

CONSUMER EXPENDITURE AND COINTEGRATION

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Abstract

In this paper we estimate the Almost Ideal Demand System (AIDS) for the Portuguese economy. The budget shares and real per capita income are found to be integrated of order one, $I(1)$, but prices seem to be better classified as $I(2)$. This raises new problems, as it is not possible to test for homogeneity and symmetry in a straightforward way. As cointegration is not rejected after the imposition of homogeneity and symmetry restrictions, we conclude that the AIDS is an acceptable characterisation of the Portuguese data on consumer expenditure.

1-Introduction

Until very recently, most empirical work on demand systems was carried out under the explicit or implicit assumption that variables were stationary (see for instance Deaton and Muellbauer (1980), Attfield (1991), Majunder (1992), Alley et al.(1992), Moschini (1998)). A recent exception is Attfield (1997) which estimates the Almost Ideal Demand System (AIDS) assuming that all the series involved are integrated of order one, $I(1)$.

This paper estimates the AIDS model for the Portuguese economy. However as prices are integrated of order two, $I(2)$, we could not resort to the so called Phillips' triangular ECM representation (see Phillips, (1991) and Phillips (1994)) to test for homogeneity as in Attfield (1997). So, we decided to impose homogeneity from the outset and estimate the model accordingly. In this framework, as relative prices are $I(1)$, one can test for cointegration and check whether the estimated system is not spurious. A similar approach is followed for the imposition of symmetry. The main conclusion is that the system with

homogeneity exhibits good cointegrating properties and sensible results for the elasticities. When, additionally, one imposes the symmetry restrictions we find that the cointegrating relations do not degrade. According to cointegration results one may say that both homogeneity and symmetry are not rejected by the data and, in this sense, the AIDS model is a cointegrating demand system.

This paper is organised as follows. In section 2 we present the model and discuss the time series properties of the data; in section 3 we estimate the model with the homogeneity restriction and discuss the cointegrating properties of the system; in section 4 we analyse the consequences of imposing symmetry; in section 5 we evaluate the income and prices elasticities of the system; finally section 6 concludes.

2-The AIDS with non-stationary variables

The AIDS model, first introduced by Deaton and Muellbauer (1980), is now widely used in the literature. The model is a set of m demand equations which relate the budget shares, w_i , to a set of prices, $p_1, p_2, \dots, p_m$, and a measure of per capita real income, y , that is

$$w_{it} = \alpha_i + \sum_{j=1}^m \beta_{ij} p_{jt} + \gamma_i y_t + u_{it} \quad i=1, \dots, m \quad (1)$$

where w_{it} is the budget share of commodity group i at time t , defined as the ratio of nominal expenditure on commodity i , say X_{it} , to total nominal expenditure on all commodities, say

$X_t = \sum_{i=1}^m X_{it}$; p_{jt} stands for the natural log of the price of the j th commodity. Real per capita

income, y_t , is defined as $y_t = \ln(X_t/N_t) - \ln P_t^*$ where N_t is the population at time t and

$\ln P_t^* = \sum_{i=1}^m w_{it} p_{it}$ is the so-called Stone price index (see Deaton and Muellbauer, 1980). In

system (1) two-stage budgeting is assumed so that consumers allocate expenditure among commodity groups given an exogenously determined total expenditure.

The data analysed were drawn from Pinheiro et al. (1997) and refer to annual data from 1958 to 1993 for the following groups of commodities: (1) food, alcoholic drink and tobacco; (2) clothing and footwear; (3) energy products; (4) other non durable goods and (5) services; so it is implicitly assumed that the demand for services and nondurable goods is separable from the demand for durables.

Table 1 presents the results of the ADF tests for unit roots in the commodity shares, w_{it} , log of prices, p_{it} , log of real per capita income, y_t , and relative prices

Table 1
Unit root tests¹

w_1	w_2	w_3	w_4	w_5
ADF(0)=-0.67 No constant	ADF(0)=-1.07 No constant	ADF(0)=-0.02 No constant	ADF(1)=-2.91*	ADF(0)=0.88 No constant
Y	p_1	p_2	p_3	p_4
ADF(1)=-2.80 With trend	ADF(1)=-1.06 With trend	ADF(0)=-3.15 With trend	ADF(0)=-4.22*	ADF(1)=-1.25 With trend
p_5	p_{15}	p_{25}	p_{35}	p_{45}
ADF(0)=-1.54 With trend	ADF(0)=-1.25 No constant	ADF(0)=-1.22 No constant	ADF(0)=-0.90 No constant	ADF(0)=-2.61 With trend

($p_{i5t}=p_{it}-p_{5t}$). According to the tests performed we may conclude that a unit root in w_{it} ($i=1,\dots,4$) and y_t cannot be rejected². Of course, by construction, budget shares are bounded and so, in the long run, one would expect them to be stationary. However, for the sample data, the w_i display the characteristics of $I(1)$ variables and so we treat them accordingly (see Graph No.1). As the first differences of prices do not exhibit mean reversion (see Graphs No.2 and No. 3) we decided to entertain the possibility of prices being $I(2)$. For these variables we performed the ADF test resorting to the so-called Dickey-Pantula strategy (see Dickey and Pantula 1987). As can be seen from the table, the null hypotheses of two unit

¹ ADF(k) is the Augmented Dickey- Fuller test statistic based on the t-statistics to test the hypothesis $H_0 : \gamma = 0$ in the model $\Delta y_t = \alpha + \beta t + \gamma y_{t-1} + \delta_1 \Delta y_{t-1} + \dots + \delta_k \Delta y_{t-k} + \varepsilon_t$. The test was carried out following a "general to specific strategy", in which k was set so that the regression did not exhibit serial correlation in the residuals. If during the simplification process, it happened that α and β were both non-significant, we have the "no constant" case, if β is non-significant we have the "no trend" case and if α and β were both significant we have the "with trend" case. The critical values for 36 observations and a 5% test are: -1.95 (no constant), -2.94 (no trend) and -3.54 (with trend). The figures with an asterisk (*) are significant at least at 5 per cent.

For the price variables, for which the Dickey-Pantula strategy was followed, the estimated model was

$$\Delta^2 y_t = \alpha + \beta t + \gamma_1 y_{t-1} + \gamma_2 \Delta y_{t-1} + \mu_1 \Delta^2 y_{t-1} \dots + \mu_k \Delta^2 y_{t-k} + \varepsilon_t$$

and the figures in the table refer to the t-statistics of γ_2 (after imposing $\gamma_1 = 0$).

² An exception, according to the ADF test result is w_4 . However by looking at the corresponding figure (see Graph No.1) it is easy to recognise that w_4 is not stationary.

roots cannot be rejected (only p_3 seems to be an exception appearing as $I(1)$). In turn, relative prices p_{i5} ($i=1,\dots,4$) appear to be $I(1)$ (see also Graphs No. 4 and No.5). As it will be discussed below this turns out to be a very important result.

3-Testing for homogeneity

Let us consider system (1). Consumer theory imposes homogeneity of degree zero in prices and nominal income for consumer demand equations. In the AIDS model the homogeneity restriction on each share equation implies that

$$\sum_{j=1}^m \beta_{ij} = 0 \quad i = 1, 2, \dots, m \quad (2)$$

As $\sum_{i=1}^m w_{it} = 1$ only $m-1$ of the shares can be independently explained/estimated. For

this reason the equation for the commodity group “(5)-services” is omitted in the estimation.

Testing for homogeneity would be straightforward if the variables were stationary or integrated of order one. In the first case one would resort to the classical F test. In the second case one could proceed by resorting to the so called Phillips triangular representation (see Phillips 1991 and 1994) as in Attfield (1997), or through the use of the fully modified Phillips-Hansen estimator (see Phillips and Hansen, 1990, Hansen (1992) and Phillips 1995). To see how this could easily be accomplished let us reparameterize model (1) in the following way:

$$w_{it} = \alpha_i + \sum_{j=1}^{m-1} \beta_{ij} p_{j5t} + \mu_i p_{5t} + \gamma_i y_t + u_{it} \quad i=1, \dots, m-1 \quad (3)$$

Notice that in equation (3) $p_{j5t} = p_{jt} - p_{5t}$ ($j=1,\dots,4$) and $\mu_i = \sum_{j=1}^m \beta_{ij}$. So, testing for

homogeneity amounts to testing $\mu_i = 0$ ($i=1,\dots,m-1$) using the corresponding t-statistics.

However as p_{5t} is $I(2)$ none of those approaches is applicable.

The Johansen approach is also not suitable as the AIDS model is derived under the assumptions underlying the consumer demand theory and these do not conform with the

hypotheses underlying the Johansen approach, namely, that all the variables are endogenous and generated by a VAR³.

Share equations in (1) or (3) are unlikely to hold in each and every period, and are best seen as defining long run “equilibrium” budget shares. In this sense we may say that system (1) is data consistent if every equation exhibits cointegration (otherwise the regressions would be spurious). So, a way of testing the homogeneity restrictions consists in estimating system (3) after imposing $\mu_i = 0$ in each equation and then testing for cointegration. If cointegration for each equation is not rejected then we may conclude that homogeneity is not inconsistent with empirical evidence. Notice that after imposing $\mu_i = 0$ in each equation, all the remaining variables in system (3) are I(1) and so the standard cointegrating analysis applies. In particular, the potential endogeneity of the real expenditure variable y_t , discussed, for instance, in Attfield (1991), is not an issue as the estimators are super-consistent in case of cointegration

Table 2 presents the results of the Augmented Dickey-Fuller (ADF) test, Phillips-Ouliaris (1990), Z_α , statistic and Shin (1994) cointegration test for the 4 equations estimated by OLS, after setting $\mu_i = 0$, so that homogeneity is imposed. The lack of power of the cointegration tests is well known. That is why we decided to compute two cointegration tests in which the null hypothesis is the absence of cointegration, i.e., the ADF and the Z_α tests and a cointegration test where the null hypothesis is cointegration, i.e., the Shin test. Both the ADF and Z_α tests were computed because the first is one of the most used cointegration tests and the second because it is generally more powerful than the ADF test, specially in moderate sample sizes or when the regressors are exogenous (see for instance Haug (1996)). In general, one tends to favour the cointegration tests that postulate cointegration as the null hypothesis. After all, the economic theory, basically, posits long run equilibrium relationships and so, a null hypothesis of cointegration is more in line with economic theory than the reverse. In this case, the economic theory will only be rejected if there is strong evidence against it, otherwise we would take the risk of rejecting a good theory just because the test has not enough power or because data are not enough informative.

According to the tables constructed by Engle and Yoo (1987) the ADF critical values for a model with five variables and 50 observations is -4.15 (for a 5% test). As the critical values increase about 0.20 points for each additional variable, the critical value for a model

³ In a recent paper Pesaran and Shin (1997) estimate the AIDS model through a general VAR model, the main difference to the Johansen approach being the way the cointegrating relations are identified. Treating prices as endogenous, however, is not a reasonable assumption and for this reason, this approach has not been adopted in our study.

with six variables must be close to -4.35 for a 5% test and to -4.05 for a 10% test. According to these critical values, non-cointegration would be rejected for equations w_1 , w_2 and w_4 for a 5% test. For the w_3 equation, the null hypothesis would be rejected for a 20% test.

As we said the Z_α test is generally more powerful than the ADF test. For a sample size of 50 observations with five explanatory variables, Haug (1992) gives 5% critical value for Z_α of -33.49 and of -30.48 for a 10% critical value (demeaned statistics). So according to the Z_α test the null of no cointegration is rejected for equations w_3 (at 5%), w_2 and w_4 (at 10%) and w_1 (at 20%).

Table 2
Cointegration tests for AIDS with homogeneity⁴

	w_1	w_2	w_3	w_4
ADF(k)	ADF(2)=-4.55	ADF(0)=-4.97	ADF(1)=-3.70	ADF(0)=-4.50
$Z_\alpha(m=3)$	-27.42	-32.95	-35.15	-32.74
SHIN(m=3)	0.056	0.045	0.135	0.114

A similar conclusion may be drawn from the Shin cointegration test. According to Shin's tables the critical value for a model with 5 explanatory variables is 0.097 for a 5% test and 0.158 for a 1% test. The null hypothesis of cointegration is not rejected for any equation. For the first two equations is not rejected for a 20% test and for equations w_4 and w_3 cointegration is not rejected at 2.5% and 1% respectively. Additionally the analysis of graph No.6 suggests that the residuals of the four equations are stationary. We may then conclude that system (3) represents a cointegrating demand system in the almost ideal form of Deaton and Muellbauer (1980). As the system was estimated with homogeneity imposed ($\mu_i = 0$, $i=1, \dots, m-1$) we may say that homogeneity in the AIDS model is not rejected by the data.

⁴ In the table ADF(k) stands for the ADF test with k lags in the first differences of the residuals of the cointegrating regressions; k was identified following a "general to specific strategy" so that the residuals from the ADF regression do not exhibit serial correlation in the residuals; $Z_\alpha(m=3)$ stands for the Phillips-Ouliaris cointegration test with m representing the number of lag windows used to compute the test; The Shin cointegration test is computed from the residuals of a cointegrating regression including lags and leads of the first differences of the regressors; because of a degrees-of-freedom problem we only added to the cointegrating regression the current first differences of the regressors; we suspect that this could have biased the results of the computed Shin test against the null of cointegration; as in Z_α test, m stands for the number of lag windows used to compute the test.

4-Testing for homogeneity and symmetry

In addition to homogeneity restrictions consumer theory also imposes the so called symmetry restrictions, i.e.,

$$\beta_{ij} = \beta_{ji} \quad \text{for all } i,j \quad (4)$$

These are cross-equation restrictions and so it is no longer possible to estimate the system by ordinary least squares. Either one uses a non-linear least squares estimation procedure or the well known SURE (seemingly unrelated regression equations) approach. The four static structural equations have been estimated through the use of the restricted SURE estimation approach, with the cross-equation restrictions imposed, using Microfit 4.0⁵. Asymptotically the estimators will be super-consistent in case of cointegration.

The restricted residuals of each equation were used to test for cointegration. Using restricted rather than unrestricted residuals (as in section 3) affects both size and power of conventional cointegration statistics. In general, size will be reduced and power increased at conventional critical values, provided the restrictions are empirically satisfied. It is difficult to know, without Monte Carlo simulation, exactly what critical values to use when testing the restricted residuals. Intuition suggests that since there are 4 equations, 5 explanatory variables and 6 symmetry restrictions, critical values associated with a cointegrating relationship with 4 explanatory variables are probably a reasonable approximation. Notwithstanding we decide to carry out our analysis as if no restrictions were imposed, i.e., resorting to the critical values used in the previous section for a regression with 5 regressors. Of course these are expected to be conservative tests, in the sense of acting against cointegration⁶.

The results of the cointegration tests are in table 3. As it can be seen the new results support the existence of cointegration. According to the ADF test the null of no cointegration is rejected for all equations (at 5% for the w_1 and w_3 equations and at 10% for the w_2 and w_4 equations). The same conclusion holds for the Z_α test, even though in case of the w_1 equation the null is only rejected for a 20% test. According to the Shin cointegration test the

⁵ We also estimated the model using the LNSYSTEM routine of RATS 4.3, but the estimates for the parameters were exactly the same.

⁶ In rigour, we do not know the true critical values, as the residuals were not obtained using OLS. Additionally one must note another potential limitation of the analysis carried out in this section. Cointegration is analysed equation by equation, but one should probably carry out a global cointegration test taking into account the fact that we are estimating a system with cross-equations restrictions. However, such a test, to our knowledge, has not yet been developed in the context of the consumer demand theory.

null of cointegration is not rejected for any equation and the results for the w_3 equation are even superior to the non-symmetry case (the same holds for the ADF test). In addition to cointegration tests, the analysis of Graph No. 7 also suggests that the residuals of the four equations are stationary.

Table 3
Cointegration tests for AIDS with homogeneity and symmetry

	w_1	w_2	w_3	w_4
ADF(k)	ADF(2)=-4.69	ADF(0)=-4.00	ADF(1)=-4.37	ADF(0)=-4.27
$Z_\alpha(m=3)$	-29.99	-37.32	-34.90	-30.83
SHIN(m=3)	0.056	0.063	0.097	0.116

In short, cointegration holds for the AIDS model with homogeneity and symmetry imposed and, in this sense, this model is an acceptable characterisation of the Portuguese data on consumer expenditure.

5-Analysing income and price elasticities

This section reports income and price elasticities for the AIDS model both with and without symmetry restrictions imposed. In the first case (model without symmetry restrictions), elasticities were computed using the so called dynamic OLS estimator, that is the OLS estimator on the triangular representation of the AIDS model in which the first differences of the regressors were added to system (3) (with $\mu_i = 0$), i.e.,

$$w_{it} = \alpha_i + \sum_{j=1}^{m-1} \beta_{ij} p_{j5t} + \gamma_i y_t + \sum_{j=1}^{m-1} \delta_{ij} \Delta p_{j5t} + \lambda_i \Delta y_t + \varepsilon_{it} \quad i=1, \dots, m-1 \quad (5)$$

In rigour this system is expected to include lags and leads of the first differences of the regressors, but the small number of observations prevented us from including them. It is known that the MLE estimators of β_{ij} and γ_i parameters reduce to the GLS estimators. Because regressors p_{j5t} and y_t are $I(1)$ it can be shown that simple OLS on (5) gives an asymptotically equivalent estimator of parameters β_{ij} and γ_i (Stock and Watson, 1993). These estimators are also asymptotically efficient (Saikkonen, 1991). In the second case

(model with both homogeneity and symmetry restrictions), elasticities were computed estimating system (5) using the SURE technique.

Table 4 presents income and uncompensated own-price elasticities for the AIDS model, computed at sample means⁷. The imposition of symmetry produces some considerable changes for food, clothing and energy. The economic reasonability of some elasticities seems to be deteriorated after the imposition of symmetry. This is the case of food, for instance. Food has an income elasticity slightly larger than one after the imposition of symmetry, whereas the imposition of homogeneity only was compatible with the usual classification of

Table 4
Elasticities

Commodities	Model with homogeneity		Model with homogeneity and symmetry	
	Income Elasticity	Own-price elasticity	Income elasticity	Own-price elasticity
Food	0.907	-0.310	1.049	-0.390
Clothing	0.857	-0.820	0.689	-0.625
Energy	0.822	-0.453	1.180	-0.282
Other goods	1.197	-0.977	1.205	-0.965
Services	1.069	-0.770	0.982	-0.835

this good as a necessity. The demand for food is relatively price inelastic. Clothing is a necessity. The income elasticity of energy is either close to 0.8, with homogeneity imposed, or to 1.2, in the system with symmetry imposed; energy is relatively price inelastic. Other goods are a luxury and display the larger own-price elasticity, close to -1 . Finally, the income elasticity of services is close to one and the respective own-price elasticity is close to -0.8 .

6. Concluding remarks

This paper uses the AIDS to analyse the allocation of consumption amongst different types of goods in Portugal, over the period 1958-1993. In contrast with most previous application of the Almost Ideal model, this paper takes into account the non-stationary properties of the data. The statistical analysis of the series indicates that budget shares and

⁷ One should note that as the budget shares are $I(1)$ they do not have a constant mean. In spite of this we decided to compute the elasticities in the usual way.

real per capita income are $I(1)$, whereas prices are $I(2)$. This result prevented us from testing the imposition of homogeneity resorting to the Phillips' triangular representation (Attfield, 1997).

As relative prices are $I(1)$, homogeneity was imposed from the outset. As cointegration was not rejected, we concluded that the AIDS with imposition of homogeneity is a data consistent model. Imposition of symmetry involves cross-equation restrictions and therefore critical values for cointegration are not known. The comparison with some 'conservative' critical values indicated that cointegration is not rejected in the symmetric AIDS. This model is, in this way, an acceptable characterisation of the Portuguese data on consumer expenditure.

7- References

Alley A.G., Ferguson D. G., Stewart K. G., 1992, "An almost ideal demand system for alcoholic beverages in British Columbia", *Empirical Economics*, 17, 401-418;

Attfield, C. L. F., 1991, "Estimation and testing when explanatory variables are endogenous: an application to a demand system" *Journal of Econometrics* 48, 395-408;

Attfield, C. L. F., 1997, "Estimating a cointegrating demand system", *European Economic Review*, 41, 61-73;

Deaton, A., Muellbauer, J., 1980, "An almost ideal demand system", *American Economic Review* 70, 312-326;

Dickey, Pantula, 1987, "Determining the order of differencing in autoregressive processes", *Journal of Business & Economic Statistics*, Vol. 5, No 4, 455-461;

Engle R. F., Yoo, B. S., 1987, "Forecasting and testing in co-integrated systems", *Journal of Econometrics*, 35, 143-159;

Hansen, B. E., 1992, "Tests for parameter instability in regressions with $I(1)$ processes", *Journal of Business & Economic Statistics*, Vol. 10, No 3, 321-335;

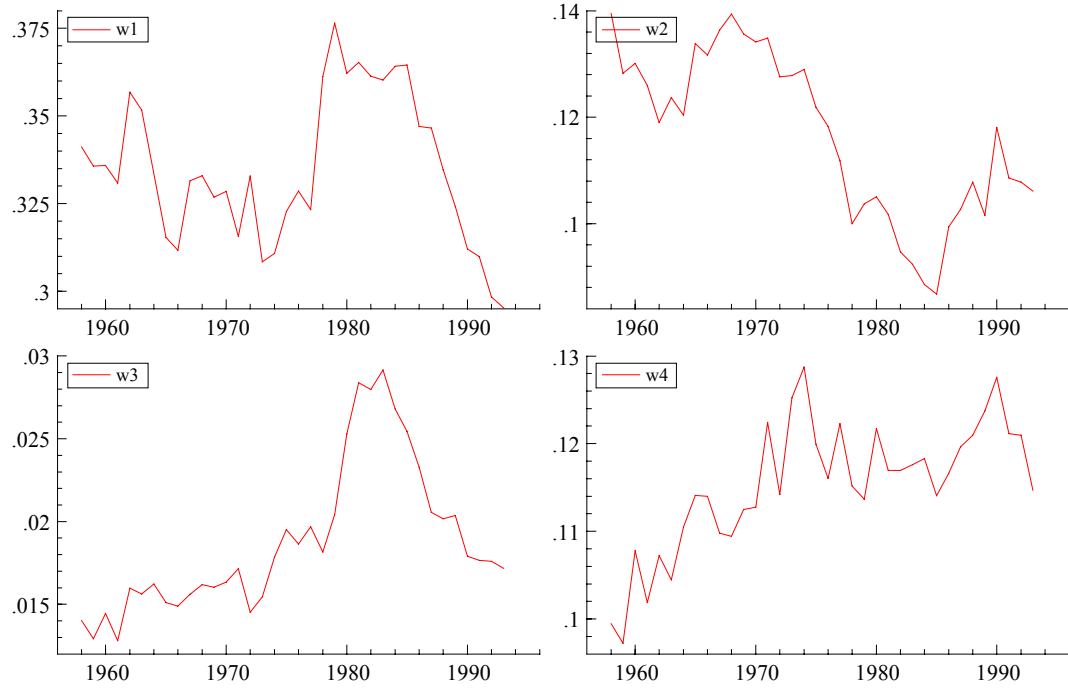
Haug, A. A., 1992, "Critical values for the Z_{α} -Phillips-Ouliaris test for cointegration", *Oxford Bulletin of Economics and Statistics*, 54, 473-480.

Haug, A. A., 1996, "Tests for cointegration, a Monte Carlo comparison", *Journal of Econometrics*, 71, 89-115;

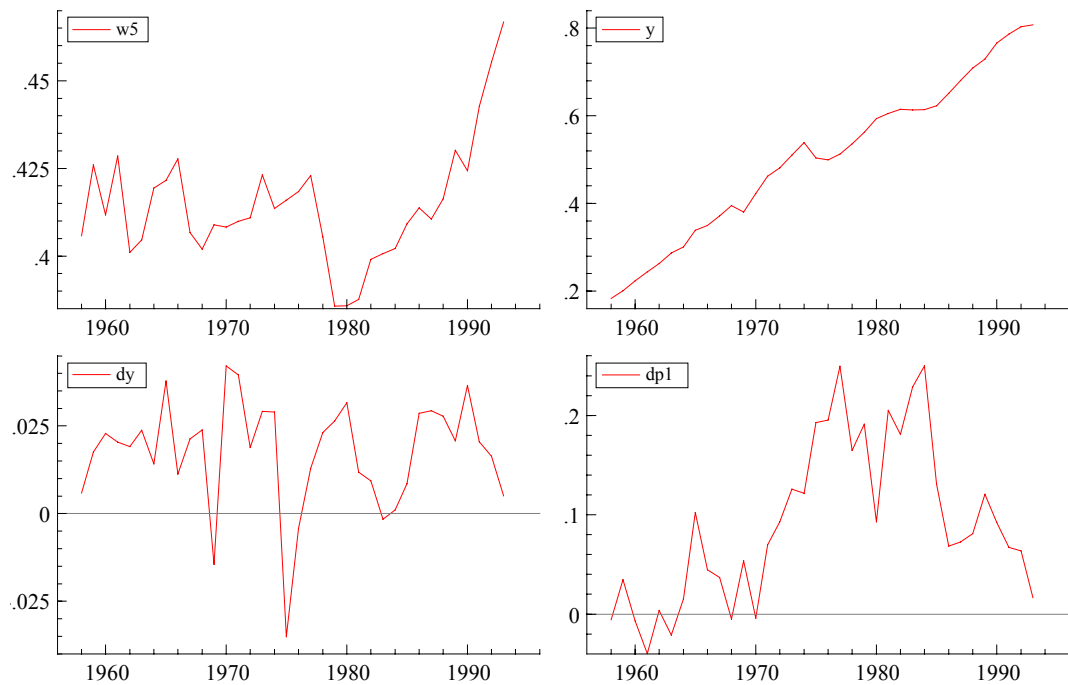
Majumder, A., 1992, "Measuring income responses: a log quadratic demand model for consumers in India", *Empirical Economics*, 17, 315-321;

- Moschini, G., 1998, "The semiflexible almost ideal demand system", *European Economic Review*, 42, 349-364;
- Pesaran, M. H., Shin Y., 1997, "Long-run structural modelling", manuscript, <http://www.econ.cam.ac.uk/faculty/pesaran/>
- Phillips, P. C. B., 1991, "Optimal inference in cointegrated systems", *Econometrica* 59, 283-306;
- Phillips, P. C. B., 1994, "Some exact distribution theory for maximum likelihood estimators of cointegrating coefficients in error correction models", *Econometrica* 62, 73-93;
- Phillips, P. C. B., 1995, "Fully modified least squares and vector autoregression", *Econometrica*, Vol.63, No 5, 1023-1078;
- Phillips, P. C. B., Hansen, B. E., 1990, "Statistical inference in instrumental variables regression with I(1) processes", *Review of Economic Studies*, 57, 99-125;
- Pinheiro et al., 1997, "Historical series for the Portuguese economy", Banco de Portugal;
- Saikkonen, P., 1991, "Asymptotically efficient estimation of cointegration regressions", *Econometric Theory*, 7, 1-21;
- Shin, Y., 1994, "A residual-based test of the null of cointegration against the alternative of no cointegration", *Econometric Theory*, 10, 91-115;
- Stock, J. H., Watson M.W., 1993, "A simple estimator of cointegrating vectors in higher order integrated systems", *Econometrica* Vol.61, No. 4, 783-820;

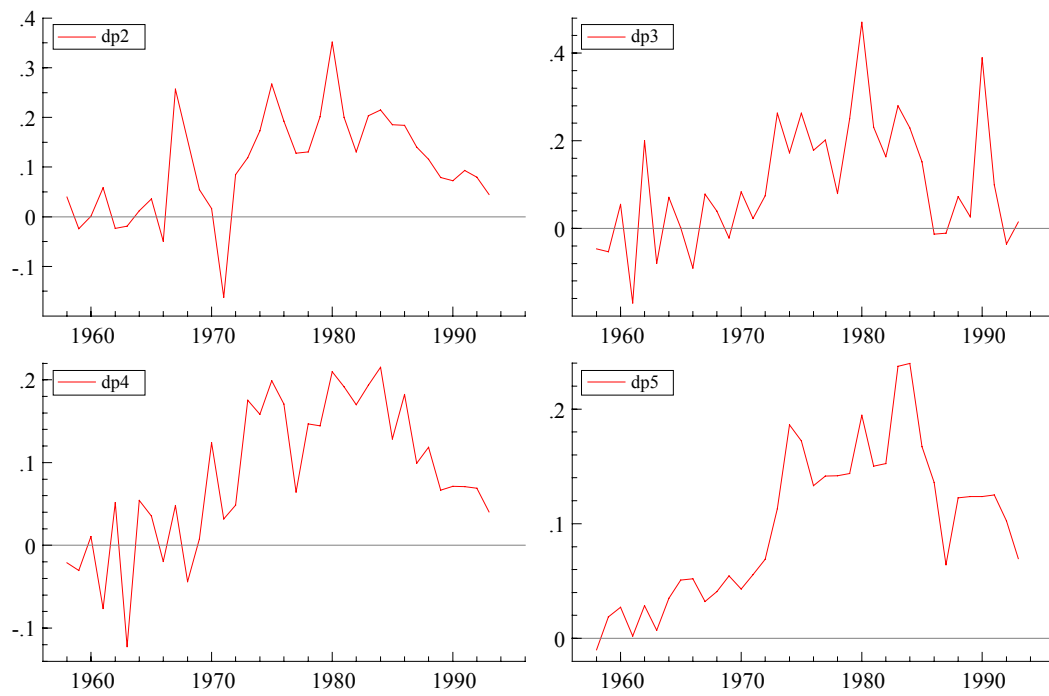
Graph No.1



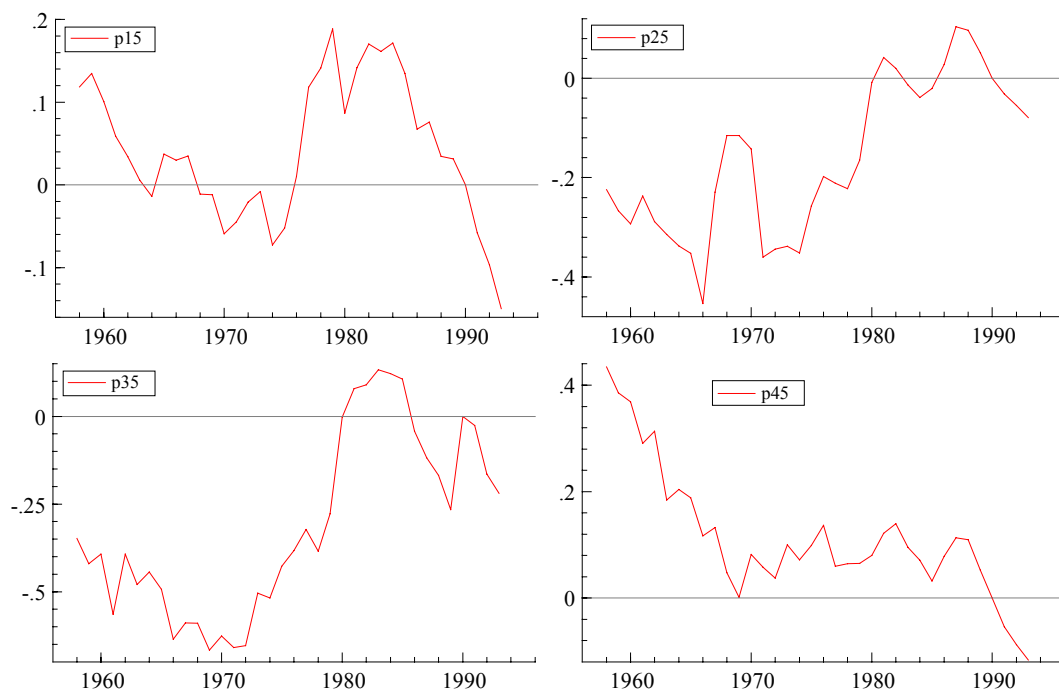
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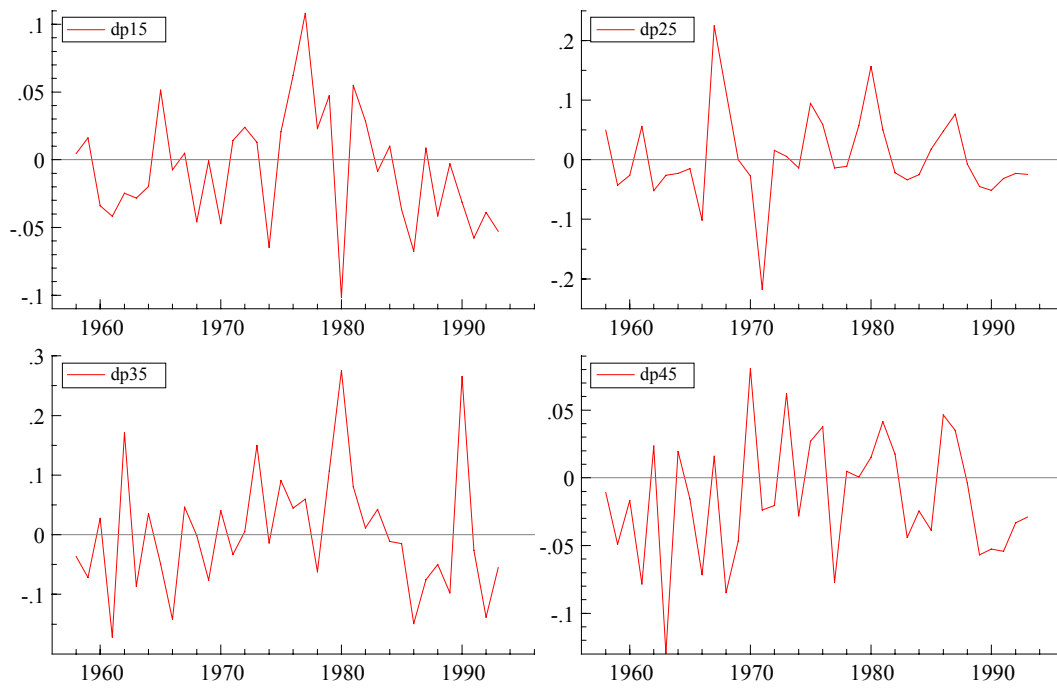
Graph No.3



Graph No.4

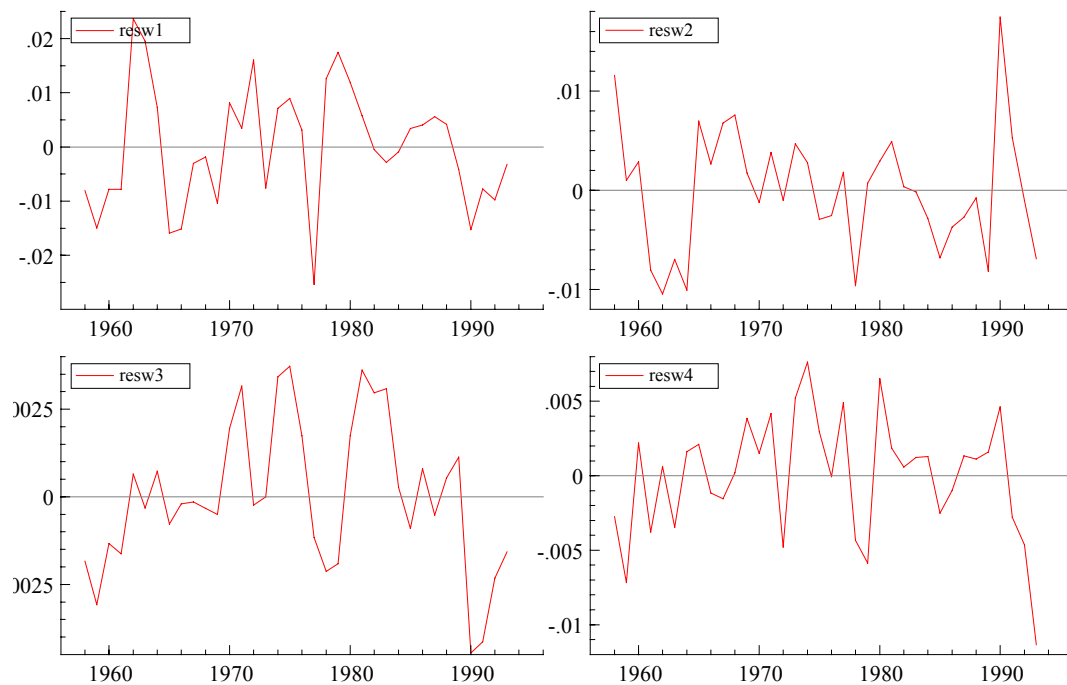


Graph No.5



Graph No.6

Residuals from cointegrating regressions
Model with homogeneity



Graph No.7
Residuals from cointegrating regressions
Model with homogeneity and symmetry

