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Financing in the Eurosystem: fixed versus variable rate tenders

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Abstract

In a three-stage game in which banks can obtain liquidity through open market operations, interbank transactions or standing facilities we compare the equilibrium outcomes of fixed and variable rate tenders in the primary market. We focus on bidding behavior, induced allotment ratios, functioning of the secondary market and resorting to standing facilities, under several scenarios, among which collateral shortage and credit rationing. It is shown that overbidding is inherent to the fixed rate auction, but can be very mitigated under a variable rate procedure. Due to the existence of a finite number of equilibria, variable rate tenders allow keeping the informational content of quantity bids, as opposed to fixed rate tenders.

Keywords: Eurosystem; Liquidity auctions; Bidding equilibria

JEL classification: D44; E52; G21

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1. Introduction

Since the beginning of Stage Three of the Economic and Monetary Union in January 1999, and until the end of June 2000, the Eurosystem conducted its main refinancing operations (MROs) for the provision of liquidity to banks through fixed rate tenders. The change to a variable rate procedure was introduced in the MRO of the 28 June 2000.

In this paper both types of tender are compared in what respects: induced bidding behavior, allocation of liquidity, functioning of the secondary market, resorting to standing facilities. Several situations are considered: collateral is binding; liquidity suppliers in the secondary market face informational problems and ration quantity; there are liquidity supply shocks in the secondary market; the ECB underestimates or overestimates the system's liquidity needs; some of these together; none of these distortions.

Banks have to comply with minimum reserve requirements. There are three markets where they can obtain/place liquidity: the primary market, through the open market operations of the European Central Bank (ECB); the secondary market or interbank market, where liquidity is traded among banks; the standing facilities - lending or deposit - of the ECB, at penalty rates. All operations with the ECB, either in the primary or in the third market, must be collateralized. In contrast, the exchange of liquidity in the secondary market is assumed to be uncollateralized.

In a fixed rate tender the ECB sets the interest rate at which MROs operations are conducted and also the lending and deposit facilities' interest rates. Only the interbank market rate is determined by demand and supply forces. In a variable rate tender procedure the prevailing primary market rate fluctuates according to market conditions in a given interval, the ECB using the amount of liquidity allotted to steer its behavior; the other two rates can be used as policy instruments too.

Two recent papers have also addressed the open market operations of the ECB. In a model with no secondary market, Nautz and Oechssler (1999) advise the use of variable rate tenders with a preannounced interest signal as a means to avoid the allotment ratio from vanishing over time. Ayuso and Repullo (2001a) assume that the ECB wants to minimize deviations of the interbank rate from the target rate that signals the stance of monetary policy; they show that when the penalty is higher for rates below the target, preannouncing the amount of the liquidity injection eliminates overbidding in a variable rate tender, but the equilibrium in a fixed rate tender is still characterized by extreme overbidding.

The paper is organized as follows. Section 2 presents the models for fixed and for variable rate liquidity auctions. In section 3 the equilibrium bidding behaviors in the fixed rate auction are derived under different conditions, while the corresponding analysis for the variable rate auction is performed in section 4. Section 5 compares results for both types of tender and section 6 concludes.

2. The two models

To make the analysis tractable, we consider two banks, one of which expects not to be a liquidity supplier in the secondary market (i), whereas the other (j) has the opposite expectation. In equilibrium these expectations are confirmed. The equilibrium behaviors of banks i and j replicate the bids of banks that expect to have these positions in the interbank market.

Banks' liquidity needs are denoted by $\bar{l}_i$ and $\bar{l}_j$. Liquidity obtained in the primary market is l_i and l_j, so bank i still needs $\bar{l}_i - l_i \geq 0$, while bank j has $l_j - \bar{l}_j \geq 0$ in excess.

The interest paid by the central bank's deposit facility is d, and the interest charged on the lending facility is denoted by c. In a fixed rate tender there is another administered interest rate, denoted by r, with $r \in]d, c[$, and which is the rate paid by the liquidity obtained in the MRO. In a variable rate tender there may exist a minimum interest rate for bids, also chosen by the ECB. The amount of liquidity that bank i (j) proposes to buy in the MRO is b_i (b_j). In a variable rate procedure the bank also makes a bid, denoted by r_i (r_j), for the price to pay for this liquidity.

The interbank rate is denoted by o. Bank j is not willing to sell liquidity at a price below d, so o is bounded from below by d; however, bank i may be willing to buy at an interest rate higher than c, in case collateral is insufficient to satisfy the remaining liquidity needs through the lending facility (remember that, contrary to operations with the ECB, transactions in the interbank market are uncollateralized).

Banks wish to minimize the unavoidable expenditure they have to face in order to comply with reserve requirements. Both games - fixed and variable rate auctions - have three stages, corresponding to the three different possibilities of financing available: MRO, interbank market, and standing facilities.[1] The

[1] We are not considering longer-term refinancing operations, fine-tuning operations, and struc-

problems are solved backwards. The solution concept employed is the subgame perfect Nash equilibrium.

The optimum bidding behavior in the first market arises from the first-order conditions of the expenditure minimization problem (assuring second-order conditions for a minimum are verified). In a fixed rate tender there is only one decision variable, the quantity bid, whereas in a variable rate tender there is an additional one, the interest rate that the bank proposes to pay for the quantity bid it makes. Banks and the monetary authority are assumed to be risk-neutral, except for the case in which it is considered that, due to informational problems, liquidity suppliers in the secondary market may choose a credit rationing strategy.

3. Fixed rate tenders

In a fixed rate tender each bank is allocated a proportion of its bid (under the implicit assumption that total bids exceed the allotted amount). The allotment ratio is defined as $\frac{\bar{l}_i+\bar{l}_j+v}{b_i+b_j}$, where $\bar{l}_i + \bar{l}_j + v$ is the total amount of liquidity that the ECB decides to allocate. A parameter $v > 0$ ($v < 0$) means an overestimation (underestimation) by the ECB of the system's liquidity needs. Experience with fixed rate tenders has shown low allotment ratios (see table in the Appendix).

Denote by α_i the proportion of bank i's bid in terms of total bids: $\alpha_i = \frac{b_i}{b_i+b_j}$. In the primary market bank i is allocated

$$l_i = \frac{\bar{l}_i + \bar{l}_j + v}{b_i + b_j} b_i = \alpha_i(\bar{l}_i + \bar{l}_j + v)$$

So, it still has to acquire

$$\bar{l}_i - l_i = (1 - \alpha_i)\bar{l}_i - \alpha_i \bar{l}_j - \alpha_i v = \alpha_j \bar{l}_i - \alpha_i \bar{l}_j - \alpha_i v$$

Bank j buys

$$l_j = \frac{\bar{l}_i + \bar{l}_j + v}{b_i + b_j} b_j = \alpha_j(\bar{l}_i + \bar{l}_j + v)$$

tural operations, which have a modest importance in terms of obtaining the liquidity needed. Indeed, total liquidity acquired since January 1999 through longer-term refinancing operations, the more relevant, accounted, on the 31 July 2001, for only 5.7 per cent of the total liquidity allocated in MROs; the corresponding figure at the end of 1999 was higher, but nevertheless of small importance (6.7 per cent); at the end of 2000 the percentage was 5.9. Including these operations in our analysis would only add further steps, not changing the main results qualitatively.

in the primary market, and so has excess liquidity equal to

$$l_j - \bar{l}_j = \alpha_j \bar{l}_i + (\alpha_j - 1)\bar{l}_j + \alpha_j v = \alpha_j \bar{l}_i - \alpha_i \bar{l}_j + \alpha_j v$$

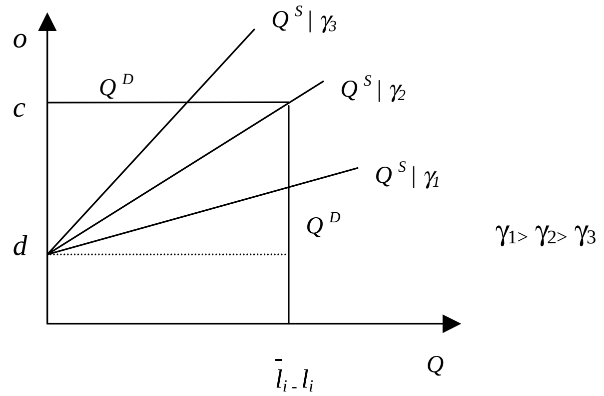
The demand curve in the interbank market is given by[2]

$$Q^D = \begin{cases} \alpha_j \bar{l}_i - \alpha_i \bar{l}_j - \alpha_i v \Leftarrow o \leq c \\ 0 \Leftarrow o > c \end{cases}$$

The supply curve can be written as

$$Q^S = \begin{cases} 0 \Leftarrow o < d \\ \gamma(\alpha_j \bar{l}_i - \alpha_i \bar{l}_j + \alpha_j v).\frac{o-d}{c-d} \Leftarrow o \geq d \end{cases}$$

where the parameter $\gamma > 0$ stands for the "willingness" of bank j to lend. A higher γ means a more elastic supply, as the following figure shows.



For low enough γ ($\gamma < \gamma_2$) bank i is not able to acquire all the liquidity needed in the secondary market and has to make use of the lending facility.

Equating demand and supply the equilibrium overnight rate arises:

$$o^* = \begin{cases} d + \frac{1-\rho}{\gamma}(c-d) \Leftarrow \gamma \geq 1-\rho \\ c \Leftarrow \gamma < 1-\rho \end{cases}$$

where $\rho = \frac{2\alpha_i v}{\alpha_j \bar{l}_i - \alpha_i(\bar{l}_j - v)}$. The quantity traded in the secondary market is

[2] If collateral is not restrictive. Otherwise the first branch is valid for all o.

$$Q^* = \begin{cases} \alpha_j \bar{l}_i - \alpha_i \bar{l}_j - \alpha_i \upsilon \Leftarrow \gamma \geq 1 - \rho \\ \gamma(\alpha_j \bar{l}_i - \alpha_i \bar{l}_j + \alpha_j \upsilon) \Leftarrow \gamma < 1 - \rho \end{cases}$$

In the third market there is no interaction between banks. Liquidity in short is obtained at c, and liquidity in excess gives rise to a deposit remunerated at d.

Liquidity in short (for bank i) is

$$\bar{l}_i - l_i - Q^* = \begin{cases} 0 \Leftarrow \gamma \geq 1 - \rho \\ (1-\gamma)(\alpha_j \bar{l}_i - \alpha_i \bar{l}_j) - (\alpha_i + \gamma \alpha_j)\upsilon \Leftarrow \gamma < 1 - \rho \end{cases}$$

and liquidity in excess (for bank j) is

$$l_j - \bar{l}_j - Q^* = \begin{cases} (\alpha_i + \alpha_j)\upsilon \Leftarrow \gamma \geq 1 - \rho \\ (1-\gamma)(\alpha_j \bar{l}_i - \alpha_i \bar{l}_j + \alpha_j \upsilon) \Leftarrow \gamma < 1 - \rho \end{cases}$$

Higher γ implies less need for i to resort to lending facilities and less need for j to deposit excess liquidity with the ECB.

The problem for bank i in the primary market (which expects not to be a liquidity supplier in the interbank market) can be stated as

$$\min_{b_i} r.\alpha_i(\bar{l}_i + \bar{l}_j + \upsilon) + (d + \frac{1-\rho}{\gamma}(c-d))(\alpha_j \bar{l}_i - \alpha_i \bar{l}_j - \alpha_i \upsilon) \Leftarrow \gamma \geq 1 - \rho$$

or

$$\min_{b_i} r.\alpha_i(\bar{l}_i + \bar{l}_j + \upsilon) + c(\alpha_j \bar{l}_i - \alpha_i \bar{l}_j - \alpha_i \upsilon) \Leftarrow \gamma < 1 - \rho$$

The problem for bank j (which expects not to be a liquidity demander in the interbank market) is

$$\min_{b_j} r.\alpha_j(\bar{l}_i + \bar{l}_j + \upsilon) - (d + \frac{1-\rho}{\gamma}(c-d))(\alpha_j \bar{l}_i - \alpha_i \bar{l}_j - \alpha_i \upsilon) - d(\alpha_i + \alpha_j)\upsilon \Leftarrow \gamma \geq 1 - \rho$$

or

$$\min_{bj} r.\alpha_j(\bar{l}_i + \bar{l}_j + \upsilon) - (\gamma c + (1-\gamma)d)(\alpha_j \bar{l}_i - \alpha_i \bar{l}_j + \alpha_j \upsilon) \Leftarrow \gamma < 1 - \rho$$

Note that ρ is a function of b_i and b_j, through α_i and α_j.[3] Actually, $\frac{\partial \rho}{\partial b_j} < 0$, so a higher bid by bank j decreases the likelihood of having $\bar{l}_i - l_i$ entirely satisfied through interbank transactions. A higher b_i has the opposite effect.

To begin with, let us consider the simple case with $\upsilon = 0$ (the estimation of liquidity needs by the ECB is acute). As the reader may have noticed in the derivations above, we are implicitly assuming that either the ECB announces the injection amount, or banks have an exact expectation about it. Since the former did not happen in any of the fixed rate tenders conducted, the latter may be a more adequate assumption.

3.1. Absence of estimation errors

When the ECB correctly estimates the system's liquidity needs ($\upsilon = 0$), liquidity in excess matches liquidity in short for the interbank market. The parameter ρ reduces to zero and the equilibrium overnight rate is

$$o^* = \begin{cases} d + \frac{c-d}{\gamma} \Leftarrow \gamma \geq 1 \\ c \Leftarrow \gamma < 1 \end{cases}$$

If $\gamma \geq 1$ (supply is sufficiently elastic so that all remaining liquidity needs of bank i are satisfied in the secondary market) the problem of bank i resumes to

$$\min_{b_i} r\alpha_i(\bar{l}_i + \bar{l}_j) + (d + \frac{c-d}{\gamma})(\alpha_j \bar{l}_i - \alpha_i \bar{l}_j)$$

and the problem of bank j to

$$\min_{b_j} r\alpha_j(\bar{l}_i + \bar{l}_j) - (d + \frac{c-d}{\gamma})(\alpha_j \bar{l}_i - \alpha_i \bar{l}_j)$$

To be consistent with the assumption that i will not be a liquidity supplier and that j will not be a liquidity demander in the interbank market, their bids have to satisfy the following condition (arising from $Q^D \geq 0$ and $Q^S \geq 0$, with $\upsilon = 0$): $b_j \bar{l}_i - b_i \bar{l}_j \geq 0 \Leftrightarrow$

$$b_i \leq \frac{\bar{l}_i}{\bar{l}_j} b_j$$

[3] $\frac{\partial \alpha_i}{\partial b_i} = -\frac{\partial \alpha_j}{\partial b_i} = \frac{1-\alpha_i}{b_i+b_j} = \frac{\alpha_j}{b_i+b_j} > 0$ and $\frac{\partial \alpha_j}{\partial b_j} = -\frac{\partial \alpha_i}{\partial b_j} = \frac{1-\alpha_j}{b_i+b_j} = \frac{\alpha_i}{b_i+b_j} > 0$

$$b_j \geq \frac{\bar{l}_j}{\bar{l}_i} b_i$$

So, $b_i \in (0, \frac{\bar{l}_i}{\bar{l}_j} b_j]$, and $b_j \in [\frac{\bar{l}_j}{\bar{l}_i} b_i, +\infty)$. From this it is clear that

Proposition 3.1. *In a fixed rate tender, bids can grow infinitely high, no matter what the expected position of the bank in the secondary market, which is at the origin of potentially low allotment ratios.*

This result, which is not specific to the scenario under analysis, is a direct consequence of the fact that bids are not limited by available collateral, only the liquidity received is.

Optimizing the two objective functions (of_i and of_j) gives

$$\frac{\partial of_i}{\partial b_i} = (1 - \alpha_i)\varepsilon(r - o)$$

$$\frac{\partial of_j}{\partial b_j} = (1 - \alpha_j)\varepsilon(r - o)$$

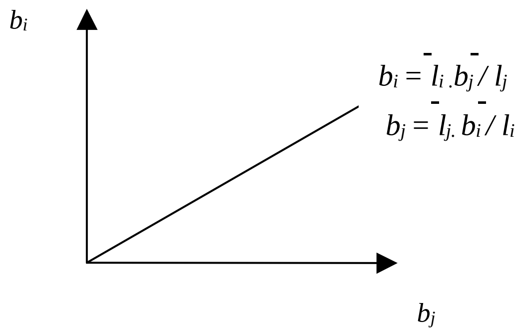
where $\varepsilon = \frac{\bar{l}_i + \bar{l}_j}{b_i + b_j}$ is the allotment ratio.

If $o > r$, which corresponds to $1 \leq \gamma < \frac{c-d}{r-d}$, the higher b_i the better for bank i, and the higher b_j the better for bank j, so bids will tend to reach very high levels. However, if γ is sufficiently high so that the equilibrium overnight rate falls below the MRO interest rate, both banks have an incentive to bid very low quantities in the MRO.

The equilibrium locus is

$$b_i = \frac{\bar{l}_i}{\bar{l}_j} b_j$$

The two reaction curves are upward-sloping and fully coincide, as the following figure shows. There are multiple equilibria. The allotment ratio can thus turn out to be very low, if the equilibrium bids locate at high levels, which corresponds to the outcome of most MROs conducted through fixed rate auctions.[4] This multiplicity of equilibria additionally reduces (or even extinguishes) the informational content of bids as regards liquidity needs.



Bids of banks i and j are strategic complements: the higher the expectation of the bid of the rival, the higher is the optimal bid of each bank. As expected, given the bid of the opponent, a bank's optimal bid is increasing in its own liquidity needs, and decreasing in the opponent's liquidity needs.

If $\gamma < 1$ ($\bar{l}_i - l_i$ is not entirely solved in the secondary market) the problem of bank i can be written as

$$\min_{b_i} r\alpha_i(\bar{l}_i + \bar{l}_j) + c(\alpha_j \bar{l}_i - \alpha_i \bar{l}_j)$$

and the problem of bank j as

$$\min_{b_j} r\alpha_j(\bar{l}_i + \bar{l}_j) - (\gamma c + (1-\gamma)d)(\alpha_j \bar{l}_i - \alpha_i \bar{l}_j)$$

The first-order conditions of these problems are

$$\begin{aligned} \frac{\partial of_i}{\partial b_i} &= \frac{b_j(\bar{l}_i + \bar{l}_j)}{(b_i + b_j)^2}(r - c) = (1 - \alpha_i)\varepsilon(r - c) < 0 \\ \frac{\partial of_j}{\partial b_j} &= (1 - \alpha_j)\varepsilon(r - d - \gamma(c - d)) \end{aligned}$$

[4] For an empirical analysis of the reasons of overbid in fixed rate tenders see Ayuso and Repullo (2001b).

For bank i the higher b_i the better, which is quite intuitive from the fact that the overnight rate will reach c. If $\frac{r-d}{c-d} < \gamma < 1$ bank j also wants to bid high in the first market, but for low enough γ this result may be reversed, as j will be left with much liquidity to deposit with the ECB. The equilibrium locus is again given by $b_i = \frac{\bar{l}_i}{\bar{l}_j} b_j$, and the allotment ratio may reach very low levels.

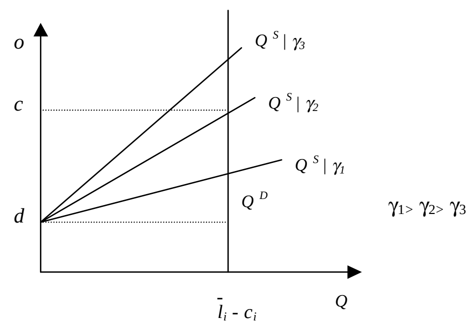
3.2. Shortage of collateral

Suppose now that bank i is limited by its amount c_i of collateral, such that $c_i < \bar{l}_i$.[5] Note that since bids are independent of the availability of collateral, banks may be faced with a situation in which the liquidity amount they were allocated can not be covered by collateral. Fixed rate tenders thus involve a high disturbing risk for each bank and for the whole system, especially when the allotment ratio is difficult to predict (and happens to be higher than expected by bidders).

Bank i will try to acquire an amount equal to c_i (the maximum possible) in the primary market.[6] In the secondary market the supply curve does not change. The demand curve is perfectly inelastic with

$$Q^D = \bar{l}_i - c_i$$

Because of collateral shortage bank i has to fully satisfy Q^D, as is patent in the following figure (for $\upsilon = 0$, $\gamma_2 = 1$). The equilibrium overnight rate may thus be higher than c.



The equilibrium o will be

[5] To keep the analysis tractable we still assume $\upsilon = 0$.

[6] There is no point in saving collateral for the third market, because the price of liquidity is higher than in the MRO.

$$o = d + \frac{(c-d)(\bar{l}_i - c_i)}{\gamma(\alpha_j \bar{l}_i - \alpha_i \bar{l}_j)}$$

which is higher than c for $\gamma < \frac{\bar{l}_i - c_i}{\alpha_j \bar{l}_i - \alpha_i \bar{l}_j}$ (a lower c_i increases the likelihood of o rising above c). The interbank rate is lower, the higher the elasticity of supply.

In the third market bank j deposits

$$\alpha_j(\bar{l}_i + \bar{l}_j) - \bar{l}_j - Q^S = c_i - \alpha_i(\bar{l}_i + \bar{l}_j)$$

which increases with c_i: the shortage of collateral intensifies trading in the interbank market and so less liquidity remains to be placed at d.

Lemma 3.2. *Lack of enough collateral to cover the allotted amount in the primary market raises price and quantity traded in the interbank market, whereas the use of the deposit facility is softened.*

In order for i to guarantee itself a liquidity amount equal to c_i in the primary market, its minimum bid is $b_i^{min} = \frac{c_i . b_j}{\bar{l}_i + \bar{l}_j - c_i}$, arising from the condition $\alpha_i(\bar{l}_i + \bar{l}_j) \geq c_i$. This is actually the optimal bid, since the objective function appears to be increasing in b_i. It does not pay bank i to make a bid above this one, since that would not guarantee it any additional liquidity in the primary market, and would increase the prevailing interest rate in the secondary market, because bank j would be allocated with less liquidity in the MRO.

So, bidding equilibria are now given by the relationship $b_i = \frac{c_i}{\bar{l}_i + \bar{l}_j - c_i} b_j$. The equilibrium locus is steeper than before, which means that in equilibrium the bidding behavior of i is more responsive to the bidding behavior of j. The shortage of funds for i, equal to $\bar{l}_i - c_i$, is solved in the secondary market, at an equilibrium interest rate equal to $d + \frac{c-d}{\gamma}$.

To fulfill its liquidity needs, bank i spends $r.c_i + (d + \frac{c-d}{\gamma})(\bar{l}_i - c_i)$, higher than in the case in which collateral is not restrictive if γ is low enough ($\gamma < \frac{c-d}{r-d}$). For this range of values of the supply elasticity bank j is able to take advantage of i's shortage of collateral, and its expenses decline. ECB's revenues, in turn, remain unchanged as compared with the case of unbinding collateral.[7] The total amount spent by i and j together is not altered (so there is no loss of efficiency), but the distribution may change in favor of the non collateral restricted banks.

[7] Although we refer to ECB's earnings, throughout the paper we are implicitly assuming that, as in Ayuso and Repullo (2001a), the Central Bank has mainly policy concerns, the revenue obtained being less important.

3.3. Credit rationing in the interbank market

Suppose now that bank i is subject to credit rationing in the secondary market. We elaborate on the case with $\upsilon = 0$, but with collateral restrictions. So, in the equilibrium bank i will use all its collateral in the primary market.

Because of informational problems bank j is not willing to lend at an overnight rate higher than t, with $d < t < c$.[8] There will be excess demand in the secondary market at t.

In this case equilibrium bidings are again given by $b_i = \frac{c_i}{\bar{l}_i + \bar{l}_j - c_i}.b_j$. Quantity traded in the interbank market, at the interest rate t, is equal to $\frac{\gamma(t-d)(\bar{l}_i - c_i)}{c-d}$. As expected, it varies positively with t: the more rationing there is (the lower t), the lower Q. Credit rationing is active if and only if $t < d + \frac{c-d}{\gamma}$, the equilibrium overnight rate obtained in subsection 3.2, which is equivalent to $\gamma < \frac{c-d}{t-d}$.

Bank i is unable to satisfy reserve requirements by an amount equal to what bank j deposits with the ECB. This amount varies positively with the level of rationing, and negatively with the elasticity of supply, as expected.

Lemma 3.3. *Due to the existence of rationing in the interbank market, banks which are binded by collateral may be unable to satisfy reserve requirements. In turn, the other banks have excess liquidity to be applied through the deposit facility.*

The revenue of the ECB declines as compared with the no credit rationing case, because banks of the j type make now use of the deposit facility.

3.4. Supply shocks in the interbank market

Let us admit that supply by bank j in the interbank market is subject to exogenous shocks, denoted by ξ, with a negative (positive) supply shock corresponding to $\xi > 0$ ($\xi < 0$).[9] Once again we elaborate on the case with $\upsilon = 0$, but with collateral restrictions. So, in the equilibrium bank i will again use all its collateral in the primary market.

[8] Recall that transactions in the interbank market are assumed to be uncollateralized.

[9] For some reason bank j wants to modify its liquidity supply. For example, in a dynamic context, these shocks might have to do with expectations of changes in administered rates, which would alter the intertemporal pattern of reserve keeping through the maintenance period and, hence, the willingness to sell in the interbank market in a given period.

Bank i is able to satisfy all its remaining liquidity needs at the equilibrium overnight rate, which is $o = d + \frac{\bar{l}_i - c_i + \xi}{\gamma(\bar{l}_i - c_i)}(c - d)$. A negative (positive) supply shock thus increases (reduces) the price paid in the interbank market. For $\gamma < 1 + \frac{\xi}{\bar{l}_i - c_i}$ the value for o is larger than c.[10]

However, if i were not collaterally restricted, the equilibrium o would not surpass c and, if $\xi > 0$, for low enough values of the supply elasticity, banks of the j-type would be left with ξ excess liquidity to deposit with the ECB.

Lemma 3.4. *A negative supply shock implies a rise in the equilibrium interbank rate, and may force banks with insufficient collateral to pay more than the lending facility rate. A positive supply shock has no such consequences.*

Bank i benefits from a positive supply shock and is harmed by a negative one. Bank j, on the contrary, benefits from negative shocks, and its expenses rise due to positive ones. However, if collateral is not restrictive, bank j may also be harmed by negative supply shocks, as it may not be able to sell all the excess liquidity in the interbank market.

ECB's revenues are not affected either by negative nor by positive interbank liquidity shocks not accounted for by monetary policy operations when banks of the i-type are collaterally restricted. These revenues, however, may be reduced when the shocks are negative and c_i is not binding, if j has to make use of the deposit facility.

3.5. Estimation errors

In this subsection the hypothesis that $\upsilon = 0$ is abandoned. The ECB may underestimate ($\upsilon < 0$) or overestimate ($\upsilon > 0$) the system's liquidity needs. This case (with no collateral restrictions) corresponds to the general problem formulated in the beginning of the section. The wrong estimation derives from an incorrect forecast of the "autonomous factors" of liquidity injection/absorption and affects banks of both types (i and j). The "autonomous factors" are Eurosystem's balance sheet items whose amount is independent of the monetary policy operations of the Central Bank.

[10] Bank j is representative of the supply side. Otherwise, if there was only one supplier, market power could be exploited when $o < c$ and the interbank rate raised to c, even in the absence of supply shocks. Nyborg and Strebulaev (2000) address (in a variable rate tender context) the case in which long players (banks of the j type) have market power and are able to exploit it. Then short players (banks of the i type) face a positive probability of being squeezed.

When $v < 0$ the equilibrium bids are given by $b_i = \frac{\bar{l}_i + v}{\bar{l}_j}.b_j$. For a given bid of j, in equilibrium bank i will thus bid lower than when no estimation errors are expected (and bank j higher, for a given b_i).[11] Bank i is allocated a quantity equal to $\bar{l}_i + v$ (below its needs); bank j receives $\bar{l}_j$. So, i is the one who bears all the burden of the ECB's estimation error. As a consequence, it will have to acquire $|v|$ through the lending facility; total expenses are higher than without estimation errors.[12] Bank j's expenses, in turn, are unaffected. The ECB benefits from the underestimation of liquidity needs, since its revenue rises by $(c - r)|v|$.

These results are reverted when liquidity needs are overestimated ($v > 0$). Then equilibrium bids satisfy $b_i = \frac{\bar{l}_i}{\bar{l}_j + v}.b_j$, bank i is allocated with its full needs and bank j receives v in excess, which it will deposit with the ECB. Expenses of bank i are left unchanged as compared with the case with $v = 0$, but bank j's expenses rise. ECB's revenues are again larger than without estimation errors. The ECB always benefits from $v \neq 0$, because the estimation error will have to be corrected through the standing facilities.

Lemma 3.5. *When the ECB underestimates (overestimates) the system's liquidity needs, bank i (j) is the one who bears all the burden of the estimation error.*

4. Variable rate tenders

In a variable rate tender each bank has two decision variables: the amount it proposes to buy (as in a fixed rate auction) and the interest rate it proposes to pay.

The bank offering the highest interest rate wins the auction and receives a liquidity amount equal to its bid. The looser is left with the remaining liquidity (or, more precisely, with the minimum between the remaining liquidity and available collateral). Each bank pays its own interest rate bid, so we have a multiple rate auction, the solution actually adopted by the Eurosystem.

Consider that the ECB wishes to allocate $\bar{l}_i + \bar{l}_j + v$, as before, with $v < 0$ corresponding to an underestimation of the system's needs. Denote by r_i (r_j) the

[11] So that expectations as to net positions in the interbank market are fulfilled, a necessary condition for an equilibrium.

[12] If i is restricted by collateral, it may be unable to comply with reserve requirements.

interest rate bid of bank i (j). Then, liquidity allocated to each counterpart is the following.[13]

$$l_i = \begin{cases} b_i \Leftarrow r_i > r_j \\ \bar{l}_i + \bar{l}_j + \upsilon - b_j \Leftarrow r_i < r_j \end{cases}$$

$$l_j = \begin{cases} \bar{l}_i + \bar{l}_j + \upsilon - b_i \Leftarrow r_i > r_j \\ b_j \Leftarrow r_i < r_j \end{cases}$$

Liquidity in short for bank i is

$$\bar{l}_i - l_i = \begin{cases} \bar{l}_i - b_i \Leftarrow r_i > r_j \\ b_j - \bar{l}_j - \upsilon \Leftarrow r_i < r_j \end{cases}$$

and in excess for bank j

$$l_j - \bar{l}_j = \begin{cases} \bar{l}_i + \upsilon - b_i \Leftarrow r_i > r_j \\ b_j - \bar{l}_j \Leftarrow r_i < r_j \end{cases}$$

When bank i wins the auction, demand and supply in the interbank market (with no collateral restrictions for i) are

$$Q^D = \begin{cases} \bar{l}_i - b_i \Leftarrow o \leq c \\ 0 \Leftarrow o > c \end{cases}$$

$$Q^S = \begin{cases} 0 \Leftarrow o < d \\ \gamma(\bar{l}_i - b_i + \upsilon)\frac{o-d}{c-d} \Leftarrow o \geq d \end{cases}$$

so the equilibrium overnight rate is

$$o^* = \begin{cases} d + \frac{\bar{l}_i - b_i}{\gamma(\bar{l}_i - b_i + \upsilon)}(c-d) \Leftarrow \gamma \geq \frac{\bar{l}_i - b_i}{\bar{l}_i - b_i + \upsilon} \\ c \Leftarrow \gamma < \frac{\bar{l}_i - b_i}{\bar{l}_i - b_i + \upsilon} \end{cases}$$

and quantity traded

[13] We will not consider the uninteresting case in which $r_i = r_j$. When both banks bid the same price, each one receives a proportion of the liquidity injection corresponding to the quantity bid made, as in a fixed rate tender.

$$Q^* = \begin{cases} \bar{l}_i - b_i \Leftarrow \gamma \geq \frac{\bar{l}_i - b_i}{\bar{l}_i - b_i + \upsilon} \\ \gamma(\bar{l}_i - b_i + \upsilon) \Leftarrow \gamma < \frac{\bar{l}_i - b_i}{\bar{l}_i - b_i + \upsilon} \end{cases}$$

When the winner is bank j $(r_i < r_j)$ the following changes occur:

$$Q^D = \begin{cases} b_j - \bar{l}_j - \upsilon \Leftarrow o \leq c \\ 0 \Leftarrow o > c \end{cases}$$

$$Q^S = \begin{cases} 0 \Leftarrow o < d \\ \gamma(b_j - \bar{l}_j)\frac{o-d}{c-d} \Leftarrow o \geq d \end{cases}$$

so the equilibrium overnight rate is

$$o^* = \begin{cases} d + \frac{b_j - \bar{l}_j - \upsilon}{\gamma(b_j - \bar{l}_j)}(c-d) \Leftarrow \gamma \geq \frac{b_j - \bar{l}_j - \upsilon}{b_j - \bar{l}_j} \\ c \Leftarrow \gamma < \frac{b_j - \bar{l}_j - \upsilon}{b_j - \bar{l}_j} \end{cases}$$

and quantity traded

$$Q^* = \begin{cases} b_j - \bar{l}_j - \upsilon \Leftarrow \gamma \geq \frac{b_j - \bar{l}_j - \upsilon}{b_j - \bar{l}_j} \\ \gamma(b_j - \bar{l}_j) \Leftarrow \gamma < \frac{b_j - \bar{l}_j - \upsilon}{b_j - \bar{l}_j} \end{cases}$$

Therefore bank i has to resort to the lending facility for

$$\bar{l}_i - b_i - Q^* = \begin{cases} 0 \Leftarrow \gamma \geq \frac{\bar{l}_i - b_i}{\bar{l}_i - b_i + \upsilon} \\ (1-\gamma)(\bar{l}_i - b_i) - \gamma\upsilon \Leftarrow \gamma < \frac{\bar{l}_i - b_i}{\bar{l}_i - b_i + \upsilon} \end{cases}$$

when $r_i > r_j$, and for

$$\bar{l}_i - b_i - Q^* = \begin{cases} 0 \Leftarrow \gamma \geq \frac{b_j - \bar{l}_j - \upsilon}{b_j - \bar{l}_j} \\ (1-\gamma)(b_j - \bar{l}_j) - \upsilon \Leftarrow \gamma < \frac{b_j - \bar{l}_j - \upsilon}{b_j - \bar{l}_j} \end{cases}$$

when $r_i < r_j$.
Bank j, in turn, deposits

$$l_j - \bar{l}_j - Q^* = \begin{cases} \upsilon \Leftarrow \gamma \geq \frac{\bar{l}_i - b_i}{\bar{l}_i - b_i + \upsilon} \\ (1-\gamma)(\bar{l}_i - b_i + \upsilon) \Leftarrow \gamma < \frac{\bar{l}_i - b_i}{\bar{l}_i - b_i + \upsilon} \end{cases}$$

when $r_i > r_j$, and

$$l_j - \bar{l}_j - Q^* = \begin{cases} \upsilon \Leftarrow \gamma \geq \frac{b_j - \bar{l}_j - \upsilon}{b_j - \bar{l}_j} \\ (1-\gamma)(b_j - \bar{l}_j) \Leftarrow \gamma < \frac{b_j - \bar{l}_j - \upsilon}{b_j - \bar{l}_j} \end{cases}$$

when $r_i < r_j$.

For the sake of tractability and without loss of the main economic intuitions, we assume an uniform distribution for interest rate bids. We first deal with the generic case of a pure variable rate tender, in which bids are distributed in the interval $[d, c]$.[14] Later on (section 4.6) we particularize these results to variable rate tenders with a minimum bid higher than d, the solution adopted by the ECB. As we will see, qualitative findings do not change.

The probability for bank i of winning the auction is $\Pr(r_j < r_i) = \frac{r_i - d}{c - d}$. For bank j, in turn, the probability of winning is $\Pr(r_i < r_j) = \frac{r_j - d}{c - d}$. Each bank minimizes in its own price and quantity bids a weighted average (where the weights are these probabilities) of the expenses in each case (winning or loosing the auction). We are now able to completely formulate the banks' problems for every relevant interval of γ. As stated before, we assume that each bank pays exactly the price it offers.

For example, for $\gamma \geq \frac{\bar{l}_i - b_i}{\bar{l}_i - b_i + \upsilon}$ and $\gamma \geq \frac{b_j - \bar{l}_j - \upsilon}{b_j - \bar{l}_j}$, bank i wishes to

$$\min_{b_i, r_i} \left(\frac{r_i - d}{c - d}\right)(r_i b_i + (d + \frac{\bar{l}_i - b_i}{\gamma(\bar{l}_i - b_i + \upsilon)}(c-d)).(\bar{l}_i - b_i)) +$$
$$\left(\frac{c - r_i}{c - d}\right)(r_i.(\bar{l}_i + \bar{l}_j + \upsilon - b_j) + (d + \frac{b_j - \bar{l}_j - \upsilon}{\gamma(b_j - \bar{l}_j)}(c-d)).(b_j - \bar{l}_j - \upsilon))$$

and bank j intends to

[14] It does not pay to bid an interest rate higher than c, because the bank can obtain liquidity at c through the lending facility. Also, the Central Bank is not willing to accept bids below d, since banks can deposit liquidity at d.

$$\min_{bj,r_j} \left(\frac{c-r_j}{c-d}\right)(r_j.(\bar{l}_i+\bar{l}_j+\upsilon-b_i)-(d+\frac{\bar{l}_i-b_i}{\gamma(\bar{l}_i-b_i+\upsilon)}(c-d)).(\bar{l}_i-b_i)-d\upsilon)$$
$$+\left(\frac{r_j-d}{c-d}\right)(r_j.b_j-(d+\frac{b_j-\bar{l}_j-\upsilon}{\gamma(b_j-\bar{l}_j)}(c-d)).(b_j-\bar{l}_j-\upsilon)-d\upsilon)$$

The solution of these two problems gives rise to the optimal b_i, b_j, r_i, r_j, which, once replaced in the equilibrium expressions above for the interbank market and for the standing facilities, allow obtaining the complete solution of the reserve keeping problem. The switch to a variable rate procedure in June 2000 was accompanied by the decision to announce ECB's estimates for aggregate liquidity needs; this fact, however, does not imply that banks know the injection amount, since the ECB is not committed to inject precisely the estimate announced. Therefore, and as for the fixed rate procedure, the assumption consistent with our formulation is that banks have an exact expectation about the amount to be allocated.

Just as we did for the analysis of fixed rate tenders, we begin with the case with $\upsilon = 0$.

4.1. Absence of estimation errors

When the ECB has a precise estimation of the liquidity needs of the system ($\upsilon = 0$), the conditions imposed on b_i and b_j such that in equilibrium bank i is not a supplier in the interbank market, nor is bank j a demander imply that $b_i \in (0, \bar{l}_i]$ and $b_j \in [\bar{l}_j, \bar{l}_i + \bar{l}_j]$. Note that, contrary to what happens in a fixed rate auction, b_i and b_j have finite superior limits. This prevents the allotment ratio from falling to very low levels.

Interior solutions for r_i and r_j require that aggregate bidding behavior exceeds aggregate liquidity needs ($b_i + b_j > \bar{l}_i + \bar{l}_j$), which corresponds to rationing at the marginal rate, a situation occurred in almost all variable rate tenders conducted sofar.

When $\upsilon = 0$ the cut-off level for γ so that the equilibrium overnight rate does not rise above c is simply equal to 1. The problems of banks i and j for $\gamma \geq 1$ ($o^* \leq c$) resume to

$$\min_{b_i,r_i} \left(\frac{r_i-d}{c-d}\right)(r_ib_i+(d+\frac{c-d}{\gamma}).(\bar{l}_i-b_i))+$$

$$\left(\frac{c-r_i}{c-d}\right)(r_i.(\bar{l}_i+\bar{l}_j-b_j)+(d+\frac{c-d}{\gamma}).(b_j-\bar{l}_j))$$

$$\underset{bj,r_j}{min} \quad \left(\frac{c-r_j}{c-d}\right)(r_j.(\bar{l}_i+\bar{l}_j-b_i)-(d+\frac{c-d}{\gamma}).(\bar{l}_i-b_i))+$$
$$\left(\frac{r_j-d}{c-d}\right)(r_j.b_j-(d+\frac{c-d}{\gamma}).(b_j-\bar{l}_j))$$

For $\gamma<1$ the problems are

$$\underset{b_i,r_i}{min} \quad \left(\frac{r_i-d}{c-d}\right)(r_ib_i+c(\bar{l}_i-b_i))+\left(\frac{c-r_i}{c-d}\right)(r_i.(\bar{l}_i+\bar{l}_j-b_j)+c(b_j-\bar{l}_j))$$

$$\underset{bj,r_j}{min} \quad \left(\frac{c-r_j}{c-d}\right)(r_j.(\bar{l}_i+\bar{l}_j-b_i)-(c\gamma+d(1-\gamma)).(\bar{l}_i-b_i))+$$
$$\left(\frac{r_j-d}{c-d}\right)(r_j.b_j-(c\gamma+d(1-\gamma)).(b_j-\bar{l}_j))$$

There are four equilibria for the variable rate tender when $\gamma\geq1$. The equilibrium with both types of banks participating in the MRO and interior solutions for the price bids is

$$\begin{aligned} b_i^* &= \bar{l}_i,\, r_i^*=d+\frac{c-d}{2\gamma} \\ b_j^* &= \bar{l}_i+\bar{l}_j,\, r_j^*=\begin{cases} d+\frac{c-d}{2\gamma}-\frac{(c-d)\bar{l}_j}{2\bar{l}_i} \Leftarrow \bar{l}_j\leq\frac{\bar{l}_i}{\gamma} \\ d\Leftarrow \bar{l}_j>\frac{\bar{l}_i}{\gamma} \end{cases} \end{aligned}$$

In this equilibrium bank i wins the auction $(r_i>r_j\ \forall\ \bar{l}_i,\bar{l}_j)$[15] and is allocated with all the liquidity it needs. Bank j also receives all the liquidity required (it receives $\bar{l}_i+\bar{l}_j-\bar{l}_i=\bar{l}_j$, even though it bids higher) but pays a lower interest rate, declining with its own liquidity needs and rising with the rival's, such that when $\bar{l}_i$ is sufficiently small relative to $\bar{l}_j$, bank j is not willing to make a price offer above the minimum (d). A rise in the elasticity of supply in the interbank

[15] The bank which has the worse expectations about its position in the secondary market is the one who bids the highest price.

market, γ, induces an equal decline in the interest rate bid of both banks. The bank that expects to have a demand position in the interbank market is the one that makes the lowest quantity bid; however, it is the one who wins, so bidding an amount equal to its needs is the optimal behavior. The bank which expects a supply position in the secondary market bids above the needs because it looses the auction and so, in equilibrium, this is the only way to guarantee that it will not to be on the demand side. This is the only equilibrium with both banks participating in the MRO and having interior solutions for the interest rate bids. In a sense, it is the most interesting one, and hence, ahead, we sometimes refer just to it.

The other equilibria are

$$\begin{aligned} b_i^* &= 0, \text{ so } i \text{ does not participate in the MRO} \\ b_j^* &= \bar{l}_i + \bar{l}_j,\ r_j^* = d, \end{aligned}$$

$$\begin{aligned} b_i^* &= 0, \text{ so } i \text{ does not participate in the MRO} \\ b_j^* &= \bar{l}_j,\ r_j^* = \frac{c+d}{2} + \frac{c-d}{2\gamma} + \frac{(c-d)\bar{l}_j}{2\bar{l}_i} \end{aligned}$$

and

$$\begin{aligned} b_i^* &= \bar{l}_i,\ r_i^* = c \\ b_j^* &= \bar{l}_j,\ r_j^* = d \end{aligned}$$

Whenever i participates, it wins.

For $\gamma < 1$ there are two equilibria:

$$\begin{aligned} b_i^* &= \bar{l}_i,\ r_i^* = \frac{c+d}{2} \\ b_j^* &= \bar{l}_i + \bar{l}_j,\ r_j^* = \begin{cases} d + \frac{(c-d)\gamma}{2} - \frac{(c-d)\bar{l}_j}{2\bar{l}_i} \Leftarrow \bar{l}_j \leq \gamma \bar{l}_i \\ d \Leftarrow \bar{l}_j > \gamma \bar{l}_i \end{cases} \end{aligned}$$

in which i is the winning bank, with an interest rate independent of γ, because unsatisfied liquidity will have to be paid at c. The price offered by j is rising with

γ ($\gamma < 1$), for the more elastic is supply, the more bank j will be able to sell in the interbank market (at a rate equal to c), and hence the less it will have to deposit at d. The other equilibrium is

$$\begin{aligned} b_i^* &= \bar{l}_i,\, r_i^* = d \\ b_j^* &= \bar{l}_j,\, r_j^* = d \end{aligned}$$

Proposition 4.1. *The equilibrium quantity bid in a variable rate tender is finite. The equilibrium allotment ratio equals either* 1 *or* $\frac{\bar{l}_i+\bar{l}_j}{2\bar{l}_i+\bar{l}_j}$*, which is growing in the liquidity needs of the bank that expects not to be a demander in the secondary market (*j*), and decreasing in the liquidity needs of the bank that expects not to be a supplier (*i*). In the context of our model, the allotment ratio is higher than* $\frac{1}{2}$.

The finite number of equilibrium quantity bids in a variable rate tender is a clear and important difference as compared with the outcome of fixed rate tenders. The informational role of quantity bids (in conjunction with price bids) regarding liquidity needs is preserved.

4.2. Shortage of collateral

Assume now that i is limited by the collateral amount c_i, such that $c_i < \bar{l}_i$. Under these circumstances bank i will bid in such a way that available collateral is fully used in the primary market, as in the fixed rate tender. The overnight rate may rise above c. We consider that collateral is restrictive both when i wins and when it looses the auction.[16] For all γ demand in the interbank market is

$$Q^D = \bar{l}_i - c_i$$

while supply is given by

$$Q^S = \begin{cases} 0 \Leftarrow o < d \\ \gamma(\bar{l}_i - c_i)\frac{o-d}{c-d} \Leftarrow o \geq d \end{cases}$$

if i wins the auction, and by

[16] So, when i looses it does not buy $\bar{l}_i + \bar{l}_j - b_j$, but just c_i, exactly the same amount bought when it wins.

$$Q^S = \begin{cases} 0 \Leftarrow o < d \\ \gamma(b_j - \bar{l}_j)\frac{o-d}{c-d} \Leftarrow o \geq d \end{cases}$$

if the winner is j.

In the equilibrium of this game the interest rate paid by j is higher the more i is restricted by collateral (and lower, the more elastic is supply in the interbank market). When i wins the auction its lack of funds is solved in the secondary market, at an interest rate equal to $d + \frac{c-d}{\gamma}$;[17] when it looses, the interbank interest rate is higher.

Lemma 4.2. *In the equilibrium of a variable rate tender with collateral restrictions for bank i, interest rate bids of j in the primary market are rising in i's shortage of collateral.*

There are multiple equilibria for the quantity bidding behavior of i. The focal equilibrium is $b_i^* = c_i$, under which the allotment ratio is higher than with no collateral restrictions, because now i bids below its needs. The choice of $b_i^* = c_i$ is also related to the fact that, in the practice of the Eurosystem, bids which show up to be impossible to cover with collateral are highly penalized.

4.3. Credit rationing in the interbank market

When the liquidity supplier in the secondary market, j, decides, due to informational problems, to ration credit at the interest rate t, with $d < t < c$, the equilibrium of the game with both banks participating in the auction and interior solutions for the interest rate bids becomes

$$\begin{aligned} b_i^* &= \bar{l}_i,\ r_i^* = \frac{c+d}{2} - \gamma\frac{(t-d)(c-t)}{2(c-d)} \\ b_j^* &= \bar{l}_i + \bar{l}_j,\ r_j^* = d + \gamma\frac{(t-d)^2}{2(c-d)} - \frac{(c-d)\bar{l}_j}{2\bar{l}_i} \end{aligned}$$

Note that $\frac{\partial r_i^*}{\partial t} < 0$ is true only for $t < \frac{c+d}{2}$. This inequality is valid whenever $\gamma > 2$, so for high enough supply elasticity the loss in terms of quantity acquired

[17] Note that the value of c_i has no influence on o^* because to the shift in demand due to collateral shortage corresponds a shift in supply, such that o^* remains unchanged.

in the interbank market (and that has to be transferred to the lending facility at a penalty rate) is important and induces a more aggressive price bidding behavior in the MRO. If supply is not so elastic, then the reduction in quantity traded is more modest and the rise in the price bid of i occurs only if rationing is sufficiently relevant. Comparing with the corresponding equilibrium with no rationing, we state the following

Lemma 4.3. *Credit rationing in the secondary market gives rise to a decline in the interest rate bid of the bank which is a supplier in that market. The price bid made by the demander may rise or fall, depending on the elasticity of supply being high or low, respectively.*

4.4. Supply shocks in the interbank market

A supply shock in the interbank market, either negative ($\xi > 0$) or positive ($\xi < 0$) does not alter the optimum strategies of banks, since it only accounts for a parallel shift in expenses.

The bidding equilibrium is given by the same expressions as in subsection 4.1 if i is not restricted by collateral, whereas the results of subsection 4.2 apply if c_i is binding. When i wins the auction, but is collaterally restricted, its expenses rise (decline) if $\xi > 0$ ($\xi < 0$), while bank j' expenses are reduced (increased) by the same amount. The lower the elasticity of supply, the more important are these variations.

Lemma 4.4. *If i is constrained by collateral, a negative supply shock in the interbank may raise its expenses and decrease those of j. The reverse is true for a positive supply shock.*

If the shock is negative and sufficiently high ($\xi \geq (\gamma - 1)(\bar{l}_i - c_i)$, and $\gamma > 1$), bank i may end up indifferent between buying to bank j or resorting to the lending facility, when it is not constrained by collateral. Then r_i^* becomes independent of γ.

4.5. Estimation errors

When the ECB fails to correctly estimate the system's liquidity needs,[18] bank i will have it more expensive to comply with reserve requirements if $\upsilon < 0$ (underestimation), and cheaper if $\upsilon > 0$ (overestimation): the interest rate paid rises in

[18] As in fixed rate tenders, this wrong estimation is related to the absorbtion or injection of liquidity by "autonomous factors".

the former situation as compared with the correct estimation case, and declines in the latter. There is an additional reason for bank i's expenses to grow in the underestimation case, the fact that it will have to resort to the lending facility for the amount $|\upsilon|$.

As to bank j, the estimation error has no effect on the price paid when it is negative, and implies a lower price offer when $\upsilon > 0$. This bank makes use of the deposit facility (for an amount equal to the estimation error) when $\upsilon > 0$.

The equilibria characterized by interior solutions for the overnight rate and by both banks participating in the MRO and making interior price bids are as follows. For $\upsilon < 0$

$$
\begin{aligned}
b_i^* &= \bar{l}_i + \upsilon,\ r_i^* = d + \frac{c-d}{2\gamma} - \frac{\upsilon(c-d)}{2\bar{l}_i} \\
b_j^* &= \bar{l}_i + \bar{l}_j,\ r_j^* = \begin{cases} d + \frac{c-d}{2\gamma} - \frac{(c-d)\bar{l}_j}{2\bar{l}_i} \Leftarrow \bar{l}_j \leq \frac{\bar{l}_i}{\gamma} \\ d \Leftarrow \bar{l}_j > \frac{\bar{l}_i}{\gamma} \end{cases}
\end{aligned}
$$

and for $\upsilon > 0$

$$
\begin{aligned}
b_i^* &= \bar{l}_i,\ r_i^* = d + \frac{(c-d)\bar{l}_i}{2\gamma(\bar{l}_i+\upsilon)} \\
b_j^* &= \bar{l}_i + \bar{l}_j + \upsilon,\ r_j^* = \begin{cases} d + \frac{(c-d)\bar{l}_i}{2\gamma(\bar{l}_i+\upsilon)} - \frac{(c-d)(\bar{l}_j+\upsilon)}{2\bar{l}_i} \Leftarrow \bar{l}_j \leq \frac{\bar{l}_i^2}{\gamma(\bar{l}_i+\upsilon)} - \upsilon \\ d \Leftarrow \bar{l}_j > \frac{\bar{l}_i^2}{\gamma(\bar{l}_i+\upsilon)} - \upsilon \end{cases}
\end{aligned}
$$

The bank which expects not to be a liquidity supplier in the secondary market reduces its quantity bid as compared with the no estimation error case when $\upsilon < 0$, and the bank which expects not to be a liquidity demander raises its bid when $\upsilon > 0$. In these circumstances expectations as to net positions in the interbank market are fulfilled, a necessary condition for an equilibrium. Deviations as to liquidity needs are fully corrected through the ECB's standing facilities. As stated above, bank i is harmed by underestimation, and benefits from overestimation. Bank j is unaffected, in terms of expenses, by underestimation; when there is overestimation results are ambiguous and depend, among other factors, on the magnitude of υ (on the one hand the price paid in the MRO is reduced as compared to the $\upsilon = 0$ case, but liquidity acquired is higher, and there is a surplus to be deposited at d).

In these equilibria the allotment ratio is $\frac{\bar{l}_i+\bar{l}_j}{2\bar{l}_i+\bar{l}_j+v}$. So, underestimation of the system's liquidity needs raises this ratio, whereas overestimation lowers it.

Lemma 4.5. *An underestimation (overestimation) by the ECB of the system's liquidity needs raises (decreases) the interest rate bid of bank i and its expenses, as well as the allotment ratio.*

4.6. Variable rate tenders with a minimum bid for the interest rate

The change to variable rate tenders in the MRO of the 28 June 2000 included the adoption of a minimum rate bid.

In the analysis conducted sofar the lower bound for the price offers has been d. When the change to variable rate tenders occurred, the minimum bid was set at the fixed rate in use in the MROs of the Eurosystem at the time of the change, r in our notation. In these circumstances r_i and r_j are now free to fluctuate in the interval $[r, c]$, instead of $[d, c]$. The equilibrium for $\gamma \geq 1$ with both types of banks participating in the MRO and with interior solutions for the price bids (and no distortions) becomes

$$
\begin{aligned}
b_i^* &= \bar{l}_i,\, r_i^* = \begin{cases} \frac{d+r}{2} + \frac{c-d}{2\gamma} \Leftarrow \gamma \leq \frac{c-d}{r-d} \\ r \Leftarrow \gamma > \frac{c-d}{r-d} \end{cases} \\
b_j^* &= \bar{l}_i + \bar{l}_j,\, r_j^* = \begin{cases} \frac{d+r}{2} + \frac{c-d}{2\gamma} - \frac{(c-r)\bar{l}_j}{2\bar{l}_i} \Leftarrow \bar{l}_j \leq \frac{(c-d-\gamma(r-d))\bar{l}_i}{\gamma(c-r)} \\ r \Leftarrow \bar{l}_j > \frac{(c-d-\gamma(r-d))\bar{l}_i}{\gamma(c-r)} \end{cases}
\end{aligned}
$$

The interest rates paid by both banks rise as compared with the no minimum bid situation. Hence their expenses rise as well, and the same do the ECB's revenues. The winning price bid is mostly influenced by the minimum level r, the interest rate on the deposit facility being the one with the lowest impact on r_i^* if the elasticity of supply is not too high ($\gamma < 2$); otherwise the lending facility rate has the lowest impact.

When $\gamma < 1$ this equilibrium is

$$
\begin{aligned}
b_i^* &= \bar{l}_i,\, r_i^* = \frac{c+r}{2} \\
b_j^* &= \bar{l}_i + \bar{l}_j,\, r_j^* = \begin{cases} \frac{d+r}{2} + \frac{(c-d)\gamma}{2} - \frac{(c-r)\bar{l}_j}{2\bar{l}_i} \Leftarrow \bar{l}_j \leq \frac{((c-d)\gamma-(r-d))\bar{l}_i}{c-r} \\ r \Leftarrow \bar{l}_j > \frac{((c-d)\gamma-(r-d))\bar{l}_i}{c-r} \end{cases}
\end{aligned}
$$

Interest bids are again higher than in the absence of a minimum bid. The winning bank offers a price which is the mid point of the allowed interval, just as in the no minimum bid case. The value of γ does not influence this bid, because the interbank rate is predetermined at c, contrary to what happens when $\gamma \geq 1$.

The results obtained for this basic case ($\upsilon = 0$, enough c_i, no rationing in the interbank market, $\xi = 0$) generalize for the other possible situations. A minimum bid higher than d increases the interest rate bids, banks' expenses, and ECB's revenues.

A variable rate tender with a minimum price bid has no qualitative difference from a pure variable rate tender. When the signaling effect of the minimum offer as to the monetary policy stance can be achieved through some other vehicle (for instance, by announcing an operational target), the choice of the instrument is irrelevant.

5. Fixed versus variable rate tenders

In this section we briefly summarize some of the results obtained for fixed and variable rate liquidity auctions in the Eurosystem, and compare these two procedures.

As we have shown, low allotment ratios are inherent to fixed rate tenders, as bids may reach very high levels. On the contrary, variable rate tenders allow for reasonable levels of this ratio. Overbidding may be present in both types of auction, however it may be a much more serious problem in the former than in the latter. Therefore, the risk of being allocated an amount which cannot be covered by available collateral is substantially higher under fixed rate auctions, which acts as a disturbing factor for the whole system. The change from fixed to variable rate procedures has indeed led to clearly higher allotment ratios, as can be seen in the table in the Appendix: the average ratio switched from 8.2 per cent to 62.5 per cent in the operations conducted sofar.

The informational content of quantity bids as regards true liquidity needs is lost in fixed rate tenders, due to multiplicity of equilibria. However, it is preserved in variable rate tenders.

In a variable rate tender banks have a higher probability of being allocated with all the liquidity needed in the MRO. This is especially important for those institutions who expect to be in a demanding position in the secondary market. Actually, in a fixed rate tender liquidity allocated to each counterpart is a function of its own quantity bid and also of the rival's, so that a perception error about the latter may leave the bank in a weak position thereafter; on the contrary, in a variable rate tender there are equilibria in which the bank may guarantee itself the desired liquidity as long as it makes a sufficiently high price bid.

Suppose that i believes j will bid b_{j1} and therefore, in a fixed rate tender (with no distortions), chooses $b_{i1} = \frac{\bar{l}_i}{\bar{l}_j} b_{j1}$, but j actually bids $b_{j2} > b_{j1}$. Then bank i will receive $\frac{\bar{l}_i+\bar{l}_j}{b_{i1}+b_{j2}} b_{i1} < \frac{\bar{l}_i+\bar{l}_j}{b_{i1}+b_{j1}} b_{i1}$ and j will obtain $\frac{\bar{l}_i+\bar{l}_j}{b_{i1}+b_{j2}} b_{j2} > \frac{\bar{l}_i+\bar{l}_j}{b_{i1}+b_{j1}} b_{j1}$ (liquidity allocated is increasing in the own bid and decreasing in the rival's bid). In the equilibrium set each bank is allocated with its full liquidity needs; in this example, which lies out of the equilibrium line, i receives less than $\bar{l}_i$ and j more than $\bar{l}_j$. What j receives in excess is exactly what i is short of; this misallocation is corrected in the secondary market at an (interior) interbank rate equal to $d + \frac{c-d}{\gamma}$. For sufficiently low γ ($\gamma < \frac{c-d}{r-d}$) j benefits in terms of expenses and i is harmed, the reverse being true for high γ. In any case ECB's revenues are unaffected by this perception error of bank i.

In a variable rate tender (again with no distortions) the winning bank is not harmed by a wrong perception of the rival's bid. The looser, however, may suffer from an incorrect perception of the other's liquidity needs (and hence quantity bid), both in what concerns allotted amount and price paid.

For banks binded by collateral, bidding mistakes (above the equilibrium) are more harmful under fixed rate than under variable rate procedures. As we have seen, the equilibrium quantity bid for bank i is $\frac{c_i}{\bar{l}_i+\bar{l}_j-c_i} b_j$ in a fixed rate tender, and c_i in a variable rate one. Bidding above the equilibrium implies less liquidity for bank j in the fixed rate auction, and hence a rise in the interbank market rate; in a variable rate auction, on the contrary, liquidity received by j is unaffected by b_i, so in that sense bank i's situation is not worsened. This is clear also from the fact that in a variable rate tender the objective function of i (expenses to minimize) is invariant with b_i.

Shortage of collateral to cover liquidity needs reduces the elasticity of demand in the secondary market, from which banks on the supply side may benefit. The revenues of the ECB are unaffected by collateral shortage in a fixed rate tender, but may rise in variable rate tenders, because price bids of unrestricted banks are higher.

The existence of credit rationing in the interbank market, accompanied by shortage of collateral of the bank which will be in a demand position in that market, leaves this institution with unsatisfied liquidity needs in both procedures.

In fixed rate tenders, as well as in variable rate ones, negative supply shocks in the interbank market imply a rise in the expenditures of banks on the demand side. Banks on the supply side usually benefit from these shocks, but may be harmed if i banks are not collaterally restricted (then ECB's revenues suffer, as well).

Estimation errors by the ECB - associated with an incorrect forecast of the autonomous factors of liquidity injection or absorption - have clearer effects on banks' expenses under a fixed rate procedure, than under a variable rate one. In the former banks with a demand position in the secondary market are hurt by underestimation of liquidity needs and are unaffected by overestimation; the reverse happens for banks with a supply position. In a variable rate auction underestimation has the same implications as in the fixed rate auction; however, banks with a demand position may benefit from an ECB's overestimation of the system's needs, while the impact of this mistake on the expenses of the banks with a supply position is ambiguous.

6. Conclusion

This paper has intended to compare fixed and variable rate auctions for the provision of liquidity in the Eurosystem in what concerns some specific issues: bidding behavior of the counterparts, induced allotment ratios, functioning of the interbank market, and resorting to standing facilities. Several scenarios have been considered, namely the possibility that collateral is binding, supply shocks or rationing by suppliers in the interbank market, estimation errors of the system's needs by the monetary authority. We have derived, under all these circumstances, the equilibrium outcomes of a three-stage game in which banks intend to minimize the unavoidable expenditure associated with the reserve keeping problem.

The results obtained favor the adoption of variable rate tenders, as a means of restricting overbidding behavior by banks. Experience with this type of procedure has confirmed this finding. Due to the finite number of equilibria, variable rate tenders also allow keeping the informational content of quantity bids, as opposed to fixed rate tenders.

References

[1] Ayuso, J. and Repullo, R. (2001a), A model of the open market operations of the European Central Bank, *mimeo.*

[2] Ayuso, J. and Repullo, R. (2001b), Why did the banks overbid? An empirical model of the fixed rate tenders of the European Central Bank, *mimeo.*

[3] European Central Bank (1998), *The Single Monetary Policy in Stage Three - General documentation on ESCB monetary policy instruments and procedures*, Frankfurt am Main.

[4] European Central Bank (2000), *Annual Report 1999*, Frankfurt am Main.

[5] European Central Bank (2001), *Annual Report 2000*, Frankfurt am Main.

[6] Favero, C., Freixas, X., Persson, T. and Wyplosz, C. (2000), *One Money, Many Countries*, Monitoring the European Central Bank 2, Centre for Economic Policy Research, London.

[7] Nautz, D. and Oechssler, J. (1999), The repo auctions of the European Central Bank and the vanishing quota puzzle, Discussion Paper #79, SFB 373, Humboldt-University Berlin.

[8] Nyborg, K. and Strebulaev, I. (2000), Multiple unit auctions and short squeezes, *mimeo.*

APPENDIX

Main Refinancing Operations of the Eurosystem
(fixed rate tenders until 21 June 2000, variable rate tenders from 28 June 2000 on)

Date	Allotment ratio
7/Jan/99	15.6%
13/Jan/99	8.5%
20/Jan/99	9.9%
27/Jan/99	10.0%
3/Feb/99	8.2%
10/Feb/99	7.1%
17/Feb/99	6.9%
24/Feb/99	7.9%
3/Mar/99	6.1%
10/Mar/99	7.9%
17/Mar/99	13.1%
24/Mar/99	27.4%
31/Mar/99	32.9%
7/Apr/99	100.0%
14/Apr/99	8.6%
21/Apr/99	8.2%
28/Apr/99	10.3%
5/May/99	6.4%
12/May/99	11.0%
19/May/99	6.7%
26/May/99	12.2%
2/Jun/99	6.2%
9/Jun/99	9.5%
16/Jun/99	4.2%
23/Jun/99	7.4%
30/Jun/99	4.7%
7/Jul/99	7.4%
14/Jul/99	4.2%
21/Jul/99	6.4%
28/Jul/99	5.4%
4/Aug/99	5.4%
11/Aug/99	5.1%
18/Aug/99	4.7%
25/Aug/99	6.0%
1/Sep/99	4.4%
8/Sep/99	6.1%
15/Sep/99	5.8%
22/Sep/99	13.9%
29/Sep/99	5.9%
6/Oct/99	5.4%
13/Oct/99	3.9%
20/Oct/99	6.8%
28/Oct/99	3.8%
3/Nov/99	2.8%
10/Nov/99	18.3%
17/Nov/99	14.2%
24/Nov/99	10.8%
1/Dec/99	7.1%
8/Dec/99	8.1%
15/Dec/99	19.9%
22/Dec/99	6.1%
30/Dec/99	14.4%

Date	Allotment ratio
12/Jan/00	3.8%
19/Jan/00	6.7%
26/Jan/00	4.5%
2/Feb/00	2.1%
9/Feb/00	6.4%
16/Feb/00	5.8%
23/Feb/00	3.0%
1/Mar/00	3.1%
8/Mar/00	2.9%
15/Mar/00	2.0%
22/Mar/00	3.1%
29/Mar/00	2.9%
5/Apr/00	1.7%
12/Apr/00	1.9%
19/Apr/00	1.4%
27/Apr/00	1.6%
4/May/00	1.4%
10/May/00	1.1%
17/May/00	1.0%
24/May/00	1.1%
31/May/00	0.9%
7/Jun/00	0.9%
15/Jun/00	1.9%
21/Jun/00	4.0%
28/Jun/00	49.1%
5/Jul/00	33.8%
12/Jul/00	51.3%
19/Jul/00	33.0%
26/Jul/00	55.8%
2/Aug/00	26.1%
9/Aug/00	55.5%
16/Aug/00	31.0%
23/Aug/00	51.8%
30/Aug/00	45.4%
6/Sep/00	56.7%
13/Sep/00	39.8%
20/Sep/00	61.8%
27/Sep/00	50.9%
3/Oct/00	56.8%
11/Oct/00	59.0%
18/Oct/00	86.4%
25/Oct/00	56.6%
1/Nov/00	59.8%
8/Nov/00	64.6%
15/Nov/00	69.1%
22/Nov/00	72.5%
29/Nov/00	62.6%
6/Dec/00	98.5%
13/Dec/00	78.4%
20/Dec/00	79.5%
23/Dec/00	86.3%

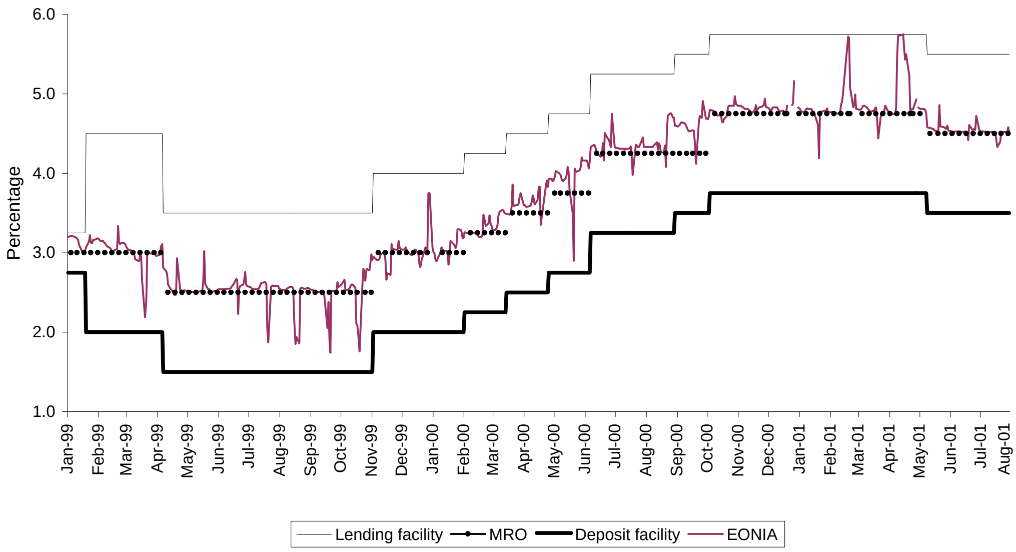
Date	Allotment ratio
3/Jan/01	74.0%
10/Jan/01	99.1%
17/Jan/01	73.4%
24/Jan/01	87.7%
31/Jan/01	61.0%
7/Feb/01	95.8%
14/Feb/01	100.0%
21/Feb/01	77.3%
24/Feb/01	24.6%
7/Mar/01	75.3%
14/Mar/01	39.2%
21/Mar/01	74.2%
28/Mar/01	86.9%
4/Apr/01	91.4%
11/Apr/01	100.0%
19/Apr/01	66.7%
24/Apr/01	6.0%
27/Apr/01	54.3%
4/May/01	47.9%
14/May/01	44.8%
22/May/01	57.0%
29/May/01	48.2%
5/Jun/01	72.9%
12/Jun/01	49.5%
19/Jun/01	61.1%
26/Jun/01	54.5%
3/Jul/01	77.6%
10/Jul/01	51.5%
17/Jul/01	58.0%
24/Jul/01	74.6%
31/Jul/01	70.5%

Administered interes rates
(percentage)

Date	MRO r (*)	Deposit facility d	Lending facility c
04-Jan-99	3.00	2.75	3.25
22-Jan-99	3.00	2.00	4.50
09-Apr-99	3.00	1.50	3.50
14-Apr-99	2.50	1.50	3.50
05-Nov-99	2.50	2.00	4.00
10-Nov-99	3.00	2.00	4.00
04-Feb-00	3.00	2.25	4.25
09-Feb-00	3.25	2.25	4.25
17-Mar-00	3.25	2.50	4.50
22-Mar-00	3.50	2.50	4.50
28-Apr-00	3.50	2.75	4.75
04-May-00	3.75	2.75	4.75
09-Jun-00	3.75	3.25	5.25
15-Jun-00	4.25	3.25	5.25
01-Sep-00	4.50	3.50	5.50
06-Oct-00	4.75	3.75	5.75
11-May-01	4.50	3.50	5.50

(*) Fixed rate until 21 June 2000, minimum rate from 28 June on.

Administered interest rates and interbank market rate



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