Banco de Portugal
Banco de Portugal Economic Studies, Vol. IX, No. 3

Risk reallocation under Central Bank's large-scale asset purchases

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July 2023

Abstract
Crises have some common features: increases in risk premia, decrease of real risk-less interest rates, and flight to quality assets, among others. This paper studies the effects of large-scale asset purchases on the market price of risk and the risk-free rate. We observe how, when the central bank buys risky assets using risk-less debt, there is a reduction of risk-taking in the economy, as the risk is transferred to non-market participants. Large-scale asset purchases by the central bank reduce the exposure of intermediaries' balance sheets to capital shocks, leading to a reduction in the risk premium and an increase in the risk-free rate. (JEL: E21, E60, F40)

1. Introduction

In recessions, risk premia increase and risk-free real interest rates decrease. Below we present figures that illustrate these stylized facts. Figure 1 shows the evolution of the TED spread, which is defined as the difference between the interest rate on interbank loans and the rate on 3-month U.S. government debt ("T-Bills"). The TED spread is an indicator of perceived credit risk in the general economy, since T-bills are considered risk-free, while LIBOR reflects the credit risk of lending to commercial banks. An increase in the TED spread is a sign that lenders believe the risk of default on interbank loans is higher. In turn, interbank lenders demand a higher rate of interest, or accept lower returns on safe investments such as T-bills.

Another risk premium measure is the difference between the return on a risky asset, like a long-term Treasury bond, and the return on a low risk asset, like a short-term Treasury bond. Figure 2 shows the evolution of the difference between the yield on the 10-year Treasury bond and the 3-month T-Bill. It shows that this variable increases in recessions periods.

The risk-free real interest rate decreases during recessions. Figure 3 shows the evolution of the risk-free real interest rate measured as the difference between the 3-month T-bill rate and an indicator of expected inflation, the Sticky Price Consumer Price

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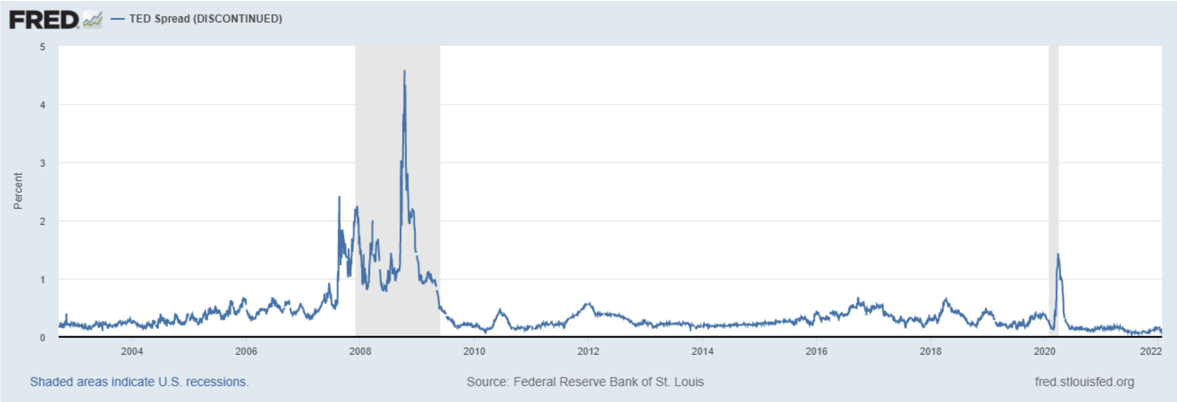

FIGURE 1: TED spread: 3M USD LIBOR – 3M Treasury Bill

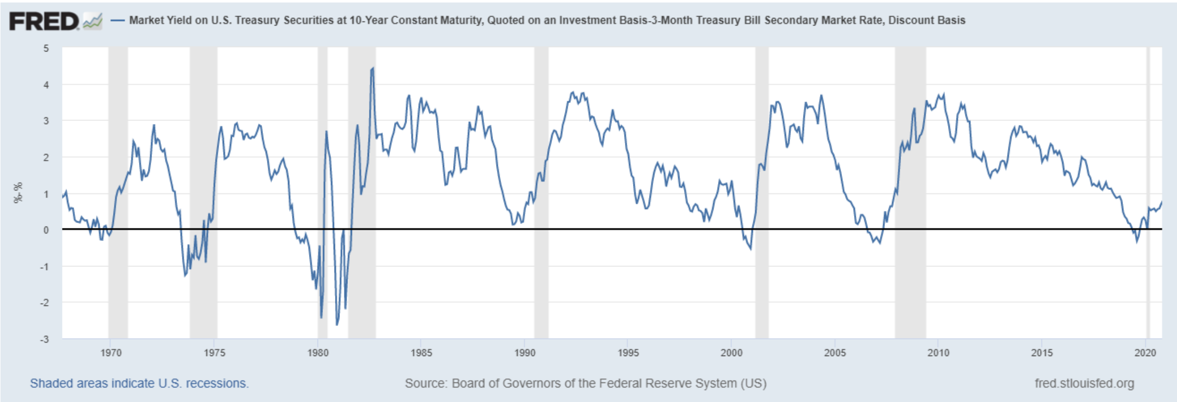

FIGURE 2: 10Y Treasury yield – 3M Treasury Bill

Index. The Sticky Price Consumer Price Index (CPI) is calculated from a subset of goods and services included in the CPI that change price relatively infrequently. These goods and services are thought to incorporate expectations about future inflation to a greater degree than goods and services whose prices change on a more frequent basis. See: Bryan and Meyer (2010).

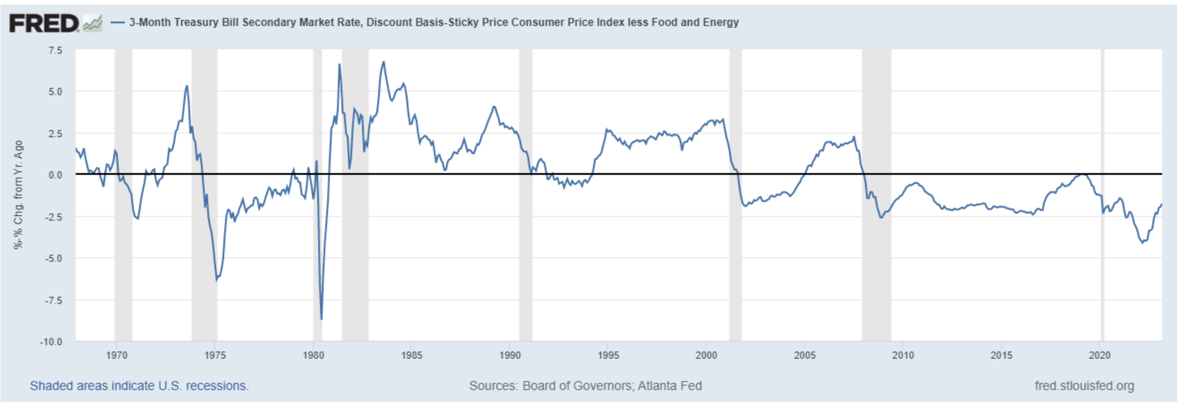

FIGURE 3: 3M T-bill – CPI (less food and energy) inflation

The crisis of 2007-08 required central banks around the globe to expand their monetary policy toolbox in an attempt to ease credit conditions and compress risk

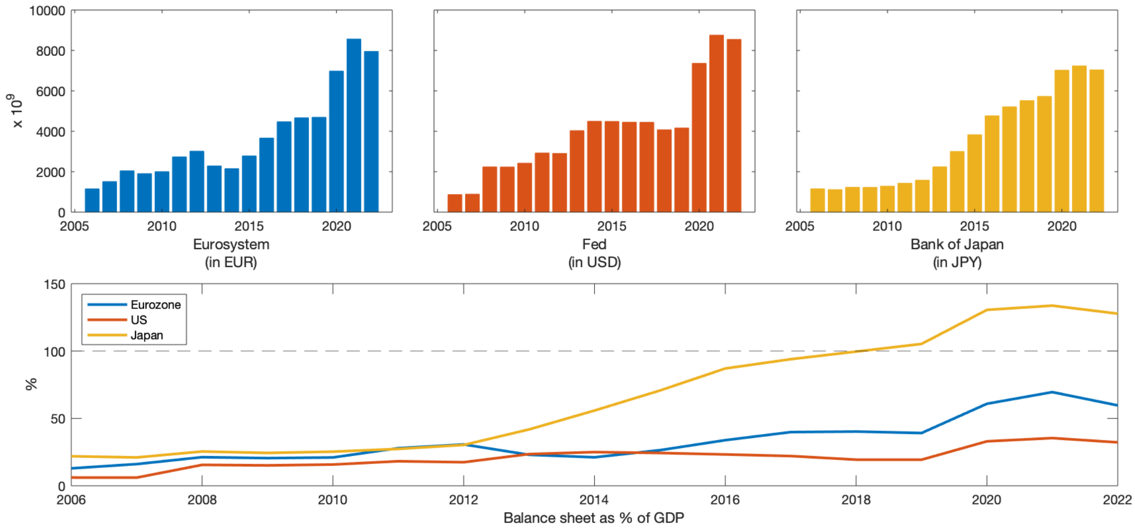

FIGURE 4: Balance sheets of the Eurosystem, the FED and the BoJ

premia. The shock in 2007-08, coupled with the sovereign debt crisis that followed, drove economies to situations in which conventional monetary policy instruments were unable to support a sustained economic recovery. Taylor rules would recommend cutting nominal interest rates well below zero but that was not possible due to the lower bound on the nominal interest rate. Moreover, close to the lower bound the previously well-functioning relationship between changes in official interest rates and market interest rates was no longer reliable.

Quantitative easing, and the purchase of large quantities of financial assets, became the tool of choice of monetary authorities facing the zero lower bound of policy rates. By reducing the supply to the private sector of risk-bearing assets (for example, due to their long maturity), and increasing the supply of less risky assets (for example, bank reserves) central banks expected to lower longer-term market interest rates, and hence channel more lending to consumers and businesses. The purchases of risky assets took many forms: long-term Treasury securities, corporate bonds, and stocks held by financial institutions, among others. Additionally, central banks provided funds to agents that normally do not have access to central bank money, like Commercial Paper issuers, Money Market Mutual Funds, agency bonds and agency Mortgage Backed Securities issuers, among others. All central banks in advanced economies implemented these kind of measures in different proportions to alleviate the effects of the global financial crisis of 2008.

Non-standard monetary policies conducted in response to the 2008 crisis continued to be used in various degrees until the recent pandemic crisis in 2020. Reflecting the net asset purchase programs in place, this led to a substantial increase in the central banks balance sheet. Figure 4 shows the evolution of the balance sheets of the Eurosystem, the FED and the BoJ (in absolute values and as a percentage of GDP), between these two landmarks.

The contribution of this paper is theoretical. We consider a simple model that relies on the model with heterogeneous agents introduced by Brunnermeier and Sannikov (2017), which, in turn follows Basak and Cuoco (1998), to unravel some of the macro effects of unconventional monetary policy measures. In particular we show how unconventional monetary policy affects the risk premium and the risk-less interest rate.

We consider an economy with two heterogeneous rational agents: a market expert, with the required know-how to accumulate a risky asset (capital), and a household, that finances the expert's purchase of risky assets by holding expert-issued risk-less debt. In addition, we introduce a central bank. The central bank redistributes risk in the economy by issuing risk-less bonds to purchase capital, and transferring dividends (via Treasury) to the households.

When solving this model, we obtain analytical expressions for the risk-free interest rate and the evolution of the expert sector's relative wealth. From here, we are able to reason how, after a negative shock to the capital of intermediaries, their ability to continue holding risky assets decreases and, as a consequence, risk premium increases. We show how, when the central bank buys risky assets using risk-less debt, there is a reduction of risk taking in the economy, as this risk is transferred to non-market participants. The asset purchases performed by the central bank change the equilibrium in the economy, leading to a decrease of the market price of risk and an increase in the risk-free interest rate.

The remainder of this paper is organized as follows. Section 2 reviews some of the relevant literature. Section 3 presents the model, describes the equilibrium in the economy and explains how non-conventional monetary policy affects the equilibrium. Section 4 concludes.

2. Literature Review

We start this section by discussing why the size and composition of the central bank's balance sheet might modify the equilibrium. Neil Wallace wrote the pioneer paper on this issue with the title "A Modigliani-Miller Theorem for Open-Market Operations". Wallace (1981) established, in the context of an overlapping generations model, that, under the assumption that markets are complete, neither the size nor the composition of the central bank's balance sheet affects the economy's equilibrium. In his model money serves only as a store of value, as it does not facilitate transactions. Other authors confirmed this result in either different or more general environments. Peled (1985) established that open market operations between money and indexed bonds do not matter in a real sense despite their different risk characteristics. Chamley and Polemarchakis (1984) extended Wallace's result to incomplete markets. Sargent and Smith (1987) extended the neutrality result to a particular economy where money is dominated in return.

The neutrality result is puzzling because open market operations instrument has been the main instrument of monetary policy, and monetary policy is taken to affect the economy. In fact, the targets of monetary policy have changed through time, by either targeting the price of reserves or the monetary aggregates, but the main instrument has always been open market operations, Bindseil (2014). Thus, this theoretical result seemed not to apply to the operations of actual central banks.

Eggertsson and Woodford (2003) explain why in general Wallace (1981) neutrality result does not apply to open market operations with reserves. Reserves (and base money more generally) are assets that have non-pecuniary returns as they help mitigate transaction frictions. Unlike other financial assets, base money provides transactions services, by relaxing constraints that would otherwise restrict the transactions that the holders of the asset can perform.

However, there is a situation where Wallace's neutrality still holds even for open market operations with reserves. When the nominal interest rate is zero, there is no longer any shortage of cash as its opportunity cost is zero. As the non-pecuniary returns of the reserves are zero, then open market operations with reserves should have no effects either. Thus, given that since 2008 until recently the nominal interest rate has been approximately zero, then the quantitative easing strategy used during this period should not have been effective in providing monetary stimulus to the economy.

The neutrality result relies on the existence of frictionless financial markets. In this case the price of an asset equals the present value of its future stochastic returns, where the present value is calculated using a standard stochastic discount factor. For example, in a simple model economy, with complete markets, the stochastic discount factor is unique and is determined by the households' marginal rate of substitution between consumption today and consumption in the different future states of nature. If the trades of assets between the central bank and the private sector do not change the real quantity of resources available for consumption in each state of nature, the households' marginal rate of substitution in the different states of nature should not change either. Thus, the discount factor should not change, and the price of the assets should not change because their returns in each state of nature have not changed.

Suppose, for instance, that the central bank decides, through an open-market operation, to hold a portfolio with more risk, which results in private investors holding a portfolio with less risk. To make things more concrete suppose that after the open-market operation the central bank's portfolio pays a low return in the event of a pandemic, while the portfolio of the private sector pays a similar return in all states of nature. This change in the central bank's portfolio does not make the risk disappear from the economy. The central bank's returns on its portfolio will be lower in the state with a pandemic, and this will imply lower dividends distributed to the Treasury, which in turn means that lower transfers will be disbursed to the private sector in that state. Therefore, the households' income after transfers will remain unchanged in that state and in all the other states of nature. This means the stochastic discount factor will be unchanged too, and thus the open-market operation will fail to change the asset prices.

The irrelevance result of the central bank's balance sheet is easier to prove if there is a representative household, but it remains true even if that is not the case. The result is true even if households are heterogeneous, they may have different risk aversion, different time profiles of income, different types of non-tradeable income risk, etc. The crucial assumption, as Chamley and Polemarchakis (1984), Cúrdia and Woodford (2011), d'Avernas *et al.* (2019) and Silva (2020) show, is that all investors can purchase or sell arbitrary quantities of the same assets at the same prices.

Under this assumption, if the central bank does an open-market operation that will change the households state-contingent income, then households should want to trade in the financial markets to undo the effects of the central bank's trade. Suppose a share x_h of the returns on the central bank's portfolio is distributed, by the Treasury, to household h. If the central bank decides, through an open-market operation, to hold a portfolio with more risk, which results in private investors holding a portfolio with less risk, then household h should choose a trade that cancels exactly fraction x_h of the central bank's trade, as a means to obtain the state-contingent consumption stream he had before the central bank's intervention.

The relevant notion of the households' wealth is the sum of their own portfolio and the present value of the discounted future transfers of the central bank to the households, via the Treasury. Hence, the relevant risk exposure of households includes both the financial risk from their portfolio as well as the risk resulting from the central bank intervention transmitted to households through transfers via the Treasury. If with an open market operation, the central bank increases its holdings of risky assets, investors have an incentive to reduce their own exposure to risk, to keep their total risk exposure constant. Because total risk exposure does not change, asset prices and macroeconomic variables do not change either. Note that this result is easier to prove with complete markets, but still holds under incomplete markets. Thus, the neutrality result holds provided transfers can be replicated (or undone) by a portfolio of tradable financial assets.

In contrast, most of the empirical literature seems to conclude that quantitative easing had effects on the economies.[1] There is a wealth of papers that study the effects of large-scale asset purchases empirically and for different jurisdictions. Krishnamurthy and Vissing-Jorgensen (2011) evaluate the effect of the Federal Reserve's quantitative easing programs on interest rates. According to the authors' findings, the Fed's influence was greater on the risk premia of the assets being purchased. For example, QE1 (2008-09) had a large effect on the reduction of mortgage rates, partly due to the fact that QE1 involved large purchases of agency backed mortgage-backed securities (MBS). In turn, QE2 (2010-11), which involved only Treasury purchases, impacted mainly Treasury and agency bond rates, and less so MBS and corporate rates.

Work has also been done in identifying different channels through which LSAPs transmit their effects to the real economy Koijen *et al.* (2017); Krishnamurthy and Vissing-Jorgensen (2011); Vayanos and Vila (2021). Krishnamurthy *et al.* (2017) focus on the effect of the European Central Bank (ECB) programs, namely the Securities Markets Programme (SMP), the Outright Monetary Transactions (OMT) framework, and the three-year long-term refinancing operations (LTROs). Their interest is on the channels through which the SMP and OMT affected sovereign bond yields of so-called GIIPS countries, as well as stock returns. A number of other authors have also studied the

1. For a skeptical interpretation of the evidence, see Stroebel and Taylor (2012) and Taylor (2009).

effect of LSAP programmes on bank lending to the private sector (Andrade *et al.* (2015); Carpinelli and Crosignani (2017); Fonseca *et al.* (2015); Garcia-Posada and Marchetti (2016)). Empirical studies have also been performed to assess the effect of QE in different asset types (De Santis and Zaghini (2021); Albertazzi *et al.* (2021); Balcilar *et al.* (2020); Farinha and Vidrago (2021); Lewis and Roth (2019); Guo *et al.* (2020)).

With financial imperfections the Wallace's neutrality result does not hold. A simple model modification assumed by many authors, like Chamley and Polemarchakis (1984), Cúrdia and Woodford (2011), d'Avernas *et al.* (2019), and Silva (2020), is that financial markets are segmented. These authors assume that investors are heterogeneous, i.e., not all investors have equal opportunities to invest in all assets on the same terms. It could be that only certain specialists have the expertise required to invest in some assets, or by law some agents do not have access to a particular market: for instance, shadow banks do not have access to central bank reserves. Thus, when the central bank purchases that specific asset, the central bank transfers to investors, via Treasury, have more income risk that is correlated with that specific asset returns. Investors may want to hold less of that risk, but since they do not have that asset in their portfolio and cannot go short on it, that is not possible.

In our model we assume that not all financial market participants can cost-less trade the same set of financial instruments. We assume that some participants lack the expertise required to directly extend credit to firms, so that they must instead deposit funds with competitive intermediaries who are in turn able to offer loan contracts to the firms. By conducting open market operations, the central bank can extend credit to firms and issue risk-free debt. As shocks affect the relative wealth of intermediaries, they affects the investment and the supply of risky assets in the economy. The central bank can modify the risk in the economy by buying the risky asset, supplying the risk-less asset and channeling the risky dividends to non market participants.

3. The model

Our model builds on the model of Brunnermeier and Sannikov (2017) which in turn is based on the paper of Basak and Cuoco (1998). The original model studies the equilibrium dynamics of an economy with financial frictions and the effect of financial disintermediation in times of crises. The model is an infinite horizon economy with production, heterogeneous agents and financial frictions. We extend the original model by introducing a central bank that does non-conventional monetary policy. In addition to the central bank, there are two other type of agents: financial experts and households. There is a continuum (with mass one) of each type of agents. The production technology uses capital to produce final goods and there are investment adjustment costs. There is one risky asset and a risk-free asset. Financial markets are segmented, only the experts and the central bank can trade the risky asset. The central bank and the experts finance their holdings of the risky asset by issuing bonds to households. Non-conventional monetary policy consists in buying capital with the revenue obtained from the issuance of bonds, and transferring the profits (or losses) associated with this operation to households.

The transfers should be interpreted as being done via Treasury after receiving the dividends from the central bank. To simplify the analysis these transfers are assumed to be received entirely by the households, but the qualitative results remain unchanged as long as the households receive some fraction of these transfers. The larger the fraction of the transfers received by the households, the larger will be the quantitative effect of the non-conventional monetary policy on the risk-free interest rate and the risk premium.

The assumption that the households cannot participate in the risky asset market is an extreme assumption that simplifies the analysis. However, in order for non-conventional policy to have effects in the economy it is just enough that households have limited access to the risky asset market, either because they do not have the necessary expertise or lack the relevant information. It is important that the actions of the agents that receive the transfers from the central bank be restricted, so that these agents cannot undo the effect of the central bank non-conventional policy. For instance, if the households were allowed to trade without any restrictions the risky asset then they would choose an efficient portfolio. In that case, whenever there is an open market operation the households would want to change their portfolio, so that the return on the new portfolio together with the central bank transfers would be equal to the return on the original portfolio.

3.1. Financial experts

Experts have a linear constant returns to scale production function of the form

$$Y_t = Ak_t, A > 0. \tag{1}$$

The capital stock evolves according to the law of motion:

$$\frac{dk_t}{k_t} = (\Phi(\iota_t) - \delta)dt + \sigma dZ_t, \tag{2}$$

where ι_t is the investment rate of capital, and the investment function Φ satisfies the usual assumptions: $\Phi(0) = 0, \Phi'(0) = 1, \Phi' > 0$, and $\Phi'' < 0$. The concavity of Φ reflects capital adjustment costs. The investment technology Φ transforms $\iota_t k_t$ units of output into $\Phi(\iota_t)k_t$ units of capital. δ is the depreciation rate of capital. The last term is a growth rate shock which follows a Brownian motion with volatility σ. This term can be interpreted as the risk of holding capital. It is the only shock in the economy.

In order to finance their holdings of the risky asset, experts issue non-contingent bonds which are bought by households. Let θ_t, $\theta_t < 0$, be the expert's short position on bonds, r_t be the interest rate on the bonds, r_t^K be the rate of return on capital, c_t^e be the expert's consumption and n_t^e be the expert's wealth. The law of motion of the expert's wealth is

$$\frac{dn_t^e}{n_t^e} = \left(-\frac{c_t^e}{n_t^e} + \theta_t r_t\right)dt + (1-\theta_t)dr_t^K. \tag{3}$$

The dynamics of r_t^K are described below in (7). The representative expert has logarithmic utility, and discounts at rate $\rho^e \geq 0$ utility flows from future consumption c_t^e:

$$\mathbb{E}_0\left[\int_0^\infty e^{-\rho^e \cdot t}\log(c_t^e)dt\right]. \tag{4}$$

The representative expert's problem amounts to deciding the investment rate ι_t, consumption flow c_t^e, and short position on bonds θ_t, so as to maximize (4) subject to the evolution of the wealth in (3) and the initial value of wealth, n_0^e.

The price of capital, denoted by q_t, follows a stochastic process described by:

$$\frac{dq_t}{q_t} = \mu_t^q dt + \sigma_t^q dZ_t, \tag{5}$$

where the drift and volatility, (μ_t^q, σ_t^q), are determined in equilibrium. The instantaneous rate of return on capital is given by

$$dr_t^K = \frac{A - \iota_t}{q_t} dt + \frac{d(q_t k_t)}{q_t k_t}, \tag{6}$$

where the first term represents the dividend yield and the second term the capital gains. We can simplify this expression to obtain:

$$dr_t^K = \left(\frac{A - \iota_t}{q_t} + \mu_t^q + \Phi(\iota_t) - \delta + \sigma_t^q \sigma \right) dt + (\sigma_t^q + \sigma)\, dZ_t, \tag{7}$$

where $\mu_t^q + \Phi(\iota_t) - \delta + \sigma_t^q \sigma$ denotes the expected capital gain and $\sigma_t^q + \sigma$ the volatility of the return. The volatility term is the total risk, composed by the fundamental risk σ and the price risk σ_t^q. For convenience in later calculations, we denote the drift of this stochastic process by $\mu_t^{r^K}$ and the volatility by $\sigma_t^{r^K}$.

As the measure of experts is one, the aggregate consumption of the expert sector is $C_t^e = \int_0^1 c_{i,t}^e di$.

3.2. *Central bank*

The central bank follows the rule of holding a share of the economy's stock of capital, which is financed by issuing risk-less debt, and distributing the dividends to households. Formally, if K_t^c is the stock of capital own by the central bank and K_t the aggregate capital in the economy then $K_t^c = \varepsilon K_t$, $0 \leq \varepsilon < 1$. To simplify the analysis we assume the central bank has zero net wealth. The central bank invests in capital, K_t^c, by issuing risk-less bonds, B_t^c, so that $q_t K_t^c = B_t^c$. These bonds, B_t^c, pay the same rate, r_t, as those issued by the financial expert. The proceeds obtained from this portfolio are rebated to households as a lump-sum transfer, T_t,

$$T_t = q_t K_t^c r_t^K - B_t^c r_t = \varepsilon q_t K_t (r_t^K - r_t). \tag{8}$$

3.3. *Households*

The financial friction in the model is the restriction that households do not have access to the market for risky securities. Households do not have the necessary expertise to hold risky securities, but may hold bonds.

The representative household can buy risk-less debt, which pays an interest rate, r_t. Additionally, the household receives a transfer, T_t, from the central bank and consumes, c_t^h. The law of motion for the household's wealth is

$$dn_t^h = (n_t^h r_t - c_t^h)dt + dT_t. \tag{9}$$

Households' preferences are also represented by a logarithmic utility function

$$\mathbb{E}_0\left[\int_0^\infty e^{-\rho^h\cdot t}\log(c_t^h)dt\right], \tag{10}$$

with $\rho^h \geq 0$.

The households' problem amounts to choosing their consumption flow c_t^h so as to maximize (10) subject to (9), and an initial value n_0^h.

As we also assume the measure of households is one, the aggregate consumption of the households sector is $C_t^h = \int_0^1 c_{i,t}^h di$.

3.4. *State variable*

Let K_t be the economy's capital. If we denote by K_t^e the experts' aggregate capital, then

$$K_t^e = \int_0^1 k_{i,t}^e di, \tag{11}$$

where $k_{i,t}^e$ denotes the i-th individual expert's capital stock. As the only risk in the model is at the aggregate level, we have that $k_{i,t}^e = k_t^e$. Capital is held by the expert sector and the central bank, so

$$K_t = K_t^e + K_t^c.$$

Let N_t be the economy's aggregate net worth. Then $N_t = q_t K_t$. Let N_t^e and N_t^h be the aggregate wealth levels of the expert and the household sectors, respectively, $N_t = N_t^e + N_t^h$ (remember that we assume the central bank's wealth, N_t^c, is zero). Since there is no idiosyncratic risk $N_t^e = n_t^e$ and $N_t^h = n_t^h$. Finally, we define the (relative) wealth of the expert sector, which we take as the aggregate state variable of the economy,

$$\eta_t \equiv \frac{N_t^e}{N_t} = \frac{N_t^e}{q_t K_t}. \tag{12}$$

As it will be useful in later calculations, we denote the drift of the stochastic process of the state variable by μ_t^η and the volatility by σ_t^η.

Households are assumed to be more patient than financial experts, $\rho^e > \rho^h$, as in Kiyotaki and Moore (1997) to prevent a degenerate stationary distribution of the relative net-worth of experts. As we will see later if $\rho^e = \rho^h$ then in the long run η_t converges to one.

3.5. *Equilibrium*

An equilibrium in this economy is defined as paths for price $\{q_t\}$, expert decisions $\{\iota_t, \theta_t, c_t^e\}$, household decisions $\{c_t^h\}$, central bank decisions $\{\varepsilon, T_t\}$, and net worths $\{n_t^e,\ n_t^h\}$ such that: (i) both agents maximize their objective functions, given relevant restrictions, and (ii) all markets clear.

3.6. *The financial expert's problem*

The relationship between the price and the replacement cost of capital, known in the literature as the Tobin's q formula is given by the expert's problem's first-order condition with respect to investment

$$\Phi'(\iota_t) = \frac{1}{q_t}. \tag{13}$$

In the case of logarithmic utility, the expert's problem can be solved analytically, yielding optimal policies for the expert's consumption and portfolio composition. The first order condition for the choice of consumption

$$c_t^e = \rho^e n_t^e. \tag{14}$$

and the condition for the choice of the optimal portfolio

$$(1-\theta_t) = \frac{\mu_t^{r^K} - r_t}{\left(\sigma_t^q + \sigma\right)^2}. \tag{15}$$

Equation (14) says that the consumption flow is a constant share of net-worth. Equation (15) says that the share of the risky asset in the expert's portfolio is the ratio between the expected excess return and the variance of the risky return. From (15) we can obtain the Sharpe ratio. This ratio is the expected return earned in excess of the risk-free rate per unit of volatility or total risk. It turns out that it is equal to the volatility of the expert's net worth:

$$\frac{\mu_t^{r^K} - r_t}{\sigma_t^{r^K}} = \frac{\frac{A-\iota_t}{q_t} + \mu_t^q + \Phi(\iota_t) - \delta + \sigma_t^q \sigma - r_t}{\sigma_t^q + \sigma} = \sigma_t^{n^e}. \tag{16}$$

3.7. *The household's problem*

From the first order condition of the household's problem we derive the consumption choice,

$$c_t^h = \rho^h n_t^h. \tag{17}$$

The consumption flow is a constant share of the net worth.

3.8. *Solving for the equilibrium*

In order to solve the model we need to specify the functions. We start by specifying the investment function. We assume the investment function:

$$\Phi(\iota_t) = \frac{1}{\varphi}\log(\varphi\iota_t + 1), \tag{18}$$

where φ is a capital's adjustment cost parameter.[2] From the solution to the expert's problem (13), we get the optimal investment rate as

$$\iota_t = \frac{q_t - 1}{\varphi}. \tag{19}$$

2. With this functional form, if φ tends to 0, then $\Phi(\iota_t)$ tends to ι_t, and there are no adjustment costs.

Using the capital market clearing condition and the first order conditions of the agents, we have that $C_t = C_t^e + C_t^h = \rho^e N_t^e + \rho^h(q_t K_t - N_t^e)$. Dividing both sides of this equation by N_t we get

$$\frac{C_t}{N_t} = \rho^e \eta_t + \rho^h(1-\eta_t) \equiv f(\eta_t), \tag{20}$$

where for convenience we define $f(x) \equiv \rho^e x + \rho^h(1-x)$.

The total supply of the consumption good is $(A - \iota_t)K_t$. Market clearing of the consumption good implies

$$f(\eta_t) = \frac{A - \iota_t}{q_t}. \tag{21}$$

From here, we obtain the price of capital,

$$q(\eta_t) = \frac{1 + \varphi A}{1 + \varphi f(\eta_t)}. \tag{22}$$

After substituting this expression back in (19) we get the expression for investment:

$$\iota(\eta_t) = \frac{A - f(\eta_t)}{1 + \varphi f(\eta_t)}. \tag{23}$$

The fraction of the experts' wealth invested in the risky asset is obtained from the clearing condition of the capital market: $N_t^e(1-\theta_t) = (1-\varepsilon)q_t K_t$ or, equivalently,

$$(1-\theta_t) = (1-\varepsilon)\frac{1}{\eta_t}. \tag{24}$$

The risk-free rate is determined from the equation that characterizes the expert's optimal portfolio (15) and the market clearing condition (24):

$$r_t = -(1-\varepsilon)\frac{1}{\eta_t}\left(\sigma_t^q + \sigma\right)^2 + \mu_t^{r^K}. \tag{25}$$

The derivation of the drifts and the volatilities of the relevant variables: the state variable, the price of capital, and the rate of return of capital are straightforward but too lengthy. As such they are derived and specified in the Appendix.

3.9. *Numerical example*

We adopt the values in the literature for the parameters, for example Silva (2020) and Brunnermeier and Sannikov (2017). We set the level of technology $A = \frac{1}{3}$, which corresponds to a capital-output ratio of 3. The depreciation rate δ equals 0.05. We will assume households to be more patient than financial experts, and so $\rho^e = 0.05 > 0.02 = \rho^h$. Finally, the fundamental risk $\sigma = 10\%$ and the capital cost parameter $\varphi = 10$.

Figure 5 depicts the price of capital, the risk-free rate, and the drift and volatility of the wealth share of experts η_t for different values of the parameters. We start by describing the case, represented by the blue line, in which households are as patients as experts, $\rho^h = \rho^e = 0.05$, and there is no intervention of the central bank ($\varepsilon = 0$). The results in this case agree with the results in Brunnermeier and Sannikov (2017). We observe in the top left panel that the price of capital, q, is constant and equal to $\frac{1+\varphi A}{1+\varphi \rho^e}$. Experts' balance sheet concentrates all risk in the economy, as in this case experts are the

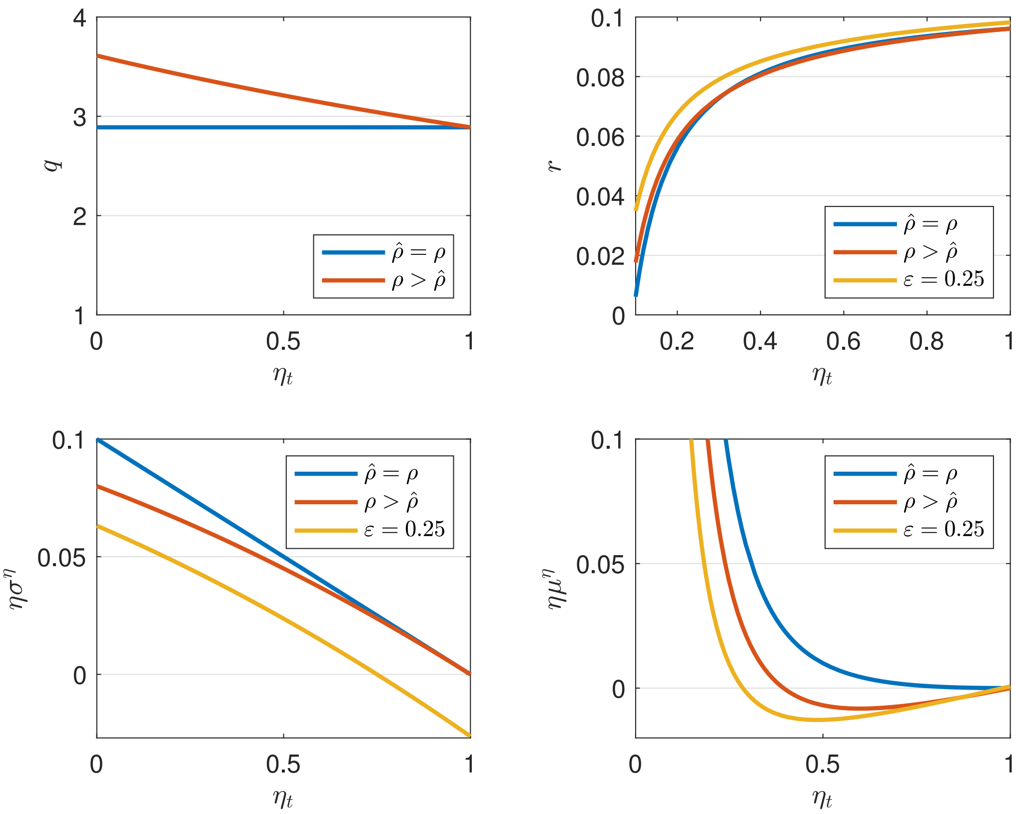

FIGURE 5: Price of capital, risk-free rate, drift and volatility of η_t.

only ones holding capital. In the event of a negative shock on capital, experts' relative wealth decreases. In that case experts demand a higher risk premium, and the risk-free rate decreases, as observed in the top right panel of Figure 5. With a lower risk-free rate, experts pay less to households on their loans, and so experts' relative wealth moves quickly back to higher values, as can be seen in the blue line in the bottom right panel. In the long run the experts relative wealth converges to one.

Let us look now at the case, represented by the red line, when households are more patient than financial experts, $\rho^e > \rho^h$, but still without central bank intervention. In this case, the price of capital is no longer constant, depends on η_t. In fact, in the top left panel of Figure 5, we observe $q'(\eta_t) < 0$. With a negative shock, as experts' relative wealth decreases, the price of capital increases. Differences in the discount rates of the agents lead to an increase in the risk-free rate, as observed when we compare the blue and red lines in the top right panel of Figure 5 and the left panel of Figure 6. As we now have differences in the consumption patterns of agents, if experts' relative wealth increases substantially, their higher consumption rate, when compared to households, will make them lose wealth over time, on average (see bottom right panel in Fig. 5). As a result, and contrary to the case above, the economy is no longer dominated in the long run by the experts' sector. Finally, in the bottom left panel, we observe how, in this case, we have a mitigation of the effect of a negative shock on the capital rate of return. As a result the volatility of the aggregate variable is lower in this case.

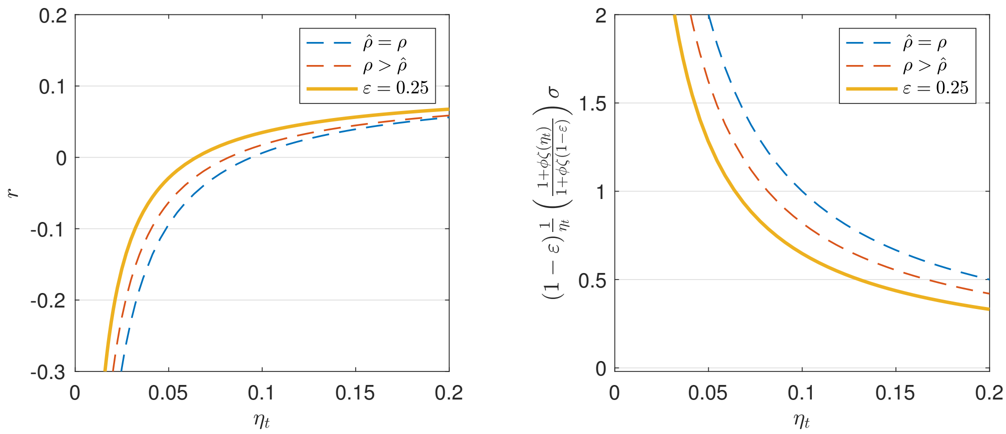

FIGURE 6: Risk-free rate and Sharpe ratio (detail, $0 < \eta_t \leq 0.2$).

At last, we look at the full model, in which the central bank owns part of the capital in the economy. This case is represented by the yellow line. Here we assume the proportion of capital owned by the central bank is $1/4$ of the aggregate capital, i.e. $\varepsilon = 0.25$, which is equivalent to $3/4$ of GDP. In this case, the price of capital is unchanged, i.e. is equal to the previous case. As we can observe in the bottom left panel, there is a lower volatility in the relative wealth of experts. As in the previous case, as experts' wealth decrease, their net-worth risk increases. However, as the experts are no longer the only agents holding capital, the increase in risk is lower than in the previous cases. As a result, the risk-free rate is higher and exhibits a more convex profile. The reduction in risk for experts has a direct impact on the risk premium.

In Figure 6, we can observe how a negative shock on capital leads to an increase in the risk premium, measured by the Sharpe ratio (right panel). In the case with central bank intervention the market price of risk increases but by less than in the case without central bank intervention, hence demonstrating the positive effect of the non conventional monetary policy. In the left panel, we observe how this dampening effect on the risk premium translates into a risk-free rate higher than in the cases without non-conventional monetary policy.

3.10. Results for the steady state

In this section we study how different levels of non-conventional policy affect the steady state equilibrium of the economy when households are more patient than financial experts. We start by determining the steady state distribution of the aggregate state variable. The steady state distribution of the state variable is obtained by simulating the discretized version of the law of motion for η for a fine time interval until convergence is achieved. A fine grid of points in the interval $(0, 1)$ was considered for the initial values of the aggregate state variable. All these initial values of the state variable converged to the same steady state distribution.

Figure 7, displays the steady state distribution of the aggregate variable for 3 different levels of non-conventional monetary policy, $\varepsilon = 0.01, 0.25$ and 0.5. It shows that an

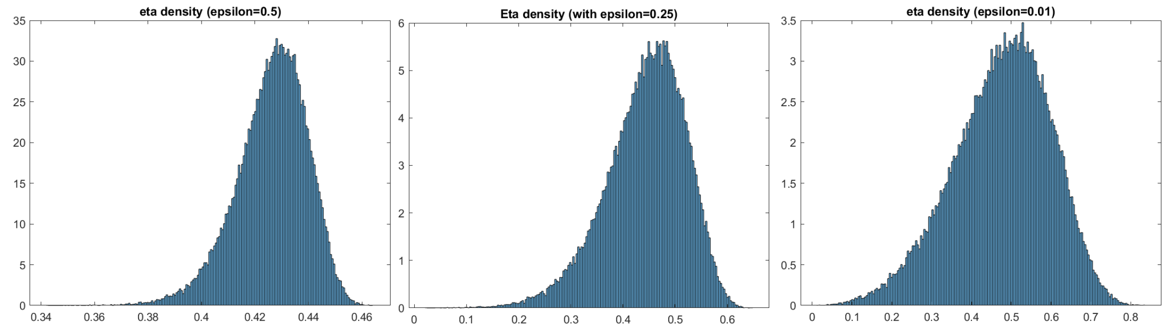

FIGURE 7: Steady state density of η_t

increase in the size of the open market operations decreases the dispersion of the aggregate state variable, which implies lower dispersion of the risk-free interest rate, price of capital, risk premium, investment rate and Sharpe ratio. Additionally, the mean of the relative wealth of the expert decreases with the size of the open market operations.

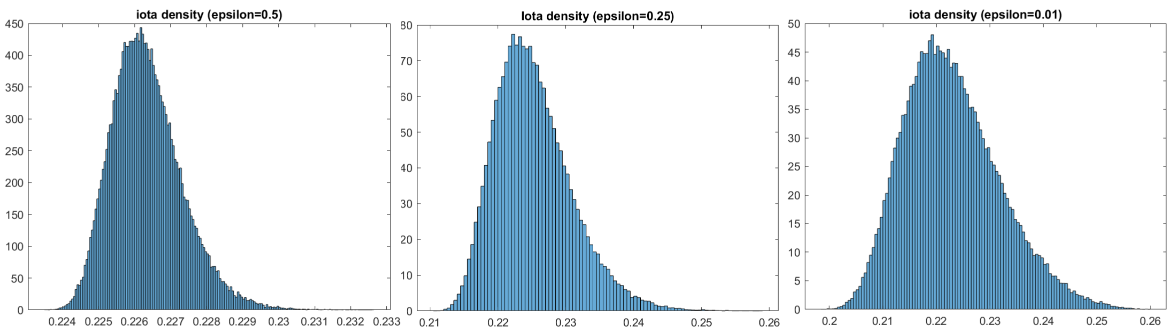

FIGURE 8: Steady state density of ι_t

Figure 8 displays the steady state distribution of the investment rate for the same 3 different levels of non-conventional monetary policy. The investment rate dispersion decreases with the size of the non-conventional policy and the mean investment rate increases with the size of the non-conventional policy. However, the impacts are quantitatively small.

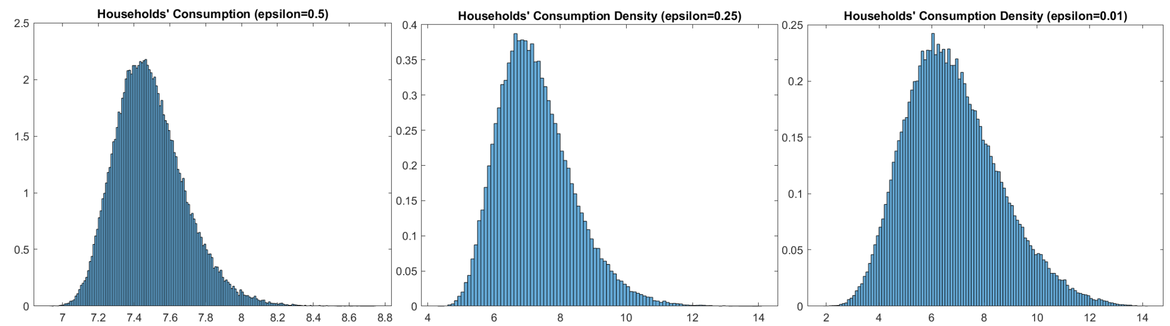

FIGURE 9: Steady state distribution of c_t^h

Finally, we investigate the effects of the non-conventional policy over the consumptions of the two types of households. Figures 9 and 10 show how the steady

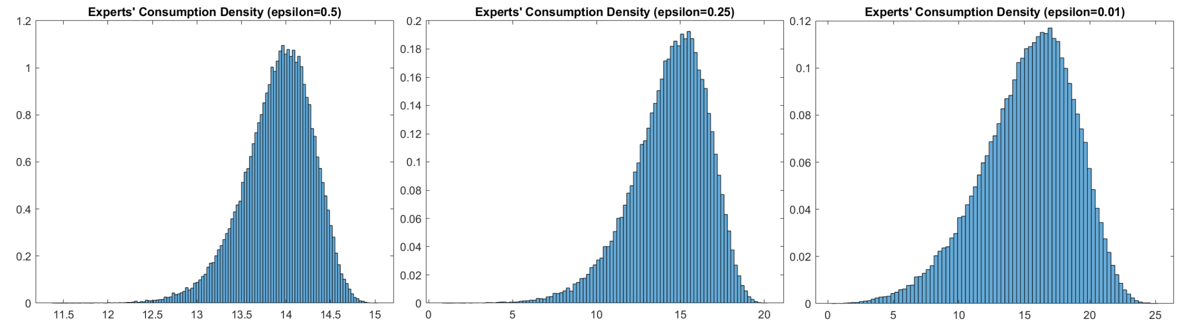

FIGURE 10: Steady state density of c_t^e

state densities of consumptions change with the non-conventional monetary policy. The dispersion of consumptions decreases substantially as the size of the non-conventional policy increases.

Using these stationary densities we compute that the expected utility of the households increases with the size of the open market operations, while the expected utility of experts decreases with the size of the open market operations. The reason for the mean investment rate to increase with the size of the open market operations is because households save more than the experts, and the average relative wealth of households increases.

4. Conclusion

After a negative shock on the capital of intermediaries, their ability to hold risky assets decreases and, as a consequence the market price of risk (or risk premium) increases and the risk-less interest rate decreases. In this paper we show that unconventional monetary policy can mitigate these effects. We unravel the effects of large-scale asset purchases (LSAPs) by central banks in a very simple model where some agents, the households are restricted from participating in risky financial markets. In this context LSAPs redistribute risk in the economy, reducing the exposure of intermediaries' balance sheets to capital shocks, leading to a reduction in the risk premium and an increase in the risk-free rate.

LSAPs stabilize the economy too: the volatilities of consumptions, investment and GDP decrease with the size of the non-conventional monetary policy. As LSAPs allow the households to indirectly access a market they did not have access to, and break the monopoly of the intermediaries in that market, the average net worth of experts decreases while the average net worth of households increases. The expected utility of households increases, expected utility of intermediaries decreases and the average investment rate increases.

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Appendix

We start by deriving the dynamics of the aggregate state variable, the share of experts' wealth, $\eta_t = \frac{N_t^e}{q_t K_t}$, with $0 \leq \eta_t \leq 1$. For that, we make use of Itô's lemma and the definition of stochastic discount factor. The stochastic discount factor for this economy is given by

$$\frac{d\xi_t}{\xi_t} = -r_t dt - \sigma_t^{n^e} dZ_t, \tag{A.1}$$

Given the absence of risks other than the aggregate risk, we have $N_t^e = n_t^e$. Then,

$$\begin{aligned}\frac{dN_t^e}{N_t^e} = \frac{dn_t^e}{n_t^e} &= \left(-\frac{c_t^e}{n_t^e} + \theta_t r_t + (1-\theta_t)\mu_t^{r^K}\right) dt + (1-\theta_t)\left(\sigma_t^q + \sigma\right) dZ_t \\ &= \left(-\frac{c_t^e}{n_t^e} + r_t + (1-\theta_t)\sigma_t^{n^e}\left(\sigma_t^q + \sigma\right)\right) dt + (1-\theta_t)\left(\sigma_t^q + \sigma\right) dZ_t,\end{aligned} \tag{A.2}$$

and using

$$\begin{aligned}\frac{d(q_t k_t)}{q_t k_t} &= \left(\mu_t^q + \Phi(\iota_t) - \delta + \sigma_t^q \sigma\right) dt + \left(\sigma_t^q + \sigma\right) dZ_t \\ &= \left[\left(r_t - \frac{A-\iota_t}{q_t}\right) + \sigma_t^{n^e}\left(\sigma_t^q + \sigma\right)\right] dt + \left(\sigma_t^q + \sigma\right) dZ_t,\end{aligned} \tag{A.3}$$

we obtain

$$\frac{d\eta_t}{\eta_t} = \left(-\frac{c_t^e}{n_t^e} + \frac{A-\iota_t}{q_t} - \theta_t\left(\sigma_t^q + \sigma\right)\left(\sigma_t^{n^e} - \left(\sigma_t^q + \sigma\right)\right)\right) dt - \theta_t\left(\sigma_t^q + \sigma\right) dZ_t. \tag{A.4}$$

Using the fact that $\sigma_t^{n^e} = (1-\theta_t)\left(\sigma_t^q + \sigma\right)$, we get an alternative expression for the law of motion of η_t:

$$\frac{d\eta_t}{\eta_t} = \left(\frac{A-\iota_t}{q_t} - \rho^e + \theta_t^2\left(\sigma_t^q + \sigma\right)^2\right) dt - \theta_t\left(\sigma_t^q + \sigma\right) dZ_t. \tag{A.5}$$

Now we proceed with the determination of the law of motion for the price of capital $q_t = q(\eta_t)$, which we assumed to follow a stochastic process with drift μ_t^q and volatility σ_t^q. By Itô's lemma we get

$$\frac{dq(\eta_t)}{q(\eta_t)} = \frac{q'(\eta_t)\mu^\eta \eta_t + \frac{1}{2}q''(\eta_t)(\sigma^\eta \eta_t)^2}{q(\eta_t)} dt + \frac{q'(\eta_t)}{q(\eta_t)}\eta_t \sigma^\eta dZ_t. \tag{A.6}$$

From the capital market equilibrium condition (24), and (A.5):

$$\sigma^q(\eta_t) = \frac{q'(\eta_t)}{q(\eta_t)}\eta_t \sigma^\eta = \frac{q'(\eta_t)}{q(\eta_t)}(1-\varepsilon-\eta_t)\left(\sigma_t^q + \sigma\right). \tag{A.7}$$

After solving (A.7) for the volatility of the price we get

$$\sigma^q(\eta_t) = \frac{(1-\varepsilon-\eta_t)\frac{q'(\eta_t)}{q(\eta_t)}}{1-(1-\varepsilon-\eta_t)\frac{q'(\eta_t)}{q(\eta_t)}}\sigma. \tag{A.8}$$

After substituting in (A.8) the expressions for $q(\eta_t)$ in (22) (and its first derivative) we get

$$\sigma^q(\eta_t) = \frac{\varphi(\rho^h - \rho^e)(1-\varepsilon-\eta_t)}{1+\varphi f(1-\varepsilon)}\sigma, \tag{A.9}$$

where $f(1-\varepsilon) = \rho^e(1-\varepsilon) + \rho^h\varepsilon$. For convenience, we use the notation $f(x) \equiv \rho^e x + \rho^h(1-x)$. The drift of the process, $\mu^q = \mu^q(\eta_t)$, can be computed also. It can be established that

$$\mu^q(\eta_t) = \varphi(\rho^h - \rho^e)\left[\frac{\eta_t(1-\eta_t)}{1+\varphi f(\eta_t)}(\rho^h - \rho^e) + \frac{(1+\varphi\rho^h)}{\eta_t}\left(\frac{(1-\varepsilon-\eta_t)\sigma}{1+\varphi f(1-\varepsilon)}\right)^2\right], \tag{A.10}$$

where $f(\eta_t) = \rho^e\eta_t + \rho^h(1-\eta_t)$. According to (25) in order to get r_t we need to determine $\mu_t^{r^K}$, which is specified in (7) as $\mu_t^{r^K} = \left(\frac{A-\iota_t}{q_t} + \mu_t^q + \Phi(\iota_t) - \delta + \sigma_t^q\sigma\right)$.

After replacing in (25) the expressions for μ_t^q, σ_t^q and $\mu_t^{r^K}$ we obtain the expression for the risk-free interest rate, which is only dependent on the model's parameters and the state variable,

$$\begin{aligned} r_t = f(\eta_t) + \frac{1}{\varphi}\log\left(\frac{1+\varphi A}{1+\varphi f(\eta_t)}\right) - \delta + \frac{\varphi(\rho^h-\rho^e)\sigma^2}{1+\varphi f(1-\varepsilon)}\left((1-\varepsilon)-\eta_t\right) \\ - (1-\varepsilon)\frac{1}{\eta_t}\left(\frac{1+\varphi f(\eta_t)}{1+\varphi f(1-\varepsilon)}\right)^2\sigma^2 \\ + \varphi(\rho^h-\rho^e)\left[\frac{\eta_t(1-\eta_t)(\rho^h-\rho^e)}{1+\varphi f(\eta_t)} + \frac{(1+\varphi\rho^h)}{\eta_t}\left(\frac{(1-\varepsilon)-\eta_t}{1+\varphi f(1-\varepsilon)}\sigma\right)^2\right]. \end{aligned} \tag{A.11}$$

Finally, after substituting in (A.5) the expression for σ_t^q we obtain the law of motion of η_t,

$$\begin{aligned} \frac{d\eta_t}{\eta_t} = \left((\rho^h-\rho^e)(1-\eta_t) + \left(\frac{(1-\varepsilon)-\eta_t}{\eta_t}\frac{1+\varphi f(\eta_t)}{1+\varphi f(1-\varepsilon)}\right)^2\sigma^2\right)dt \\ + \left(\frac{1+\varphi f(\eta_t)}{1+\varphi f(1-\varepsilon)}\right)\frac{(1-\varepsilon)-\eta_t}{\eta_t}\sigma dZ_t. \end{aligned} \tag{A.12}$$

The Sharpe ratio of risky investment is

$$\begin{aligned} \frac{\frac{A-\iota_t}{q_t} + \mu_t^q + \Phi(\iota_t) - \delta + \sigma_t^q\sigma - r_t}{\sigma_t^q + \sigma} &= (1-\varepsilon)\frac{1}{\eta_t}\left(\sigma_t^q + \sigma\right) \\ &= (1-\varepsilon)\frac{1}{\eta_t}\left(\frac{1+\varphi f(\eta_t)}{1+\varphi f(1-\varepsilon)}\right)\sigma. \end{aligned} \tag{A.13}$$