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A circular business cycle clock for Portugal

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Abstract
The assessment and tracking of business cycles have always played an important role in policymaking and economic decisions in general. In this respect, the use and development of business cycle clocks have long attracted a lot of attention among national and international organisations. Building on circular statistics, a novel approach is pursued to depict the business cycle momentum in Portugal resorting to a large monthly dataset. We show that such approach allows for a timely and reliable tracking of business cycle developments, namely during peaks and troughs which typically constitute a challenge for economic monitoring. In addition, a circular clock for the year-on-year growth rate cycle is put forward. (JEL: C30, C55, E32.)

1. Introduction

Understanding how an economy evolves over time is essential to derive sound economic policies. This is why the analysis of the business cycle stance is considered a critical instrument not only to identify the current state of the economy, but also to formulate macroeconomic policies to influence prospects for economic growth.

In fact, the seminal contributions by Burns and Mitchell (1946) for the United States laid down the foundations for the empirical assessment of the business cycle stance. In particular, it was acknowledged the pervasive nature of economic fluctuations, thus motivating the assessment of a wide range of economic indicators. The increasing data dissemination by statistical authorities has made this comprehensive assessment easier, as a wide range of disaggregated indicators are now regularly published, resulting in the so-called data-rich environments.

In the past few decades, a lot of attention has been devoted to the development of tools to enhance the visualisation of the cyclical stance of selected or economy-wide indicators. In this regards, a widely proposed instrument concerns the so-called business cycle clock, which serves as basis for many economic analyses of national and international institutions that monitor economic conditions regularly.

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Previous work on business cycle clocks dates back to the 1990s, when the Ifo institute developed a business cycle clock by plotting the current business situation of German firms against their expectations for the next six months (see Nerb (2004) and Abberger (2006) for a critical review).

As the amount of statistical data released has become progressively larger, new tools to visualise the cyclical stance of a wide range of indicators have been proposed. In this spirit, the Statistics Netherlands developed a tool named the business cycle tracer consisting of the graphical display of thirteen lagging, coincident and leading indicators for the Dutch economy (see Ruth *et al.* (2005) for details).

Advancing on the initially proposed survey watch, the European Commission has been using for several years in its regular publications an economic climate tracer (Gayer (2008)).[1] This draws on principal component analysis of balance series stemming from the monthly business and consumer surveys conducted by the European Commission. Other examples of business cycle clocks include Destatis (2010) for Germany or Statistics Denmark (2013) for Denmark.

Drawing on the novel methodology introduced by Lourenço and Rua (2022), we develop a business cycle clock for Portugal relying on circular statistics. This approach allows taking on board a large number of series. As each individual series may potentially point to a different direction regarding the business cycle momentum, it is also important to summarise those signals in a single direction. Hence, the proposed circular business cycle clock not only depicts the circular histogram providing information on the dispersion of the signals, but also displays the mean direction along with its corresponding confidence interval. Such features allow for a more encompassing assessment of the cyclical stance of the economy than previous approaches.

Resorting to a large monthly dataset for the Portuguese economy, we assess the behaviour of the circular business cycle clock, namely during the turning points in economic activity. Such episodes are of particular interest from a policymaking point of view and constitute a challenge in terms of macroeconomic monitoring. We also assess its overall performance vis-à-vis other alternative procedures in tracking the business cycle. The results are supportive of its informational content, even in a pseudo real-time context.

Furthermore, we develop a circular clock for the growth rate cycle given the past interest in tracking yearly changes in the Portuguese economy. We find that the circular clock performs well during the turning points in economic activity and can constitute a valuable tool for tracking current economic developments.

The remainder of this paper is organised as follows. Section 2 introduces the methodology underlying the circular business cycle clock. Section 3 describes the dataset. In Section 4, the business cycle clock and the economic growth clock for Portugal are presented. Finally, section 5 concludes.

1. The survey watch relied on the representation of manufacturers' current business perceptions against their production expectations for the next three months.

2. Methodology

The idea underlying the business cycle clock is to depict the cyclical stance of economic activity through a four-quadrant representation, based on the cyclical component of selected series. This is often obtained as the deviation from its long-term trend, with the resulting level being plotted against its month-on-month change (Figure 1). Thus, it is possible to evaluate if a given indicator lies above or below its long-term trend, while assessing if it is improving or worsening in the short-run. The business cycle clock has a counterclockwise reading along the four phases of the business cycle, where the peaks are identified at the top centre of the diagram and the troughs at the bottom centre.

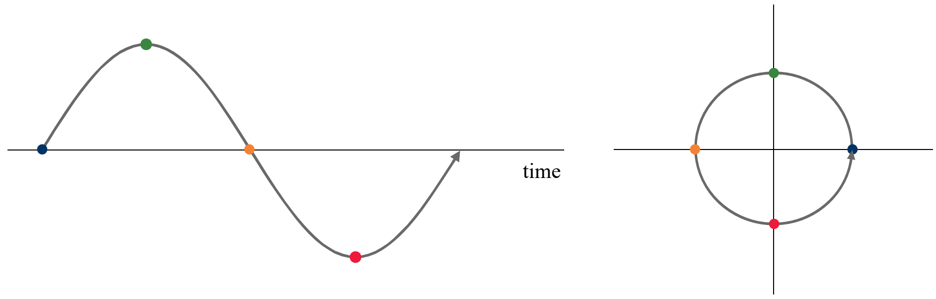

FIGURE 1: Stylised business cycle and corresponding clock.

Recently, Lourenço and Rua (2022) proposed a novel business cycle clock resorting to circular statistics. The use of circular statistics in this context seems natural given the recurrent nature of the business cycle. In contrast with previous literature, where the business cycle momentum is displayed in the Cartesian coordinate plane, Lourenço and Rua (2022) suggest its representation through polar coordinates. In fact, any point displayed in the Cartesian plane can be defined by a distance from the origin and an angle θ_i (Figure 2).

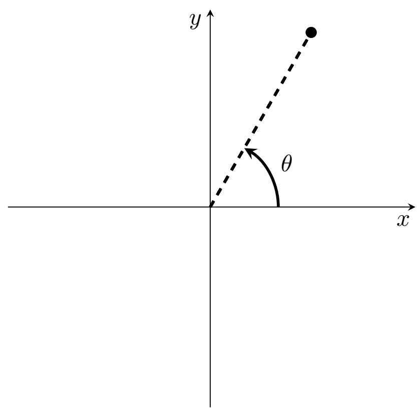

FIGURE 2: Illustrative angle computation.

Given N indicators, one ends up with N directions defined by the corresponding angle associated with each indicator. This implies that we have N possible indications about the current stance of the business cycle. In a data-rich environment, where dozens or hundreds of indicators are available, it becomes critical to summarise those indications in a single direction. Hence, the circular business cycle clock proposed by Lourenço and Rua (2022), besides displaying the histogram for the sample of angles which provides information on the dispersion of the directions, also conveys the mean angle along with its corresponding confidence interval.

Succinctly and following Lourenço and Rua (2022), consider a random angle θ and a sample of N angles, $\theta_1, \theta_2, ..., \theta_N$. The sample uncentred p^{th} trigonometric moment is given by

$$t_{p,0} = \frac{1}{N} \sum_{i=1}^{N} (\cos p\theta_i + i \sin p\theta_i) = a_p + ib_p, \tag{1}$$

where $a_p = \frac{1}{N} \sum_{i=1}^{N} \cos p\theta_i$ and $b_p = \frac{1}{N} \sum_{i=1}^{N} \sin p\theta_i$.

The sample centred p^{th} trigonometric moment reads as

$$t_{p,\overline{\theta}} = \frac{1}{N} \sum_{i=1}^{N} \left[\cos p \left(\theta_i - \overline{\theta}\right) + i \sin p \left(\theta_i - \overline{\theta}\right)\right] = \overline{a}_p + i\overline{b}_p, \tag{2}$$

where $\overline{a}_p = \frac{1}{N} \sum_{i=1}^{N} \cos p \left(\theta_i - \overline{\theta}\right)$, $\overline{b}_p = \frac{1}{N} \sum_{i=1}^{N} \sin p \left(\theta_i - \overline{\theta}\right)$ and $\overline{\theta}$ denotes the mean angle. A confidence band for the mean angle is given by $\pm z_{\frac{\alpha}{2}} \sqrt{\frac{1-\overline{a}_2}{2N\overline{R}^2}}$, where $\overline{R} = \sqrt{a_1^2 + b_1^2}$ and $z_{\frac{\alpha}{2}}$ denotes the upper quantile $\frac{\alpha}{2}$ of the $N\left(0,1\right)$ distribution.

3. Data

The monthly dataset for Portugal used in the empirical application draws extensively on previous research on forecasting or business cycle dating in a data-rich environment (Dias *et al.* (2015), Rua (2017) and Dias *et al.* (2018b)). It covers the period from January 1995 up to December 2021 and comprises both hard and soft data, amounting to 126 series overall.

The panel of series includes the following broad categories: industrial production, turnover in industry and services, retail trade sales, employment, hours worked and wage indices in industry and services, overnight stays in tourist accommodation establishments, car registrations, cement sales, vacancies and registered unemployment, energy consumption, international trade of goods, real effective exchange rate, Portuguese stock market index, ATM/POS series and business and consumer surveys. The latter conveys sectoral information (e.g. assessment of recent and future trends in production) in the industry (manufacturing), services, retail trade and construction, as well as consumers, who are asked, inter alia, about their spending and savings intentions.

As followed by the European Commission in the business and consumer surveys compilation, the series on the unemployment over the next 12 months (consumers survey) and on the assessment of stocks (manufacturing and retail trade surveys) were sign-inverted. The registered unemployment and the unemployment rate series have also been sign-adjusted. Finally, excluding the survey data and the unemployment rate, all series were log-transformed.

4. Empirical results

4.1. The circular business cycle clock

As alluded before, the development of business cycle clocks has been centred on the concept of growth cycles, that is, deviations from a long-term trend. In this respect, it has been current practice in previous work on business cycle clocks to use the well-known Hodrick and Prescott (1997) filter for detrending and smoothing the economic indicators.

Following Lourenço and Rua (2022), to assess the usefulness of the resulting business cycle clock for Portugal in tracking turning points in economic activity, we resort to the monthly growth cycle chronology put forward by the OECD.[2] One should note that the focus herein is on the growth cycle and not the classical cycle, which draws on the evolution of the level of economic activity. For dating classical business cycles for Portugal see, for example, Rua (2017). To make such comparison feasible, we adopt the same approach pursued by the OECD embedded in dating the growth cycle namely by using the HP filter as a band-pass filter. In particular, the HP filter is run twice, firstly to remove the trend and secondly to smooth the series by discarding the high frequencies. This implies using a high value for the HP parameter λ in the first step and a small value in the latter stage. In particular, such values are chosen so that one retains the cycles with duration between 12 and 120 months.[3] In practice, it corresponds setting λ to 133107.94 and 13.93, respectively. One should mention that, as stressed by Lourenço and Rua (2022), the circular business cycle clock can be computed with any filtering method which renders series stationary.[4]

The circular business cycle clocks for Portugal around all the turning points since 2003 are reported in Figure 3. Each row in Figure 3 corresponds to a turning point, either a peak or a trough, and for each turning point, we present the business cycle clock for the month identified by the OECD as a peak or a trough as well as for the previous and subsequent months. Each business cycle clock displays the histogram of the sample angles, where the histogram bins are plotted as blue straight bars with a width of 10 degrees. That is, the heigth of each blue bar denotes the number of indicators that have the corresponding direction falling within a 10 degrees range. Hence, the sum of the height of all blue bars corresponds to the number of series. The red arrow represents the mean direction which corresponds to the average of the directions given by all the indicators. On top of that, the corresponding confidence intervals are shown on the outside edge of the circle as arcs. In particular, the gray and black lines indicate the 95 and 99 per cent confidence intervals, respectively.

2. See `https://www.oecd.org/sdd/leading-indicators/CLI-components-and-turning-points.csv`

3. Although the business cycle literature, in particular for the United States, tends to consider cycles of duration between 1.5 and 8 years, there is evidence that business cycles may last longer in Europe as argued by the OECD.

4. In this respect, we also considered the band-pass filter suggested by Christiano and Fitzgerald (2003) and the results are similar.

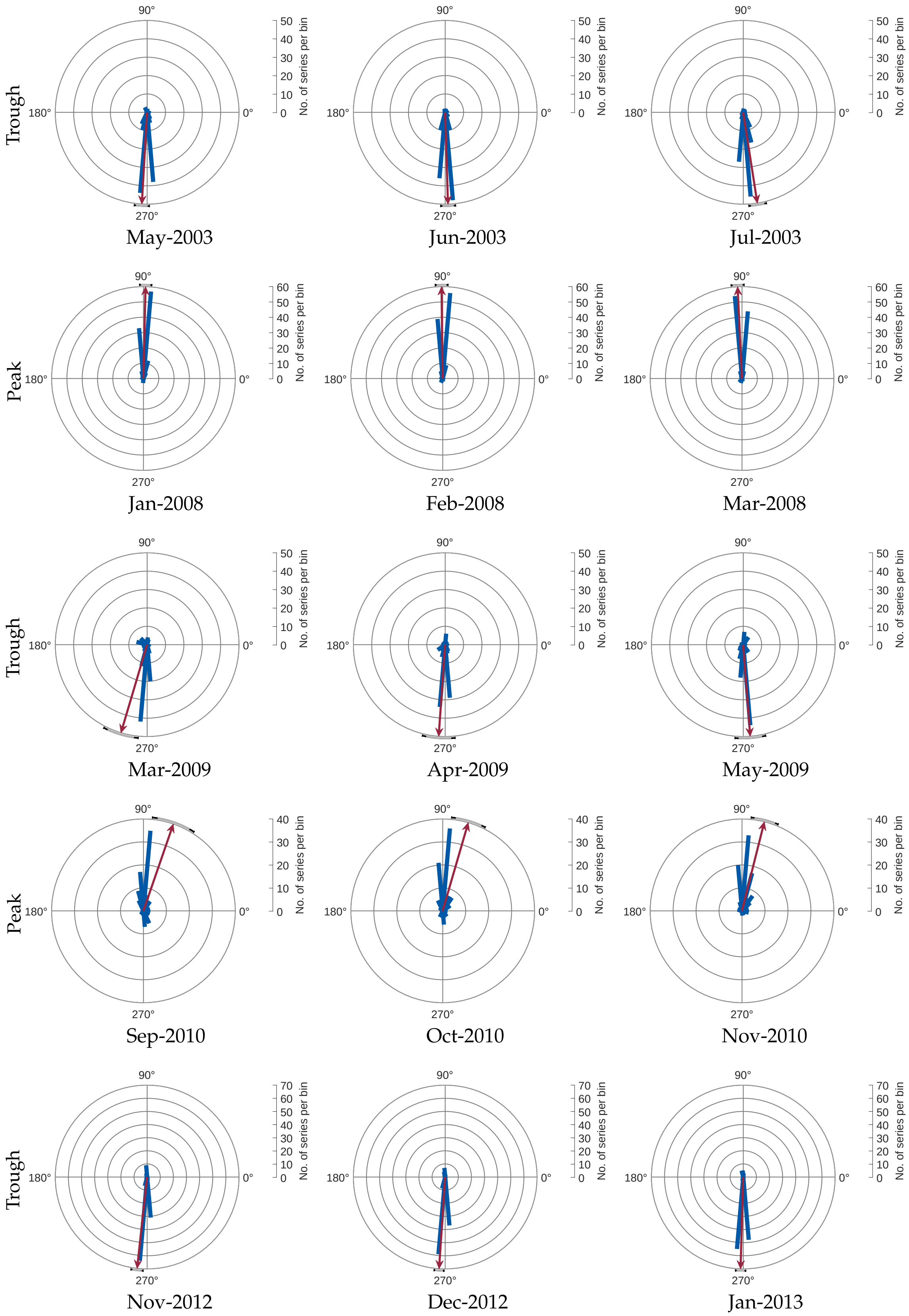

FIGURE 3: Business cycle clock around the turning points.

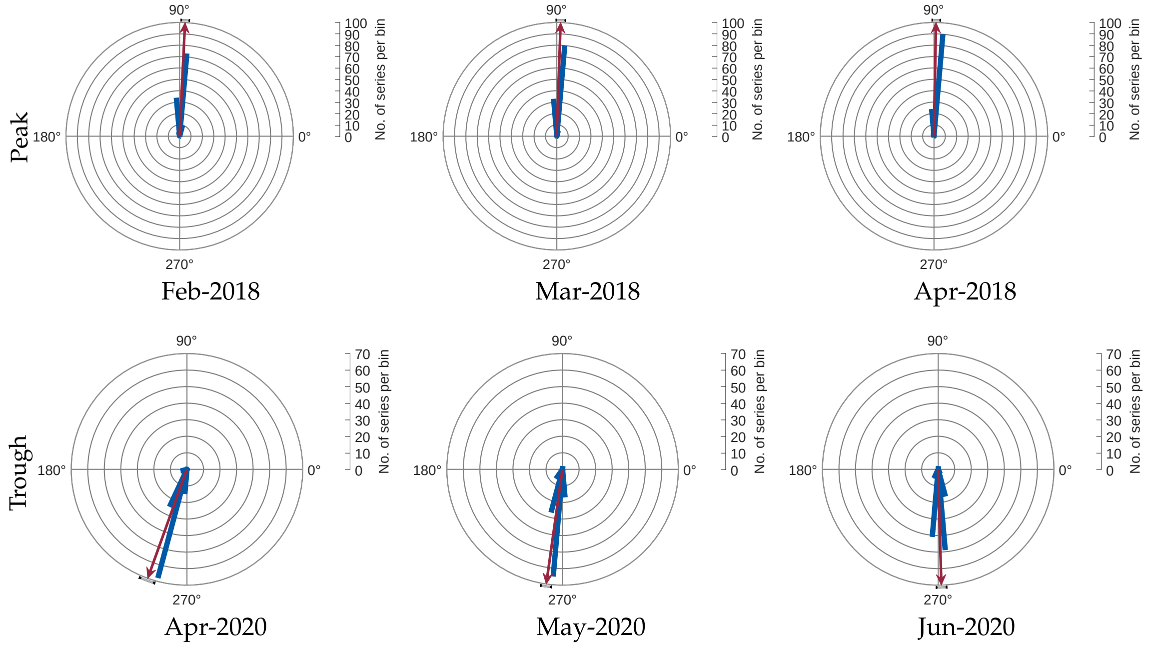

FIGURE 3: Business cycle clock around the turning points (continued).

One can conclude that, in general, the mean direction always points clearly to the bottom (270 degrees) during the troughs and to the top (90 degrees) during the peaks as desired. In detail, one can see that both in the trough in 2003 and in the peak in 2008, the circular business cycle clock reading coincides with the OECD chronology. During the Great Recession in 2009, the mean direction crosses the 270 degrees direction between April and May. In both months, based on the confidence interval, one cannot reject that the trough has been attained, although the mean direction is closer to 270 degrees in April, which matches the OECD monthly dating. Regarding the peak at the end of 2010, the business cycle clock readings suggest that the peak has been reached a few months after the month identified by the OECD. According to the OECD chronology, the peak was reached in October 2010 whereas the mean direction points to February 2011 (although one cannot reject that the peak could also have been reached in January 2011). Concerning the trough at the end of 2012, the circular business cycle clock points to its occurrence at the beginning of 2013 while the peak in 2018 cannot be rejected to have occurred in March. In what concerns the trough during the COVID-19 pandemic, the mean direction points to June, which corresponds to one month after the month in the OECD chronology.

The above analysis has been focused on the evaluation of the behaviour of the circular business cycle clock during the turning points. Such analysis can be complemented with the assessment of the informational content of the mean direction regarding the overall evolution of the business cycle. That is, instead of focusing solely on turning points, this type of analysis provides further insights on how the mean direction tracks business cycle developments every month. To conduct such evaluation, a monthly measure of the business cycle for Portugal is required. A proxy can be obtained by resorting to the monthly coincident indicator for the Portuguese economic activity which has been released by Banco de Portugal on a monthly basis since 2004 (see Rua (2004, 2015)). The monthly coincident indicator for the economic activity is a composite indicator that merges information from quarterly real GDP as well as from higher frequency variables and is available at a monthly frequency. Taking its trend-cycle format, one can apply the same filtering procedure discussed earlier to obtain the cyclical component.[5]

The correlation between such cyclical component and the mean direction can be computed by

$$r_{z\theta} = \sqrt{\frac{r_{zc}^2 + r_{zs}^2 - 2r_{zc}r_{zs}r_{cs}}{1 - r_{cs}^2}} \tag{3}$$

where $r_{zc} = corr(z, \cos\theta)$, $r_{zs} = corr(z, \sin\theta)$ and $r_{cs} = corr(\cos\theta, \sin\theta)$, with $corr$ denoting the usual Pearson correlation coefficient.

In Figure 4, we report the contemporaneous correlation as well as the correlation for the leads and lags up to 6 months. One can see that the correlations are basically symmetric around the contemporaneous one, supporting the coincident nature of the proposed business cycle clock. To provide a term of comparison to the performance of the mean direction, we considered two alternative approaches. Firstly, instead of the mean direction we computed the median direction. That is, instead of using the average of the sample angles, we consider the corresponding median. Secondly, we resorted to a factor model to extract the common factor underlying the dataset and computed the corresponding direction. The results are also displayed in Figure 4. One can conclude that the clock based on the factor outperforms the median direction but both are worse than the mean direction.

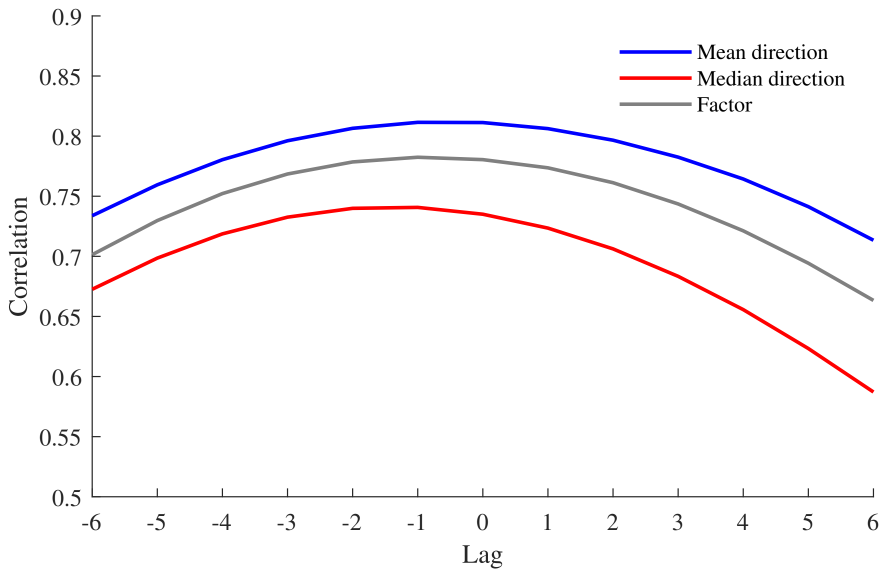

FIGURE 4: Correlations with the cyclical component of the monthly coincident indicator.

5. Alternatively, one could take the quarterly GDP series and perform a monthly disaggregation with to the well-known Chow-Lin method, as pursued by Lourenço and Rua (2022). We find that the results are qualitatively similar.

As extensively discussed in the literature, the use of any filtering method entails revisions namely at the end of the sample. In fact, decomposing a series in trend, cycle and irregular components is particularly challenging at the end of the sample and only as time goes by such distinction can be made more reliable and less prone to revisions. Being an issue for any statistical filter, it also applies to the HP filter. In the current context, we assess how such revisions affect the circular business cycle clock namely during the turning points where the reading is more critical. To assess the reliability of the reading of the business cycle clock during the turning points, we computed the clock for every month using data only up to that month, thus mimicking a pseudo real-time exercise. Furthermore, as revisions can be mitigated by extending the series prior to filtering as suggested in related literature, we extend the series resorting to a univariate autoregressive process. In this respect, Nilsson and Gyomai (2011) show that extending the series for a couple of months when applying the HP filter delivers the best performance in terms of revisions around the business cycle turning points.

Figure A.1 in the Appendix displays the circular business cycle clock in a pseudo real-time context to illustrate the impact of the revisions induced by the filtering procedure. Despite the inherent difficulties in detecting the turning points in real-time, we find that the readings in pseudo real-time are in general similar to the ones presented in Figure 3. In fact, the mean absolute revision during such episodes is close to eight degrees.

4.2. A circular clock for the growth rate cycle

In the case of Portugal, there has been a long tradition to put forward tools that allow tracking and monitoring the year-on-year evolution of the economy. In this respect, one should mention the early work by Dias (1993) who proposed a quarterly coincident indicator for the Portuguese economy, followed by Gomes (1995) who resorted to the same approach to deliver a composite indicator for private consumption. The compilation and release of these two indicators have been replaced by the monthly coincident indicators developed by Rua (2004, 2005) drawing on the methodology proposed by Azevedo *et al.* (2006). More recently, a daily economic indicator for the year-on-year evolution has been put forward by Lourenço and Rua (2021). In what concerns short-term forecasting the Portuguese economy, there is also a body of work devoted to forecasting year-on-year growth rates. See, for example, Esteves and Rua (2012) for a GDP bottom-up approach forecast, Dias *et al.* (2018a) for forecasting exports of goods and Lourenço *et al.* (2021) for tourism forecasting.

As stressed earlier, the suggested circular approach can be applied to any filtering technique, including the yearly difference.[6] Hence, in the detrending step, instead of using a HP filter, we consider the yearly difference. On top of that, as discussed by Gayer (2008), it is desirable to smooth the series so as to remove erratic fluctuations and improve the reliability of the business cycle clock. In particular, likewise Gayer

6. If the variable is expressed in logs, the yearly difference basically corresponds to the year-on-year growth rate.

(2008), we apply the HP filter to remove short-term movements of duration less than 18 months.[7]

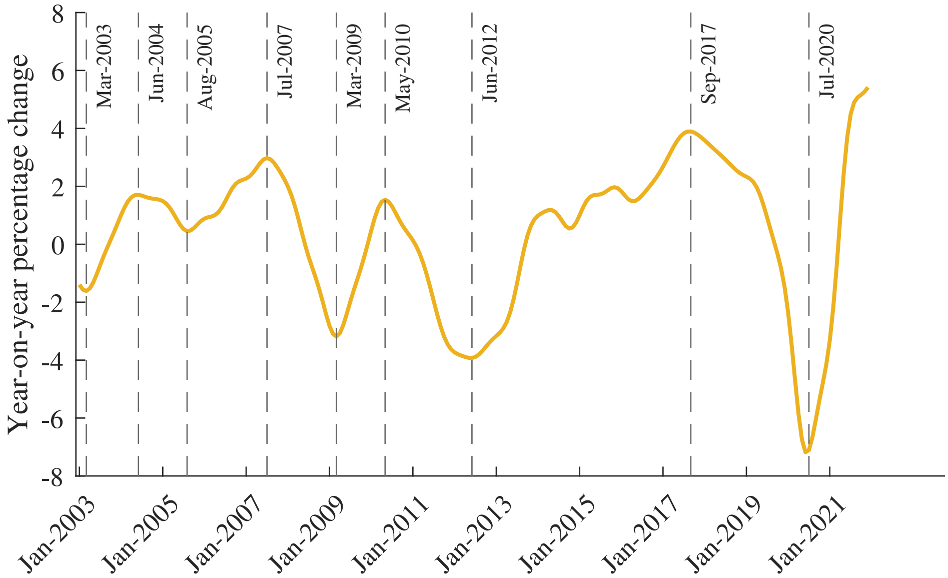

FIGURE 5: Monthly coincident indicator for economic activity.

7. Note that the removal of those short-term fluctuations is also present in the methodology underlying the monthly coincident indicator which is used as term of comparison.

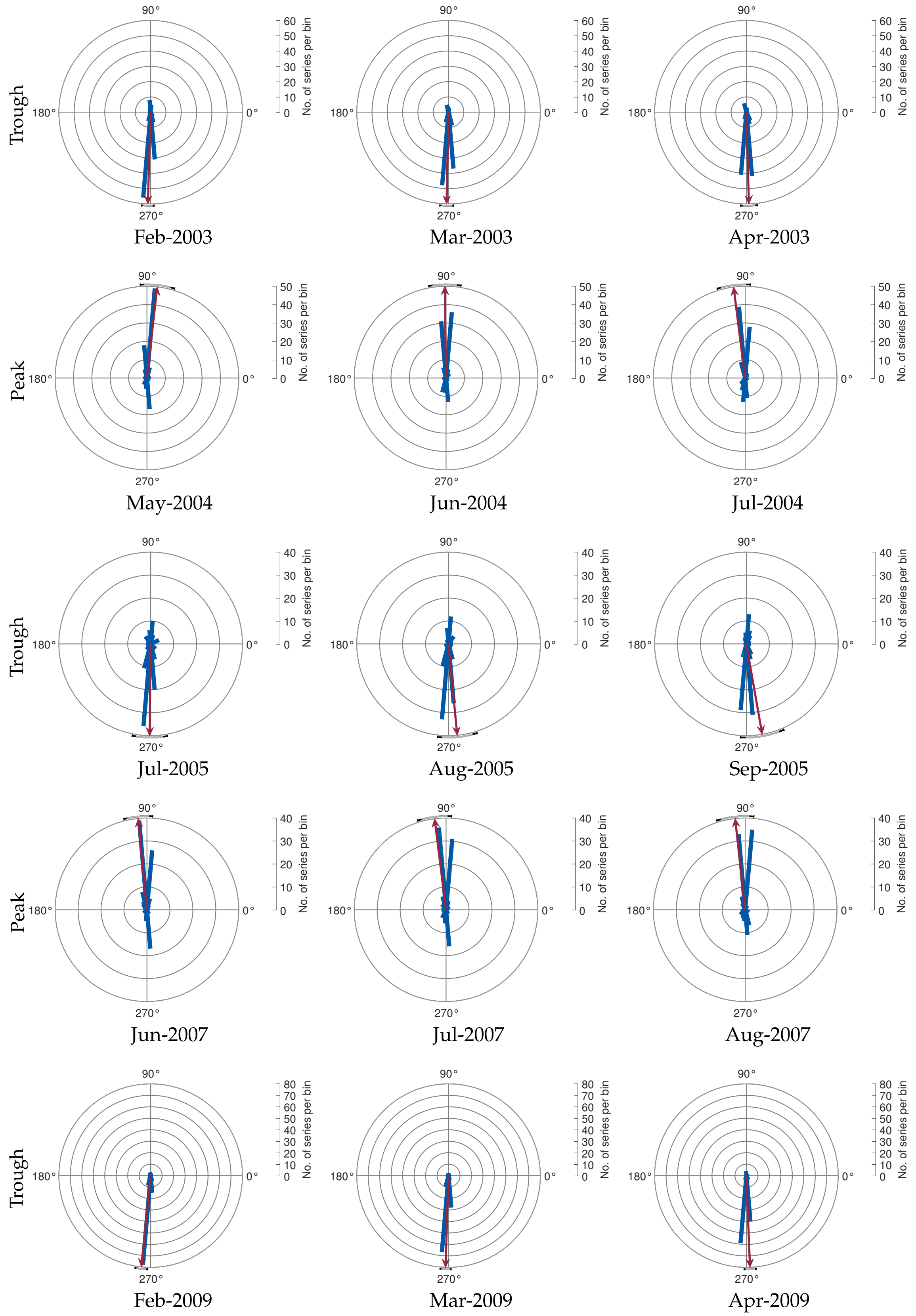

FIGURE 6: Circular clock for the growth rate around the turning points.

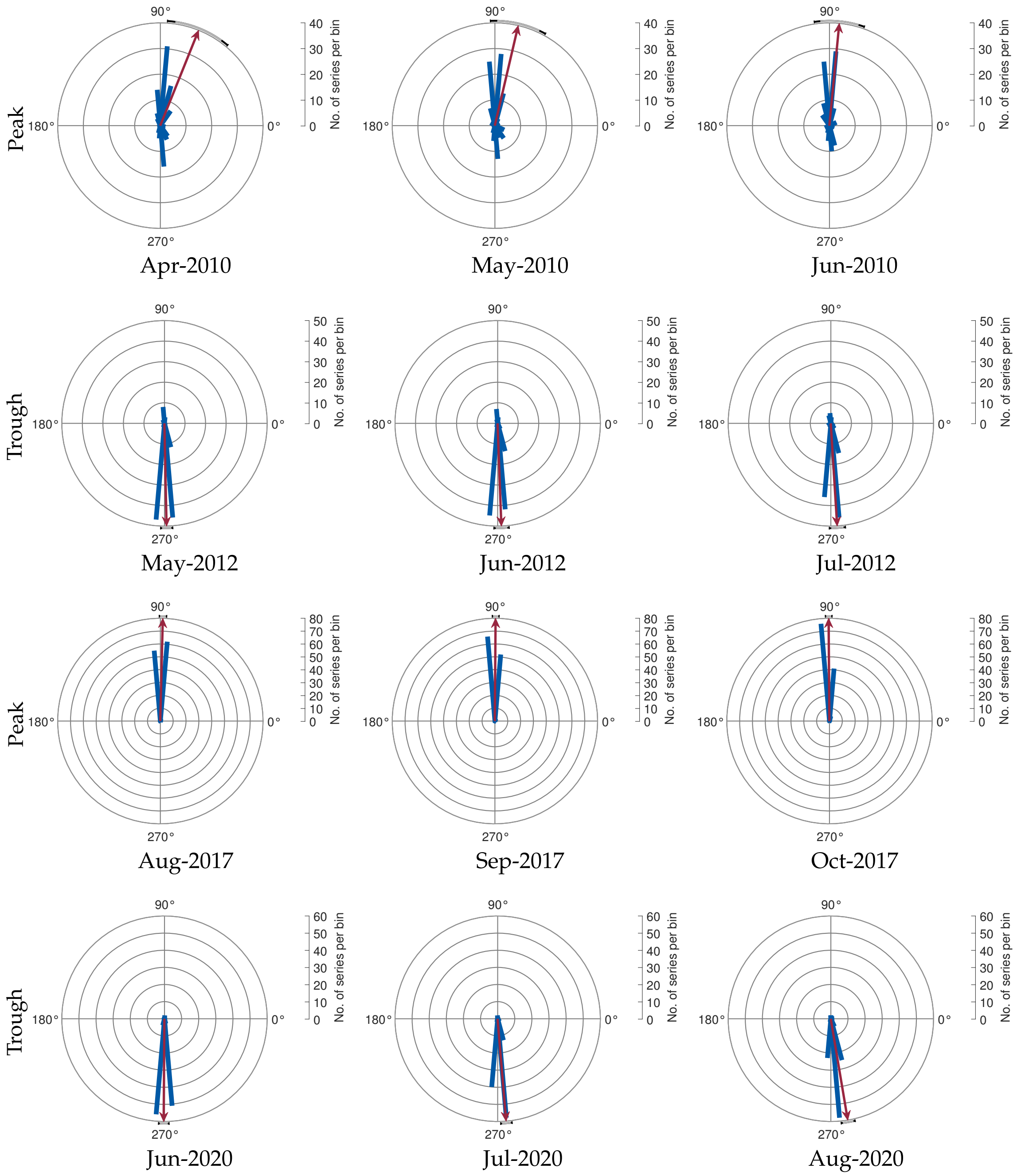

FIGURE 6: Circular clock for the growth rate around the turning points (continued).

As the reference series, we take again the monthly coincident indicator for the Portuguese economy, but now as it is released to the general public, that is, on a year-on-year format, and assess the resulting clock around the corresponding turning points. In particular, the months corresponding to the turning points of the monthly coincident indicator since January 2003 are highlighted in Figure 5, while the clocks for those months as well as for the previous and following months are reported in Figure 6. One can see that for eight out of the nine turning points, the circular clock does not reject the timing of those turning points. That is, the confidence interval includes either the 90 degrees in the case of a peak or the 270 degrees in the case of a trough. Only in the case of the trough during 2020, the circular clock suggests a different month, namely June instead of July.

In a similar fashion to the analysis conducted earlier, we also consider as alternative procedures the median angle as well as the factor model. In Figure 7, we present the contemporaneous correlation as well as the correlation for the leads and lags up to 6 months vis-à-vis the monthly coincident indicator. The results seem to suggest a slightly leading behaviour of the mean angle but one should note that the correlations for lags between -4 and 0 are all very close to 0.75. More noteworthy is the fact that the correlations for the mean angle are always higher than those obtained with the median and the factor model.

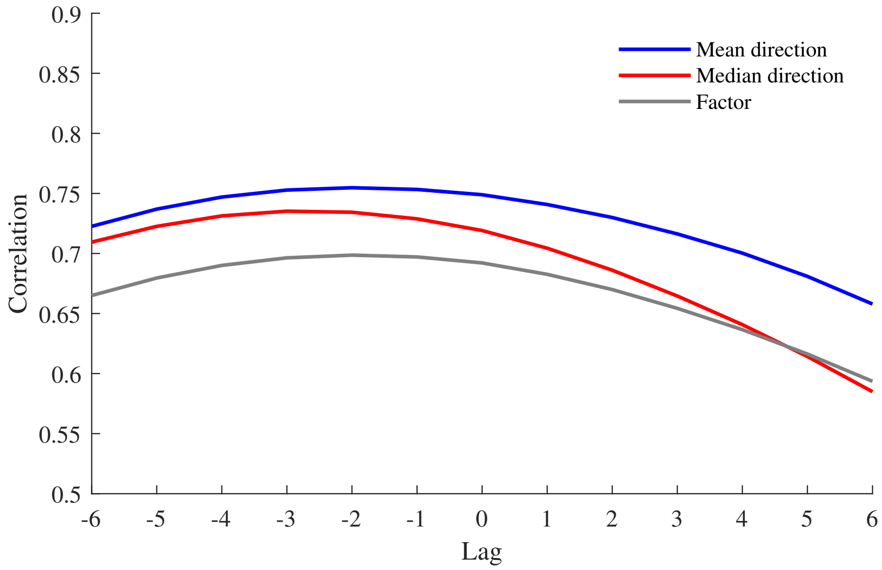

FIGURE 7: Correlations with the monthly coincident indicator.

Although the yearly difference filter does not imply any revisions to the circular clock, the smoothing of the series may lead to revisions. In line with the procedure described earlier, these revisions can also be reduced by extending the series prior to smoothing. Figure B.1 in the Appendix presents the corresponding pseudo real-time clocks. Again, we find that the readings in pseudo real-time are broadly unchanged. In fact, the revisions in this case are lower than in the case of the circular business cycle clock discussed above. In particular, the mean absolute revision around the turning points is less than five degrees.

5. Concluding remarks

The analysis and monitoring of the current business cycle momentum have always been at the core of economic analysis. For instance, this led to the development of the so-called business cycle clocks, which have been used regularly in publications by several national and international organisations. In particular, the business cycle clock depicts the cyclical stance of the economy through a four-quadrant visualisation, based on the cyclical component of selected series.

Building on previous work, we propose a business cycle clock resorting to circular statistics. Such an approach retains the appealing features of the clock representation, while making it possible to depict numerous indicators in a data-rich environment. In fact, the circular clock enables such information to be succinctly displayed by conveying the circular histogram and the mean direction along with its confidence interval.

We find that the resulting business cycle clock is quite informative regarding the peaks and troughs in economic activity as well as business cycle developments in general. We also document its behaviour in a pseudo real-time context during the turning points to assess its reliability in such challenging episodes.

Furthermore, in a similar fashion, we also lay out a circular clock for the year-on-year growth rate cycle given the interest on such developments when monitoring the Portuguese economy. We also find that the circular clock enables a timely and reliable tracking of economic developments, namely during the turning points.

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Appendix A: Pseudo real-time business cycle clock

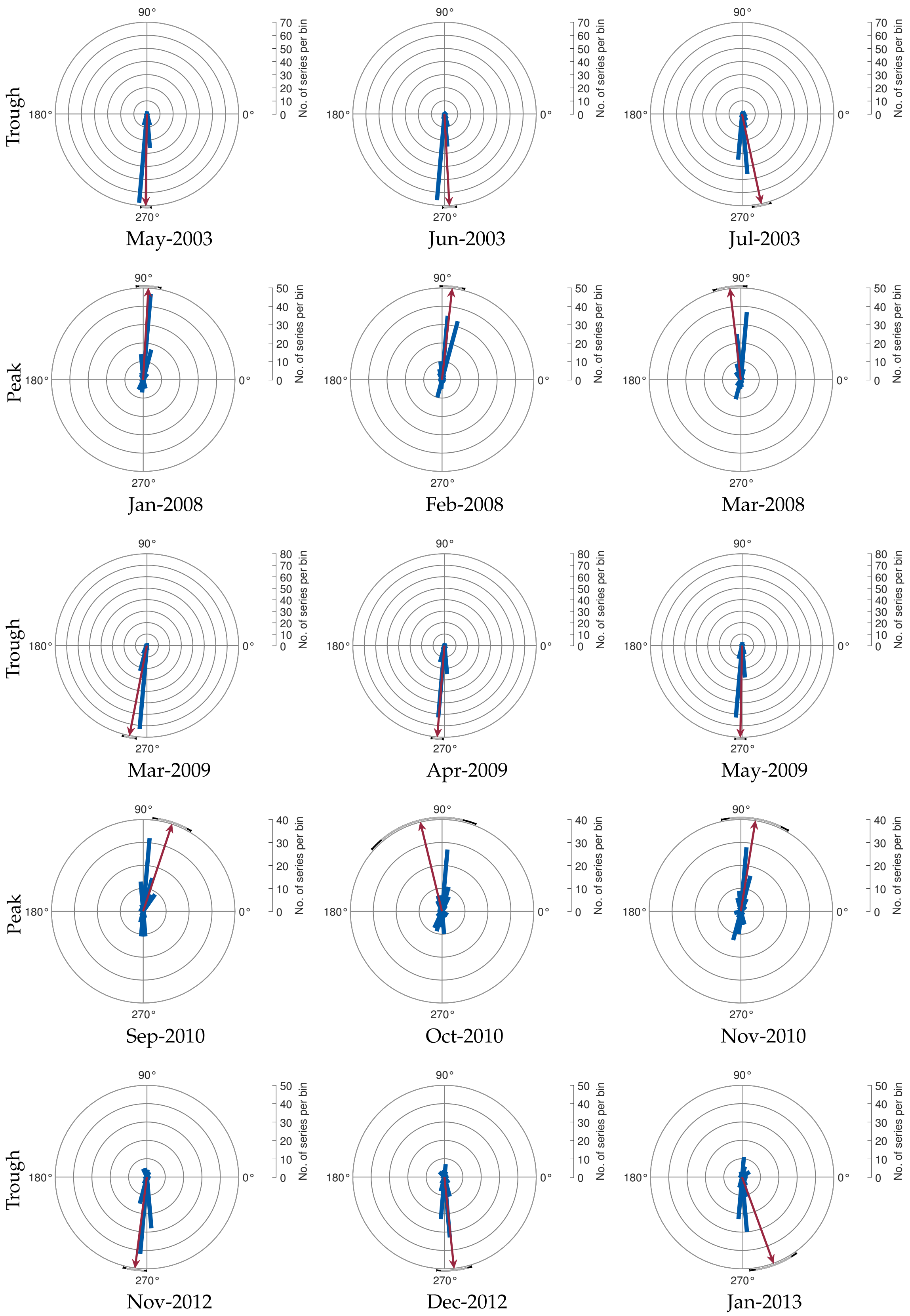

FIGURE A.1: Pseudo real-time business cycle clock.

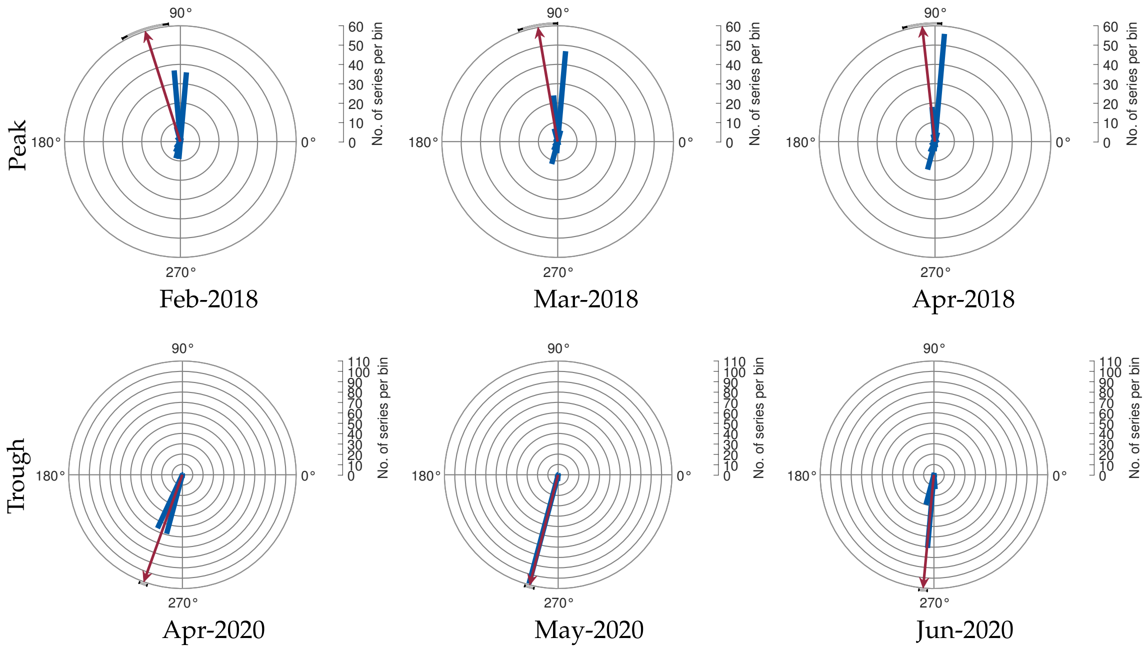

FIGURE A.1: Pseudo real-time business cycle clock (continued).

Appendix B: Pseudo real-time circular clock for the growth rate

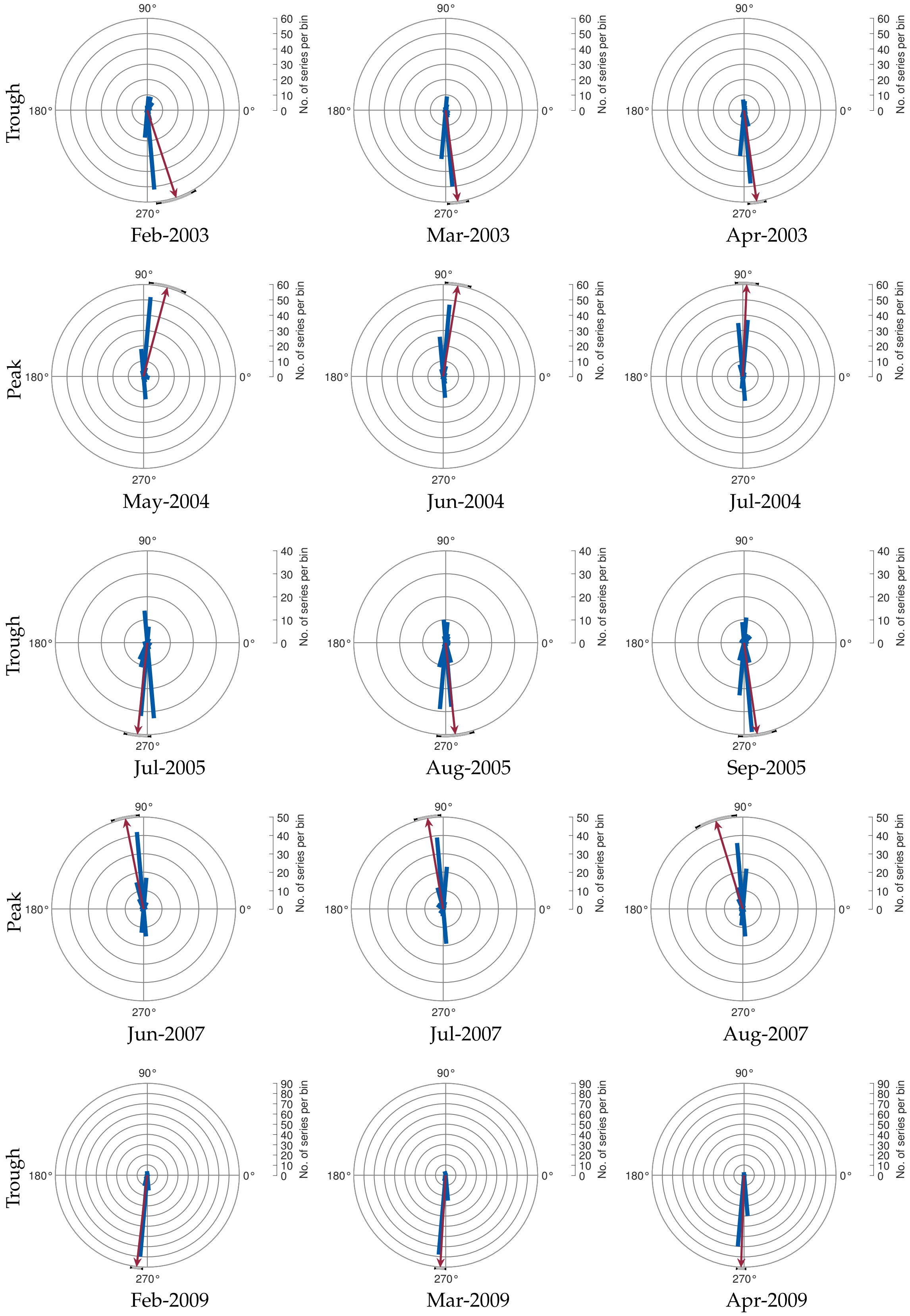

FIGURE B.1: Pseudo real-time circular clock for the growth rate.

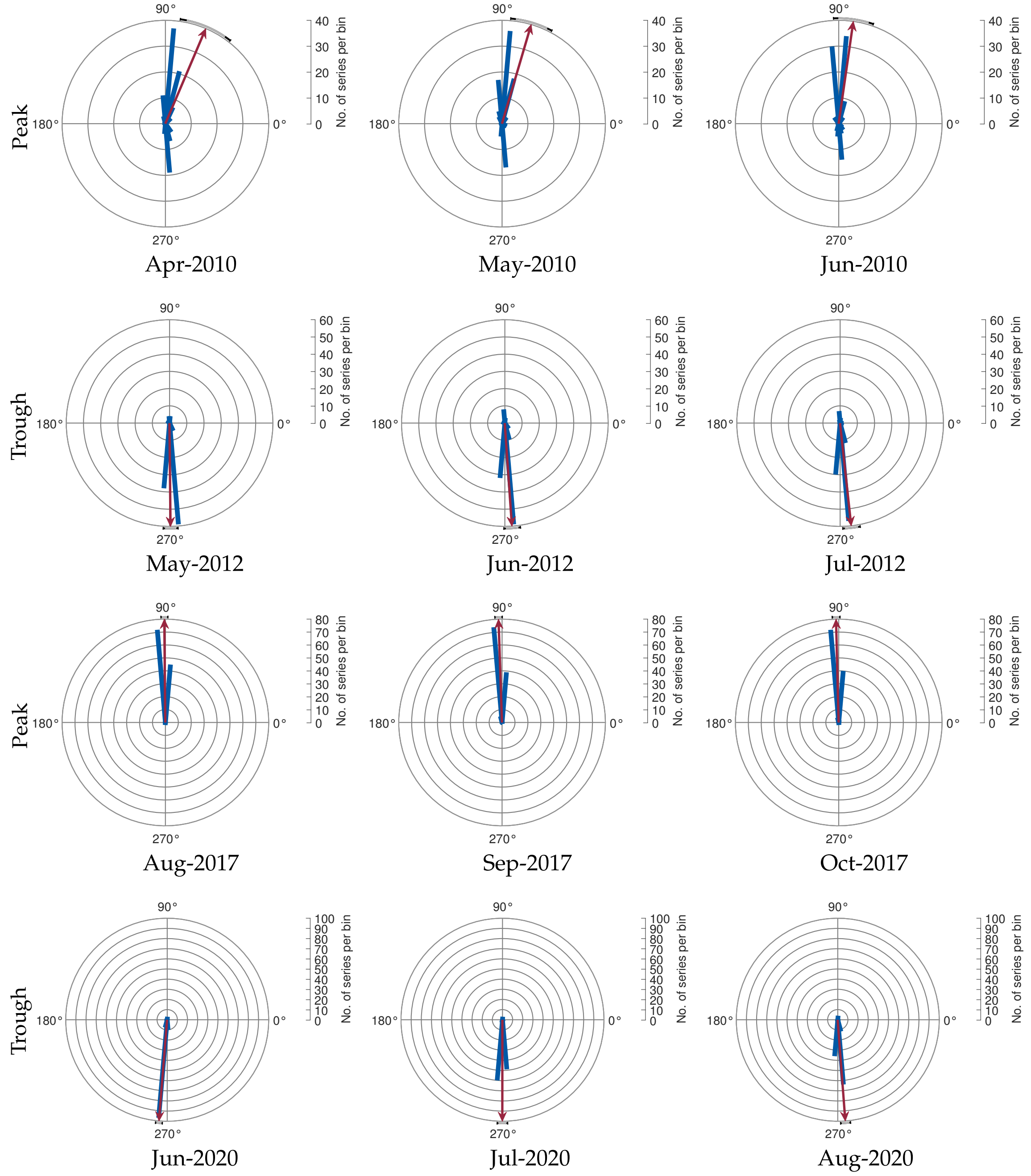

FIGURE B.1: Pseudo real-time circular clock for the growth rate (continued).