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# Modelling Taylor Rule Uncertainty

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## Abstract

In recent years, one has witnessed a widespread attention on the way monetary policy is conducted and in particular on the role of the so-called monetary policy rules. The conventional approach in the literature consists in estimating reaction functions for a monetary authority (the Federal Reserve, in most cases) in which a nominal interest rate, directly or indirectly controlled by that monetary authority, is adjusted in response to deviations of inflation (current or expected) from target and of output from potential. These reaction functions, usually called Taylor rules, following John Taylor's seminal paper published in 1993, match a number of normative principles set forth in the literature for optimal monetary policy. This provides a good reason for the growing prominence of indications given by Taylor rule estimations in debates about current and prospective monetary policy stance. However, they are usually presented as point estimates for the interest rate, giving a sense of accuracy that can be misleading. Typically, no emphasis is placed on the risks of those estimates and, at least to a certain extent, the reader is encouraged to concentrate on an apparently precise central projection, ignoring the wide degree of uncertainty and operational difficulties surrounding the estimates. As in any forecasting exercise, there is uncertainty regarding both the estimated parameters and the way the explanatory variables evolve during the forecasting horizon. Our work presents a methodology to estimate a probability density function for the interest rate resulting from the application of a Taylor rule (the Taylor interest rate) which acknowledges that not only the explanatory variables but also the parameters of the rule are random variables.

# 1 Introduction

In recent years, one has witnessed a widespread attention on the way monetary policy is conducted and in particular on the role of the so-called monetary policy rules. Several reasons seem to underlie this renewed interest. Perhaps the most important is that since the second half of the 1980s, a number of studies have concluded that monetary policy significantly influences the short-term performance of the real economy. Part of this strand of literature tries to identify simple monetary policy rules that could reduce the likelihood of inflationary shocks similar to those of the 1970s.

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This provided a good reason for the growing prominence of indications given by Taylor rule estimations in debates about current and prospective monetary policy stance. However, these indications should be interpreted with prudence. Indeed, they are usually presented as point estimates for the interest rate, giving a sense of accuracy that can be misleading. Typically, no emphasis is placed on the risks of those estimates and, at least to a certain extent, the reader is encouraged to concentrate on an apparently precise central projection, ignoring the wide degree of uncertainty and operational difficulties surrounding the estimates. As in any forecasting exercise, there is uncertainty regarding both the estimated parameters and the way the explanatory variables evolve during the forecasting horizon [see Martins (2000)].

Our work presents a methodology to estimate a probability density function for the interest rate resulting from the application of a Taylor rule (the Taylor interest rate) which acknowledges that not only the explanatory variables but also the parameters of the rule are random variables. The approach builds on work by the Bank of England [see Whitley (1999) and Britton et al (1998)] and the Sveriges Riksbank [see Blix and Sellin (1998)] produced in the context of their inflation forecasting exercises. The method has a Bayesian flavour in that involves a subjective component, through a permanent assessment of the state of the economy, based on a central projection and the risks surrounding it. This assessment gives rise to the adoption of asymmetric distributions both for the explanatory variables and the parameters of the Taylor rule. However, unlike the approach followed by the aforementioned central banks, the resulting distribution for the Taylor interest rate is obtained by numerical simulation.

This article is structured as follows. Section 2 presents a brief outline of the Taylor rule and describes the procedure used to compute a probability density function for the Taylor interest rate. In section 3, we present a simple statistical model for the dependence between the inflation

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<sup>1</sup>The usefulness of Taylor rules as instruments for monetary policy analysis can be sustained not only on normative grounds, with many studies concluding that simple monetary rules have stabilising properties which are close to those of optimal policy rules, but also on positive grounds, since rules with this kind of formulation seem to depict fairly well the way major monetary authorities have conducted monetary policy. Martins (2000) provides a summary of the empirical literature on Taylor rules as well as a discussion on the operational difficulties and limitations associated with this kind of instrument.

forecast and the output gap. This procedure is then applied to the euro area in section 4. Finally, section 5 presents some concluding remarks.

## 2 Taylor rule: a distribution for the explanatory variables

The original formulation of the Taylor rule is the following:

$$i_{Tt} = r^* + \pi^* + \beta(\pi_t - \pi^*) + \theta X_t, \quad (1a)$$

where  $i_T$  is the interest rate recommended using a Taylor rule (the Taylor interest rate),  $\pi_t$  the average inflation rate over the previous four quarters (measured by the GDP deflator),  $\pi^*$  the inflation rate target,  $X_t$  the output gap and  $r^*$  the equilibrium (or neutral) real interest rate<sup>2</sup>.

Formulation (1a) by taking into account only the contemporaneous inflation rate and output gap overlooks the forward-looking nature of monetary policy. To overcome this problem, a forward-looking version of the Taylor rule is used in line with Clarida, Gali and Gertler (1997):

$$i_{Tt} = r^* + \pi^* + \beta(\pi_{t+2}^e - \pi^*) + \theta X_{t+1}^e. \quad (1b)$$

The different time horizons considered for the output gap and the inflation forecast (one and two years, respectively) has implicit the stylised fact that, at least in large and relatively closed economies, monetary policy affects economic activity faster than it affects inflation [see Ball (1997)].

The estimation of Taylor rules involves uncertainty regarding not only the estimated parameters ( $\beta$  and  $\theta$ ) but also the way in which the explanatory variables evolve over the forecasting horizon. As a result, in this article all the arguments of the Taylor rule, excluding the inflation target, are assumed to be random variables. It is also considered that the probabilistic behaviour for each of these variables is characterised by a two-piece normal distribution (TPN)<sup>3</sup>. This distribution, which is also used by the Bank of England and the Sveriges Riksbank in their inflation forecasting exercises, is a simple way to introduce asymmetry considerations in the analysis.

A random variable  $W$  has a TPN distribution if its probability density function is given by:

$$f(W; \mu_w; \sigma_{w,1}) = C \exp \left[ -\frac{1}{2\sigma_{w,1}^2} (W - \mu_w)^2 \right], W \leq \mu_w, \quad (2a)$$

$$f(W; \mu_w; \sigma_{w,2}) = C \exp \left[ -\frac{1}{2\sigma_{w,2}^2} (W - \mu_w)^2 \right], W \geq \mu_w, \quad (2b)$$

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<sup>2</sup>Note that if  $\beta > 1$  and  $\theta > 0$ , the real interest rate adjusts in a way that stabilises both inflation and output; if  $\beta < 1$ , some inflation is accommodated. In this case, the nominal interest rate change is not sufficient to cause the real interest rate to move in the same direction. This also applies to  $\theta$ , which has to be non-negative for the rule to be stabilising. In Taylor's seminal paper (1993) the rule arguments were set at  $\beta = 1.5$ ,  $\theta = 0.5$ ,  $\pi^* = 2.0$  and  $r^* = 2.0$ .

<sup>3</sup>See Johnson, Kotz and Balakrishnan (1994) for a brief description of this distribution.

with  $C = \sqrt{\frac{2}{\pi}}(\sigma_{w,1} + \sigma_{w,2})^{-1}$ .

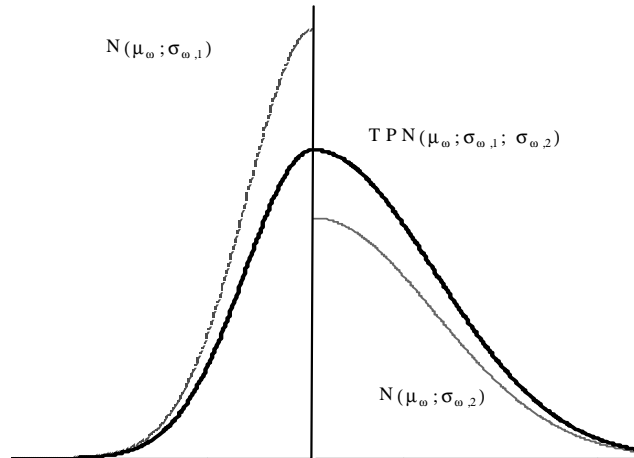
This distribution can be understood as a merging of the left and right halves of two standard normal distributions with the same mode ( $\mu_w$ ) but with different standard deviations ( $\sigma_{w,1}; \sigma_{w,2}$ )<sup>4</sup>. Figure 1 provides an illustration with  $\sigma_{w,1} < \sigma_{w,2}$ . In this example, the probability mass to the left of the mode is smaller than the probability mass to its right, so that both the mean and the median exceed the mode (positive asymmetry).

The mean and the variance for a random variable with this distribution are given by:

$$E(W) = \mu_w + \sqrt{\frac{2}{\pi}}(\sigma_{w,2} - \sigma_{w,1}) \quad (3)$$

$$Var(W) = (1 - \frac{2}{\pi})(\sigma_{w,2} - \sigma_{w,1})^2 + \sigma_{w,2}\sigma_{w,1} \quad (4)$$

Figure 1 - Probability density function of a two-piece normal distribution  
 $\sigma_{w,1} < \sigma_{w,2}$



In our analysis,  $W$  denotes each of the arguments of the Taylor rule (inflation forecast, output gap. . .). In order to obtain the three parameters of the distribution ( $\mu_w, \sigma_{w,1}; \sigma_{w,2}$ ), it is necessary to assign values to:

- (i)  $\mu_w$ , which represents the central projection (i.e. the single most likely outcome);

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<sup>4</sup>The factors of adjustment applied to the normal distribution are  $2\sigma_{w,1}/(\sigma_{w,1} + \sigma_{w,2})$  to the left of the mode and  $2\sigma_{w,2}/(\sigma_{w,1} + \sigma_{w,2})$  to its right. This ensures that the probability density function is continuous and the integral adds to 1.

(ii)  $\omega_w = h_w \sigma_w$ , which represents the standard deviation calculated using historical data ( $\sigma_w$ ) and adjusted by a factor of additional uncertainty ( $h_w$ );

(iii)  $P_w$ , represents the subjective probability of  $W$  being below the central projection – the downside risk<sup>5</sup>. This parameter plays a key role in the analysis, since the distribution asymmetry builds on the particular value of  $P_w$ . If  $P_w = 0.5$ , the distribution collapses to the standard normal distribution.

Given the distribution for the arguments of the Taylor rule, the question that arises is how to determine the distribution of the Taylor interest rate itself<sup>6</sup>. The main difficulty is that the aggregation of random variables with a TPN distribution does not result in a new variable with a TPN or any other known distribution. In our work the Taylor interest rate distribution is obtained by numerical simulation.

One of the problems to be solved before numerical simulation is made is the likely statistical dependence among the Taylor rule arguments. Whereas regarding most of them it seems reasonable to assume independence, that would be little realistic vis-à-vis the inflation forecast and the output gap. In the next section, we present a simple model for the dependence between these two variables.

### 3 A simple statistical model of dependence between output gap and inflation forecasts

Let the random variable  $X^e$  denote the output gap forecast and assume that  $X^e$  follows a TPN distribution,

$$X \sim TPN(\mu_{X^e}; \sigma_{X^e,1}; \sigma_{X^e,2}).$$

Let  $\pi_i$ , for  $i = 1, 2$  be two TPN random variables, independent of  $X^e$ , with the following specification

$$\pi_i^e \sim TPN(\mu_{\pi^e}; \sigma_{\pi^e,1}; \sigma_{\pi^e,2})$$

and

$$P_{\pi_1^e} < P_{\pi_2^e}.$$

The latter is equivalent to imposing that

$$\frac{\sigma_{\pi_1^e,1}}{\sigma_{\pi_1^e,1} + \sigma_{\pi_1^e,2}} < \frac{\sigma_{\pi_2^e,1}}{\sigma_{\pi_2^e,1} + \sigma_{\pi_2^e,2}}$$

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<sup>5</sup>Bearing in mind that  $P_w = \int_{-\infty}^{\mu} f(x)dx = \frac{\sigma_1}{\sigma_1 + \sigma_2}$ , it is possible to show that the standard deviations of the TPN distribution are given by:  $\sigma_{w,1}^2(\sigma_w, P_w) = h_w^2 \sigma_w^2 \left[ \left(1 - \frac{2}{\pi}\right) \left(\frac{1-2P_w}{P_w}\right)^2 + \left(\frac{1-P_w}{P_w}\right) \right]^{-1}$  and  $\sigma_{w,2}^2(\sigma_w, P_w) = h_w^2 \sigma_w^2 \left[ \left(1 - \frac{2}{\pi}\right) \left(\frac{1-2P_w}{1-P_w}\right)^2 + \left(\frac{P_w}{1-P_w}\right) \right]^{-1}$ .

<sup>6</sup>The Bank of England and the Sveriges Riksbank solve this problem by assuming that the resulting distribution is also a TPN.

or

$$\frac{\sigma_{\pi_1^e,2}}{\sigma_{\pi_1^e,1}} > \frac{\sigma_{\pi_2^e,2}}{\sigma_{\pi_2^e,1}}.$$

This means that the distribution of  $\pi_1^e$  is relatively more skewed to the right than that of  $\pi_2^e$ .

The inflation forecast ( $\pi^e$ ) will be modelled as a mixture of  $\pi_1^e$  and  $\pi_2^e$ , where the mixing coefficients depend on the output gap. The idea is the following: if the output gap outcome exceeds (falls behind) its central projection, a higher (lower) proportion of agents will be pessimistic (optimistic) about the inflation prospects, that is the mass of probability to the right (left) of the modal inflation forecast will be greater. In other words, “good news” in terms of output/unemployment will lead to a less favourable outlook for inflation. Formally,

$$\pi^e = \pi_1^e I(X > \mu_{X^e}) + \pi_2^e I(X < \mu_{X^e}) \quad (5)$$

where  $I(A)$  is the indicator of event  $A$ .

A few (obvious) remarks about the distribution of  $\pi^e$  are in order,

1. The marginal probability density function (pdf) of  $\pi^e$  is

$$f_{\pi^e}(y) = (1 - P_{X^e})f_{\pi_1^e}(y) + P_{X^e}f_{\pi_2^e}(y)$$

where  $f_{\pi_i}(y)$  is the pdf of a variable with a TPN distribution.

2. The mode of  $f_{\pi^e}$  is  $\mu_{\pi^e}$ .
3. The downside risk is given by

$$P_{\pi^e} = \int_{-\infty}^{\mu_{\pi^e}} f_{\pi^e}(y)dy = P_{X^e}P_{\pi_2^e} + (1 - P_{X^e})P_{\pi_1^e}, \quad (6)$$

and thus,  $\pi^e$  has a downside risk that is between those of  $\pi_1^e$  and  $\pi_2^e$ .

### 3.1 Moments

For calibration purposes it is useful to compute the first moments of the distribution of  $\pi^e$  as well as the cross-moments of  $X^e$  and  $\pi^e$ .

#### 3.1.1 Mean

Using the independence of  $X^e$  and  $\pi_i^e$  and the expression for the mean of a TPN distribution, it is easy to see that

$$\begin{aligned} E(\pi^e) &= P_{X^e}E(\pi_2^e) + (1 - P_{X^e})E(\pi_1^e) \\ &= \mu_{\pi^e} + \sqrt{2/\pi}[P_{X^e}(\sigma_{\pi_2^e,2} - \sigma_{\pi_2^e,1}) + (1 - P_{X^e})(\sigma_{\pi_1^e,2} - \sigma_{\pi_1^e,1})]. \end{aligned}$$

### 3.1.2 Variance

Recall that if a variable  $W$  follows a TPN distribution, then

$$E(W^2) = (\sigma_{W,2} - \sigma_{W,1})^2 + \sigma_{W,1}\sigma_{W,2} + 2\sqrt{2/\pi}\mu_W(\sigma_{W,2} - \sigma_{W,1}) + \mu_W^2.$$

Bearing in mind that

$$E(\pi^{e2}) = P_{X^e}E\pi_2^{e2} + (1 - P_{X^e})E\pi_1^{e2}$$

the second moment of  $\pi^e$  can be computed by an easy (albeit cumbersome) substitution.

An interesting special case is when one assumes that

$$\sigma_{\pi_1^e,1} = \sigma_{\pi_2^e,1} \equiv v.$$

Then, writing

$$\kappa_i \equiv \frac{1 - P_{\pi_i^e}}{P_{\pi_i^e}}, \quad i = 1, 2$$

and noticing that

$$\sigma_{\pi_i^e,2} = \kappa_i \sigma_{\pi_i^e,1} = \kappa_i v \tag{7}$$

one has

$$E(\pi^{e2}) = \mu_{\pi^e}^2 + [P_{X^e}(\kappa_2^2 - \kappa_2 + 1) + (1 - P_{X^e})(\kappa_1^2 - \kappa_1 + 1)]v^2 + 2\sqrt{2/\pi}[P_{X^e}\kappa_2 + (1 - P_{X^e})\kappa_1 - 1]\mu_{\pi^e}v$$

and

$$V(\pi^e) = v^2\{[P_{X^e}(\kappa_2^2 - \kappa_2 + 1) + (1 - P_{X^e})(\kappa_1^2 - \kappa_1 + 1)] - (2/\pi)[P_{X^e}\kappa_2 + (1 - P_{X^e})\kappa_1 - 1]^2\}. \tag{8}$$

### 3.1.3 Cross-moments

$$E(X^e \pi^e) = E(\pi_1^e)E[X^e I(X^e > \mu_{X^e})] + E(\pi_2^e)E[X^e I(X^e < \mu_{X^e})].$$

Using the well known results on the moments of truncated normal distributions [see, for instance, Green (1993), page 683]

$$E[X^e | X^e > \mu_{X^e}] = \mu_{X^e} + \sqrt{2/\pi}\sigma_{X^e,2}$$

and

$$E[X^e | X^e < \mu_{X^e}] = \mu_{X^e} - \sqrt{2/\pi}\sigma_{X^e,1}.$$

Consequently,

$$E[X^e I(X^e > \mu_{X^e})] = (1 - P_{X^e})(\mu_{X^e} + \sqrt{2/\pi}\sigma_{X^e,2})$$

and

$$E[X^e I(X^e < \mu_{X^e})] = P_{X^e}(\mu_{X^e} - \sqrt{2/\pi}\sigma_{X^e,1}).$$

Therefore,

$$\begin{aligned} E(X^e \pi^e) &= (1 - P_{X^e})(\mu_{X^e} + \sqrt{2/\pi}\sigma_{X^e,2})[\mu_{\pi^e} + \sqrt{2/\pi}(\sigma_{\pi_1^e,2} - \sigma_{\pi_1^e,1})] \\ &\quad + P_{X^e}(\mu_{X^e} - \sqrt{2/\pi}\sigma_{X^e,1})[\mu_{\pi^e} + \sqrt{2/\pi}(\sigma_{\pi_2^e,2} - \sigma_{\pi_2^e,1})]. \end{aligned} \tag{9}$$



### 3.2 A calibration strategy

The distribution for the inflation forecast can be computed by numerical simulation on the basis of equation (5). To do that, we may use point mass priors for the distribution parameters. For the distribution of  $X^e$ , we have to assign values for the central projection ( $\mu_{X^e}$ ); the historical standard deviation ( $\sigma_{X^e}$ ) and the factor of additional uncertainty ( $h_{X^e}$ ); and the downside risk ( $P_{X^e}$ ). Concerning the inflation forecast, we have to specify values for the central projection ( $\mu_{\pi^e}$ ); the historical standard deviation ( $\sigma_{\pi^e}$ ) and the factor of additional uncertainty ( $h_{\pi^e}$ ); and the downside risks  $P_{\pi_1^e}$  and  $P_{\pi_2^e}$ <sup>7</sup>. The simplest approach is to make

$$\sigma_{\pi_1^e,1} = \sigma_{\pi_2^e,1} \equiv v,$$

a constant to be determined by the aforementioned assumptions for the parameters. As,

$$\sigma_{\pi_2,2} = \kappa_2 v \text{ and } \sigma_{\pi_1,2} = \kappa_1 v,$$

this amounts to fixing  $\sigma_{\pi_2^e,2}$  and  $\sigma_{\pi_1^e,2}$  up to a constant. Without the aforementioned simplifying assumption, the determination of the standard deviations would involve the cross moments restrictions (9).

Once the distribution of the inflation forecast is defined, on the basis of equation (1b), it becomes possible to obtain by numerical simulation a probability density function for the Taylor interest rate that takes into account the statistical dependence between the inflation forecast and the output gap.

## 4 An application to the euro area

Taking into account the above procedure, this section provides an assessment of the euro area monetary policy stance on basis of data available in December 2000. Table 1 presents the central projections for each of the relevant variables of the Taylor rule, as well as the degree of uncertainty and the balance of risks. The underlying assumptions as well as the remaining calculations needed to compute the Taylor interest distribution are listed below.

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<sup>7</sup> Assuming values for  $P_{\pi_1^e}$  and  $P_{\pi_2^e}$  is, of course, equivalent to assume a value for  $P_{\pi^e}$  and another for  $P_{\pi_1^e}$  or  $P_{\pi_2^e}$ .

Table 1 – The euro area Taylor interest rate: central projections and balance of risks  
(as of December 2000)

	$r^*$	$\pi^*$	$\pi^e$		$X^e$	$\beta$	$\theta$
<b>Main assumptions</b>							
Central projections ( $\mu_w$ )	3.00	1.50	1.90		0.30	1.31	0.25
Adjusted standard deviation ( $\omega_w$ )	0.21	-	0.75		1.21	0.09	0.04
Historical standard deviation ( $\sigma_w$ )	0.19	-	0.75		1.10	0.09	0.04
Additional uncertainty factor ( $h_w$ )	1.10	-	1.00		1.10	1.00	1.00
<b>Balance of risks</b>							
			$\pi_1^e$	$\pi_2^e$			
Upside	0.60	-	0.60	0.40	0.45	0.60	0.70
Downside	0.40	-	0.40	0.60	0.55	0.40	0.30
<b>Memorandum item</b>							
Mean	3.07	1.50	1.88		0.11	1.34	0.27

#### 4.1 Coefficients

Table 2 presents different estimated values for coefficients  $\beta$  and  $\theta$ . Even though the results are not qualitatively very distinct, the conclusions drawn in each model could be quantitatively different. To define the baseline scenario and the standard deviations, we took the values estimated for Germany in Clarida, Galí and Gertler (1997), which are similar to those estimated in Peersman and Smets (1998). We did not consider any additional uncertainty. Against the backdrop that most estimates for the United States point to higher coefficients than those of Germany, we considered upside risks greater than 50 percent (60 and 70 percent, respectively for  $\beta$  and  $\theta$ ).

Table 2 – Estimated coefficients  $\beta$  and  $\theta$

		$\beta$	$\theta$
Taylor (1993)	USA	1.50	0.50
Taylor (1999)	USA	1.50	1.00
Ball (1997)	USA	1.50	1.00
Christiano (1999)	USA	3.00	0.50
Clarida, Galí and Gertler (1998)	USA	1.80	0.12
Clarida, Galí and Gertler (1997)	Germany	1.31	0.25
Peersman and Smets (1998)	Germany	1.30	0.28

#### 4.2 Equilibrium real interest rate

We assume a value of 3.0 percent for the euro area equilibrium real interest rate. This figure is consistent with estimates derived from a reaction function for the Bundesbank over the last two decades and with the average real interest rates in G7 during the 1990s<sup>8</sup>. Moreover, according

<sup>8</sup>The calculation of the equilibrium real interest rate for the euro area on the basis of the average real interest rate prevailing, for example, over the last decade is likely to show an upward bias. Indeed, over this period, the

to the well-known golden rule of capital accumulation, the marginal product of capital, which in equilibrium equals the real interest rate should not be less than the growth rate of output (otherwise, the economy would be dynamically inefficient). Current estimates for the euro area potential growth rate suggest a lower bound for the real interest rate of around 2.0 to 2.5 percent. For the historical standard deviation, we take the value derived by Smets (1999) from a forward-looking reaction function for the Bundesbank over the period 1979-1997. In this work, the implicit equilibrium real interest rate for an inflation target of 1.5 percent was 3.0 percent as well. As regards the balance of risks, different estimates put forward by the literature for the equilibrium real interest rate for Germany fall overwhelmingly in the range of 2.5 to 3.5 percent, with a slight bias on the upper half. This last evidence seems to indicate that the balance of risks is on the upside, thus justifying the attribution of a 60 percent probability to upside risks. Finally, the possible effects of the so-called “New Economy” induce some uncertainty over the current potential output estimates and consequently over the equilibrium real interest rate. As a result, we decided to include a factor of additional uncertainty of 10 percent.

### 4.3 Inflation target

We assumed an inflation target of 1.5 percent. Recall that within the ECB monetary policy strategy, adopted in October 1998, price stability was defined as an annual increase in the Harmonised Index of Consumer Prices (HICP) of below 2 percent. In addition, the derivation of the reference value for the growth rate of the M3 monetary aggregate had implicit an inflation rate of 1.5 percent. We did not consider any uncertainty regarding this target.

### 4.4 Inflation forecast

The central projection for the inflation forecast corresponds to the mid-point of the Eurosystem’s forecasting interval for the HICP growth in 2002 published in December 2000 – i.e. 1.9 percent. The historical standard deviation was computed taking into account that the Eurosystem’s forecasting interval (1.3, 2.5) is equal to twice the absolute mean error of the forecasting exercises undertaken over the last years [ECB (2000)]. Considering a normal distribution, this leads to a standard deviation of 0.75. We did not assume any additional uncertainty vis-à-vis the historical standard deviation. Regarding the balance of risks, we took downside risks of 60 percent, if the output gap realisation falls behind the modal forecast, and of 40 percent, if the output gap realisation exceeds the modal forecast.

### 4.5 Output gap

The central projection for the output gap in 2001 (0.3 percent) was obtained with the Hodrick-Prescott filter, using quarterly data since 1977. This estimate is in line with the European Commission and the OECD projections for 2001, published in October and November 2000, respectively. The historical standard deviation was computed bearing in mind that the Eurosystem’s forecasting interval (2.6;3.6) for the GDP growth rate is equal to twice the absolute

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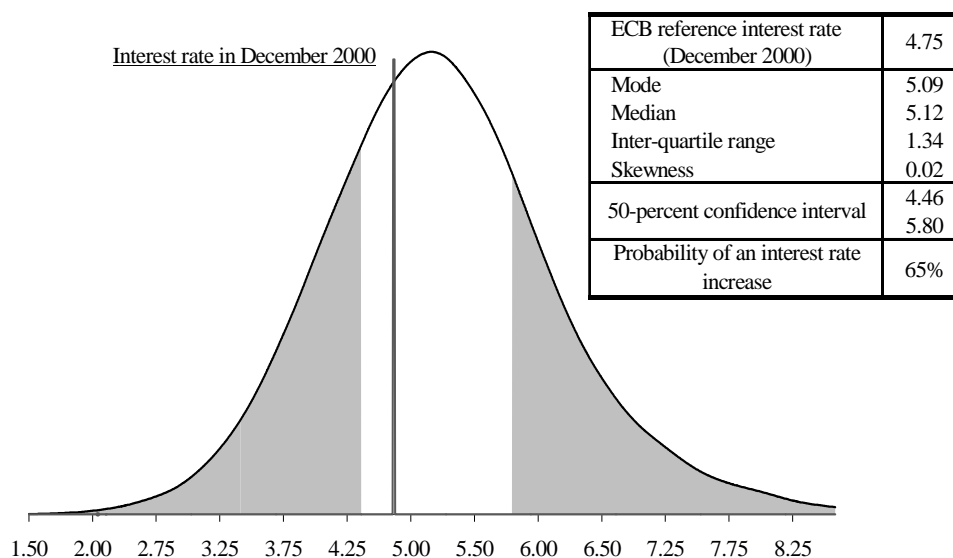
disinflation process in the current euro area countries may have caused real interest rates to stand above their equilibrium level. Against this background, it seems more appropriate to take past German interest rates as a benchmark for comparison with today’s euro area.

mean error of the forecasting exercises undertaken over several years. Taking into account the effects of the so-called “New Economy”<sup>9</sup>, we decided to include a factor of additional uncertainty of 1.1. As to the balance of risks, we admitted a downside risk of 55 percent, given the prospect that the economic slowdown in the United States could be more pronounced than the available projections.

## 4.6 Results

With the aforementioned assumptions, all the central statistical measures for the Taylor interest rate would be above the ECB reference interest rate in December 2000 (see Figure 2). Nevertheless, given the significant variance implied by the Taylor interest rate distribution, these indications are surrounded by considerable uncertainty, which are confirmed by the width of the confidence interval. Indeed, the 50-percent confidence interval for the Taylor interest rate is (4.46;5.80). The ECB interest rate in December 2000 lied inside this interval.

Figure 2 - Probability density function of the Taylor interest rate



## 5 Concluding remarks

Empirical evidence suggests that Taylor rules depict fairly well the way major monetary authorities (in particular, the Federal Reserve and the Bundesbank) have conducted monetary policy

<sup>9</sup>Estimates for the current output gap are particularly uncertain both because recent output figures are in most cases preliminar or because many estimation techniques, in particular univariate methods such as the HP filter, pose some end-of-sample problems.

over the last two decades – a period during which monetary policy is generally considered to have been rather successful in reducing inflation. In this context, it seems reasonable to sustain that indications given by Taylor rule estimations could be a useful reference when assessing monetary policy stance.

However, the conventional approach, which consists in presenting these indications as point estimates for the interest rate, seems to be little prudent, given the high degree of uncertainty and operational difficulties surrounding the derivation of a Taylor interest rate. In particular, the use of Taylor rules in a forward-looking perspective requires the inclusion of macroeconomic forecasts over the period relevant for the monetary policy transmission mechanism. Given the forecasting errors of the past, that requirement provides an important source of uncertainty.

In our work, the informative content of the Taylor rule was presented as a probability density function for the interest rate. This approach makes clear that monetary policy decisions are taken in an uncertain environment, which has to be taken into account explicitly in the context of monetary policy assessment.

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