determine how far BVAR coefficients are allowed to deviate from their prior means and how much is the model allowed to approach a non-Bayesian VAR model, that is, an unrestricted VAR. A BVAR model gets closer to an unrestricted VAR model as λ and θ go to infinity; conversely, as these hyperparameters approach zero, a BVAR model gets closer to the random walk prior mean. When λ goes to infinity and θ goes to zero, a BVAR model approaches a set of univariate autoregressive models.

Of course, as the number of equations gets larger, the number of hyperparameters continues to increase. However, as shown below, using an appropriate hyperparameterization scheme that takes into account the economic nature of the variables, it is possible to further reduce the number of hyperparameters.

For the intercept and seasonal dummy terms, variance is set to infinity, that is, a completely diffuse prior for the deterministic terms is considered.

In spite of being Bayesian in its philosophy, a BVAR model is not completely Bayesian since hyperparameters are usually calibrated using an optimisation algorithm based on an objective function that depends on out-of-sample forecast errors. In the case of the quarterly models for the euro area, 12 quarters ahead out-of-sample forecast errors are used.

3. Bayesian VECM models

A criticism pointed to VAR and BVAR models is the fact that these do not take into account explicitly eventual long-run, or cointegrating, relationships among the variables. As argued by Engle and Yoo (1987), in the presence of cointegration, a VAR model with an error correction mechanism (VECM) should outperform a VAR and a BVAR over longer forecasting horizons. A BVAR model with an error correction mechanism (also known as a Bayesian VECM or BECM model) can be used to combine BVAR models' advantages with the benefits of taking into account explicitly long-run relationships in forecasting exercises.

A VECM model can be represented in general terms as follows:

$$\Delta Y_t = CD_t + A_1 \Delta Y_{t-1} + \cdots + A_p \Delta Y_{t-p} + B\beta' Y_{t-1} + \varepsilon_t ,$$

where Δ denotes the difference operator, β is the $(n \times r)$ matrix of cointegrating vectors, B denotes the $(n \times r)$ matrix of coefficients associated with the error correction terms, $\beta' Y_{t-1}$, also called factor loadings, and r denotes the dimension of the cointegration space $(r < n)$.

In this work, estimation of the BECM model is done in two steps.[3] First, long run relationships are estimated using either Engle-Granger[4] or Johansen[5] methodologies. In the Engle-Granger approach it is possible to test for and estimate a single cointegrating vector, which can be interpreted as a linear combination of all the cointegrating vectors in the cointegration space. In the Johansen approach, after testing for the rank, or dimension, of the cointegration space, it is possible that more than one cointegrating vectors are estimated. Secondly, the resulting estimated error correction terms are then plugged in the VECM model to be estimated. Since in a BECM model all regressors are stationary, prior means for all the coefficients are set to zero. Prior variances for the coefficients in C, A_1, ... , A_p follow the same hyperparameterization scheme used in the BVAR model.

The factor loadings, B, have an increased relevance in a BECM model since they determine the importance of long-run relationships and how fast variables converge to their long-run levels. As discussed in Amisano and Serati (1999) in the context of a small BVAR model for the Italian economy, an uninformative prior on factor loadings combined with an informative prior on the short-run dynamics may confer an exaggerated weight to the long-run relative to the short-run (since only short-run dynamics would be restricted by the prior), thereby greatly endangering forecasting performance. The empirical application presented in the next section considers a BECM

[3] This two-step estimation procedure was initially proposed in LeSage (1990), and was also used in Shoesmith (1992) and Amisano and Serati (1999). Alvarez and Ballabriga (1994) propose an alternative two-step approach based on FIML estimation of $\Pi = B\beta'$.
[4] See Engle and Granger (1987).
[5] See Johansen (1988).

model with an uninformative prior on factor loadings and, as an alternative, a BECM model with an informative prior on factor loadings in order to compare the forecasting performances of both models.

In summary, two alternative methods of estimating cointegrating relationships and two alternative priors on the coefficients associated with the error correction terms are considered, giving rise to the four types of BECM models considered below.

When there are no cointegrating vectors or if the prior variance for the factor loadings is very small, the BECM model reduces to a BVAR model in first differences.

4. BVAR models for the euro area

This section presents the results obtained with several alternative models in terms of forecasting accuracy regarding a set of economic variables that usually play an important role in euro area forecasting exercises. The variables considered are: real GDP, unemployment rate, consumer prices, nominal wage rate, long term interest rate and nominal effective exchange rate.

A set of exogenous variables has also been considered: external real GDP, external prices and the short term interest rate. All the variables used in the VAR, BVAR and BECM models for the euro area are presented in Table 1 and plotted in Appendix D.[6]

The database used in the empirical application for the euro area was built by recovering country series from a variety of sources (BIS, AMECO, IMF, OECD and Eurostat). The sample covers a period from 1977:1 to 1997:4 on a quarterly basis. Euro area variables were obtained by aggregation of country variables using the so-called "index method" suggested by Fagan and Henry (1997). Appendix A presents a more detailed description of the aggregation method.

[6] ADF tests for the presence of unit-roots confirm that all the variables in Table 1 can be considered as I(1). See Appendix B for details.

Table 1. Description of variables

Variable	Description	Status	Block
Y	Log GDP at constant prices - measure of economic activity in the euro area (index)	Endogenous	Real
U	Unemployment rate - measure of labour market conditions in the euro area (in percentage of labour force)	Endogenous	Real
P	First difference of log private consumption deflator - measure of inflation rate in the euro area (index)	Endogenous	Price
W	First difference of log nominal wage rate - measure of labour force nominal earnings in the euro area (index)	Endogenous	Price
ILT	Long term interest rate - measure of capital and investment costs in the euro area (in percentage)	Endogenous	Price
S	Log effective nominal exchange rate of euro - measure of currency market conditions (index)	Endogenous	Price
YW	Log external GDP at constant prices - measure of activity outside the euro area (index)	Exogenous	Real
PW	First difference of log external GDP deflator - measure of external price inflation (index)	Exogenous	Price
IST	Short term interest rate - measure of the monetary authority policy instrument (in percentage)	Exogenous	Price

4.1. Hyperparameterization scheme

In this work, the hyperparameterization scheme used is somewhat different from the one in Doan *et al*. (1984) since a special treatment of the hyperparameters governing cross-variable relationships is considered. In addition to the prior assumptions discussed in Section 2, the hyperparameterization scheme relies on a classification of the variables into two blocks: real variables and price variables (see Table 1). Based on this classification additional prior assumptions are made. The chosen specification is able to reduce the number of hyperparameters while keeping the flexibility of the BVAR model. A list of the hyperparameters is presented in Table 2, and the hyperparameterization scheme is illustrated in Figure 1.

Table 2. Description of hyperparameters

	Description
λ_1	Overall tightness for real variables equations
λ_2	Overall tightness for price equations
θ_1	Tightness of parameters of real (price) variables in real (price) variables equations
θ_2	Tightness of parameters of price (real) variables in real (price) variables equations
θ_3	Tightness of parameters of exogenous real (price) variables in real (price) variables equations
θ_4	Tightness of parameters of exogenous price (real) variables in real (price) variables equations
θ_5	Tightness of parameters of monetary instrument in real variables equations
θ_6	Tightness of parameters of monetary instrument in price equations
Ω	Tightness of ECM factor loadings

Different overall tightness hyperparameters are considered for the endogenous variables in each block (λ_1 and λ_2), thereby allowing restrictions in the equations for variables in the real block to differ from those in the prices block.

Regarding exogenous variables, the forecasting exercises considered below are not made conditional on specific macroeconomic scenarios. Therefore, the prior specification was chosen so that exogenous variables are influenced only by their own past values and not by any other variables. In fact, these variables are projected into the future using univariate autoregressive processes.

It is considered that cross-variable relationships involving endogenous variables in the same block can have a different degree of tightness around prior means (θ_1) relative to cross-relations between endogenous variables in different blocks (θ_2). This way, equations for variables in the prices block may be more influenced by variables in the same block than by variables in the real block, and vice-versa.

<table>
<tr><td></td><td></td><td></td><td colspan="2">Real variables</td><td colspan="4">Price variables</td><td colspan="3">Exogenous variables</td></tr>
<tr><td></td><td></td><td></td><td>Y</td><td>U</td><td>P</td><td>W</td><td>ILT</td><td>S</td><td>YW</td><td>PW</td><td>IST</td></tr>
<tr><td rowspan="2">Real variables equations</td><td rowspan="2">λ_1</td><td>Y</td><td>1</td><td>θ_1</td><td rowspan="2" colspan="4">θ_2</td><td rowspan="2">θ_3</td><td rowspan="2">θ_4</td><td rowspan="2">θ_5</td></tr>
<tr><td>U</td><td>θ_1</td><td>1</td></tr>
<tr><td rowspan="4">Prices equations</td><td rowspan="4">λ_2</td><td>P</td><td rowspan="4" colspan="2">θ_2</td><td>1</td><td>θ_1</td><td>θ_1</td><td>θ_1</td><td rowspan="4">θ_4</td><td rowspan="4">θ_3</td><td rowspan="4">θ_6</td></tr>
<tr><td>W</td><td>θ_1</td><td>1</td><td>θ_1</td><td>θ_1</td></tr>
<tr><td>ILT</td><td>θ_1</td><td>θ_1</td><td>1</td><td>θ_1</td></tr>
<tr><td>S</td><td>θ_1</td><td>θ_1</td><td>θ_1</td><td>1</td></tr>
<tr><td rowspan="3">Exogenous variables equations</td><td rowspan="3">∞</td><td>YW</td><td colspan="2">0</td><td colspan="4">0</td><td>1</td><td>0</td><td>0</td></tr>
<tr><td>PW</td><td colspan="2">0</td><td colspan="4">0</td><td>0</td><td>1</td><td>0</td></tr>
<tr><td>IST</td><td colspan="2">0</td><td colspan="4">0</td><td>0</td><td>0</td><td>1</td></tr>
</table>

Figure 1. Hyperparameterization scheme

In the same manner, it is considered that the coefficients of exogenous variables can have different degrees of tightness around their prior means if they appear in an equation for a variable in the same block (θ_3) or in the other block (θ_4). Therefore, an exogenous variable on the block of prices may have more influence on the equations for the variables in that block than on equations for variables in the real block, and vice-versa.

In what concerns the monetary policy instrument, it is considered that the degree of tightness of the associated parameters can be different across equations for variables in the real block (θ_5) and in the price block (θ_6). For instance, it is possible that the short-term interest rate can have a larger influence in variables such as prices or long-term interest rates than on economic activity.

Finally, the additional hyperparameter Ω controls the priors on factor loadings in BECM models. Factor loadings have zero prior means in order to ensure consistency with the random walk prior mean. Prior variances are all set equal to the Ω hyperparameter. An informative prior corresponds to the case $0 < \Omega < \infty$. As discussed in Section 3, the use of a diffuse prior on factor loadings, $\Omega = \infty$, raises the problem of an excessive weight given to long-run relationships relative to short-run dynamics, thereby endangering the forecasting performance of the models. When $\Omega = 0$, the BECM model reduces to a BVAR model in first differences without cointegrating relationships.

4.2. Hyperparameter calibration

Since BVAR models are used for forecasting purposes, hyperparameter calibration usually proceeds by optimising an objective function based on out-of-sample forecast errors. In the approach followed in this work, the sample is split in two sub-samples: the first one, 1977:1-1991:4, is used to estimate the BVAR parameters; the second one, 1992:1-1997:4, is used to compute out-of-sample forecast errors. The model is first estimated using only the first sub-sample. For each additional observation in the second sub-sample, the model is re-estimated and dynamic *h*-steps ahead forecasts are computed. The process continues adding-up observations up to the point where there are not enough observations available in the second sub-sample to compute the *h*-steps ahead forecast errors.

The root mean squared error (RMSE) is the most common measure used to evaluate the quality of the forecasts for a single variable. Since *n* variables are included, the optimisation criterion combines the RMSE for all variables in the form of a weighted average:

$$RMSE^h = \sum_{i=1}^{n} \frac{1}{n\sigma_i} \left[\sum_{t=1}^{T} \frac{1}{T} (\varepsilon_{it}^h)^2 \right]^{1/2}$$

where ε_{it}^{h} is the h steps-ahead forecast error for variable i in the t-th iteration, T is the total number of h steps-ahead forecast errors, and σ_i is set equal to the estimated standard error of a univariate autoregressive model for each variable. In the calibration of the hyperparameters, a simple average of the 1 to 12 quarters-ahead $RMSE^h$, $h = 1, 2, \dots,$ 12, was considered. The choice of three years as the horizon to calibrate the hyperparameters seems reasonable given the sample size available and the need of having enough observations to evaluate the forecasting performance of the models. Also, some of the models considered include long-run relationships that are more likely to operate in longer forecast horizons.

5. Forecasting results

Several models were compared in terms of their forecasting performance for the euro area. These include the random walk model, five non-Bayesian models and six Bayesian models. Bayesian models are compared with their non-Bayesian counterparts. Bayesian models with and without ECM are also compared with each other in order to evaluate the role played by the inclusion of long-run relationships. Both Engle-Granger and Johansen approaches were used to estimate the cointegrating vectors. The first methodology points to the existence of cointegration, while the second points to the existence of four cointegrating vectors.[7] We considered a lag length of 4 for all models except the random walk. Table 3 lists all models considered.

[7] See Appendix C for more details.

Table 3. Description of the models under analysis

Model	Description	Status
RW	Random-walk model	Non Bayesian
AR	Univariate AR model with variables in levels	Non Bayesian
VAR	VAR model with variables in levels	Non Bayesian
VAR–1st dif.	VAR model with variables in first differences	Non Bayesian
VECM (EG)	VECM model with variables in first differences and ECM estimated by Engle-Granger methodology	Non Bayesian
VECM (J)	VECM model with variables in first differences and ECM estimated by Johansen methodology	Non Bayesian
BVAR	BVAR model with variables in levels	Bayesian
BVAR–1st dif.	BVAR model with variables in first differences	Bayesian
BECM(EG)–FP	BVAR model with variables in first differences, ECM estimated by Engle-Granger methodology and flat prior on factor loadings	Bayesian
BECM(J)–FP	BVAR model with variables in first differences, ECM estimated by Johansen methodology and flat prior on factor loadings	Bayesian
BECM(EG)–IP	BVAR model with variables in first differences, ECM estimated by Engle-Granger methodology and informative prior on factor loadings	Bayesian
BECM(J)–IP	BVAR model with variables in first differences, ECM estimated by Johansen methodology and informative prior on factor loadings	Bayesian

Table 4 presents the values for the average RMSE over all endogenous variables for each model and its comparison with the random-walk prior mean model (c.w.p.). A value smaller (larger) than unity points to a better (worse) performance than that obtained with the random walk model.

The clearest evidence from the comparison is that all Bayesian models, except for the BECM(J)-FP, perform better than the random-walk prior. Also, non-Bayesian models always perform worse than the random walk in terms of forecasting.

Table 4. Averaged 1 to 12 quarters-ahead RMSE for competing models

Models	Avg.RMSE 1-12	c.w.p
RW	6.938	1.000
AR	7.612	1.097
VAR	14.396	2.075
VAR - 1st dif.	11.406	1.644
VECM (EG)	10.074	1.452
VECM (J)	21.508	3.100
BVAR	5.071	0.731
BVAR - 1st dif.	5.820	0.839
BECM (EG) - FP	5.948	0.857
BECM (J) - FP	8.867	1.278
BECM (EG) - IP	5.229	0.754
BECM (J) - IP	5.587	0.805

Additionally, it is also clear that all Bayesian models perform better than their non-Bayesian counterparts, which supports the evidence that a Bayesian approach to VAR modelling delivers better forecast accuracy, overcoming the overfitting problems of VAR models.

Regarding long-run relationships, BECM models do not perform better than BVAR models in levels; thus, the explicit modelling of long-run relationships does not seem to improve forecasts. In the literature, there are other cases where this kind of results can also be found, namely in Joutz *et al*. (1995).

It is interesting to note that although the BVAR-1st dif. performs worse than the BVAR in levels, when an ECM term is included in a BVAR-1st dif. using an informative prior, as in the BECM(EG)-IP model, then it performs almost as well as a BVAR in levels.

The role played by the hyperparameter Ω is clear when comparing the results for the BECM models with informative priors (IP) and with flat priors (FP) on factor loadings. When incorporating several cointegrating relations, the forecasting performance of the BECM(J)-FP model using a flat prior is very poor when compared with all the other models. By using an informative prior, as in the BECM(J)-IP model, the forecasting performance is greatly enhanced. There are also some gains when using an informative prior in the case of a single cointegrating relation (the BECM(EG)-IP model) but not so dramatic as in the previous case. Moreover, both models (BECM(EG)-IP and BECM(J)-IP) incorporating one or more cointegrating relations using an informative prior on factor loadings perform better than the BVAR model in first differences, suggesting the inappropriateness of fully excluding long-run relationships.

Table 5. Averaged 1 to 16 quarters-ahead RMSE for each variable and model

Models	Y	U	P	W	ILT	S
RW	1.977	0.782	1.761	5.191	1.724	5.127
AR	2.841	0.998	1.868	4.591	2.697	5.815
VAR	5.040	2.224	4.581	4.564	3.316	14.990
VAR - 1st dif.	3.521	1.165	4.945	5.137	3.217	11.521
VECM (EG)	7.723	1.602	6.934	14.598	6.964	16.079
VECM (J)	4.587	1.339	10.424	11.603	4.551	16.361
BVAR	1.594	0.516	1.256	2.836	1.436	4.941
BVAR - 1st dif.	1.799	1.033	1.291	3.873	1.754	6.247
BECM (EG) - FP	1.498	0.801	1.675	4.410	1.888	6.944
BECM (J) - FP	1.979	0.774	2.996	7.304	2.033	8.041
BECM (EG) - IP	1.716	0.650	1.341	3.961	1.699	6.249
BECM (J) - IP	1.711	0.621	1.311	4.157	1.700	6.720

A detailed analysis of the forecasting performance variable-by-variable reveals a heterogeneity of results not captured by the global criterion function used above. Results using an average of 1 to 16 quarters-ahead RMSE for each variable are presented in Table 5.

The BVAR model in levels is the best model in almost every case; the exception being real GDP that is best predicted by the BECM (EG)-FP model but followed closely by the BVAR model in levels. BECM (J)-IP is the second best model to forecast the euro area unemployment rate and the third best model to forecast the long-term nominal interest rate, real GDP and the price index. The BECM (EG)-IP model is the second best model to predict the long-term nominal interest rate and the third best model to forecast the euro area unemployment rate and the nominal wage rate. The BVAR-1st dif. is the second best model to predict the price index and the nominal wage rate.

As usual, nominal effective exchange rate exhibits a random-walk behaviour. This feature is best captured by the BVAR model, with the second best model to predict this variable being the random-walk model.

To uncover eventual differences in forecasting performance not captured by the average RMSE used above, it is important to examine the profile of the $RMSE^h$ for each variable across the forecasting horizon. In the following figures only the most relevant models for each variable are presented.

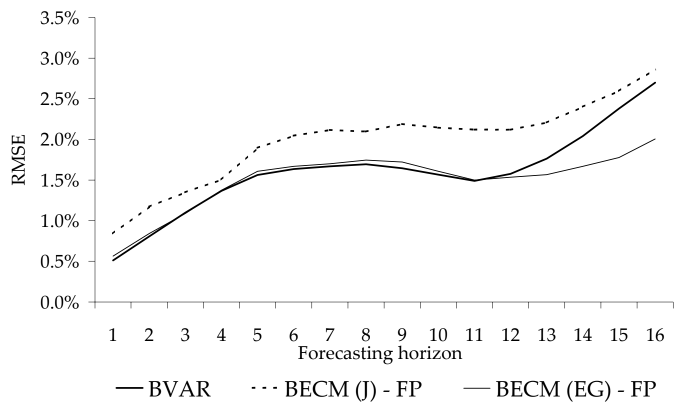


Figure 2. RMSE profiles for Y

The best performing model in terms of real GDP forecasting for the euro area is the BECM model with one cointegrating vector and a flat prior on factor loadings (BECM (EG)-FP), followed by the BVAR in levels (BVAR) model (see Figure 2). The BECM (EG)-FP model beats the BVAR when forecasting at more than 12-quarters ahead; in this case there is some evidence that including one long-run relationship improves forecasts at longer horizons. The BECM (J)-FP model performs worse at all forecasting horizons confirming that including a larger number of long-run relationships with an uninformative prior on factor loadings endangers forecast accuracy.

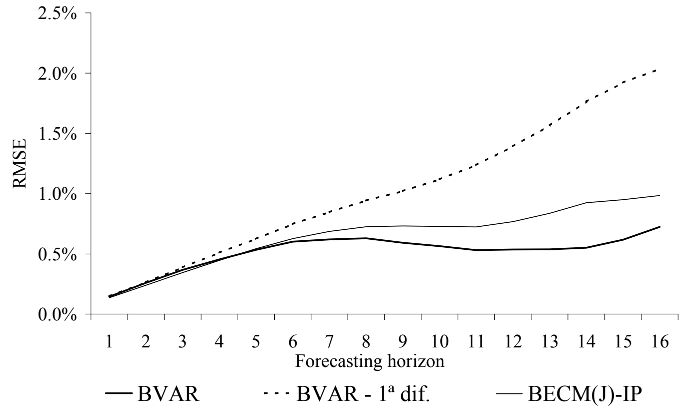


Figure 3. RMSE profiles for U

Both BVAR and BECM (EG)-IP models reveal a good forecasting accuracy when forecasting the unemployment rate (see Figure 3). The BVAR-1st dif. model performs badly, specially at longer horizons, suggesting that this may be caused by the omission of cointegrating relations in this model. Nonetheless, the model incorporating long-run relationships performs slightly worse than the BVAR, even at longer forecast horizons.

The BVAR model is the one that delivers the best forecasts for the price consumption deflator. Both BECM (J)-FP and BECM (EG)-FP models perform worse (see Figure 4). Again, a larger number of long-run relationships endangers forecast accuracy leading to poor performances. The same analysis is valid for the nominal wage rate given that these variables exhibit a very similar behaviour (see Figure 5).

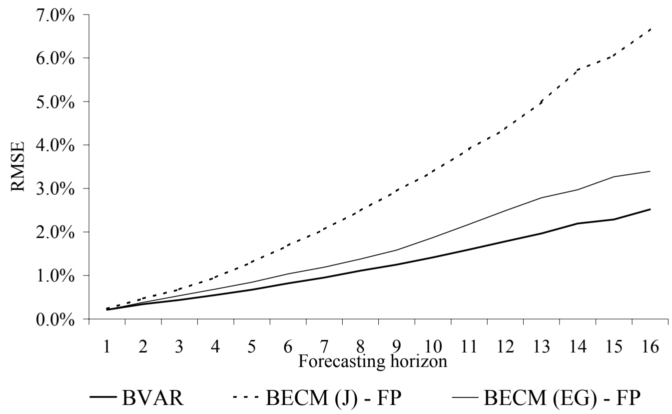


Figure 4. RMSE profiles for P

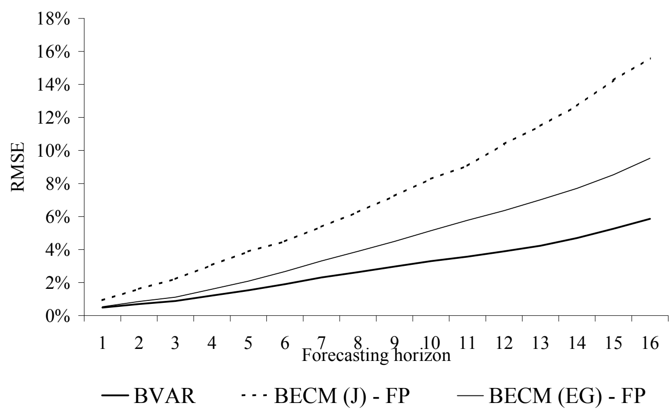


Figure 5. RMSE profiles for W

The model delivering the best forecasts for the long-term nominal interest rate is the BVAR in levels (see Figure 6). If a BECM (EG)-FP model is considered then forecast performance becomes poorer. Again, this may be due to the uninformative prior used on factor loadings.

Finally, for all models considered, nominal effective exchange rate forecasts are very poor even at short-term horizons (see Figure 7). Again, this is not an unexpected result given the random walk behaviour of the exchange rate series.

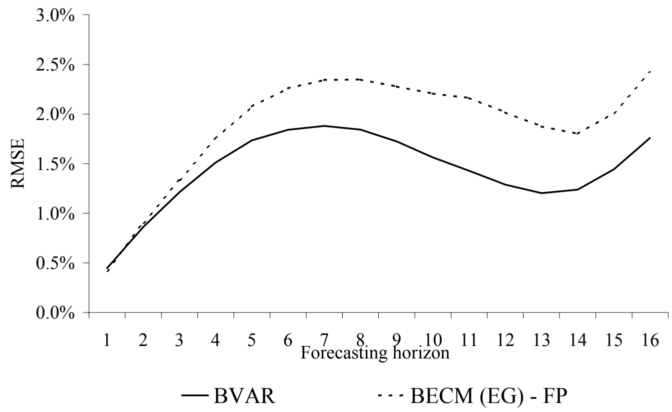


Figure 6. RMSE profiles for ILT

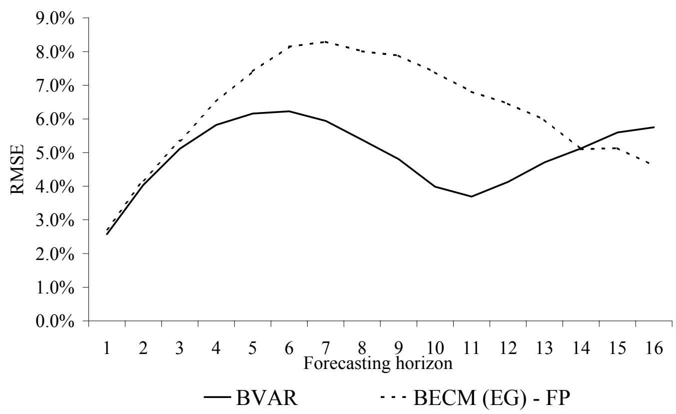


Figure 7. RMSE profiles for S

Overall the results confirm the superiority of the BVAR model in levels over competing models across variables and forecasting horizons. This is in contrast with other empirical studies, as mentioned in Section 1, where some improvements could be obtained using some form of BECM model. The negative consequences of using a flat prior on factor loadings in a BECM model confirms the results also obtained by Amisano and Serati (1999) for the Italian economy.

6. Conclusions

This paper presents a comparison of alternative BVAR models in terms of forecasting euro area macroeconomic aggregates. The proposed modified hyperparameterization scheme, based on a classification of the variables in terms of real/price variables, and endogenous/exogenous variables, avoids having an excessive number of hyperparameters while keeping the flexibility of BVAR models. The merits of incorporating long-run relationships are also discussed. Alternative methods to estimate eventual cointegrating relations in the variables are considered, and the problem of choice of appropriate prior distributions for the factor loadings in BECM models is addressed.

The first conclusion is that Bayesian models perform better than their non-Bayesian counterparts in terms of forecasting accuracy. It is worth mentioning that only Bayesian models perform better than the random walk.

A second conclusion arising from the analysis of the results is that modelling long-run relationships with BECM models leads to a poorer forecast accuracy when compared to BVAR models in levels, even at longer forecast horizons. This is in contrast with other empirical studies where some gains could be obtained by taking into account cointegrating relationships. However, when BECM models are compared with BVAR models in first differences, a better forecast accuracy is obtained. These results are consistent with the existence of misspecification problems in BVAR models in first differences where eventual long-run relationships are not taken into account.

Finally, the use of an uninformative or flat prior on factor loadings leads to an unbalanced prior treatment of short-run and long-run dynamics. The negative consequences of this are more serious when the model incorporates a large number of cointegrating relations as suggested by Johansen's method.

In this paper, cointegrating vectors were estimated and selected using Engle-Granger and Johansen methodologies, which are non-Bayesian. An issue that deserves further research is the use of an alternative Bayesian method to estimate the cointegrating relations in BECM models. A possible approach would be to use Bayesian reduced rank regression techniques as in Geweke (1996).

References

Alvarez, L.J., & Ballabriga, F.C. (1994). BVAR Models in the Context of Cointegration: A Monte Carlo Experiment. Working Paper No. 9405, Bank of Spain.

Amisano, G., & Serati, M. (1999). Forecasting Cointegrated Series with BVAR Models. *Journal of Forecasting* 18, 463-476.

Bikker, J.A. (1998). Inflation Forecasting for Aggregates of the EU-7 and EU-14 with Bayesian VAR models. *Journal of Forecasting* 17, 147-165.

Doan, T.A., Litterman, R.B., & Sims, C.A. (1984). Forecasting and conditional projections using realistic prior distributions. *Econometric Reviews* 1, 1-100.

Engle, R.F., & Granger, C.W.J. (1987). Co-integration and error correction: representation, estimation and testing. *Econometrica* 55, 251-276.

Engle, R.F., & Yoo, B.S. (1987). Forecasting and Testing in Co-integrated Systems. *Journal of Econometrics* 35, 143-159.

Fagan, G., & Henry, J. (1997). Long-run money demand in the EU: evidence for area wide aggregates. Paper presented in the Workshop "Money Demand in Europe", Berlin.

Geweke, J. (1996). Bayesian Reduced Rank Regression in Econometrics. *Journal of Econometrics* 75, 121-146.

Johansen, S. (1988). Statistical analysis of co-integration vectors. *Journal of Economic Dynamics and Control* 12, 231-254.

Joutz, F.L., Maddala, G.S., & Trost, R.P. (1995). An integrated Bayesian Vector Autoregression and error correction model for forecasting electricity, consumption and prices. *Journal of Forecasting* 14, 287-310.

LeSage, J.P. (1990). A comparison of the forecasting ability of ECM and VAR models. *Review of Economics and Statistics* 72, 664-671.

Lütkepohl, H. (1993). Introduction to Multiple Time Series Analysis, 2nd ed., Springer-Verlag, N.Y.

Shoesmith, G.L. (1992). Cointegration, error correction, and improved regional VAR forecasting. *Journal of Forecasting* 11, 91-109.

Shoesmith, G.L. (1995). Multiple cointegrating vectors, error correction, and forecasting with Litterman's model. *International Journal of Forecasting* 11, 557-567.

Appendix A. Aggregation method

Time series for the euro area variables were obtained by aggregating data from the eleven original constituent countries: Austria, Belgium, Finland, France, Germany, Ireland, Italy, Luxembourg, Netherlands, Portugal and Spain. All series, except interest rates and the unemployment rate, were transformed to indices based on 1990. Nominal and price variables were built by taking a geometric weighted average of national variables. The aggregation formula used is given by:

$$Y_t^{EU} = \prod_i \left(\frac{Y_t^i}{Y_{1990}^i} \right)^{w_i}$$

where Y_t^i denotes the value taken by the variable in country *i* at time *t*, and w_i is the weight of country *i* in the euro area measured by GDP at PPP exchange rates in ECU's for 1993. Taking the logarithm of the above formula, it follows that:

$$\ln(Y_t^{EU}) = \sum_i w_i \ln\left(\frac{Y_t^i}{Y_{1990}^i} \right)$$

Therefore, euro area aggregates can be built as arithmetic weighted averages of the logarithms of country variables. The same applies for the rates of change of euro area variables, which can be approximated by arithmetic weighted averages of the rates of changes of country variables. Since fixed weights are used, real variables can also be obtained in this way or derived by deflating nominal variables.

Appendix B. Unit root tests

Unit root tests were conducted to identify the order of integration of the euro area aggregates prior to specification and estimation of all models. The augmented Dickey-Fuller (ADF) test for a unit-root in y_t is based on the following specification:

$$\Delta y_t = \alpha + \beta t + (\rho - 1) y_{t-1} + \sum_{j=1}^{k} \gamma_j \Delta y_{t-j} + \varepsilon_t ,$$

where k is the number of lags needed to eliminate autocorrelation from the residuals. The results presented in Tables B.1 and B.2 suggest that all the variables considered contain at least one unit-root.

Table B.1. ADF tests for a unit root: constant and trend included

Variable	$\hat{\rho}$	t-statistic	k
Log GDP at constant prices	0.90766	-3.2796	2
Unemployment rate	0.98314	-1.3447	1
Log private consumption deflator	0.99712	-1.2142	2
Log nominal wage rate	0.99374	-1.6141	4
Log external GDP at constant prices	0.89869	-3.0969	3
Log external GDP deflator	0.99679	-0.7097	5

Notes: Critical values at 5%: -3.452, at 1%: -4.047.

Table B.2. ADF tests for a unit root: constant included

Variable	$\hat{\rho}$	t-statistic	k
Long term interest rate	0.97049	-1.4714	1
Log effective nominal exchange rate	0.92866	-2.4693	1
Short term interest rate	0.94039	-2.4843	1

Notes: Critical values at 5%: -2.889, at 1%: -3.493.

Table B.3. ADF tests for a unit root in the first differences: constant included

Variable	$\hat{\rho}$	t-statistic	k
First difference of log GDP at constant prices	0.44155	-4.8439*	1
First difference of unemployment rate	0.69136	-3.0452*	4
First difference of log private consumption deflator	0.96442	-1.0414	1
First difference of log nominal wage rate	0.95688	-0.7415	3
First difference of long term interest rate	0.36456	-4.9220*	3
First difference of log effective nominal exchange rate	0.33913	-7.0186*	0
First difference of log external GDP at constant prices	0.16043	-8.6101*	0
First difference of log external GDP deflator	0.87813	-1.9919	4
First difference of short term interest rate	0.31303	-5.0064*	4

Notes: * Significant at the 5% level.

Table B.4. ADF tests for a unit root in the second differences: constant included

Variable	$\hat{\rho}$	t-statistic	k
Second difference of log private consumption deflator	-0.28910	-13.631*	0
Second difference of log nominal wage rate	-0.54257	-18.597*	0
Second difference of log external GDP deflator	-0.43322	-4.8353*	3

Notes: * Significant at the 5% level.

Results for unit-root tests for the first and second differences presented in Tables B.3 and B.4 suggest that we can consider all variables as I(1) except for the log private consumption deflator, the log nominal wage rate, and the log external GDP deflator which appear to be I(2). Based on these results we include I(2) variables in first differences (second-differences) in models where I(1) variables enter in levels (first-differences). See Table 1 above for a description of the variables and Appendix D for their corresponding time-series plot.

Appendix C. Estimating cointegrating relationships

Following the Engle-Granger (1987) methodology, a static long-run regression was estimated by least squares. The estimated residuals are plotted in Figure C.1. An Engle-Granger cointegration test based on these residuals confirms that the null hypothesis of no cointegration is rejected.

Johansen (1988) approach allows for more than one cointegrating relationship. Using a system with a lag order equal to four, the trace test results presented in Table C.1 point to the existence of five or six cointegrating relationships. The corresponding first five estimated long-run relationships are plotted in Figures C.2 – C.6. Since the plot of the fifth cointegrating relationship revealed a non-stationary behavior, we only allowed for four cointegrating relationships in the BECM models considered above.

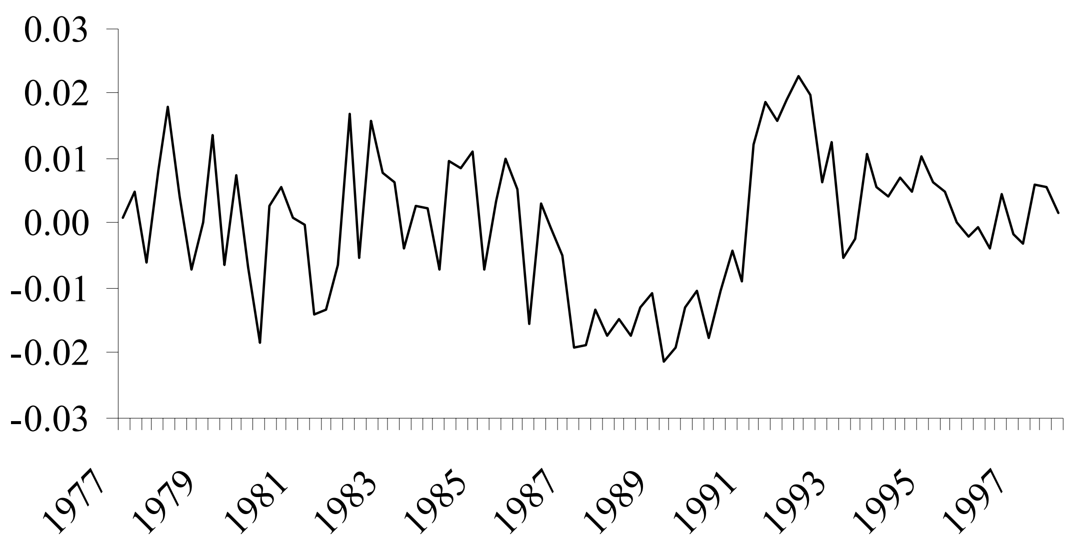


Figure C.1. Estimated Engle-Granger error correction term

Table C.1. Johansen's trace test statistics for cointegration

H_0: cointegrating rank = r	Trace	5% critical value
$r = 0$	272.4**	192.9
$r = 1$	200.6**	156.0
$r = 2$	150.5**	124.2
$r = 3$	109.9**	94.2
$r = 4$	76.5**	68.5
$r = 5$	48.5*	47.2
$r = 6$	26.5	29.7
$r = 7$	11.4	15.4
$r = 8$	1.2	3.8

Notes: ** Denotes rejection at the 1% significance level; * denotes rejection at the 5% significance level.

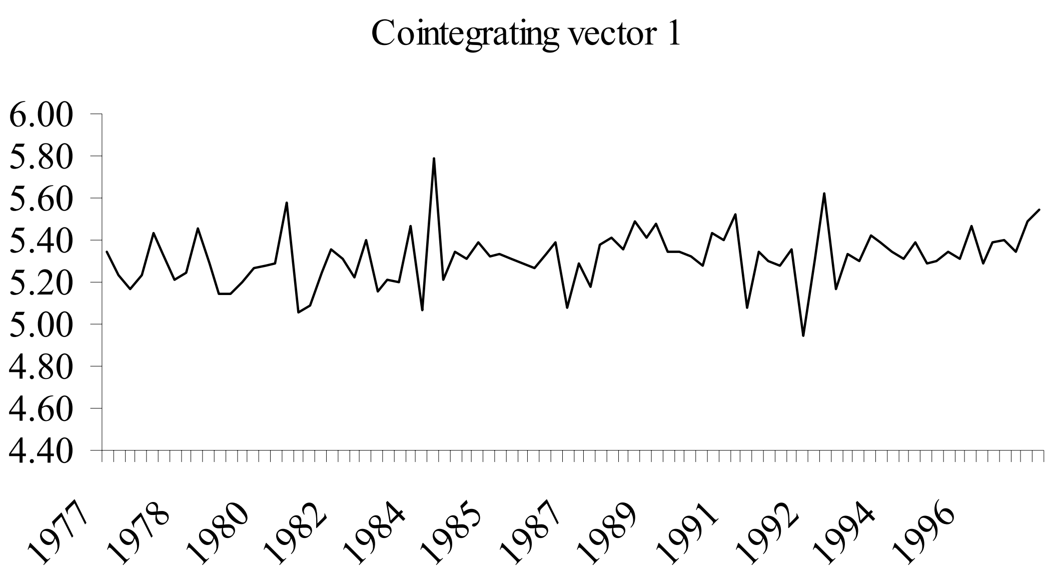


Figure C.2. First estimated error correction term

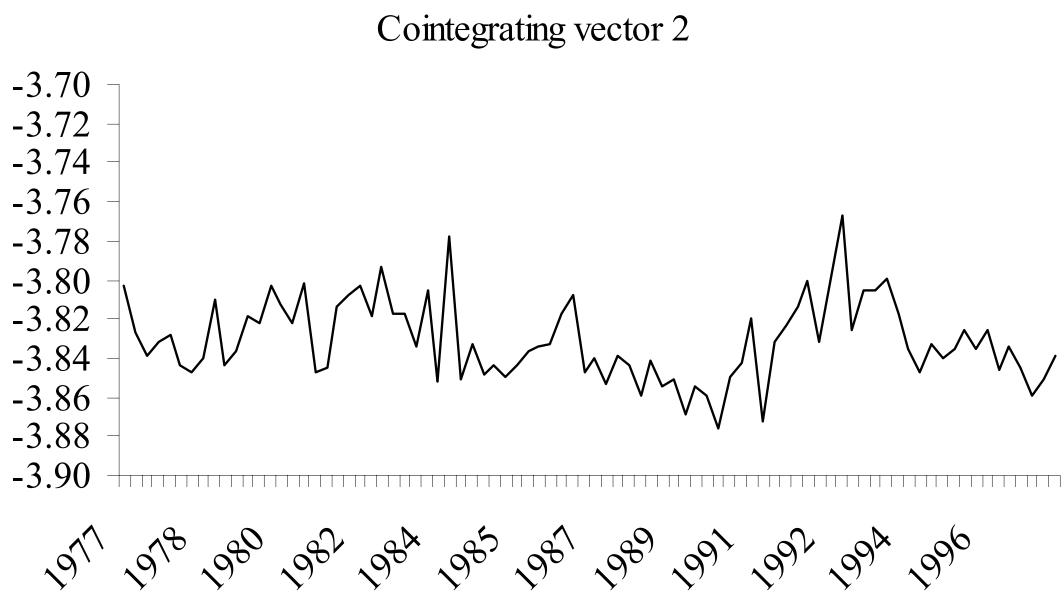


Figure C.3. Second estimated error correction term

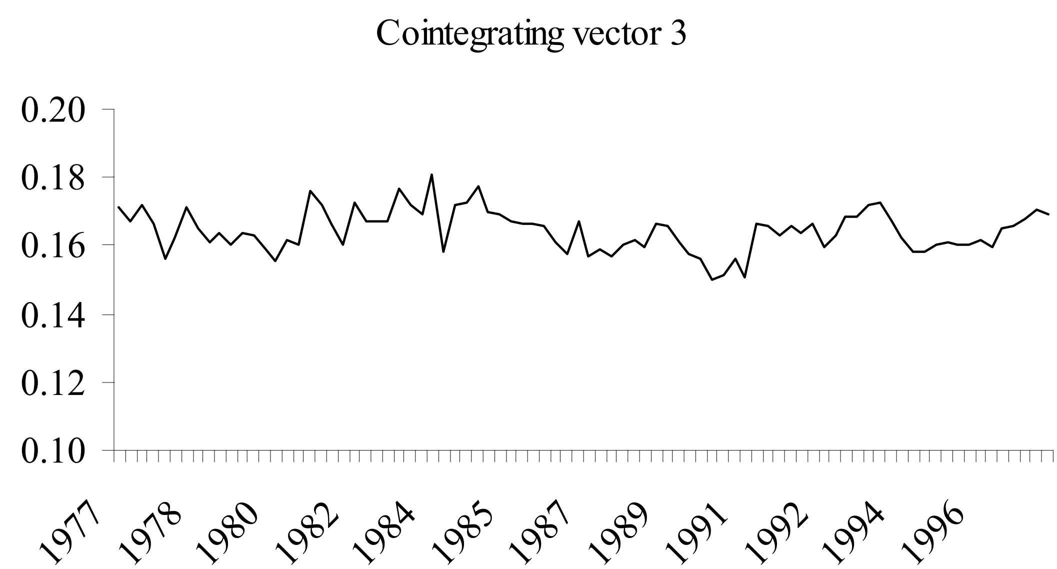


Figure C.4. Third estimated error correction term

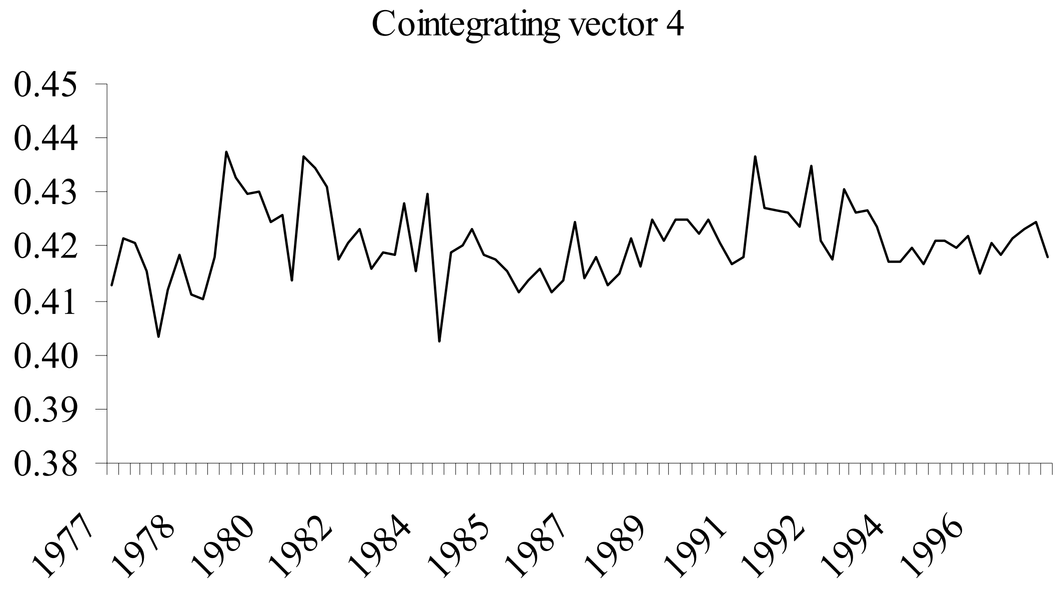


Figure C.5. Fourth estimated error correction term

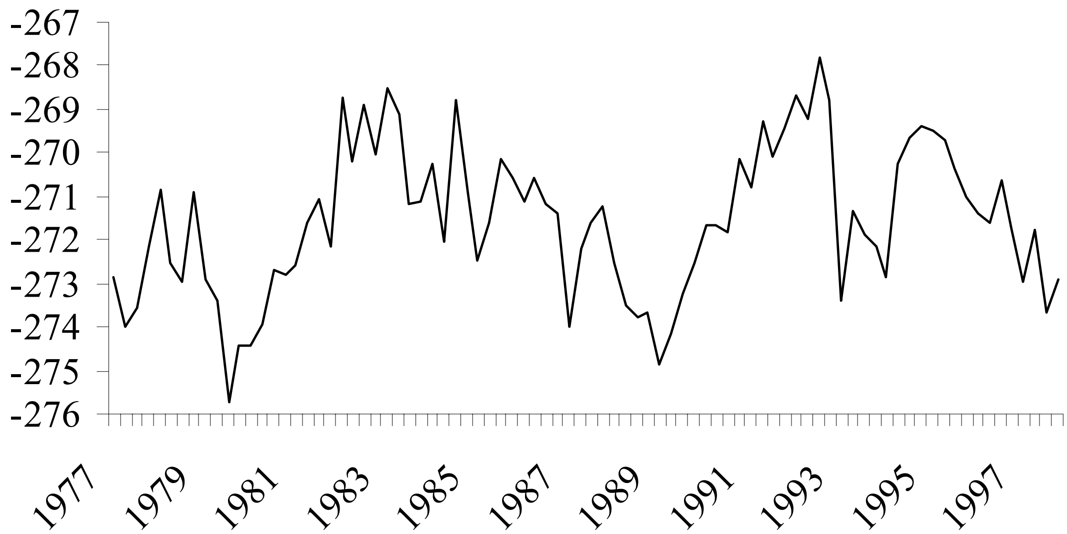
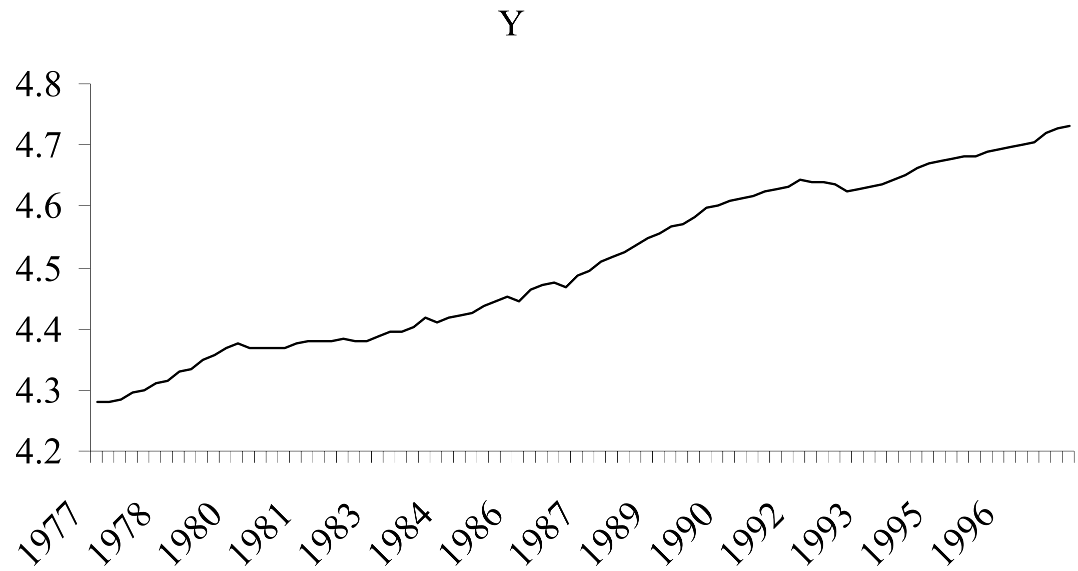
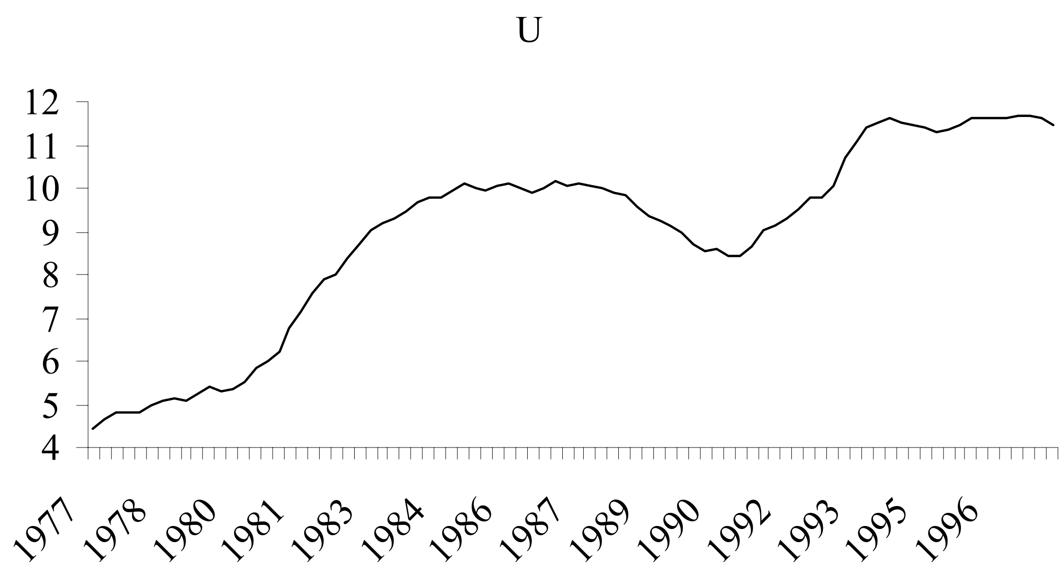


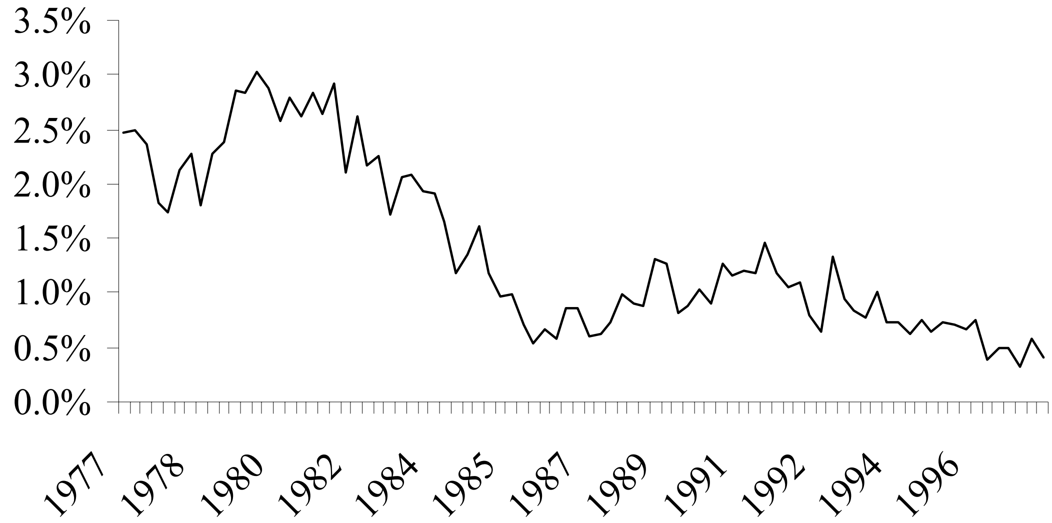
Figure C.6. Fifth estimated error correction term

Appendix D. Time-series data, 1977:1 - 1997:4



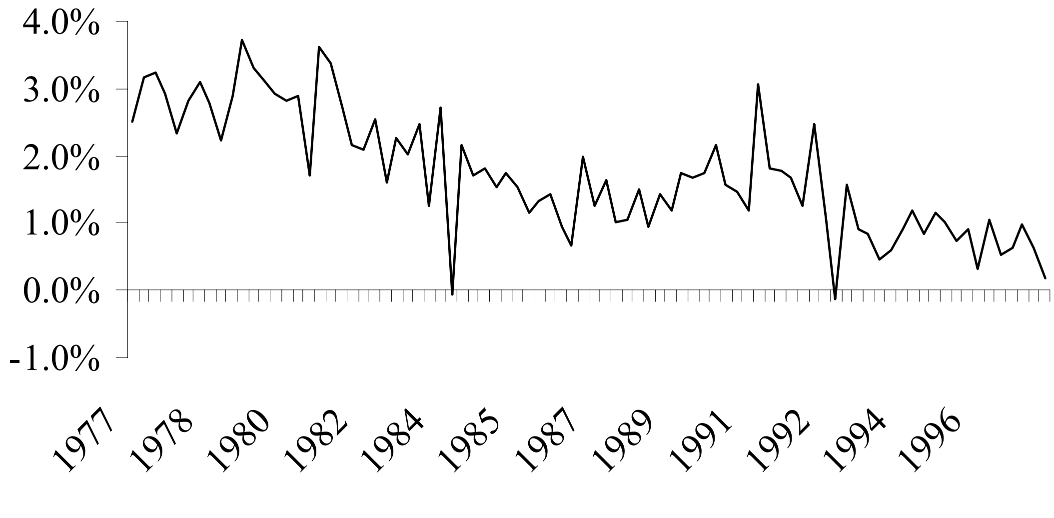


P



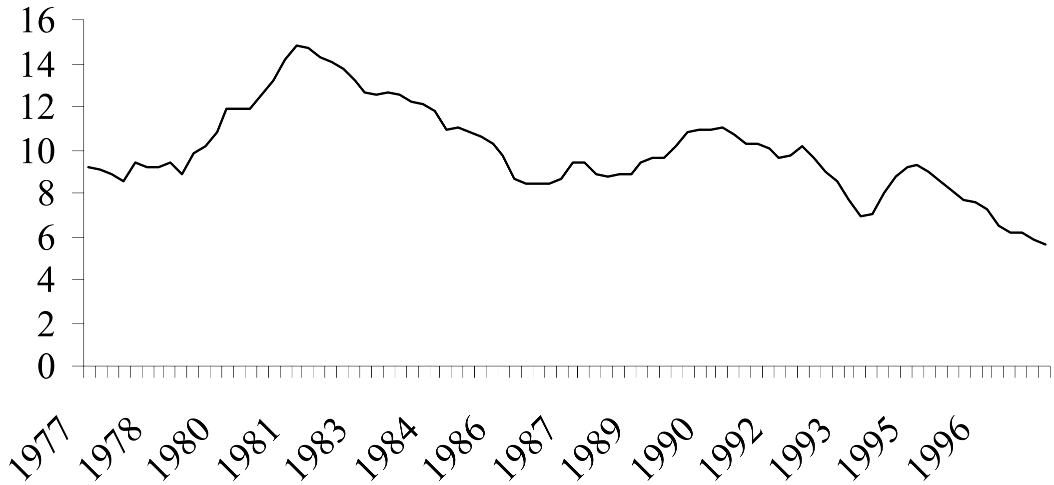
3.5%
3.0%
2.5%
2.0%
1.5%
1.0%
0.5%
0.0%
1977
1978
1980
1982
1984
1985
1987
1989
1991
1992
1994
1996

W



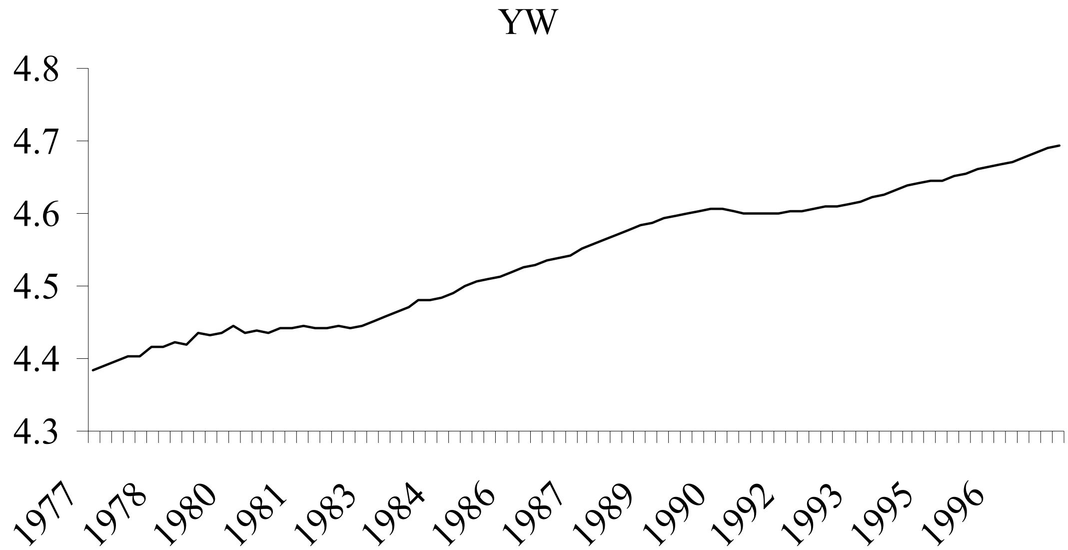
4.0%
3.0%
2.0%
1.0%
0.0%
-1.0%
1977
1978
1980
1982
1984
1985
1987
1989
1991
1992
1994
1996

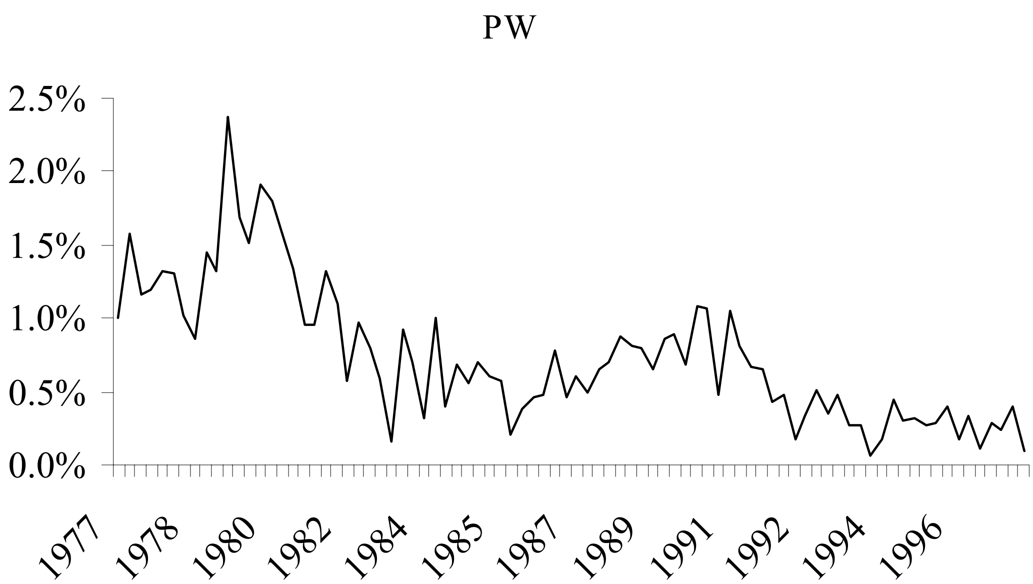
ILT

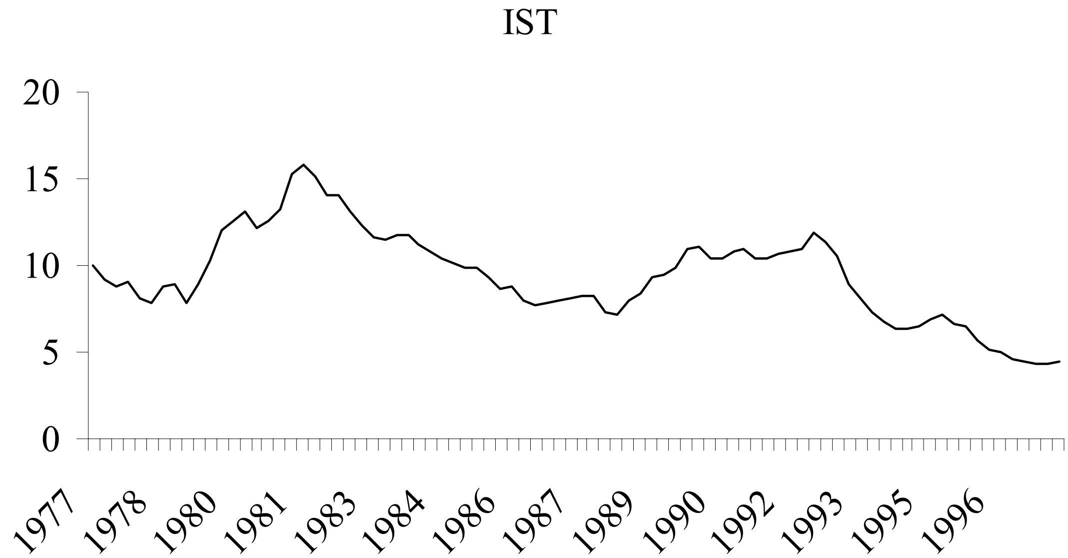

16
14
12
10
8
6
4
2
0
1977
1978
1980
1981
1983
1984
1986
1987
1989
1990
1992
1993
1995
1996

S


5.0
4.9
4.8
4.7
4.6
4.5
4.4
4.3
1977
1978
1980
1981
1983
1984
1986
1987
1989
1990
1992
1993
1995
1996


YW
4.8
4.7
4.6
4.5
4.4
4.3
1977
1978
1980
1981
1983
1984
1986
1987
1989
1990
1992
1993
1995
1996


PW
2.5%
2.0%
1.5%
1.0%
0.5%
0.0%
1977
1978
1980
1982
1984
1985
1987
1989
1991
1992
1994
1996


IST
20
15
10
5
0
1977
1978
1980
1981
1983
1984
1986
1987
1989
1990
1992
1993
1995
1996

WORKING PAPERS

2000

1/00 UNEMPLOYMENT DURATION: COMPETING AND DEFECTIVE RISKS
— *John T. Addison, Pedro Portugal*

2/00 THE ESTIMATION OF RISK PREMIUM IMPLICIT IN OIL PRICES
— *Jorge Barros Luís*

3/00 EVALUATING CORE INFLATION INDICATORS
— *Carlos Robalo Marques, Pedro Duarte Neves, Luís Morais Sarmento*

4/00 LABOR MARKETS AND KALEIDOSCOPIC COMPARATIVE ADVANTAGE
— *Daniel A. Traça*

5/00 WHY SHOULD CENTRAL BANKS AVOID THE USE OF THE UNDERLYING INFLATION INDICATOR?
— *Carlos Robalo Marques, Pedro Duarte Neves, Afonso Gonçalves da Silva*

6/00 USING THE ASYMMETRIC TRIMMED MEAN AS A CORE INFLATION INDICATOR
— *Carlos Robalo Marques, João Machado Mota*

2001

1/01 THE SURVIVAL OF NEW DOMESTIC AND FOREIGN OWNED FIRMS
— *José Mata, Pedro Portugal*

2/01 GAPS AND TRIANGLES
— *Bernardino Adão, Isabel Correia, Pedro Teles*

3/01 A NEW REPRESENTATION FOR THE FOREIGN CURRENCY RISK PREMIUM
— *Bernardino Adão, Fátima Silva*

4/01 ENTRY MISTAKES WITH STRATEGIC PRICING
— *Bernardino Adão*

5/01 FINANCING IN THE EUROSYSTEM: FIXED VERSUS VARIABLE RATE TENDERS
— *Margarida Catalão-Lopes*

6/01 AGGREGATION, PERSISTENCE AND VOLATILITY IN A MACROMODEL
— *Karim Abadir, Gabriel Talmain*

7/01 SOME FACTS ABOUT THE CYCLICAL CONVERGENCE IN THE EURO ZONE
— *Frederico Belo*

8/01 TENURE, BUSINESS CYCLE AND THE WAGE-SETTING PROCESS
— *Leandro Arozamena, Mário Centeno*

9/01 USING THE FIRST PRINCIPAL COMPONENT AS A CORE INFLATION INDICATOR
— *José Ferreira Machado, Carlos Robalo Marques, Pedro Duarte Neves, Afonso Gonçalves da Silva*

10/01 IDENTIFICATION WITH AVERAGED DATA AND IMPLICATIONS FOR HEDONIC REGRESSION STUDIES
— *José A.F. Machado, João M.C. Santos Silva*

2002

1/02 QUANTILE REGRESSION ANALYSIS OF TRANSITION DATA
— *José A.F. Machado, Pedro Portugal*

2/02 SHOULD WE DISTINGUISH BETWEEN STATIC AND DYNAMIC LONG RUN EQUILIBRIUM IN ERROR CORRECTION MODELS?
— Susana Botas, Carlos Robalo Marques

3/02 MODELLING TAYLOR RULE UNCERTAINTY
— Fernando Martins, José A. F. Machado, Paulo Soares Esteves

4/02 PATTERNS OF ENTRY, POST-ENTRY GROWTH AND SURVIVAL: A COMPARISON BETWEEN DOMESTIC AND FOREIGN OWNED FIRMS
— José Mata, Pedro Portugal

5/02 BUSINESS CYCLES: CYCLICAL COMOVEMENT WITHIN THE EUROPEAN UNION IN THE PERIOD 1960-1999. A FREQUENCY DOMAIN APPROACH
— João Valle e Azevedo

6/02 AN "ART", NOT A "SCIENCE"? CENTRAL BANK MANAGEMENT IN PORTUGAL UNDER THE GOLD STANDARD, 1854-1891
— Jaime Reis

7/02 MERGE OR CONCENTRATE? SOME INSIGHTS FOR ANTITRUST POLICY
— Margarida Catalão-Lopes

8/02 DISENTANGLING THE MINIMUM WAGE PUZZLE: ANALYSIS OF WORKER ACCESSIONS AND SEPARATIONS FROM A LONGITUDINAL MATCHED EMPLOYER-EMPLOYEE DATA SET
— Pedro Portugal, Ana Rute Cardoso

9/02 THE MATCH QUALITY GAINS FROM UNEMPLOYMENT INSURANCE
— Mário Centeno

10/02 HEDONIC PRICES INDEXES FOR NEW PASSENGER CARS IN PORTUGAL (1997-2001)
— Hugo J. Reis, J.M.C. Santos Silva

11/02 THE ANALYSIS OF SEASONAL RETURN ANOMALIES IN THE PORTUGUESE STOCK MARKET
— Miguel Balbina, Nuno C. Martins

12/02 DOES MONEY GRANGER CAUSE INFLATION IN THE EURO AREA?
— Carlos Robalo Marques, Joaquim Pina

13/02 INSTITUTIONS AND ECONOMIC DEVELOPMENT: HOW STRONG IS THE RELATION?
— Tiago V. de V. Cavalcanti, Álvaro A. Novo

2003

1/03 FOUNDING CONDITIONS AND THE SURVIVAL OF NEW FIRMS
— P.A. Geroski, José Mata, Pedro Portugal

2/03 THE TIMING AND PROBABILITY OF FDI:
An Application to the United States Multinational Enterprises
— José Brandão de Brito, Felipa de Mello Sampayo

3/03 OPTIMAL FISCAL AND MONETARY POLICY: EQUIVALENCE RESULTS
— Isabel Correia, Juan Pablo Nicolini, Pedro Teles

4/03 FORECASTING EURO AREA AGGREGATES WITH BAYESIAN VAR AND VECM MODELS
— Ricardo Mourinho Félix, Luís C. Nunes