EMU, Exchange Rate Volatility and Bid-Ask Spreads

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Comments are welcome

Summary:

This study deals with two issues related to the determination of exchange rate bid-ask spreads in the transition to EMU. First, we discuss how a credible announcement of conversion rates affects exchange rate volatility in the run up to the introduction of the Euro. Second, we discuss the theoretical relation that exists between exchange rate uncertainty and the bid-ask spread. The theory suggests that there is a positive association between exchange rate uncertainty and transaction costs and that we should observe a gradual reduction of exchange rate volatility in the transition to EMU. This theory implies a gradual shrinking of the bid-ask spread during the transition period. These conjectures are subject to empirical testing in the case of the exchange rate of the Portuguese escudo against the Deutsche Mark.

Key words: Exchange rates; Market Microstructure
JEL classification:

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1 Introduction

On 1 January 1999, the currencies of the European Union Member States participating in the third phase of EMU[1] will cease to exist and a new currency, the Euro, will be born. The provisions of the Maastricht Treaty and other resolutions relating to the introduction of the Euro imply that the conversion rates between the Euro and the currency of any of the participating Member States will be known only on the last trading day in December 1998.

However, as announced in 3 May 1998, the current ERM bilateral central rates will be used in determining the irrevocable conversion rates for the Euro[2]. Assuming the full credibility of this announcement the bilateral market exchange rates will converge (or remain close) to the conversion rates by (until) the end of 1998. Meanwhile, the EU central banks participating in the Euro area will stand ready to ensure this equality, if necessary, through the use of appropriate market techniques.

Although the introduction of the Euro will eliminate all exchange rate related conversion costs, it is not clear to what extent these costs will decline as exchange rates become increasingly fixed[3]. In the literature relating to the transition to EMU the determination and evolution of bid-ask spreads, the major component of overall exchange rate costs, has received little attention.

Our study deals with two issues related to the determination of bid-ask spreads in the transition to EMU. Firstly, we discuss how a credible announcement of conversion rates affects exchange rate volatility in the run up to the introduction of the Euro (section 2.1). Secondly, we discuss the theoretical relation that exists between exchange rate uncertainty and the bid-ask spread (section 2.2). The theory suggests that we should observe a gradual reduction of exchange rate volatility in the transition to EMU and also that there is a positive association between exchange rate uncertainty and transaction costs thus pointing to a gradual shrinking of the bid-ask spread during the transition period (section 2.3). These conjectures are subject to empirical testing in

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[1] Austria, Belgium, Finland, France, Germany, Ireland, Italy, Luxembourg, Netherlands, Portugal and Spain.

[2] On the advantages of pre-announced rates see Begg *et al.*(1997).

[3] This issue was first raised in Emerson *et al*. (1992).

the case of the exchange rate of the Portuguese escudo against the Deutsche Mark (section 3). At the end of the paper we present the major conclusions.

2 Theoretical framework

2.1 Exchange rate volatility and anticipations of fixed exchange rates

In its most general form the dynamics of the exchange rate is given by[4]:

$$s_t = f_t - \alpha \frac{E_t(ds_t)}{dt}, \quad (1)$$

where: s_t is the log of the spot exchange rate; f_t is the log of an index of macroeconomic fundamentals; $\alpha < 0$ is the semi-interest elasticity of the demand for money; and $E_t(.)$ is the conditional expectation operator.

Expectations are conditional on the information set available at time t, which includes the current value of fundamentals as well as any explicit or implicit restrictions the authorities have placed on the future evolution of fundamentals. The dynamics of the exchange rate when the authorities are committed to keep the exchange rate within a band (credible or not) has been thoroughly analysed in the literature, following Krugman (1988). The dynamics of the exchange rate when the authorities promise to fix the exchange-rate once it reaches a predetermined future level, i.e. a state-dependent regime switch, was studied by Froot and Obstfeld (1991). The analysis of a time-dependent switch from floating to managed exchange-rates was carried out by Ichikawa, Miller and Sutherland (1990)[5].

Suppose that fundamentals obey the following stochastic differential equation:

$$df_t = \sigma_t dW_t, \quad (2)$$

where: σ_t is the instantaneous standard deviation of df_t and W_t is a standardised Wiener process. Under floating the solution to (1) is given by:

$$s_t = f_t. \quad (3)$$

When the authorities announce fixed exchange rates after time T the solution after the announcement, $t \in [0, T]$, is given by:

$$s_t = f_t.\left(1 - e^{(T-t)/\alpha}\right) + s^*.e^{(T-t)/\alpha}, \quad (4)$$

[4] For a recent survey of the literature on exchange rate determination see Bertola (1994).
[5] See also the papers collected in Krugman and Miller (1992).

where: s^* is the level at which the exchange rate will be pegged; and $(T - t)$ is the time to the peg.

Equation (4) states that the exchange rate, after the announcement of the future peg, follows a trajectory that is a weighted average of its floating value and of the value of the peg. The weights are time varying and, as date T approaches, the first term vanishes ensuring that the exchange rate will not jump at the moment of the peg. At time T, $s_T = s^*$, which is the *smooth pasting* condition for solving (1) given (2) and the announcement of the peg.

The conditional expected value and variance of exchange rate changes after the announcement of the peg are given by:

$$\frac{E_t(ds_t)}{dt} = \frac{1}{\alpha}.(f_t - s^*).e^{(T-t)/\alpha}. \qquad (5)$$

$$\frac{Var_t(ds_t)}{dt} = \left[1 - e^{(T-t)/\alpha}\right]^2.\sigma^2. \qquad (6)$$

Equation (5) states that the conditional expected change in the log exchange rate is proportional to the distance between the current value of fundamentals and the future value of the peg. If $f_t > s^*$ the exchange rate is expected to appreciate ($E_t(ds_t)<0$ because $\alpha < 0$) and if $f_t < s^*$ the exchange rate is expected to depreciate. When $t = T$, that is, at the time of the peg, using (5) and (1), we obtain:

$$\begin{aligned} s_T &= f_T - \alpha\frac{E_T(ds_t)}{dt} \\ &= f_T - \alpha.\frac{1}{\alpha}.(f_T - s^*).e^{(T-T)/\alpha} \\ &= s^* \end{aligned}$$

and $E_T(ds_t) = 0$.

The conditional standard deviation of the changes in the log exchange-rate, derived from (6), is equal to $\sqrt{Var_t(ds_t)} = \left[1 - e^{(T-t)/\alpha}\right]\sigma.\sqrt{dt}$, that is, it is exponentially decaying in the time to the peg (T-t). When $t = T$, the time of the peg, $\sqrt{Var_T(ds_t)} = 0$.

Figure 1 shows four simulated trajectories of the log of a floating exchange rate which, starting all from the same value, *walk away* randomly thereafter as stochastic shocks drive the fundamentals.

Figure 2 shows the simulated paths of the exchange rate after the announcement of the future peg, for the same sample of fundamentals.

In the beginning the trajectories free and constrained floating are very similar. As time *T* approaches the dynamics of the exchange rate in Figure 2 becomes determined, essentially, by the terminal condition and all trajectories converge to the same value irrespective of the shocks that drive the fundamentals.

In Figure 3 we compare the conditional volatility of the free (thin lines) and constrained floating (thick lines) for each sample of fundamentals[6]. It is apparent that conditional volatility in the constrained floating simulations die out gradually as time *T* approaches, in sharp contrast with the free floating cases.

[6] Conditional volatility measured by the absolute change in the log of the exchange rate.

Figure 1 - Simulated trajectories of the floating log exchange rate

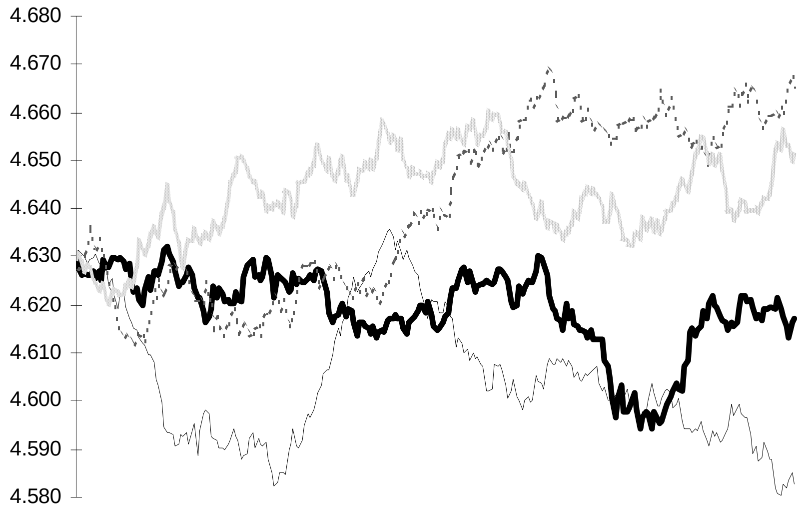


Figure 2 – Simulated trajectories after the announcement of the future peg

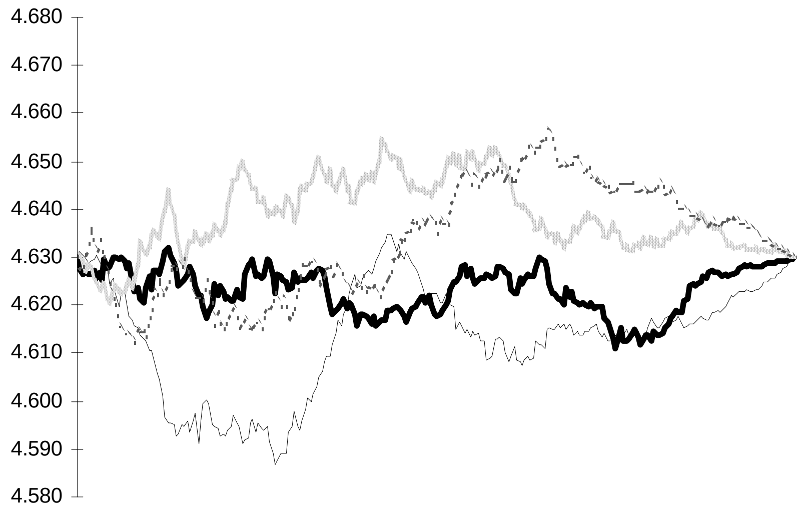
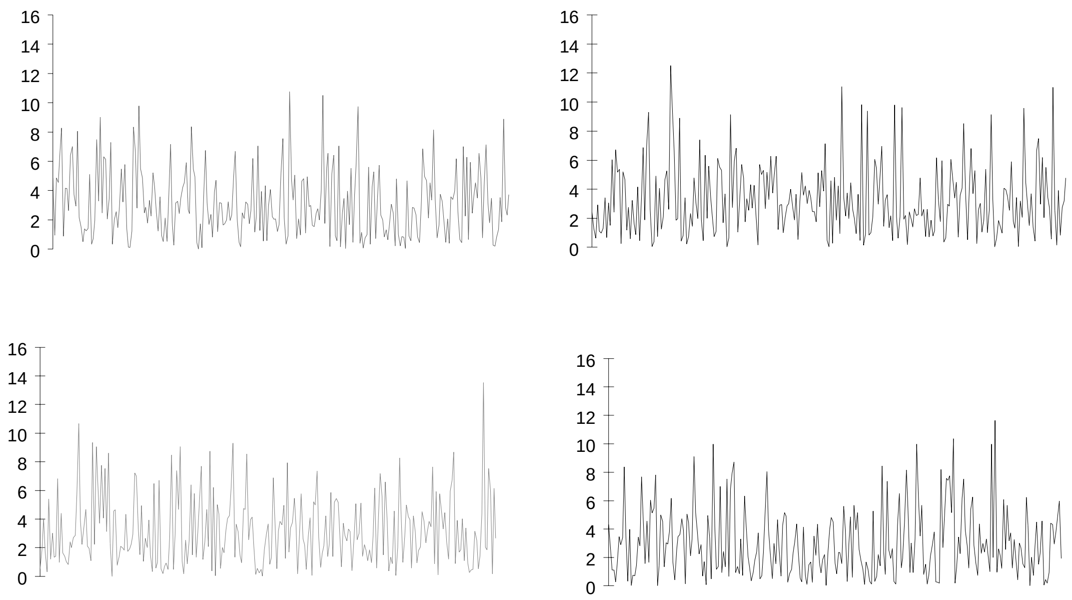
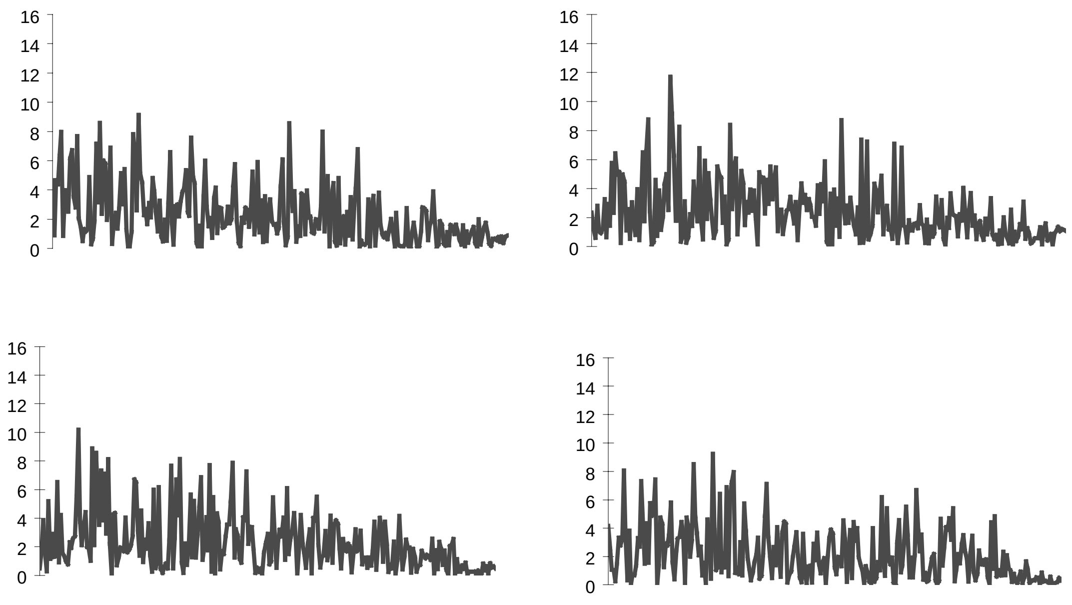


Figure 3 – Volatility of exchange rate changes

Free floating simulations



Constrained floating simulations



2.2 The bid-ask spread and exchange rate volatility

The theoretical relation between exchange rate uncertainty and transaction costs can be derived using a market microstructure model due to Copeland and Galai (1983) and applied to the foreign exchange market by Bollerslev and Melvin (1994).[7]

The basic assumption of the model is that the foreign exchange market is a decentralised anonymous market[8] where the market makers (dealers) face two kinds of traders: *liquidity traders* and *informed traders*. Liquidity traders' transactions are driven by the needs for buying and selling goods and services and do not speculate. They buy or sell currencies due to the financing needs of their normal business activity, and are willing to pay a "fee" in order to obtain immediacy. Informed traders take positions in the foreign exchange market based on information asymmetries regarding fundamentals underlying the determination of the spot exchange rate. Because these traders are endowed with special information, and have the option of not trading with the dealers, these will lose (on average) in trading with them. Therefore, assuming a competitive dealers' market, the market makers' bid-ask spread is chosen so as to equalise the difference between the expected gains obtained in dealing with liquidity traders and the expected loses arising from trading with informed traders. Formally, this may be expressed as:[9, 10]

$$\max_{K_A, K_B} \left\{ \begin{aligned} &(1-p_I)\left[p_{BL}(K_A - S_0) + p_{SL}(S_0 - K_B)\right] \\ &- p_I\left[\int_{K_A}^{+\infty}(S-K_A)q(S)dS + \int_0^{K_B}(K_B - S)q(S)dS\right] \end{aligned} \right\} = 0, \qquad (7)$$

where: S_0 - is the current "true" exchange rate as perceived by the dealer.[11] The dealer makes a short-lived commitment to buy a fixed quantity (one unit) at the bid price, K_B, or to sell at the ask price, K_A; S - is the "true" underlying exchange rate which follows a stochastic process which is known (*ex ante*) to all market participants. For example, if S follows a geometric Brownian motion $q(S)$ is the lognormal density; p_I - is the

[7] For a comprehensive survey of market microstructure theory see O'Hara (1995) and for extensions and applications to the foreign exchange market see Frankel *et al.* (1996).

[8] The foreign exchange market is anonymous in the sense that dealers do not know, *ex ante*, whether or not the other side of the transaction possesses superior information. The anonymity of the foreign exchange market is a sufficient condition for information to have private value. The role of the brokers is to maintain the anonymity in the market (Copeland and Galai (1983)).

[9] Assuming risk neutrality.

[10] Discounting is not considered because the quote is valid only for a small time period.

[11] It is the equilibrium price of foreign currency which would exist in a world without any demand for immediacy and where all market participants are equally well informed (Copeland and Galai (1983)).

probability (exogenously determined) that the next request for a quote is motivated by information trading; and p_L ($p_L = 1 - p_I$), is the probability that it is motivated by liquidity trading; p_L is decomposed in two parts: p_{TL} and p_{NL} the probabilities of trading and non-trading, respectively, given that the trader is a liquidity trader; p_{BL} - is the probability of buying given that the trade is liquidity motivated and p_{SL} is the probability of selling given that the trade is liquidity motivated ($p_{TL}=p_{BL}+p_{SL}$).

Dealers and liquidity traders know *q(S)* but are uninformed about the *realisations* of *q(S)*. These realisations are conveyed to the market by informed traders. The trader arrival to the market is a stochastic process independent of the price change process and the quote interval is of fixed lenght. Dealers' quotes are limited to the first trader to call.

The market makers' gains from trading with liquidity traders are $K_A - S_0$ or $S_0 - K_B$ if the trades are, respectively, a sell or a buy. Thus, the first term in (7) is the market makers' expected gain from trading with liquidity traders:

$$(1-p_I)\left[p_{BL}(K_A - S_0) + p_{SL}(S_0 - K_B)\right],$$

and the second term is the expected loss to an informed trader:

$$p_I\left[\int_{K_A}^{+\infty}(S-K_A)q(S)dS + \int_0^{K_B}(K_B - S)q(S)dS\right].$$

It is convenient to look at the market makers' problem from the perspective of a (short) *strangle* option. A strangle is a combination of a put and a call on the same underlying asset with two different exercise prices. In fact, the dealer gives a prospective trader a call option to buy at the asking price, $K_A > S_0$, and also a put option to sell at the bid price, $K_B < S_0$. Both options are out-of-the-money, from the perspective of the dealers and liquidity traders. A liquidity trader will be willing to suffer a certain loss by exercising the out-of-the-money option. His loss is the price of immediacy. The informed trader will be trading for a gain either if $S > K_A$ or if $S < K_B$. Therefore (7) can be rewritten as:

$$\max_{K_A,K_B}\left\{\begin{matrix}(1-p_I)\left[p_{BL}(K_A - S_0) + p_{SL}(S_0 - K_B)\right] \\ = \\ p_I\left[C(K_A) + P(K_B)\right]\end{matrix}\right\}, \quad (8)$$

where $C(K_A)$ and $P(K_B)$ are the call and the put premium, respectively.

The values of the call and the put increase with the volatility of the exchange rate and therefore it follows immediately that the bid-ask spread is affected by changes in the foreign exchange risk. However, the sign of the effect is not clear from the direct examination of (8).

The first order conditions for optimality are given by:

$$\begin{cases}(1-p_I)p_{BL} = p_I \dfrac{\partial C(K_A)}{\partial K_A} \\ (1-p_I)p_{SL} = -p_I \dfrac{\partial P(K_B)}{\partial K_B}\end{cases} \Rightarrow -\frac{\partial C(K_A)}{\partial K_A} p_{SL} = \frac{\partial P(K_B)}{\partial K_B} p_{BL}, \tag{9}$$

and noting that:

$$\begin{cases}\dfrac{\partial C(K_A)}{\partial K_A} + 1 = \mathrm{P}(S \le K_A) \\ \dfrac{\partial P(K_B)}{\partial K_B} = \mathrm{P}(S \le K_B)\end{cases}, \tag{10}$$

the first order conditions (9) can be rewritten as:

$$\begin{gathered}\left(1-\mathrm{P}(S \le K_A)\right).\,p_{SL} = \mathrm{P}(S \le K_B).\,p_{BL} \\ \Leftrightarrow \\ \mathrm{P}(S \ge K_A).\,p_{SL} = \mathrm{P}(S \le K_B).\,p_{BL}\end{gathered} \;. \tag{11}$$

Equation (11) states that the marker maker equalises the (expected) marginal cost to the (expected) marginal benefit of dealing with liquidity traders. The left hand side of (11), the product of the probability of selling, given that a trader is a liquidity trader, p_{SL}, by the probability that the next realisation of S lies above the ask price, $P(S\exists K_A)$, is the expected marginal cost. The right hand side of (11), the product of the probability of buying, given that a trader is a liquidity trader, p_{BL}, by the probability that the next realisation of S lies below the bid price, $P(S\# K_B)$, is the expected marginal benefit. Defining the spread as $\theta = K_A/S_0 \geq 1$, and considering that it is centred on S_0 we can rewrite equation (11) as:

$$\mathrm{P}(S \ge \theta S_0).p_{SL} = \mathrm{P}(S \le (2-\theta).S_0).p_{BL}, \tag{12}$$

and dealers' problem is to find θ that satisfies (12).

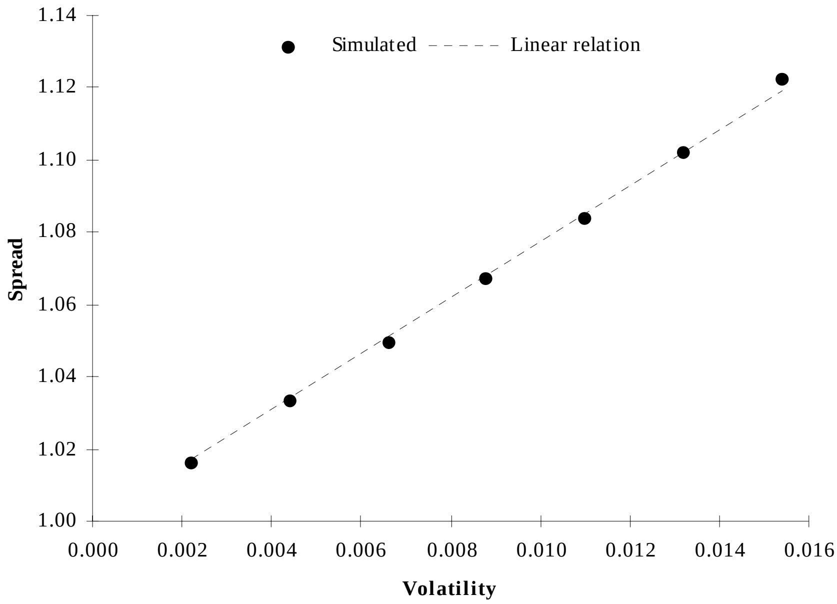
This problem does not have, in general, a closed form solution. If S follows, for example, a geometric Brownian motion, $q(S)$ is the lognormal density. In this case it is easy to evaluate, by simulation, how the bid-ask spread is affected by changes in exchange rate volatility. Suppose that p_{SL} = p_{BL}. In this case the market makers' problem is to find θ that satisfies:

$$\Lambda(\theta S_0) = \frac{1-\Lambda(2S_0)}{2} \tag{13}$$

where $\Lambda(.)$ is the cumulative lognormal density function.

In Figure 4 we plot several values of the spread (θ), obtained by solving equation (13) for different values of volatility[12], with $S_0 = 102.505$. The main conclusion is that the bid-ask spread increases (almost) linearly with the exchange rate risk (volatility).

Figure 4 - Exchange rate volatility and the bid-ask spread



2.3 EMU, exchange rate volatility and the bid-ask spread

Consider the last trading minute on the foreign exchange markets in 1998 and suppose that S_0 is an announced bilateral conversion rate. The model presented in section 2.2 implies that the bid-ask spread for the bilateral exchange rates among the EU Member States participating in the Euro area will be set at $\theta=1$ (i.e. no spread). This result comes from two different although related factors affecting the behaviour of market makers: the elimination of foreign exchange risk and the absence of informed traders.

Using the options analogy the elimination of volatility (foreign exchange risk) makes the call and the put worthless in problem (8) and so the *strangle* has zero value.

[12] Volatility calculated for a time interval between transactions of 10 minutes. For example, a standard deviation of 0.2 in annual terms is equivalent to a 10 min. standard deviation of ≈ 0.0022 (82,500 trading minutes per year).

For non-trivial values of p_I, p_{BL} and p_{SL}, the solution to (8) implies K_A=K_B=S_0. Or, from another angle, θ=1 is the only viable solution to (12) when S=S_0 with certainty:

$$1 = \mathrm{P}(S \geq S_0) = \mathrm{P}(S \leq S_0) \Rightarrow \mathrm{P}(S \geq \theta S_0) = \mathrm{P}(S \leq (2-\theta).S_0) \wedge \theta = 1 .$$

As mentioned in section 2.2, informed traders take positions in the foreign exchange market based on information asymmetries regarding fundamentals underlying the determination of the spot exchange rate. Assuming the credibility of announced bilateral conversion rates there will be no information asymmetries hanging over in the last trading minute of 1998. Therefore p_I = 0 (certain absence of informed traders) and for non-trivial values of p_{BL} and p_{SL}, the solution to (8) implies, again, K_A=K_B=S_0.

Now consider the transition period to EMU. According to the analysis in section 2.1, in the run up to 1999 exchange rate volatility will be decaying which implies that the bid-ask spreads of bilateral exchange rates among the countries participating in the Euro area will also be shrinking during the transition.

This idea and the positive relation between the bid-ask spread and volatility are empirically testable hypothesis, to which we turn next.

3 Empirical evidence

In section 2 we derived the proposition that in the transition to EMU the bid-ask spreads of the bilateral exchange rates of the currencies participating in the Euro area will be decreasing and eventually eliminated (at least) on the last trading day in December 1998.

A definitive test of the theory cannot be performed yet. However the empirical evidence presented in this section suggests that the data does not reject the major implications of the theory.

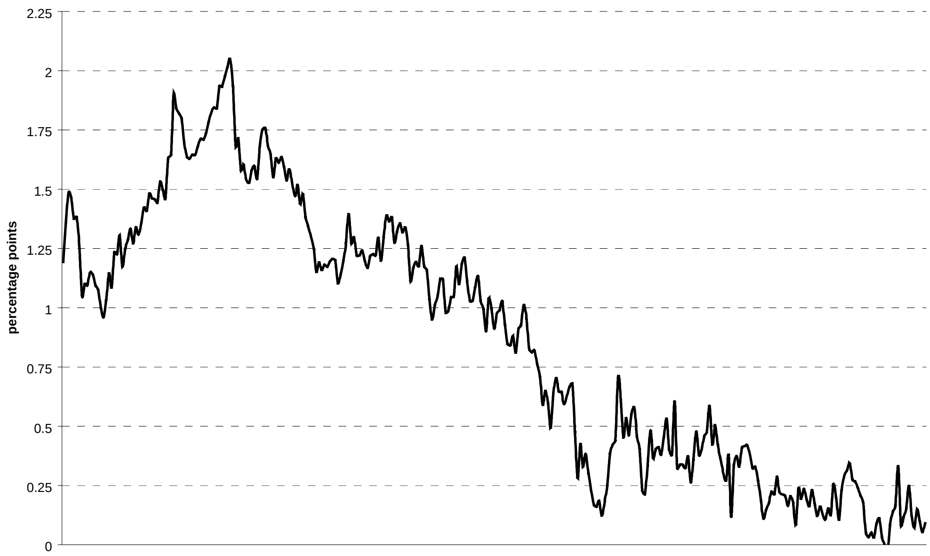
3.1 Credibility of EMU participation

In section 2 it was assumed the full credibility of the future peg. A necessary condition for the credibility of EMU participation is the convergence of forward overnight interest rates of the participating currencies, with settlement date in *T*=1/1/1999. In fact, using the discrete-time counterpart of equation (5) we have:

$$Credibility \Rightarrow E_T\,(s_{T+1} - s_T) \approx i_{T,T+1} - i^*{}_{T,T+1} = 0\,, \qquad (14)$$

where $i_{T,T+1}$ and $i^*{}_{T,T+1}$ are, respectively, the forward overnight interest rates of the Portuguese Escudo and the Deutsche Mark[13].

Figure 5 – Forward overnight interest-rate differential PTE-DEM. Settlement date: 01/01/1999
Jan.1997-Mar.1998



[13] Assuming that EMU goes ahead timely and that the Deutsche Mark is a sure participant.

Figure 5 shows the daily evolution over the period January 1997-March 1998, of this measure of the credibility of EMU participation for the Portuguese Escudo. It is apparent that full credibility can be rejected and that it is only recently that the forward interest differential shows greater expected convergence. It is also noteworthy the gradual improvement in the credibility of EMU participation.

This suggests that both the conditional volatility and the bid-ask spread of the PTE/DEM should have declined, in particular towards the end of the sample period.

3.2 Exchange rate volatility

The implied volatility from over the counter (OTC) foreign exchange (FX) options[14] seems to be the most appropriate way of measuring conditional expected volatility for the purposes of our study. In contrast with volatility estimated using GARCH models, the implied volatility should reflect very quickly any changes in market expectations of the future course of foreign exchange risk.

Figure 6 – Implied volatility from OTC PTE/DEM FX options
Jan.1997-Mar.1998

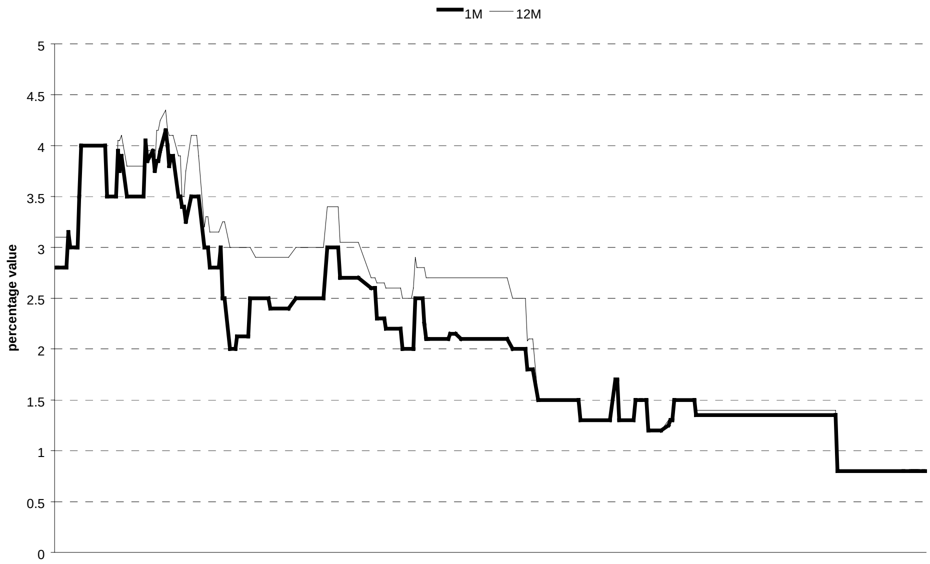


Figure 6 shows the daily evolution, over the period January 1997-March 1998, of this measure of volatility for 1 month and 12 months OTC FX contracts for the PTE/DEM. Two facts are noteworthy. Firstly, the gradual decline in implied volatility, accompanying very closely the improvement in the credibility of EMU participation of the Portuguese Escudo, noted in section 3.1. Secondly, the evolution of the term

[14] At-the-money forward options quoted by BPA.

structure of volatility[15], changing from positively sloped to flat towards the end of the sample period. This confirms our conjecture about the evolution of the conditional volatility and reinforces the idea that the bid-ask spread of the PTE/DEM should have also declined.

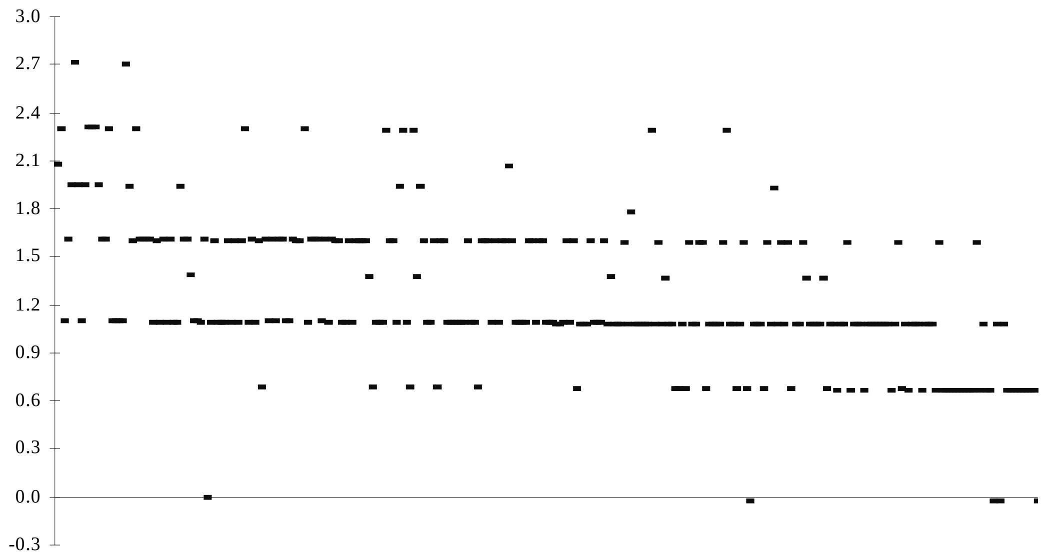
3.3 The bid-ask spread

In this section we review the recent evolution of the bid-ask spread of the Portuguese Escudo against the Deutsche Mark. Firstly, we take a look at the empirical evidence in order to see whether the spread has been declining, as the theory suggests. Secondly, using an appropriate econometric methodology we test the hypothesis that the spread is positively related to the volatility of the exchange rate. Thirdly, we carry out a simulation exercise illustrating a possible evolution of the spread in the run up to a fixed exchange rate regime.

3.3.1 Descriptive statistics

Let us first look at the evidence on the evolution of the bid-ask spread. We use daily bid-ask quotations[16] for the PTE/DEM (close, spot rate values) extracted from *Reuters*. The sample period goes from 2 January 1997 to 18 March 1998 with a total of 289 observations.

Figure 7 – (Log) Bid-ask spread on the DEM/PTE



[15] Measured by the difference between the 12 months over the 1 month implied volatility.
[16] We use quotations because, unfortunately, transaction data is not available.

Figure 7 shows the daily evolution of the spread. Two facts are noteworthy. Firstly, the observed spread, k_t [17],tends to assume only a limited number of discrete values (a_1, a_2, ..., a_J). Secondly, the higher frequency of lower values of the spread towards the end of the sample period. Thus it is apparent that the spread has been shrinking.

The histogram of relative frequencies shown in Figure 8 and the values in Table 1 confirm the nature of the observed spread.

Figure 8 - Histogram of relative frequencies

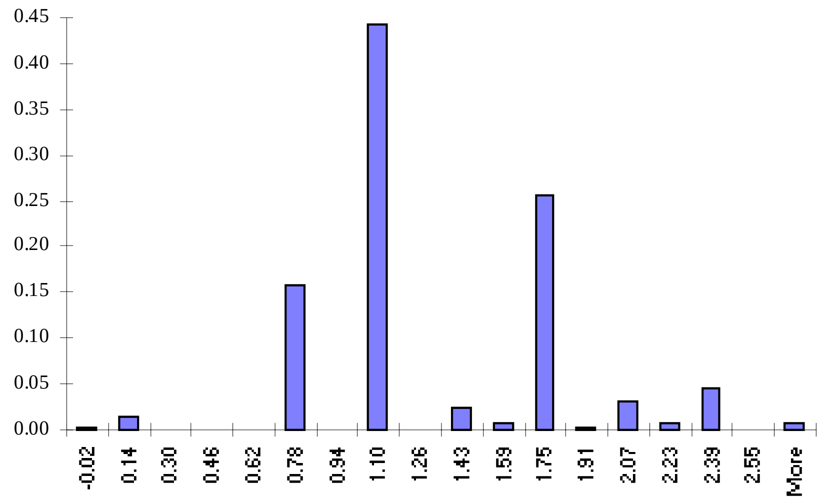


Table 1 - Relative distribution of the (log) spread

Up limit	**-0.024**	**0.137**	**0.298**	**0.459**	**0.620**	**0.781**	**0.942**	**1.103**	**1.264**
Frequency	0.35%	1.38%	0.00%	0.00%	0.00%	15.92%	0.00%	44.29%	0.00%
Up limit	**1.425**	**1.586**	**1.747**	**1.908**	**2.069**	**2.230**	**2.391**	**2.552**	**More**
Frequency	2.42%	0.69%	25.61%	0.35%	3.11%	0.69%	4.50%	0.00%	0.69%

3.3.2 Ordered probit analysis

As the empirical evidence shows the observed spread lacks continuity. Thus in testing the hypothesis of the existence of a positive relation between the spread and exchange rate volatility an appropriate econometric methodology has to be chosen. To deal with this problem Hausman *et al*. (1992) and Bollerslev and Melvin (1994) propose using ordered probit analysis. Here we follow their methodology briefly described in the Annex.

Consider the unobservable continuous random variable, k_t^*, defined as:

$$k_t^* = \delta' X_t + \varepsilon_{k,t}. \qquad (15)$$

[17] k_t is the *log* (*10 000*.($K_{A,t} - K_{B,t}$) /S_t), where $K_{A,t}$, $K_{B,t}$ and S_t have the meaning defined in section 2.

The vector X_t denotes a set of predetermined variables that affect the conditional mean of k_t^*; δ is a vector of parameters and $\varepsilon_{k,t}$ is conditionally normally distributed with mean zero and variance, $\sigma^2_{k,t}$,

$$\varepsilon_{k,t}.|I_{t-1} \sim N(0, \sigma^2_{k,t}) \qquad (16)$$

The ordered probit model relates the observed spreads to k_t^* via

$$k_t = a_j, \text{ iff } k_t^* \in A_j, \ j = 1, 2, \ldots, J, \qquad (17)$$

where the A_j's form an ordered partition of the real line into J disjoint intervals. The probability that the spread takes on the value a_j is equal to the probability that k_t^* falls into the appropriate partition, A_j.

A test for heteroskedasticity is carried out by testing the significance of γ' in:

$$\sigma^2_{k,t} = [\exp(\gamma' X_t)]^2 \qquad (18)$$

The empirical analysis is based on a classification of the spread into four different categories. The corresponding intervals for the unobservable latent variable k_t^* are defined by:

$$\begin{aligned} A_1 &\equiv]-\infty, \mu_1] \\ A_2 &\equiv]\mu_1, \mu_2] \\ A_3 &\equiv]\mu_2, \mu_3] \\ A_4 &\equiv]\mu_3, +\infty[\end{aligned} \qquad (19)$$

The partition parameters, μ_i, are estimated jointly with the other parameters of the model.

With the model defined by equations (15) - (19) we can estimate the probability of a particular spread being observed, as a function of the predetermined variables, X_t. To test the hypothesis that the spread is significantly affected by exchange rate volatility, the implied volatility is included as one of the elements in X_t.

Given the partitions boundaries determined by the data, if a higher conditional mean $\delta' X_t$ is caused by a larger conditional variance of the exchange rate, and this raises the probability of observing a higher spread, we will infer that the hypothesised theoretical link is supported by the empirical analysis.

The model was estimated by Maximum Likelihood[18], considering first as explanatory variable only exchange rate volatility (and a constant). The results obtained are presented in Table 2.

Table 2 - Ordered probit model (PTE/DEM) [Initial specification]

$$LSPTE^* = \delta_0 + \delta_1 LVPTE + \varepsilon_{k,t}$$

$$\sigma^2_{k,t} = [exp(\gamma.LVPTE)]^2$$

Coefficient	Estimate	Standard Error	t-ratio	Probability
δ_0	-0.7602	0.2405	-3.160	[.001]
δ_1	2.2469	0.4637	4.845	[.000]
γ	0.1192	0.2059	0.579	[.562]

289 observations $\xi_1 = 4.15$ Log Likelihood = -316.1965

LSPTE is the log of the spread; *LVPTE* is the log of exchange rate volatility.

The *score statistic* ($\xi_1 \sim \chi^2_{(1)}$) suggests that the residuals are serially correlated due to the omission of (at least) the first lag of the dependent variable (see the Annex for technical details). The inclusion of some lags of the (log) spread in the ordered probit model should capture the observed daily persistence in the spread.

The results obtained considering the inclusion of lagged values of the (log) spread are presented in Table 3.

[18] The estimates were obtained using the LIMDEP computer program and confirmed with TSP/V.4.4.

Table 3 - Ordered probit model (PTE/DEM) [Final specification]

$$LSPTE^* = \delta_0 + \delta_1 LVPTE + \delta_2 LSPTE(-1) + \delta_3 LSPTE(-2) + \varepsilon_{k,t}$$

Coefficient	Estimate	Standard Error	t-ratio	Probability
δ_0	-1.1356	0.2485	-4.568	[.000]
δ_1	1.5339	0.2476	6.195	[.000]
δ_2	0.2125	0.1646	1.291	[.196]
δ_3	0.5131	0.1606	3.195	[.001]
μ_1	1.5334	0.1182	12.963	[.000]
μ_2	2.7400	0.1588	17.246	[.000]

289 observations $\xi_3 = 0.1815$ Log Likelihood = -310.3972 $\chi^2_{(1)}$ (HET) = -0.508

LSPTE is the log of the spread; *LVPTE* is the log of exchange rate volatility.

Now the *score statistic* ($\xi_3 \sim \chi^2_{(1)}$) suggests that the residuals are not serially correlated and the $\chi^2_{(1)}$ test suggests that the residuals are homoskedastic.

The magnitude and the statistical significance of volatility in the determination of the spread (δ_1) is the most important conclusion of the analysis. The magnitude and significance of lagged spreads supports the idea of persistence in the evolution of the spread. However, it should be noted that the spread exhibits an oscillatory pattern[19]. Although an interesting issue in itself an explanation for this pattern is beyond the scope of our paper.

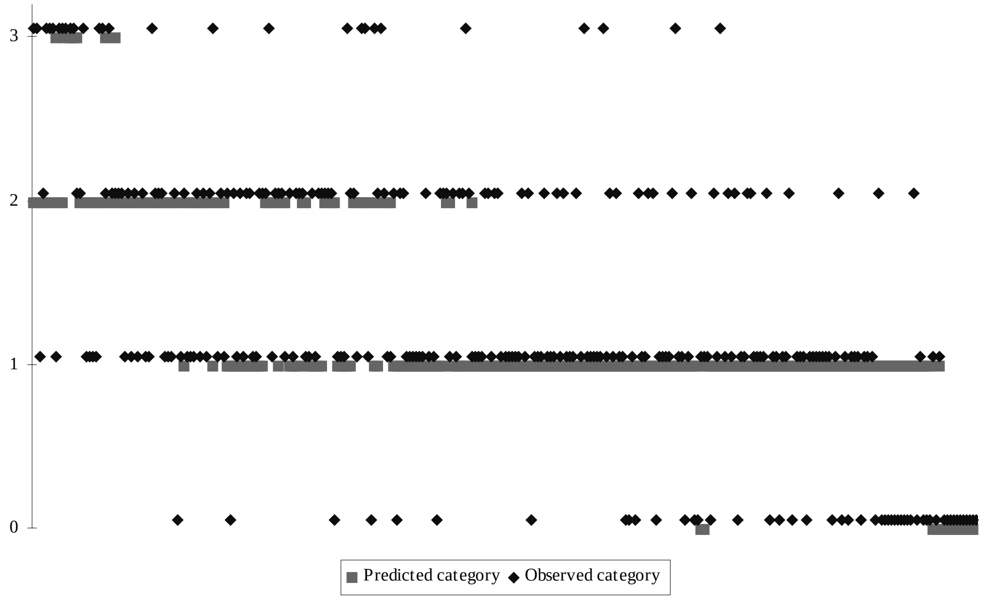
In Table 4 and in Figure 9 we compare the actual values of the spread with the predicted values, using the ordered probit model. Despite some differences, the model captures reasonably well the observed trend in the spread.

[19] In over 60% of the sample changes in the spread alternate in sign (i.e. follow the pattern + - + -). This pattern is less evident as we approach the end of the sample. A possible explanation for this behaviour could be related to the evolution of information asymmetries that became, perhaps, less marked towards the end of the sample.

Table 4 - Predicted *versus* actual values

Actual	Predicted				TOTAL
	0	**1**	**2**	**3**	
0	12	36	3	0	**51**
1	3	94	30	1	**128**
2	0	54	26	3	**83**
3	0	8	16	3	**27**
TOTAL	**15**	**192**	**75**	**7**	**289**

Figure 9 - Observed and predicted categories



The marginal effects are presented in Table 5. These effects represent the change in the probabilities of the spread falling into each of the four different categories resulting from a unit increase in the mean of the explanatory variables.

Table 5 - Marginal Effects for Ordered Probit

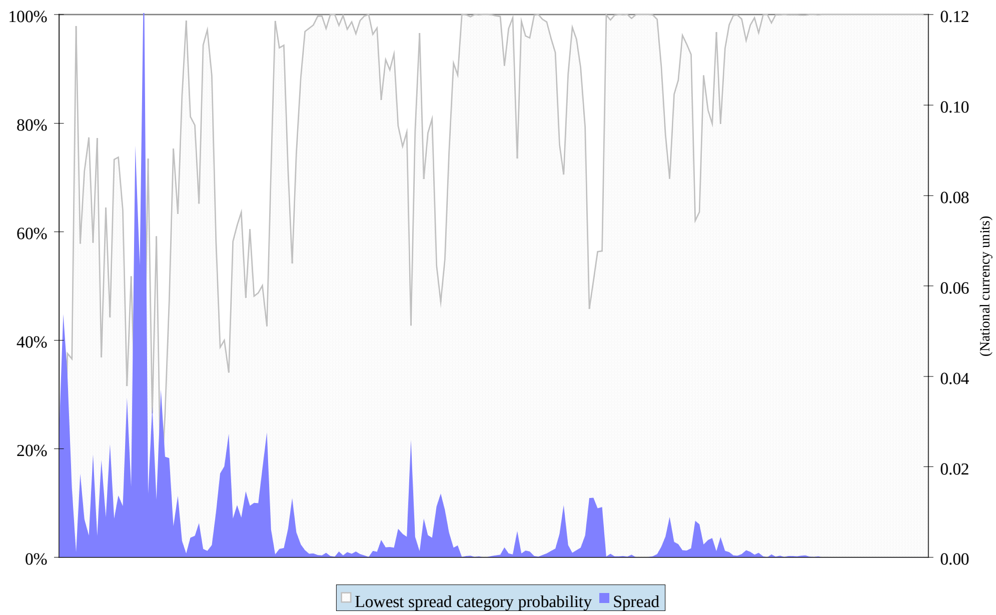
Variable	LSPTE = 0	LSPTE = 1	LSPTE = 2	LSPTE = 3
LVPTE	**-0.3122**	-0.2586	**0.3951**	0.1757
LSPTE(-1)	-0.0433	-0.0358	0.0548	0.0243
LSPTE(-2)	-0.1044	-0.0865	0.1322	0.0588

According to the figures shown in Table 5 an increase of one unit in the (log) of volatility increases the probability of the spread falling into the third category by nearly 40%, whilst decreasing by approximately 31% the probability of the spread falling into the lowest category. This confirms the relevance of changes in the perception of exchange rate risk (as captured by implicit volatility) in the determination of transaction costs.

3.3.3 Simulation exercise

Consider now for illustrative purposes the average volatility taken from the simulations carried out in section 2.1. Starting from a spread in the upper category we simulate a possible evolution of the spread using the estimated parameters from the ordered probit model.[20] We calculate also the probability associated with the category of minimum spread.

Figure 10 - Probability associated with the lowest spread category and spread



The simulation is shown in Figure 10. The spread declines over time, with an increasing probability associated with lower values of the spread. Thus in a scenario of full credibility we see, after the announcement of the future peg, volatility declining and the spread shrinking and, eventually, disappear.

[20] The simulations were carried out using values that are compatible with the PTE/DEM quotations, namely setting as initial and terminal values for the simulations s* = log (102.505).

4 Conclusions

In this study we present a theory that establishes two main propositions. First that there is a positive association between exchange rate uncertainty and transaction costs. Second that we should observe a gradual reduction of exchange rate volatility in the transition to EMU. These propositions imply that we should observe a gradual shrinking of the bid-ask spread during the transition period for the bilateral exchange rates of the participating countries.

The main theoretical propositions are subject to a first empirical testing in the case of the exchange rate of the Portuguese escudo against the Deutsche Mark. In particular, using an ordered probit model for the spread, we find a statistically significant effect of volatility in the determination of the spread. This provides some (early) support for the theory.

In addition to the conventional credibility indicators, based on forward interest rate differentials, our paper suggests also some indicators that should be monitored in the transition to EMU. These indicators are, namely, the bid-ask spread against the DEM, the evolution of implied volatility, extracted from OTC FX options, and the probability associated with the lowest spread category.

Annex:

Econometric methodology

Consider the unobservable continuous random variable, k_t^*, defined as:

$$k_t^* = \delta' X_t + \varepsilon_{k,t}. \qquad \text{A 1}$$

The vector X_t denotes a set of predetermined variables that affect the conditional mean of k_t^*, δ is a vector of parameters, and $\varepsilon_{k,t}$ is conditionally normally distributed with mean zero and variance, $\sigma^2_{k,t}$,

$$\varepsilon_{k,t}. | I_{t-1} \sim N(0, \sigma^2_{k,t}). \qquad \text{A 2}$$

The ordered probit model relates the observed spreads to k_t^* via

$$k_t = a_j, \text{ iff } k_t^* \in A_j, \; j = 1, 2, ..., J, \qquad \text{A 3}$$

where the A_j's form an ordered partition of the real line into J disjoint intervals. The probability that the spread takes on the value a_j is equal to the probability that k_t^* falls into the appropriate partition, A_j.

A test for heteroskedasticity is carried out by testing the significance of γ' in:

$$\sigma^2_{k,t} = [\exp(\gamma' X_t)]^2. \qquad \text{A 4}$$

The empirical analysis is based on a classification of the spread into four different categories. The corresponding intervals for the unobservable latent variable k_t^* are defined by:

$$\begin{aligned} A_1 &\equiv]-\infty, \mu_1] \\ A_2 &\equiv]\mu_1, \mu_2] \\ A_3 &\equiv]\mu_2, \mu_3] \\ A_4 &\equiv]\mu_3, +\infty[\end{aligned} \qquad \text{A 5}$$

The partition parameters, μ_i, are estimated jointly with the other parameters of the model.

The ordered probit model defined by equations A 1 - A 5 allow us to estimate the probability of a particular spread being observed as a function of the predetermined variables, X_t. To test the hypothesis that the spread is significantly affected by exchange rate volatility, the implied volatility is included as one of the elements in X_t.

Given the partitions boundaries determined by the data, if a higher conditional mean $\delta' X_t$ is caused by a larger conditional variance of the exchange rate, and this raises the probability of observing a higher spread, we will infer that the hypothesised theoretical link is supported by the empirical analysis.

The conditional distribution of observed spreads k_t, conditioned on X_t, is determined by the partition boundaries and the particular distribution of $\varepsilon_{k,t}$. For Gaussian ε's the conditional distribution is:

$$P(k_t = a_i | X_t) = P(\delta' X_t + \varepsilon_{k,t} \in A_i | X_t),$$

$$= \begin{cases} P(\delta' X_t + \varepsilon_{k,t} \le \mu_1 | X_t) \ldots i = 1 \\ P(\mu_{i-1} \le \delta' X_t + \varepsilon_{k,t} \le \mu_i | X_t) \ldots 1 < i < 3 \\ P(\mu_3 < \delta' X_t + \varepsilon_{k,t} | X_t) \ldots i = 3 \end{cases},$$

$$= \begin{cases} \Phi(\frac{\mu_1 - \delta' X_t}{\sigma_{k,t}}) \ldots i = 1 \\ \Phi(\frac{\mu_i - \delta' X_t}{\sigma_{k,t}}) - \Phi(\frac{\mu_{i-1} - \delta' X_t}{\sigma_{k,t}}) \ldots 1 < i < 3 \\ 1 - \Phi(\frac{\mu_3 - \delta' X_t}{\sigma_{k,t}}) \ldots i = 3 \end{cases}, \qquad \text{A 6}$$

where Φ is the standard normal cumulative distribution function.

The likelihood function is given by:

$$L(k|X) = \sum_{t=1}^{n} \left\{ \begin{array}{l} y_1 . \ln \Phi(\frac{\mu_1 - \delta' X_t}{\sigma_{k,t}}) + \\ + \sum_{i=2}^{3} y_i . \ln \left[\Phi(\frac{\mu_i - \delta' X_t}{\sigma_{k,t}}) - \Phi(\frac{\mu_{i-1} - \delta' X_t}{\sigma_{k,t}}) \right] + \\ + y_4 . \ln \left[1 - \Phi(\frac{\mu_3 - \delta' X_t}{\sigma_{k,t}}) \right] \end{array} \right\}, \qquad \text{A 7}$$

where: y_i is an indicator variable which takes on the value 1 if the realisation of the m^{th} observation of k_t is in the i^{th} satate a_i and zero otherwise.

The ordered probit model allows us to estimate the probability of a particular spread being observed as a function of the predetermined variables, X_t. However, the estimated coefficients do not necessarily coincide with the marginal effects.

The marginal effects are given by:

$$\frac{\partial Prob(k_t = a_1)}{\partial X_t} = -\frac{1}{\sigma_{k,t}}\phi(\frac{\mu_1 - \delta' X_t}{\sigma_{k,t}})\delta$$

$$\frac{\partial Prob(k_t = a_2)}{\partial X_t} = \frac{1}{\sigma_{k,t}}\left[\phi(\frac{\mu_2 - \delta' X_t}{\sigma_{k,t}}) - \phi(\frac{\mu_1 - \delta' X_t}{\sigma_{k,t}})\right]\delta \qquad \text{A 8}$$

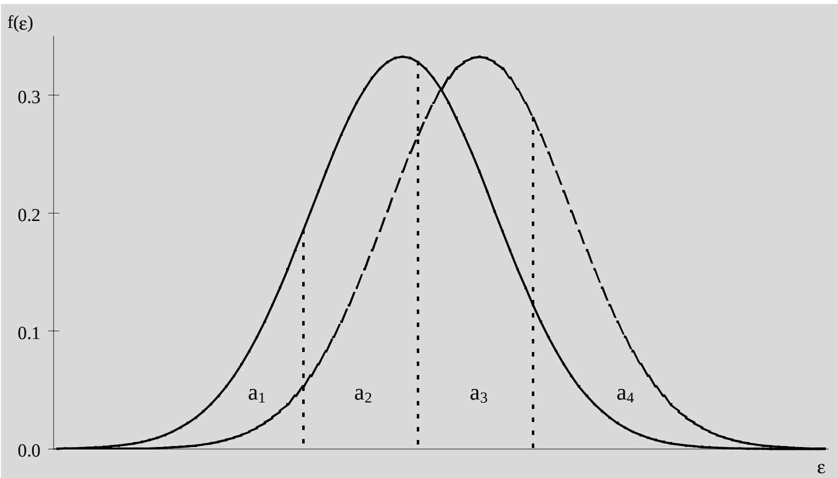
$$\ldots$$

$$\frac{\partial Prob(k_t = a_4)}{\partial X_t} = \frac{1}{\sigma_{k,t}}\phi(\frac{\mu_3 - \delta' X_t}{\sigma_{k,t}})\delta ,$$

where ϕ is the standard normal density function.

Figure A 1 illustrates the effect of a change in *X* on the probabilities. An increase in one of the *X*'s while holding δ and μ_i constant shifts the density function to the right. This means that the $Prob(k_t = a_1)$ must decline for this *X* if the corresponding element in δ is positive. By a similar logic, the change in the $Prob(k_t=a_4)$ will have the same sign as the relevant element in δ. These effects are unambiguous. What happens in the middle categories will depend on the densities, being an ambiguous outcome.

Figure A 1 - Effects of a change in X on predicted probabilities



Testing for autocorrelation is carried out as suggested by Hausman *et al.* (1992). In the case of the ordered probit, it is not possible to calculate the residuals directly since the latent dependent variable, k_t^*, is not observable. Therefore, we cannot compute $k_t^* - \hat{\delta}' X_t$. However, we do have an estimate of the conditional distribution of k_t^*, conditioned on X_t, based on the ordered probit specification and the maximum likelihood parameter estimates. From this we can obtain an estimate of the conditional distribution of $\varepsilon_{k,t}$, from which we can construct generalised residuals $\hat{\varepsilon}_{k,t}$, along the lines suggested by Gourieroux *et al*. (1985).

$$\varepsilon_t \equiv E\left[\varepsilon_t \mid k_t, X_t; \hat{\theta}_{ml}\right] \qquad \text{A 9}$$

where: $\hat{\theta}_{ml}$ is the maximum likelihood estimator of the unknown parameter vector containing $\hat{\mu}, \hat{\gamma}$ and $\hat{\delta}$. In the case of the ordered probit, if k_t is in the j^{th} partition, i.e., $k_t = a_j$, then the generalised residual $\hat{\varepsilon}_{k,t}$ may be expressed explicitly using the moments of the truncated normal distribution as:

$$\hat{\varepsilon}_t = E\left[\varepsilon_t \mid k_t = a_j, X_t; \hat{\theta}_{ml}\right] = \hat{\sigma}_t . \frac{\phi(c_1) - \phi(c_2)}{\Phi(c_2) - \Phi(c_1)}, \qquad \text{A 10}$$

$$c_1 = \frac{1}{\hat{\sigma}_t}\left(\hat{\mu}_{j-1} - \hat{\delta}' X_t\right),$$

$$c_2 = \frac{1}{\hat{\sigma}_t}\left(\hat{\mu}_j - \hat{\delta}' X_t\right),$$

$$\sigma_t = \exp(\gamma' X_t),$$

Gourieroux *et al.* (1985) show that these generalised residuals may be used to test for misspecification in a variety of ways and derive valid tests for serial correlation due to the omission of lagged endogenous variables using the *score statistic*.

This statistic is essentially the derivative of the likelihood function with respect to an autocorrelation parameter, evaluated at the maximum likelihood estimates under the null hypothesis of no serial correlation.

More specifically, consider the following model for k_t^*:

$$k_t^* = \varphi k_{t-1}^* + \delta' X_t + \varepsilon_t, |\varphi| < 1. \qquad \text{A 11}$$

In this case, the score statistic, ξ_1, is the derivative of the likelihood function with respect to φ, evaluated at the maximum likelihood estimates. Under the null hypothesis that $\varphi = 0$, it simplifies to the following expression:

$$\hat{\xi}_1 \equiv \left(\sum_{t=2}^{n} \hat{k}_{t-1}\hat{\varepsilon}_t\right)^2 \Big/ \sum_{t=2}^{n} \hat{k}_{t-1}^2 \hat{\varepsilon}_t^2, \qquad \text{A 12}$$

where:

$$\hat{k}_t \equiv E\left[k_t^* \mid k_t, X_t; \hat{\theta}_{ml}\right] = \hat{\delta}' X_t + \hat{\varepsilon}_t \qquad \text{A 13}$$

When $\varphi = 0$, ξ_t is asymptotically distributed as a χ_1^2 variate. Therefore, using ξ_t, we can test for autocorrelation induced by the omission of the variable k_{t-1}^*. This test can be generalised for any lag of k_t^*.

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