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The *M* Model: a macroeconomic model for the Portuguese economy

Gabriela Castro
Banco de Portugal

Cláudia Duarte
Banco de Portugal

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Abstract
Macroeconomic models offer valuable insights to improve economic reasoning and to help interpreting agents' behaviour, albeit being an imperfect representation of reality. This article presents a general description of the current version of the *M* model, a macroeconomic quarterly model for the Portuguese economy, which has been developed at the Economics and Research Department of Banco de Portugal since the early 2000's. This semi-structural model embodies a compromise between theoretical foundations, anchored on the so-called neoclassical synthesis, and a more flexible approach to better fit the data. This type of models remains a common and useful tool, due to its pragmatic approach to the changing economic reality. The interconnectedness of economic relations is considered through linkages between the several blocks of the model, which include demand, supply, wages, prices, labour market, financial and fiscal variables. The *M* model is used for different purposes and is part of the toolkit for projection exercises and scenario analyses. An illustration of its dynamic properties in the short and medium term is provided through the simulation of five shocks: foreign demand, public consumption, exchange rate, oil price and interest rate. (JEL: C32, C53, E17)

1. Introduction

This article presents a general description of the key features of the *M* model, a macroeconomic quarterly model for the Portuguese economy. This model has been developed at the Economics and Research Department of Banco de Portugal and its initial version dates back to the early 2000's. The *M* model is a semi-structural model, which embodies the so-called neoclassical synthesis. Its structure builds on the experiences of other central banks and international institutions with macroeconometric models, although some specificities of the Portuguese economy are explicitly accounted for, such as the participation in the euro area.

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E-mail: ggcastro@bportugal.pt; cfduarte@bportugal.pt

The model is one of the tools used in the projection exercises published in Banco de Portugal's Economic Bulletin. There are four projection exercises in each year—March, June, October and December. The June and December projections are produced within the Eurosystem framework, with the collaboration of the European Central Bank (ECB) and the National Central Banks, as described in ECB (2016). The projection horizon varies across exercises, covering between two to three years ahead of the current year.

In addition to medium-term projections, the *M* model is also used for scenario analyses. These analyses can include the assessment of risks, for instance the stress testing of the financial sector, as well as economic policy simulations. The *M* model also underlies a tool called Basic Model Elasticities (BME), which provides mechanical impacts of shocks on the economy, such as changes in the exogenous assumptions underlying the projection exercises. This tool has been used in several sensitivity analyses, such as Banco de Portugal (2019, 2022a,b).

Given the variety of utilisations, the *M* model is a flexible, though quite detailed, tool that is focused on both short- and medium-term purposes. The model incorporates the interdependencies of a broad set of variables, which includes aggregate demand, international trade, potential output, wage and price formation, labour market developments, financial and fiscal variables.

The model design also accounts for the need to track recent developments in macroeconomic data. This requires a compromise between theoretical foundations, which anchor the supply block, and a more flexible specification of the demand block, to better fit the data. Moreover, the model is fitted to the data through a combination of econometric estimation, based on error-correction equations, and calibration.

This type of macroeconomic models remains a common and useful tool for macroeconomic analyses and projections, as well as for assessing policy interventions (Pareja *et al.* 2017; Bulligan *et al.* 2017; Berben *et al.* 2018). Flexibility is one of the key factors for its longevity, which facilitates a pragmatic approach to the changing economic environment. The *M* model has undergone several changes over time and is regularly re-estimated, as new data for the Portuguese economy becomes available (e.g. new National Accounts series).

The remainder of this article is organised as follows. Section 2 provides an overview of the model. Section 3 explains key details about the model's blocks—supply, demand, prices and wages, labour market, financial sector and public sector. In Section 4 the dynamic properties of the model are illustrated with some simulation results. Section 5 concludes.

2. A bird's eye view of the model

The *M* model is a medium-sized semi-structural model, with almost 30 behavioural equations. The basic theoretical elements are fairly standard and are in line with the ECB's Area-Wide Model, presented in Fagan *et al.* (2001) and Fagan *et al.* (2005), and the country blocks of the European System of Central Banks's Multi-Country Model, such as Villetelle and Boissay (2005) and Warmedinger and Vetlov (2006). The theoretical foundation of the behavioural equations combines both neoclassical and Keynesian elements. The time dimension plays a crucial role in this synthetic approach.

The supply block is the anchor for the long-run properties of the model, which essentially correspond to the basic framework of neoclassical economic theory. In particular, aggregate supply is determined by available resources and technological progress, through a production function. Firms' decisions on prices and production factors are derived from a profit maximization problem, assuming perfect competition in input markets and monopolistic competition in output markets, where producers set a markup over the marginal cost. In the long run, the aggregate supply curve is vertical, so that prices do not matter for the supply level. Inflation is, therefore, a nominal phenomenon.

A key concept of the supply block is potential output, which corresponds to the maximum level of production, with full employment that does not trigger excessive inflationary pressures (Okun 1962). The concept of full employment is linked to the Non-Accelerating Wage Rate of Unemployment (NAWRU), i.e. the unobserved unemployment rate that does not trigger excessive wage pressures. Both these elements are exogenous to the model. In the long run, actual production equals potential output, i.e., the output gap is nil, which implies full capacity utilisation. In the labour market, the actual unemployment rate equals the NAWRU in the long run, i.e., the unemployment gap is nil. Long-run potential output growth reflects exogenous assumptions on technology and population developments and all real variables must grow at this rate.

In the short run, prices and wages are sluggish to adjust, reflecting, for instance, the existence of contracts and transaction costs. In this context, output is determined by demand and, therefore, mismatches between potential and actual output can arise. The over-utilisation (underutilisation) of production capacities translates into positive (negative) output gaps in the product market and into negative (positive) unemployment gaps in the labour market. These mismatches on product and labour markets are key factors in the price setting by firms and wage setting through bargaining, in a framework akin to price and wage Phillips curves. Deviations of aggregate demand from potential supply trigger wage and price adjustments, which contribute to move the economy towards the equilibrium.

The *M* model explicitly accounts for the fact that the Portuguese economy participates in the euro area. Consistent with this framework, all variables describing the world economic conditions, such as commodity prices, market interest rates and nominal exchange rates, are treated as exogenous variables, and are included in the set of common external assumptions within the Eurosystem projection exercises (Figure 1). This set of assumptions also includes foreign demand and competitors' prices. The Portuguese economy is sufficiently small for domestic shocks to have no effect on the rest of the world and for domestic conditions to have no influence on international prices and output. In the long run, the domestic inflation rate is driven by the exogenous growth rate of foreign prices, and all prices must grow at this rate.

The expectation formation process of agents is assumed to be backward looking, i.e. adaptive. The use of backward-looking expectations in semi-structural models is quite common, as in, for example, Banca d'Italia econometric model (Bulligan *et al.* 2017) and De Nederlandsche Bank macroeconomic policy model (Berben *et al.* 2018). In this context, equations may include lagged terms to proxy expected variables. Moreover, lags may also signal that agents take some time to react. As a consequence, product and labour markets mismatches may persist for long periods of time.

2.1. Data and estimation strategy

The dataset used to estimate the M model spans the period from $1999Q1$ to $2019Q4$ and mainly covers the Quarterly National Accounts released by Statistics Portugal. Whenever needed, this information set is complemented by the quarterly historical series presented in Cardoso and Sequeira (2015), which are regularly updated. In projection exercises, observed data is prolonged with short-term projections for the current quarter and one quarter ahead. These short-term projections draw heavily on a set of bridge models, which use a comprehensive dataset of high-frequency indicators, as described in Esteves and Rua (2012).

The specification of the behavioural equations follows an error-correction model. This type of specification preserves structural links between variables that share common trends, due to embodied cointegrating relationships, to which they converge in the long run. In this framework, both short- and long-run features of economic agents' behaviour are taken into account. Developments of a given variable depend in the long run on the levels of its determinants (i.e., the cointegration relationship), but are influenced by changes in variables included in the short-term dynamics. Equation (1) presents a generic and simplified version of this model:

$$\Delta y_t = A(L)\Delta y_{t-1} + B(L)\Delta x_t + C(L)\Delta z_t + \lambda(y_{t-1} - C - \varphi x_{t-1}), \quad (1)$$

where y_t is the variable of interest, x_t is the determinant in the long run, z_t influences the short-run dynamics, C is a constant, $A(L), B(L)$ and $C(L)$ are lag polynomials, Δ denotes first differences and lower-case variables denote logarithms, i.e. $y_t = \log(Y_t)$ and, conversely, $Y_t = \exp(y_t)$. Extended versions of this specification can include more than one variable in the long-run relationship and in the short-run dynamics. The coefficient λ is the error-correction coefficient, which reflects the speed of convergence back to the long-run relationship. Cointegration exists if this coefficient is significantly different from zero and negative. In some cases, deterministic terms, such as step dummies or trends, must be included in the long-run relationship in order to obtain a stationary error-correction term. Moreover, dummy variables may be added to the estimated equations to account for outliers. These deterministic terms play an in-sample role, because they contribute to improve the statistical fit of the model. However, they remain constant in the last figure of the estimation sample over the projection horizon. For the sake of simplicity, deterministic variables are not presented in this article.

The equations are estimated separately with ordinary least squares, using the two-step procedure proposed by Engle and Granger (1987). The first step consists in estimating the long-run, co-integrated relationships. In the second step, the dynamic equations are estimated in the error-correction format.

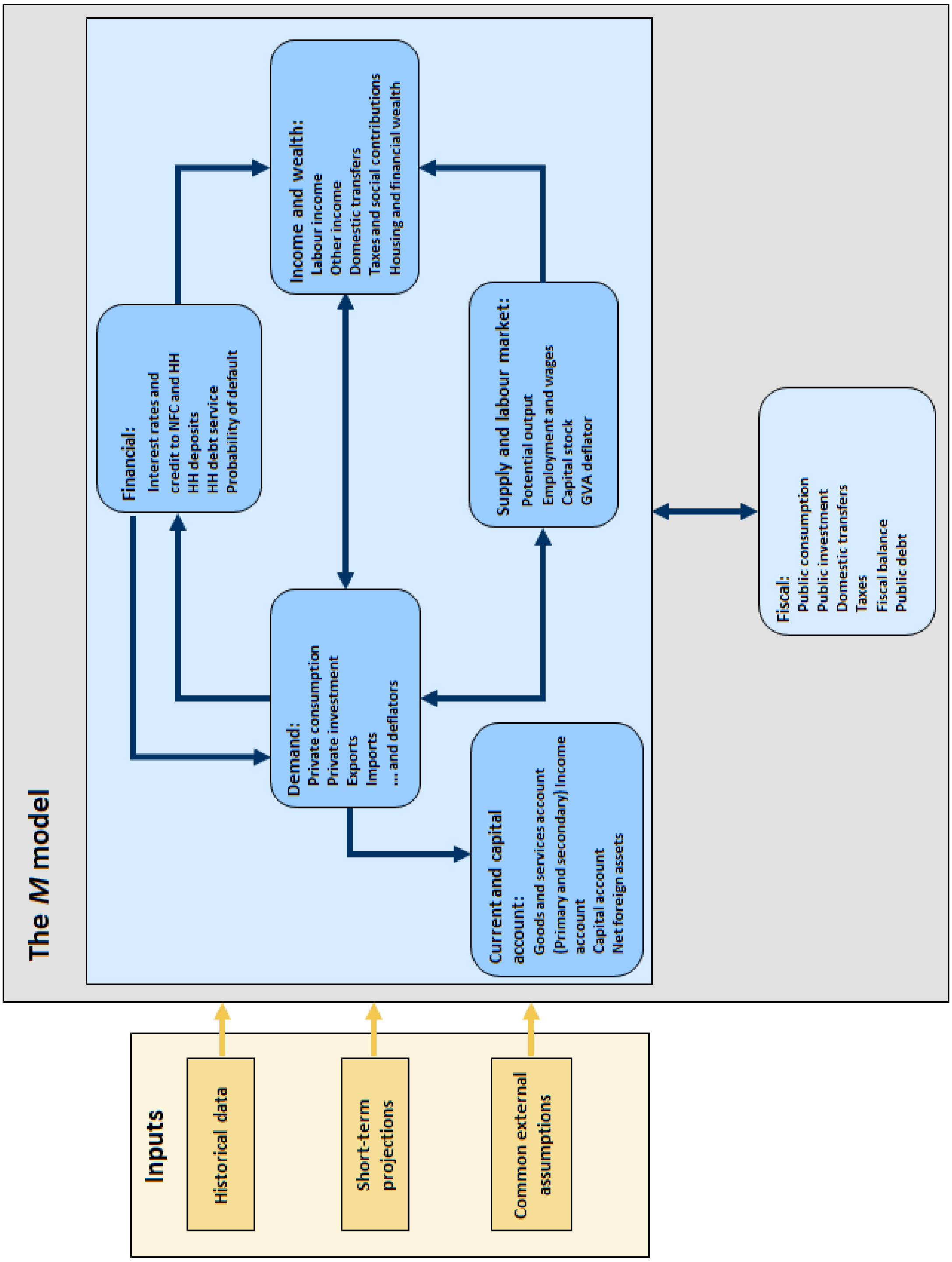

FIGURE 1: Summary of *M* model's structure

Source: The authors.

Notes: GVA - Gross value added. HH - Households. NFC - Non-financial corporations.

The short-term dynamics is estimated more freely and is not heavily conditioned by theory. Initially, all relevant variables and lags are included, but only those that are statistically significant remain in the final specification. The duration of short-term effects, as well as the speed of adjustment to the long-run equilibrium, depends exclusively on the estimation outcome.

The extent of short-term effects and the coefficients in the long-run relationship result from a mix of estimation and calibration. The criteria for calibration involve theoretical restrictions and assumptions that yield more plausible projections. Theoretical restrictions mainly result from the need to impose that the model converges to a stable solution in the long run, i.e. a balanced-growth path (static homogeneity) and that the static and dynamic equilibrium solutions of the model coincide (dynamic homogeneity). When the data clearly reject this restriction, dynamic homogeneity is imposed in its simplest form, which involves adding a short-term constant that guarantees consistency between static and dynamic steady-state growth rates for the dependant variable (Stoevsky and Consolo 2016). For the sake of brevity, these constants are omitted from the equations.

3. Properties of the main equations

This section presents the most important equations grouped according to the main blocks in the model. It also provides information on the most relevant parameters of interest. A comprehensive listing of variable names and corresponding description is provided in the Online Appendix A. The Online Appendix B contains the full set of equations. In addition to the estimated equations, the model also includes a large set of identities, which reflect the national accounts framework, and definitions, such as bottom-up aggregation procedures in order to obtain broader concepts from more detailed elements.

3.1. *Supply*

The M model splits the economy into three sectors, namely (i) private non-housing, (ii) housing and (iii) public sector. Potential gross value added (GVA) in the private non-housing sector ($yftp_t$) features a Cobb-Douglas production function with potential labour supply ($lntp_t$) and capital stock ($ksrp_t$) as production factors, as well as trend total factor productivity (tft_t), formulated as Hicks-neutral technical progress:

$$yftp_t = tft_t + \alpha \cdot lntp_t + (1 - \alpha) \cdot ksrp_t, \tag{2}$$

where the parameter α represents the elasticity of $yftp_t$ with respect to $lntp_t$ and lower-case variables denote logarithms. This elasticity is proxied by the share of labour compensation on GVA. Currently, the α parameter is calibrated to around 60%, corresponding to the average share of labour compensation over 1999-2019.

Potential private sector labour supply is exogenous to the model, and is calculated by multiplying actual labour force by the NAWRU estimates, obtained using the methodology described in Duarte *et al.* (2020), and subtracting public sector employment. Over the projection horizon, labour force projections take on board the demographic projections released by the Eurostat and Statistics Portugal, while the NAWRU follows a random walk process, i.e., remains constant in the last quarter of the estimation sample. The capital stock is obtained through the standard capital accumulation equation, in which the depreciation rate is exogenous to the model (see equation B.3 in the Online Appendix B). Trend total factor productivity is calculated by applying the Hoddrick-Prescott filter to the Solow residual, obtained by applying equation (2) to actual GVA and employment in the private non-housing sector (equation B.4). Over the projection horizon, tft_t is assumed to follow the steady-state growth path.

The supply side anchors the long run of the model. Thus, long-run relationships for the capital stock ($ksrp_t^*$), price level ($yfdp_t^*$) and real wages ($wrnp_t^*$) follow from the first-order conditions of profit maximising producers, in combination with exogenous labour supply, technology and capital financing costs ($RCCP_t$). In particular,

$$wrnp_t^* = \log(\alpha) + yftp_t - lntp_t \tag{3}$$

$$yfdp_t^* = wunp_t + lntp_t - yftp_t - log(\alpha) \tag{4}$$

$$ksrp_t^* = \alpha \cdot \log\left(\frac{1-\alpha}{\alpha}\right) + \alpha \cdot (wunp_t - ccp_t) + yftp_t - tft_t, \tag{5}$$

where $wunp_t$ denotes the nominal wage and $ccp_t = yfdp_t + \log(RCCP_t)$.

The M model considers a simplified approach to account for housing and public sectors. Output of the housing sector corresponds to imputed and actual rents ($pcrr_t$), which are also included in the households' disposable income account (see Subsection 3.2 for more details). The public sector output ($yerg_t$) corresponds to compensation to public employees plus gross operating surplus and received subsidies on production (see Subsection 3.6 for more details). The housing stock ($ksrh_t$) and the public capital stock ($ksrg_t$) are obtained through standard capital accumulation equations.

3.2. *Demand*

The production function framework holds in the long run only. In the short run, output is determined by aggregate demand, which results from combining the main components of gross domestic product (GDP, YER_t) on the expenditure side—private consumption (PCR_t), public consumption (GCR_t, see Subsection 3.6 for more details), gross fixed capital formation (ITR_t), changes in inventories (SCR_t), exports (XTR_t) and imports (MTR_t).

$$YER_t = PCR_t + GCR_t + ITR_t + SCR_t + XTR_t - MTR_t \tag{6}$$

In the M model, the specification of private consumption is influenced by the life-cycle and permanent income hypotheses. Households try to smooth the path of consumption over time based on their expected lifetime income with a more persistent nature. In the long run, consumption depends on both adjusted real disposable income ($pyra_t$) and real (financial fwr_t and housing hwr_t) wealth.

Private consumption is disaggregated into four components, namely housing services (i.e., imputed and actual rents), durables, fuels, and other goods and services. The latter component is the most important in terms of spending and has the following equation:

$$\Delta pcro_t = \beta^{pcro} \cdot \Delta pyra\_mm_t + \gamma^{pcro} \cdot \Delta URX_t + \varphi^{pcro} \cdot \Delta STR^D_{t-3} + \lambda^{pcro}(pcro_{t-1} - C^{pcro} - \psi^{pcro} \cdot pyra_{t-1} - \zeta^{pcro} \cdot fwr_{t-1} - (1 - \psi^{pcro} - \zeta^{pcro}) \cdot hwr_{t-1}), \quad (7)$$

where $pyra_t$ comprises net compensation of labour, transfers and other income, which is adjusted by the debt servicing, housing rents and financial intermediation services indirectly measured. Income and wealth in real terms are obtained by deflating the nominal aggregates with the private consumption deflator (see Subsection 3.3). Nominal housing wealth is computed as the real housing stock valued at current house prices (equations B.20 and B.31).

The long-run parameter ψ^{pcro} represents the marginal propensity to consume out of income, while ζ^{pcro} and $(1 - \psi^{pcro} - \zeta^{pcro})$ are the marginal propensities to consume out of financial and housing wealth, respectively. These parameters add up to one in order to impose the static homogeneity condition. This specification is also compatible with the assumption that there are two types of households in the economy—those who have liquidity constraints (consuming a fraction of their current disposable income) and those who have not (consuming a fraction of their wealth).

The short-run dynamics is influenced by developments in the real interest rate of deposits (STR^D_t), which proxies an opportunity cost, and the unemployment rate (URX_t), as a measure of households' uncertainty, both with a negative impact on $pcro_t$, in line with economic theory. Moreover, a moving average of the adjusted real disposable income ($pyra\_mm_t$) is also included, which allows to smooth the reaction to transitory shocks in income. This formulation contributes to a procyclical savings rate, in line with historical patterns (Alves and Cardoso 2010).

The consumption of durable goods comprises the adjusted real disposable income and financial wealth, as well as STR^D_t and URX_t in the long-run relationship, while the short run is governed by $\Delta pyra_t$ (equation B.12). Thus, the consumption of durable goods shows a more cyclical pattern, in accordance with historical evidence. Consumption of housing services and of fuels evolve according to simple rules, the former taking into account the housing stock.

Regarding gross fixed capital formation, the model considers a breakdown by institutional sector—private non-housing (ipr_t), housing (ihr_t) and public investment (see Subsection 3.6 for more details). In the long run, private non-housing investment is pinned down by the capital stock derived from the first-order conditions of the profit maximising producers ($ksrp^*_t$ in equation 5). Thus, the actual capital stock converges to its equilibrium level and ipr_t matches capital depreciation adjusted for the exogenous assumptions on technology and population growth, so that the investment to capital stock ratio is constant.

The short-run dynamics of ipr_t is influenced by fluctuations in private GDP ($yerpr_t$) and some inertia. Additionally, investment decisions also depend (negatively) on financing conditions and on entrepreneurs' assessment of the economic outlook and its uncertainty. Uncertainty is proxied by the composite indicator of financial stress ($ICSF_t$), introduced by Braga *et al.* (2014):

$$\Delta ipr_t = \beta^{ipr} \cdot \Delta yerpr_t + (1 - \beta^{ipr}) \cdot \Delta ipr_{t-1} + \gamma^{ipr} \cdot \Delta STR^B_{t-4} + \varphi^{ipr} \cdot ICSF_{t-2} + \lambda^{ipr}(ipr_{t-1} - ksrp^*_{t-1} - \log\left(\frac{g+\delta}{1+g}\right)), \quad (8)$$

where δ is the exogenous depreciation rate of the capital stock in the private non-housing sector and g denotes the exogenous assumption on technology and population growth, both in the steady state. Parameters β^{ipr} and $(1 - \beta^{ipr})$ add up to one in order to impose the dynamic homogeneity condition. Moreover, the estimated value of β^{ipr} is above one, in line with the investment accelerator effect.

Housing investment mainly reflects demand conditions from domestic households, which are similar to those that determine private consumption. The long-run level of ihr_t is influenced by the adjusted real disposable income and the unemployment rate. The short-run dynamics depends on developments in real disposable income, in the real interest rate of housing mortgages, in the unemployment rate and in Tobin's q for housing investment. As discussed in Mankiw (2002), Tobin's q compares the market price of housing (ihx_t) with its replacement cost (proxied by the housing gross fixed capital formation deflator, ihd_t). In addition to housing demand by households who plan to live in it, this variable tries to capture the incentives to invest in housing, as a non-financial asset:

$$\Delta ihr_t = \beta^{ihr} \cdot \Delta pyra\_mm_t + \gamma^{ihr} \cdot \Delta URX_{t-2} + \varphi^{ihr} \cdot \Delta STR^H_t + \xi^{ihr} \cdot \Delta(ihx_{t-2} - ihd_{t-2}) + \lambda^{ihr}(ihr_{t-1} - C^{ihr} - pyra_{t-1} - \psi^{ihr} \cdot URX_{t-1}), \quad (9)$$

where γ^{ihr} and φ^{ihr} have negative values.

The observed data for changes in inventories are extremely volatile and do not seem to be correlated with demand, supply or financial conditions. Thus, a simplifying technical assumption is used—the evolution of changes in inventories, as well as the statistical discrepancy associated with chain-linked data, is assumed to be neutral in terms of its impact on real GDP growth.

Turning to external flows, exports (xtr_t) and imports (mtr_t) are modelled in a standard way, where market shares—computed in relation to aggregate foreign demand and final demand, respectively—are a function of a price competitiveness indicator, which compares domestic and foreign prices. Exports and imports of goods excluding fuels, of fuels and of services are modelled separately. In the latter case, the model also accounts for a further disaggregation of exports into tourism and other services.

Exports of goods excluding fuels ($xtro_t$) are largely determined by a foreign demand indicator (wdr_t), which is a trade-weighted average of imports by trading partners (Hubrich and Karlsson 2010). In the long run, the equation also accounts for a price competitiveness indicator, which compares developments in GDP deflators of main trading partners (yed_t^F) with the domestic value-added deflator ($yfdp_t$). The parameters associated with wdr_t are set to one, due to the static and dynamic homogeneity restrictions.

$$\Delta xtro_t = \Delta wdr_t + \lambda^{xtro}(xtro_{t-1} - C^{xtro} - wdr_{t-1} - \psi^{xtro}(yed_{t-1}^F - yfdp_{t-1})) \qquad (10)$$

The equation for tourism exports ($xtrt_t$) is quite similar, with specific tourism-related foreign demand and price competitiveness indicators (equation B.16). Simple rules are used for exports of fuels ($xtre_t$), depending on foreign demand developments, and for exports of other services ($xtrs_t$), considering an average of developments in exports and imports of goods excluding fuels and exports of tourism. This weighted average tries to capture insurance and transportation services, which represent a significant share of $xtrs_t$.

Imports of goods excluding fuels ($mtro_t$) depend on the weighted final demand indicator, excluding fuels (wer_t) and on a price competitiveness indicator, which compares imports deflator ($mtdo_t$) with the deflator of domestically produced goods in the private sector ($yfdpr_t$). This specification assumes (imperfect) substitutability between imported and domestically produced goods. The wer_t indicator is calculated as a weighted average of final demand components taking into account their import content, which is calculated following the methodology in Cardoso and Rua (2021).

$$\begin{aligned}\Delta mtro_t =& \beta^{mtro} \cdot \Delta wer_t + (1 - \beta^{mtro}) \cdot \Delta wer_{t-1} + \\ & \lambda^{mtro}(mtro_{t-1} - C^{mtro} - wer_{t-1} - \psi^{mtro}(mtdo_{t-1} - yfdpr_{t-1}))\end{aligned} \qquad (11)$$

Imports of fuels ($mtre_t$) are modelled in a similar way to $mtro_t$, depending on a weighted final demand for fuels and import prices of fuels in real terms (equation B.18). The long-run parameter on relative prices is smaller than in equation (11), reflecting a less elastic demand for imports of fuels. Finally, imports of services ($mtrs_t$) are driven by exports and imports of goods excluding fuels. This simple rule tries to capture the need for financing, insurance and transportation services.

The trade balance is computed by combining real developments for exports and imports with their respective prices (see Subsection 3.3). This balance is an important element of the current and capital account, which reflects the net lending/borrowing position of the economy. The other elements are the (primary and secondary) income and capital accounts. These accounts are modelled as a mix between simple rules for the interest-bearing and -paying items, including public debt, and exogenous information (e.g. related with EU transfers, with the exception of taxes paid and the national contribution for the EU budget). Current and capital account balances cumulatively change the net international investment position. Every year the net flow is added to the previous year stock of assets, ignoring changes in valuation.

3.3. *Prices and wages*

The two main equations in this block refer to the GVA deflator ($yfdp_t$) and nominal wages ($wunp_t$) in the private non-housing sector. Starting with the GVA deflator, the long-run relationship ($yfdp_t^*$) is derived from the supply block (see Subsection 3.1 for more details), while the short-run dynamics is driven by the deviation between actual demand and potential supply in the product market, i.e., the output gap ($ygat_t$), and by developments in trend unit labour costs ($ultp_t$), with some lags:

$$\Delta yfdp_t = \beta^{yfdp} \cdot ygat_t + \Sigma_{i=1}^{4} \gamma_i^{yfdp} \cdot \Delta ultp_{t-i} + \lambda^{yfdp}(yfdp_{t-1} - yfdp_{t-1}^*) \tag{12}$$

where $\Sigma_{i=1}^{4} \gamma_i^{yfdp} = 1$ in order to ensure dynamic homogeneity. The GDP deflator is obtained through the accounting identities linking market prices to basic prices.

Long-run nominal wages are also derived from the supply block, using the GVA deflator and real wages ($wrnp_t^*$). The short-run dynamics includes deviations of the unemployment rate from the NAWRU, i.e., the unemployment gap (UGA_t) and current and lagged developments in output per worker ($yerp_t - lnnp_t$). Moreover, expected inflation is also included, measured by lags of changes in the private consumption deflator (pcd_t):

$$\begin{aligned}\Delta wunp_t =& \beta^{wunp} \cdot UGA_{t-1} + \Sigma_{i=0}^{3} \gamma_i^{wunp} \cdot \Delta(yerp_{t-i} - lnnp_{t-i}) + \Sigma_{i=0}^{2} \varphi_i^{wunp} \cdot \Delta pcd_{t-i} + \\ & \lambda^{wunp}(wunp_{t-1} - yfdp_{t-1} - wrnp_{t-1}^*),\end{aligned} \tag{13}$$

where $\Sigma_{i=0}^{3} \gamma_i^{wunp} = 1$ and $\Sigma_{i=0}^{2} \varphi_i^{wunp} = 1$ in order to ensure dynamic homogeneity.

Prices and wages are crucial for the model to move towards the long-run equilibrium. Positive (negative) pressures in the output and labour markets are associated with a positive (negative) output gap and a negative (positive) unemployment gap, which translate into positive (negative) price and wage pressures (with a positive β^{yfdp} and a negative β^{wunp}) until the long-run equilibrium is again achieved. Moreover, the so-called "output per worker gap" represents an additional implicit channel connecting demand-driven pressures to inflation—the equation for $yfdp_t$ considers trend output per worker, while the equation for $wrnp_t$ includes actual output per worker.

Given that private domestic demand combines domestic inputs with imports, its deflators—following the same breakdown as in Subsection 3.2—result from a combination of the GVA deflator with the imports deflator. In the long run, the shares of these two components are calibrated according to the specific import content (Cardoso and Rua 2021). The dynamic equations include the same variables as in the long run, extended with lags of the dependent variable whenever deemed necessary, coupled with the restriction that dynamic homogeneity holds (equations B.24 to B.27).

The three exceptions to the standard formulation of domestic demand deflators are the consumption of housing services, the consumption of fuels and changes in inventories. In the first case, a simple rule is used. In the second case, oil prices and specific taxes are taken into account. Lastly, the deflator of changes in inventories is such that the national accounts aggregation condition for GDP in nominal terms holds.

Regarding external deflators, the M model assumes that the Portuguese economy is a price taker in international markets. Thus, export and import deflators—with the same breakdown as in Subsection 3.2—mainly follow competitors' prices on the export and import side in the long run, which are taken from the set of common assumptions underlying Eurosystem's projection exercises. The short run captures the pass-through dynamics of a shock in foreign prices, which may reflect, for instance, time and contractual lags or composition effects (equations B.28 and B.30). In the case of fuels, export and import deflators are linked to developments in oil prices. The deflators of exports of other services and of imports of services use simple rules. The exception to the price-taker framework is the deflator of exports of tourism, which is determined by domestic consumer prices (equation B.29).

3.4. *Labour*

In the labour block, the main equation refers to employment in the private non-housing sector, measured in full-time equivalents. In the long run, the level of employment is determined by the exogenous labour supply ($lntp_t$). Employment growth in the short run depends on developments in real wages and output, both adjusted for trend total factor productivity.

$$\Delta lnnp_t = \beta^{lnnp} \cdot \Delta(wunp_t - yfdp_t - (tft_t/alpha)) + \Sigma_{i=0}^{1}\gamma_i^{lnnp} \cdot \Delta(yerp_{t-i} - (tft_{t-i}/alpha)) + \Sigma_{i=1}^{3}\varphi_i^{lnnp} \cdot \Delta lnnp_{t-i} + \lambda^{lnnp}(lnnp_{t-1} - lntp_{t-1}) \quad (14)$$

As expected from theory, employment growth is negatively related to real wage growth ($\beta^{lnnp} < 0$). The short-run elasticity of employment growth with respect to real output is about 0.4, split over two quarters ($\Sigma_{i=0}^{1}\gamma_i^{lnnp} \simeq 0.4$). The equation also includes lags of the growth rate of the dependent variable, to mimic employment inertia.

By assumption, no employment is considered in the housing sector, while employment developments are exogenous in the public sector (see Subsection 3.6 for more details).

3.5. *Financial sector*

The financial block accounts for interactions between the real economy and the financial sector in a stylised way. Under normal financing conditions and complete markets, interest rates clear the market for any credit demand level and, therefore, credit to the private sector can be obtained from the demand curve, under the assumption of a perfectly elastic credit supply. Changes in interest rates affect (i) the cost of capital and capital formation by firms, (ii) the housing market, both through real investment and house prices, and (iii) the intertemporal substitution underlying consumption decisions. Moreover, households' consumption and investment decisions are also influenced by income and wealth effects triggered by developments in the financial markets.

This block includes equations for bank lending rates and outstanding credit to the private sector in three different segments: non-financial corporations, mortgage lending to households and consumer credit. Households' debt service can be obtained by combining households' credit and lending rates.

The specification for bank lending rates is very similar across the different segments. Previous work on this topic can be found in Castro and Santos (2010). The long-run relationship is determined by the reference interbank interest rate (the 3-month EURIBOR, $EURIBOR_t$) and by an aggregate default probability indicator (PD_t). In the short run, developments in the lending rates reflect the evolution of $EURIBOR_t$, an assessment of the uncertainty outlook, proxied by changes in URX_t, and some inertia:

$$\begin{aligned}\Delta STN_t^k =&\beta^{STN^k} \cdot \Delta STN_{t-1}^k + \Sigma_{i=0}^1 \gamma_i^{STN^k} \cdot \Delta EURIBOR_{t-i} + \varphi^{STN^k} \cdot \Delta URX_{t-1} + \\ &\lambda^{STN^k} (STN_{t-1}^k - C^{STN^k} - \psi^{STN^k} . EURIBOR_{t-1} - \zeta^{STN^k} \cdot PD_{t-1}),\end{aligned} \quad (15)$$

where $k = \{B, C, H\}$ denotes the segment, namely non-financial corporations, consumer credit and mortgage lending to households, respectively, and $\varphi^{STN^H} = 0$. The pass-through of $EURIBOR_t$ to financing costs is complete in the long run, except in consumer credit ($\psi^{STN^B} = \psi^{STN^H} = 1$ and $\psi^{STN^C} < 1$).

The default probability indicator mimics the financial accelerator effect discussed in Bernanke *et al.* (1996), through which credit market conditions contribute to amplify the effects of real or monetary shocks. In an adverse shock financial markets are also negatively hit and it becomes harder and more expensive to borrow money. In turn, this leads to further decreases in investment and spending, amplifying the slowdown in the economy. The default probability is modelled in a simple way, evolving with real GDP growth and an average nominal lending rate (STN_t^A), with some inertia in the short run (equation B.39).

Turning to outstanding amounts, credit to non-financial corporations (cre_t^B) depends positively on the nominal private non-housing capital stock ($ksnp_t$) and negatively on the financing costs, in the long run. The short-run dynamics is governed by fluctuations in STN_t^B, URX_t and in the underlying type of financed expenditure—nominal private non-housing gross fixed capital formation (ipn_t)—in addition to some inertia:

$$\begin{aligned}\Delta cre_t^B =&\beta^{cre^B} \cdot \Delta cre_{t-1}^B + (1 - \beta^{cre^B}) \cdot \Delta ipn_t + \gamma^{cre^B} \cdot \Delta STN_{t-1}^B + \varphi^{cre^B} \Delta URX_{t-2} + \\ &\lambda^{cre^B} (cre_{t-1}^B - C^{cre^B} - ksnp_{t-1} - \psi^{cre^B} \cdot STN_{t-1}^B - \zeta^{cre^B} \cdot EURIBOR_{t-1}),\end{aligned} \quad (16)$$

where static and dynamic homogeneity restrictions are assumed to hold. The equations for credit to households for housing acquisition (cre_t^H) and for consumption (cre_t^C) have similar specifications, where the underlying expenditures and financing costs are replaced by the relevant ones (equations B.41 and B.42).

Households' deposits and the fixed-term deposit rate are also included in the model. The fixed-term deposit rate is determined by $EURIBOR_t$ in the long run, while its developments in the short run are a function of banks' financing costs, proxied by changes in $EURIBOR_t$ and a simple average of bank lending rates (equation B.36).

The households' fixed-term deposit stock depends on households adjusted nominal disposable income and on the fixed-term deposit rate (equation B.43). Demand deposits use a simple rule involving adjusted nominal disposable income.

3.6. Public sector

The fiscal block describes public revenues and expenditures—for the general government sector as a whole, according to national accounts—and public debt. The macro-fiscal linkages contribute to enrich the M model and allow for a better understanding of (i) the reaction of fiscal aggregates to changes in the macroeconomic scenario and, conversely, of (ii) the reaction of the macroeconomic environment to changes in fiscal policies. For example, fiscal and demand blocks are linked through the impact of social transfers and households' income taxes on disposable income.

This block offers a quite extensive disaggregation of revenues and expenditures. Although many variables are modelled endogenously, there are some exogenous elements, especially on the expenditure side, given its more discretionary nature. For the current year, these exogenous elements are based on information included in official documents, such as the State Budget.

The revenue side includes a myriad of tax and non-tax revenues. In general, tax revenues evolve in line with the underlying tax base, acting in many cases as automatic stabilisers. According to Eurosystem's rules, tax rates are assumed to remain unchanged over the projection horizon, unless changes to statutory taxes or to taxation schemes have been approved or have been specified with enough detail and are likely to pass the legislative process.

The expenditure side is also quite detailed. Public consumption (gcr_t) results from a bottom-up approach, aggregating several variables. Its main items are compensation to public employees plus intermediate consumption and social payments in kind, to which the proceeds from the sale of goods and services are subtracted. Real compensation to public employees evolves in line with the exogenous dynamics of public employment. The evolution of the other components is mostly linked to activity, while in some cases, particularly for the current year, they may follow an exogenous path.

Developments in social transfers in cash depend on demographic and labour market developments, as well as on legislation in force, which includes, among other things, the pensions update formula. Social transfers also include the unemployment benefits, which act as an automatic stabiliser and mainly reflect the evolution of the unemployment rate. Public investment (gir_t) is either set exogenously, on the basis of information included in official documents, or evolves in line with GDP. Its profile also takes into account the expected absorption of European Union funds.

In general, prices and wages in the fiscal block take into account inflation developments. The main exception is the public gross fixed capital formation deflator (gid_t), with a specification close to those of private domestic demand deflators (equation B.44). Moreover, in the short term, the evolution of public wages takes into account the approved updates of the wage scale plus a wage drift due to promotions and progressions in careers. When relevant, composition effects related with the change in the wage structure are also taken on board.

Interest payments are calculated using the structure of public debt stock and the evolution of short- and long-term yields. Public debt results from a standard debt accumulation condition—debt in previous period plus fiscal balance (i.e., difference between total revenues and total expenditures) and any exogenous deficit-debt adjustment. No fiscal rule is incorporated over the projection horizon. With the exception of approved or well specified policies, the projection follows a no-policy-change scenario.

4. Simulations

This section reports the results of some key simulations to illustrate the main properties of the model in the short and medium run. The simulations compare a baseline scenario with an alternative scenario in which all exogenous assumptions are kept unchanged except one, the variable that is shocked. Thus, the simulations use a stylised approach to calculate, *ceteris paribus*, the mechanical impact of shocks.

The article presents simulations of five standard shocks, namely a foreign demand shock, a public consumption shock, an exchange rate shock, an oil price shock and an interest rate shock. The shocks are implemented in the variable level, starting in $2022Q1$ and lasting over a horizon of six years (i.e., 24 quarters). Note that the timing of the shock may affect the results, in particular because the baseline scenario is conditional on the shares of demand aggregates, which may change over time. For example, *ceteris paribus*, the impact of a foreign demand shock depends on the share of exports in GDP. These shares are constant only in the steady state. Moreover, it is important to bear in mind that this type of models may take some years to reach the steady state, as their speed of convergence tends to be low (Fagan *et al.* 2005; Berben *et al.* 2018).

The charts present the difference in percentages or percentage points (for the trade and current plus capital accounts) between the level of a variable in the shocked and the baseline scenarios, in each quarter.

4.1. Foreign demand

This shock corresponds to an increase of 1% in foreign demand for Portuguese goods and services. In response to this shock, Portuguese exports increase by 1% in the short run, implying a rise of 0.3% in GDP by the end of the first year (Figure 2).

Higher exports translate into demand pressures, boosting the demand for capital and labour services, and this triggers an increase in investment and employment, and a decline in the unemployment rate. In response to higher output per worker and to labour market tightness, wages increase. Households' purchasing power and, thus, private consumption rise as a result. The impact of higher exports in GDP is attenuated by the increase in imports, due to the high import content of exports.

In the medium run, positive spillovers to domestic demand increase and by year four GDP is 0.4% above the baseline. However, demand pressures implies an increase

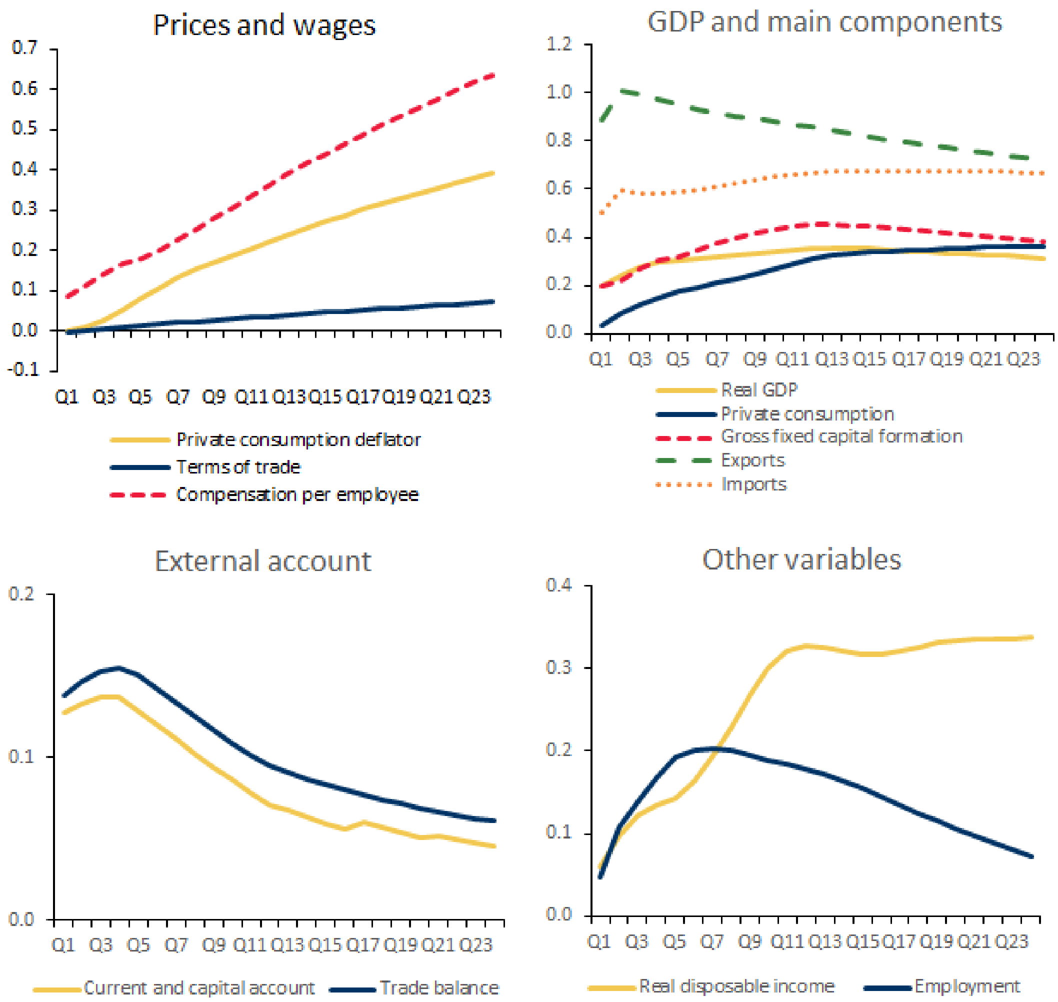

FIGURE 2: Effects of an increase of foreign demand by 1% | In percentage differences from baseline

Source: Authors' calculations.

in prices leading to a loss competitiveness, which attenuates the initial boost in exports. Over the horizon, exports slow down, standing at 0.7% above the baseline in the sixth year. Trade and current and capital account balances improve in the medium term due to higher exports and marginal gains in the terms of trade. The positive developments in domestic activity lead to higher tax revenues and lower unemployment benefits, which have a positive effect on the fiscal balance.

4.2. *Public consumption*

This shock corresponds to an increase in real public consumption by 1% of initial GDP driven by a change to real purchases of goods and services. The immediate and positive effect on GDP is amplified by the increase in business investment and private consumption (Figure 3). The former reflects the accelerator effect, while the latter derives from higher employment and real wages. Due to higher aggregate demand and a tighter labour market, wages and production costs rise, placing upward pressure

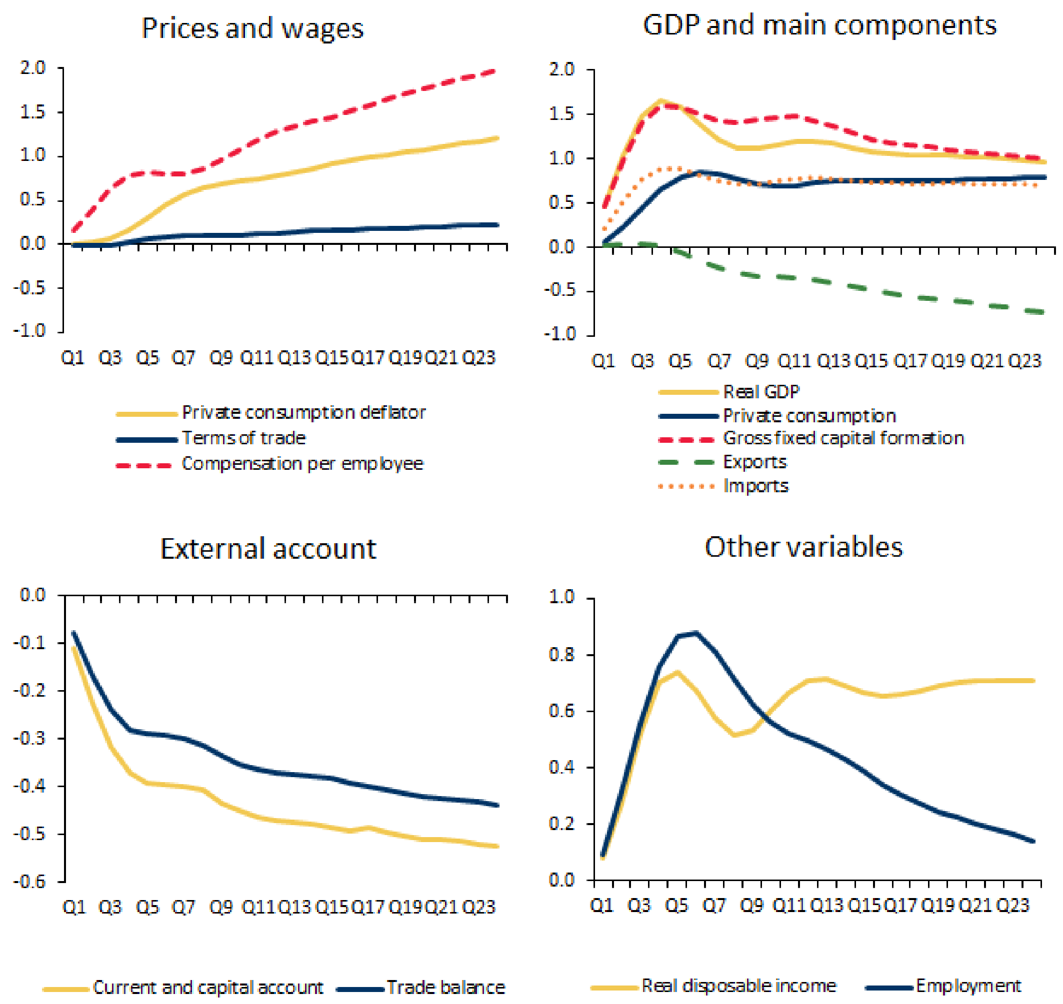

FIGURE 3: Effects of an increase in public consumption by 1% of initial GDP | In percentage differences from baseline

Source: Authors' calculations.

on consumer prices. Since nominal rates are kept unchanged, the rise in inflation results in a fall of real interest rates, amplifying the positive effect on consumption and investment. Higher prices also trigger a competitiveness loss and, thus, a decline in exports. The trade balance steadily deteriorates, reflecting the increase in imports, driven by higher domestic demand, and the competitiveness loss, triggered by the increase in domestic prices. Lower exports and the fading of the initial accelerator effect in business investment lead to a gradual downward adjustment in GDP.

4.3. Nominal exchange rate

This shock corresponds to an appreciation of the euro against all other currencies by 10%. This shock propagates through the model mainly *via* its effect on price competitiveness of exports and imports. Additionally, there is also an important channel *via* the impact on oil prices, since oil is traded on international markets in US dollars.

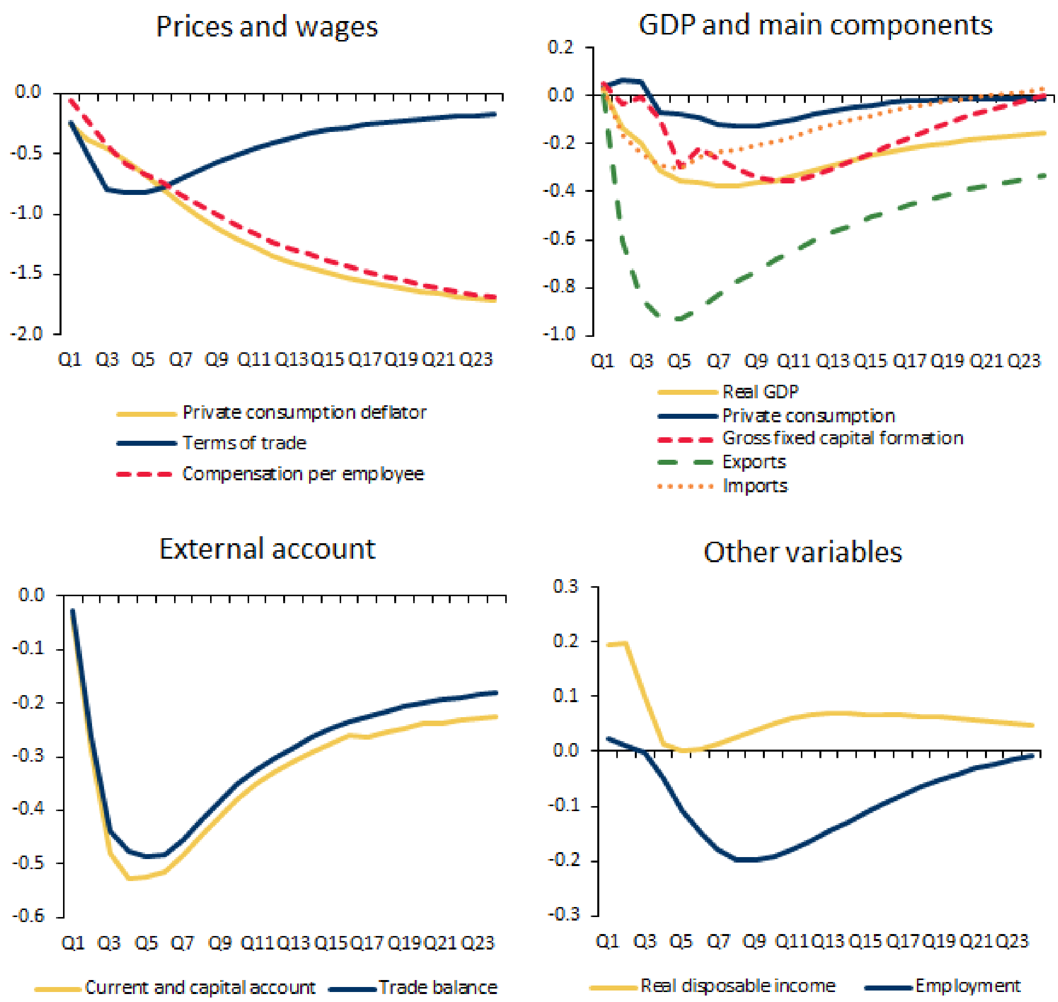

FIGURE 4: Effects of an appreciation of euro nominal exchange rate by 1% | In percentage differences from baseline

Source: Authors' calculations.

The pass-through effect on import and export prices is almost complete after six years. The appreciation of the euro makes Portuguese exports more expensive to consumers and firms outside the euro area. This translates into a reduction of exports and consequently of GDP, implying also a decline in imports due to lower demand (Figure 4). By the end of the first year GDP is 0.3% below the baseline level. The negative impact on imports is partially offset by the substitution of domestic production by imports, which are now cheaper. Lower demand triggers less investment and lower employment, implying an increase in the economy's slack. As wages and prices decrease, and real interest rate and unemployment increase, private consumption adjusts downwards. The lagged effect of a higher real interest rate leads to some volatility in the second year of the investment profile.

The adverse economic developments lead to lower tax revenues and higher unemployment benefits, both having a negative effect on the fiscal balance. The nominal trade balance deteriorates slightly, reflecting the volume effect that offsets the nominal impact of euro appreciation.

Over the medium term, the negative effect on exports due to the price competitiveness loss is gradually attenuated and after six years GDP is 0.2% below the baseline. This effect reflects the decline in domestic deflators, reinforced as lower import prices pass-through the production chain.

4.4. Oil prices

This shock corresponds to an increase of 10% in oil prices in international markets, in US dollars. The simulation assumes that the oil price is about 85 dollars per barrel in the baseline scenario. An increase in the price of imported oil products leads immediately to higher consumer prices, implying also an increase in wages and domestic production costs (Figure 5). Notwithstanding the increase in nominal wages, real wages decline, implying a reduction in real disposable income and, hence, in private consumption.

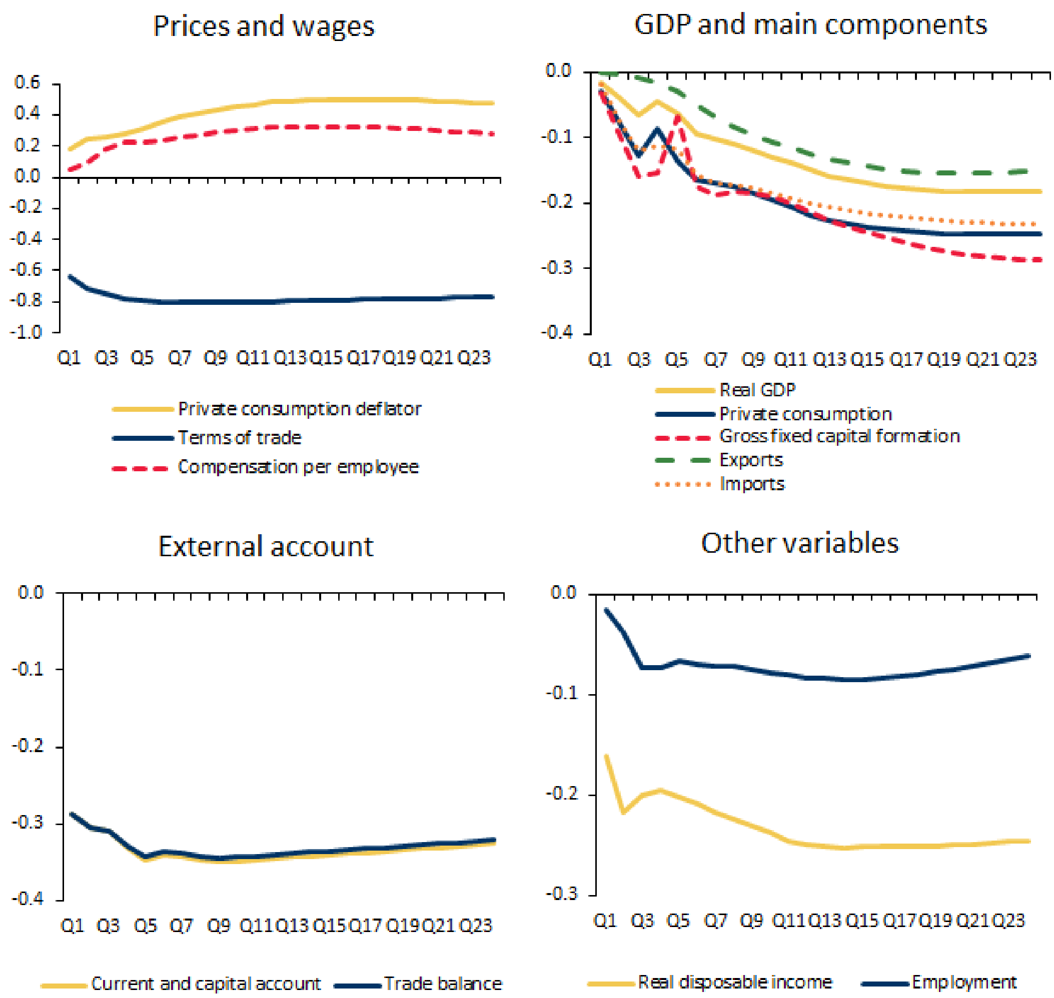

FIGURE 5: Effects of an increase in oil prices by 10% | In percentage differences from baseline
Source: Authors' calculations.

The increase in the production costs causes a decline in competitiveness and, therefore, exports fall. Less production leads to lower input demand, which affects negatively investment and employment. There is some volatility in the second year of the investment profile, which is linked to the lagged effect of the decrease in the real interest rate. In this context, terms of trade and the trade balance deteriorate. The fiscal balance also deteriorates, reflecting lower production and employment, which lead to lower tax revenues and higher unemployment benefits.

4.5. *Interest rates*

This shock corresponds to an increase in short- and long-term domestic interest rates by 100 basis points. Higher interest rates affect the economy through various channels and have a negative impact on the activity of firms, households, and the government. The borrowing costs of firms increase, affecting the cost of capital and the optimal capital stock. Lower demand for capital goods implies lower production and lower business investment and employment (Figure 6).

Real wages also decline due to lower productivity, since employment adjusts more slowly than activity. These developments result in lower real disposable income and consumption. Households' debt service increases following the interest rate hike, which contributes to a further decline in private consumption. Additionally, higher interest rates increase the opportunity cost of consuming, favouring a delay in spending and an increase in savings. The increase in interest rates also translates into lower demand for housing, placing a downward pressure on house prices and triggering a negative wealth effect that reinforces the reduction in private consumption. In turn, the negative impact on activity translates into lower domestic prices, implying marginal gains in competitiveness, which favours exports and contributes to slightly attenuate the negative impact on activity.

The trade balance steadily improves, mainly reflecting the decrease in imports, driven by lower domestic demand, especially investment and private consumption. Overall, the adverse impact on activity and labour market implies lower tax revenues and higher unemployment benefits, reducing the fiscal balance and increasing public debt. The increase in interest rates also implies a larger interest bill on public debt.

5. Final remarks

Models should be seen as a valuable tool to improve our reasoning and to help interpreting agents' behaviour. The *M* model has been a useful tool for empirical purposes over the years, especially in the context of projection and simulation exercises.

However, three important points underlying its utilisation must be stressed. Firstly, all models are probabilistic in nature, i.e. they generate results with some degree of uncertainty. This arises partly from uncertainty about estimated parameters and partly from the structure of the model itself. Therefore, the *M* model is one tool within an eclectic framework that includes other models and indicators, all contributing to inform the projection and simulation exercises. This broader set of tools includes large-scale models, such as the PESSOA model (Júlio and Maria 2017), and satellite models addressing specific topics, such as short-term developments (Lourenço and Rua 2021), inflationary pressures (Serra 2018) and potential output (Duarte *et al.* 2020).

Secondly, the information plugged into the model is also subject to uncertainty, especially the exogenous set of assumptions. The M model should not be seen as a tool for obtaining the best prediction of future developments, but rather as a way of building consistent projections, if a set of exogenous assumptions follows a certain path.

Finally, the M model has been extended and adjusted over the years, and will continue to be developed and improved, in order to better respond to the challenges of an ever-changing reality.

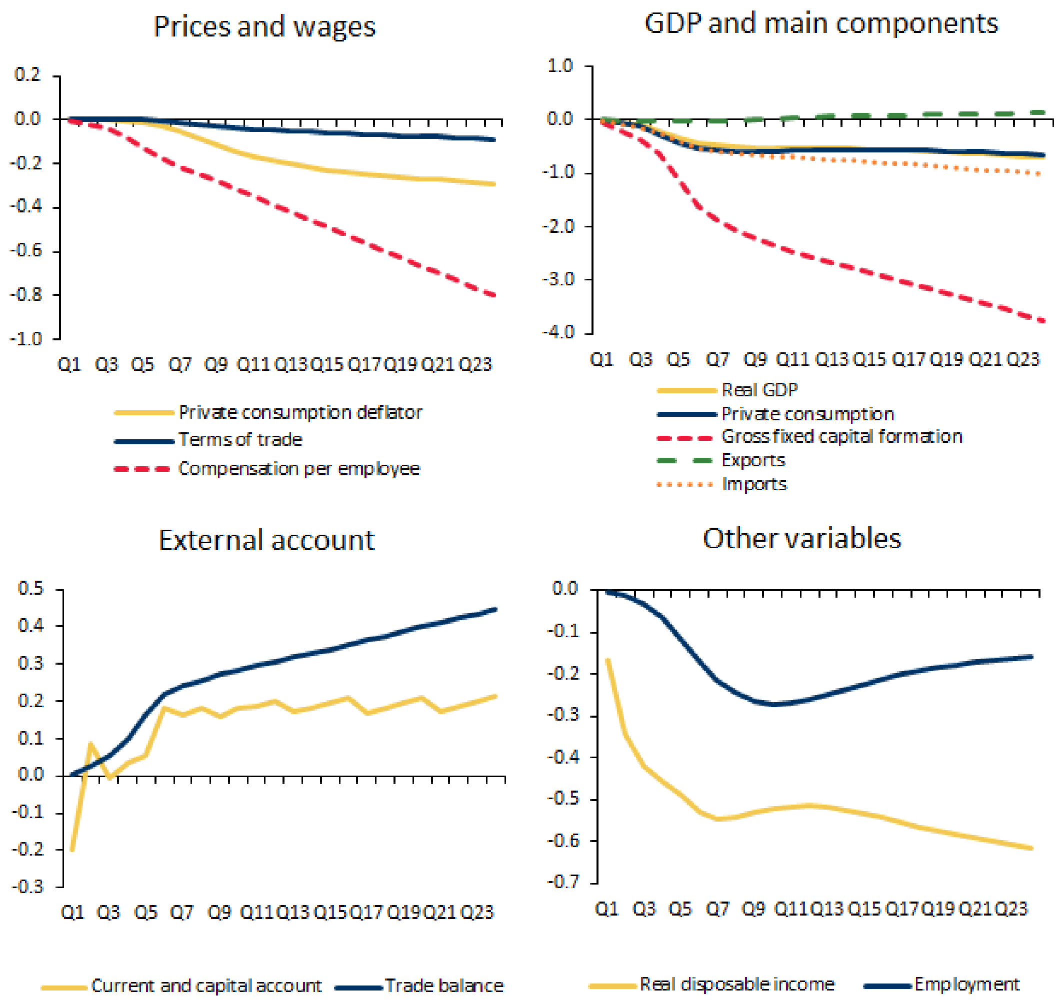

FIGURE 6: Effects of an increase in short-term and long-term interest rates by 100 basis points | In percentage differences from baseline

Source: Authors' calculations.

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