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and Dynamic Long Run Equilibrium
in Error Correction Models?**

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Should we distinguish between static and dynamic long run equilibrium in error correction models?*

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Abstract

This paper shows that there is no theoretical foundation to distinguish between static and dynamic long run equilibrium in error correction models with deterministically cointegrated variables, and so, that the so-called dynamic homogeneity restriction aimed at guaranteeing that the two solutions coincide, also lacks a theoretical justification. Examples in which dynamic homogeneity cannot hold are also discussed.

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1 Introduction

Some literature distinguishes between the long run static and dynamic equilibrium solutions of dynamic econometric models. For instance, the econometric textbook “Econometric methods” by Johnston and Dinardo (1997), distinguishes between the long run dynamic equilibrium and the long run static equilibrium of a single equation dynamic model claiming that the two solutions differ in the constant. Similarly, De Brouwer and Ericsson (1998) discuss these two apparently different concepts and compute the “two” long run solutions for an error correction model for inflation in Australia.

A related issue concerns the so-called dynamic homogeneity condition. In macroeconometric modelling it is often argued that in order to guarantee that the long run equilibrium solution of a dynamic model does not depend on the growth rates of the variables in the model, the so-called dynamic homogeneity restriction on the estimated model needs to be imposed.

This paper claims that, from a theoretical point of view, there is no reason to distinguish between the static and the dynamic long run equilibrium of an error correction model with cointegrated variables and thus, in particular, the need to impose dynamic homogeneity in the estimated models lacks theoretical foundations. Besides, there are situations in which dynamic homogeneity cannot hold.

The paper is organized as follows. Section 2 reviews the definitions of dynamic versus static long run equilibrium solutions for a single equation dynamic model and section 3 reviews the formal definition of dynamic homogeneity. Section 4 shows that there is no theoretical reason to distinguish between static and dynamic equilibrium solutions in error correction models with cointegrated variables and thus there is also no need to impose the dynamic homogeneity restriction. Section 5 discusses the case under which dynamic homogeneity should not be expected to hold and section 6 concludes.

2 Static versus dynamic equilibrium

To motivate the problem let us start by considering the simple ADL (1,1) model as in Johnston and Dinardo (1997), section 8.1, pag. 244:

$$y_t = m + \alpha_1 y_{t-1} + \beta_0 x_t + \beta_1 x_{t-1} + \varepsilon_t \quad (1)$$

where y_t and x_t are best seen as the natural logs of original variables Y_t and X_t . Model (1) may be written in the error correction form as

$$\Delta y_t = \beta_0 \Delta x_t - (1 - \alpha_1)(y_{t-1} - \delta - \gamma x_{t-1}) + \varepsilon_t \quad (2)$$

with

$$\delta = \frac{m}{1 - \alpha_1}; \quad \gamma = \frac{\beta_0 + \beta_1}{1 - \alpha_1} \quad (3)$$

Let us start by reviewing the Johnston and Dinardo's definition of static and dynamic long run equilibrium. Suppose that x is held constant at some level \bar{x} indefinitely. Then, assuming that the stability condition $|\alpha_1| < 1$ holds and setting the innovations at their expected value of zero, y will tend to a constant value \bar{y} , given by

$$\bar{y} = \frac{m}{1 - \alpha_1} + \frac{\beta_0 + \beta_1}{1 - \alpha_1} \bar{x} = \delta + \gamma \bar{x} \quad (4)$$

This is the so-called **static equilibrium** equation, which corresponds to equation (8.2) in Johnston and Dinardo (1997). Instead of the static assumption suppose now that X grows at a steady rate k so that $\Delta x_t = k$ for all t . Given the constant elasticity γ , the growth rate in Y will be γk . Substituting in equation (2) gives the **dynamic equilibrium** as

$$y = \frac{m + (\beta_0 - \gamma)k}{1 - \alpha_1} + \gamma x \quad (5)$$

which is equation (8.7) in Johnston and Dinardo (1997). Thus according to these authors the long run static and dynamic equilibrium solutions would differ in the constant of the equation.

In a similar vein De Brouwer and Ericsson (1998), which estimate an error correction model for inflation in Australia, also distinguish between static and dynamic equilibrium solutions and

they in fact report two empirical equilibrium equations, which apparently differ in the constant (their equations (11) and (12)). Let us now address the related dynamic homogeneity issue.

3 Dynamic homogeneity

In the literature concerning macroeconometric models it is often claimed that in order to guarantee that the long run equilibrium solution of a dynamic model does not depend on the growth rates of the variables in the model, one needs to impose the so-called **dynamic homogeneity** restriction. In other words this restriction aims at preventing the steady state solution of the model from changing in response to a shift in the “average” growth rates of the variables of the model brought about, say, by changes in monetary policy or by an exogenous shock.

The need for such a restriction, even though discussed in the context of a single equation dynamic model, usually concerns some specific equations in a general structural macro-econometric model, such as the price and wage equations. The economic argument for such a restriction is that otherwise the model is bound to exhibit some unpleasant inflation nonneutrality in the long run. For instance, Nickell (1988) argues that neutrality with respect to inflation rate is an important issue because “if the model does not possess this kind of neutrality, then unemployment can be shifted, even in the long run simply by changing the level of inflation”. One can find the same argument in some of the Bank of England’s recent publications, where we can read: “in order to ensure (inflation neutrality) equations containing nominal variables are restricted to satisfy dynamic homogeneity” (see Bank of England (1999 and 2000)). Thus within this framework dynamic homogeneity is seen as a way of introducing the neoclassical view of the world in econometric models, whereby the level of real activity is independent of the steady state inflation rate¹.

In terms of our simple model it is readily seen that the dynamic homogeneity issue arises

¹On the need or the convenience for the dynamic homogeneity restriction see also Church et al. (1998), Fagan et al. (2001) and Pierce (1991 and 1992)

in the context of the dynamic equilibrium solution of the model given by equation (5) and may be stated as follows. In order to prevent the dynamic long run solution from depending on the growth rates of the variables in the model we need to impose the restriction $\gamma = \beta_0$. This would be the homogeneity restriction for the simple ADL(1,1) model with a single regressor. We note that, in the context of model (1), the dynamic homogeneity restriction can be seen as a condition for (4) and (5) to coincide.

In order to better understand the inflation neutrality argument let us investigate the dynamic homogeneity issue in the equation that is at the very heart of this matter: the wage equation. Let y_t stand for the log of the wage rate, x_t for the log of the price index and z_t for the log of labour productivity. A very general ADL for the wage equation in which the long run elasticities are both equal to one, may be reparameterised in the error correction form as:

$$A(L)\Delta y_t = m + B(L)\Delta x_t + C(L)\Delta z_t - \mu(y_{t-1} - x_{t-1} - z_{t-1}) + \varepsilon_t \quad (6)$$

where $A(L)$, $B(L)$ and $C(L)$ are scalar polynomials in the lag operator L , such that:

$$\begin{aligned} A(L) &= 1 - \alpha_1 L - \dots - \alpha_p L^p \\ B(L) &= \beta_0 + \beta_1 L + \dots + \beta_r L^r \\ C(L) &= c_0 + c_1 L + \dots + c_s L^s \end{aligned}$$

It is well known that any polynomial $D(L) = \sum_{j=0}^m d_j L^j$, may be written as:

$$D(L) = D^*(L)(1 - L) + D(1)L \quad (7)$$

where

$$\begin{aligned} D(1) &= \sum_{j=0}^m d_j \\ d_0^* &= d_0 \quad j = 1, 2, \dots, m-1 \\ d_j^* &= - \sum_{i=j+1}^m d_i \end{aligned}$$

and so, (6) may be re-written as:

$$\begin{aligned} A^*(L)\Delta^2 y_t &= m + B^*(L)\Delta^2 x_t + C^*(L)\Delta^2 z_t - A(1)\Delta y_{t-1} + B(1)\Delta x_{t-1} + C(1)\Delta z_{t-1} \\ &\quad - \mu(y_{t-1} - x_{t-1} - z_{t-1}) + \varepsilon_t \end{aligned} \quad (8)$$

Notice that in this case, as we will show below, the error correction term in levels (if properly defined) implies that the steady state wage growth rate is equal to the steady state price growth rate plus the steady state productivity growth rate. If we assume that in “steady state” $\Delta x_t = k_2$ and $\Delta z_t = k_3$, the steady state growth rate for Y_t is $\Delta y_t = \Delta x_t + \Delta z_t = k_2 + k_3$. But since in steady state we have $\Delta^2 y_t = \Delta^2 x_t = \Delta^2 z_t = 0$, the dynamic equilibrium solution would be:

$$y_t = \frac{m}{\mu} + \frac{1}{\mu}[(B(1) - A(1))k_2 + (C(1) - A(1))k_3] + x_t + z_t \quad (9)$$

Now, if we follow the literature, we would conclude from (9) that the dynamic homogeneity restriction needed to guarantee that the long run solution of the model does not depend on the growth rate of the variables would be given by $A(1) = B(1) = C(1)$. From (9), the model with imposed dynamic homogeneity would become:

$$\begin{aligned} A^*(L)\Delta^2 y_t &= m + B^*(L)\Delta^2 x_t + C^*(L)\Delta^2 z_t - A(1) [\Delta y_{t-1} - \Delta x_{t-1} - \Delta z_{t-1}] - \\ &\quad - \mu [y_{t-1} - x_{t-1} - z_{t-1}] \end{aligned} \quad (10)$$

and, obviously, in this case, the long run (static or dynamic) solution would be given by:

$$y_t = \frac{m}{\mu} + x_t + z_t \quad (11)$$

So, again the so-called dynamic homogeneity restriction can be seen as a condition for the static and dynamic equilibrium solutions of the model to coincide.

4 Dynamic equilibrium in error correction models

The need to consider the dynamic equilibrium condition as opposed to the “conventional” static equilibrium is usually invoked in the context of models with non-stationary variables, as the hypothesis of constant levels for the variables in steady state is realistic only if the variables are stationary. For an integrated series it is not meaningful to talk about a long run or steady state

level of the series². If the variables are integrated of order one, $I(1)$, with a non-zero drift, then it is more realistic to assume that in steady state the variables grow at a constant rate (which must equal the drift).

For this reason the discussion that follows assumes that the variables in the model are \sum

integrated of order one with a non-zero drift, since this is the only case where it makes sense to talk about long run constant growth rates³.

Under this assumption, by definition, we may write:

$$\begin{aligned} y_t &= k_1 + y_{t-1} + v_{1t} \\ x_t &= k_2 + x_{t-1} + v_{2t} \\ z_t &= k_3 + z_{t-1} + v_{3t} \end{aligned} \tag{12}$$

where v_{1t} , v_{2t} and v_{3t} are $I(0)$ variables with $E[v_{1t}] = E[v_{2t}] = E[v_{3t}] = 0$ and k_i ($i = 1, 2, 3$) are three (possibly different) constants. Solving (12) recursively, the three integrated processes can equivalently be written as⁴:

$$\begin{aligned} y_t &= y_0 + k_1 t + \sum_{i=1}^t \varepsilon_{1i} + r_t^0 = y_0 + k_1 t + y_t^0 + r_t^0 \\ x_t &= x_0 + k_2 t + \sum_{i=1}^t \varepsilon_{2i} + r_t^1 = x_0 + k_2 t + x_t^0 + r_t^1 \\ z_t &= z_0 + k_3 t + \sum_{i=1}^t \varepsilon_{3i} + r_t^2 = z_0 + k_3 t + z_t^0 + r_t^2 \end{aligned} \tag{13}$$

where y_0 , x_0 and z_0 are the starting values of the stochastic processes (usually assumed constant); y_t^0 , x_t^0 and z_t^0 are three pure random walks with no deterministic components and r_t^0 ,

²This argument appears, for instance, in Johnston and Dinardo (1997), pg.262 and in Schwert (1987).

³Notice that if the variables are $I(1)$ with a zero drift there is no reason to argue that the growth rates can appear in the long-run solution of the model since in this case $E[\Delta y_t] = E[\Delta x_t] = E[\Delta z_t] = 0$.

⁴Notice, for instance, that by the Wold representation theorem v_{1t} may be written as an infinite moving average and thus we have $y_t = k_1 + y_{t-1} + \sum_{j=0}^{\infty} c_{1j} \varepsilon_{1,t-j}$ where ε_{1t} is a white noise. Equation (13) follows from the Beveridge-Nelson decomposition theorem. Similarly for x_t and z_t .

r_t^1 and r_t^2 are $I(0)$ variables. Therefore the $I(1)$ variables with a non-zero drift can be seen as the sum of a deterministic linear trend plus a pure stochastic $I(1)$ variable. In this case, from (12) we have:

$$E[\Delta y_t] = k_1, E[\Delta x_t] = k_2 \text{ and } E[\Delta z_t] = k_3 \quad (14)$$

Let us now go back to our wage equation (6). For the model to be statistically well defined we must have y_t , x_t and z_t cointegrated with cointegrating vector $(1, -1, -1)$. In other words, $(y_t - x_t - z_t)$ must be a stationary variable around a (possibly) non-zero constant. Therefore we may write:

$$y_t = \delta + x_t + z_t + u_t \quad (15)$$

where δ is a constant and u_t a stationary variable with no deterministic components such that $E[u_t] = 0$. No one would dispute that equation (15) corresponds to the static equilibrium equation of model (6). We now proceed to demonstrate that (15) also necessarily represents the dynamic equilibrium solution, providing some conditions are met.

In order to have y_t , x_t and z_t cointegrated with unitary coefficients, we need to impose some restrictions on the stochastic processes in (13). From (13) it follows that

$$\begin{aligned} y_t - x_t - z_t &= (y_0 + k_1 t + y_t^0 + r_t^0) - (x_0 + k_2 t + x_t^0 + r_t^1) - (z_0 + k_3 t + z_t^0 + r_t^2) \\ &= (y_0 - x_0 - z_0) + (k_1 - k_2 - k_3) t + (y_t^0 - x_t^0 - z_t^0) + (r_t^0 - r_t^1 - r_t^2) \end{aligned} \quad (16)$$

and in order to satisfy (15) we must have $\delta = y_0 - x_0 - z_0$, $k_1 = k_2 + k_3$, $y_t^0 = x_t^0 + z_t^0$ and $u_t = r_t^0 - r_t^1 - r_t^2$.

Thus equation (15) implies that y_t , x_t and z_t are *deterministically cointegrated*. This means that the cointegrating vector $(1, -1, -1)$ eliminates the deterministic trends as well as the stochastic trends exhibited by the three series. This definition of cointegration should not be mistaken with the conventional definition of cointegration first introduced by Engle and Granger, also known as *stochastic cointegration*, which requires cointegration to eliminate solely the stochastic trends exhibited by the series⁵.

⁵On the difference between stochastic and deterministic cointegration see, for instance, Campbell and Perron (1991), Park (1992) or, more recently, Hassler (1999).

Notice also that equation (15) implies that $k_1 = k_2 + k_3$ in (16) so that the "average" growth rate exhibited by Y_t is equal to the sum of the "average" growth rates of X_t and Z_t in the long run, as we have assumed in the derivation of (9).

As we have seen in the previous sections, the literature distinguishes between the long run static and dynamic equilibrium solutions and supporters of the dynamic homogeneity restriction would say that to guarantee that the long run relation does not depend on the growth rates of the variables entering the model one has to impose the restriction $\beta_0 = \gamma$ in case of model (1) or $A(1) = B(1) = C(1)$ in case of model (6).

To see that this is not the case, let us take another look at the cointegrating regression or the long run equilibrium relationship (15). In this equation $E[u_t] = 0$ by assumption so it follows that:

$$E[y_t - x_t - z_t] = \delta \tag{17}$$

that is, in the long run, the expected value of the stationary productivity adjusted real wage rate equals the constant δ .

Here an interesting question arises: why should one consider two different long run equilibrium relationships, one given by (15) and one given by (9)? The truth is that if equation (15) (or (17)) holds (and it must, by definition of cointegration) then necessarily it imposes an important restriction on equation (9). And that restriction is just that the right hand side of (15) and (9) must be equal, or in other words we must have $A(1) = B(1) = C(1)$.

We then have the following important result: in model (6), with y_t , x_t and z_t integrated of order one with a non-zero drift and deterministically cointegrated, the static and dynamic long run equilibrium solutions necessarily coincide⁶.

Let us elaborate a little further on this issue. By the well-known Granger's representation theorem, we know that if (15) holds there exists an error correction representation for y_t , which

⁶The case of stochastic cointegration will be analysed in section 5.

may be written as:

$$A(L)\Delta y_t = B(L)\Delta x_t + C(L)\Delta z_t - \mu(y_{t-1} - x_{t-1} - z_{t-1} - \delta) + \varepsilon_t \quad (18)$$

Solving this equation for the error correction term and noting that $y_{t-1} = y_t - \Delta y_t$, $x_{t-1} = x_t - \Delta x_t$ and $z_{t-1} = z_t - \Delta z_t$, we may re-write (18) as:

$$y_t - x_t - z_t = \delta + (\Delta y_t - \Delta x_t - \Delta z_t) + \frac{1}{\mu} [B(L)\Delta x_t + C(L)\Delta z_t - A(L)\Delta y_t + \varepsilon_t] \quad (19)$$

which, similarly to (8), may further be reparametrised as:

$$\begin{aligned} y_t - x_t - z_t &= \delta + (\Delta y_t - \Delta x_t - \Delta z_t) + \frac{1}{\mu} [B^*(L)\Delta^2 x_t + C^*(L)\Delta^2 z_t - A^*(L)\Delta^2 y_t] \\ &\quad + \frac{1}{\mu} [B(1)\Delta x_{t-1} + C(1)\Delta z_{t-1} - A(1)\Delta y_{t-1} + \varepsilon_t] \end{aligned} \quad (20)$$

We have seen that, deterministic cointegration implies that $E[\Delta y_t] = E[\Delta x_t] + E[\Delta z_t] = k_2 + k_3$ and thus, from (20) we have:

$$E[y_t - x_t - z_t] = \delta + \frac{1}{\mu} [(B(1) - A(1))k_2 + (C(1) - A(1))k_3] \quad (21)$$

This equation is identical to (9) since we must have $\delta = m/\mu$. By definition, (9) or (15) and (21) represent the same equation and therefore we must have $A(1) = B(1) + C(1)$. Notice that this outcome is a direct consequence of the fact that the so-called short term dynamics in the ECM model only captures the autocorrelation in the residuals u_t pertaining to the static cointegrating regression (15), which is a zero mean stationary variable.

It should be stressed that these conclusions are not specific to our wage equation (6). Rather they carry over to quite general error correction models with I(1) variables, providing deterministic cointegration holds. In particular, we note that for the general error correction model with two arbitrary explanatory variables and arbitrary long run coefficients, reparametrised in a similar form to (8)

$$\begin{aligned} A^*(L)\Delta^2 y_t &= B^*(L)\Delta^2 x_t + C^*(L)\Delta^2 z_t - A(1)\Delta y_{t-1} + B(1)\Delta x_{t-1} + C(1)\Delta z_{t-1} \\ &\quad - \mu(y_{t-1} - \gamma_1 x_{t-1} - \gamma_2 z_{t-1} - \delta) \end{aligned} \quad (22)$$

we have

$$A(1)E[\Delta y_{t-1}] = B(1)E[\Delta x_{t-1}] + C(1)E[\Delta z_{t-1}] \quad (23)$$

providing we assume

$$E[(y_{t-1} - \gamma_1 x_{t-1} - \gamma_2 z_{t-1} - \delta)] = 0 \quad (24)$$

and $E[\Delta^2 y_t] = E[\Delta^2 x_t] = E[\Delta^2 z_t] = 0$, as it is customary in the literature. Given (23) it is clear that the long run equilibrium solution of (22) boils down to

$$y_t = \delta + \gamma_1 x_t + \gamma_2 z_t \quad (25)$$

regardless of the assumed steady state behaviour for the variables x_t and z_t .

We thus conclude that in error correction models, deterministic cointegration, on its own, implies that the long run solution of the model does not depend on the growth rates of the variables or, in other words there is no theoretical ground to distinguish between static and dynamic long run equilibrium in such models. As an immediate corollary it follows that the so called dynamic homogeneity restriction aiming at guaranteeing that the two solutions coincide also lacks theoretical justification.

Of course one must be aware that this conclusion does not imply that when freely estimating model (6) one necessarily gets the exact equality $\hat{A}(1) = \hat{B}(1) + \hat{C}(1)$, as we also do not exactly have in the sample $mean(\Delta y) = mean(\Delta x) + mean(\Delta z) = \hat{k}_2 + \hat{k}_3$. However if in the sample the equality $mean(\Delta y) = \hat{k}_2 + \hat{k}_3$ holds approximately one must expect $\hat{A}(1) \approx \hat{B}(1) + \hat{C}(1)$ and, in practical terms, it does not matter whether one imposes this restriction or not, as it does not have significant implications for the long run behaviour of the model. The cointegration relation, on its own, ensures that in the long run the productivity adjusted real wage will evolve around the constant δ even if the steady state growth rate of x_t or z_t , or both, change⁷.

⁷If this is not the case then we must have a structural break in the constant of the cointegrating relation, but this is a different issue which (supposedly) the dynamic homogeneity restriction does not intend to address.

5 Cases where dynamic homogeneity should not be expected to hold

The conclusions on dynamic homogeneity drawn above must however be qualified as it is the case that the dynamic homogeneity condition may not hold in the estimated version of some ECMs, or in other words that the computed static and dynamic equilibrium may differ.

One important case in which dynamic homogeneity may not hold occurs when the so-called short-term dynamics additionally includes non-zero mean stationary variables that are not included in the error correction term⁸. The most notable example is probably (again) the wage equation. Usually the wage equation includes the unemployment rate in the short-term dynamics, under the (implicit) assumption that this is stationary variable. In this case there is no reason to expect dynamic homogeneity to hold empirically. To see that consider the following standard dynamic wage equation obtained from (6) by considering additionally in the equation the unemployment rate U_t :

$$\begin{aligned} A^*(L)\Delta^2 y_t &= m + B^*(L)\Delta^2 x_t + C^*(L)\Delta^2 z_t + D^*(L)\Delta U_t + D(1)U_{t-1} \\ &\quad - A(1)\Delta y_{t-1} + B(1)\Delta x_{t-1} + C(1)\Delta z_{t-1} - \mu [y_{t-1} - x_{t-1} - z_{t-1}] + \epsilon_t \end{aligned} \quad (26)$$

Taking expectations we get:

$$E[y_t - x_t - z_t] = \frac{m}{\mu} - \frac{1}{\mu} \left[D(1)\bar{U} - A(1)k_1 + B(1)k_2 + C(1)k_3 \right] \quad (27)$$

where $\bar{U} = E[U_t]$ and so cointegration implies that

$$D(1)\bar{U} - A(1)k_1 + B(1)k_2 + C(1)k_3 = 0 \quad (28)$$

where as before $k_1 = k_2 + k_3$. Using this condition in (28) we may finally write

$$D(1)\bar{U} = [A(1) - B(1)]k_2 + [A(1) - C(1)]k_3 \quad (29)$$

⁸This is apparently the case in Brouwer and Ericsson (1998), whose estimated model, besides the output gap, also includes an impulse dummy in the short term dynamics .

and so one should *not* expect the homogeneity restriction $A(1) = B(1) = C(1)$ to hold unless $\overline{U} = 0$ ⁹.

Another example occurs when the variables are stochastically but not deterministically cointegrated. Of course, if the variables are stochastically but not deterministically cointegrated, the correct specification of the model would require the introduction of a linear time trend in model (22). In this case the long run static solution reads as

$$y_t = \delta + \lambda t + \gamma_1 x_t + \gamma_2 z_t \quad (30)$$

Obviously also in this case there is no theoretical foundation to distinguish between static and dynamic long run equilibrium. However, such a situation raises new issues in what concerns the dynamic homogeneity restriction. To see that let us resume again our equation (6). Under static but not deterministic cointegration it turns out that in (16) we have $k_1 \neq k_2 + k_3$, so that instead of (15) we would have the long run equilibrium solution

$$y_t = \delta + \lambda t + x_t + z_t \quad (31)$$

where $\lambda = k_1 - k_2 - k_3$. The equivalent to (18) would be given by

$$A(L)\Delta y_t = B(L)\Delta x_t + C(L)\Delta z_t - \mu(y_{t-1} - x_{t-1} - z_{t-1} - \lambda(t-1) - \delta) + \varepsilon_t \quad (32)$$

and now it is straightforward to show that equation (23) still holds for this model. The important difference is that we no longer have the condition $k_1 = k_2 + k_3$, but rather the condition $k_1 = \lambda + k_2 + k_3$. Substituting this condition into (23) we get

$$[B(1) - A(1)]k_2 + [C(1) - A(1)]k_3 = A(1)\lambda \quad (33)$$

which shows that the dynamic homogeneity condition $A(1) = B(1) = C(1)$ is no longer valid.

⁹Notice that in the case of this example the problem may be overcome by specifying the model with the unemployment gap $U_t - U_t^*$ provided the NAIRU U_t^* is defined in a way such that $E[U_t - U_t^*] = 0$.

6 Conclusions

This paper shows that there are no theoretical grounds to distinguish between the static and the long run equilibrium solutions in error correction models with $I(1)$ deterministically cointegrated variables. In other words, and against conventional wisdom, the paper shows that the dynamic equilibrium solution of such models does not depend on the growth rates of the variables in the model. As a side product it is also shown that there are no valid reasons to argue for the need to impose the so-called dynamic homogeneity restriction in the estimated models. Examples in which dynamic homogeneity cannot hold are also discussed.

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