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**Business Cycles: Cyclical Comovement  
Within the European Union in the  
Period 1960-1999.  
A frequency domain approach**

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# Business Cycles:

## *Cyclical comovement within the European Union in the period 1960 – 1999. A frequency domain approach.*

João Valle e Azevedo \*

### **Abstract**

This paper provides a descriptive analysis of the business cycles of the European Union countries and of the two main industrialised countries outside the Union, the United States and Japan. We use the spectral analysis to identify three main features of the business cycles:

- 1- The duration of the business cycle.
- 2- The degree of correlation, in the frequency domain, of the business cycles.
- 3- The identification of leading and lagging countries with respect to the business cycles of a reference series.

We conclude that the United States, Italy and Greece have the shortest cycles, with an average duration around eight years. Japan, Spain and Austria have the longest cycles, lasting more than ten years. All the other countries lie in between with an average duration ranging from eight to nine years. By comparing the business cycles of the various countries with the Euro Area business cycle we conclude that Sweden, Finland, Great Britain and the United States lead the Euro Area by more than one year. The Netherlands, Italy, Japan and Spain are also leading countries but with a lead of no more than one year. There is evidence of counter-cyclical behaviour for Denmark in a sub-period of the sample and no reliable conclusions can be stated for Greece and Ireland. The remaining countries exhibit a high degree of correlation with the Euro Area business cycles and with a lag of no more than three-quarters, with the exception of Austria.

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## 1. Introduction

To study the properties and relations among business cycles we need first to define and measure the business cycle. Since the influential work of Burns and Mitchell at the NBER (1946), the literature on this issue has grown increasingly. There are two main ways to describe the business cycle. One refers to the peaks and troughs in the *level* of the series. The dating of these peaks and troughs results in a “classical” cycle and follows the definition and methodology of Burns and Mitchell. Business cycles are defined as:

“... a type of fluctuations found in the aggregate economic activity of nations that organise their work mainly in business enterprises: a cycle consists of expansions occurring at about the same time in economic activities, followed by similarly general recessions, contractions and revivals which merge into the expansion phase of the next cycle; this sequence of changes is recurrent but not periodic; in duration business cycles vary from more than one year to ten or twelve years; they are not divisible into shorter cycles of similar character with amplitudes approximating their own.”

With this definition, one cannot determine business cycles with a single time series, such as GDP. Comovement among economic variables and a division of business cycles into separate phases or regimes are the key features of this approach. It implies a strong element of judgement and it is being replaced by various methods that follow the seminal contribution of Lucas (1977) who defined business cycles as “movements about trend in gross national product”. *Detrended* series are used in order to obtain a “growth” cycle, which is the deviation of output from its long-term trend, or potential output. Potential output is associated with the permanent component of output while the business cycles, or output gap, correspond to a transitory component.

Since the important work of Nelson and Plosser (1982), suggesting that output is best characterised as an integrated series, it is recognised that measuring potential output is a difficult task. Potential output cannot be treated as a deterministic trend. Linear detrending of an integrated economic time series neglects the changes in the growth component of the series and leads to an overestimation of the variance and persistence of the cyclical component.

The methods used to recover the two components still continue being proposed. Two related problems emerge in these proposals. The first is the definition of business cycle fluctuations. Are these fluctuations the deviations of the path that output would follow if price and wage rigidities were absent? What are the properties of the trend and its relations with the cycle? Are the trend and cycle innovations correlated? The second problem is the question of a statistical vs. an economic-based decomposition. Should we measure without theory? There is clearly a trade-off between the operationality of the methods and the comprehension of the mechanisms that generate business cycle dynamics. But, as argued by Canova (1998), there is still room for improvements in the economic-based decompositions. They still exhibit a considerable lack of definition of the trend and cyclical components and little discussion is undergoing concerning the models and its links to the actual fluctuations.

Our purpose is to compare the business cycles of a set of countries. If we used an economic-based decomposition we would have to keep in mind the specification errors of more than a dozen of models, one for each country. Also, a



statistical approach can support a definition of business cycle that comprises some *a priori* information. For instance, if there is information from other macroeconomic variables suggesting that business cycles are fluctuations within a range of periodicities, a natural definition arises in terms of these fluctuations. It is clear that in this case the option is to detrend the series, trying to isolate the desired fluctuations, using the wide range of filtering methods available. These are the two-sided moving averages, first differencing, Hodrick-Prescott filter, the band-pass filters and so on. We will justify our use of the Hodrick-Prescott filter in obtaining the business cycles.

This paper provides a descriptive analysis of the cyclical behaviour of the European Union economies. The exercise is useful, but leads us to state the need of developments in the construction of statistical indicators on cyclical divergence. In a context of single currency and common monetary policies in the Euro Area, the synchronisation of the business cycles of the participant countries should be a major concern.

We stress the idea that the purpose of our analysis is merely descriptive. There is no attempt to state that the main features of the recent past can be relevant in future analysis or judgement.

The paper is organised as follows. In section 2 we present some concepts that introduce the spectral analysis and the consequences of filtering a time series. Section 3 provides our justification for the use of the Hodrick-Prescott filter, showing that many criticisms against it are unwarranted. Section 4 gives an insight into the tools of the spectral analysis that we need to describe the business cycles and section 5 presents the main results. Section 6 concludes.

## 2. The frequency domain analysis

In this section we introduce some important definitions used in the frequency domain analysis. We discuss the consequences of filtering a time series, relating these consequences to the measurement of the business cycle.

### 2.1 Spectral representation

In the frequency domain approach, a series can be interpreted as being constituted of infinite components with different periods and amplitudes. Namely, the trend corresponds to a component with an infinite period (that component is not repeated over time) and the “noise” corresponds to components with very short periods. In the between, we can find all the other components, the “cyclical components”, that account for all the other movements in a series. With these ideas in mind it is better understood that it is possible to give a representation of a covariance-stationary real process in terms of all these components, the so-called spectral representation or *Cramér’s representation* (see e.g. Harvey 1992):

$$y_t = \mu + \int_{-\pi}^{\pi} e^{i\omega t} z(\omega) d\omega \quad (1)$$

where  $i = \sqrt{-1}$ ,  $\mu$  is the mean of the process,  $\omega$  is measured in radians and ranging from  $-\pi$  to  $\pi$  and  $z(\omega)d\omega$  are complex orthogonal increments with variance  $f_y(\omega)$ , which is a continuous function.



That is, we can decompose a series in an infinite weighted sum of orthogonal periodic functions, each with random amplitude coefficient. It can be proven that

$$f_y(\omega) = \frac{1}{2\pi} \left( \gamma(0) + 2 \sum_{\tau=1}^{\infty} \gamma(\tau) \cos(\omega\tau) \right) \quad (2)$$

where  $\gamma(\tau)$  is the autocovariance function. Setting  $\tau = 0$  in (2) gives:

$$\gamma(0) = 2 \int_0^{\pi} f_y(\omega) d\omega. \quad (3)$$

$f_y(\omega)$  is called the *spectrum* of a series and it represents the decomposition of the variance of the process in terms of its components. Roughly, if there is a maximum in  $f_y(\omega)$  corresponding to some frequency  $\omega^*$ , this means that the component corresponding to that frequency is of significant importance in the behaviour of a series, that is, we will probably identify a dominant wave in the series with period  $\frac{2\pi}{\omega^*}$ , corresponding to that frequency. Instead of the spectrum one could use the *spectral density*, which is the spectrum divided by the variance of the process.

## 2.2 Filtering a time series

When we filter a series what we are doing is to change the relative importance of its components and possibly, the timing of their relations. If we apply a linear time-invariant filter  $W(L) = w_{-r}L^{-r} + \dots + w_{-1}L + w_0 + w_1L + \dots + w_sL^s$  to the stationary series  $y_t$  in (1), we obtain a transformed series  $y_t^*$ , given by

$$y_t^* = \sum_{j=-r}^s w_j y_{t-j} = W(L)y_t, \text{ which we assume that is still stationary.}$$

It is easy to verify that the spectrum of the transformed series is related to that of the original one by the expression:

$$f_{y^*}(\omega) = |W(e^{-i\omega})|^2 f_y(\omega), \quad (4)$$

where  $|W(e^{-i\omega})|^2$  is called the *transfer function* of the filter  $W(L)$  and it represents the changes in the contribution of the original components to the variance of the new process. So, in general, the variance and the autocovariances of the series are changed. This is easy to conclude by looking to expression (2). If the spectrum changes, the autocovariances also change.

In terms of the spectral representation of the process it follows from (1) that

$$y_t^* = \int_{-\pi}^{\pi} e^{i\omega t} W(e^{-i\omega}) z(\omega) d\omega, \quad (5)$$



where  $W(e^{-i\omega}) = \sum_{j=-r}^s w_j e^{-i\omega j}$  denotes the *frequency response function*. To interpret the new representation, we may write the frequency response function in polar form. Letting  $W(e^{-i\omega}) = G(\omega)e^{-iPh(\omega)}$ , where  $G(\omega)$  is the *gain* and  $Ph(\omega)$  the *phase*, we obtain:

$$y_t^* = \int_{-\pi}^{\pi} e^{i\omega(t - \frac{Ph(\omega)}{\omega})} G(\omega) z(\omega) d\omega, \quad (6)$$

that is, the amplitude of the components is altered by the factor  $G(\omega)$  while there is a shift in time if  $Ph(\omega) \neq 0$ . If a filter is symmetric ( $w_j = w_{-j}$ ,  $j = 1, 2, \dots, r = s$ ) there is no phase effect and the transfer function is simply the squared gain.

We should note that the interpretation of the gain function also applies when an integrated series is filtered. We will precise this statement in section 3. The commonly used filters in business cycle research render stationary time series when applied to integrated series. This is an important point, since literature in business cycle filters often leads to misleading interpretations of filtered series.

### 2.3 Definition of business cycles

We follow Baxter and King (1999), who define business cycles as “fluctuations with a specified range of periodicities” (in their case this range is from six to thirty two quarters). The researcher provides the characteristics of the business cycle by retaining only the components of interest. We do not provide an economic interpretation of the fluctuations and we do not need to explain them. They simply exist and we extract them.

We want to choose a filter such that it eliminates the low frequencies in the series, the slowly evolving components of that series, and preserves the components that account for the short-run major fluctuations, fluctuations that we will define as lasting less than twelve years (we use annual data). But why twelve years? Our purpose is to impose less rigidity in the definition of business cycle frequencies while considering the wide range of series being treated. One of the purposes of this paper is to identify differences in the duration (or in the persistence) of the different cycles. With this broader definition those differences and some patterns can be better distinguished.

We impose only an upper bound in the periodicity of the isolated fluctuations, that is, high frequencies are not disregarded. This means that we want to use a *high-pass filter*. The Hodrick-Prescott (HP) filter, which we will use, is an *high-pass filter*. The *ideal high-pass filter* should retain the desired frequencies and perfectly eliminate the remaining while inducing no phase shift<sup>1</sup>

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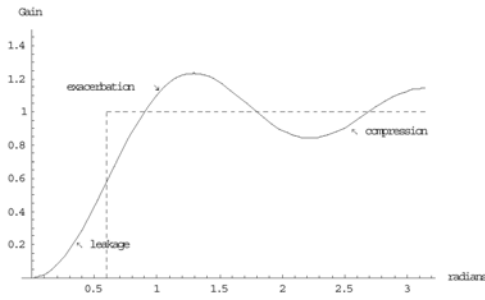
<sup>1</sup> A *high-pass filter* retains high frequency components and attenuates fluctuations at low frequencies. If we want to eliminate fluctuations below some specified frequency  $\omega_h$  (the cut-off frequency), the frequency response function of the *ideal high-pass filter* in the range  $[-\pi, \pi]$  is,

$$W_h(\omega) = \begin{cases} 1, & |\omega| \geq \omega_h \\ 0, & |\omega| < \omega_h \end{cases}$$



Ideal filters cannot be constructed to apply in finite samples. Every filter fails to retain perfectly the desired frequencies (*compression* and/or *exacerbation* effect) and frequencies that it should suppress pass the filter (*leakage* effect). We show in figure 1 the gain function of a high-pass filter that was supposed to isolate fluctuations above the cut-off frequency determined by the vertical dashed line. If the gain function is higher than one in a range of frequencies above the cut-off frequency, the fluctuations corresponding to those frequencies are being expanded relative to the original series (*exacerbation*). Others are being attenuated (*compression*), in which case the gain function is lower than one. There is also a *leakage* effect, frequencies below the desired cut-off frequency are not completely eliminated.

**Figure 1- The distortionary effects of a filter**



Baxter and King (1999) construct symmetric linear time-invariant filters that approximate the ideal filters by choosing the filter weights so as to minimise

$$Q = \frac{1}{2\pi} \cdot \int_{-\pi}^{\pi} |W^*(\omega) - W_k(\omega)|^2 d\omega, \quad (7)$$

where  $W^*(\omega)$  is the frequency response function of the ideal filter,  $W_k(\omega)$  is the frequency response function of a filter within the referred class and  $k$  denotes the maximum lag length of the resulting two-sided moving-average. We recognise that the Baxter-King (BK) filters are a good and justified attempt to approximate the ideal filters, but we will not use them for a practical reason. We would lose too many observations. A good approximation to the ideal filters implies a large lag length in the resulting two-sided moving-average, which means loss of observations in the endpoints of the sample.

### 3. HP-filter - unwarranted scepticism

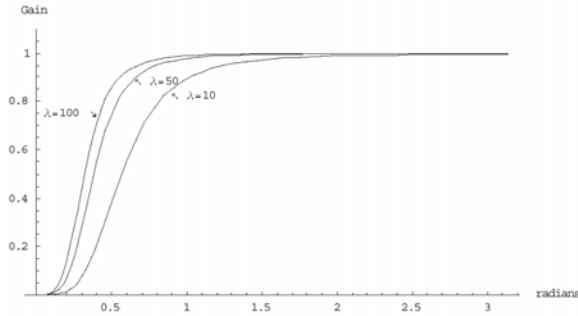
In this section we review the main properties of the HP filter, including its gain function, and also analyse the spurious cyclicity problem and that of the smoothing parameter choice. We show that the criticisms of Cogley and Nason (1995) and Harvey and Jaeger (1993) are unjustified, which does not imply that we should not interpret results from HP filtered data carefully.



### 3.1 Gain function of the HP filter<sup>2</sup>

The HP filter is, as deduced by King and Rebelo (1993), equivalent to an infinite moving-average symmetric filter with time-varying coefficients. It does not induce phase shift and removes unit root components up to the fourth order. In figure 2 we plot the gain function of the cyclical HP filter for values of  $\lambda$  commonly used when applying the filter to annual data.

**Figure 2 – Gain function of the HP filter**



This function ( $C(\omega) = \frac{4\lambda(1 - \cos(\omega))^2}{1 + 4\lambda(1 - \cos(\omega))^2}$ ), along with the already referred

properties, completely characterises the HP filter. High frequencies are almost unaffected and *exacerbation* effects are absent at every frequency. Moreover, there is no cycle in the gain function of the HP filter. It behaves as a very effective high-pass filter and this is the reason why we use it. Notice however that there are always some *leakage* and *compression* effects, independently of the considered cut-off frequency.

At this point it should be clear that our emphasis is in the frequency domain properties of the HP filter. We only analyse the properties of the filter by comparing it with the referred ideal filters. We do not use the HP filter as an operational definition of the trend and cyclical components, we view it as an approximation to the ideal filters. We turn now to the analysis of some criticisms made to the HP filter and see that most of them are unwarranted.

### 3.2 The HP filter does not induce spurious cycles

The comprehension of the consequences of filtering a time series is crucial to this analysis. Many criticisms to the HP filter forget such consequences. For instance, differences between second moments (variances, autocorrelations and

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<sup>2</sup> The HP filter decomposes a time series  $y_t$  into a cyclical component  $y_c$  and a growth component  $y_{Tr}$ .

$$y_t = y_{Tr,t} + y_{c,t}$$

The method used to recover the two components requires the minimisation of the following problem:

$$\min \sum_{t=1}^T (y_t - y_{Tr,t})^2 + \lambda \sum_{t=2}^{T-1} [(y_{Tr,t+1} - y_{Tr,t}) - (y_{Tr,t} - y_{Tr,t-1})]^2$$

$$\{y_{Tr,t}\}_{t=1}^T$$



cross-correlations) of filtered and unfiltered series cannot be taken as an evidence of a weak performance of the filter as do King and Rebelo (1993), Guay and St-Amant (1997) and Canova (1998). The differences are a consequence of filtering, as pointed in section 2, and would arise even if an ideal filter were used.

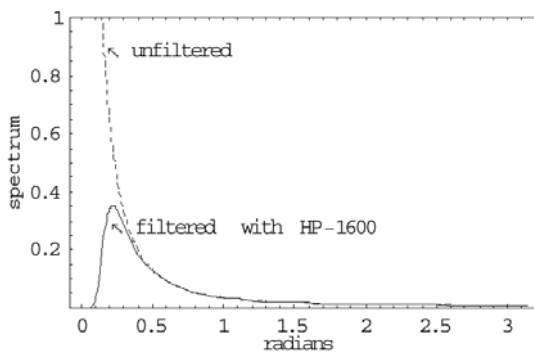
As noted before the HP filter, like any other applicable filter, fails to isolate perfectly a range of frequencies. It has some distortionary effects that often lead some authors to conclude that it induces spurious cycles. We will borrow from Pedersen (1998), who analyses only stationary processes, to show that these conclusions are unjustified and extend the arguments to the case of non-stationary processes.

One classical example of the distortionary effects of filtering is the Slutsky-effect, which describes the spurious ciclicity induced by some filters. The Slutsky-effect can in general be characterised by a peak in the transfer function of the considered filter.

Harvey and Jaeger (1993), Cogley and Nason (1995) and Guay and St-Amant (1997) criticise the HP filter for inducing business cycle periodicity in integrated or near-integrated time series, those with the “typical spectral shape” of Granger (1966). A peak in the spectrum of HP filtered series, which is absent in the spectrum of the original series, is taken as an evidence of Slutsky-effect (or spurious ciclicity). Pedersen (1998) shows that these critics rest on an “inadequate definition of the Slutsky-effect – a definition which has the unfortunate consequence that even an ideal high-pass filter induces a Slutsky effect”. If we define the Slutsky-effect as a cycle in the transfer function of a filter, then the HP filter does not induce a Slutsky effect. There is clearly no cycle in the transfer function of the HP filter.

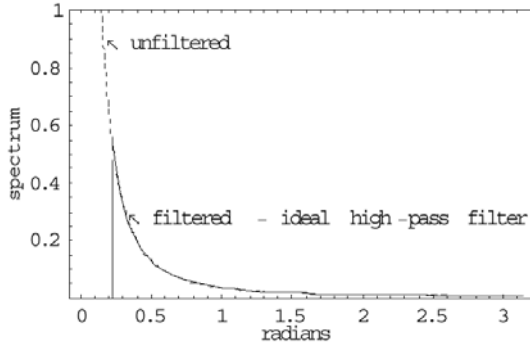
Consider a highly persistent stationary series,  $y_t = 0.9y_{t-1} + \varepsilon_t$ . The spectrum of this process is shown in figure 3 as well as the spectrum of the process filtered with the HP ( $\lambda=1600$ ). The spectrum of the filtered series is exactly what we should expect. We are using a high-pass filter with some *leakage* and *compression* effects relative to some ideal filter. If we had applied an ideal high-pass filter with a cut-off frequency  $\omega_c = \pi/16$  (corresponding to 32 time periods) the spectrum of the filtered process would have a spiked peak at the cut-off frequency (figure 4). The differences arise because the HP is not an ideal filter.

**Figure 3 – Spectrum of filtered (HP-1600) and unfiltered AR(1)**





**Figure 4 – Spectrum of filtered (ideal high-pass filter) and unfiltered AR(1)**



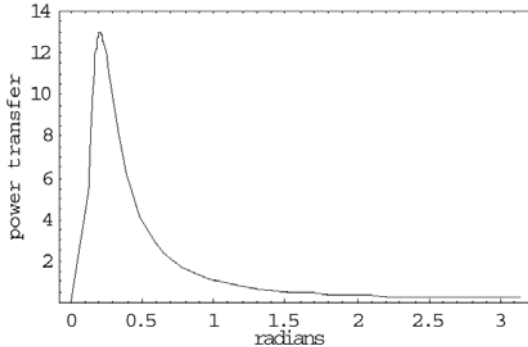
All the theory presented so far is based on the assumption of covariance stationarity. Most macroeconomic time series are clearly non-stationary. A non-stationary process cannot be represented as a “sum” of sine and cosine waves with orthogonal amplitude coefficients. The traditional approach is to “transform” the process into a stationary form. Differencing the data a sufficient number of times is the most common procedure. This can often be very useful but in many cases it may hide or distort relevant characteristics of the original process. We should analyse what we are losing or transforming relative to the original series, not analyse only the stationarised series. We are introducing these ideas because there is a common mistake when the HP filter is analysed, concerning its application to integrated series.

Harvey and Jaeger (1993) and Cogley and Nason (1995) show that there is a peak in the power transfer function of a *subcomponent* of the HP filter. This subcomponent is isolated to interpret the effects of the filter in  $I(1)$  time series. Let the frequency response function of the HP cyclical filter be decomposed as:

$$C(\omega) = (1 - e^{-i\omega}).C_1(e^{-i\omega}), \quad \text{with } C_1(\omega) = \frac{\lambda e^{2i\omega} (1 - e^{-i\omega})^3}{\lambda e^{2i\omega} (1 - e^{-i\omega})^4 + 1} \quad (8)$$

According to this decomposition, applying the HP filter to an  $I(1)$  time series is equivalent to: First, filter the non-stationary time series with the first difference filter and then filter the remaining stationary component with the asymmetric filter determined by  $C_1(\omega)$ . The transfer function of this second filter is plotted in figure 5. It has a clear peak at business cycle frequencies that Harvey and Jaeger classify as “a classical example of the Yule-Slutsky-effect”.

**Figure 5 – Power transfer function of the  $C_1(\omega)$  filter ( $\lambda=1600$ )**

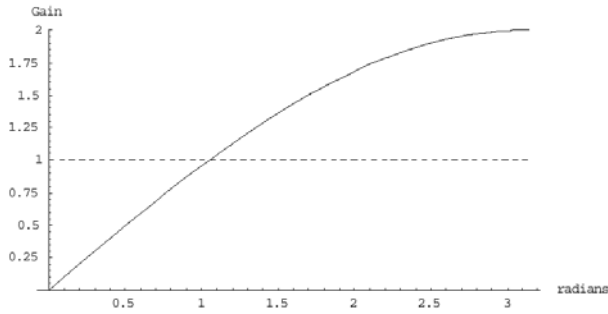




An ideal high-pass filter can be decomposed in the same way and generate a spiked peak in the transfer function of the equivalent subcomponent, as Pedersen (1998) shows. But he does not interpret what this implies in terms of the spectral characteristics of the resulting stationary series. We think that in order to analyse the consequences of the filtering process we must look to the total frequency response function  $C(\omega)$  and not only to the subcomponent  $C_1(\omega)$ . In fact, by looking solely to the effects of  $C_1(\omega)$ , we are overlooking the consequences of the filter  $(1-L)$ . We know that this filter, in the stationary case, expands the existing high-frequency fluctuations (see figure 6).

So, the real issue is to interpret the total effects of the filter on an integrated series. Even if we could not understand the effects of the first-difference filter on an integrated series, it would be misleading to analyse only the consequences of the *subcomponent* of the HP filter on the differenced series.

**Figure 6 – Gain function of the First -Difference filter**



To start answering these questions let us first introduce the concept of *pseudo-spectrum* of an integrated series. The pseudo-spectrum of a general ARIMA process is defined as (see Harvey (1993))

$$f_y^p(\omega) = \frac{|\theta(e^{-i\omega})|^2 \sigma_\varepsilon^2}{|(\Delta(e^{-i\omega}))^s \varphi(e^{-i\omega})|^2}, \quad (9)$$

where  $\theta(L)$  is the moving-average polynomial,  $\varphi(L)$  the auto-regressive polynomial,  $\Delta(L) = 1 - L$  the first-difference operator,  $\sigma_\varepsilon^2$  the variance of the white-noise innovations and  $s$  the order of integration of the series. The pseudo-spectrum can be thought of as the “limit” of the spectrum of a stationary process, when the largest autoregressive roots tend to 1. It has the same interpretation as the spectrum except at zero frequency, which is associated with unit-root components. The existence of a vertical asymptote at zero frequency reflects the fact that components with infinite period, or non-periodic components, dominate the behaviour of the series. Of course, the autocorrelation function does not exist so the pseudo-spectrum is not the Fourier transform of an autocorrelation function. Nor the integral of the pseudo-spectrum in the range  $[-\pi, \pi]$  equals the variance of the process, because this variance does not exist. However, an extension of the relation stated in (4) holds:

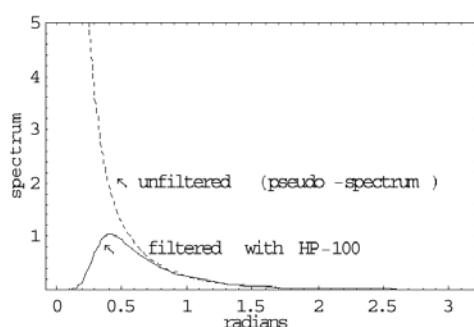
$$f_{y^*}(\omega) = |W(e^{-i\omega})|^2 f_y^p(\omega), \quad (10)$$



where  $y^*$  is a filtered series, which we may assume that is stationary,  $|W(e^{-i\omega})|^2$  is the transfer function of the correspondent filter and  $y$  is an integrated series. In figure 7 we show the “typical” pseudo-spectrum of an economic integrated time series.

Simply by looking to the pseudo-spectrum, it is almost impossible to identify a dominant stationary component in the series. This means that business cycle fluctuations will hardly be identified through the pseudo-spectrum. The spectrum of this process filtered with the HP( $\lambda=100$ ) is also shown in figure 7. Clearly, the filter attenuates fluctuations at low frequencies and retains high frequency components. An ideal high-pass filter would define a sharp cut in the spectrum of the resulting stationary series.

**Figure 7– Typical Pseudo-Spectrum of an economic integrated time series**



Using the HP filter we are not expanding fluctuations of the original series, as a “Slutzky” filter would do. We are just approximately isolating them. The existence of a peak in the spectrum of the filtered series cannot be interpreted as the creation of a spurious cycle. It is just a consequence of the dominant role played by the low frequencies in the spectrum. Cogley and Nason (1995) do not resort to the concept of pseudo-spectrum. They only analyse the effects of a part of the filter on the first-differences of a difference-stationary (DS) process.

All these arguments do not imply that we should unquestionably use the HP filter. For instance, similar spectral patterns would arise if the original series were a random walk. By applying the HP filter to a random walk process, we are again defining a trend and allowing specific frequencies to remain in the cyclical component. But does it make sense to filter a random walk? HP filtering is certainly “a sensible way to look at the data” (Kaiser and Maravall (1999)), just as is ideal high-pass filtering<sup>3</sup>.

Should these similarities concern us, if we know that more or less regular fluctuations affect output and other economic variables? The fact is that these fluctuations are not identified in a spectral sense. Sargent (1987) and more recently Pagan (1999), point this fact. This is the reason why increasing the smoothing

<sup>3</sup> We should also note that all the criticisms made to the HP filter, concerning its application to DS processes, can be applied to the BK filters (and in fact, to any moving-average high-pass filter). Murray (2000) decomposes the frequency response function of a general BK filter as  $W_{BK}(\omega) = (1 - e^{-i\omega}) \cdot W_{BK}^1(\omega)$  and one can now easily guess the arguments that follow. The reasoning presented in this paper can also be applied to weaken these arguments that start appearing against the BK filters.



parameter results in a spectral peak located at a lower frequency. There is no “truth” in any isolated cyclical component. Prior information must be added through the specification of business cycle fluctuations. An important element of definition is therefore needed.

Unfortunately, the preceding discussion is based on the infinite sample version of the HP filter. Some problems may arise when it is applied to the data. The gain function of the filter is not exactly the one showed above for some of the observations. Namely, in the endpoints of the sample it provides a distorted picture of the theoretical infinite sample filter. This happens because in the endpoints the HP filter must become a one-sided moving-average. For that reason some authors recommend dropping three observations near the endpoints of the sample. This is a valid argument that Baxter and King (1999) use to support the use of their filter. However, Kaiser and Maravall (1999) show how the cyclical signal can be improved in these points by applying the filter to the series extended with proper backcasts and forecasts.

### ***3.4 Duration of the cycle***

As we have seen before, the peak in the spectral density, which can be interpreted as the modal period (or duration) of the cycle, is mostly determined by the value of the smoothing parameter. Given this important element of definition we share the view of Kaiser and Maravall (1999): “The analyst should first decide the length of the period around which he wishes to measure economic activity.” The variance of the cyclical component will therefore be mostly justified by fluctuations around the critical length. Given a desired cut-off frequency this criterion is equivalent to choose the smoothing parameter so as to obtain a peak in the spectrum at that frequency.

It is perfectly reasonable to ask how do we want to identify differences in the duration of the cycles when that duration is clearly dependent on the smoothing parameter. What happens is that if we apply the HP filter with the same  $\lambda$  to different series, the differences in the duration of the cycles will be exclusively determined by the stochastic structure of the various series. Is this information relevant, or are the differences just an artifact? Viewed in this context, an ideal filter would be undesirable, because it would deterministically fix the modal period of the cycle.

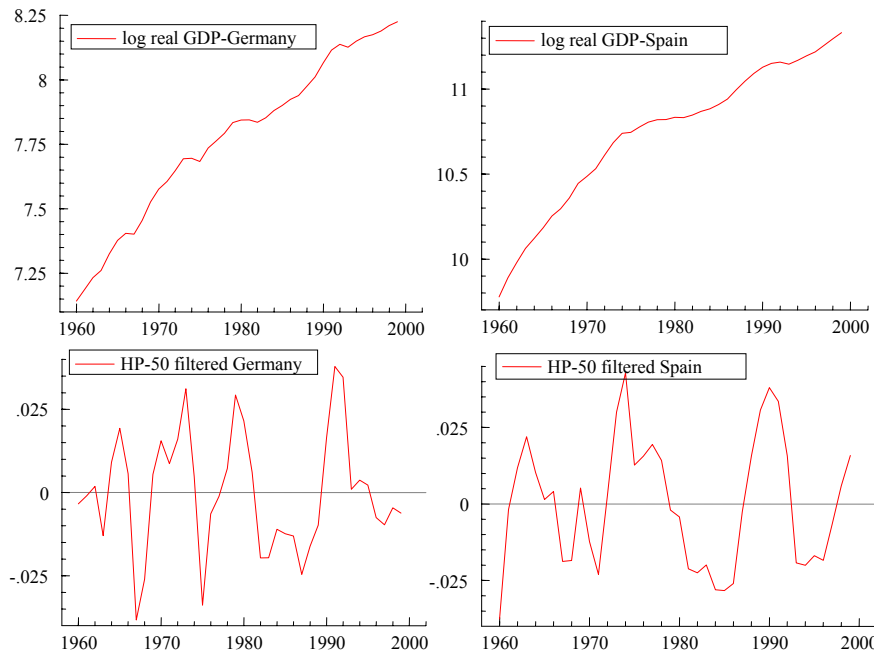
We do not claim that the HP filter is preferable when compared to an ideal filter. We just argue that the use of the same  $\lambda$  in all the series can give us a sign of what are the differences that we should impose in the definition of cyclical fluctuations, or equivalently, what is the modal period of a “well-defined” cyclical component. Observing some empirical regularities we are tempted to believe that the differences that arise can really be a measure of the differences in the persistence of the various business cycles. For instance, using values of  $\lambda$  from 10 to 400 and estimating the modal period of the cycles for some of the countries considered, the order relations are remarkably maintained<sup>4</sup>.

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<sup>4</sup> For example, in the case of Italy the estimated modal period of the cycle ranged from 6.96 to 9.35 years (always the lowest except with  $\lambda=10$ ), from 6.69 to 11.73 in the case of Germany and from 8.83 to 16.4 in the case of Spain (always the highest).



**Figure 8 – Filtered and unfiltered log of real GDP –Germany and Spain**



According to figure 8, the cyclical component of Spain appears to be more persistent than the one of Germany. This suggests that the definition of business cycle frequencies should be different in these two countries. In order to allow for different business cycles frequencies in different countries, we must choose  $\lambda$  such that no constraints are imposed on the “well-defined” modal period of the cycle for any country. That is why we chose  $\lambda=50$ , since it allows for a modal period of up to 12 years, a reasonable upper bound given the wide range of series in analysis. We will also use the popular value of  $\lambda=100$  in order to verify how the results are affected when we consider lower frequencies. It must be emphasised that our intention is not to provide a true value of the duration of the business cycles. Given the needed element of definition the results obtained must be viewed as relative measures of the business cycles persistence.

Increasing the smoothing parameter will also result in a higher regularity of the cycle around the critical length. This happens because the variance of the cyclical component becomes more concentrated around the critical length. It turns that in this case the regularity would not be equal in all the series, even if we used the same ideal filter. The arguments presented before can therefore be applied with less stringent assumptions to support the interpretation of the measure of regularity of the cycle that we will present below.

#### **4. Spectrum, Coherency and Phase effect**

In this section we review the concepts of coherency and phase. We also propose indicators of the persistence and regularity of the business cycle as well as indicators of cyclical correlation and lead/lag relation between business cycles.



#### 4.1 Estimation of the Modal and average duration of the cycle

The concept of spectrum was presented in section 2. Recall that the spectrum gives us the relative importance of the constituent components of a series. We will estimate the spectral density<sup>5</sup> (the standardised spectrum) of the business cycles obtained with the HP filter in order to determine the most relevant components of those series. It will then be possible, for each series, to determine the period corresponding to the peak in the spectral density. This period, as already referred, can be interpreted as the modal duration of the business cycle.

We can also define the *mean frequency* over a frequency band that includes the modal frequency. We will define the *mean frequency* as:

$$\bar{\omega} = \frac{\int_{\omega_m - \varepsilon_1}^{\omega_m + \varepsilon_2} \omega f_c(\omega) d\omega}{\int_{\omega_m - \varepsilon_1}^{\omega_m + \varepsilon_2} f_c(\omega) d\omega}, \quad (11)$$

where  $\omega_m$  is the modal frequency and  $\varepsilon_1, \varepsilon_2 > 0$  define an interval around the modal frequency. The denominator transforms the weighting function into a density in the interval  $[\omega_m - \varepsilon_1, \omega_m + \varepsilon_2]$ . This interval should only contain frequencies that are important to the variability of the cycle series and that were supposed to be isolated by the HP filter. Corresponding to this *mean frequency* there is a periodicity  $2\pi / \bar{\omega}$  that we call the *average duration* of the business cycle. Of course, if  $f_c(\omega)$  is symmetric around  $\omega_m$  and the considered band is centred in  $\omega_m$ , then the average duration and the modal duration coincide.

We recognise that the filtering procedure is constraining our interpretation of this *average duration*, but our intention is not to provide a “true” value of the duration of the business cycles. What is interesting is the comparison of this measure across countries, given the same definition of business cycle fluctuations.

#### 4.2 Cycle regularity

Another quantity of interest is the concentration of power in the vicinity of the modal frequency ( $\omega_m$ ). That quantity is a measure of the *regularity* of the cycle in terms of its duration. We define such quantity as:

$$R = \int_{\omega_m - \Delta\omega_m}^{\omega_m + \Delta\omega_m} f_c(\omega) d\omega, \quad (12)$$

where  $\Delta\omega_m$  is the length of a small interval around  $\omega_m$ . However, this measure of regularity for a particular country has little meaning if not compared with the same

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<sup>5</sup> The discussion of the various estimation methods of the spectral density is beyond the scope of this analysis (for a detailed discussion see e.g. Priestley (1981)). We used two methods. In the first, a *Parzen* “window” was used to weight the correlogram. The second is a pre-whitening technique, which implies the estimation of an AR(4). It is a fully non-parametric approach and much more reliable, given our forty observations sample.



measure for other countries. So, we will divide it by the value obtained with the Euro Area cycle series in order to have a “*regularity index*”.

#### 4.3 Multivariate Spectrum, Coherency and phase effect

The analogue of (3) in the multivariate case is the *spectral matrix* or *multivariate spectrum* (see Priestley (1981) and Harvey (1982)):

$$f_Y(\omega) = \frac{1}{2\pi} \cdot \sum_{\tau=-\infty}^{\infty} \Gamma(\tau) e^{-i\omega\tau} \quad (13)$$

where  $\Gamma(\tau)$  is the  $\tau^{\text{th}}$  order autocovariance. Let  $n=2$  and  $Y = [y \ x]^T$ . The diagonal elements of the spectral matrix are the spectrum of the individual processes and the element 1,2 is the *cross-spectrum* between  $y$  and  $x$  which is expressed as:

$$f_{yx}(\omega) = \frac{1}{2\pi} \cdot \sum_{\tau=-\infty}^{\infty} \gamma_{yx}(\tau) \cdot e^{-i\omega\tau} \quad (14)$$

with  $\gamma_{yx}(\tau) = \text{Cov}[y_t, x_{t-\tau}]$ . The *cross-spectrum* contains all the information concerning the relations between the two series in the frequency domain and it is, in general, complex valued. It can therefore be decomposed in its imaginary and real parts:

$$f_{yx}(\omega) = co_{yx}(\omega) - i \cdot qu_{yx}(\omega),$$

$co_{yx}(\omega)$  is the *cospectrum* between the two variables and it represents the covariance between  $y$  and  $x$  attributable to the fluctuations determined by  $\omega$ . It can also be interpreted as the covariance between the “in-phase” components of the two processes, components whose phases are matched in time.  $qu_{yx}(\omega)$  is the *quadrature spectrum* and it represents the covariance between the “out of phase” components of the two processes. Fluctuations of significant importance in the series, as captured by large values of  $f_y(\omega)$  and  $f_x(\omega)$ , may not have an important contribution to the contemporaneous covariance between the variables, simply because they are in different phases of the implied cycle. The *quadrature spectrum* searches for these unmatched fluctuations.

The analogue of the autocorrelation function in the frequency domain is the spectral density. The analogue of the cross-correlation function is *coherency*. *Coherency* is defined as:

$$C_{yx}(\omega) = \frac{|f_{yx}(\omega)|}{\sqrt{f_x(\omega)f_y(\omega)}} \quad (15)$$

Coherency can be viewed as a measure of the correlation between the components of two series at different frequencies. It ranges from 0 to 1. If coherency is high at a particular frequency, this means that the components of each series corresponding to that frequency, are highly correlated. But this measure is completely independent of the position in time of the two series. Coherency adjusts the series in time so that the components phases match. For



example, if the business cycles obtained from two series had exactly the same shape, but one of the business cycles series was lagged with respect to the other, coherency would be high for every frequency. What matters is the cyclical behaviour. If it is similar, coherency is high.

We are not interested in the value of coherency at every frequency. The important frequencies are the dominant ones, those that define the cyclical pattern or, in other words, account for the major fluctuations in the derived cycles. What we will do is to estimate coherency between various cycle series and then compute the “*mean coherency*” at the relevant frequencies. It is now clear that these relevant frequencies must be an interval containing the dominant frequencies of the cycle series considered, which are the frequencies around the peak in the spectrum of those series. So, we propose that the *mean coherency* between two cycle series be defined as:

$$\hat{C}_{yx} = \frac{\int_{\hat{C}_{yx}}^{\text{Max}(\omega_{m,x}, \omega_{m,y}) + \Delta} \hat{C}_{yx}(\omega) d\omega}{\text{Max}(\omega_{m,x}, \omega_{m,y}) - \text{Min}(\omega_{m,x}, \omega_{m,y}) + 2\Delta}, \quad (16)$$

where  $\hat{C}_{xy}(\omega)$  is the estimated coherency,  $\omega_{m,x}$  and  $\omega_{m,y}$  are the frequencies corresponding to the peaks in the spectrum of the cycle series and  $\Delta$  is the length of a small interval. If the *mean coherency* between two cycle series is high, this means that the major movements in one cycle series are highly correlated with the major movements of the other cycle series.

But coherency, as said, is independent of the position in time of the series. It tells us only whether or not two series have the same pattern. Fortunately, there is a concept that will help us to characterise much better the relations between the series: the *phase effect*. Mathematically its definition is:

$$Ph_{yx}(\omega) = \text{ArcTan}\left(\frac{-\text{Im}[f_{yx}(\omega)]}{\text{Re}[f_{yx}(\omega)]}\right) = \text{ArcTan}\left(\frac{qu_{yx}(\omega)}{co_{yx}(\omega)}\right), \quad (17)$$

where Im and Re denote, respectively, the imaginary and real parts of a number.

The phase effect in time units is simply  $\frac{Ph_{yx}(\omega)}{\omega}$  and it represents, for each

frequency, the shift in time between the corresponding components of two series. Once again, we will only be interested in this phase effect at the relevant frequencies, the frequencies that define the dominant waves of the cycles. Therefore, we propose an indicator of the discrepancies in time of the main components of two series, the “*mean phase effect*”, defined as:

$$\hat{Ph}_{yx} = \frac{\int_{\hat{Ph}_{yx}}^{\text{Max}(\omega_{m,x}, \omega_{m,y}) + \Delta} \frac{\hat{Ph}_{yx}(\omega)}{\omega} d\omega}{\text{Max}(\omega_{m,x}, \omega_{m,y}) - \text{Min}(\omega_{m,x}, \omega_{m,y}) + 2\Delta}, \quad (18)$$

with the already known notation. We should note that to have a consistent interpretation of this value, the dominant frequencies of the cycle series considered should not be very different. As we will see, the differences are quite



acceptable and will allow us to interpret the *mean phase effect* as the average lead or lag of the business cycle of country A when compared with the business cycle of country B.

Based on the interpretation of the *cospectrum* Croux, Forni and Reichlin (1998) proposed a measure of correlation in the frequency domain, which they called *dynamic correlation*:

$$\rho_{yx}(\omega) = \frac{co_{yx}(\omega)}{\sqrt{f_y(\omega) \cdot f_x(\omega)}} \quad (19)$$

This is a measure of the correlation between the components of two series, but it depends on the phase shift between those components. *Dynamic correlation on a frequency band* defined by the interval  $\Lambda$  is determined as follows:

$$\rho_{yx}(\Lambda) = \frac{\int_{\Lambda} co_{yx}(\omega) d\omega}{\sqrt{\int_{\Lambda} f_y(\omega) d\omega \int_{\Lambda} f_x(\omega) d\omega}} \quad (20)$$

Having the values of coherency and phase it will be easy to guess the results provided by these measures. If the phase effect is negligible in a range of frequencies, the absolute value of dynamic correlation will be almost equal to coherency in that range. Relevant phase effect will result in important differences between the two measures.

We have finally all the concepts needed to characterise the features of the defined business cycles that we had in mind<sup>6</sup>.

## 5. Results

In this section we present the main empirical results, based on the estimation of the spectrum (univariate and bivariate). We present the estimated modal and average duration of the various cycles as well as the proposed regularity index. In order to analyse the relations among business cycles, we present the estimated mean coherency, mean phase-effect and dynamic correlation between the cycle series of the various countries.

To anticipate some results one should look to the plots of the obtained cycle series in appendix B.

In Table 1 we present the estimated modal duration of the various cycles, obtained with the three methods described and with the two smoothing parameters considered ( $\lambda=50$  and  $\lambda=100$ ). With the VAR estimation, we used the cycle series of the considered country and the cycle series of the Euro Area (EU11). To calculate the VAR estimate of the EU11 cycle series spectral density, we chose the Belgium cycle series as second variable, because it has a small weight in the Euro Area and exhibits a high degree of correlation with it.

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<sup>6</sup> As in the case of the spectral density we present the results from two estimation methods, in order to give some robustness to our findings. In the first, a *Parzen* “window” was used to weight the multivariate correlogram. The second is a pre-whitening technique, which implies the estimation of a VAR(2). This pre-whitening estimation provides also another estimate of the spectral density of the cycle series, an estimate that uses the information of the two series considered. Having estimated the three relevant quantities of the spectral matrix, the estimation of coherency and phase is straightforward.



**Table 1**  
*modal duration of the cycle (years)*

	$\lambda=50$			$\lambda=100$		
	Pre-whitening AR(4)	Parzen window	Pre-whitening VAR(2)	Pre-whitening AR(4)	Parzen window	Pre-whitening VAR(2)
Bel	9.04	9.98	9.18	10.19	11.42	11.04
Dnk	8.33	9.18	8.60	8.88	9.78	9.55
Deu	8.28	8.81	8.31	9.26	10.09	9.20
Grc	7.56	6.95	7.42	8.39	7.98	9.02
Fra	9.07	9.51	9.02	9.69	10.21	11.54
Esp	11.39	12.30	11.86	12.58	14.15	13.61
Ire	8.18	8.48	7.28	9.41	10.21	7.95
Ita	7.72	8.26	8.14	8.03	8.26	8.81
Nld	8.61	10.61	8.30	13.7	19.01	11.31
Aut	10.46	11.46	9.49	11.95	13.62	12.21
Prt	8.81	8.93	8.40	9.36	9.06	9.42
Fin	9.41	9.82	8.44	9.90	10.34	9.09
Swe	8.30	9.60	7.80	9.20	10.77	9.05
Gbr	8.32	8.99	7.47	8.87	9.75	8.15
Jpn	9.98	10.96	10.03	11.54	14.49	14.57
Usa	7.38	7.84	7.01	7.72	8.44	7.54
EU11	9.28	9.98	8.76	10.34	11.39	10.40

The most reliable estimate is, as noted before, the one referred as “Pre-whitening AR(4)”. Identifying sharp peaks in the spectral density with the usual estimation methods can be a difficult task. The pre-whitening techniques identify these patterns much better and are recommended in short samples (see Priestley (1981)). The Parzen window estimate is larger in most cases but the order relations are generally maintained across the methods.

Obviously, the modal duration of the cycles is smaller with  $\lambda=50$ . The maximum modal duration allowed in this case is 12,3 years (Esp). With  $\lambda=100$  14,5 years is the limit if the “outlier” corresponding to The Netherlands were not considered.

The order relations are in general maintained if we use  $\lambda=100$ . However, with Austria, The Netherlands, Japan and Spain, the countries with the longer periods, the use of  $\lambda=100$  clearly changes the modal duration of the cycles. On the opposite, the countries with short cycles have the modal period almost unaffected across the  $\lambda$ 's. In all the others the use of  $\lambda=100$  increases the modal duration of the cycles in half/one year.

We present in Table 2 the defined average duration of the cycle based on the pre-whitening spectral density estimate. With  $\lambda=50$ , periodicities from 6 to 12 years were considered and with  $\lambda=100$  the upper bound was increased to 15 years. The upper bound is defined according to the desired maximum modal duration allowed. Frequencies that remain are a consequence of the leakage effect of the HP-filter. We chose six years as the lower bound because there was no evidence of significant variability in the series attributable to lower periodicities. The referred intervals justify in most cases the important movements in the series.



**Table 2**  
**average duration of the cycle (years)**

<b>Pre-whitening AR(4)</b>		
	$\lambda=50$	$\lambda=100$
Bel	8.72	9.14
Dnk	8.31	8.76
Deu	8.28	8.86
Grc	8.02	8.45
Fra	8.85	9.43
Esp	9.66	10.52
Ire	8.19	9.18
Ita	8.06	8.25
Nld	8.36	8.97
Aut	9.31	10.17
Prt	8.72	8.76
Fin	9.03	9.52
Swe	8.27	8.77
Gbr	8.28	8.69
Jpn	9.16	10.01
Usa	7.77	8.07
EU11	8.79	9.36

The differences between the modal and the average duration of the cycles are more pronounced when we use  $\lambda=100$  and in the countries with a high modal duration of the cycle. This happens with Spain, Austria and Japan and in a lower degree with the Euro Area. This was predictable since in these cases the spectral density is not nearly symmetric around the modal frequency in the relevant intervals, as is with the other series. We are disregarding relevant low frequencies that have passed the filter and consequently determining the value of the mean period with a clear dependence on its definition. The interpretation should therefore be made carefully in these cases.

As we will see The Netherlands denotes an irregular cyclical behaviour. When we use  $\lambda=100$  it is difficult to identify a dominant component in the series. This can explain in this case the apparent inconsistency between the modal and the average duration of the cycle.

One could classify the countries according to the modal and average duration of the cycles ( $\lambda=50$ ):

*Long cycles* - Spain, Japan and Austria.

*Medium-Long cycles* - EU11, France, Belgium, Finland and Portugal

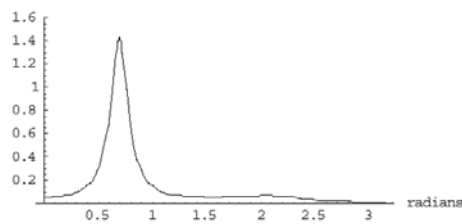
*Medium cycles* -Sweden, United Kingdom, The Netherlands, Germany, Denmark and Ireland

*Short cycles* - United States, Italy and Greece.

Let us now address the regularity issue. To better understand the measure proposed in section 4, let us start by comparing two extreme cases (France and The Netherlands) that have arisen when the spectral density and the regularity measure were estimated. In the case of France there is a very sharp peak in the spectral density of the derived cycle series (figure9).

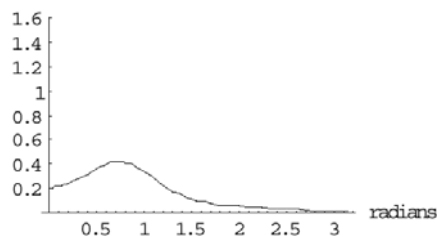


**Figure 9 – France- cycle series spectral density  $\lambda=50$**



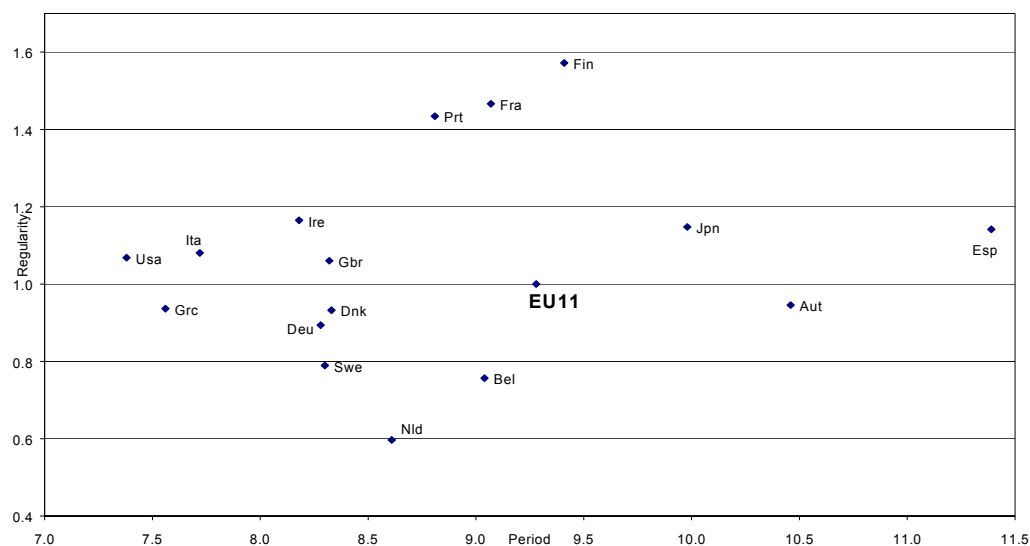
The components that account for the variability in this series are clearly defined, and around frequency  $\omega = 0,7$ . This means that the various cycles obtained have on average approximately nine years. But we can also say that there is little variability in the duration of each one. The power of the spectral density is concentrated around the referred frequency, i.e., no other components can explain the major movements in the cycle series. Then, the integral of the spectral density in a small interval centred on the dominant frequency should be high (with  $\Delta\omega_m = \pi/25$  in (12),  $R=0.25$ ). If the spectral density is very smooth, the identification of the dominant component can also be done. However, a wide range of frequencies accounts with almost equal importance to the movements in the series. This is the case for The Netherlands (with  $\Delta\omega_m = \pi/25$  in (12),  $R=0.1$ ). See figure10

**Figure 10 -The Netherlands- cycle series spectral density  $\lambda=50$**



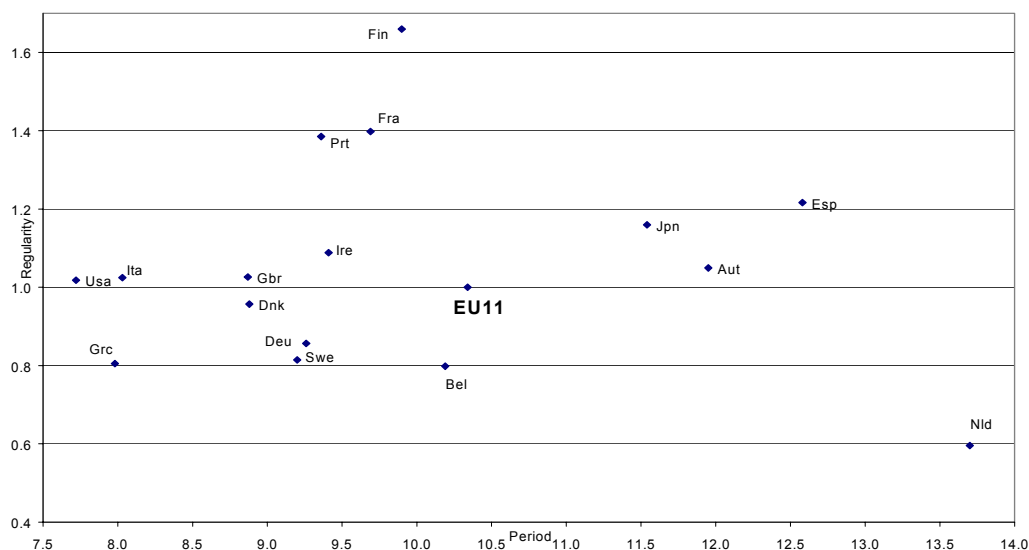
In figures 11 and 12 we present the proposed regularity index plotted against the modal period of the cycle, for the estimate based on the pre-whitening.

**Figure 11 -Modal Period-Regularity index with  $\lambda=50$**





**Figure 12 -Modal Period-Regularity index with  $\lambda=100$**



The countries with the highest regularity in the duration of its cycles are clearly Finland, France and Portugal. On the opposite, The Netherlands has an extremely low value in the regularity index. Remember also that for this country the difference in the modal/mean duration of the cycle across the  $\lambda$ 's was very pronounced. These results tell us that for The Netherlands, the cyclical pattern is not well defined.

In the cases of Japan and Spain we can say that the long duration of its cycles is not due to a poor cyclical behaviour. There is evidence that the cycles of these two countries are in fact long.

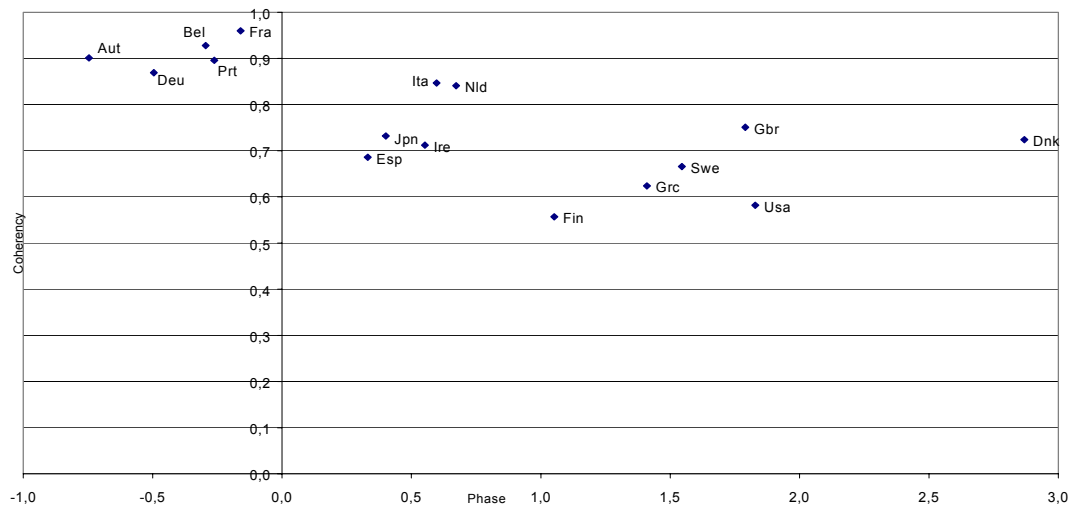
For the other countries (probably with the exception of Belgium and Sweden) the regularity in the duration of the cycles is similar to that of the Euro Area.

In figures 13 to 16 we present the estimates of the *mean phase effect* and *mean coherency* between each country and the Euro Area. We plot them together in order to assess the relative cyclical position of the various countries when compared with the Euro Area. We should note that the estimation of these quantities is less accurate when the true coherency is low. High coherency means higher precision in the estimation. This will be empirically verified when comparing the estimates given by the two methods described.

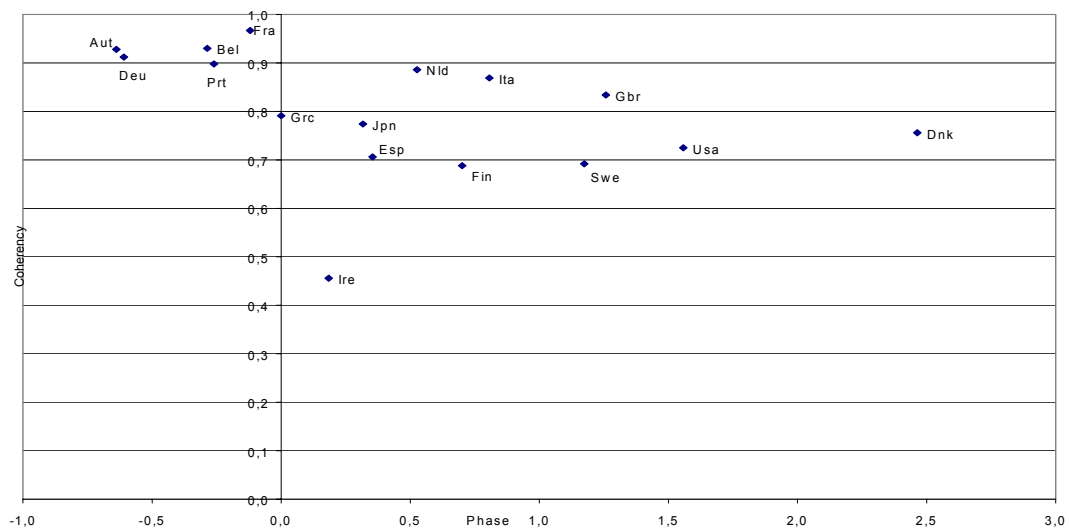
The interpretation of the *mean phase effect* should be made carefully. When we say that country A leads country B by one year, this means that the phases of country A cycles, lead on average the phases of country B cycles by one year.



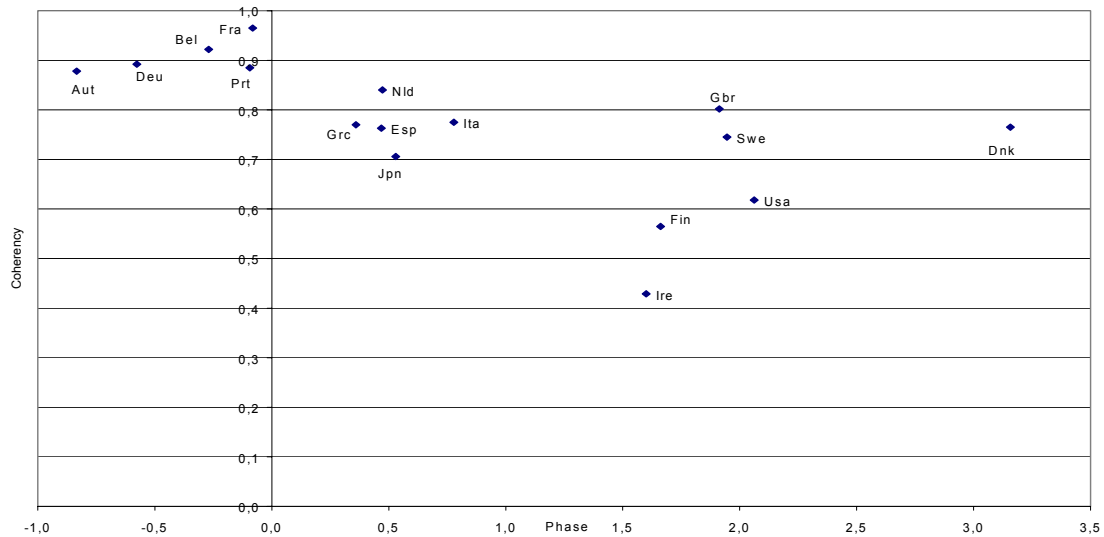
**Fig. 13 - Mean-phase effect (years) – Mean coherency (country vs. EU11, Parzen window -  $\lambda=50$ )**



**Fig. 14- Mean-phase effect (years) – Mean coherency (country vs. EU11, VAR(2) -  $\lambda=50$ )**

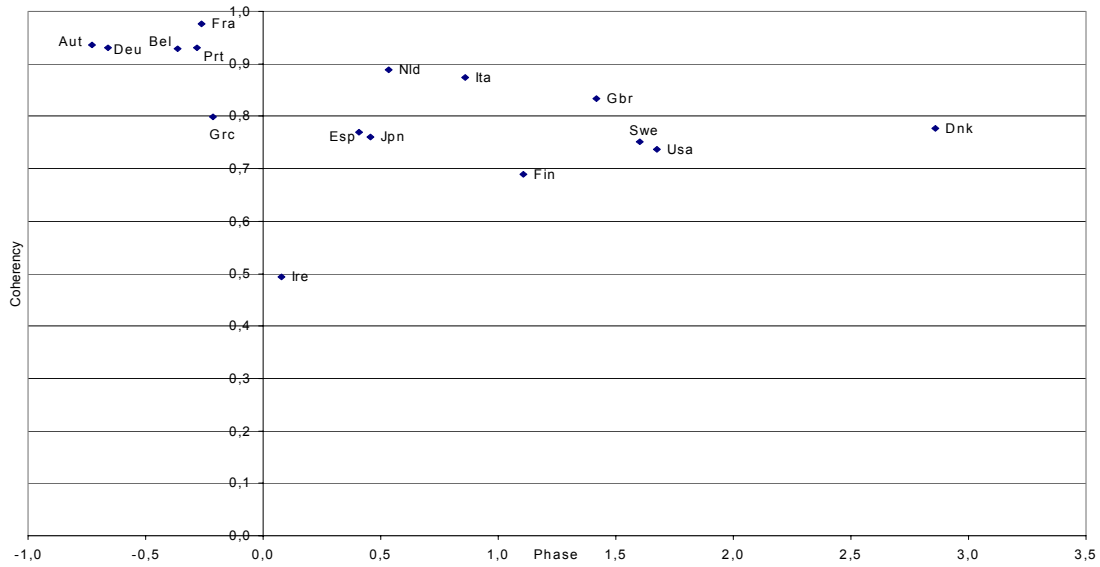


**Fig. 15 - Mean-phase effect (years) – Mean coherency (country vs. EU11, Parzen window -  $\lambda=100$ )**





**Fig. 16 - Mean-phase effect (years)– Mean coherency (country vs. EU11, VAR(2) -  $\lambda=100$ )**



The mean-phase effect is naturally smaller with  $\lambda=50$  (the modal duration of the cycles is smaller). For that reason, some patterns are better distinguished with  $\lambda=100$ .

There is significant robustness in the results for five countries (Austria, Germany, Belgium, Portugal and France), across the estimation methods and across the  $\lambda$ 's. Austria and Germany are a bit more lagged while the others have a cycle whose dominant components are almost contemporaneous with the Euro Area ones. Mean coherency is very high, i.e., the cyclical pattern is very similar when compared with the Euro Area one.

A second cluster could include The Netherlands, Italy, Spain and Japan. They are all leading countries in cyclical terms (but always no more than one year), with The Netherlands and Italy having higher coherency and higher mean phase effect.

Sweden, Great Britain, Finland and the United States can be considered the “strongly leading countries”. Mean coherency is high in the case of Great Britain (around 0.75-0.82) and consistently the lowest in the case of Finland. Finland has also the lowest mean phase effect of this group, even lower than Italy in the case of the VAR estimate with  $\lambda=50$ . The United States have always the highest mean phase effect (always higher than 1.5 years).

The result for Denmark is due to the counter-cyclical behaviour of this country in the last eighteen-twenty years (see Appendix B).

Finally, no accurate conclusions can be stated for Greece and Ireland. This can be explained by the somewhat “noisy” behaviour of their cyclical components. However, it should be noted that Ireland has a very low coherency, which is not the case of Greece (it is always around 0.8 except for the Parzen window estimate with  $\lambda=50$ ). There is however some robustness for Greece if we consider only the VAR estimates. We may cautiously conclude that Greece has a near zero cyclical lag when compared with the Euro Area.

In table 3 we present the estimated dynamic correlation between each country and the Euro Area (a Parzen window was used to estimate the cospectrum and the



individual spectrums). With  $\lambda=50$  the frequencies considered correspond to periodicities from 6 to 12 years. With  $\lambda=100$  the upper bound was extended to 15 years. The results are what we should expect. The importance of the phase effect determines the difference between mean coherency and the absolute value of dynamic correlation.

**Table 3**  
**Dynamic correlation (Parzen window)**  
**Country vs. EU11**

	$\lambda=50$	$\lambda=100$
Bel	0.91	0.91
Dnk	-0.08	-0.14
Deu	0.81	0.83
Grc	0.14	0.74
Fra	0.95	0.96
Esp	0.65	0.69
Ire	0.69	0.27
Ita	0.78	0.65
Nld	0.74	0.76
Aut	0.80	0.80
Prt	0.85	0.84
Fin	0.49	0.40
Swe	0.44	0.41
Gbr	0.33	0.37
Jpn	0.74	0.69
Usa	0.22	0.22

The sensitivity of results for Greece and Ireland to the value of  $\lambda$  is again evident. The use of a different  $\lambda$  clearly alters the dynamic properties of the cycle series of these two countries.

The high average lead of Denmark in the period considered is reflected in the very low value of this measure of contemporaneous correlation.

The remaining results are consistent with the obvious consequences of the phase-effect in this measure.

There could be some interest in extending this analysis to the comparison between neighbour countries or countries with relevant economic relations. Only the estimates based on the Parzen window method and with  $\lambda=50$  are presented (see appendix C). The results from the VAR estimation are very similar. Given the referred sensibility to  $\lambda$  of Ireland and Greece, we do not present the results for these cases.

We should note that there is only approximate transitivity in the mean phase effect when one compares the various combinations of results provided by three countries. The assessment of this transitivity and the comparison of coherency considering the Euro Area and any two countries lead us to conclude *a posteriori* that the use of the Euro Area cyclical component as a reference series is justified. One could be sceptical about the results obtained with large countries such as France or Germany, but it is easily verified that similar conclusions would be



drawn if we had not a reference series. The cyclical relations hold independently of the reference series.

Analysing the large European Union countries we can conclude that Great Britain is the one denoting the higher degree of cyclical correlation with the United States, followed by Italy. In the case of Great Britain contemporaneous movements can explain that correlation while for Italy there is a lag of more than one year. For Germany and France mean coherency is low and the cyclical lag is higher, reflected in the low value of dynamic correlation.

Finland and Sweden are more correlated with the Euro Area than with the United States, but leading the Euro Area and led by the United States.

The Netherlands and Austria present a high mean coherency when compared with Germany. France denotes a similar behaviour, but in a lower degree. We also note that for some countries mean coherency is high when they are compared with France and lowers substantially when Germany is considered. This happens with Belgium, Denmark, Italy, Finland, Sweden, Great Britain and less obviously with Spain.

Mean coherency is moderately high between Great Britain and Belgium, The Netherlands, Spain, Italy, and Sweden but low between Great Britain and Germany.

We cannot state in general that geographical proximity implies a high degree of cyclical correlation, even adjusting the lags. This can be true for Sweden-Finland, Sweden-Denmark, Austria-Germany, Austria-France, The Netherlands-Germany, Belgium-France, Germany-France, Italy-France and possibly Great Britain-France but fails to be evident in cases like Portugal-Spain, Denmark-Germany or Austria-Italy.

## **6. Conclusions**

Using well-known concepts from the spectral analysis we have described some properties of the business cycles of the European Union countries and also of the United States and Japan.

We conclude that the United States, Italy and Greece have the shortest cycles and Japan, Spain and Austria the longest cycles. Sweden, Finland, Great Britain and the United States lead the Euro Area by more than one year while The Netherlands, Italy, Japan and Spain are also leading countries but with a lead of no more than one year. There is evidence of counter-cyclical behaviour for Denmark in a sub-period of the sample and inconclusive results are found in the cases of Greece and Ireland. The remaining countries exhibit a high degree of correlation with the Euro Area business cycles and with a lag of no more than three-quarters, with the exception of Austria.

We have analysed the frequency domain properties of the HP filter and argued in its favour, showing that many criticisms against it are unwarranted. However, and like Kaiser and Maravall (1999), we have recognised that the HP filter is a “sensible way to look at the data”.



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## Appendix A - Data

Annual series of the GDP (log transformed) from 16 countries, plus the Euro Area (EU11):

- EU11: Germany (**Deu**), France (**Fra**), Italy (**Ita**), Spain (**Esp**), The Netherlands (**Nld**), Belgium (**Bel**), Austria (**Aut**), Finland (**Fin**), Portugal (**Prt**), Ireland (**Ire**) and **EU11** itself.
- United Kingdom (**Gbr**), Denmark (**Dnk**), Sweden (**Swe**) and Greece\* (**Grc**).
- United States (**Usa**) and Japan (**Jpn**).

Period: 1960-1999

Source: European Commission -Ameco

The choice of annual data is imposed by the insufficient or non-existent quarterly series in some of the countries.

With Germany and the Euro Area, there was the need to construct a series due to the obvious lack of information from the unified Germany before 1991. As the data for the unified Germany is available since 1992, we worked backwards with the rate of growth of West Germany in order to “reconstruct” a series for the whole Germany.

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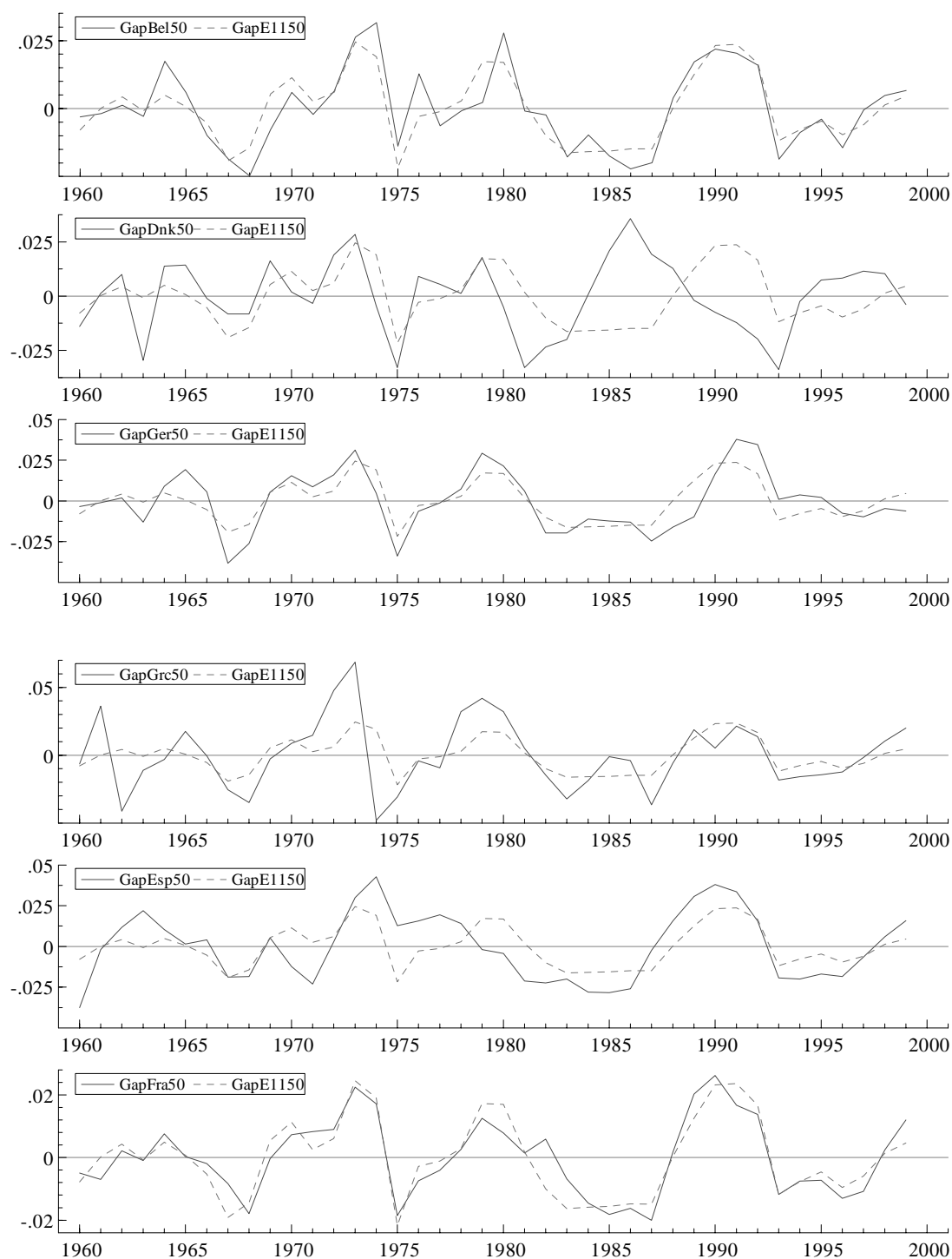
\* Greece is considered a non-participant country since it joined the Euro Area in January 2001



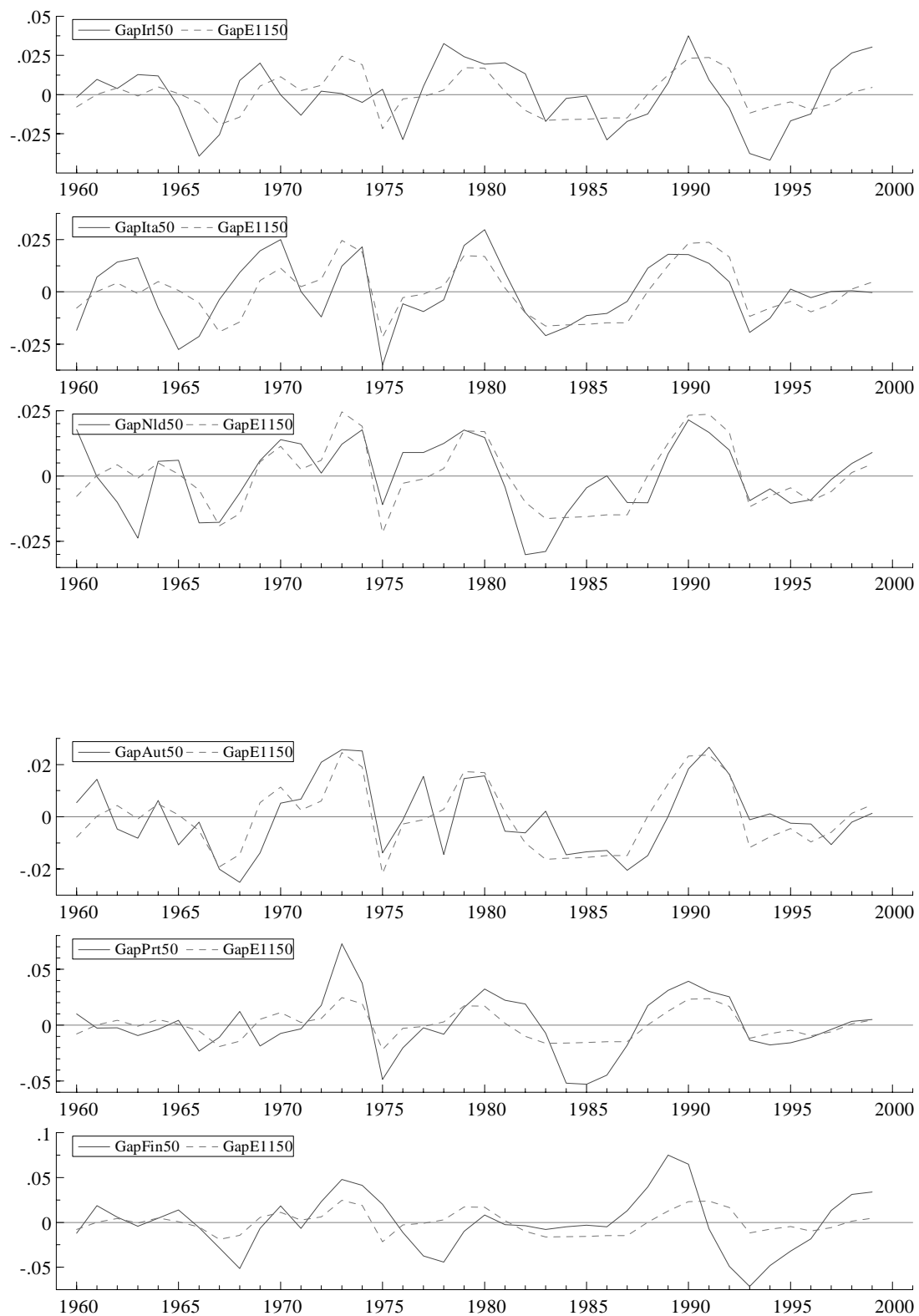
## Appendix B – Plots of the derived cycles

### HP( $\lambda=50$ )

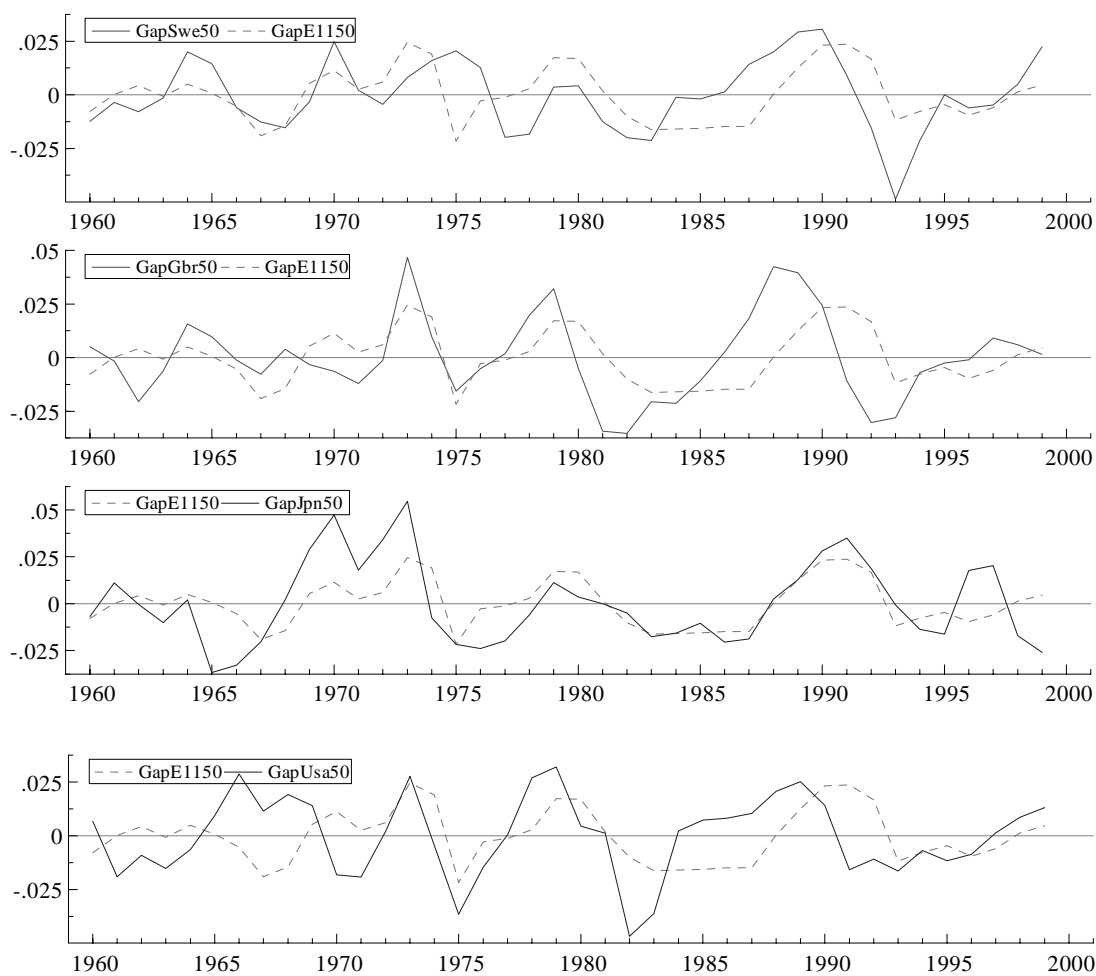
(In the legends, Gapseries $\lambda$  means some log GDP series filtered with HP- $\lambda$ )



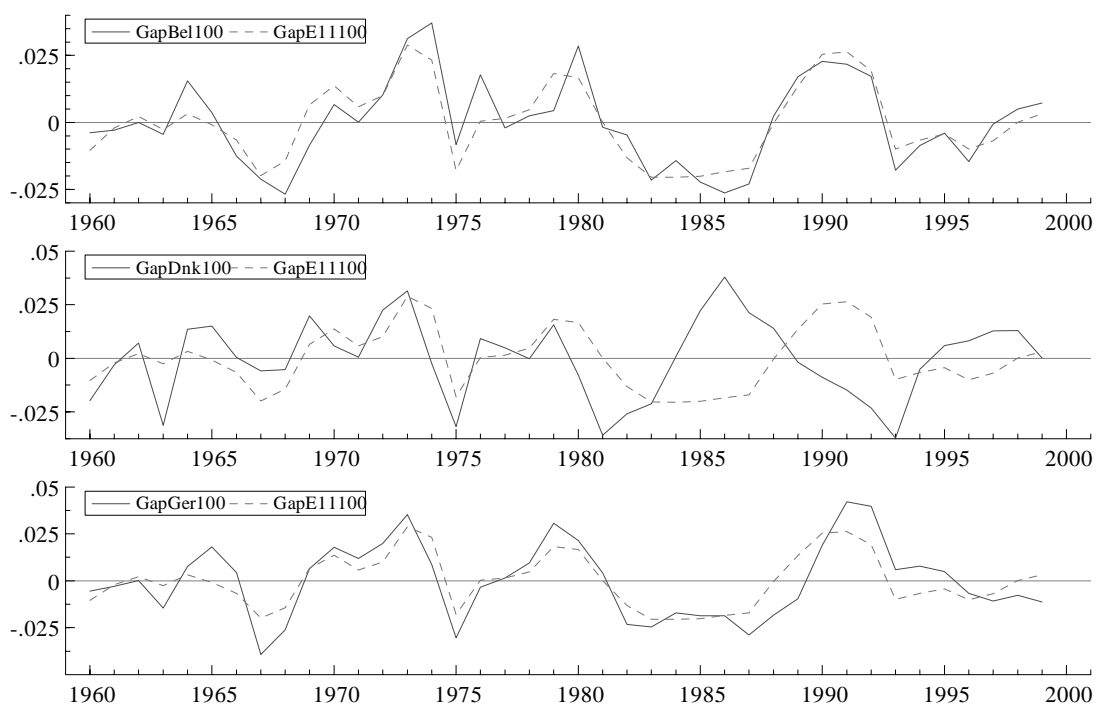




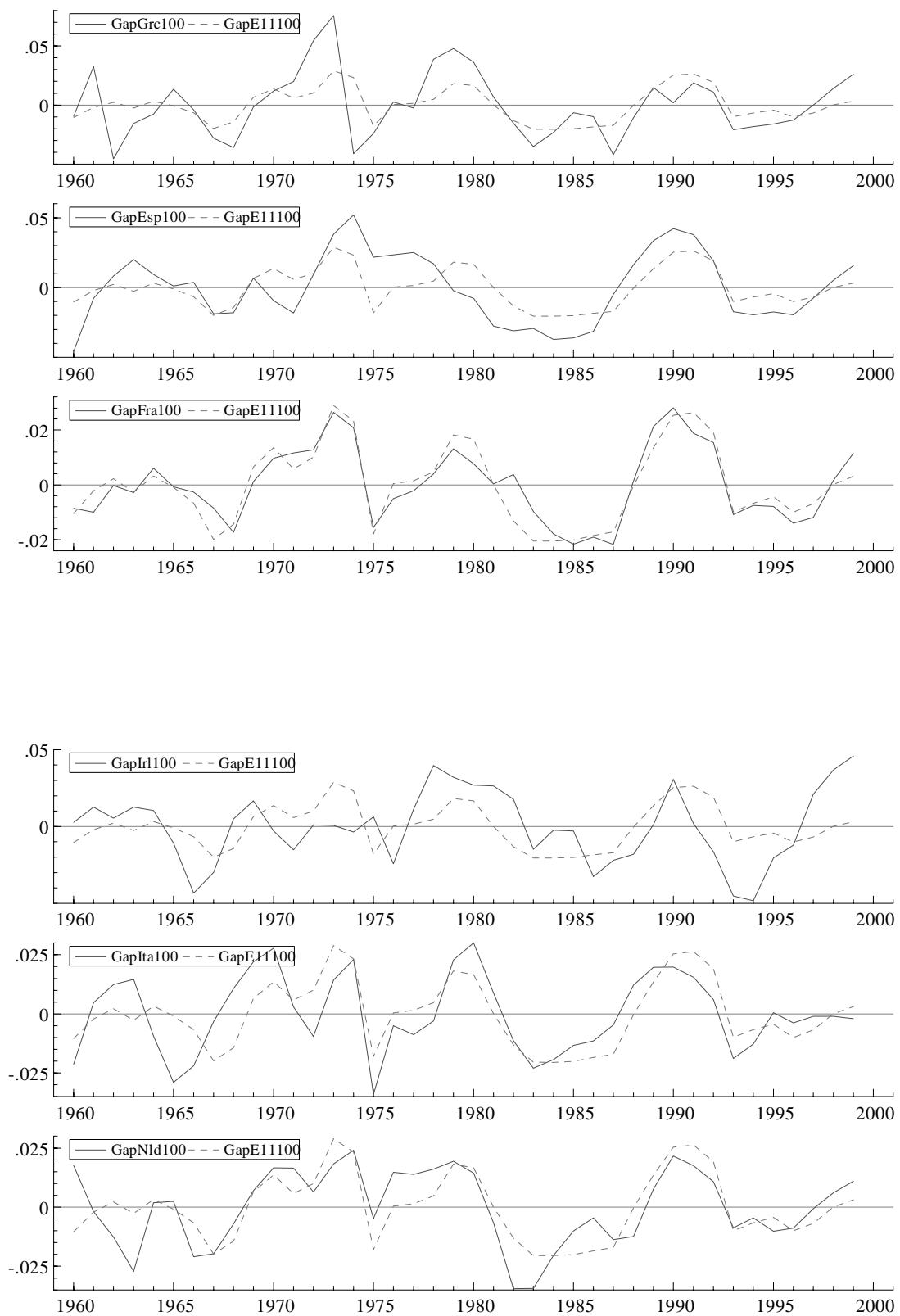




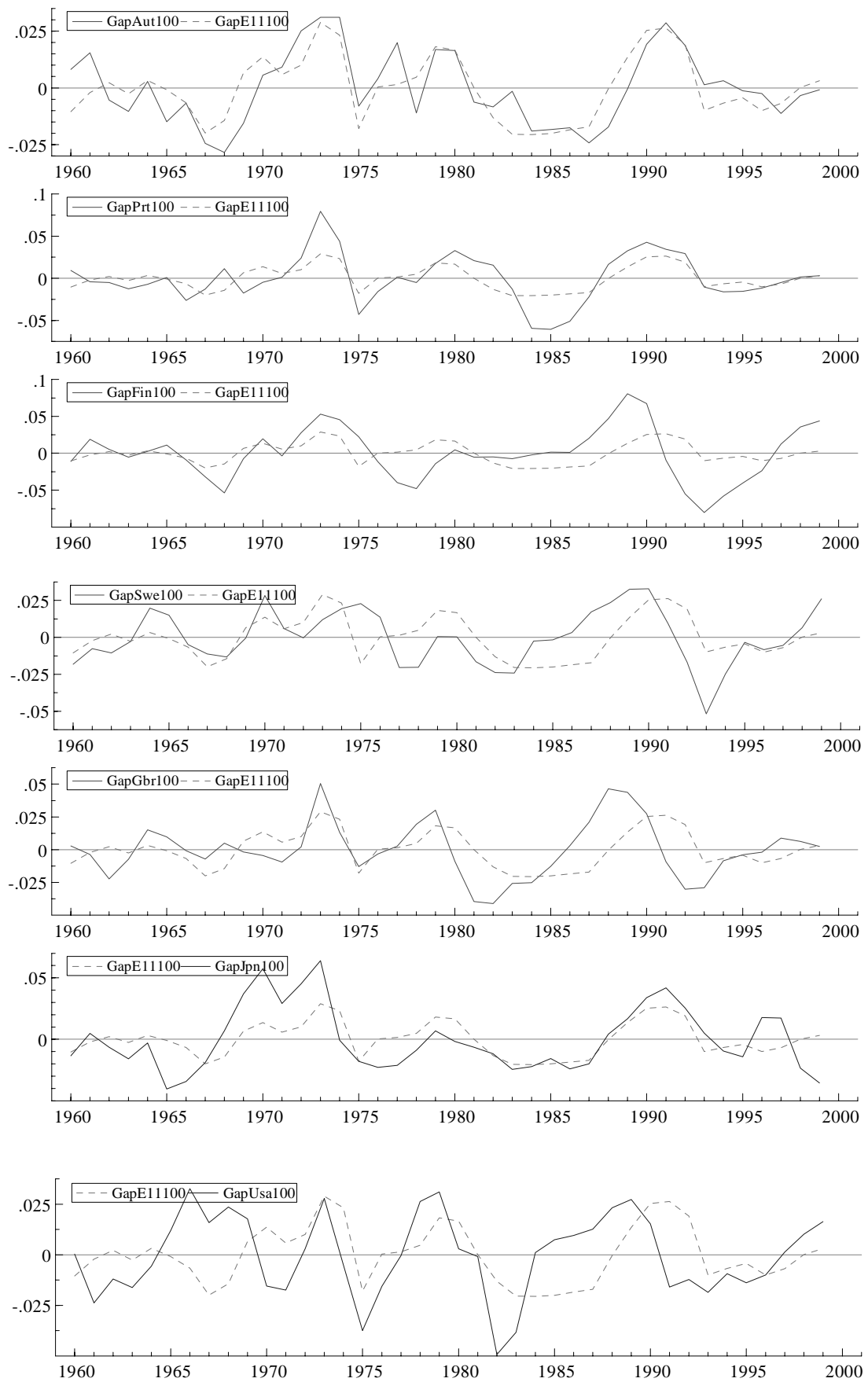
## HP( $\lambda=100$ )













## Appendix C – mean coherency, mean phase effect and dynamic correlation between countries.

**Table C.1**

*mean coherency ,mean phase effect and dynamic correlation at relevant frequencies*

<b>Belgium vs country</b>		<b>HP Filter(<math>\lambda=50</math>)</b>	
	<b>mean coherency</b>	<b>mean phase effect</b>	<b>Dynamic correlation</b>
Deu	0.69	0.27	0.65
Fra	0.94	-0.11	0.93
Nld	0.72	-0.90	0.56
Gbr	0.74	-1.77	0.31
EU11	0.93	-0.29	0.91

**Table C.2**

*mean coherency ,mean phase effect and dynamic correlation at relevant frequencies*

<b>Denmark vs country</b>		<b>HP Filter(<math>\lambda=50</math>)</b>	
	<b>mean coherency</b>	<b>mean phase effect</b>	<b>Dynamic correlation</b>
Deu	0.36	3.01	-0.02
Fra	0.73	2.91	-0.23
Gbr	0.92	1.17	0.65
Swe	0.74	1.58	0.31
Fin	0.65	1.89	0.14
EU11	0.72	2.87	-0.08

**Table C.3**

*mean coherency ,mean phase effect and dynamic correlation at relevant frequencies*

<b>Germany vs country</b>		<b>HP Filter (<math>\lambda=50</math>)</b>	
	<b>mean coherency</b>	<b>mean phase effect</b>	<b>Dynamic correlation</b>
Fra	0.77	-0.37	0.72
Gbr	0.46	-2.21	0.05
Usa	0.31	-2.45	0.12
EU11	0.87	-0.50	0.81

**Table C.4**

*mean coherency ,mean phase effect and dynamic correlation at relevant frequencies*

<b>France vs country</b>		<b>HP Filter (<math>\lambda=50</math>)</b>	
	<b>mean coherency</b>	<b>mean phase effect</b>	<b>Dynamic correlation</b>
Deu	0.77	0.37	0.72
Gbr	0.74	-1.88	0.28
Usa	0.53	-1.94	0.13
EU11	0.96	-0.16	0.95



**Table C.5**

*mean coherency ,mean phase effect and dynamic correlation at relevant frequencies*

<b>Spain vs country</b>		<b>HP Filter (<math>\lambda=50</math>)</b>	
	<b>mean coherency</b>	<b>mean phase effect</b>	<b>Dynamic correlation</b>
Deu	0.45	1.48	0.01
Fra	0.67	0.33	0.26
Gbr	0.74	-1.27	0.46
Ita	0.42	-0.21	0.41
EU11	0.69	0.33	0.65

**Table C.6**

*mean coherency ,mean phase effect and dynamic correlation at relevant frequencies*

<b>Italy vs country</b>		<b>HP Filter (<math>\lambda=50</math>)</b>	
	<b>mean coherency</b>	<b>mean phase effect</b>	<b>Dynamic correlation</b>
Fra	0.83	0.88	0.66
Deu	0.64	0.96	0.48
Gbr	0.71	-1.32	0.44
Usa	0.70	-1.39	0.36
EU11	0.85	0.60	0.78

**Table C.7**

*mean coherency ,mean phase effect and dynamic correlation at relevant frequencies*

<b>Netherlands vs country</b>		<b>HP Filter (<math>\lambda=50</math>)</b>	
	<b>mean coherency</b>	<b>mean phase effect</b>	<b>Dynamic correlation</b>
Fra	0.71	0.79	0.57
Deu	0.82	1.13	0.68
Gbr	0.67	-1.30	0.42
Bel	0.72	0.90	0.56
EU11	0.84	0.67	0.74

**Table C.8**

*mean coherency ,mean phase effect and dynamic correlation at relevant frequencies*

<b>Austria vs country</b>		<b>HP Filter (<math>\lambda=50</math>)</b>	
	<b>mean coherency</b>	<b>mean phase effect</b>	<b>Dynamic correlation</b>
Fra	0.87	-0.68	0.81
Deu	0.84	-0.18	0.83
Ita	0.65	-1.71	0.32
EU11	0.90	-0.75	0.80



**Table C.9**

*mean coherency ,mean phase effect and dynamic correlation at relevant frequencies*

<b>Portugal vs country</b>		<b>HP Filter (<math>\lambda=50</math>)</b>	
	<b>mean coherency</b>	<b>mean phase effect</b>	<b>Dynamic correlation</b>
Fra	0.92	-0.03	0.91
Esp	0.60	-0.32	0.59
EU11	0.90	-0.26	0.85

**Table C.10**

*mean coherency ,mean phase effect and dynamic correlation at relevant frequencies*

<b>Finland vs country</b>		<b>HP Filter (<math>\lambda=50</math>)</b>	
	<b>mean coherency</b>	<b>mean phase effect</b>	<b>Dynamic correlation</b>
Fra	0.67	0.99	0.58
Deu	0.26	2.16	0.04
Gbr	0.62	-0.46	0.55
Usa	0.29	-1.14	0.18
Swe	0.89	-0.14	0.87
Dnk	0.65	-1.88	0.14
EU11	0.56	1.05	0.49

**Table C.11**

*mean coherency ,mean phase effect and dynamic correlation at relevant frequencies*

<b>Sweden vs country</b>		<b>HP Filter (<math>\lambda=50</math>)</b>	
	<b>mean coherency</b>	<b>mean phase effect</b>	<b>Dynamic correlation</b>
Fra	0.70	1.48	0.45
Deu	0.49	2.55	0.04
Gbr	0.77	-0.21	0.65
Usa	0.37	-0.25	0.33
Fin	0.89	0.14	0.87
Dnk	0.74	-1.58	0.31
EU11	0.67	1.55	0.44

**Table C.12**

*mean coherency ,mean phase effect and dynamic correlation at relevant frequencies*

<b>Great Britain vs country</b>		<b>HP Filter (<math>\lambda=50</math>)</b>	
	<b>mean coherency</b>	<b>mean phase effect</b>	<b>Dynamic correlation</b>
Fra	0.74	1.88	0.28
Deu	0.46	2.21	0.05
Usa	0.79	0.13	0.77
EU11	0.75	1.79	0.33



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