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as a Core Inflation Indicator**

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USING THE ASYMMETRIC TRIMMED MEAN AS A CORE INFLATION INDICATOR*

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This paper discusses the use of the trimmed mean as a core inflation indicator when the price changes distribution is fat tailed and asymmetric and computes several asymmetric trimmed means that meet all the conditions suggested in Marques *et al.* (2000). It turns out that the 10 per cent trimmed mean centred on the 51.5th percentile is the one with the lowest volatility and so, its use, as a core inflation indicator, is recommended.

1 - INTRODUCTION

Marques *et al.* (2000) have recently introduced new criteria to evaluate potential core inflation measures and showed that, for the Portuguese case, the symmetric trimmed mean exhibits a systematic bias relative to the average inflation level. This finding strongly reduces the usefulness of this estimator as a core inflation measure.

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This paper discusses the use of trimmed means as core inflation indicators, when the price changes distribution is skewed and shows that the previous finding stems from the fact that the price changes distribution, in the Portuguese case, is on average, right skewed.

Using this finding as a starting point the paper computes several trimmed means which meet all the conditions set out in Marques *et al.* (2000) and so, in particular, do not exhibit any systematic bias.

Among these, the 10 per cent asymmetric trimmed mean centred on the 51.5th percentile, i.e., the one that trims 11.5 per cent from the left-hand tail and 8.5 per cent from the right-hand tail of the (ordered) price changes distribution is the less volatile. For this reason we recommend it as a core inflation indicator. One should note however that this indicator is not very smooth and so it may be difficult to draw definite conclusions from its short run behaviour.

The remaining part of this paper is organised as follows. In section 2 the main economic and statistical theoretical arguments for the use of trimmed means as core inflation indicators are briefly reviewed. In section 3 the skewness and kurtosis of the Portuguese data are computed and the price changes distribution is characterised. In section 4 the use and computation of the trimmed mean when the price changes distribution is fat-tailed and asymmetric is discussed and, in section 5, asymmetric trimmed means that do not exhibit any systematic bias are evaluated according to the criteria set out in Marques *et al.* (2000). Finally, in section 5 the main conclusions are put forward.

2-THEORETICAL MOTIVATIONS FOR USING TRIMMED MEANS AS CORE INFLATION INDICATORS

This section shortly reviews the main arguments for the use of trimmed means as core inflation indicators. Let P_{it} stand for the price index of the i^{th} basic item of the consumer price index (CPI), and α_i for its weight. By definition, we have:

$$P_t = \sum_{i=1}^N \alpha_i P_{it} \quad (1)$$

where P_t stands for the CPI and N for the number of basic items¹. It is easy to demonstrate that (1) can be written as:

$$\pi_t = \sum_{i=1}^N \omega_{it} \pi_{it} \quad (2)$$

where

$$\begin{aligned} \pi_t &= \left(\frac{P_t}{P_{t-12}} - 1 \right) * 100 \\ \pi_{it} &= \left(\frac{P_{i,t}}{P_{i,t-12}} - 1 \right) * 100 \\ \omega_{it} &= \alpha_i \cdot \frac{P_{i,t-12}}{P_{t-12}} \end{aligned} \quad (3)$$

Notice that equation (2) represents the year-on-year inflation rate, π_t , as a weighted average of the year-on-year rates of change of prices of all basic CPI items. Although the sum of the weights ω_{it} is 1, the weights are time varying. So, according to equation (2), the year-on-year inflation rate in period t can be seen as the sample mean of the price changes distribution in period t.

Economic literature sees inflation as the outcome of two effects. The first one arises from changes in the general price level and is connected with monetary factors. This is usually referred to as core inflation. The second effect stems from changes in relative prices of one or more of the CPI items, as a consequence of phenomena restricted to individual markets, caused by short-term economic factors or measurement problems. Based on these effects, price changes of one CPI item can be written as:

$$\pi_{it} = \pi_t^* + v_{it} \quad (4)$$

¹ The current CPI index comprises 189 basic items.

where π_t^* denotes core inflation and v_{it} the deviations between prices changes of item i and core inflation, in period t . Multiplying (4) by ω_{it} , adding and taking into account that $\sum_{i=1}^N \omega_{it} = 1$, we get:

$$\pi - \pi_t^* = \sum_{i=1}^N \omega_{it} v_{it} = u_t \quad (5)$$

where u_t is, by assumption, a stationary variable with zero mean. Equation (5) corresponds to the first condition set out in Marques *et al.* (1999, 2000) as a requirement for a core inflation measure.

Monetary policy is supposed to be concerned with long-run inflation (π_t^*) and not with current inflation. This explains why the building and testing of core inflation indicators has recently become an active area of research especially in some Central Banks. By definition, a core inflation indicator should disregard changes of prices arising from short-term phenomena that induce random changes on inflation, as measured by the year-on-year rate of change of the CPI. In order to build such indicators several methods have been proposed in the literature. These run from the very simple and intuitive “underlying inflation indicator” (computed excluding the most volatile CPI items, namely food and fuel), to the more elaborated Quah and Vahey (1995) indicator based on monetary theory and computed with a bivariate VAR².

One of the more appealing methods, due to its simplicity and potential, is based on the use of trimmed means. Several authors have recently argued in favour of trimmed means, using both economic and statistical arguments.

The trimmed mean can be computed from equation (2) by excluding a given percentage of the highest and lowest price changes. For instance, the 10 per cent trimmed mean is obtained by excluding from (2) the 10 per cent lowest price changes (π_{it}) and the 10 per cent highest price changes (taking into account the weights ω_{it}), that is, it only considers 80 per cent of the central price changes.

² A description of the different approaches that have been developed in the literature to compute core inflation indicators is out of the scope of this paper. The interested reader may refer to, for instance, Roger (1998), Marques *et al.* (1999) or Álvarez and Matea (1999).

The economic motivations, described, for instance, in Bryan and Cecchetti (1999) are based on theoretical models of price setting, namely on the Ball and Mankiw (1995) model. The use of trimmed means in these models is useful because there is some price rigidity, arising from menu costs. As a consequence, idiosyncratic shocks on some prices cause some short-term change in the prices of these products. The complete adjustment of the relative prices will only take place once firms are able to change their prices without costs or when the general price level changes enough. So, idiosyncratic shocks only have a short-term impact on inflation. As trimmed means disregard the CPI items with a behaviour strongly different from the mean, they just exclude the effects of those shocks, and so they potentially record only trend inflation.

Bryan and Cecchetti (1999) also invoke the Balke and Wynne (1996) model to justify the use of trimmed means. In this multisectoral model, with a fixed supply of money and a cash-in-advance constraint, a shock on aggregate supply increases production and the general price level but different sectors are differently affected, namely as regards prices. So, in the model, these shocks give rise to an asymmetric price distribution. Again, trimmed means by disregarding the CPI items with behaviour strongly different from the mean, disregard the effects of those shocks and record only trend inflation.

Bakhshi and Yates (1999) challenged the economic arguments in favour of trimmed means. They develop two versions of a model in which the best indicator of core inflation do not give more but less weight to central observations of price changes. In the first version of their model the prices can only be changed at regular intervals (eventhough at no cost), the growth rate of money supply is stable (aggregate demand shocks) but there are no supply shocks, and consequently, no desired changes in relative prices. If, in this context, the dates of adjustments of prices are evenly distributed in time, a trimmed mean will disregard all price changes, systematically underestimating inflation. The second version of the model differs from the first in that the prices are changed only when the distance between the desired price and the actual price is large enough. Also in this context a trimmed mean will underestimate trend inflation.

Both the Ball and Mankiw (1995) and the Balke and Wynne (1996) models imply a positive correlation between inflation (the sample mean of the price changes distribution) and the skewness coefficient, which has been referred to as one of the most accepted empirical facts regarding aggregate prices. Recently Bryan and Cecchetti (1999) tried to demonstrate, using Monte Carlo simulations, that we may explain this correlation as a consequence of a small-sample bias, brought about by a significant level of kurtosis. The conclusions of this paper are however disputed by Ball and Mankiw (1999) and Verbrugge (1999).

It is important to stress the idea that the two above mentioned economic models invoked to justify the use of trimmed means imply the existence of an error correction mechanism that causes inflation to converge to its trend (“desired” inflation).

The statistical arguments in favour of trimmed means are based on the idea, supported by empirical evidence, that the kurtosis of the distribution of price changes of the CPI items is larger than the kurtosis of a normal distribution. If the price changes distribution is leptokurtic it can be shown that, in general, an estimator for the mean that puts more weight on central price changes, is more efficient than the sample mean. The reason is that in a leptokurtic distribution the probability of a large contribution to the inflation of an extreme observation not to be offset by equally extreme observation on the other side of the distribution is larger than in the normal distribution³.

In this case the most efficient estimator for the mean inflation (the mean of the price changes distribution) may not be the year-on-year rate of change of the CPI, but any estimator that gives more weight to central observations. The trimmed mean and the median are examples of such estimators. Bryan *et al.* (1997) show that the more leptokurtic the distribution the less efficient is the sample mean. They also show that the more leptokurtic the distribution the larger the ideal trim. The idea is that the more leptokurtic the distribution the larger the proportion of unrepresentative price changes that must be eliminated in order to identify the trend of inflation.

These statistical arguments, as presented in Cecchetti (1997), are based on the idea that the underlying price distribution is symmetric. As shown below, these results do not hold when the price changes distribution is also asymmetric as in this case the trim changes the estimator expected value. Due to this fact, trimmed means used in the next sections, should be seen mainly as a statistical device which allows one to build core inflation indicators potentially meeting the conditions set out in Marques *et al.* (2000).

3 – CHARACTERISTICS OF PRICE CHANGES DISTRIBUTIONS

This section analyses the main characteristics of the price changes distribution resorting to some of the indicators conventionally used to compute the kurtosis and skewness of a distribution. A similar analysis was carried out in Coimbra and Neves (1997).

³ See Bryan *et al.* (1997).

The k^{th} central moment of a cross-section distribution may be written as:

$$m_{kt} = \sum_{i=1}^N \omega_{it} (\pi_{it} - \pi_t)^k \quad (6)$$

and the skewness (S_t) and kurtosis (K_t) coefficients are the scaled third and fourth moments, respectively:

$$S_t = \frac{m_{3t}}{(m_{2t})^{\frac{3}{2}}} \quad (7)$$

$$K_t = \frac{m_{4t}}{(m_{2t})^2} - 3 \quad (8)$$

The upper side of Figure 1 depicts the S_t series over the period 1993/7 to 2000/5. It is seen that S_t exhibits relatively long periods in which it is positive followed by similar long periods in which it is negative. On average S_t is positive and equal to 0.83 (the S_m line in Figure 1). This figure is higher than the one found for the USA (0.2) by Bryan *et al.* (1997), but similar to the ones found for the Australia (0.7) by Kearns (1998), or for Ireland (0.8) by Meyler (1999). However one must bear in mind that these figures may be sensitive to the disaggregation level of the CPI, to the frequency of price measurement as well as to the sample period.

Another skewness indicator is the so-called mean percentile (the percentile which corresponds to the sample mean of the distribution)⁴. If the distribution is symmetric and mesokurtic ($S_t = 0$ in (7) and $K_t = 0$ in (8)) one expects the average mean percentile to be the 50th percentile. This result is not to be expected if the distribution is skewed. Figure 2 shows the evolution of the mean percentile of the price changes distribution (β) and the corresponding average (β_m) obtained by averaging the monthly mean percentiles over the sample period⁵.

⁴ See Roger (1997).

⁵ The β_x series in Figure 2 is an exponential smoothing of β and will be analysed in the next section.

Figure 1

Skewness and Kurtosis

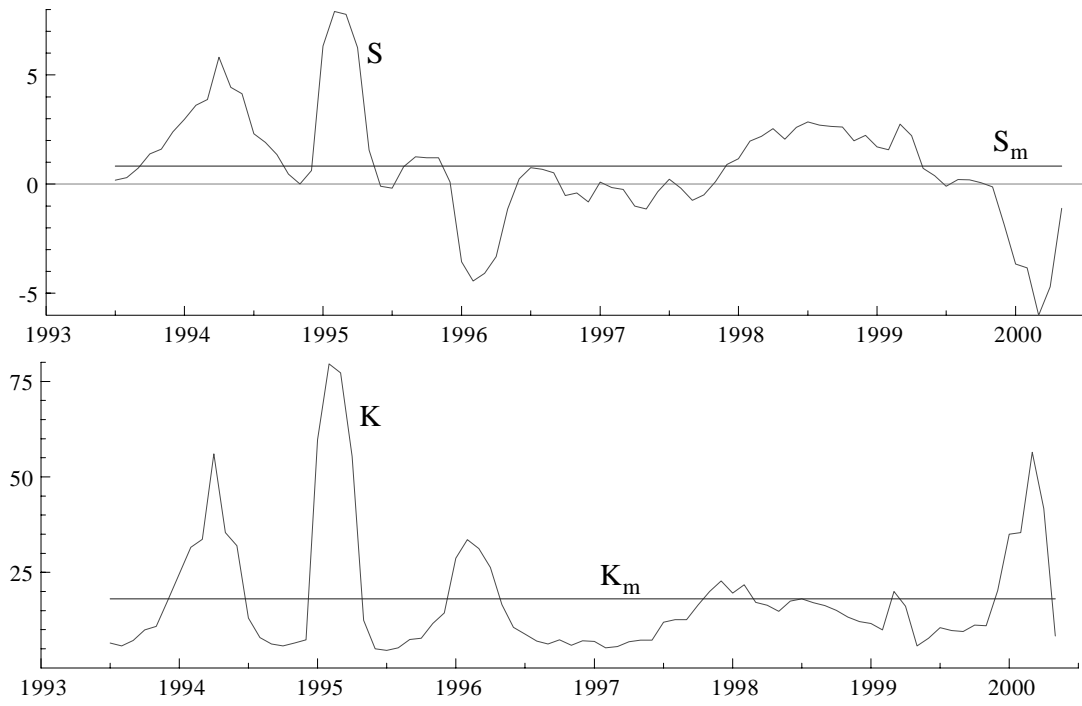
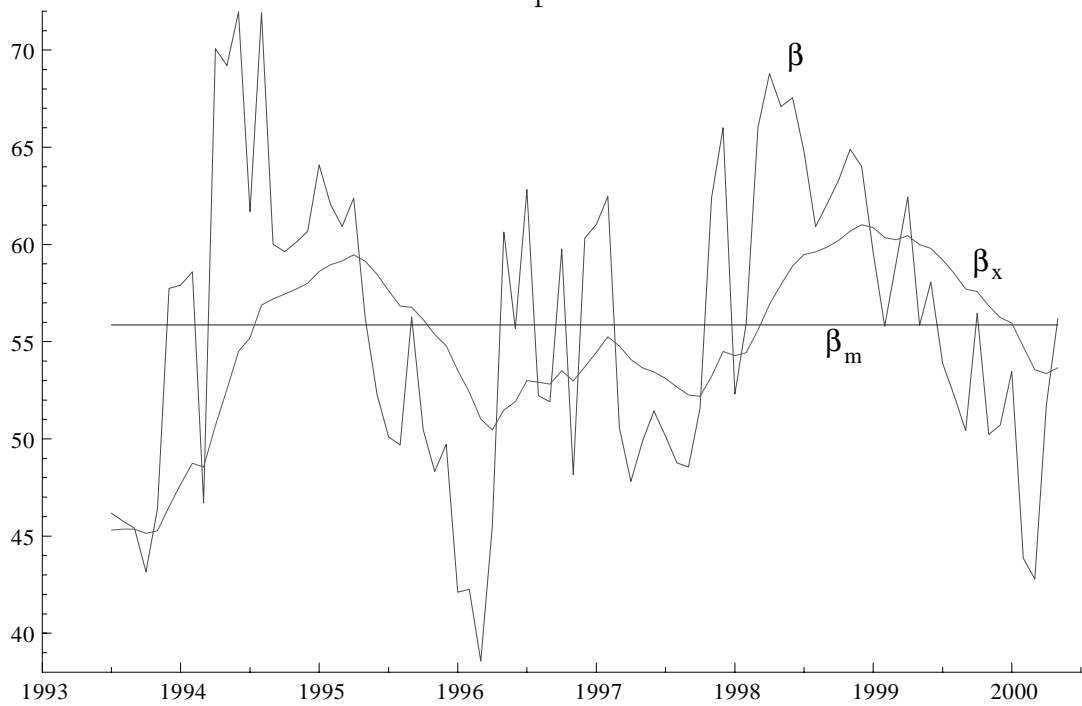


Figure 2

Mean percentile



The main feature to note is that the mean percentile is almost always above the 50th percentile, which is another strong piece of evidence of the chronic right skewness of the price changes distribution. For the sample period the average mean percentile β_m is 56. This figure is similar to the one computed to Ireland (58) by Meyler (1999), or to France (60) by Bihan and Sédillot (1999), but differs from the one computed to Australia (52) by Kearns (1998). One should notice however that there is some evidence that the average mean percentile depends critically on the aggregation level at which the consumer price index is recorded, i.e., the number of items in the basket. Bihan and Sédillot (1999), for instance, report a figure of 55 for France computed from a more aggregate price index, i.e., with a smaller number of items⁶.

The lower part of Figure 1 depicts K_t the kurtosis coefficient. Notice that K_t in equation (8) measures the “excess kurtosis” relative to the Normal distribution, so that any figure above zero means that the distribution is leptokurtic (platykurtic if $K_t < 0$). We readily see that K_t is always positive over the sample period. Furthermore its average is as high as 15.10, so that we may conclude that the price changes distribution is highly leptokurtic (fat-tailed)⁷. This indicates that in a typical month, a large proportion of the CPI items may experience price changes that differ significantly from the mean inflation rate. As we saw in the previous section, this fact justifies the use of the trimmed mean as an estimator of the population mean, as it is a more efficient estimator than the sample mean.

Before proceeding it seems important to notice that the conclusions drawn on the asymmetry and kurtosis of the price changes distribution were obtained with non-robust estimators. In fact neither S_t nor K_t are robust estimators, as they tend to underestimate the true importance of the tails of the distribution. This is the so-called masking phenomenon, which is basically due to the fact that both S_t and K_t are computed using the sample mean and sample standard deviation, which are themselves influenced by the occurrence of outliers. Such occurrence tends to decrease the numerator of (7) and to raise the denominator of (7) and (8)⁸.

⁶ Similar results were obtained in Aucremanne (2000) with different levels of aggregation. Computations carried out at Banco de Portugal with a less disaggregated price index, for a similar time period, led to an average mean percentile of 51.8, which is substantially below 56.

The average mean percentile is also likely to be very sensitive to the frequency of price measurement. With longer intervals the distribution of the price changes will tend to become more symmetric, not simply because of the law of large numbers, but also because the frequency of price measurement will come closer to the frequency of adjustment of regulated prices.

⁷ The median of K_t , which may be a better indicator of excess kurtosis, is 9.48.

⁸ See, for instance, Aucremanne (2000), where some alternative measures for skewness and tail weight are suggested. Roger (2000) also includes formulae for the calculation of unbiased sample moments.

So, if anything, one should expect the true skewness and kurtosis coefficients (particularly this one) to be even higher than Figure 1 suggests.

There is still an additional point worth noticing. From Figure 1 we can see that periods characterised by strong asymmetry⁹ (positive or negative) are also periods in which the kurtosis is higher. There seems to be two main explanations for this correlation between skewness and kurtosis. On one hand, if the distribution is positively (negatively) skewed the samples tend to be skewed to the right (left) indicating that the right (left) hand tail is the longest. As the kurtosis evaluates the relative importance of the tails it will tend to be higher the stronger (in absolute terms) the asymmetry. But this can also work the other way around. According to Bryan *et al.* (1997), when the distribution is fat-tailed one is more likely to obtain a draw from one of the tails of the distribution that is not balanced by an equally extreme observation in the opposite tail. That is to say, the higher the kurtosis the higher the probability of getting a skewed sample, even if the underlying distribution is symmetric. Therefore some proportion of the sample skewness may simply be generated by the kurtosis of the distribution.

This finding has important practical consequences as it implies that it is not possible to isolate or separately correct the effects of skewness and kurtosis for a given sequence of samples.

4 – THE USE OF THE ASYMMETRIC TRIMMED MEAN WHEN THE DISTRIBUTION IS SKEWED AND LEPTOKURTIC.

We saw in section 2 that the use of the trimmed mean as a core inflation indicator stems from the fact that it is a more efficient estimator of the population mean than the sample mean, when the distribution is symmetric but leptokurtic.

The starting hypothesis is that, on a given date, the price change of one of the items in the CPI basket is a particular draw from a distribution whose (unknown) population mean is the core inflation prevailing at that date. However given that the distribution is leptokurtic, the recorded inflation (the sample mean) is an unbiased but relatively inefficient estimator of the core

⁹ In this paper we use the terms symmetry and skewness as synonymous. However we must stress that some authors use to distinguish between skewness and asymmetry as, in rigour, they are different concepts. Symmetry implies that the left and right sides of the distribution are mirror images, while the coefficient of skewness compares the density of the tails in distance relative with the mean. It is possible that a distribution may be asymmetric and yet has a zero coefficient of skewness. However if the distribution has a non-zero coefficient of skewness it must be asymmetric.

inflation (population mean) and this justifies using the trimmed mean as the estimator of the population mean.

When the distribution is symmetric the median coincides with the mean and the mode so that the question of which one of these three central tendency measures should be seen as representing core inflation is not very important (even though it can be relevant to define the properties of the estimators). However, this question becomes relevant when the price changes distribution is skewed. Let us take, for instance, the case of a positively skewed distribution. In this case, in general, $\text{mode} < \text{median} < \text{mean}$, and so it is necessary to decide from the start which central tendency measure is to be seen as the core inflation indicator, as in this case the estimator to be used depends on the choice made.

Besides the trimmed mean, the use of a (weighted) median as a core inflation indicator has also been suggested in the literature (Bryan and Cecchetti (1994) and Cecchetti (1997)). Under the symmetry assumption, using the median (or even the mode) as a core inflation indicator is probably worth trying. However if the price changes distribution is asymmetric the median or the mode cannot be used as core inflation indicators. And this is simply because recorded inflation – the variable of interest- corresponds to the sample mean of the price changes distribution, which, in case of asymmetry, should not be confused with the median and the mode. So, if we wish to build a core inflation indicator complying with condition (5) in section (2) we have to use an unbiased estimator of the population mean of the price changes distribution.

This conclusion suggests that the use of the trimmed mean may be warranted even when the price changes distribution is skewed. However it seems now pretty obvious that the trimmed mean must be such that it is an unbiased estimator of the population mean. As it will be seen below, this condition helps us to give the right answer to an interesting question that arises in this context: should one trim more on the right-hand tail or on the left-hand tail?

The correct answer depends on whether one is putting more weight on the bias or on the variance of the resulting estimator. If our interest resides in minimising the variance of the resulting estimator the correct answer is that one should trim more from the right-hand tail of the distribution. In fact, if the distribution is skewed to the right, the draws of individual price changes with large absolute values will most often come from the right side of the distribution, so minimising the variance implies trimming a larger percentage from the right side of the distribution. Another argument to justify trimming more from the right side of the distribution was put forward by Bakhshi and Yates (1999). “In a distribution that is positively skewed the largest 5% of prices changes will have a greater moment about the mean (will lie farther way from

it in inflation units) than will the smallest 5% of price changes; in turn, sampling errors in the highest part of the distribution will also play a more important role in incorrectly estimating the mean than those in the lowest part. So in a positively skewed distribution, a larger proportion of the top of the distribution should be trimmed in order that over repeated draws, the expected impact of sampling errors on the mean is zero”.

However it is easy to anticipate what happens to the resulting trimmed mean if one trims more from the right hand side of the distribution. In a the distribution that is positively skewed, the largest, let us say, 10% prices changes will have a larger contribution to the inflation rate (sample mean) than the smallest 10% of price changes. So, if we trim the same percentage from the two tails of the distribution, the resulting trimmed mean will systematically underestimate the inflation rate and so the core inflation rate¹⁰. And condition (5) of section (2) cannot be met. So, if one is interested in minimising the bias we must trim less from the right hand tail of the distribution¹¹.

Figure 3 depicts the 10% symmetric trimmed mean that has been computed by Banco de Portugal, for some years now. To harmonise the notation it shall be denoted by TM(50,10) where TM stands for trimmed mean, 50 is the percentile in which the trimmed mean is centred on and 10 is the percentage trimmed on each side of the distribution. From Figure 3 it is readily seen that the “average level” of the time series TM(50,10) during the sample period is lower than the “average level” of the inflation rate reflecting the fact that the symmetric trimmed mean is below the inflation rate most of the time. In other words TM(50,10), as already shown in Marques *et al.* (1999,2000), does not meet condition (5) of section 2, and so, it is not able to define appropriately the level of the trend component of the inflation rate. This obviously reduces the usefulness of TM(50,10) as a core inflation indicator. It is now clear however why this is so: the symmetric trimmed mean systematically underestimates the inflation rate because it trims too much from the right hand tail (or equivalently because it trims too little from the left hand tail).

This evidence suggests that in order to get a trimmed mean which does not exhibit a systematic bias relative to the recorded inflation, one must find the right proportion that has to be trimmed from each tail of the distribution. To do so, we have to centre the trimmed mean on

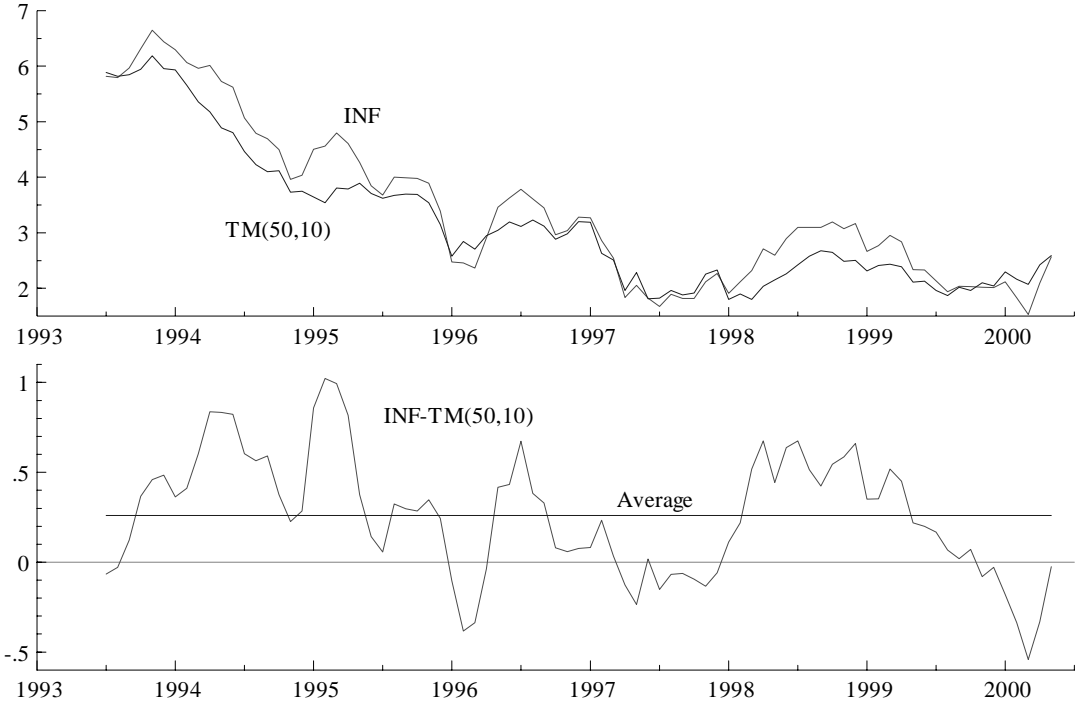
¹⁰ As the recorded inflation rate is the sample mean of the price changes distribution, which is an unbiased estimator of the population mean (the core inflation) we need an estimator that does not systematically differ from the sample mean in order to guarantee that it is also unbiased relative to the population mean.

¹¹ A very interesting discussion of this issue can be seen in Bihan and Sédillot (1999). These authors compute several trimmed means using as selection criteria: i) the minimum bias ii) the minimal variance and iii) minimal (weighted) sum of bias and variance. The conclusion is that as the (positive) asymmetry increases the

a percentile different from the 50th percentile. For instance, the 10 per cent trimmed mean centred on the 55th percentile is obtained by taking (only) 80 per cent of the price changes centred

Figure 3

The inflation rate (INF) and the 10 % symmetric trimmed mean TM(50,10)



on this percentile, i.e., by trimming the smallest (weighted) 15 per cent and the largest 5 per cent price changes.

In practical terms the question of how to find an asymmetric trimmed mean, which is not systematically biased relative to inflation, i.e., that meets equation (5) of section (2), arises. The literature on this issue is working out this question in an apparently unordered way. For instance, Kearns (1998) and Meyler (1999) compute an (almost) infinity of trimmed means changing both the trimming percentage from each tail (between zero and 50 per cent) as well as the percentile in which the trimmed mean is centred on. Kearns (1998) computes the asymmetric trimmed means

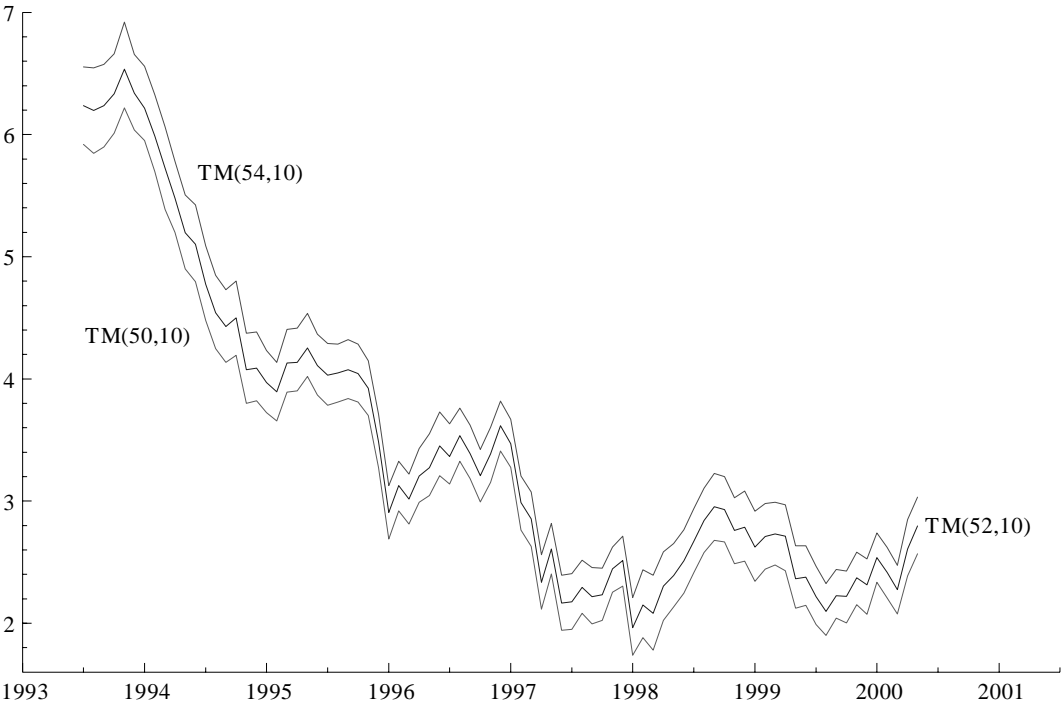
trimming from the left hand tail must increase under criterion i), but under criterion ii) is the trimming from the right hand side that has to increase. The third criterion gives intermediate results.

for all the percentiles between the 40th and the 60th¹², and Meyler (1999) between the 40th and the 70th. Both authors select the asymmetric trimmed mean that minimises the average absolute deviation and/or the mean square error relative to some predefined “inflation reference measure” (usually a centred moving average). More recently, Aucremanne (2000) used a similar but somewhat more elaborated process. Even though he also computed the trimmed means for all the percentiles between the 50th and the 60th, as a first step he selected as the optimal trimmed means the ones for which the null of Normality was not rejected according to the Bera-Jarque statistics. Among these, the optimum trimmed mean is chosen as the one that minimises the average absolute error relative to the inflation rate.

The shortcomings of using a so-called “inflation reference measure” as a device to select core inflation indicators were put forward in Marques *et al.* (2000) so that they would not be resumed herein. At this stage we are only concerned with the issue of defining some useful criteria in order to simplify and accelerate the searching process.

Figure 4

10 per cent trimmed means centred on the 50th, 52nd, and 54th percentiles

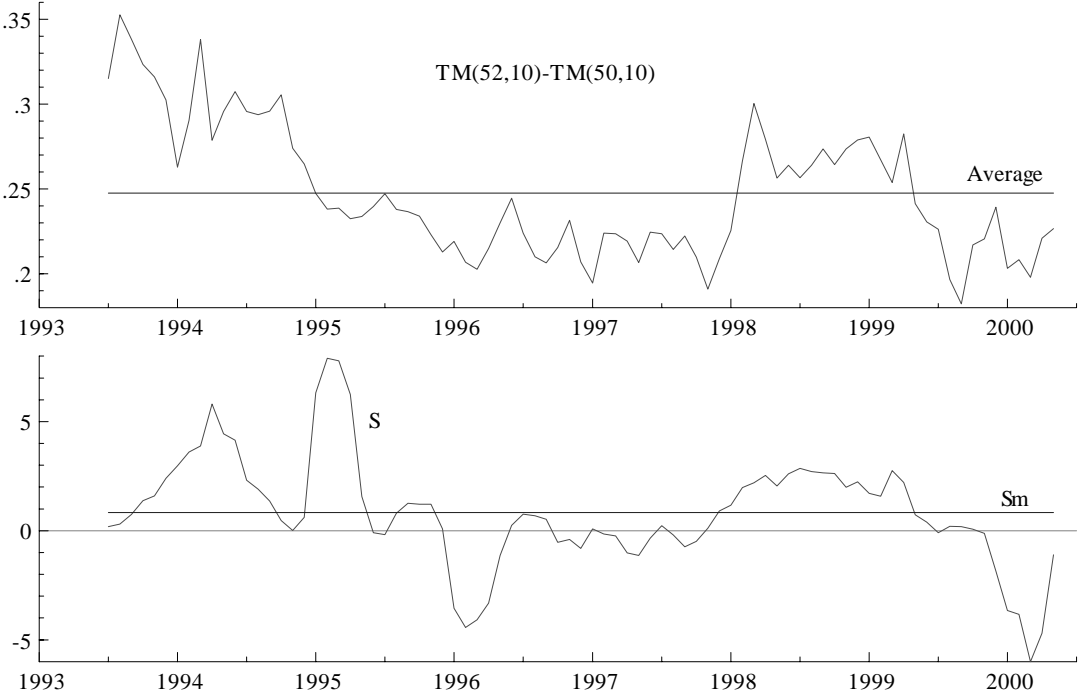


¹² In fact Kearns has computed the corresponding trimmed means for a larger interval, but he only reports the results for the interval 40-60 as “outside this range, large trimmed mean rates of inflation differ markedly from the moving average rate of inflation”

Figure 4 depicts three 10 % trimmed means centred on three different percentiles: the 50th, 52nd, and 54th percentile, denoted TM(50,10), TM(52,10) and TM(54,10) respectively. It is apparent that as we move from the smallest to the largest percentile, the corresponding 10% trimmed means move upward, thus exhibiting larger “average levels”.

This is the outcome one should expect if the price changes distribution were symmetric. However in our case the upward shift is probably even larger as a consequence of the price changes distribution being on average skewed to the right. For instance, in order to compute the 10% trimmed mean centred on the 52nd percentile, one excludes the smallest 12 per cent price changes from the left hand tail and the largest 8 per cent from the right hand tail of the distribution. But for the calculation of the 10 per cent trimmed mean centred on the 54th percentile we trim 14 per cent from the left hand tail and only 6 per cent from the right hand tail. So, when moving from TM(52,10) to TM(54,10), we are including 2 additional percentage points in the right hand tail of the distribution (between the 92nd and the 94th percentile in the ordered sample) which have a larger than average contribution to the resulting trimmed mean as they lie farther away to the right of the centre of the distribution and we are also excluding the 2 percentage points from the left hand side (between the 12th and the 14th percentiles) which were shifting TM(52,10) downwards as they have a smaller than average contribution to the trimmed mean. Thus, the overall result is a trimmed mean with a larger average level.

Figure 5



Notice, however that this upward movement is not parallel as the distribution is not symmetric. Figure 5 shows that the difference between TM(52,10) and TM(50,10) is generally above the average difference when the skewness coefficient is above the average skewness (for instance between the start of the sample and late 1994, or during 1998) and is below the average difference when the skewness coefficient is below the average skewness (for instance from late 1997 until late 1997 or during the last year of the sample).

Given these facts, we may suggest a very simple rule that, under the assumption that the kurtosis and skewness of the price changes distribution are time invariant, allows us to find, in a very simple way, one or several trimmed means that do not exhibit any systematic bias relative to the inflation rate. For a positively skewed distribution we have the following rule:

- i) For a given trimming level, compute the trimmed means centred on the 50th, the 51st, the 52nd percentile, etc.;
- ii) Stop whenever the resulting trimmed mean satisfies the condition $\sum_1^T (\pi_i - \pi_i^*) = 0$, (where π_i^* stands for the trimmed mean and π_i for the inflation rate).

Condition i) tell us that if the distribution is positively skewed then there is no point in computing trimmed means centred on percentiles below the 50th percentile, as it will never be possible to find any trimmed mean satisfying condition (5) of section 2, i.e., which is not systematically biased. Condition ii), which is the empirical counterpart of condition (5) in section 2, sets the upper limit for the searching procedure. For instance, let us assume that the condition for the absence of a systematic bias for a 20% trimmed mean is met for the 53rd percentile. This condition tell us that it is not useful to compute additional trimmed means for higher percentiles as it will not be possible to find another 20% trimmed mean satisfying that property (as we have seen, all the 20% trimmed means centred on higher percentiles will exhibit higher average levels).

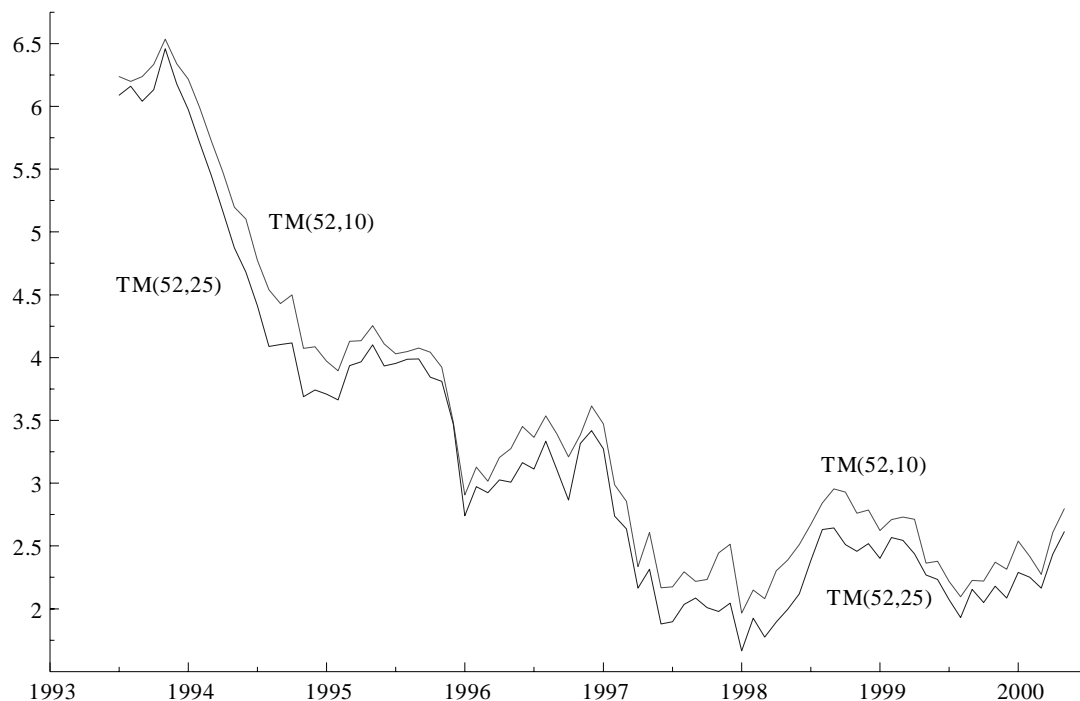
Using this rule for the 5% trimmed mean, with the Portuguese data, condition ii) was met for the 51st percentile. Notice that this trimmed mean is obtained excluding 4 per cent from the right hand tail of the distribution and 6 per cent from the left-hand tail.

What if one wants to compute another trimmed mean using a different amount of trimming? One could think that the solution (the 51st percentile) obtained for the 5% trimmed mean would also be the correct answer for a 10% or a 25% trimmed mean, but this is not so.

We know that if the price changes distribution is leptokurtic the trimmed mean is a more efficient estimator of the population mean than the sample mean. Furthermore we have seen that the more leptokurtic the distribution the larger the proportion of the ideal trim¹³. This rule, put forward by Bryan *et al.*(1997), is valid under the assumption of symmetry so that varying the percentage of trimming only changes the efficiency of the estimator but not its expected value. This rule is however not true if the distribution, besides being leptokurtic, is also skewed. In fact it is easy to show that if the distribution is skewed varying the amount of trimming will also alter the expected value of the resulting trimmed mean. Figure 6 depicts the 10% and the 25% trimmed means both centred on the 52nd percentile. As can be seen, when we change the amount of trimming we basically change the expected value and not so much the volatility of the corresponding trimmed mean.

Figure 6

10% and 25% trimmed means centred on the 52nd percentile



¹³ We have seen that the kurtosis coefficient is a non-robust measure of the tail weight of the distribution as it is vulnerable to the masking phenomenon. For instance, a single outlier very far away from the centre of the sample may produce a higher kurtosis than several outliers, because the variance which is used in the computation of the kurtosis in (8) may be higher in the second case. So, one must be careful when establishing a relation between kurtosis and optimal trimming percentage. We owe this point to Luc Aucremanne.

The explanation for such an outcome is very simple, as when we increase the amount of trimming we change the proportion trimmed from each side of the distribution. For instance, to compute the 10% trimmed mean we just trim 10% from each tail, but to compute the 25% trimmed mean we have to trim more 15% from each tail. Now, as the distribution is positively skewed the additional 15% trimmed from the right hand tail have a larger contribution to the resulting trimmed mean than the additional 15% trimmed from the left tail. The final result is a series with a lower average level. Notice however that, as in Figure 5, the downward shift is not exactly parallel. The difference between the two series is generally larger the larger the skewness of the distribution¹⁴.

This exercise allow us a better understanding of condition ii) of the practical rule suggested above, as it shows that if we wish to find an asymmetric trimmed mean not systematically diverging from the inflation rate, the amount of trimming from each tail must be such that their contribution to the mean (inflation rate) is identical to each other, over the sample period. Notice that it is this type of relation we are looking for when we initiate the process of computing, let us say, the 5% trimmed mean centred around successive percentiles in accordance with condition i). The fact that we obtain the desired result for the 51st percentile just means that the degree of asymmetry is such that the contribution to inflation of the largest 4% price changes is on average equal (in absolute value) to contribution of the smallest 6% price changes.

Let us now show how to define the set of relevant percentiles over which it is worth carrying out a search procedure. As we have seen, Figure 2 depicts both the mean percentile (β) and the average mean percentile (β_m). By definition, current inflation is the inflation rate corresponding to the mean percentile. Therefore the time series of inflation rates corresponding to average mean percentile (β_m in Figure 2) satisfies, by construction, equation (5) in section 2, that is it does not exhibit any systematic bias. Notice that this series is the 50% trimmed mean centred on the percentile β_m and it appears as a natural candidate for a core inflation measure. In our case we have $\beta_m = 56$ and so, we know that the 56th percentile is the highest one for which there exists an asymmetric trimmed mean with at least one interesting property¹⁵. And we also

¹⁴ Implicit in this discussion is of course the idea that the asymmetry in the distribution is throughout the distribution and not just in the tails. If the distribution is symmetric after some arbitrary trim, then the expected additional symmetric trim would not change the expected value of the trimmed mean.

¹⁵ Note that the 50% asymmetric trimmed mean represents the maximum off-centre trim in “well-behaved” situations. This will not be the case if, for example, the distribution between the 40th and 60th percentiles is left-

know that this is the 50 % trimmed mean. On the other hand, the percentile on which we have centred the trimmed mean with the least amount of trimming we have decided to entertain (5% in our case) gives us the lowest searching bound for β . Let us exemplify. In our case the 5% trimmed mean (the least amount of trimming we decided to admit) was obtained with the 51st percentile. This means that all the other asymmetric trimmed means (for levels of trimming higher than 5% and lower than 50%) must be searched for in the open interval (51,56).

Some approaches, developed in the literature, do not suggest a searching procedure. They rather resort to the above-mentioned property of β_m . In this vein Roger (1997) was probably a pioneer. This author just noticed that, in the case of New Zealand, the right skewness of the price changes distribution tended to bias the median measure of inflation downwards relative to the mean, and to overcome the problem he computed the 50% asymmetric trimmed mean centred on the average mean percentile (the 57th)¹⁶. For France, Bihan e Sédillot (1999) followed a similar procedure and computed the 50% asymmetric trimmed mean centred on the 60th percentile and, for Australia, Kearns (1998) concluded that the 50% asymmetric trimmed mean centred on the 51st percentile is the best core inflation indicator out of all the other trimmed means, as it minimised the average absolute error or the mean square error vis-à-vis a centred moving average of the inflation rate.

In the next section, besides the 50% trimmed mean centred on the 56th percentile, we will also evaluate the other trimmed means that do not exhibit any systematic bias relative to the inflation rate: the 5%, 10%, 15% 20% and 25% asymmetric trimmed means centred on the 51st, 51.5th, 52.5th, 53rd and 54th percentile, respectively.

Now let us face the question of deciding whether one should assume that the asymmetry is constant or time variable during the sample period.¹⁷ This is a very difficult question for which there not seems to be a definite answer. The difficulty arrives because the kurtosis and skewness coefficients are not independent as we have seen. This means that it is not possible, from a given sample, to definitely say which is which, and so, to separately correct for these two phenomena.

skewed, while the tails of the distribution were right-skewed. In this case a 50% trim might be less asymmetric than a 25% trim. We thank Scott Roger for making us this point.

¹⁶ Roger called this measure the “median-based measure of core inflation”. Notice however that it is not an estimator of the median of the price change distribution but rather an estimator of the mean, as we have seen. To avoid this kind of potential misunderstanding it shall simply be denoted hereafter as the 50% trimmed mean centred on β_m . In practical terms it is, of course, the inflation rate corresponding to the β_m percentile.

¹⁷ The discussion that follows implicitly assumes that the level of the kurtosis is constant across the sample.

In some cases it may be however possible to say something more about this issue. Let us once again take a look at the time profile of the mean percentile β in Figure 2. The fact that, for a given month, β is significantly higher than 50, does not allow us to infer anything about the skewness of the price changes distribution as this may be the outcome of a symmetric but leptokurtic distribution or just a consequence of the sampling error. On the other hand, if we assume that the distribution is leptokurtic and skewed, and in a given month, β is, for instance, equal to 60, we are not able to tell which of the three phenomena (kurtosis, skewness or sampling error) explains which part of the difference between 60 and 50.

One way of overcoming this difficulty is to look for potential regularities in the time series dimension of the data. In other words we are supposed to solve a signal extraction problem. Let us assume that the price changes distribution is leptokurtic, but not skewed. Then we may expect, month after month, the mean percentile to differ from the 50th percentile (both because the distribution is leptokurtic and because of sampling errors), but these deviations to be temporary and to cancel each other after a while. If not, we may identify the skewness as the responsible for the systematic nature of these deviations¹⁸.

The question of how to identify these two characteristics and how to best account for them is however an empirical matter and the answer will depend on the time series properties of β (the mean percentile). In the Portuguese case, it happens that this series is stationary around a constant (the average mean percentile β_m) and so, there is empirical evidence that the asymmetry of the price changes distribution may be assumed as constant across the sample period. However one should avoid the temptation of identifying the level of this systematic component as a measure of the degree of skewness¹⁹. The idea is simply that if β is stationary about a constant (β_m), there is evidence that potential time variability of the skewness is not a problem and we may well take it as constant.

Additionally notice that if we compute the asymmetric trimmed mean under the assumption of constant asymmetry when in fact it varies through time (in a significantly manner to be defined in below) then we may expect this fact to be uncovered with the tests carried out in

¹⁸ One may argue that if the distribution is leptokurtic but symmetric the β coefficient is expected to behave like a white noise around the 50th percentile. The point to be made however is that asymmetry is not a problem if β is not a white noise, provide it is a stationary variable around the 50th percentile, as the resulting (symmetric) trimmed mean would be an unbiased estimator of the mean.

the next section. Particularly, if the asymmetry is time varying and we compute the asymmetric trimmed mean assuming it is constant then we should expect the null hypothesis of non-cointegration between this series and the inflation rate not to be rejected. In other words, condition i) suggested in Marques *et al.* (2000) and evaluated in the next section can be used as an indirect test for the time constancy of the asymmetry.

The problem is much more complicated if β is not stationary around β_m as in this case we can no longer assume that asymmetry is constant through time. The difficulty arises because we need to previously identify some sort of non-constant trend for β , which (by definition) cointegrates with β . Furthermore this trend for β must be computable timely and once and for all, otherwise the core inflation measure will be such that history could change each time a new observation becomes available. However once such trend for β is identified we can proceed by computing the 50% trimmed mean centred on this trend.

A very simple and potentially useful procedure that allow us to find a “smoothed β ” that satisfies the above requisites is the conventional exponential smoothing process. If we let $\bar{\beta}_t$ to denote the smoothed β we have $\bar{\beta}_t = \lambda \bar{\beta}_{t-1} + (1 - \lambda) \beta_t$, $0 \leq \lambda \leq 1$, where the level of smoothness depends on the parameter λ . The value of λ must be chosen taking into account that there is a trade-off between the smoothness and the bias of $\bar{\beta}_t$. If $\lambda = 0$ then $\bar{\beta}_t = \beta_t$. In this limiting case there is no bias but there is no smoothness either. On the other hand when λ tends to 1 the smoothed series converges to a constant (that equals the starting level). In this other limiting case the smoothness and the bias are the highest. By construction the 50% trimmed mean centred on $\bar{\beta}$ is an unbiased estimator in the sense that it meets condition (5) of section 2 (at least approximately).

Figure 2, in the previous section, also depicts the smoothed β , denoted $\bar{\beta}$, for $\lambda = 0.9$. In the Portuguese case the 50% trimmed mean centred on $\bar{\beta}$, as expected, does not exhibit any systematic bias. However, also as expected, it fluctuates around the inflation rate more than the 50% trimmed mean centred on β . For this reason it will not be further analysed in the next section.

¹⁹ The fact that the percentile were the trimmed mean is centred on changes with the amount of trimming, shows that we cannot take the face value of β as a measure of the degree of skewness of the price changes distribution.

We may of course ask ourselves how to proceed in order to compute trimmed means for a trimming less than 50%. Notice once again that we cannot just centre the trimmed means on $\bar{\beta}_i$ as they will exhibit a systematic bias relative to the inflation rate. By analogy with the constant asymmetry case we can however compute alternative trimmed means for different levels of $\bar{\beta}_i$. Suppose for instance we are interested in computing the 25% trimmed mean under the assumption that symmetry is time variable. We know that if we centre this trimmed mean on $\bar{\beta}_i$ it will exhibit a systematic positive bias relative to inflation. We can however compute the 25% trimmed mean centred on $\bar{\beta}_i^*$ were $\bar{\beta}_i^* = -\alpha + \bar{\beta}_i$ for different positive values of the constant α ($\alpha = 1, 2, 3, \dots$) and just stop when the unbiasedness condition is met. We also know that if this condition is met let us say for $\alpha = 2$ then in order to compute the 20% trimmed mean, for instance, we should start by trying values for α (only) larger than 2. Of course it may also be case that one has to use non-integer values for α .

The computation of the different trimmed means centred on $\bar{\beta}_i^*$, for the Portuguese case, will not be carried out in this paper because, as we have seen, there is no empirical evidence of a time varying asymmetry.

5 – THE ASYMMETRIC TRIMMED MEAN FOR THE PORTUGUESE DATA

This section evaluates the different unbiased asymmetric trimmed means computed in the previous section. We aim at establishing if those are core inflation indicators with nice properties, assuming that inflation is measured by the year-on-year rate of change of the consumer price index.

The period under analysis runs from July 1993 to May 2000. This period has been chosen to exclude the effects of major changes introduced in the VAT rates in the middle of 1992, which strongly increased the year-on-year inflation rate during one year.

We analyse the following 7 trimmed means: i) the 10% symmetric trimmed mean, that has been computed and released by Banco de Portugal on a regular basis for some years now and is denoted TM(50,10) and ii) the asymmetric trimmed means TM(51,05), TM(51.5,10), TM(52.5,15), TM(53,20), TM(54,25) and TM(56,50). Notice that the last one is just the sample

inflation rate of the 56th percentile²⁰. These asymmetric trimmed means are all unbiased estimators as they were defined so that the condition $\sum_1^T (\pi_t - \pi_t^*) = 0$ is met, in statistical terms.

According to the conditions set out in Marques *et al.* (2000), when inflation, π_t , is I(1), we say that π_t^* is a core inflation measure if:

i) π_t^* is I(1) and π_t and π_t^* are cointegrated with unitary coefficient, i.e., $\pi_t - \pi_t^*$ is a stationary variable with zero mean;

ii) there is an error correction mechanism given by $z_{t-1} = (\pi_{t-1} - \pi_{t-1}^*)$ for $\Delta\pi_t$, i.e., $\Delta\pi_t$ may be written as

$$\Delta\pi_t = \sum_{j=1}^m \alpha_j \Delta\pi_{t-j} + \sum_{j=1}^n \beta_j \Delta\pi_{t-j}^* - \gamma(\pi_{t-1} - \pi_{t-1}^*) + \varepsilon_t \quad (9)$$

iii) π_t^* is strongly exogenous for the parameters of equation (9).

The rationale for these conditions may be seen in Marques *et al.*(1999, 2000). To test condition i) we may just start by carrying out a cointegration test on the static regression

$$\pi_t = \alpha + \beta\pi_t^* + u_t \quad (10)$$

and demonstrate that $\beta=1$ and $\alpha=0$. The complete procedure may be carried out in two steps. Firstly we run a unit root test on $z_t = (\pi_t - \pi_t^*)$ in order to establish that z_t is a stationary variable. The results of this test are reported in the first column of Table 1. In the second step we test the null $\alpha=0$, given that z_t is stationary. The outcome of these tests is reported in the second column²¹.

Condition i) having been established the verification of ii) is simple, just requiring the specification of a model like (9) and the testing of the hypothesis $\gamma=0$, using the conventional t-

²⁰ In practice this variable was computed as the arithmetic average of the price changes of the following two central items: the one that last occurs before the 56th percentile and the one that first occurs after this percentile.

²¹ Notice that the condition used in the previous section to identify the unbiased asymmetric trimmed means guaranties that we get $\alpha=0$. However this condition does not imply the stationarity of z_t .

ratio of $\hat{\gamma}$. The results of this test are reported in column 3 of Table 1. Remember that condition ii) may be interpreted as the requirement of π_t^* to be an attractor for π_t and is the empirical counterpart of the implication of the economic models reviewed in section 2 according to which an error correction mechanism must exist to force inflation to converge towards its trend.

Condition iii) implies that in the error correction model for π_t^* :

$$\Delta\pi_t^* = \sum_{j=1}^r \delta_j \Delta\pi_{t-j}^* + \sum_{j=1}^s \theta_j \Delta\pi_{t-j} - \lambda(\pi_{t-1}^* - \pi_{t-1}) + \eta_t \quad (11)$$

we should not reject the null hypothesis $\lambda = \theta_1 = \dots = \theta_s = 0$. This condition guarantees that the time profile of π_t^* is not determined by past values of π_t . The results of this test are reported in column 5 of Table 1. Instead of iii) one may just require the less stringent condition of weak exogeneity of π_t^* . To test this condition we just need to run the test $\lambda=0$ in equation (11). Column 4 of Table 1 reports the results of this test.

Several important conclusions may be drawn from Table 1. First, as expected, given the results obtained in Marques *et al.* (1999, 2000), the 10% symmetric trimmed mean does not meet condition i). Thus it is not an unbiased estimator for the trend of inflation. In fact this trimmed mean systematically underestimates inflation ($\alpha \neq 0$ in column 2). The explanations for such an outcome were put forward in the previous section and stem from the fact that the price changes distribution is right-skewed, on average.

On the other hand the 50% asymmetric trimmed mean centred on the 56th percentile, TM(56,50), fails condition iii) and so it depends on lagged inflation. Figure 7 and the results of Table 2 show why we get such a result: TM(56,50) is too much volatile to be an useful core inflation indicator. It is even more volatile than the inflation rate itself. This outcome is a clear indication that we are trimming too much, this way excluding information, which is fundamental for the definition of a trend measure of inflation. Remember, however, that this measure has been suggested as a good core inflation indicator for New Zealand (Roger (1997)) or even selected as the best indicator among all the trimmed means in case of Australia (Kearns(1998)).

Figure 7

Inflation rate (INF) and 50% trimmed mean
centred on the 56th percentile, TM(56,50)



The last important conclusion is that all the remaining asymmetric trimmed means meet all the three conditions and so they can be used as core inflation indicators. In particular, notice that, as we saw in the previous section, as we increase the total amount of trimming we have also to increase the percentile on which the trimmed mean is centred on in order to get an unbiased estimator. For instance, for the 10% asymmetric mean to be an unbiased estimator we need to centre it on the 51.5th percentile. In turn, the 15%, the 20% and 25% asymmetric trimmed means have to be centred on the 52.5th, the 53rd and the 54th percentile respectively.

One should notice that the percentile that ensures that the resulting trimmed mean is an unbiased estimator is not truly unique. For instance it is possible to compute a large number of 10% trimmed means centred on a small neighbourhood of the 51.5th percentile, all of them being unbiased estimators of the trend of inflation. Of course all these 10% trimmed means are statistically equivalent. That is why the searching procedure for the right percentile, carried in this paper, only considers integer numbers and the average of two consecutive integers. We think however it is important to stress the idea that an infinity of asymmetric trimmed means exists that, similarly to the five above mentioned ones, are expected to meet the three conditions analysed in Table 1.

As could be expected, all these 5 indicators exhibit a very similar time profile, as can be seen from Figure 8, which depicts both the $TM(51.5,10)$ and $TM(54,25)$. However, once all these 5 indicators meet the necessary conditions for a core inflation measure, we need an additional criterion to further choose among them. It seems that a good additional criterion is the degree of smoothness. In fact, for two otherwise identical indicators we surely prefer the smoothest one as it will exhibit a smaller short-run volatility and so will allow a clearer interpretation of the most recent inflation developments. So, in order to select the best indicator we compute the variance for each estimator and pick up the one with the smallest relative variance.

Figure 8

10% trimmed mean centred on the 51.5th percentile, $TM(51.5,10)$, and
 25% trimmed mean centred on the 54th percentile, $TM(54,25)$

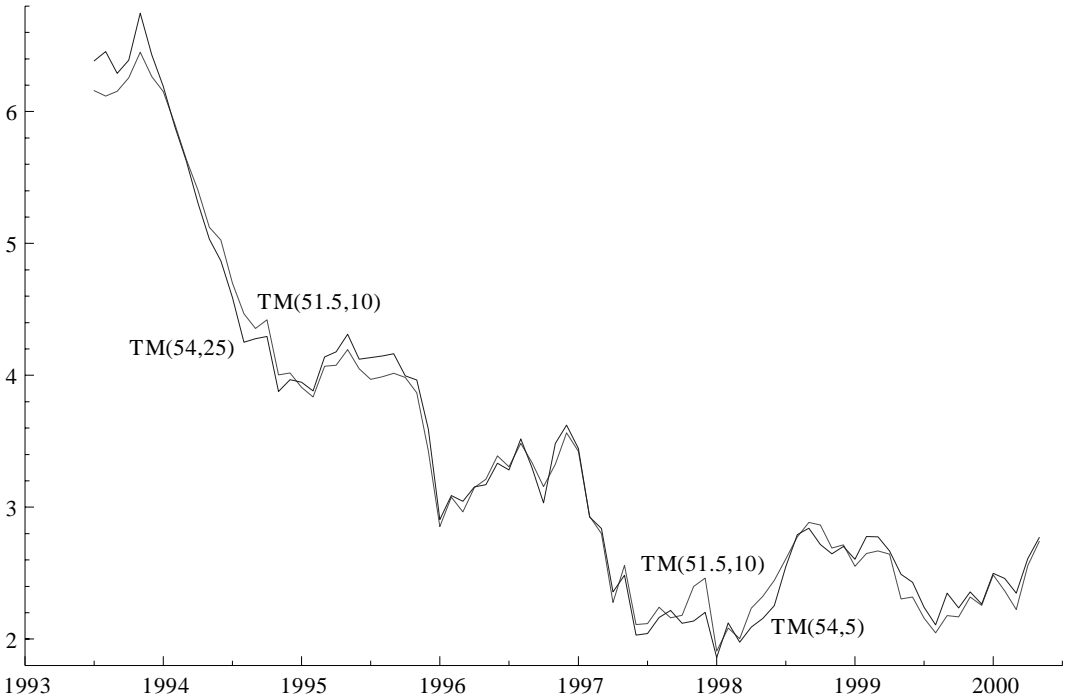


Table 2 reports the quotient between the variance of the first difference of each indicator and the variance of the first difference of inflation²². This statistic is a good indicator of the relative smoothness of each indicator.

²² It is interesting to notice that all the five indicators are well described by pure random walks, i.e., their first difference behaves as a white noise.

TABLE 2

RELATIVE VARIANCE OF CORE INFLATION INDICATORS						
TM(50,10)	TM(51,05)	TM(51.5,10)	TM(52.5,15)	TM(53,20)	TM(54,25)	TM(56,50)
0.503	0.522	0.497	0.510	0.540	0.541	2.238

For all five above-mentioned indicators it turns out that the corresponding variance is lower than the variance of inflation. This nice property is also a consequence of conditions ii) and iii). Out of these indicators the 10% asymmetric trimmed mean centred on the 51.5th percentile, i.e., TM(51.5,10), is the smoothest one. So this is the best core inflation indicator in the class of the trimmed mean core inflation indicators.

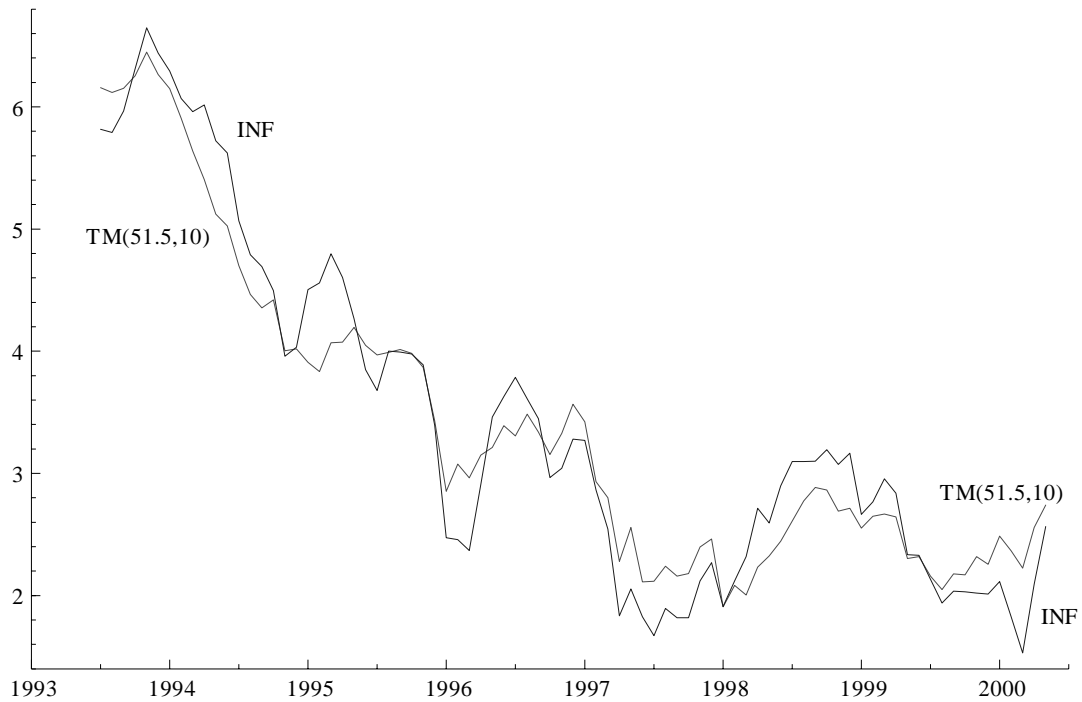
Figure 9 depicts TM(51.5,10) as well as the inflation rate. We can see that the time profile of the indicator accords with what should be expected from a core inflation indicator. The indicator is below inflation when this is particularly high (most of period between 93 and 95, 98 and early 99) and is above inflation when this is abnormally low (early 96, 1997, late 99 and early 2000). Additionally it turns out that, in general, after TM(51.5,10) being above inflation for a while, inflation increases and the opposite occurs after TM(51.5,10) being below inflation for a while.

This said, and notwithstanding the fact that this indicator satisfies all the necessary conditions for a core inflation indicator and is the smoothest one among these, it evidences some limitations. Firstly, it is not smooth enough as can be seen from Figure 9. There are several small changes in the inflation rate that are passed on to the core inflation indicator. This originates some noise in the indicator that makes its interpretation very difficult in the short run.

Secondly, this core inflation indicator suffers from the limitation that affects all the indicators based on trimmed means. It is not able to deal with simultaneous temporary shocks on all the price index components as, for instance, in the case of a VAT rate increase. In this case the indicator will also show an increase similar to the one exhibited by the year-on-year rate of change of the CPI. This is the reason why our sample period only starts in July 1993 as a major change in the VAT rates occurred in the first half of 1992. During the second half of 1992 and the first half of 1993, the period for which the effect of the VAT rates increase on the year-on-year rate of change of the CPI has lasted, the core inflation indicator stood above the inflation

Figure 9

Inflation rate, INF and 10% trimmed mean centred
on the 51.5th percentile, TM(51.5,10)



rate. However, if anything, one should expect, in this period, the “true” trend of inflation to lie somewhere below the inflation rate, not above it. The indicator TM(51.5,10) stood above inflation because during this period the price changes distribution is left-skewed (which, in turn, may be due, at least in part, to the changes in VAT rates). This limitation which is common to all indicators based on trimmed means must be borne in mind when interpreting the behaviour of TM(51.5,10).

6 - CONCLUSIONS

This paper addresses the issue of how to define and compute trimmed means if they are to be core inflation indicators with nice properties, when the price changes distribution is leptokurtic and asymmetric.

It is shown that when the price changes distribution is asymmetric the conventional (symmetric) trimmed means are biased estimators, and also that simply changing the total amount

of trimming in a symmetric way, changes not so much the efficiency of the estimator, as one would expect should the distribution be symmetric, but above all the expected value of the estimator.

The issue of whether one should treat asymmetry as constant or time varying is addressed in the general case and an easy to implement empirical approach is suggested. Simple rules in order to facilitate the searching procedure of an unbiased asymmetric trimmed mean are also put forward.

For the Portuguese data it is shown that, in line with studies carried out in many other countries, the price changes distribution is not only leptokurtic (has heavy tails) but it is also right-skewed, on average. This fact explains why the symmetric trimmed mean, released by Banco de Portugal on a regular basis, chronically underestimates the inflation rate.

Following the practical rules suggested in the paper several unbiased trimmed means are computed and tested against the three conditions suggested in Marques *et al.* (2000). It is found that all but one of these asymmetric trimmed means meet these conditions. The exception is the asymmetric trimmed mean, which trims 100% of the distribution around the average mean percentile, that is the sample value of this percentile. The reason is that it is too volatile probably because it trims too much. Strangely enough this estimator has been suggested in the literature as a good core inflation indicator for some countries.

Among the unbiased asymmetric trimmed means that meet the three conditions in Marques *et al.* (2000) we select the smoothest one, i.e., the one exhibiting the smallest variance in the first differences. This is the 10% trimmed mean centred around the 51.5th percentile, that is the one that trims 11.5% from the left-hand tail and 8.5% from the right-hand tail.

This core inflation indicator despite all its properties must be used cautiously not only because it exhibits some volatility which may render its interpretation somewhat difficult in the short run, but also because trimmed means as such are not able to deal with general price increases brought about, for instance, by value added tax rate changes, which have permanent effects on the general price level but only temporary effects on the inflation rate.

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TABLE 1
EVALUATING ASYMMETRIC TRIMMED MEANS

Trimmed Mean	ADF test on $(\pi-\pi^*)^{(a)}$	$\alpha=0$ given $\beta=1$	$\gamma=0$	$\lambda=0$	Strong exogeneity $\lambda=\theta_1=\theta_2=\dots=\theta_s=0$	Conclusion
Column	(1)	(2)	(3)	(4)	(5)	(6)
TM(50,10)	Yes ADF(1)=-3.59	No P=0.02	No P=0.00	Yes ^(b) P=0.12	Yes F(2,77)=1.77 P=0.18	Fails condition i)
TM(51,05)	Yes ADF(1)=-3.92	Yes P=0.81	No P=0.00	Yes P=0.22	Yes F(2,77)=0.18 P=0.32	OK
TM(51.5,10)	Yes ADF(1)=-3.70	Yes P=0.69	No P=0.00	Yes P=0.33	Yes F(2,77)=0.80 P=0.45	OK
TM(52.5,15)	Yes ADF(1)=-3.57	Yes P=0.87	No P=0.01	Yes P=0.47	Yes F(2,77)=0.39 P=0.68	OK
TM(53,20)	Yes ADF(1)=-3.55	Yes P=0.62	No P=0.01	Yes P=0.61	Yes F(2,77)=0.24 P=0.79	OK
TM(54,25)	Yes ADF(1)=-3.63	Yes P=0.72	No P=0.01	Yes P=0.85	Yes F(2,77)=0.22 P=0.81	OK
TM(56,50)	Yes ADF(1)=-2.78	Yes P=0.61	No P=0.04	No P=0.00	No F(2,77)=5.68 P=0.01	Fails condition iii)

(a) The critical values for the ADF test with 83 observations (model with nonzero constant) are: -3.51 (1% test), -2.90 (5% test) and -2.59 (10% test)

(b) We have $\lambda \neq 0$ in the model with no constant term

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