Inputs, technology and efficiency: The Portuguese economy in the last three decades

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Abstract
This article estimates a common dynamic stochastic production frontier for the European Union countries taking several decades within the period 1990–2017. These frontiers are the starting point to analyse developments in the Portuguese economy, notably through a growth accounting exercise that disentangles the total contribution of inputs' accumulation and total factor productivity (TFP) to GDP growth. In addition, TFP contribution is broken down into technological progress and changes in efficiency. Moreover, the computation of the elasticities of capital and labour to GDP make it possible to disentangle total input's accumulation into the contributions of capital and labour. Results reflect a modest performance of the Portuguese economy along the last decades, particularly in terms of the contribution of efficiency developments. (JEL: C11, O47, O52)

1. Introduction

The expansion of total factor productivity (TFP) reflects the ability of an economy to grow over and above the accumulation of inputs like capital and labour and it is typically obtained as part of a growth accounting exercise. Therefore, the analysis of TFP developments is a relevant part of the debate on Portuguese and European economic growth. However, in order to better understand economic performance, GDP growth must be disentangled in such a way that TFP is not obtained as a simple residual, i.e., not just in terms of what is not explained by the accumulation of inputs. If we assume that every economy can have access to the world technology, which evolves over time for different capital-labour combinations, it is possible to estimate an international stochastic production frontier and decompose TFP as the contribution of technological progress (shifts in the frontier) and efficiency developments (changes in the distance to the frontier).

These two components represent different dimensions to be considered in TFP developments. In conceptual terms, technological progress corresponds to more *productive* techniques, for example associated with innovations, which are not

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captured by the conventional methods of accounting the stock of inputs. In parallel, improvements in efficiency correspond to improved institutional and organizational arrangements, i.e., more *efficient* use of the current level of inputs and technology. Therefore, for given levels of capital and labour, an economy benefits from the world technological progress, though these gains may not entirely materialize due to efficiency developments. In practical terms, the best performers within the set of countries in the sample determine the international frontier, also meaning that technology can deteriorate if all countries perform worse for each combination of inputs. Although the direct causes for efficiency developments are not identified in this type of methodological approach, growth accounting exercises based on stochastic technological frontiers are a step forward in understanding the drivers of economic developments in different periods of time.

The seminal contribution in empirical growth literature is that of Solow (1957), which decomposes GDP growth along input's accumulation and TFP. The application of dynamic stochastic production frontiers to growth accounting, notably through Bayesian statistical methods, was suggested by Koop *et al.* (1999), using a set of developed economies. Our exercise closely follows this methodological approach and updates the work of Amador and Coimbra (2007), maintaining all its priors and assumptions, while using another database and a different set of countries. The time period covered is 1990–2017 and the set of countries corresponds to the EU28. This group of countries faces a similar set of institutional constraints, making it even more likely that they potentially have access to a common technology. Amador *et al.* (2019) present a short exercise using this methodology and database for the same set of countries but for a shorter period, 1995–2014.

The main question underlying this article concerns the relative performance of the Portuguese economy comparatively to the EU28 average along the last three decades with an emphasis on the role of inputs, technology and efficiency. The results suggest that the experience of EU28 countries is quite different, with Portugal showing a modest performance in relative terms. This is associated with decreasing contributions from capital and labour accumulation and basically no positive effects emerging from TFP developments. Most strikingly, efficiency developments have been systematically negative, hinting at the existence of structural problems that have not been solved.

The article is organized as follows. The next section briefly describes the methodology and the database used. Section 3 presents the contributions of the growth accounting components to Portuguese and EU28 GDP growth in the five 10-year periods considered. Moreover, we highlight the low efficiency levels in Portugal and their low contributions to GDP growth, also in comparison with similar EU28 countries in terms of capital stock and employment. Section 4 offers some concluding remarks.

2. Methodology and data

The estimation of stochastic production functions (or stochastic frontiers) is standard in the literature, though the utilization of Bayesian methods is somewhat less common. In this section we briefly overview the estimation methodology and present the database. As in all empirical work, results are dependent on the data available, which is a challenge when, as in our study, long comparable time series for different countries are required. The data for the most recent years are necessarily prone to revisions.

2.1. The stochastic frontier approach

The basic underlying hypothesis in the methodology is the existence of a common EU28 stochastic dynamic production frontier, which can be statistically identified because there are countries lying in its different segments. Conceptually, it means that, since all countries have access to the same technology, if two of them have equal labour and capital endowments the one with higher GDP is more efficient, i.e., it stands closer to the frontier.

The validity of the assumption on the existence of a EU28 production frontier is worthwhile discussing. Although it is accepted that knowledge about production techniques and about the relative value of goods and services is widely accessible across Member States, its dissemination may take a long time to materialize, for example due to specificities of domestic institutions or other barriers. For instance, heavy licensing procedures or other regulatory costs may deter the entry of firms with new technologies. In this vein, Basu and Weil (1998) discuss the speed of international dissemination of technological progress and its implications in terms of growth, arguing that it occurs at a slower pace than the diffusion of knowledge. In this context, the time that elapses until a country effectively adopts the technological innovations in the production systems is reflected in its relative productive efficiency. Overall, the dissemination of knowledge is faster within a group of countries that is homogeneous in terms of institutional setup and with geographical proximity, which supports the decision to estimate the EU28 stochastic production frontier.

Another issue is the assumption regarding the pace of technological progress. The assumption taken is that it evolves in a linear way. In connection with what was referred in the previous paragraph, this implicitly means that there is an average speed for the adoption of new technologies across countries and specific lags or leads are captured by the efficiency component. Koop *et al.* (1999) tested alternative formulations for the dynamics of the production function, namely a time specific model, where frontiers are totally independent in time, a quadratic trend model and a linear trend model under constant returns to scale. Each of these alternatives presents advantages and limitations. The time specific model is very flexible but implies the sampling of numerous parameters, which is computationally heavy. The linear and quadratic trend models are less demanding in terms of parameters but impose some rigidity in the dynamics of technical progress. The quadratic trend is more flexible than the linear one, which makes it preferable if long periods of time are analysed. In turn, the linear trend constrained to a constant returns technology imposes too much structure. Taking the set of alternatives, the linear trend model seems to offer the best compromise, with good results in terms of the in-sample fit and useful balance between flexibility and parsimony.

Our article considers five separate 10-year periods (9 annual rates) and results for the growth accounting exercises are presented in terms of average contributions to GDP growth. In order to increase the robustness of conclusions, we take a rolling window perspective with partial overlapping, considering periods [1990–1999], [1995–2004], [1999–2008], [2004–2013], [2008–2017]. It should be noted that the length of the periods considered is sufficient to average out short-run fluctuations in the macroeconomic variables.

Regarding the functional form of the production function, we use a translog specification. This formulation encompasses, as a special case, the logarithmic transformation of the Cobb-Douglas production function and it is more flexible than the latter. In fact, a major limitation of the logarithmic transformation of the Cobb-Douglas production function is the absence of interaction terms between labour and capital. Temple (2006) argues that the assumption of a Cobb-Douglas specification may lead to spurious results in economical and statistical terms. The problem is magnified because traditional growth accounting exercises treat TFP as unobservable (omitted variable). Conversely, if the researcher identifies a good proxy for TFP and the data are actually generated by a translog, a suitable specification accurately recovers the original parameters and rejects the Cobb-Douglas. Nevertheless, as a robustness test, we estimated the frontier as a Cobb-Douglas production function and results remain qualitatively unchanged.

Econometric principles allow for the estimation of stochastic production functions through maximum likelihood methods.[1] However, the Bayesian methods are suitable when samples are small because they allow for inferences without relying on asymptotic approximations. In addition, most importantly, Bayesian methods make it possible to rationally combine observed data with economically meaningful initial assumptions (priors). In practice, observed data is combined with priors to generate a posterior distribution function.

In our exercise, the prior for the efficiency parameter is an asymmetric positive distribution. The rationale behind this assumption is twofold. Firstly, this parameter measures the distance relatively to the production frontier so it should be positive. Secondly, there is a smaller probability of finding observations as we move further inside the production frontier. This assumption is common for the estimation of stochastic frontier functions but the specification of the asymmetric distribution remains an open question. We opted for a normal-gamma model (normal distribution of the residual component and gamma distribution for the efficiency component). Its relative advantages versus other alternatives, such as normal-half normal and normal-exponential models, are discussed in Greene (2000) and Tsionas (2000).

1. For references on non-bayesian estimation methods of stochastic production functions see, for example, Aigner *et al.* (1977), Meeusen and der Broeck (1977) and Kumbhakar and Lovell (2004).

2.2. *The model*

The model considered for the decomposition of the GDP growth closely follows Koop *et al.* (1999) and takes the form:

$$Y_{ti} = f_t \left(K_{ti}, L_{ti}\right) \tau_{ti} w_{ti}, \tag{1}$$

where Y_{ti}, K_{ti} and L_{ti} stand for the real output, the real capital stock and labour in period t $(t = 1, ..., T)$ in country i $(i = 1, ..., N)$, respectively. Furthermore, τ_{ti} $(0 < \tau_{ti} \leqslant 1)$ is the efficiency parameter and w_{ti} represents the measurement error in the identification and its stochastic nature. As mentioned above, the basic model assumes a flexible translog production function:

$$y_{ti} = x'_{ti}\beta_t + v_{ti} - u_{ti} \tag{2}$$

where:

$$x'_{ti} = \left(1, k_{ti}, l_{ti}, k_{ti}l_{ti}, k_{ti}^2, l_{ti}^2\right) \tag{3}$$

$$\beta_t = \left(\beta_{t1}, ..., \beta_{t6}\right)' \tag{4}$$

and lower case letters indicate natural logs of upper case letters. The logarithm of the measurement error v_{ti} is iid $N(0, \sigma_t^2)$ and the logarithm of the efficiency parameter is one sided to ensure that $\tau_{ti} = \exp\left(-u_{ti}\right)$ lies between zero and one. The prior for u_{ti} is taken to be a gamma function with a time specific mean λ_t.

The contribution of input endowment, technology change and efficiency change to GDP growth are defined in a simple way. The GDP growth rate in country i in period $t + 1$ is:

$$y_{t+1,i} - y_{t,i} = \left(x'_{t+1,i}\beta_{t+1} - x'_{t,i}\beta_t\right) + \left(u_{t,i} - u_{t+1,i}\right), \tag{5}$$

where the first term includes technical progress and factor accumulation and the second term represents efficiency change. In practice, shifts in the frontier correspond to changes in the betas between two moments in time, while changes in inputs correspond to changes in k and l, which are considered in vector x. Therefore, the first term can be further broken down as:

$$\frac{1}{2}\left(x_{t+1,i} + x_{ti}\right)'\left(\beta_{t+1} - \beta_t\right) + \frac{1}{2}\left(\beta_{t+1} + \beta_t\right)'\left(x_{t+1,i} - x_{ti}\right) \tag{6}$$

The technical change for a given level of inputs results from the first term of the previous equation and is defined as:

$$TC_{t+1,i} = \exp\left[\frac{1}{2}\left(x_{t+1,i} + x_{ti}\right)'\left(\beta_{t+1} - \beta_t\right)\right] \tag{7}$$

and the input change defined as the geometric average of two pure input change effects, relatively to the frontiers in consecutive years:

$$IC_{t+1,i} = \exp\left[\frac{1}{2}\left(\beta_{t+1} + \beta_t\right)'\left(x_{t+1,i} - x_{ti}\right)\right] \tag{8}$$

The efficiency developments (change in the distance to the frontier) are defined as:

$$EC_{t+1,i} = \exp(u_{ti} - u_{t+1,i}) = \frac{\tau_{t+1,i}}{\tau_{t,i}} \tag{9}$$

As previously mentioned, the structure of technological change is assumed to evolve in a linear way. Therefore:

$$\beta_t = \beta^* + t\beta^{**} \tag{10}$$

$$\sigma_t^2 = ... = \sigma_T^2 = \sigma^2 \tag{11}$$

Thus the model is written as:

$$y = X^* \times \beta - u + v \tag{12}$$

with

$$y = \left(y_1^{'}...y_T^{'}\right), u = \left(u_1^{'}...u_T^{'}\right), v = (v_1...v_T)', \beta = \left(\beta^{*'}\beta^{**'}\right)', \tag{13}$$

where β is a 12×1 vector and:

$$X^* = \begin{bmatrix} X_1 & X_1 \\ . & . \\ X_t & tX_t \\ . & . \\ X_T & TX_T \end{bmatrix} \tag{14}$$

where X_t is a 28 (countries) by 6 vector.

The sequential Gibbs sampling algorithm defined by equations A.2 to A.6 was run with 1,020,000 iterations for each of the five 10 year periods considered, with a burn-in of the first 20,000 iterations to eliminate possible start-up effects (see Casella and George 1992). In addition, the results for the contributions of technology (ATC), input (AIC) and efficiency (AEC) are presented in terms of geometric averages for each period. The details of the likelihood function and the formulas for capital and labour elasticities are presented in Appendix A.

An important methodological aspect is to verify that the algorithm converges to a stable distribution for each parameter, thus providing robust posterior estimates. In this context, we have computed the classic Geweke (1992) algorithm convergence criteria. Geweke's statistic is a convergence diagnostic for Markov chains based on a test for equality of the means of the initial and final parts of the chain, which has an asymptotically standard normal distribution. More specifically, if the two samples are drawn from the stationary distribution of the chain, the corresponding means should equalize. In our exercise, the Z scores for all parameters reject the probability of the difference between the means of the samples associated with the first and second half of the iterations to be different from zero.

2.3. *Database*

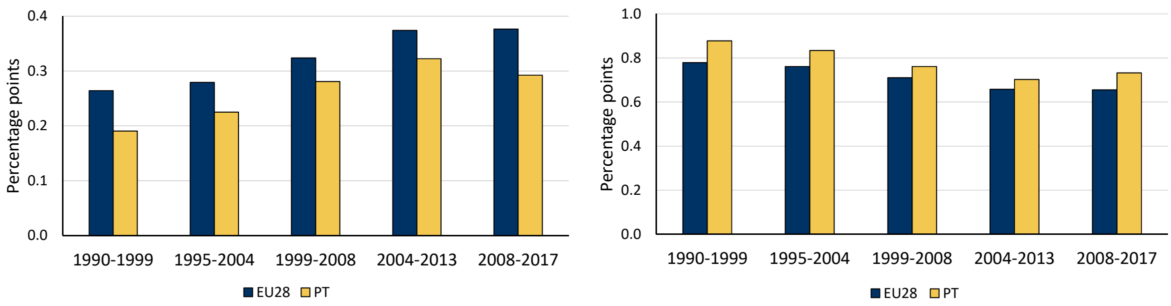
The data set used collect information for employment in number of persons, capital stock and GDP for the overall economy from 1990 until 2017 for the set of EU28 countries collected from the 9.1 version of the Penn World Table (Feenstra *et al.* 2015). Growth accounting exercises depend upon reliable data and, when the aim is to estimate a stochastic production frontier, this data have to be comparable across countries.

The Penn World Table has set a standard for high quality in historical cross-country economic aggregates, thus it is suitable to provide an accurate insight into the size and contributions to income differences in the EU28. The latest version of the database is more robust and has expanded the scope of information available relatively to the previous ones, notably in what concerns measures of physical capital. Nevertheless, GDP growth rates for Portugal in 2016 and 2017 in this version of the database are distant from those recorded in the national accounts data. Therefore, we replaced GDP growth rates for these years. As for all other years and for employment levels the database is very close to the national accounts, which is also the case for the remaining countries considered in the exercise.

3. Results

The basic results of the sequential Gibbs sampler are the posterior means and medians for the set of 12 parameters in the production function. These sets of parameters in the five periods considered can be used to compute the elasticity of capital and labour for Portugal and for the remaining EU28 countries (Figure 1). The values obtained for the elasticities of capital and labour, used to breakdown the contribution of inputs to GDP growth, are different from those usually considered in classic growth accounting exercises, with the elasticity of capital being higher in our exercise. Some facts concur to explain these differences. The translog specification is more flexible than the Cobb-Douglas as elasticities depend on the specific levels of capital and labour, thus deviating from the fixed share of inputs on GDP. The estimation of the EU28 production frontier with a Cobb-Douglas production function conveys capital elasticities close to those of labour, in a background where labour shares have been lower in recent decades. Furthermore, the estimated production functions exhibit mild increasing returns to scale. The sum of the elasticities of capital and labour is approximately 1.03 for all decades considered.

Labour elasticities increased in Portugal and in the EU28 until the decade 2004–2013, while capital elasticities decreased up to this period. In the decade 2008–2017 the Portuguese economy was characterized by capital elasticities slightly higher than those of the EU28 average (0.73 and 0.66, respectively), meaning that, in the segment of the international production function where Portugal stands, further capital accumulation has an impact on GDP levels that is not distant from that of the EU28 but smaller than in the past. In practice, this result highlights the importance of investment as a driver for Portuguese economic growth.



(A) Labour (B) Capital

FIGURE 1: Estimated labour and capital elasticities for Portugal and EU28 average

In this context, it should be added that capital-labour ratios in the Portuguese economy have been relatively low in the context of the EU15 and slightly above those prevailing in the EU28 average. Nevertheless, the Portuguese capital-labour ratio became closer to the EU15 average in the decade 2008–2017, also via the reduction of the denominator, driven by the strong job destruction that took place during the 2011–2014 economic and financial adjustment program. Amongst other drivers, the relatively low capital-labour ratios cannot be dissociated from the reduced qualifications of the Portuguese labour force. These features put the Portuguese economy in a less favourable segment of the international production function, posting lower GDP per worker and expanding relatively less in a context of capital biased technological progress.

Tables 1 and 2 report the results of the detailed growth accounting decomposition for Portugal and for the average of the EU28, respectively. The Portuguese economy presented a modest performance in the decade 2008–2017 with an average GDP growth of -0.01 per cent. The average posterior Bayesian estimate is very close to this number (0.03 per cent). Economic growth in this period was affected by the 2008 global economic and financial crisis and by the following euro area sovereign debt crisis. The subsequent recovery starting in 2014 puts Portugal's GDP in 2017 approximately at the same level reported in 2008. The sharp correction in the macroeconomic imbalances prevailing in the Portuguese economy, associated with the sudden stop in external financing, had a negative impact on investment and led to a strong destruction of jobs. Therefore, the contribution of total input accumulation was small, -0.13 percentage points (p.p.), with capital representing 0.59 p.p. and labour -0.72 p.p. These contributions were the lowest of the five 10-year periods considered.

It is also worth noting that the contribution of capital and labour to Portuguese economic growth has decreased along the five 10-year periods considered. Although the contribution of labour to inputs' accumulation is affected by cyclical developments, the long-term trend reflects adverse demographic developments associated with the ageing of the population. This is one of the important challenges affecting the Portuguese economy going forward, as well as several other European countries. As for capital accumulation in Portugal, the progressive lower contribution to GDP growth results from the combination of lower elasticities and subdued investment associated with the high debt levels that have been prevailing in the economy. The drop in investment

Decades ending	Observed GDP	Expected GDP	Input			Total Factor Productivity		
			Total	Capital	Labour	Total	Technology	Efficiency
1999	2.75	3.20	3.21	2.98	0.23	-0.01	0.46	-0.47
2004	2.61	3.26	3.01	2.75	0.26	0.25	1.54	-1.29
2008	1.36	1.73	2.03	1.94	0.09	-0.30	1.30	-1.60
2013	-0.36	-0.34	0.32	0.75	-0.43	-0.67	-0.52	-0.14
2017	-0.01	0.03	-0.13	0.59	-0.72	0.16	0.26	-0.10

TABLE 1. Growth accounting results for **Portugal**

Note: Observed and expected GDP are presented as percentage average decade growth rates, while inputs and total factor productivity are presented as percentage points (geometric) average decade contributions. Expected GDP and contributions from inputs and total factor productivity result from the bayesian estimation.

Decades ending	Observed GDP	Expected GDP	Input			Total Factor Productivity		
			Total	Capital	Labour	Total	Technology	Efficiency
1999	1.33	1.57	1.24	1.58	-0.35	0.33	0.71	-0.38
2004	3.50	3.58	1.81	1.86	-0.05	1.77	1.58	0.19
2008	3.65	3.76	2.14	1.87	0.28	1.62	1.34	0.28
2013	1.41	1.31	1.40	1.37	0.03	-0.09	-0.24	0.15
2017	1.07	1.25	0.86	0.90	-0.05	0.40	0.60	-0.21

TABLE 2. Growth accounting results for the **European Union 28**

Note: Observed and expected GDP are presented as percentage average decade growth rates, while inputs and total factor productivity are presented as percentage points (geometric) average decade contributions. Expected GDP and contributions from inputs and total factor productivity result from the bayesian estimation.

was particularly strong during the economic and financial assistance program, reaching levels lower than those of the depreciations and thus leading to reductions in the level of the capital stock. The rebound of investment is another key challenge in the Portuguese economy, a task made harder by the negative prospects related with the prevailing indebtedness and the expected negative impacts of the COVID-19 pandemic.

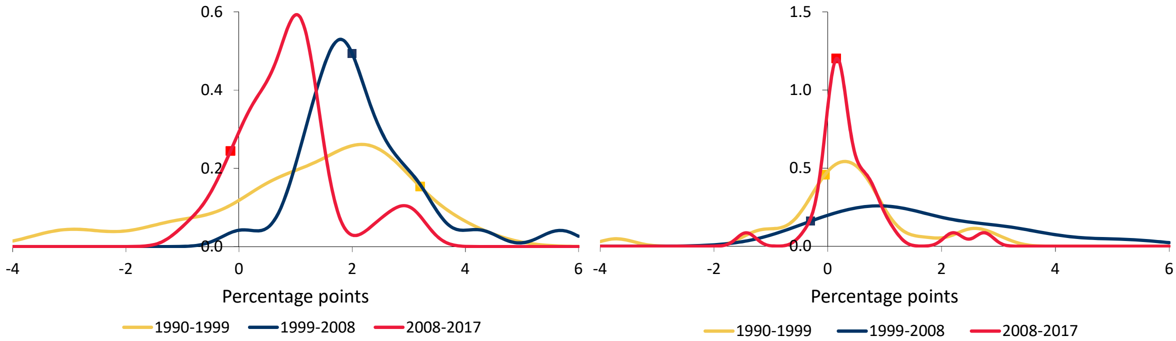
Developments in the TFP contribution to Portuguese GDP growth were different in the five 10-year periods considered, also in terms of contributions from technological progress and efficiency developments. This information is contained in the last three columns of Table 1. TFP contributions have been systematically low in Portugal, reaching values of -0.30 and -0.67 p.p in the decades ending in 2013 and 2017, respectively. In addition, contributions in Portugal were always lower than those observed in the average of the EU28, where the TFP was only contributed negatively in the decade 2004–2013 (-0.09 p.p.). Therefore, the increase in TFP, at least in terms of keeping up with the EU28 developments, constitutes another important challenge for the Portuguese economy in the next years.[2]

The comparison of the Portuguese growth accounting results with those for the average of the EU28 are very useful for benchmarking purposes. In this latter region, the decade 2008–2017 also featured the lowest average GDP growth rate of the five 10-year periods considered, with a weakening of inputs accumulation but a small improvement in TFP contribution relatively to the decade ending in 2013. Nevertheless, the EU28 average combines realities from two distinct sets of countries. Although with some differences, the countries whose accession took place until 1995 (EU15) started with an underlying situation that was quite different from that of the countries that joined later, in the aftermath of the fall in the Berlin wall (EU13). These differences in performance are visible in Tables B.1 and B.2 presented in Appendix B.

The EU13 countries experienced a negative performance in the decade 1990–1999, associated with the transition from central planning to market economies. In this period there was a strongly negative contribution from labour accumulation, due to job destruction and emigration, coupled with sizeable efficiency losses. However, in the other decades the EU13 countries performed much better than the EU15 group. Almost all the new EU member states have been catching up strongly and converging towards the EU15. The largest contributions for this good performance are attributed to TFP developments, which reached 2.79 and 2.86 p.p. in the decades ending in 2004 and 2008, respectively. These results highlight both the existence of cross-country differences but also the key role of TFP for economic growth.

Figure 2 illustrates cross-country differences in the EU28 by plotting the kernel distributions of total inputs and TFP contributions to GDP growth rates in the decades 1990–1999, 1999–2008 and 2008–2017, with the position of the Portuguese economy signalled by coloured squares. The distribution of input's contributions (panel A) presents a heavier left tail in the decade ending in 1999, due to what occurred in the Central and Eastern European countries. In the remaining decades the distribution is closer to a Gaussian curve and it has shifted left in the most recent period. The Portuguese position deteriorates, starting at the right tail of the 1990-1999 distribution and finishing at the left tail of the most recent distribution. Furthermore, the distribution of TFP contributions (panel B) reflects different underlying structural conditions in terms of quality of inputs and functioning of markets and institutions. In the decade 2008–2017 most of the density in the distribution lies between zero and 1 p.p., with Portugal posting a marginally positive figure (0.16 p.p.). In fact, Portugal reports contributions close to zero in all decades.

2. The results are in line with those in the Special Issue "Real convergence in the European Union and the relative performance of the Portuguese economy" (Banco de Portugal 2019).



(A) Total input accumulation

(B) Total factor productivity

FIGURE 2: Distribution of growth accounting contributions in the EU28

Note: The squares indicate Portugal's position in the respective distribution. Kernel density estimation is a non-parametric way of estimating the probability density function of a variable.

As previously mentioned, one of the strengths of the methodology is the possibility of breaking down TFP contributions along technological progress and efficiency developments, which are conditional on the position of the estimated international stochastic frontier. In this respect, what strikingly stands out in the Portuguese case is the systematic negative contribution from efficiency developments to average GDP growth in the five 10-year periods considered. These contributions reached -1.29 and -1.60 p.p. in the decades ending in 2004 and 2008, respectively. Moreover, these contributions were mostly compensated by the positive contribution of technological progress, which is associated with the performance of best performing countries with similar levels of inputs. Next, in the decade that ends in 2013, as previously mentioned, since most countries were negatively affected by the global economic and financial crisis, the contribution of technological progress is negative, which adds up to a smaller contribution from efficiency developments (-0.14 p.p.). Finally, in the decade 2008–2017 the contribution of technology returned to positive territory but that was not the case for efficiency developments.

The change in TFP contributions in the average of the EU28 followed a pattern similar to that of Portugal, but the levels of contributions were clearly higher. Again, most of the difference is attributed to efficiency developments. The analysis of the subgroups of countries in the EU15 and EU13 provides further detail. Comparing the last column of Tables B.1 and B.2 in Appendix B shows that, except in the decade 1990–1999, efficiency developments in the group of the most recent EU member countries were better than in the EU15, where they were marginally negative.

A complementary view consists in computing the level of efficiency of the economy. This is defined as the percentage of output actually produced relatively to the output level at the frontier, measured at the exact combination of employment and capital stock levels existing in the country. Figure 3 plots the efficiency levels in Portugal and in the average of the EU28 in the five 10-year periods considered. Efficiency levels in the average of the EU28 were relatively constant in all periods around 88 percent. In contrast, Portuguese efficiency levels stood close to 80 percent in the decade 1990–1999 and dropped to 64 in the period 2008–2017, which is a very low number. In fact, the striking result is that Portugal stands as the country with the lowest efficiency level in the set of the EU28 countries in the two final 10-year periods considered. This is in accordance with the efficiency developments reported in the growth accounting exercise and highlights the existence of ample room for improvement in the utilization and allocation of resources available in the Portuguese economy.[3]

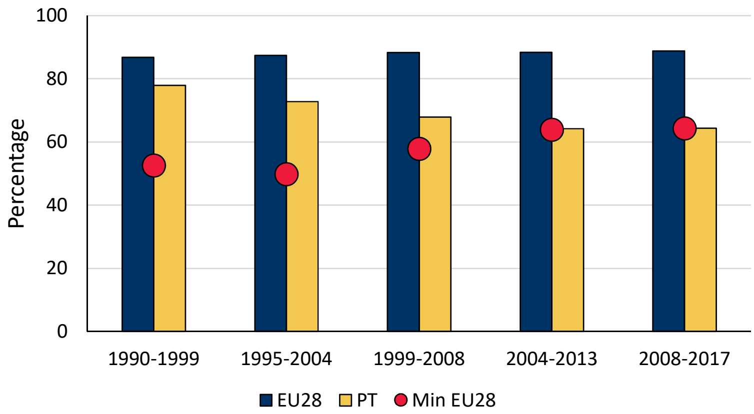
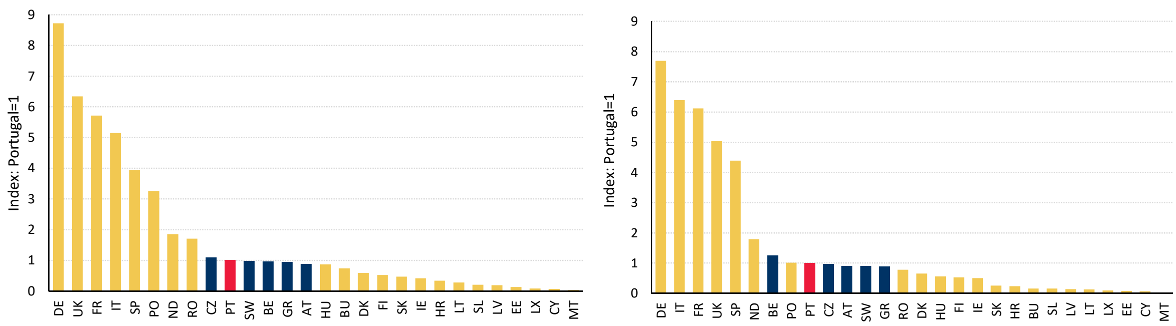


FIGURE 3: Efficiency levels in Portugal and in the EU28

In order to further document the unfavourable efficiency developments in the Portuguese economy we chose a set of five EU28 countries with a similar size in terms of employment and capital stock. Figure 4 motivates the choice of Austria, Belgium, Sweden, the Czech Republic and Greece as "peer size" countries. This group contains countries that joined the EU at different moments in time and covers different geographical areas. As a side result, Figure 4 also highlights the gap between the six largest EU28 economies and the remaining members. By choosing countries with similar levels of labour and capital we focus the analysis on a comparable segment of the EU28 production frontier.



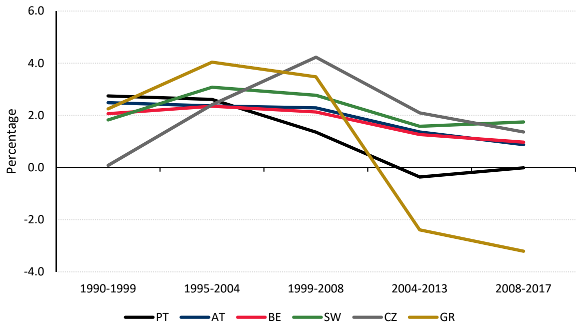
(A) Labour (B) Capital

FIGURE 4: Selected peer countries basing on total employment and capital stock
Note: Average values for the decade 2008–2017.

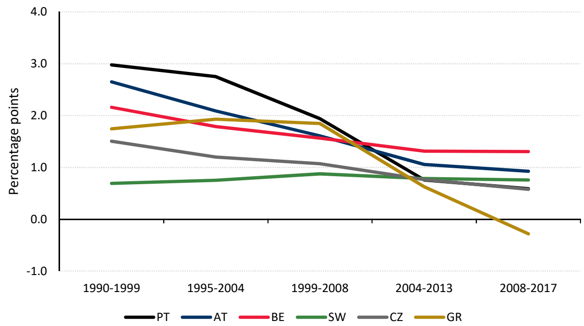
3. A robustness check using the Cobb-Douglas production function leads to the same relative efficiency levels for Portugal.

There are relevant results emerging from the comparison across peer size countries in panels of Figure 5. In the last three 10-year periods the overall GDP growth performance in Portugal was worse than in peer size countries, except for Greece in the last two periods (panel A). Conversely, the Czech Republic has shown a good GDP performance. In addition, in contrast with other peer countries, there is a sharp reduction of the contribution of capital accumulation to GDP growth in Portugal across decades (panel B). This development is paired with the negative contributions from labour in the last two decades, a feature also visible in Greece and in contrast with positive contributions in other peer countries (panel C). As for the contribution of total factor productivity in the different periods (panel D), differences are visible across countries. The Czech Republic has recorded good contributions along the time, except for the initial decade, while Portugal has posted low contributions, which were only more unfavourable for the case of Greece in the last two 10-year periods. Next, as for the contribution of technological progress for GDP growth, it is quite similar in all peer countries (panel E). This is exactly the expected result because peer countries were selected for being placed in the same segment of the stochastic production frontier. Therefore, by construction, shifts in this function affect them all in the same way. Finally, as regards the contribution from efficiency developments to GDP growth (panel F), as previously highlighted, we identify a modest performance for Portugal, translated into negative contributions, accompanied by Greece in the last two 10-year periods. These developments contrast with very small positive contributions in other peer countries, and especially with the remarkable progress observed in the Czech Republic.

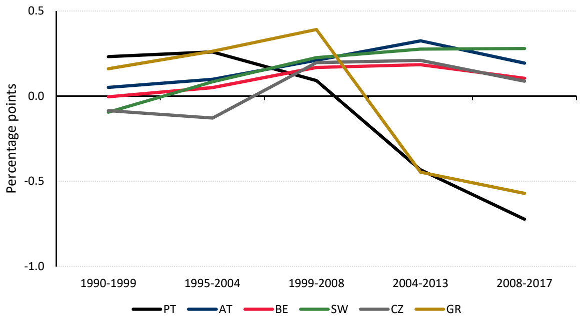
Overall, these comparisons show that countries with similar levels of inputs, which also share the EU institutional setup, may present quite different outcomes. This is related with structural conditions that go beyond the EU setup and stresses the role for good national policies and for the sharing of best practices, notably in the context of benchmarking exercises which are regularly carried out by international organizations. Moreover, the quality of inputs, related with human capital levels and types of investment made, certainly play a major role in explaining different efficiency performances.



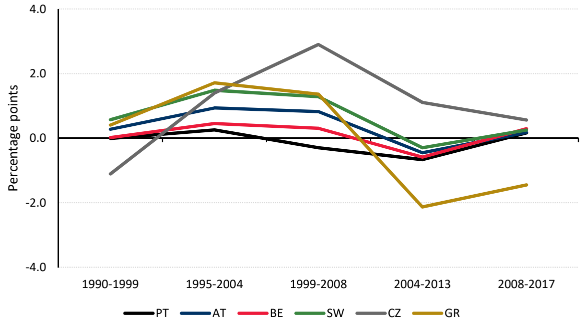
(A) GDP growth



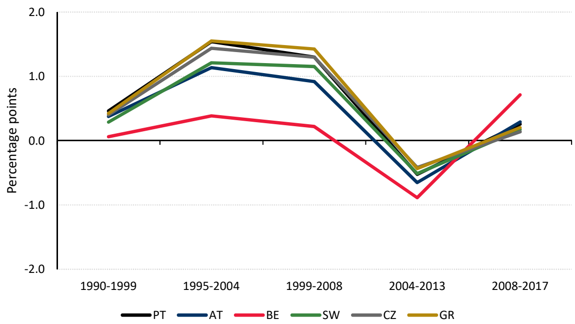
(B) Capital



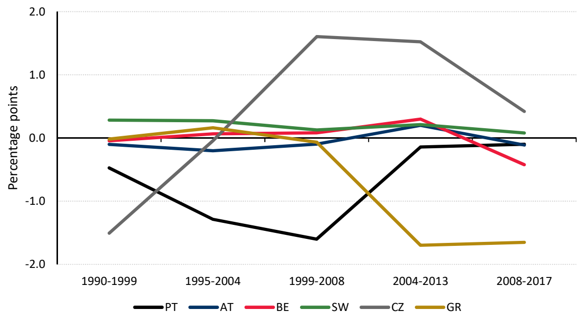
(C) Labour



(D) TFP



(E) Technology



(F) Efficiency

FIGURE 5: Growth decomposition: Portugal and peer countries regarding size

4. Final remarks

In this article we conduct a growth accounting exercise based on the estimation of a dynamic stochastic production frontier for the EU28 countries between 1990 and 2017. Growth accounting exercises are mechanical by nature but offer a useful assessment of economic performance, especially if other countries are explicitly taken as benchmarks. This comparison is possible to achieve with the stochastic production frontier approach, notably in terms of detailing TFP developments.

The results suggest a modest performance of the Portuguese economy relatively to the EU28 average. The contribution of total input accumulation in Portugal has decreased during the period considered, reflecting adverse demographic developments and low levels of investment. The relatively low capital-labour ratios and high capital elasticities highlight the importance of investment as a driver for Portuguese economic growth. Moreover, the change in TFP contributions in Portugal was qualitatively similar to that of the average of the EU28, but contributions to GDP growth were clearly lower in levels. This performance mostly resulted from efficiency developments, which had negative contributions in all decades. Moreover, efficiency levels in the Portuguese economy were lower than those for the average of the EU28 in the five 10-year periods considered, with Portugal standing as the country with the lowest efficiency level in the two final 10-year periods.

It must always be born in mind that results are sensitive to hypothesis taken and statistical data. In this latter respect, the international data for the capital stock trends are affected by different accounting measures and deflation procedures. International databases like the Penn World Tables try to offer harmonized series, though they may sometimes deviate from national sources. The alternative of fully replicating the exercise with official data for all EU28 countries is not viable due to numerous series breaks and limited time horizon. As for methodological hypothesis, it is important to underline that, although the translog production function offers substantial flexibility, this choice and the assumption of a linear trend for technological progress in each decade affects the results.

As for policy prescriptions, it is hard to go beyond the long standing references to the need to intensify the accumulation of capital and to allocate it properly, as well as to maintain progress in terms of human capital. In this article we highlight the fact that there is large room for efficiency improvements, which may materialize through the removal of unjustified regulatory barriers and the improvement of inputs' quality. The extension of the exercise that we carried out in order to explicitly consider the quality of inputs in the estimation of the technological frontier is a promising avenue for future research.

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Appendix A: The likelihood function

The full likelihood function of the model presented in subsection 2.2 can be written as:

$$f_N^{TN}\left(y\left|X^*\beta-u,\sigma^2 I_{TN}\right.\right)p\left(\beta\right)p\left(\sigma^{-2}\right)p\left(\lambda^{-1}\right)\prod_{t=1}^{T}\prod_{i=1}^{N}f_G\left(u_{ti}\left|1,\lambda^{-1}\right.\right), \tag{A.1}$$

where f_N^{TN} stands for a multivariate $T\times N$ normal probability distribution function, f_G stands for a gamma probability distribution function and:

$$\begin{aligned} p\left(\lambda^{-1}\right) &= f_G\left(\lambda^{-1}\left|1,-\ln\left(\tau^*\right)\right.\right) \\ p\left(\sigma^{-2}\right) &= \sigma^2\exp-\frac{10^{-6}}{2\sigma^2} \end{aligned}$$

The prior for λ^{-1} assumes a gamma distribution with the first parameter equal to 1 and second parameter equal to $-ln(\tau^*)^{-1}$ such that τ^* is the prior median efficiency. Typically τ^* is chosen based on a priori expectations for the median of the efficient distribution. However, in a very heterogeneous sample of countries, the existence of large deviations from the frontier increases the sum of errors and places the randomized algorithm that generates a sequence of posteriors - the sequential Gibbs sampler - in an unstable path. For the algorithm to accommodate such a sample, this has to be compensated by a low τ^*. We assume a starting point for τ^* near zero and check the posterior median efficiencies. As for σ^{-2}, we assume the usual flat prior.

Given this prior structure, the posterior marginal distributions that compose the Gibbs sampler can be easily derived. The conditional for β is:

$$p\left(\beta\left|Data,u,\sigma^{-2},\lambda^{-1}\right.\right)\sim f_N^{2J}\left(\beta\left|\widehat{\beta},\ \sigma^2\left(X^{*\prime}X^*\right)^{-1}\right.\right), \tag{A.2}$$

where

$$\widehat{\beta}=\left(X^{*\prime}X^*\right)^{-1}X^{*\prime}\left(y+u\right) \tag{A.3}$$

The conditional for σ^{-2} to be used in the Gibbs sampler is:

$$p\left(\sigma^{-2}\left|Data,\beta,u,\lambda^{-1}\right.\right)\sim f_G \left(\sigma^{-2}\left|\frac{n_0+TN}{2},\frac{1}{2}\left[a_0+\left(y-X^*\beta+u\right)'\left(y-X^*\beta+u\right)\right]\right.\right) \tag{A.4}$$

Next, the conditional for u is a left truncated normal at zero:

$$p\left(u\left|Data,\beta,\sigma^{-2},\lambda^{-1}\right.\right)\sim f_N^{TN}\left(u\left|X^*\beta-y-\frac{\sigma^2}{\lambda}\iota,\sigma^2 I_{NT}\right.\right)\prod_{t=1}^{T}\prod_{i=1}^{N}I(u_{it}\geqslant 0), \tag{A.5}$$

whose mean is forced to be higher or equal to zero in the algorithm and ι is a $TN\times 1$ vector of ones. Finally, the marginal posterior distribution for the λ^{-1} is:

$$p\left(\lambda^{-1}\left|Data,\beta,u,\sigma^{-2}\right.\right)=f_G\left(\lambda^{-1}\left|1+TN,-\ln\left(\tau^*\right)+\sum_{t=1}^{T}\sum_{i=1}^{N}u_{it}\right.\right) \tag{A.6}$$

A final important element in the methodology is verification of regularity constraints regarding the elasticities of capital (EK_{ti}) and labour (EL_{ti}). Given the matricial formulation, these generic elements are:

$$EK_{ti} = (\beta_2^* + t\beta_8^{**}) + (\beta_4^* + t\beta_{10}^{**})l_{ti} + 2(\beta_5^* + t\beta_{11}^{**})k_{ti} \tag{A.7}$$

$$EL_{ti} = (\beta_3^* + t\beta_9^{**}) + (\beta_4^* + t\beta_{10}^{**})k_{ti} + 2(\beta_6^{**} + t\beta_{12}^{**})l_{ti} \tag{A.8}$$

Therefore, we only accept a set of posterior β parameters that translate into non-negative elasticities for all countries and periods.

Appendix B: Additional growth accounting results - UE15 and UE13

Decades ending	Observed GDP	Expected GDP	Input			Total Factor Productivity		
			Total	Capital	Labour	Total	Technology	Efficiency
1999	2.67	2.73	2.30	2.22	0.08	0.43	0.55	-0.12
2004	3.13	3.25	2.37	2.14	0.23	0.87	0.94	-0.06
2008	2.53	2.78	2.23	1.91	0.32	0.54	0.63	-0.09
2013	0.73	0.66	1.28	1.21	0.07	-0.62	-0.58	-0.04
2017	0.80	1.02	0.92	0.93	-0.01	0.10	0.37	-0.28

TABLE B.1. Growth accounting results for the **European Union 15**

Note: Observed and expected GDP are presented as percentage average decade growth rates, while inputs and total factor productivity are presented as percentage points (geometric) average decade contributions. Expected GDP and contributions from inputs and total factor productivity result from the bayesian estimation. The EU15 countries are Austria, Belgium, Denmark, Finland, France, Germany, Greece, Ireland, Italy, Luxembourg, Netherlands, Portugal, Spain, Sweden and United Kingdom.

Decades ending	Observed GDP	Expected GDP	Input			Total Factor Productivity		
			Total	Capital	Labour	Total	Technology	Efficiency
1999	-0.22	0.23	0.02	0.85	-0.83	0.21	0.89	-0.68
2004	3.91	3.95	1.16	1.54	-0.38	2.79	2.32	0.47
2008	4.93	4.90	2.04	1.82	0.22	2.86	2.15	0.71
2013	2.19	2.07	1.55	1.56	-0.01	0.52	0.16	0.36
2017	1.38	1.52	0.78	0.87	-0.10	0.75	0.87	-0.12

TABLE B.2. Growth accounting results for the **European Union 13**

Note: Observed and expected GDP are presented as percentage average decade growth rates, while inputs and total factor productivity are presented as percentage points (geometric) average decade contributions. Expected GDP and contributions from inputs and total factor productivity result from the bayesian estimation. The EU13 countries are Bulgaria, Croatia, Cyprus, Czech Republic, Estonia, Hungary, Latvia, Lithuania, Malta, Poland, Romania, Slovakia and Slovenia.