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The euro area natural interest rate – Estimation and importance for monetary policy

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Abstract
This study presents estimates of the euro area natural interest rate for the last 50 years. The results are obtained using a more general version of the Holston *et al.* (2017) model adjusted for the pandemic period that incorporates the role of inflation expectations. The estimated natural interest rate shows a downward trend from around 3% in the early 1970s to levels around 0.5% in 2022. The real interest rate gap – defined as the difference between the observed interest rate and the natural interest rate – allows an assesment on the degree of monetary policy accommodation. The estimates obtained indicate an accommodative monetary policy after the start of the global financial crisis, in a progressive way until the recent sharp rise in interest rates. The communication of the level of the natural interest rate by central banks can be relevant to influence expectations of long-term interest rates. **Keywords:** Natural interest rate, semi-structural models, Kalman filter, monetary policy. (JEL: C32, E43, E52)

1. Introduction

Monetary policy since 2022 has been, in most advanced economies, marked by the increase in policy interest rates to ensure the return of inflation to levels consistent with the price stability objectives. Monetary policy uses interest rates to restrict demand and activity and consequently reduce the pressure on prices. However, a correct calibration of monetary policy implies a correct measurement of the degree of accommodation generated by the policy interest rate. At the same time, and after the prolonged period of low interest rates registered in most advanced economies, it is also relevant to assess the level at which interest rates should converge when inflation returns to levels consistent with the objectives of central banks.

The concept of natural interest rate — defined as the real interest rate that neither stimulates nor reduces economic activity or inflation — is particularly useful for the conduct of monetary policy. By comparing this rate with the real interest rate observed

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in the economy, it is possible to assess the degree of monetary policy accommodation. At the same time, the natural interest rate can be seen as the level to which interest rates should converge in the long term.

Since the natural interest rate is an unobservable variable, it is necessary to use estimates to assess its evolution. Among the various methodologies presented in the literature, the model developed by Laubach and Williams (2003) and extended in Holston *et al.* (2017) — henceforth HLW — is among the most used. This semi-structural model estimates the natural interest rate based on the relationship between the real interest rate gap (measured by the difference between the observed real interest rate and the natural interest rate), the output gap and inflation. The model estimates a Phillips curve and an IS curve and uses the Kalman filter to obtain estimates of the natural interest rate.

This study discusses and presents estimates of the euro area natural interest rate for the last 50 years based on two modifications to the Holston *et al.* (2017) model. Firstly, the original HLW model is adapted for the pandemic period, as presented by the authors in Holston *et al.* (2020), including a variable in the measurement of the level of potential output to capture part of the fluctuations recorded in this period. The Covid-19 pandemic and the imposed lock-down measures caused quite sharp fluctuations in the main euro area macroeconomic aggregates that may not be correctly captured by the model.

Secondly, the original HLW model does not include inflation expectations. This study discusses a modification to the original structure of the model to explicitly include the role of inflation expectations. This modification follows closely the work developed by Lopez-Salido *et al.* (2020) that explores the role of inflation expectations in the HLW model for the United States.

This study presents estimates for the natural interest rate based on three specifications: The first includes only the adaptation to the pandemic period, while the second and third combine the adaptation for the pandemic period with forward-looking inflation expectations over two different horizons. Despite the uncertainty associated with the results, estimates indicate a downward trend in the natural interest rate in the euro area since the early 1970s, moving from levels around 3% to levels around 0.5% in 2022.

The study is organised as follows. Section 2 presents a summary of the literature related to the natural interest rate, discussing the concept, the different methodologies used in its estimation, its evolution over time and the main determinants of this evolution. Section 3 begins by presenting the original HLW methodology, followed by a discussion of the two modifications explored in the study. Section 4 presents the results of the three estimated specifications. Section 5 seeks to draw some conclusions about the use of the natural interest rate in the conduct of monetary policy and Section 6 concludes.

2. Literature Review

2.1. *Concept*

The natural interest rate is a central concept in macroeconomics. Originally presented by Wicksell (1898) who discussed the existence of *"a certain rate of interest on loans which is neutral in respect to commodity prices, and tends neither to raise nor lower them. This is necessarily the same as the rate of interest which would be determined by supply and demand if no use were made of money and all lending were effected in the form of real capital goods"*.

Woodford (2003) introduced the concept of the natural interest rate into modern macroeconomics in the Neo-Keynesian framework, characterizing it as a short-term real interest rate that guarantees price stability every period in a model without nominal frictions. Laubach and Williams (2003) present the natural interest rate as the short-term real interest rate consistent with output at its potential level and with stable inflation over the medium term. This interpretation for the natural interest rate can be seen as the result from monetary policy rules, such as the Taylor rule (Taylor (1993)), when the output level is at its potential level and inflation is at its defined target by the central bank.

Regardless of the definition considered, the concept of natural interest rate can be seen as the real interest rate that stabilises the level of production and inflation. If the real interest rate observed in an economy is above (below) the natural interest rate then demand is expected to decrease (increase) and consequently inflation will decrease (increase). As a result, and since the main monetary policy instrument in advanced economies is the interest rate, the natural interest rate can be used as a metric for the degree of monetary policy accommodation: if the real interest rate resulting from monetary policy is above (below) the natural interest rate then monetary policy can be classified as contractionary (expansionary).

This study seeks to contribute to the understanding and estimation of the natural interest rate in the euro area according to the Laubach and Williams (2003) definition. This assessment is particularly relevant in the current context of high inflation in the euro area where monetary policy is using interest rates to control inflation, after a significant period of low interest rates.

2.2. *Estimation*

The natural interest rate is an unobservable variable, so it has to be estimated. There are different methodologies, depending on the type of model and the definition of the inherent natural interest rate. This literature can be divided into two groups. The first uses structural general equilibrium models, such as overlapping generation models — OLG — or dynamic stochastic general equilibrium models — DSGE — to estimate the natural interest rate. These two types of models employ the definition of natural interest rate presented in Woodford (2003). The OLG models estimate the natural interest rate as the real rate of return on capital that balances saving and investment in an economy in the absence of nominal frictions and reallocation shocks. Since this type of model does not allow an evaluation of the evolution of the natural interest rate over the economic cycle, the results can be interpreted as the long-term trend of the natural interest rate. These models are well suited to directly assess the role of demographic trends (such as average life expectancy, birth rates, etc.) in the evolution of the natural interest rate (see Bielecki *et al.* (forthcoming) and Papetti (2021) for results for the euro area). In the DSGE models, the estimation seeks to recover the real interest rate that would prevail in a counterfactual economy with flexible prices and wages, subject to shocks. This methodology makes it possible to assess the evolution of the natural interest rate over the economic cycle (see Del Negro *et al.* (2017) or Barsky *et al.* (2014) for estimates for the United States and Neri and Gerali (2019) for the euro area).

The second group of methodologies for estimating the natural interest rate uses semi-structural models, which impose restrictions on the data based on relationships present in macroeconomic theory and, through econometric techniques, estimate the natural interest rate. In this group, the main reference is the Laubach and Williams (2003) model where a Phillips curve and an intertemporal IS curve are estimated and, using the Kalman filter, obtain an estimate for the natural interest rate in the United States. This model seeks to extract the natural interest rate through low-frequency movements in the data that are consistent with the imposed macroeconomic structure, so the results should be interpreted as the level of the natural interest rate in the medium term where changes in the economic cycle have faded. The Laubach and Williams (2003) model was subject to several adaptations with the aim of improving its structure. Lopez-Salido *et al.* (2020) estimate the model for the United States using measures of forward looking inflation expectations rather than adaptive expectations. Brand and Mazelis (2019) apply the model to the euro area by adding a Taylor rule, a block for the labour market and relaxing the assumption of a unit root in the Phillips curve. Krustev (2019) extends the model in order to capture the impact that the financial cycle has on the determination of the natural interest rate. Pedersen (2015) estimates the natural interest rate for Denmark by modifying the model for a small open economy.

The present study seeks to contribute to this literature by estimating the natural interest rate in the euro area by modifying the Laubach and Williams (2003) model to better capture the pandemic shock in 2020 and considering measures of forward looking inflation expectations, in line with Lopez-Salido *et al.* (2020).

2.3. *Evolution and determinants*

Despite the existence of several interpretations for the natural interest rate and different methodologies for its estimation, the various models consistently point to a reduction in the natural interest rate in the euro area (as well as in most advanced economies[1]) since the 1970s, in particular after the global financial crisis in 2007/08 and the sovereign debt crisis in 2012/13. Since then, the natural interest rate has remained at relatively

1. IMF (2023) presents a revision of natural interest rate estimates for a wide range of advanced and emerging economies.

low values, around 0%[2]. The impact of the pandemic on the natural interest rate is still uncertain, since it was a shock of extreme magnitude that makes it difficult to estimate any unobservable variable, with authors arguing that it may have contributed to its rise or fall (see Jorda *et al.* (2022) and Luzzetti *et al.* (2022)).

The downward trend of the natural interest rate is consistent with the determinants of this variable identified in the literature[3]. The natural interest rate is usually seen as a variable that is determined by real factors, and therefore independent of nominal factors such as monetary policy. Among the main factors, the evolution of productivity and the level of potential output are considered the most relevant. In neoclassical models of economic growth with agents with constant relative risk aversion — CRRA — the equilibrium real interest rate in the savings market is given by the following equation:

$$r^* = \frac{1}{\sigma}g + \theta \tag{1}$$

where r^* is the equilibrium real interest rate, σ represents the intertemporal elasticity of substitution of consumption, g is the potential output growth and θ represents the agents' time preference. As can be seen in equation (1), the natural interest rate is intrinsically linked to the growth of potential output, and consequently to productivity. The intuition of this result is obtained through the intertemporal consumption decision: if the growth of potential output increases, consumers will save less to carry out the same level of consumption in the future, which implies an increase in the interest rate to guarantee the equilibrium between savings and investment. Thus, part of the decline in the natural interest rate recorded in the euro area may be related to the slowdown in potential output and productivity growth, especially after the global financial crisis of 2007/08.

Another factor often presented as a determinant for the evolution of the natural interest rate is the demographic trend. This has a direct impact on the evolution of the workforce in an economy, thereby influencing the growth of potential output, and also influence agents' savings decisions. Thus, the ageing population registered in most advanced economies — characterised by a decrease in birth rates and an increase in average life expectancy — is identified as a factor that led to a decrease in the natural interest rate, since it determines a lower growth of potential output and leads to a greater propensity for agents to save to cope with a longer retirement. Despite the fact that the total number of working-age agents making savings decreases with the ageing of the population (which could lead to an increase in the natural interest rate), the previous effect tends to be identified as dominant.

2. The methodologies discussed so far usually start from some measure of risk-free market interest rates. However, the natural rate of interest should also reflect the real rate of return on capital. Reis (2022) argues that measures of return on capital have shown a relatively constant evolution in contrast to measures of market rates, highlighting the importance of evaluating other ways of estimating the natural interest rate.

3. Brand *et al.* (2018) discuss several factors that influence the natural interest rate in the euro area. Rachel and Smith (2015) provide a global assessment of factors that influence the natural interest rate, as well as a quantification of the contribution of various factors to the evolution of the global natural interest rate.

A final set of factors identified as having an impact on the natural interest rate is related to financial factors. The global financial crisis of 2007/08 and the subsequent sovereign debt crisis in the euro area led to a significant increase in uncertainty and demand for safe assets by economic agents. This leads simultaneously to an increase in savings and a decrease in investment, potentially resulting in a reduction in the natural interest rate. At the same time, changes in banking regulation and supervision adopted also led to an increase in demand for safe assets by banking institutions, which also reinforces this trend.

All these factors contribute to explaining the reduction in the natural interest rate and its permanence at substantially low levels until 2019[4]. The pandemic crisis in 2020 generated a set of shocks of an extreme nature that make it difficult to assess the evolution of the natural interest rate in the recent period. The climate transition, which has been at the centre of discussions of policy makers, may also have an (still uncertain) impact on the natural interest rate (see Mongelli *et al.* (2022)). This study seeks to contribute with estimates for the natural interest rate in the euro area, especially for the period after the pandemic crisis.

3. Methodology

3.1. *Laubach and Williams Model*

This study adopts a modified version of the model developed in Laubach and Williams (2003) and later extended in Holston *et al.* (2017). Thus, throughout the study, the definition that will be considered for the natural interest rate is a short-term real interest rate consistent with the level of production at its potential level and with stable inflation in the medium term, as the impact of the shocks dissipates.

The structure imposed by the model starts from the equilibrium conditions for the level of inflation and the output gap present in Neo-Keynesian models, as presented in Gali (2008). These conditions can be described by a Phillips curve:

$$\pi_{H,t} = \beta E_t[\pi_{H,t+1}] + \kappa \bar{y}_t \tag{2}$$

And by an IS equation:

$$\bar{y}_t = E_t[\bar{y}_{t+1}] - \frac{1}{\sigma}(i_t - E_t[\pi_{H,t+1}] - r_t^n) \tag{3}$$

where $\pi_{H,t}$ represents the inflation of domestically produced goods and services, $\bar{y}_t$ represents the gap between the observed output level and the level of output which would prevail in a counterfactual economy without nominal frictions, i_t the nominal risk-free interest rate and r_t^n the natural equilibrium interest rate in a counterfactual

4. Other factors are also pointed out in the literature for the reduction of the natural interest rate, such as the increase in income inequality, the fall in the relative price of capital goods and the increase in demand for safe asset (*savings glut*) of some emerging economies, but the estimated impact of these factors tends to be smaller than those mentioned above (see Rachel and Smith (2015)).

economy without nominal frictions. β, κ and σ represent the discount factor, consumer preferences and technology factor. Based on these equilibrium conditions, HLW use the following equations as measurement equations for estimating the natural interest rate:

$$\pi_t = b_\pi \pi_{t-1} + (1 - b_\pi)\pi_{t-2,4} + b_y \tilde{y}_{t-1} + \varepsilon_{\pi,t} \tag{4}$$

$$\tilde{y}_t = a_{y,1}\tilde{y}_{t-1} + a_{y,2}\tilde{y}_{t-2} + \frac{a_r}{2}\sum_{j=1}^{2}(r_{t-j} - r^*_{t-j}) + \varepsilon_{\tilde{y},t} \tag{5}$$

where π_t represents inflation in the period, $\pi_{t-2,4}$ represents the average inflation in the three previous periods, $\tilde{y}$ represents the output gap, that is, the difference between the observed level of output y_t and the estimated measure of potential output y^*_t. r_t represents the short-term real interest rate *ex-post* and r^*_t the natural interest rate. Comparing equations (2) and (3) with equations (4) and (5) one can see the distinction between the interpretations for the natural interest rate in Neo-Keynesian models and the one used by HLW. In equations (2) and (3) the natural interest rate captures all shocks[5] that affect inflation and the level of output. Once the real interest rate gap is closed (in the equation (3): $i_t - E_t[\pi_{H,t+1}] - r^n_t$) in each period, inflation and the level of output stabilise. In equations (4) and (5) there can be shocks that affect inflation and the level of output but that do not necessarily affect the natural interest rate (captured in the equations above by $\varepsilon_{\pi,t}$ and $\varepsilon_{\tilde{y},t}$). Thus, the natural interest rate will only be affected by more persistent shocks in the relationship between the real rate gap and the output gap.

Based on these equations, the HLW model uses the Kalman filter to estimate the natural interest rate and the level and growth of potential output. The transition equations of the model are:

$$r^*_t = g_t + z_t \tag{6}$$

$$y^*_t = y^*_{t-1} + g_{t-1} + \varepsilon_{y^*,t} \tag{7}$$

$$g_t = g_{t-1} + \varepsilon_{g,t} \tag{8}$$

$$z_t = z_{t-1} + \varepsilon_{z,t} \tag{9}$$

where g is the growth rate of potential output and z captures other determinants that can influence the natural interest rate[6]. The transition equations assume that potential

5. Usually, in structural models, the natural interest rate is seen as the real interest rate that would prevail in an economy without nominal price and wage frictions, in the absence of *mark-up* shocks on goods and on labour.

6. Note that equation (6) is based on equation (1) assuming that the coefficient of elasticity of consumption substitution is equal to 1. Laubach and Williams (2003) estimate this parameter and conclude that it is very close to 1 for the United States. Holston *et al.* (2017) also assume this value for the parameter in the euro area.

output growth and the variable z follow a random walk and potential output in level follows a random walk with *drift*. It is assumed that the variables $\varepsilon_{y^*,t}$, $\varepsilon_{g,t}$ and $\varepsilon_{z,t}$ follow a normal distribution with standard deviations σ_{y^*}, σ_g and σ_z and are not correlated. As observed variables, the model uses a measure of output (GDP), inflation and a measure of the real interest rate. HLW uses inflation excluding food and energy (hereafter, core inflation) and calculates the observed real interest rate as an *ex-post* rate given by the difference between a short-term (three-month) interest rate and the inflation observed on average over the last four quarters.

3.2. *Adjustment for the pandemic crisis*

The years 2020 and 2021 were marked by the pandemic crisis associated with Covid-19. During this period, social distancing measures were imposed, which led to very sharp fluctuations in the variables used in the model[7]. A shock of this magnitude is not correctly captured by the HLW methodology, since the hypothesis that the errors in the measurement and transition equations follow a normal distribution and are uncorrelated is not respected. Recognizing this limitation, Holston *et al.* (2020) propose a correction to the model to mitigate this impact. This modification involves adding the stringency index developed by the University of Oxford[8] in the model with the objective of capturing the effects of the measures imposed to contain the pandemic. This variable is added to the model when measuring the level of potential output, which is now expressed as follows:

$$y_t^{*'} = \begin{cases} y_t^* + \varphi d_t, & \text{if } t = 2020Q1 \text{ or } 2020Q2 \\ y_t^*, & \text{otherwise} \end{cases} \tag{10}$$

And equation (5) is now rewritten as:

$$(y_t - y_t^{*'}) = a_{y,1}(y_{t-1} - y_{t-1}^{*'}) + a_{y,2}(y_{t-2} - y_{t-2}^{*'}) + \frac{a_r}{2}\sum_{j=1}^{2}(r_{t-j} - r_{t-j}^*) + \varepsilon_{\tilde{y}',t} \tag{11}$$

Where d_t is the level of the stringency indicator for the quarter t calculated as the aggregation of the quarterly averages of daily data from the euro area countries weighted by the weight of the population of each country in 2019. With this modification, the strong changes recorded in GDP throughout 2020 that the original model would classify as marked changes in the level of potential output are partly captured by the variable d_t. Figure 1 shows the estimates for potential output growth generated by the model with and without adjustment for the pandemic period.

7. In the second quarter of 2020, GDP in the euro area declined by 11.5% quarter on quarter and grew by 12.4% in the following quarter.

8. This indicator was developed to measure the intensity of the measures imposed to contain the Covid-19 pandemic. A higher level of this indicator means that the containment measures were more restrictive and, therefore, may have led to a greater disruption of economic activity. For more information on this indicator, consult Hale *et al.* (2021)

As expected, the results of the model without the adjustment for the pandemic period show a sharp drop in potential GDP growth, while in the model with the adjustment the drop is less pronounced. Thus, and since the potential output growth rate is a determinant of the natural interest rate in the model, the adaptation of the model tries to capture potential output growth without it being strongly influenced by the pandemic period. It should be noted that the results of the adjusted model are highly dependent on the periods chosen for the inclusion of the stringency index. As can be seen in the equation (10), in the study it is considered that the adjustment variable takes the value of the stringency index in the first and second quarters of 2020.[9]

9. Since the authors in Holston *et al.* (2020) omit the periods in which the variable is present, the choice becomes somewhat arbitrary. The choice made in this study involved comparing the model's estimates of the output gap level with estimates from other international organizations, trying to find the one that best replicates the level and evolution of these estimates.

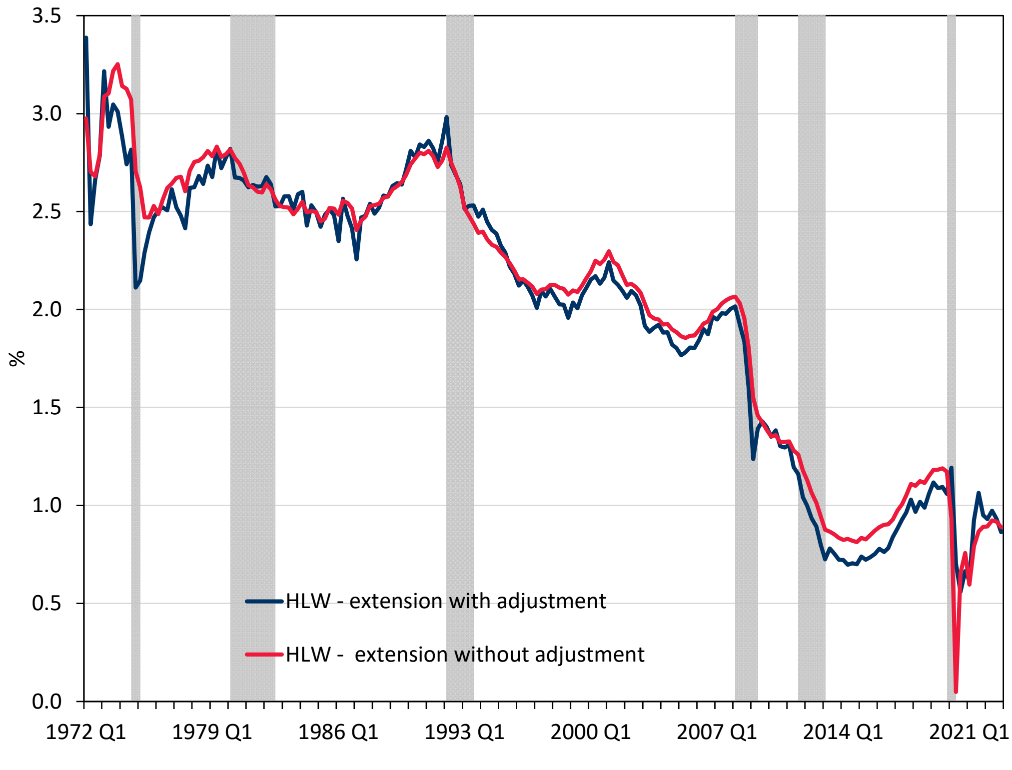

FIGURE 1: Estimates for the euro area potential output growth (g_t)

Notes: Estimates obtained through the HLW model. Extension without adjustment corresponds to the estimated replication of the HLW model until 2022 Q4. Adjusted estimate corresponds to the model discussed in Section 3.2, which includes the stringency index in the euro area in the first and second quarters of 2020. Bars in gray mark the periods of recession according to the dating of the Euro Area Business Cycle Network — EABCN.
Sources: Eurostat, EABCN, Refinitiv and author's calculations.

3.3. *Model with inflation expectations*

As previously described, the equations that serve as the basis for the specification used by HLW come from two equations from the standard Neo-Keynesian model (equations (2) and (3)). However, making a comparison between these equations and the measurement equations adopted by HLW it is possible to observe a difference in the role of inflation expectations. In equation (2) the level of inflation for the period t is dependent on the inflation expectation formed in that period for the inflation in the period $t+1$. In equation (4), a linear combination between the observed inflation in the period $t-1$ and an average of the observations in $t-2$, $t-3$ and $t-4$ is used. Thus, while in equation (2) inflation expectations are forward looking, in equation (4) expectations are adaptive.

In this study, the model is adapted to take into account forward looking inflation expectations. This adaptation consists of considering a measure of inflation expectations in the equation (4)[10]. Thus, the new Phillips curve is now written as:

$$\pi_t = b_\pi \pi_{t-1} + (1 - b_\pi) E_t[\pi_{t+h}] + b_y \tilde{y}_{t-1} + \varepsilon_{\pi,t} \tag{12}$$

Where h represents the horizon to which inflation expectations correspond. In this way, the evolution of inflation expectations starts to play a role in estimating the natural interest rate.

In addition to the role of inflation expectations, this formulation has an additional advantage. In the original HLW model, when the output gap is zero, inflation remains constant. However, the model does not specify the level around which inflation stabilises. This formulation may not be consistent with economies where central banks have well-defined inflation targets. With the inclusion of forward-looking inflation expectations, when the output gap is zero, inflation, as described by equation (12), converges to the expected level of inflation. Assuming the credibility of the central bank's inflation target, longer-term inflation expectations should approach these inflation targets. So, the equation (12) can be rewritten as:

$$\pi_t = b_\pi \pi_{t-1} + (1 - b_\pi) \pi^* + b_y \tilde{y}_{t-1} + \varepsilon_{\pi,t} \tag{13}$$

where π^* is the inflation target. This equation can also be rewritten as a function of the gap between observed inflation and π^*:

$$\tilde{\pi}_t = b_\pi \tilde{\pi}_{t-1} + b_y \tilde{y}_{t-1} + \varepsilon_{\pi,t} \tag{14}$$

Where $\tilde{\pi}_t$ equals $(\pi_t - \pi^*)$. With this formulation, the measurement of the natural interest rate is not only dependent on the evolution of the inflation level, as in the original HLW model, but also becomes dependent on the evolution of deviations of inflation from the central banks' objective.

10. A similar exercise is carried out by Lopez-Salido *et al.* (2020) for estimating the natural interest rate in the United States.

4. Results

4.1. Data

This section presents the results of estimates for the natural interest rate in the euro area based on three modifications of the original HLW model. The first is the model with adaptation for the pandemic period — Model without expectations — discussed in Section 3.2. and the second and third consist of estimates based on the model presented in Section 3.3 with two different measures of inflation expectations — Model with expectations T+2 and Model with expectations T+5 — also using the adjustment for the pandemic period. Quarterly data for GDP, core inflation and three-month nominal interest rate from 1970 to 2022 are used for the estimation. The data is obtained from the ECB's Euro Area-Wide Model database (Fagan *et al.* (2001)) updated with data from Eurostat and Refinitiv. For the euro area stringency index it is used the quarterly average of daily data calculated as the weighted average of the stringency indices of the euro area countries weighted by the population share in 2019. The inflation expectations considered are the expectations for total inflation from the ECB's Survey of Professional Forecasters — SPF — at fixed horizons two and four/five years ahead[11]. A detailed description of the variables used is provided in the appendix[12].

4.2. Estimates

Table 1 presents the estimated parameters of the three specifications mentioned in the previous section. The parameters a_y and b_π are close to one and statistically significant, which indicated that both the output gap and the inflation exhibit high persistence. The parameter φ associated with the control variable for the pandemic period is also significant and with a negative sign. This result is expected since this variable reflects part of the sharp drop in GDP in the first two quarters of 2020.

The parameters a_r and b_y are particularly relevant for estimating the natural interest rate as they measure the sensitivity of the output gap to the real interest rate gap and of inflation to the output gap, respectively. The estimates show the expected sign. If the real rate gap is positive — that is, if the observed real interest rate is above the natural interest rate — the output gap becomes more negative, and a more negative

11. The use of expectations for total inflation and not for core inflation has to do with sample availability, as core inflation expectations in the SPF have only started to be released since 2016. At the same time, it should be noted that expectations for total and core inflation at the horizons considered in the study follow a similar behaviour. It would also be possible to use inflation expectations one year ahead of the SPF, without significant change in the results. However, this series shows greater volatility than expectations at longer horizons, which translates in higher volatility of the natural interest rate. In addition, the interpretation of inflation expectations as a central bank's inflation target described in section 3.3. is more obvious with longer term expectations.

12. The choice of core inflation over total inflation is also made by HLW. The objective is to use an inflation measure that responds more directly to the output gap and the real interest rate gap so that the natural interest rate estimate is more robust. Total inflation, by including the components of energy and food, is more subject to external shocks that may not be well captured by the structure of the model.

Parameter	Model without expectations	Expectations T+2	Expectations T+5
λ_g	0.059	0.065	0.060
λ_z	0.005	0.012	0.015
$\sum a_y$	0.94	0.94	0.94
a_r	-0.05^{*}	-0.05^{*}	-0.05^{*}
	(0.03)	(0.03)	(0.03)
b_y	0.06^{*}	0.06^{**}	0.06^{**}
	(0.03)	(0.03)	(0.03)
b_π	0.81^{***}	0.80^{***}	0.80^{***}
	(0.05)	(0.05)	(0.05)
φ	-0.19^{***}	-0.19^{***}	-0.19^{***}
	(0.00)	(0.00)	(0.00)
$\sigma_{\tilde{y}}$	0.42	0.43	0.43
σ_π	0.84	0.84	0.84
σ_{y^*}	0.37	0.36	0.37
σ_g	0.09	0.09	0.09
σ_z	0.05	0.11	0.14
σ_{r^*}	0.10	0.14	0.16
Monte Carlo simulations Standard error (mean)			
r^*	1.2	1.8	2.3
g	0.3	0.3	0.3
y^*	1.5	1.5	1.5
Monte Carlo simulations Standard error (2022 Q4)			
r^*	1.6	2.6	3.3
g	0.4	0.5	0.4
y^*	2.0	2.1	2.1

TABLE 1. Parameter estimates

Notes: Estimates based on quarterly data from 1971 Q1 to 2022 Q4. Numbers in parentheses correspond to estimated standard deviations. $***$, $**$ and $*$ next to coefficients significantly different from zero at a confidence level of 99%, 95% and 90%, respectively. Estimation performed using the maximum likelihood method (MLE). Mean and 2022 Q4 standard errors calculated with the Monte Carlo procedure according to Hamilton (1986) which captures the uncertainty of the parameters and the Kalman filter.

output gap translates into less inflation. However, these parameters have a much smaller magnitude when compared to the other parameters. In the first specification of the model these coefficients are only statistically significant at a confidence level of 90%. In other specifications, the coefficient b_y slightly increases its significance. Holston *et al.* (2017) also report a lower significance of the parameters a_r and b_y in the models for the euro area, Canada and the United Kingdom.

The higher uncertainty around a_r and b_y justifies the high uncertainty associated with the estimates of the natural interest rate. The bottom part of the Table 1 shows the standard errors of Monte Carlo simulations for the level of the model's unobservable variables. As in Holston *et al.* (2017), these standard errors are high, which calls for caution when interpreting the natural interest rate estimates obtained. Uncertainty around the natural interest rate is higher in the model specifications with inflation expectations. This fact is associated with the greater variation in the errors of the transition equation of the variable z, as can be seen in the table.

Figure 2 presents the estimates of the natural interest rate in the euro area based on the three specifications considered. As identified in the literature, the natural interest rate in the euro area shows a downward trend since 1970, going from levels of 3% to values around 0.5% until the beginning of 2020. The natural interest rate tends to decreasing after periods of recession, registering particularly significant declines after the global financial crisis of 2007/08 and the sovereign debt crisis in 2013/14. These results are in line with the determinants identified in the literature, where the decline in potential output growth and the aging of the population support the downward trend in the level of natural interest rate, accentuated by the search for risk-free assets and changes in rules of banking supervision after the global financial crisis of 2007/08.

Estimates of the natural interest rate based on the different specifications begin to diverge more significantly from 2008 onwards. This divergence can be explained by two reasons. The first is directly related to data for SPF inflation expectations that only begin in 1999 (two-year ahead expectations) and 2001 (four/five-year ahead expectations), so before these dates the three models are being effectively estimated with the same data.

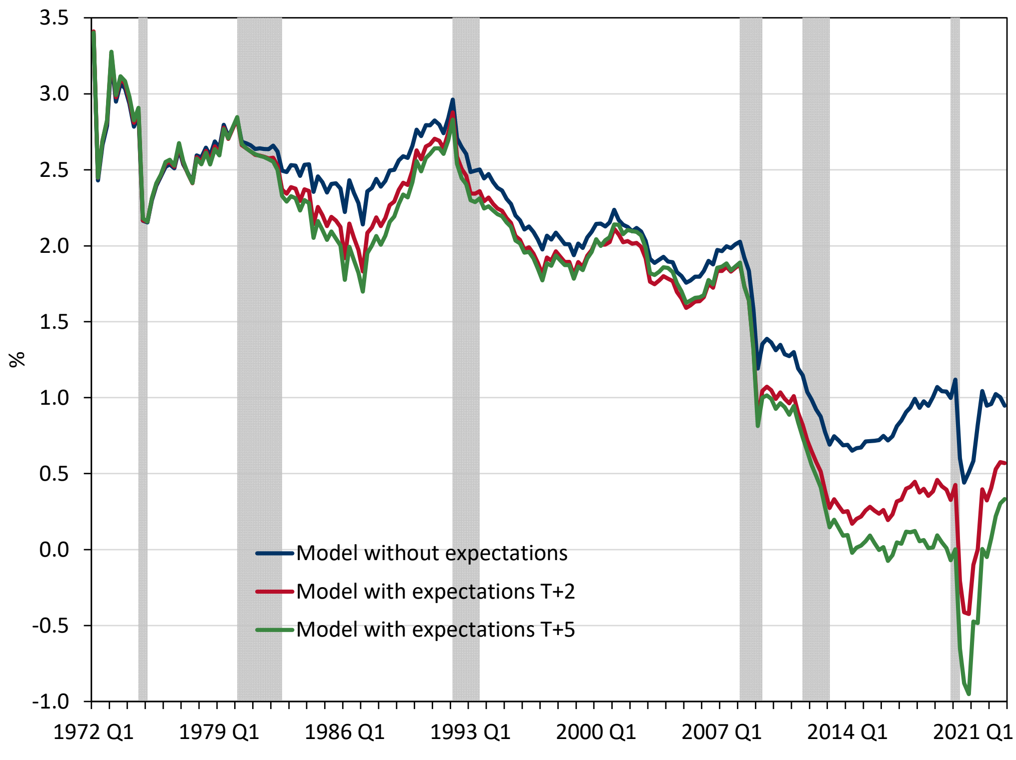

FIGURE 2: Euro area natural interest rate estimates (r_t^*)

Notes: Estimates obtained based on the HLW model. Model without expectations corresponds to the model discussed in Section 3.2 which includes the stringency index in the euro area in the first and second quarter of 2020. Models with expectations T+2 and T+5 correspond to the model discussed in Section 3.3 with inflation expectations two and four/five years ahead of the SPF. Bars in gray mark the periods of recession according to the dating of the Euro Area Business Cycle Network — EABCN.
Sources: ECB, Eurostat, EABCN, Refinitiv and author's calculations.

However, this does not in itself explain the difference from 2008. The second reason is directly related to the evolution of inflation in the euro area. After 2008 headline inflation and core inflation in the euro area declined and remained at relatively low levels until 2019[13]. This decrease leads to a greater difference between the model's inflation measure and the level of inflation expectations considered, which impacts the natural interest rate estimates in the specifications that consider inflation expectations. In fact, estimates based on these specifications show a lower level than those obtained only with the adjustment for the pandemic from 2008 onwards. With the inflation gap having an explicit impact on these specifications and with inflation standing below of the measures of inflation expectations, the model identifies these periods as less accommodative, which would imply a natural interest rate closer to the observed level of the real interest rate[14]. Since the inflation gap is even larger in the model with expectations four/five years ahead, the estimate of the natural interest rate is even lower.

Estimates show that during 2020 the natural interest rate decreased. This evolution is explained by the reduction in potential output growth, since part of the drop in GDP recorded in the euro area during this period continues to be passed on to the natural interest rate, despite the adjustment for the pandemic. In the models with inflation expectations, the drop is even more significant, since in addition to the potential output effect, inflation during this period also dropped significantly, creating another downward pressure on the natural interest rate. Despite this impact, the estimate of the natural interest rate quickly returned to its pre-pandemic level. It is important to note that this model is not suited to fully capture the dynamics of potential output after a shock such as the pandemic. Thus, a more robust assessment of the impact of the pandemic on the natural interest rate will require a model capable of fully capturing its effects.

The most recent estimates at the end of 2022 point to a natural interest rate level in the euro area around 0.5%[15].While the estimate of the model with the adjustment for the pandemic appears to be stable, the estimates based on the models with expectations of inflation have been showing an upward trend. This effect is the opposite of that observed after the 2008 crisis. With the significant rise in inflation observed in the euro area since the beginning of 2022, it is currently above the level of inflation expectations, which the model identifies as a period of greater monetary accommodation, which is converted in the model into an estimate of the natural interest rate above the observed real interest rate.

13. Average core inflation measured by the year-on-year change in the HICP excluding food and energy in the euro area between 2001 and 2007 was 1.8% while between 2008 and 2019 this average was 1.2%.

14. The period after 2008 and up to 2019 was marked by a significant reduction in interest rates from the European Central Bank's policy to fight low inflation. Thus, the *ex-post* real interest rate measure used in the model is negative during this period, on average -0.8%.

15. These results are in line with recent estimates, namely those made by IMF (2023), which put the natural interest rate in Germany and France between 0% and 0.5%.

The assessment of the evolution of the natural interest rate can be complemented by the evolution of the real interest rate gap, measured as the difference between the observed real interest rate and the natural interest rate.

Figure 3 displays the estimates for this gap [16]. The estimated gap was positive from 1980 to 2002 and temporarily between 2006 and 2008. Simultaneously, it is also visible that during periods of recession, the gap tends to decrease. In other words, monetary authorities tend to make monetary policy more accommodative in times of recession. Since 2008, the gap in the euro area has been negative, reflecting the ECB's monetary policy which kept interest rates low in view of the low inflation observed during this

16. The results for the real interest rate gap are not only dependent on the estimate of the natural interest rate used, but also on the calculation method used for the observed real interest rate. In the Graph 3 the measure of the real interest rate is an ex-post rate, which is the one used in the estimation of the model, for consistency.

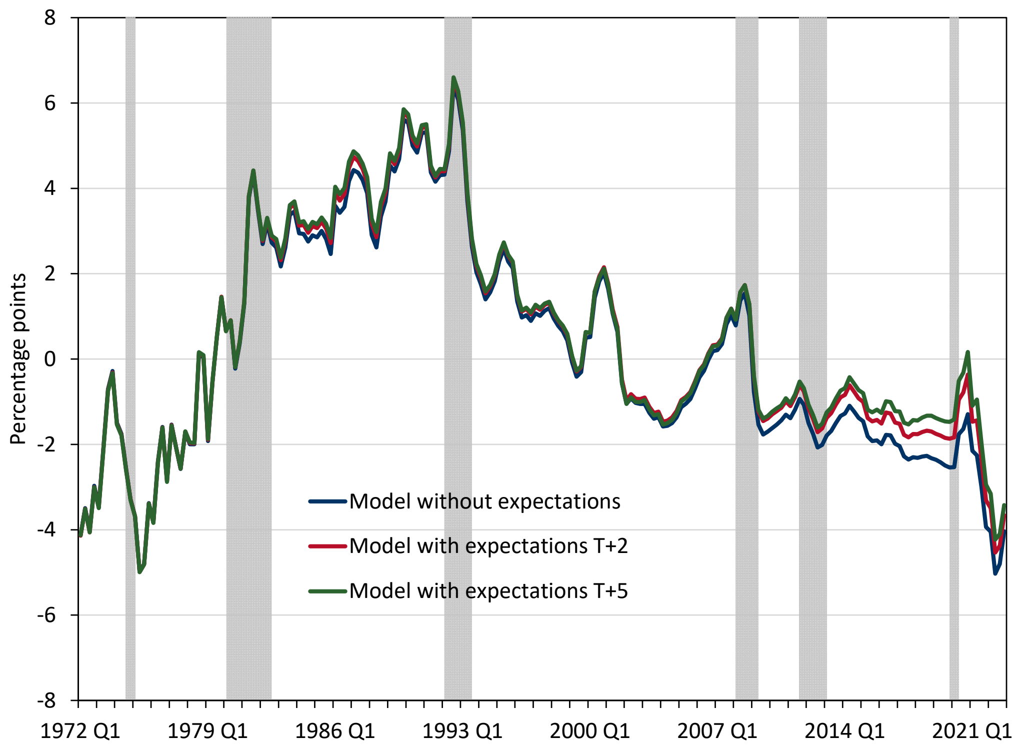

FIGURE 3: Estimates for the euro area ex-post real rate gap $(r_t - r_t^*)$

Notes: Estimates obtained based on the HLW model. Model without expectations corresponds to the model discussed in Section 3.2 which includes the stringency index in the euro area in the first and second quarter of 2020. Models with expectations T+2 and T+5 correspond to the model discussed in Section 3.3 with inflation expectations two and four/five years ahead of the SPF. The real interest rate ex-post is calculated as the difference between the nominal 3-month interbank interest rate in the euro area and average inflation observed in the previous four quarters. Bars in gray mark the periods of recession according to the dating of the Euro Area Business Cycle Network — EABCN.
Sources: ECB, Eurostat, EABCN, Refinitiv and author's calculations.

period. However, it is possible to observe differences between the models, in line with the previous discussion. In the models with inflation expectations, the real rate gap is less negative, taking into account the role of the inflation gap in the model estimation.

In 2020, and taking into account the drop observed in the natural interest rate estimates, the real interest rate gap became less negative, indicating a less accommodative monetary policy. This result was reversed almost instantly, with this gap registering a very significant reduction since 2021. This drop is not just the result of the evolution of nominal interest rates, given that the ECB has in fact raised policy interest rates since July 2022, but it is due to a substantial rise in inflation which leads to a strong decrease in the observed real interest rate. Thus, according to this measure, monetary policy remained accommodative until 2022.

This measure of the real interest rate gap can also be calculated using an *ex-ante* measure of the real interest rate instead of an *ex-post*. Figure 4 shows the real rate gap where real interest rates are calculated as the difference between the three-month nominal interest rate and inflation expectations two years ahead. Comparing the two, it is visible that the gap calculated with real rates *ex-ante* and *ex-post* present very similar behaviours. However, in 2022, the measure with *ex-ante* rates registers a strong rise, despite still remaining at negative levels. This rise reflects not only the less pronounced rise in longer-term inflation expectations when compared to actual inflation, but also the action of the European Central Bank, which has raised interest rates to remove monetary accommodation in order to guarantee the return inflation to its medium-term objective.

It should be noted that measuring the degree of monetary policy accommodation through these measures of the real interest rate gap only measures the accommodation generated by the policy interest rate. This measure does not capture the additional impact that unconventional monetary policy measures have on the monetary policy stance[17]. This distinction should be particularly taken into account in the euro area, where unconventional monetary policy was widely used in the period of low inflation.

5. Considerations for monetary policy

The estimation of the natural interest rate is particularly relevant for monetary policy. Central banks use nominal interest rates as their main instrument to achieve their mandates, with the aim of stimulating or slowing down economic activity and, as a consequence, influence the behaviour of inflation. The natural interest rate makes it possible to infer the levels at which rates should be situated in order to bring about the desired degree of accommodation. However, the practical applicability of the natural interest rate presents some challenges. First, as discussed earlier, the natural interest rate is an unobservable variable with highly uncertain estimates that use observations of economic variables with some time lag. Second, estimates of the natural interest rate

17. Some authors present methodologies to measure the degree of monetary policy accommodation that explicitly capture the impact of unconventional monetary policy measures. See for example Wu and Xia (2016) and Krippner (2013).

show some variability, so if central banks wanted to base their policy decisions on the natural interest rate, they might have to make relatively sharp increases and decreases of interest rates regardless of economic and financial conditions. Thus, the uncertainty surrounding the estimation of the natural interest rate makes its use difficult.

However, the natural interest rate can play an important role in central bank communication to anchor long-term interest rate expectations. By definition, observed interest rates should tend towards levels around the natural interest rate in the long term. This leads to several considerations relevant to monetary policy.

First, it allows an assessment of the ability of the policy interest rates to stimulate the economy. The downward trend observed in the natural interest rate in the euro area and in other advanced economies implies that policy interest rates are at lower levels than those recorded in the past, which implies that the margin available for central banks to lower rates rate will be lower given the existence of an effective lower bound.

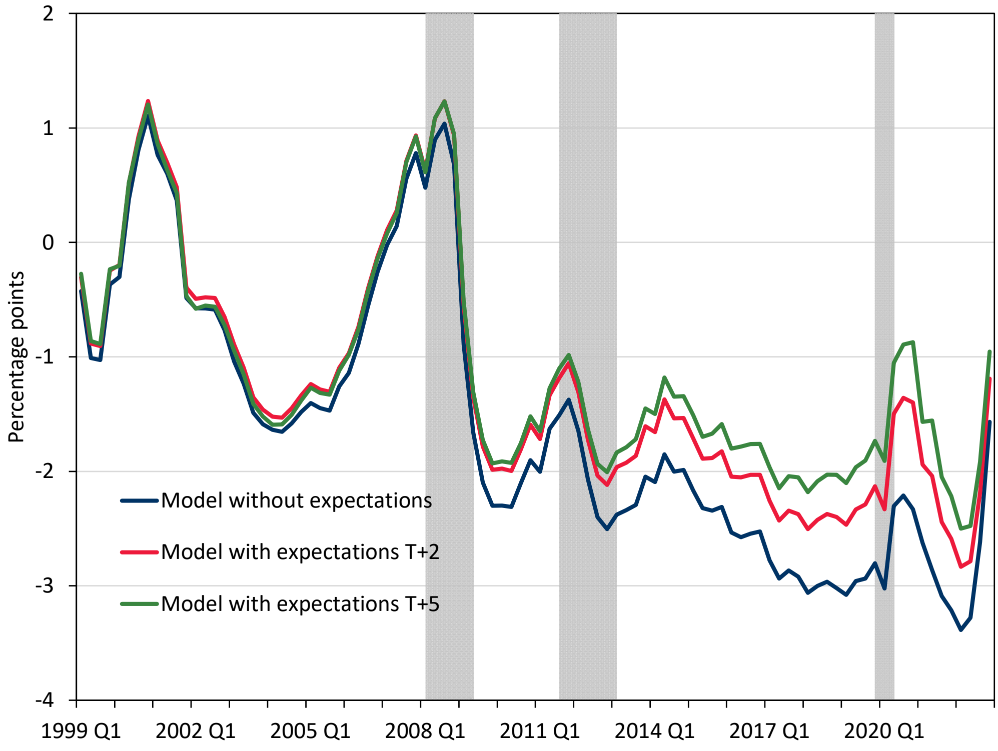

FIGURE 4: Estimates for the euro area ex-ante real rate gap ($r_t - r_t^*$)

Notes: Estimates obtained based on the HLW model. Model without expectations corresponds to the model discussed in Section 3.2 which includes the stringency index in the euro area in the first and second quarter of 2020. Models with expectations T+2 and T+5 correspond to the model discussed in Section 3.3 with inflation expectations two and four/five years ahead of the SPF. The ex-ante real interest rate is calculated as the difference between the 3-month interbank nominal interest rate in the euro area and the total inflation expectation two years ahead of the SPF. Data since 1999, in line with the availability of SPF expectations. Bars in gray mark the periods of recession according to the dating of the Euro Area Business Cycle Network — EABCN.
Sources: ECB, Eurostat, EABCN, Refinitiv and author's calculations.

In fact, the implications of a lower natural interest rate on the performance of central banks is a widely discussed topic[18] and that partly motivated the Federal Reserve's monetary policy strategy review in 2019 and the ECB's monetary policy strategy review in 2020/21[19].

Second, the natural interest rate can anchor the level of interest rates in the long run, which in turn is important to ensure the stabilisation of inflation expectations at a level consistent with the inflation targets of central banks. The long-term relationship between the level of inflation and the interest rate is usually described by the Fisher relation:

$$i_t = r_t + E_t[\pi_{t+1}] \tag{15}$$

where i_t represents a risk-free nominal interest rate, r_t the equilibrium real interest rate (natural) and π_{t+1} the expected inflation rate in the following period. Under the assumption that central banks cannot influence the natural interest rate, there is a direct relationship between the level of inflation and the nominal interest rate in the long run. Thus, and using the natural interest rate estimates presented in Section 4 together with the ECB's medium-term inflation objective of 2%, we obtain that long-term nominal interest rates consistent with this relationship should be situated around 2.5%. This result is particularly important in the current context in which the ECB is raising its policy rates to control the sharp rise in inflation. Any path chosen for the policy interest rate must imply a return to levels consistent with the level of the natural interest rate and the inflation target. If the expectation is created that interest rates will remain at a higher level for a long period of time, it may be the case that inflation expectations adjust upwards to ensure long-term consistency. Thus, a communication that clarifies the level of interest rates in the long term can support the action of central banks to achieve their price stability objectives.

Although the ECB does not explicitly do so, other central banks communicate their expectations for the level of interest rates in the long term. For example, since 2012 the Federal Reserve has released together with the Summary of Economic Projections the perspectives of the members of the Federal Open Market Committee (FOMC) for the level of the policy rate (federal funds rate) in the long run. Although this level of rates is not directly an estimate for the US natural interest rate, it is usually interpreted as the FOMC members' estimate of the natural interest rate plus the Federal Reserve's 2% inflation target. There is evidence (Hillenbrand (2023)) that revisions to the FOMC members' outlook for the long-term policy rate have a significant impact on long-term

18. Andrade *et al.* (2021) discuss the optimal inflation target taking into account the level of the natural interest rate in the area of the euro. The conclusion supports that a lower natural interest rate should lead to a higher inflation target in order to prevent the economy from hitting the effective lower bound of interest rates.

19. In the launch of the ECB's strategy review it is said "Since 2003 the euro area and the world economy have been undergoing profound structural changes. Declining trend growth, on the back of slowing productivity and an ageing population, as well as the legacy of the financial crisis, have driven interest rates down, reducing the scope for the ECB and other central banks to ease monetary policy by conventional instruments in the face of adverse cyclical developments".

market rates. This result may indicate that central banks may benefit from providing some type of guidance on longer-term interest rate levels with a view to ensuring stability in inflation expectations and, thus, facilitating the pursuit of their inflation targets.

6. Conclusion

The estimation of the natural interest rate is relevant for the conduct of monetary policy. The natural interest rate not only allows assessing the degree of monetary policy accommodation, but can also be seen as the level to which interest rates in an economy should converge in the long term. However, as it is an unobservable variable, any inference about its evolution has to be made based on estimates.

This study presents estimates for the natural interest rate in the euro area based on the methodology developed by Holston *et al.* (2017), adapting it for the pandemic period and adding an explicit role for inflation expectations. Despite the uncertainty inherent in this methodology, estimates indicate that the natural interest rate in the euro area has shown a downward trend from the early 1970s until 2019, in line with estimates for other advanced economies. Downward movements tend to follow periods of economic crisis, especially after the global financial crisis in 2007/08. The results indicate that at the end of 2022 the natural interest rate in the euro area was around 0.5%.

Evaluating the results for the real interest rate gap, it is possible to conclude that monetary policy in the euro area tended to become more accommodative after periods of economic crisis. In particular, between 2008 and 2019, the estimate of this gap was always negative, in line with the policy of the European Central Bank. Recently, this measure evaluated with real rates ex-post is at very negative levels, signalling a high degree of accommodation. This accommodation is not directly related to monetary policy decisions, but to the rapid and significant rise in inflation registered in the euro area, which leads to a significant drop in observed real rates. Evaluating the real rate gap with real rates ex-ante it is visible a much more significant increase in the real rate gap during 2022, in line with the policy followed by the European Central Bank which is reducing the degree of monetary policy accommodation to reduce inflation and achieve its objective of price stability.

The use of the natural interest rate in the explicit conduct of monetary policy presents some challenges. A monetary policy that seeks to bring the interest rate to the level of the natural interest rate may not be desirable, since there is high uncertainty about any estimate and may lead to decisions that may not be the most appropriate in the given economic context. However, the use of the natural interest rate in monetary policy communication can play a crucial role in ensuring the consistency of interest rate expectations with inflation targets. Any policy path that does not imply the return of the policy interest rate to levels consistent with the natural interest rate and with the inflation target may lead to losses in the credibility of central banks. The example of the Federal Reserve is evidence that communication about the level of long-term interest rate can influence long-term market interest rate expectations.

References

Andrade, P., J. Gali, and H. Bihan e J. Matheron (2021). "Should the ECB adjust its strategy in the face of a lower r*." *The Journal of Economic Dynamics and Control*, 132(104207).

Barsky, R., A. Justiniano, and L. Melosi (2014). "The Natural Rate of Interest and Its Usefulness for Monetary Policy." *The American Economic Review*, 104(5), 37–43.

Bielecki, M., M. Brzoza-Brzezina, and M. Kolasa (forthcoming). "Demographics, Monetary Policy and the Zero Lower Bound." *Journal of Money, Credit and Banking*.

Brand, C., M. Bielecki, and A. Penalver (2018). "The natural rate of interest: estimates, drivers and challenges to monetary policy." *ECB Occasional Paper Series*, (217).

Brand, C. and F. Mazelis (2019). "Taylor-rule consistent estimates of the natural rate of interest." *ECB Working Paper Series*, (2257).

Del Negro, M., D. Giannone, M. P. Giannoni, and A. Tambalotti (2017). "Safety, Liquidity and the Natural Rate of Interest." *Brookings Papers on Economic Activity*, Spring(48), 235–316.

Fagan, G., J. Henry, and R. Mestre (2001). "An Area-wide Model (AWM) for the Euro Area." *ECB Working Paper Series*, (42).

Gali, J. (2008). *Monetary Policy, Inflation, and the Business Cycle: An Introduction to the New Keynesian Framework*. Princeton University Press, Princeton.

Hale, T., N. Angrist, R. Goldszmidt, B. Kira, A. Petherick, T. Phillips, S. Webster, E. Cameron-Blake, L. Hallas, S. Majumdar, and H. Tatlow (2021). "A global panel database of pandemic policies (Oxford COVID-19 Government Response Tracker." *Nature Human Behaviour*, 5, 529–538.

Hamilton, J. (1986). "A Standard Error for the Estimated State Vector of a State-Space Model." *Journal of Econometrics*, 33, 387–397.

Hillenbrand, S. (2023). "The Fed and the Secular Decline in Interest Rates." *Working Paper*.

Holston, K., T. Laubach, and J. C. Williams (2017). "Measuring the Natural Rate of Interest: International trends and determinants." *Journal of International Economics*, 108, S59–S75.

Holston, K., T. Laubach, and J. C. Williams (2020). "Adapting the Laubach and Wiliamns and the Holston, Laubach and Williams Models to the COVID-19 Pandemic." *Methodological Note*.

IMF (2023). *World Economic Outlook: A Rocky Recovery*, chap. The Natural rate of Interest: Drivers and Implications for Policy, pp. 45–67. IMF Library, Washington DC., April.

Jorda, O., S. R. Singh, and A. M. Taylor (2022). "Longer-run Economic Consequences of Pandemics." *The Review of Economics and Statistics*, 104(1), 166–175.

Krippner, L. (2013). "Measuring the stance of monetary policy in zero lower bound environments." *Economic Letters*, 118, 135–138.

Krustev, G. (2019). "Journal of Economic Behavior and Organization." *The natural rate of interest and the financial cycle*, 162, 193–210.

Laubach, T. and J. C. Williams (2003). "Measuring the Natural Rate of Interest." *The Review of Economics and Statistics*, 85(4), 1063–1070.

Lopez-Salido, D., G. Sanz-Maldonado, C. Schippits, and M. Wei (2020). "Measuring the Natural Rate of Interest: The Role of Inflation Expectations." *FEDS notes - Board of Governors of the Federal Reserve System*.

Luzzetti, M., B. Ryan, J. Weidner, and A. Yang (2022). "(R-)Star gazing: Macro drivers suggest real neutral rate may have risen." *SUERF Policy Brief*, (472).

Mongelli, F., W. Pointner, and M. Van den End (2022). "The effects of climate change on the natural rate of interest: a critical survey." *ECB Working Paper Series*, (2744).

Neri, S. and A. Gerali (2019). "Natural rates across the Atlantic." *Journal of Macroeconomics*, 62(103019).

Papetti, A. (2021). "Demographics and the natural real interest Rate: historical and projected paths for the euro area." *Journal of Economic Dynamics and Control*, 132(104209).

Pedersen, J. (2015). "The Danish Natural Real Rate of Interest and Secular Stagnation." *Danmarks Nationalbank Working Paper*, (94).

Rachel, L. and T. D. Smith (2015). "Secular drivers of the global real interest rate." *Bank of England Staff working Paper*, (571).

Reis, R. (2022). "Which r-star, public bonds or private investment? Measurement and Policy implications." *Working Paper*.

Taylor, J. B. (1993). "Discretion versus Policy Rules in Practice." *Carnegie Rochester Conference Series on Public Policy*, 39, 195–214.

Wicksell, K. (1898). *Interest and Prices: A study of the causes regulating the value of money*. Macmillan, London - English translation by R. F. Kahn in 1936.

Woodford, M. (2003). *Interest and Prices: Foundations of a Theory of Monetary Policy*. Princeton University Press, Princeton.

Wu, J. and F. Xia (2016). "Measuring the Macroeconomic Impact of Monetary Policy at the Zero Lower Bound." *Journal of Money, Credit and Banking*, 48, 253–291.

Appendix: Data description

This appendix presents a summary table of the data used to estimate the natural interest rate according to the methodologies discussed in the study.

Variable	Data	Source
GDP	Euro area GDP (1970-2022)	Area Wide Model and Eurostat
Inflation	HICP headline (1970-1987); HICP exc. energy (1988-1995) and HICP exc. energy and food (1996-2022) in the euro area	Area Wide Model and Eurostat
Nominal interest rate	3 month Euribor (1970-2022)	Area Wide Model and Refinitiv
Stringency	Weighted quarterly average of the OSI of euro area countries based on population	Oxford Stringency Index
Inflation expectations		
Model without expectations	-	-
Model with expectations T+2	Average observed inflation in the past four quarters (1970-1998); Headline inflation expectations 2 year ahead (1999-2022)	Area Wide Model and ECB (SPF)
Model with expectations T+5	Average observed inflation in the past four quarters (1970-2000); Headline inflation expectation 4/5 year ahead (2001-2022)	Area Wide Model and ECB (SPF)

TABLE A.1. Data description

As shown in the table, the inflation series considered uses data from three different HICP series. This occur sdue to the lack of data since 1970 for core inflation in the Euro Area-Wide Model. Thus, an index is constructed based on the available aggregates. From 1970 to 1987 the measure considered is total inflation, between 1988 and 1995 the HICP measure excluding energy goods is used and since 1996 the HICP measure excluding food and energy. It is important to note that this aggregation implies that the inflation series used in the model potentially has two structural breaks. This treatment is also used by Holston *et al.* (2017) in estimating the natural interest rate in the euro area.