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Firm lifecycles

Nicholas Kozeniauskas
Banco de Portugal

Pedro Moreira
Banco de Portugal

António R. dos Santos
Banco de Portugal
Nova School of Business and Economics

Laszlo Tetenyi
Banco de Portugal

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Abstract
The factors determining firm size and growth are central to understanding aggregate economic growth, and the design of firm-level government policies that aim to promote growth, and smooth business cycles. Motivated by this we study the relative importance of initial conditions versus post-entry shocks in determining firm outcomes, and the relative size of capital and labor market friction that firms face. Using detailed Portuguese firm data and a simple model to aid interpretation, we find that initial conditions play an important role in determining firm outcomes, and that capital market frictions are larger than labor market frictions. (JEL: D24, E22, E23, E24, J23)

1. Introduction

Firm growth is at the center of many macroeconomic questions being considered by both researchers and policy makers. For some time governments have been interested in promoting the creation and growth of firms, and research has recently focused on the link between the dynamics of firms and aggregate economic growth (e.g. Klette and Kortum 2004; Akcigit and Kerr 2018). Understanding what drives firm growth, and what hinders it, is essential to these endeavors. A second area of interest has been support policies for firms during recessions. These featured during the Great Recession, for example with the US government bailing out major car companies, and were adopted on a large scale in response to the initial impact of Covid-19. Understanding the dynamics of firms is useful for improving the formulation of such policies.

We investigate two particular issues that are important to our understanding of firm dynamics. The first question is to what extent a firm's outcomes are determined by

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E-mail: nkozeniauskas@bportugal.pt; pmoreira@bportugal.pt; ammsanto@bportugal.pt; ltetenyi@bportugal.pt

their initial conditions or their post-entry shocks. Sterk *et al.* (2021) have shown that the answer to this question is important for understanding the impact of frictions in the economy, such as frictions in the accumulation of demand and financial frictions. It is also potentially important for policies aimed at promoting firm creation as a way of driving economic growth. If firm outcomes are largely determined by post-entry shocks, then a policy-maker may want to encourage a high level of firm creation, and exit among the less successful firms. If, instead, initial conditions are very important, then a better policy might focus on encouraging the entry of firms that are strong from the start.

The second question is about the relevance of different frictions to firm growth. There are theories under which firm growth is a result of a range of frictions, including adjustment costs to capital (e.g. Clementi and Palazzo 2016), financial frictions (e.g. Cooley and Quadrini 2001; Moll 2014; Midrigan and Xu 2014), labor market frictions (e.g. Kaas and Kircher 2015; Bilal *et al.* 2022), and frictions in demand accumulation (e.g. Gourio and Rudanko 2014). However, evidence on the relative importance of these theories is limited. If one wants to design policies to encourage firm creation and growth, or support firms in a downturn, then understanding the frictions that firms face matters.

To study these issues we start with a simple two period model of the firm lifecycle. In each period firms pay a fixed cost to operate and hire capital and labor to produce. Firm productivity is determined by a permanent component that grows as the firm ages, and a transitory shock each period. The permanent component is a firm's initial condition, and the transitory component is its post-entry shocks. We study the decisions of firms for three versions of the economy: a frictionless economy in which firms can choose their optimal levels of capital and labor in both periods; an economy with frictions that constrain capital choice when firms are young; and an economy in which frictions constrain labor choice when firms are young. We are agnostic about the sources of capital and labor constraints, modeling them in a general way that can capture many potential sources.

The model has several empirical predictions that are useful for addressing our two questions. Regarding the distinction between initial conditions and transitory post-entry shocks, we focus on two predictions. The first relates to the persistence of firm size rankings, in a similar spirit to Sterk *et al.* (2021). When productivity is purely determined by transitory post-entry shocks, there is no correlation between the size of firms when young and old. In the transition matrix for firm size rankings, all elements are equal. The greater the role of initial conditions, the closer this transition matrix is to being diagonal. The second prediction is about the correlation between the size of firms when young, and their exit probability when old. If productivity is determined purely by transitory post-entry shocks, then this correlation is zero, and there is no persistence in size and profitability. Initial conditions with permanent effects generate such persistence, causing a negative correlation between these moments, that is stronger the more important initial conditions are. We show that these predictions hold in all three versions of the economy that we consider.

For understanding the sources of firm growth, our first prediction describes the relationship between productivity growth, and capital and labor growth, in the frictionless version of the economy. This is useful as a way to consider how much the economy departs from a benchmark without frictions on capital and labor accumulation by young firms. The second prediction shows that the evolution of the capital to labor ratio with firm age differs across the three versions of the economy. This ratio is flat in the frictionless economy, increasing when there is a capital constraint on young firms, and decreasing when the constraint is on labor. The intuition for this is simple. When capital is constrained, firms use less capital than would be optimal when young, and compensate with more labor. This depresses the capital to labor ratio, causing it to increase when firms outgrow their constraint. For a constraint on labor, the intuition is analogous.

To evaluate these predictions we use panel data on the balance sheets of the universe of Portuguese firms. We focus on cohorts born in 2009–12 and track the same firms over time. This has the advantage of ensuring that, conditional on age, firms have been subject to the same conditions during their life, so that age effects are not contaminated by different cohorts having experienced different economic histories.

The data provide clear evidence of the importance of initial conditions. Transition matrices for both labor and capital rankings between ages one and six have heavy diagonals, and there is a clear correlation between the size of firms at age one, and their probability of surviving to age six. This evidence is in line with the results of Sterk *et al.* (2021), which are based on the evolution of employment of US firms. Our results on the relationship between initial size and exit are consistent with Geroski *et al.* (2010).

In assessing the extent to which firm growth is a result of productivity growth, a key component is the returns to scale in production. Our results illustrate that the predicted paths for capital and labor in a frictionless economy are very sensitive to the value of returns to scale. Estimates for this parameter vary widely in the literature depending on the methodology, making it difficult to confidently estimates the size of frictions. The paths for capital, productivity and labor in the data suggest returns to scale in the region of 0.8, if frictions to inputs are not large, and less than this otherwise. For the capital to labor ratio, the data clearly show that this moment increases with age, on average. We infer from this that frictions on capital are *relatively* larger than those on labor. This result is particularly interesting since Portugal has relatively rigid labor markets, implying that the capital market frictions are significant.

The remainder of the paper is organized as follows. Section 2 introduces the model and presents the theoretical results. Section 3 describes the data and Section 4 contains the empirical analysis. Section 5 concludes.

2. Model

For guiding our approach to the data we start with a simple model of firm production. A firm exists for up to two periods and each period can produce using the following production function:

$$y_t = (\lambda a_t + (1-\lambda) z_t) k_t^{\varphi\alpha} l_t^{\varphi(1-\alpha)}. \tag{1}$$

$a_t > 0$ is the predictable component of the firm's productivity that captures permanent differences between it and other firms. There will be no uncertainty about this once a firm is born, so we will refer to this as a firm's "initial condition" or "permanent productivity." z_t is the transitory component of productivity that is realized each period after entry, $\lambda \in [0,1]$ determines the relative importance of initial conditions and post-entry shocks, k_t and l_t are the capital and labor inputs, $\varphi \in (0,1)$ determines the returns to scale, and $\alpha \in (0,1)$ determines the relative weights on capital and labor as production inputs. In the extreme case of $\lambda = 0$, productivity is purely transitory and has no persistence. A firm draw a new value of productivity each period. In the other extreme in which $\lambda = 1$, a firm draws a value of a_t at birth and has this constant value of productivity for its whole life. A higher value of α implies that capital has more weight in production than labor. A value around 0.3 is typical, as this implies a labor share that is similar to the data. For φ, a higher value results in firms being larger because they produce more for any given level of inputs. This value also determines the profit share of firms. Values around 0.9 are commonly used in the literature (e.g. Atkeson and Kehoe 2005). For present purposes, the exact value of φ and α within the specifies ranges are not material for the results.

To simplify notation wherever the distinction between the temporary and permanent components of productivity is not important, let $\tilde{z}_t \equiv \lambda a_t + (1-\lambda) z_t$. To operate each period the firm needs to pay a fixed costs $\gamma > 0$, which must be covered by current period profits.[1]

There is a continuum of firms. Each period they receive a draw of z_t. These draws are i.i.d. across firms and over time, are independent of a, and their distribution satisfies $\mathbb{E}[z_t] = 0$. At birth, each firm draws a permanent productivity factor $a > 0$, which determines the value of a_t each period in the following way:

$$a_t = (1+\psi)^{t-1} a,$$

where $\psi > 0$ is the growth rate of the permanent component of productivity between periods one and two.

The term *productivity* should be interpreted loosely. It is not intended to captured purely technological factors determining the relationship between capital and labor, and physical output. Rather, it should be interpreted to capture all factors that affect the sales of a firm beyond these two inputs. This includes changes in demand side factors like the number of customers or prices. This point will be particularly important when the model is taken to the data.

The two periods represent the two halves of a firm's life. In the first period, a firm is young and may be subject to constraints on its input choices (which we will be specific about shortly). In the second half it is mature and is assumed to have outgrown any constraints or frictions, so that it can choose the optimal level for its inputs. To simplify the analysis we assume that both capital and labor can be rented period by period, at exogenous prices $r > 0$ and $w > 0$, respectively.

1. We are ruling out the possibility of firms borrowing, in the expectation of profits in period 2, to cover the fixed cost in period one. This simplifies the analysis of the extensive margin decision of whether to operate each period or not.

In order to think about the effects of frictions on firms, we consider three cases for the firm's problem. In case one the economy is frictionless. In both periods, so long as profits are large enough to cover the fixed cost, the firm's problem is to choose capital and labor to maximize profit:

$$\max_{k_t, l_t} \tilde{z}_t k_t^{\varphi\alpha} l_t^{\varphi(1-\alpha)} - k_t r - l_t w. \tag{2}$$

In case two, there is assumed to be a friction or constraint affecting the capital choice in period one. We model this as an upper bound on the capital choice. The problem of the firm for period one in this case is:

$$\max_{k_1, l_1} \tilde{z}_1 k_1^{\varphi\alpha} l_1^{\varphi(1-\alpha)} - k_1 r - l_1 w,$$
$$\text{s.t. } k_1 \leq \bar{k},$$

and in period two the problem is the same as in equation (2). In case three, the constraint is on labor rather than capital, with the production problem in period one being

$$\max_{k_1, l_1} \tilde{z}_1 k_1^{\varphi\alpha} l_1^{\varphi(1-\alpha)} - k_1 r - l_1 w,$$
$$\text{s.t. } l_1 \leq \bar{l}.$$

Again, there is no constraint in period two and the problem is given by equation (2). For some of the analysis we will need to specify a level for the constraint on capital or labor. For these cases we assume that the constraint is proportional to each firm's optimal input choices in the frictionless world. Where distributional assumptions for productivity are needed, we assume normality.[2]

In this framework we are interested in thinking about the predictions of the model for firm behavior, which we can test in the data. The analysis focuses on addressing two questions. The first is how important are initial conditions for the success of firms, compared to post-entry shocks. A popular way to model firm dynamics is to assume that firms have idiosyncratic productivities that evolve stochastically over time. Furthermore, all heterogeneity is typically due to transitory shocks, so that in the long run all firms expect to have the same productivity, regardless of their initial productivity level. In the present model, this corresponds to $\lambda = 0$ so that productivity is purely transitory. In the opposite extreme, initial conditions determine everything when $\lambda = 1$.

The second question is about the frictions that firms need to overcome to grow to their optimal size. There is a wide range of theories about frictions that impede firm growth. Ideas include that firm face financial constraints, that there are non-linear adjustments

2. See the Appendix B for the formal statements of these assumptions. The first assumption is useful for analytical tractability but is more restrictive than is necessary. The only condition that the constraint needs to satisfy is that it does not change the firm size ranking. Similarly, the results will hold for other distributions of the productivity shocks, but normal distributions simplify the analysis. It is true that, under this distributional assumption, it is possible for firms to have negative productivity. This is not a problem because such firms will choose to exit.

cost for capital, that labor market frictions mean that it takes time to accumulate the right workers, that there is imperfect information that takes time to resolve, and that there are frictions to accumulating customers. While there is some evidence in support of all of these, we do not have a clear idea of their relative quantitative importance. While it is beyond the scope of this paper to investigate this question in general, we make a contribution to this by investigating the relative importance of frictions impeding the accumulation of capital, and those impeding the growth of labor.

To start we consider what the evolution of capital and labor with firm age tells us about initial conditions and post-entry shocks. We focus on the transition matrix of firm size rankings for this purpose.

PROPOSITION 1. *If productivity is permanent ($\lambda = 1$) then the transition matrix for firm size (measured with labor or capital) rankings between periods one and two is diagonal. If productivity is transitory ($\lambda = 0$), every element of the transition matrix is identical. This holds for all three cases that we consider.*

The intuition for this result is as follows. If productivity is fully determined at birth ($\lambda = 1$), then firms that are larger in period one will also be larger in period two. If there is a constraint on labor or capital choice in period one, then decisions in that period will be distorted. However, since the constraint applies proportionally to all firms, it does not disturb the firm size ranking. If productivity is purely transitory ($\lambda = 0$), then productivity rankings in period one are uncorrelated with productivity rankings in period two. Whether or not there are constraints on period one input choices, firm size is monotonically increasing in productivity in both periods, so the independence of productivity between periods one and two carries over to firm size. Therefore the distribution of weights in the transition matrix are informative about the relative importance of initial conditions and post-entry shocks.

Next consider the relationship between firm size in period one and the probability of exiting in period two.

PROPOSITION 2. *If productivity is transitory ($\lambda = 0$), then, for firms that operate in period one, there is no correlation between size in that period and exit probability in period two. If productivity has a permanent component ($0 < \lambda < 1$) the correlation is negative.*

Exit in period two is determined by whether or not a firm can cover the fixed operating cost with its period two profits. These are determined by the firms productivity. Productivity also determines firm size. If productivity is purely transitory, then productivity is uncorrelated between periods one and two. It follows that firm size in period one is independent of whether the firm operates in period two. As λ increases towards one, the persistence of productivity increases. With more persistence, there is a greater correlation between size in period one, and size and profits in period two. This generates a negative correlation between size in period one and the probability of exit in period two. This proposition allows us to make inferences about the strength of initial conditions from the correlation between the size of firms at birth and their exit rates.

We next turn to considering the effects of frictions on firm growth.

PROPOSITION 3. *In the frictionless case capital and labor are proportional to a measure of productivity adjusted for returns to scale ($\tilde{z}^{\frac{1}{1-\varphi}}$).*

In the frictionless case, labor and capital inputs are fully determined by productivity. This proposition summarizes exactly how capital and labor would evolve over time for a firm that is experiencing productivity growth. This is useful because, given a value for φ and a measure of productivity, it can be used to quantify how much firm growth departs from the frictionless case. Alternatively, given a path for labor or capital, and a path for productivity, the proposition implies a value for φ that would be consistent with this behavior in a frictionless world. The difference between this value of φ and the true value, is a measure of the size of frictions to input accumulation. Given that measuring returns to scale and productivity is difficult, there will be limitations to implementing this proposition, but we will nevertheless make some use of it for interpreting the data.

A second implication for firm growth focuses on predictions for the evolution of the capital to labor ratio.

PROPOSITION 4. *If productivity has a permanent component ($\lambda > 0$) and capital is restricted in period one, then the capital to labor ratio increases with firm age. If labor is restricted, the opposite is true. In a frictionless economy this ratio is constant in firm age.*

In the frictionless case, the capital to labor ratio is independent of firm productivity. This implies that this ratio will be constant over the life of each firm. This is not the case if capital or labor are restricted in period one. If capital is restricted, then firms compensate by hiring more labor, pushing the capital to labor ratio below its frictionless value. Consequently, when the firm ages out of its constraint, the capital to labor ratio rises. If it is labor that is restricted, the same process plays out, except that it is capital that is higher in period one. So, the capital to labor ratio decreases with firm age.[3] The evolution of the capital to labor ratio with firm age is therefore informative about the strength of the frictions to accumulating these inputs.

3. Data

For the empirical analysis we use the financial statements reported under the fulfillment of the Simplified Corporate Information – IES (Informação Empresarial Simplificada). These statements generate a dataset, Central Balance Sheet Database (CBS), that covers the population of all Portuguese non-financial corporations. Firms report non-consolidated mandatory annual economic, financial, and accounting information, and the CBS is available for 2006 to 2019.

This dataset allows us to follow non-financial firms from the year in which they are born until the end of the dataset or until they close. In constructing our sample we

3. For consideration of capital and labor market frictions jointly see David and Venkateswaran (2019). While that paper studies these frictions in the cross-section, the present paper evaluates how they evolve with firm age.

only consider firms that have employees and capital. We define the birth year to be the first year in which a firm has employees and capital, and their exit year to be the last year in which this condition is satisfied. We define the sample, and the entry and exit points in this way since we want to exclude firms that may only exist for accounting, taxation or administrative purposes, and to exclude very small firms that do not matter much for aggregate economic outcomes. We impose two additional restrictions on the sample. First, a firm is only be included in the sample if it has positive labor and capital within its first three years in the dataset. We allow three years since firms may take some time to raise capital and hire workers after it is formal established. For firms that have more than two years without capital and labor, we are concerned about what their economic purpose is, and whether we would be defining their age accurately. Second, we exclude firms that have gaps in employment and capital (i.e. cases where a firm has zero employees or no capital at an age > 0, and then returns to having employees and capital) since they represent a very small share of firms or any economic outcome.

We focus the analysis on cohorts born in 2009–12 so that we can track the same firms over time. We limit our sample to firms that were created after 2009, since there was a change in the Portuguese Generally Accepted Accounting Principles (GAAP) at this time, that could create time series breaks in some variables. We do not include firms born after 2012, so that we have firms with at least eight years of data. For all of the analysis we omit the observations of all firms in the first and last years, to avoid partial year effects. As an example, if one firm is born on January 1^{st} and another firm is born on December 1^{st}, then the first firm will have 12 times the sales of the second firm, all else being equal. By ignoring data for the first and last year, we ensure that we have full years for all firm-year observations. Our age convention will be that firms are aged zero in the year in which they are born, so that the first observation that we use for each firm is at age one.

Since 2009–12 was a particularly volatile period for the Portuguese economy, the reader should be careful in extrapolating the results to other periods. This period includes part of the global financial crisis that started in 2008, and the sovereign debt crisis. While there are data limitations that affect our ability to extend the analysis to firms born in earlier years, we provide analysis to confirm some of the results for firms born in 2005–08.

Throughout the article we analyze how the productivity, capital and labor of firms changed over time. Labor is measured as the total hours worked by paid employees. Since there is a large mass of employment equal to either one or two employees, hours worked brings more variation and granularity to the data. To measure the capital stock we follow Hsieh and Klenow (2009) and use the average book value of capital from the start and end of each period. The book value of capital includes intangible assets, fixed tangible assets and productive biological assets. In order to measure productivity, we follow the approach of Foster *et al.* (2001) and Foster *et al.* (2016) by assuming a Cobb-Douglas production function with labor, capital and materials as inputs and measure the TFP of firm i in sector s as:

$$\ln TFP_i = \ln Y_i - \alpha_K^s \ln K_i - \alpha_L^s \ln L_i - \alpha_M^s \ln M_i. \tag{3}$$

Y_i is the value of output, K_i is the value of capital stock, L_i is hours of paid employees and M_i is the value of intermediate inputs (i.e., cost of goods sold and supplies and external services). The weights on the inputs in equation (3) are measured for the 224 sectors (three digit CAE code) present in the data. Given the assumption of a Cobb-Douglas production function, the weights are equal to the share of revenue spent on each input. Specifically, α_L^s is the total wage bill as a share of output in sector s; α_M^s is the total cost of intermediates as a share of output; and, assuming constant returns to scale, $\alpha_K^s = 1 - \alpha_L^s - \alpha_M^s$.[4] We estimate these using sector cost shares.[5]

Table 1 provides descriptive statistics for the firms born in 2009–12 in our sample against a pre-cleaning sample (a sample containing all firms born in this period, regardless of the values for labor and capital). Turning to the characteristics of the sample, our cleaning removes approximately 55% of firms. It covers approximately 73% of total sales, 81% of labor (hours worked and number of workers) and 54% of capital. The low number for capital is mostly a result of 20% of the capital stock being held by firms that do not produce.[6] The median sales and hours worked of firms are €117,000 and 4,400, respectively, compared with €56,000 and 2,100 for the pre-cleaning sample. The industry composition of the two samples is similar, with the main difference being that manufacturing and wholesale are slightly overrepresented (14.1% and 42.2.% of total sales compared to 12.8% and 40.0% for the pre-cleaning sample, respectively) and construction and real estate activities are slightly underrepresented (9.5% of total sales compared to 11.4% for the pre-cleaning sample).

4. Empirical results

This section provides empirical tests of the propositions presented in Section 2. We find evidence supporting the importance of initial conditions, investigate the relevance of productivity growth in accounting for increases in firm size, and show that capital market frictions appear to be more prevalent in the data than labor market frictions.

4.1. Proposition 1

The first proposition from the model is that if initial conditions are more important for firms, then firm size rankings are more stable over time. To investigate this, we compute a firm-size transition matrix. We restrict the sample to firms that live to at least age six, so that we track the same firms over time. We treat their observations at age one as the period in which they are "young," and their observation at age six as when they are

4. We assume constant returns to scale for simplicity, since measuring returns to scale is difficult. If capital to labor ratios are similar across firms within a sector, the productivity rankings – which are what we use in the analysis – should not be affected much by this assumption.

5. The weights are estimated using all firms with employees and capital and with a production value at least half of the sum of the value of the intermediate inputs and employee expenses, which prevents unreasonable reported values from significantly impacting the estimates.

6. Specifically, these are firms that have zero sales and zero employees.

	Our Sample	Pre-cleaning Sample
Firms	48,355	107,275
Sales (thousands of €)		
Average	337	220
Median	117	56
Sample share (%)	73	
Labor (thousands of hours)		
Average	8.3	5.2
Median	4.4	2.1
Sample share (%)	81	
Capital (thousands of €)		
Average	75	66
Median	15	7
Sample share (%)	54	

TABLE 1. Descriptive statistics.

Notes: This table presents descriptive statistics for the firms born in 2009–12 in our sample and in the pre-cleaning sample. The pre-cleaning sample considers all firms born in this period, regardless of the values for labor and capital. Except for the number of firms, both samples exclude first and last years values for each firm such that all measures are comparable. The data used to compute the average is winsorized at 98% to prevent the effect of outliers.

"old."[7] Given this sample, we sort firms into size deciles (size will be measured with labor or capital) at ages one and six, and compute the transition matrix between deciles at these two ages. A typical element (i, j) of the matrix is therefore the probability that a firm in decile i at age one, transitions to decile j at age six. In Figure 1 we show the transition matrix when size is measured with capital. According to Proposition 1, the closer that this matrix is to the identity matrix, the more important initial conditions are for firms. In contrast, the closer the matrix is to having all elements with the same value, the more important transitory post-entry shocks are.

Figure 1 provides the transition matrix for capital. Values are sequentially colored in red, with lighter colors corresponding to smaller probabilities and darker colors to larger ones. If shocks were purely transitory, then every value would be equal to 10%. The matrix shows clear departures from this. Values are larger the closer they are to the main diagonal. All values on the main diagonal and the adjacent elements are above 10%. Moreover, there is particularly high persistence for small and large firms. Firms born in the smallest capital decile have a 59% chance of remaining in the smallest 30% of firms when they reach age six. The top of the distribution is even more stable: firms born in the top decile have a 95% chance of staying in the top 30% of firms at age six. When we look at the transition matrix for labor (Figure 2), we find that it has even greater persistence, with virtually all of the diagonal elements being larger than for capital.

7. Ideally we would have more years of data so that we can track firms until they stop growing, as this would be closest to firms reaching their unconstrained size in the model. We are restricted by the time period that the data is available for, so we use the oldest observation for each firm that is in our sample. Our robustness exercises in Appendix A extend the analysis for labor to firms born in 2005–08, so that we can track firms to age 10. The results hold for this sample.

		Age 6									
		D1	D2	D3	D4	D5	D6	D7	D8	D9	D10
Age 1	D1	**30**	17	12	12	9	7	5	3	3	2
	D2	21	**21**	15	12	10	7	6	4	3	2
	D3	16	18	**16**	14	11	9	6	5	4	2
	D4	12	13	16	**15**	13	11	7	6	5	3
	D5	9	12	14	14	**14**	11	9	7	6	3
	D6	5	9	12	13	15	**14**	12	8	7	4
	D7	3	6	8	11	15	17	**15**	13	9	4
	D8	2	3	5	7	10	15	20	**18**	13	7
	D9	1	1	1	2	3	8	17	29	**23**	15
	D10	0	0	0	0	0	1	3	8	29	**58**

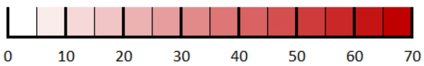


FIGURE 1: Transition matrix for capital.

Notes: This figure presents the firm-size transition matrix computed using firms' capital deciles at age 1 and age 6. Deciles range from D1 (decile one) to D10 (decile ten). Element (i, j) of the matrix is the probability that a firm in decile i at age 1, transitions to decile j at age 6. Values are sequentially colored in red, where lighter colors correspond to smaller transition probabilities and darker colors to larger values.

Labor also has particularly strong persistence in the tails of the size distribution. Firms in the top decile, for example, have nearly 90% probability of remaining in the top two deciles of the size distribution at age six. Overall, the message from these matrices is that the initial conditions for firms play an important role in determining their future outcomes.

To provide statistical evidence about persistence, we test whether the matrix is distinguishable from the extreme cases discussed in the theory: the matrix with identical elements, and the identity matrix. Formally, we perform a joint significance test for whether each element in the empirical transition matrix equals to the element of the relevant matrix in the hypothesis. Standard errors are based on Taylor-linearized variance. The data is able to reject both cases, indicating that $\lambda \in (0, 1)$ so that productivity contains both permanent and transitory components (see Appendix C.1 for additional details).

4.2. *Proposition 2*

The second prediction of our theory is about the correlation between the size of young firms and their exit probability. If shocks are purely transitory then firm size when young has no correlation with the exit rate. Persistence of productivity across periods leads to a negative correlation between size and the exit rate.

		Age 6									
		D1	D2	D3	D4	D5	D6	D7	D8	D9	D10
Age 1	D1	**33**	15	16	8	8	7	5	4	3	1
	D2	24	**26**	13	12	8	7	5	3	2	1
	D3	15	26	**16**	12	10	8	5	4	3	2
	D4	11	8	19	**16**	15	11	8	6	4	2
	D5	7	8	13	25	**13**	12	10	6	4	2
	D6	5	5	11	11	17	**15**	14	10	7	3
	D7	4	3	7	10	15	19	**17**	13	9	4
	D8	2	2	4	4	9	14	22	**20**	16	7
	D9	1	1	2	2	4	6	14	27	**27**	15
	D10	1	1	1	0	1	1	3	7	24	**61**

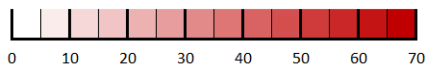


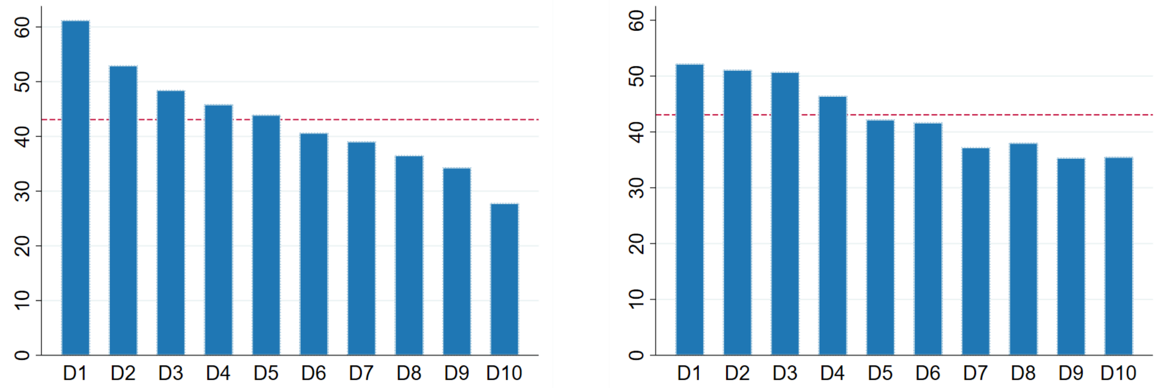
FIGURE 2: Transition matrix for labor.

Notes: This figure presents the firm-size transition matrix computed using firms' labor (hours worked) deciles at age 1 and age 6. For details on how to read the figure, see Figure 1.

To assess the relationship between initial size and exit rates, we start by dividing the firm size distribution into deciles at age one. For each decile we then compute the exit rate between ages 1 and 6. The results for the case in which size is measured with capital are presented in Figure 3a. The exit rate monotonically decreases with initial size. Firms in the smallest decile experience a 33 percentage point higher exit rate than firms in the highest decile. Formal tests reject the null hypothesis that all exit rates are the same (see Appendix C.2 for details). When we measure size with labor in Figure 3b, we find results that are qualitatively the same, but with smaller differences across the size distribution. For this case the difference between the exit rates of the lowest and the highest deciles is 17 percentage points. Since a large mass of firms have only one or two employees (which implies a lower variability of hours worked) the difference between the exit rate of some adjacent deciles is not as pronounced as for capital.

These results share some similarities with those of Geroski *et al.* (2010). Using data for Portuguese firms that lived between 1983 and 1993, that paper shows, *inter alia*, that there is a negative relationship between initial employment and the subsequent exit rates of firms. We confirm that this remains a feature of the economy approximately twenty years later, despite the changes in the business environment of Portuguese firms, and show that the result holds when size is measured with capital instead of labor. The agreement between the results also partially addresses the concern that the result from our sample could be specific to the particular years of data that we are working with.

To provide another angle on the relationship between size and exit, and also to introduce the data on firm growth that will be the focus of the rest of the analysis,



(A) Capital (B) Labor

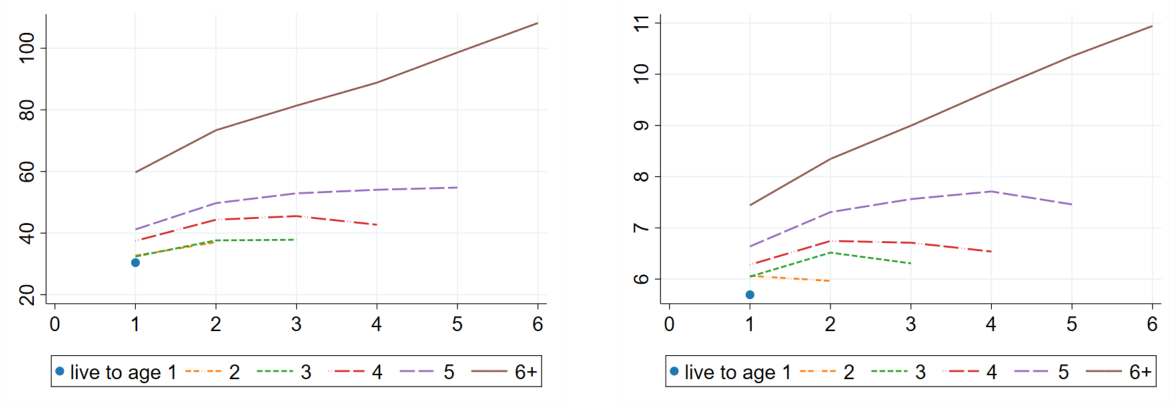
FIGURE 3: Exit rate (percentage) by size decile at age one.

Note: The exit rate is the share of firms at age 1 that close by age 6, measured in percentage. Panel (A) reports the values for each capital decile. Panel (B) reports the values for each labor (hours worked) decile. Deciles range from D1 (decile one) to D10 (decile ten). The dashed red line represents the average exit rate.

we now consider how the survival of firms is related to their initial size and their subsequent growth. Specifically, in Figure 4a we show the evolution of average capital stock conditional on how long firms live for. Firms are divided up into six groups, determined by how many years they live for, and the figure reports the average value at each age for firms in each group. For example, the blue dot is the average capital stock at age one for firms that die at age two, the orange line is the average capital stock at ages one and two for firms that die at age three, *et cetera*. The figure clearly shows that firms that live for longer start larger, that these differences are persistent, and that all groups of firms grow in size on average. This also holds when size is measured with labor, as shown in Figure 4b.

We perform a number of robustness exercises to address questions about what is driving these results. One possibility is that there could be differences in the composition of firms in each group with respect to their sector or year of birth, such that the results are at least partially a result of between group effects rather than true age effects. The results could also be contaminated by time series trends, causing all firms to grow over the period being studied. To address these concerns we perform regression analysis that allows us to control for year, year of birth and sector of activity. The results hold for the inclusion of these controls individually and jointly.[8] A related concern may be that the growth of firms is only present in sectors with relatively large firms, like manufacturing, and not in sectors with small firms, like services. To assess this we replicate the analysis in Figure 4 for these two sectors separately. While firms in manufacturing are significantly bigger than services firms, the pattern that firms that live longer start larger, and stay larger, is present in both sectors. Full results are in Appendix D.

8. See Appendix D for the details of the regression specification.



(A) Capital, thousands of euros

(B) Labor, thousands of hours worked

FIGURE 4: Average capital and labor by firm age, conditional on survival.

Notes: This figure presents the average capital and labor by firm age for six groups of firms. The blue dot represents the average value at age 1 for firms that "live up to age 1" and close at age 2. The brown line represents the average value at each age for firms that live up to age 6 and might or might not close at age 7 - "live up to age 6+". The dashed lines represent the average value at each age for firms that "live up to age t" and close at age $t+1$ ($t = 2, 3, 4, 5$). Panel (A) reports the values for capital, measured in thousands of euros. Panel (B) reports the values for labor, measured in thousands of hours worked. Values are winsorized at 98% within each age of each firm group.

As a final check, we consider the possibility that the results are particular to the years that we are studying. To address this we have repeated the analysis for labor using firms born in 2005 to 2008, and find very similar results.[9] Firms that are larger at age one are generally less likely to have exited by age 10, and the patterns in Figure 3b hold: firms that live longer tend to start out larger, and these size differences are persistent.

Overall, the empirical results for Propositions 1 and 2 provide consistent evidence of the important role that the initial conditions of firms play in determining their outcomes. This has implications that warrant consideration in the design of policies related to firm creation and growth. A common policy is to encourage the creation of firms in order to promote economic growth. If initial conditions of firms are important for their outcomes, then the types of firms that enter matters. A policy that subsidizes firm entry, for example, and encourages the creation of relatively small firms may not contribute much to the economy since, on average, these firms don't grow much. One might be particularly concerned about this since subsidies may be most attractive to small marginal firms.

4.3. *Proposition 3*

Proposition 3 tells us that capital and labor should be proportional to $\tilde{z}^{\frac{1}{1-\varphi}}$ over time if there are no frictions. By comparing the implied paths with the actual paths for capital and labor provides a measure of the size of frictions in the economy. To the extent that there are frictions to an input, it will grow at a faster rate than is implied by productivity

9. See Appendix A for the details of the 2005–2008 cohort.

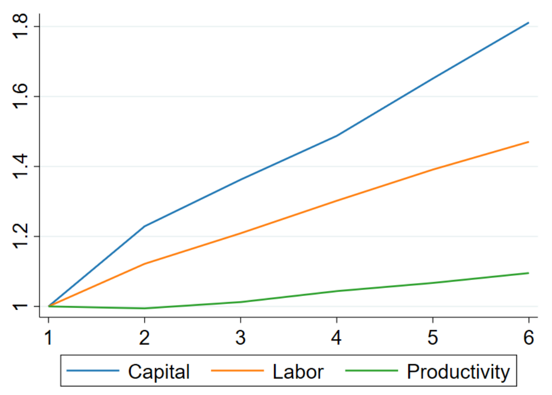


FIGURE 5: Average values by firm age relative to age 1.

Notes: This figure presents the average values for capital, labor and productivity by firm age relative to its respective value at age 1. The sample only includes firms that live to at least age six, so that we track the same firms over time. Values are winsorized at 98% within each age.

growth. Therefore, in the data, we need a measure of productivity, $\tilde{z}$, and a measure of the returns to scale of the production technology, φ, to measure frictions.

The difficulty with implementing this exercise in practice is that the returns to scale parameter is difficult to estimate, and the results are very sensitive to it. For example, consider two plausible values for φ, 0.8 and 0.9, that cover a reasonably narrow range relative to the breadth of empirical estimates. For these values, consider the effect of a 10% increase in productivity. $\varphi = 0.8$ implies a 60% increase in k and l, whereas $\varphi = 0.9$ implies a 160% increase in them. These two values will therefore imply very different levels of frictions in the economy.

Given this issue we perform a more modest exercise. We start by plotting the evolution of average capital, labor and productivity for firms that live until at least age six in Figure 5. Taking the value of these paths at age six, we can infer the values of φ that would be implied by the paths for capital and labor if they were observed in a frictionless economy. This produces $\hat{\varphi} = 0.85$ for capital and $\hat{\varphi} = 0.76$ for labor. These estimates suggest that there are relatively more frictions impeding capital when firms are young, than labor, because capital is growing faster. We explore this more in relation to Proposition 4 in the next subsection.

4.4. Proposition 4

The final part of the empirical analysis focuses on assessing the relative strengths of frictions to labor and capital. Based on Proposition 4, we study how the capital to labor ratio evolves with firm age. We interpret an increase in this ratio with age as evidence of capital being relatively more constrained than labor, and a decrease as evidence of labor being relatively more constrained.

Figure 6 presents the evolution of the capital to labor ratio in the same format as the results for capital and labor in Figure 4. That is, we separate firms into groups determined by how long they live for, so that age effects are not contaminated by selection effects due to exit. The figure shows that the capital to labor ratio tends to

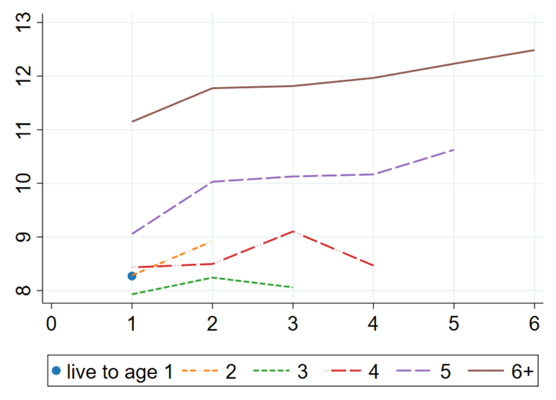


FIGURE 6: Capital to labor ratio by firm age, conditional on survival.

Notes: This figure presents the average capital to labor ratio by firm age for six groups of firms, measured in euros per hour worked. The figure is constructed in the same way as Figure 4.

increase over time, on average. Firms that live longer, in particular those that live for at least 5 or 6 years, clearly increase the relative size of their capital input. This result suggests that frictions to capital accumulation are *relatively* large compared to those on labor. Given that Portugal has quite rigid labor markets, this suggests that frictions to capital accumulation are significant.[10]

To check the robustness of these results we perform the same regression analysis as we did for the results for capital and labor growth in Figure 4, and assess whether the patterns are different in manufacturing and services. From the regressions we find that controlling for birth year, year and sector do not affect the results. Regarding manufacturing versus services, one might think that capital market frictions would be more important in the former. However, the results are quantitatively similar for these sectors.[11]

An additional interesting feature of Figure 6 is that firms that live for longer tend to have higher capital to labor ratios, and these differences are persistent. In the frictionless version of the model, all firms have the same capital to labor ratio. An extension to the model that would generate variation in these ratios is if firms have production technologies with different capital intensities, as determined by α. Interpreted in this way, the data suggest that firms that live for longer choose more intensive capital technologies. Another possible extension is that firms that live for longer are less financially constrained (higher values of θ), allowing them to start with more capital. However, if this were true, then the model says that the capital to labor ratio will grow

10. For evidence on the relative flexibility of labor markets across countries see OECD (2020) and Institute (2019). In OECD (2020) Portugal ranks in the lower half of OECD countries for most measures of labor market flexibility. In Institute (2019) Portugal ranks 38 out of 41 for employment flexibility among the EU and OECD countries studied.

11. See Appendix D for details.

more, in percentage terms, for firms that start with a lower ratio.[12] While there is some evidence of this in Figure 6 with the ratio growing more for firms that live to age five than firms that live to age six or beyond, the figure does not provide clear support for this theory.

One limitation of the analysis that is important to note is that we are measuring capital with the stock of owned capital. This excludes rented capital. This could be playing a role in the increasing capital to labor ratio if younger firms systematically rent a higher share of their capital. To investigate this we have measured the ratio of expenses on rented capital to the value of the stock of capital, and assessed how it evolves with firm age. We find that younger firms do have a higher value for this ratio, indicating that this issue is worth considering further.[13] There are several measurement issues with quantifying the effect of this, including estimating a suitable implied rental rate for owned capital, which we leave to future research.

5. Conclusion

Firm behavior is central to answering many modern macroeconomic questions, including the role of firms is driving aggregate growth, the design of government policies to promote growth, and what type of support a government should provide firms during a recession, if any. This paper contributes to our understanding of firms by studying the relative significance of initial conditions vs post-entry shocks for firm outcomes, and by evaluating the relative importance of capital versus labor market conditions.

We present a simple theory of firm production over the lifecycle that provides a number of predictions to disentangle initial conditions from post-entry shocks, and to distinguish labor market frictions from frictions to capital accumulation. We find evidence for the important role of initial conditions from the persistence of firm size and the relationship between initial firm size and exit rates. The evolution of capital to labor ratios are shown to be informative about the relative sizes of frictions to capital and labor, and the evidence points to capital market frictions being relatively more important.

While the analysis does not explicitly consider policies, the results have suggestive implications. The role of initial conditions in firm outcomes raises the question of whether a general policy promoting firm creation to drive economic growth will be effective, or whether it is important to consider the types of firms being created. The finding that frictions affecting capital are larger than those affecting labor provides motivation for further investigation into the nature of these frictions, and whether there are ways to ease them.

12. In the model, capital to labor ratio in period two divided by the capital to labor ratio in period one, in the case in which capital is constrained, is $\theta^{-(1-\varphi)/(1-\varphi(1-\alpha))}$.

13. See Appendix E for more details.

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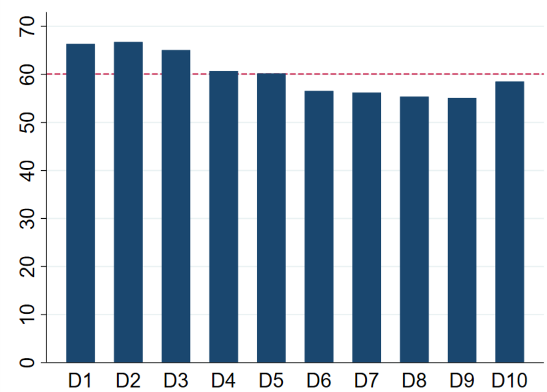
Appendix A: Labor for 2005–08 cohort

This section replicates the analysis performed in Sections 4.1 and 4.2 for firms born in 2005-–08. Since 2005 is the first year of the dataset, we need to take a different approach to defining the birth year of firms for this analysis. We define a firm to be born in year t if this is the year in which the firm was constituted, it is the first year that the firm is in the data, and the firm has at least one employee in this year. For this sample we can follow firms up to age 10 instead of age 6.

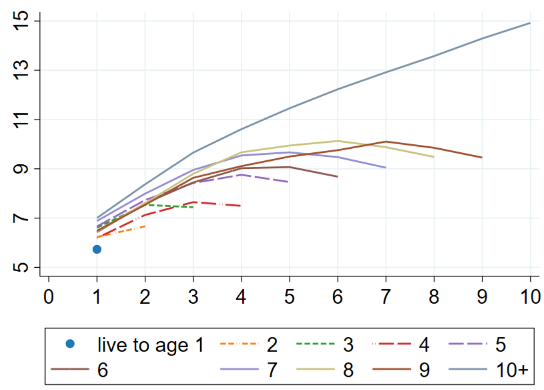
Age 1 \ Age 10	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10
D1	**30**	19	14	9	8	6	5	4	3	2
D2	19	**27**	13	12	8	7	5	4	3	2
D3	13	13	**17**	14	12	9	8	6	4	3
D4	11	12	17	**18**	13	9	8	5	4	2
D5	9	10	13	16	**14**	12	9	7	5	3
D6	6	6	11	12	17	**16**	11	10	8	4
D7	5	4	7	8	12	16	**17**	13	10	6
D8	4	4	4	6	9	14	18	**17**	15	8
D9	2	2	3	3	5	8	14	23	**25**	15
D10	2	1	1	2	2	2	4	9	22	**55**

0 10 20 30 40 50 60 70

(A) Transition matrix in the data for labor.



(B) Exit rate (percentage) by size decile at age one.



(C) Labor by firm age, conditional on survival.

FIGURE A.1: Labor figures for 2005–08 cohort.

Notes: This figure repeats the analysis in Sections 4.1 and 4.2 for labor using firms born in 2005 to 2008. Panel (A) This figure presents the firm-size transition matrix computed using firms' labor deciles at age 1 and age 10. Panel (B) presents the share of firms at age 1 that close by age 10, measured in percentage. Panel (C) presents the average labor ratio (hours worked) by firm age for ten groups of firms, measured in thousands of hours. The figure is obtained after controlling of year fixed effects. For details on how to read the figures, see Figures 1, 3 and 4.

Appendix B: Assumptions and Proofs

Two assumptions for the model are discussed in the main text. Their formal statements are as follows.

ASSUMPTION B.1. If $\bar{x} < \infty$, then $\bar{x} = \theta x^*$, where x^* is the solution for input $x \in \{l, k\}$ to problem (2).

ASSUMPTION B.2. $a \sim N(\mu_a, \sigma_a^2)$ and $z_t \sim N(0, \sigma_z^2)$ with $\mu_a, \sigma_a, \sigma_z > 0$.

The proofs of the propositions are below.

Proposition 1. If $\lambda = 1$ then firm level productivity is perfectly correlated over time and if $\lambda = 0$ the correlation is zero. This means that in the first case the transition matrix between periods one and two of productivity rankings is diagonal in the first case, and in the second case every element of the matrix is identical.

This property carries over to capital and labor. For all cases of the model that we consider,

$$k_t \propto \tilde{z}_t^{\frac{1}{1-\varphi}} \text{ and } l_t \propto \tilde{z}_t^{\frac{1}{1-\varphi}}. \tag{B.1}$$

Therefore firm rankings for capital and labor are the same as firm rankings for productivity.

Proposition 2. Whether a firm exits or not in period two depends on period two productivity, $\tilde{z}_2$. There exits a threshold $\underline{\tilde{z}}_2$ such that a firm exits if $\tilde{z}_2 < \underline{\tilde{z}}_2$. In period one, labor and capital choices are strictly increasing in productivity according to equation (B.1).

When $\lambda = 0$ productivity in period one is uncorrelated with productivity in period two. Therefore there is no correlation between labor and capital in period one, and exit in period two.

When $\lambda > 0$, this correlation is negative. The probability of a firm with permanent productivity level a exiting in period two is the probability that $a_2 + z_2 < \underline{\tilde{z}}_2$. This can be expressed as

$$\Pr[e_2 = 1|a] = \Phi\left(\frac{\underline{\tilde{z}}_2 - (1+\psi)a}{\sigma_z}\right),$$

where e_2 is an indicator for whether a firm exits in period two and Φ is the c.d.f. for the standard normal distribution. This probability is decreasing in a For all three cases of the model, it is possible to rearrange output in period one, y_1, to get an expression for productivity, $\tilde{z}_1$. Therefore y_1 generates a signal about a of the form

$$s_1 \equiv a + z_1.$$

s_1 has distribution $N(a, \sigma_z^2)$ and is increasing in y_1. Bayes rule then implies that the distribution of a, conditional on y_1, is normally distributed with mean and variance

$$\mu_p \equiv \frac{\sigma_z^2 \mu_a + \sigma_a^2 s_1}{\sigma_z^2 + \sigma_a^2}, \quad \sigma_p^2 \equiv \frac{\sigma_z^2 \sigma_a^2}{\sigma_z^2 + \sigma_a^2},$$

where the p subscript indicates that these are parameters of the posterior distribution. The probability of exiting in period two, conditional on y_1, is:

$$\Pr[e_2 = 1|y_1] = \int_{-\infty}^{\infty} \Pr[e_2 = 1|a] \varphi\left(\frac{a - \mu_p}{\sigma_p}\right) da,$$

where φ is the p.d.f. of the standard normal distribution. This probability is decreasing in y_1 since μ_p increases in y_1, this shifts the density in the φ term to higher values of a, and $\Pr[e_2 = 1|a]$ is decreasing in a.

Proposition 3. In the frictionless case, the optimal input choices satisfy equation (B.1) and the constants of proportionality are the same in periods one and two for labor, and also for capital. It follows that $k/\tilde{z}^{1-\varphi}$ and $l/\tilde{z}^{1-\varphi}$ are constant in firm age.

Proposition 4. In the frictionless case, it follows from the first order conditions of a firm's problem that

$$\frac{k}{l} = \frac{\alpha w}{(1-\alpha)r}.$$

Since this is independent of $\tilde{z}$, the capital to labor ratio is constant in firm age.

For the case in which the choice of capital is constrained in period one, the labor and capital input choices in this period can be expressed as

$$\begin{aligned} l_1 &= \left[\frac{\varphi(1-\alpha)\tilde{z}_1(\theta k^*)^{\varphi\alpha}}{w}\right]^{\frac{1}{1-\varphi(1-\alpha)}}, \\ k_1 &= \theta k^*, \end{aligned}$$

where k^* is the solution for capital for the frictionless case:

$$k^*(\tilde{z}) = \left[\varphi\tilde{z}\left(\frac{\alpha}{r}\right)^{1-\varphi(1-\alpha)}\left(\frac{1-\alpha}{w}\right)^{\varphi(1-\alpha)}\right]^{\frac{1}{1-\varphi}}.$$

Taking the labor to capital ratio delivers

$$\frac{k_1}{l_1} = \theta^{\frac{1-\varphi}{1-\varphi(1-\alpha)}}\left(\frac{\alpha w}{(1-\alpha)r}\right) < \frac{\alpha w}{(1-\alpha)r}.$$

The inequality arises because $\theta < 1$ and $(1-\varphi)/(1-\varphi(1-\alpha)) > 0$. It follows that when capital is constrained in period one, the capital to labor ratio is lower in period one than in period two.

The symmetry of the problem implies that when labor is restricted in period one,

$$\frac{k_1}{l_1} > \frac{\alpha w}{(1-\alpha)r},$$

and therefore the capital to labor ratio is higher in period one than in period two.

Appendix C: Statistical Tests

This section provides additional details on the statistical tests of whether the transition matrix is the diagonal matrix, or has identical elements; and for the hypothesis that all exit rates are the same.

C.1. Test for transition size matrix

$$H_0 : \beta_{ij} = q$$
$$H_a : \beta_{ij} \neq q$$

where β_{ij} is the element in row i and column j.

Test statistic: $(\hat{\beta} - q)'[var(\hat{\beta})]^{-1}(\hat{\beta} - q) \sim \chi^2_{100}$

Persistent $\lambda = 1$ shocks

$q = 1$ if $i = j \wedge q = 0$ if $i \neq j$

Statistic for Capital $= 119,232$ p-value $= 0$
Statistic for Labor $= 109,732$ p-value$= 0$

Transitory $\lambda = 0$ shocks

$q = 0.1 \quad \forall i, j$

Statistic for Capital $= 72,277$ p-value $= 0$
Statistic for Labor $= 47,169$ p-value $= 0$

C.2. Test for equality of exit rate

$$H_0 : \zeta_1 = \zeta_2 = ... = \zeta_{10}$$
$$H_a : \zeta_i \neq \zeta_j \quad \text{for at least one } i \text{ and } j$$

where ζ_i is exit rate for firms in decile i.

Test statistic: $(R\hat{\beta} - q)'[Rvar(\hat{\beta})R']^{-1}(R\hat{\beta} - q) \sim \chi^2_{10}$

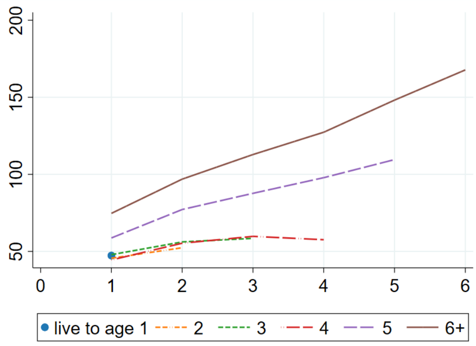
Statistic for Capital $= 1,793$ p-value $= 0$
Statistic for Labor $=$ 803 p-value $= 0$

Appendix D: Robustness exercises for section 4

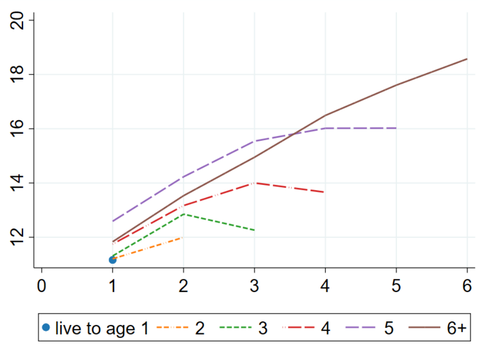
This section presents the robustness exercises referenced in Section 4. It provides additional details on how the regression analysis is performed and replicates the analysis in Figures 4 and 6 for manufacturing and services separately. For the regression analysis, the following equation is used:

$$Y_{it} = \boldsymbol{\beta}_1'(\mathbf{Age}_{it} \otimes \mathbf{MaxAge}_i) + \boldsymbol{\beta}_2'\boldsymbol{\omega}_t + \boldsymbol{\beta}_3'\boldsymbol{\omega}_{birth,i} + \boldsymbol{\beta}_4'\boldsymbol{\omega}_{sector,i} + \varepsilon_{it}$$

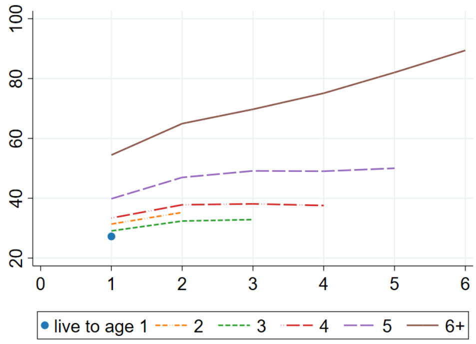
where Y_{it} is the value of either capital, labor, or the capital-labor ratio for firm i in year t, $\mathbf{Age}_{it}$ is vector of indicator variables for firm age, $\mathbf{MaxAge}_i$ is a vector of indicator variables for the age that a firm survives to in the sample, and ε_{it} is the error term. $\boldsymbol{\omega}_t$, $\boldsymbol{\omega}_{birth,i}$ and $\boldsymbol{\omega}_{sector,i}$ are vectors of indicator variables capturing year, birth year and activity sector fixed effects. The tensor product $\mathbf{Age}_{it} \otimes \mathbf{MaxAge}_i$ generates a vector containing all possible pairs of the elements of $\mathbf{Age}_{it}$ and $\mathbf{MaxAge}_i$ multiplied by each other. The coefficients of interest in $\boldsymbol{\beta}_1$ measure what the average effect of each age is on either capital, labor or capital-labor ratio, conditional on how long firms live for. If no fixed effects are included, the results of the regression are exactly the same as in the main text. We estimate the regression with all possible combinations of the fixed effects, except those that include year and birth year simultaneously since that makes $\mathbf{MaxAge}_i$ redundant (it cannot be identified). The main results hold, with results available upon request.



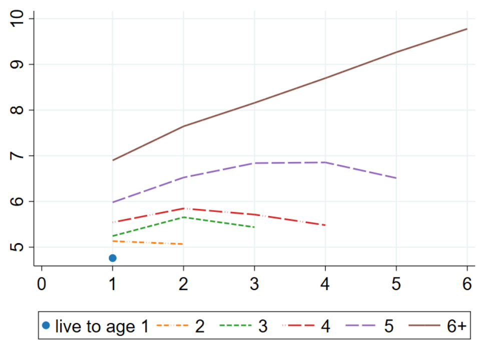
(A) Manufacturing: capital



(B) Manufacturing: labor



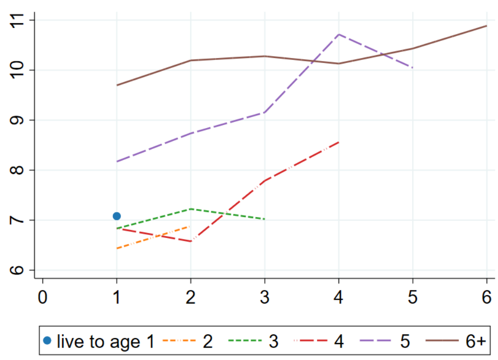
(C) Services: capital



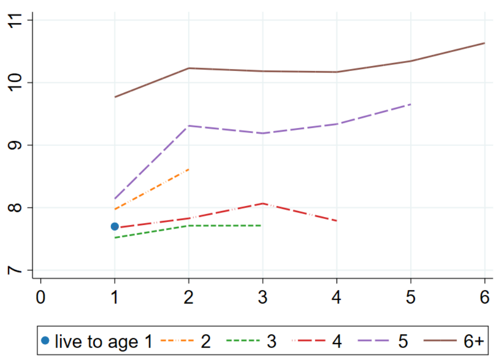
(D) Services: labor

FIGURE D.1: Average capital and labor by firm age, conditional on survival.

Notes: This figure presents the average capital and labor by firm age for six groups of firms, distinguishing between manufacturing firms (code C) and services firms (codes G,I,M-S). Values for capital are measured in thousands of euros and values for labor are measured in thousands of hours worked. The figure is constructed in the same way as Figure 4.



(A) Manufacturing



(B) Services

FIGURE D.2: Capital to labor ratio by firm age, conditional on survival.

Notes: This figure presents the average capital to labor ratio by firm age for six groups of firms, measured in euros per hour worked. Panel (A) considers Manufacturing firms (code C). Panel (B) considers Services firms (codes G,I,M-S). The figure is constructed in the same way as Figure 4.

Appendix E: Rentals and capital

This section presents the figure for rentals as a percentage of capital. Rentals are reported in the P&L of each firm (account code 6261) as a part of supplies and external services. They include operating lease expenses such as land, buildings, equipment (office, factory, sales, etc.) and vehicles.

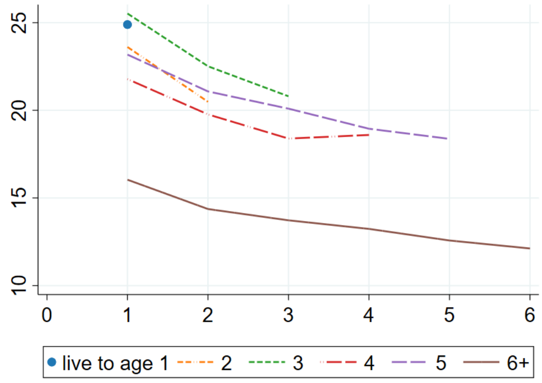


FIGURE E.1: Rentals as a percentage of capital.

Notes: This figure presents the ratio between total rentals and total capital for each age in each group of firms. The figure is constructed in the same way as Figure 4.