The relation between PD and LGD: an application to a corporate loan portfolio

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Abstract
This article performs a conceptual credit risk exercise for the Portuguese banks' aggregate loan portfolio of non-financial corporations within the Basel IRB framework and that takes into account that default rates and loss given default rates vary together systematically. The article estimates the loss distribution and several credit risk metrics for each year between 2006 and 2019 using a one-year simulation-based single-factor model. The results suggest that, except for very high LGD values, assuming a constant LGD leads to a significant underestimation of credit risk. This conclusion is in line with the Basel recommendation to use a downturn LGD instead of expected LGD to compensate for not explicitly modeling the PD/LGD relation. In the base case it is found that, in order to account for downturn conditions, expected LGD should have an add-on of approximately 15 percentage points. A sensitivity analysis points to an add-on below 10 percentage points for only high levels of expected LGD. (JEL: G17, G21, G32)

1. Introduction

Credit risk is the risk of a loss that may occur from a borrower's failure to repay its debt. The likelihood of loss materialization is tied to the borrower's probability of default (PD) while the severity of loss in the event of default is accounted for the loss given default (LGD).

Empirical evidence has shown that default rates and loss given default rates vary together systematically (Frye 2000b; Düllmann and Trapp 2004; Altman *et al.* 2005). During economic downturns, defaults occur more frequently, assets decrease in value and recovery rates tend to be smaller. Failing to account for this relationship may lead to a significant underestimation of credit losses and necessary capital in adverse macro-economic conditions. Conventional portfolio credit risk models have focused on default risk, neglecting its relation with loss given default rates. These models treat LGD either as a constant parameter (Boston 1997: *Creditrisk+*) or as a stochastic variable independent of the probability of default (Wilson 1997: *CreditPortfolioView*; Gupton *et al.*

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1997: *CreditMetrics*; Crosbie and Bohn 2003: *PortfolioManager*). Assuming an infinitely granular portfolio, both assumptions lead to identical loss distributions regardless of the assumed distribution of individual LGD and are unable to integrate the relationship between PD and LGD.

Recent models try to address the PD/LGD relationship within the Merton (1974) structural model framework (see for example Frye 2000a; Pykhtin 2003; Tasche 2004; Giese 2005). These models provide distinct LGD specifications that arise from different premises on the functional form linking PD and LGD. However, due to rare default events, the calibration of these models is nontrivial. In addition, insufficient data undermine the ability of practitioners to distinguish between theories.

Frye and Jacobs Jr (2012) suggest an LGD function that expresses a moderate, positive relationship between default and loss given default rates using only parameters that are already part of regular credit loss models. Frye (2013) argues that risk managers can use this function to avoid introducing unnecessary parameters into their models and unneeded noise into their predictions. The author shows via simulation studies that this model works well under different scenarios and can be easily implemented for stress testing.

This article performs a conceptual exercise adopting the Frye-Jacobs LGD function within the Internal Ratings-Based (IRB) framework for the Portuguese banks' aggregate loan portfolio of non-financial corporations. The loss distribution and several one-period credit risk metrics are estimated for each year between 2006 and 2019 using a simulated-based single-factor model. These results are then compared with the ones assuming a constant LGD or a stochastic LGD independent from the PD and independent across borrowers. This exercise allows to evaluate by how much credit risk can be underestimated when adopting the static assumption and the necessary add-on to the expected LGD that reflects downturn conditions. The exercise benefits from a very rich dataset with information for PD and exposure at the firm level. The objective is to provide reference values for the add-on to be applied to the long-run expected LGD to obtain the downturn LGD and to help micro and macro-prudential authorities to have a complementary tool to assess credit risk for the banking system.

2. Methodology

This section is organized as follows. Section 2.1 presents the Asymptotic Single Risk Factor (ASRF) model, a methodologically framework that transforms unconditional PDs into conditional PDs which reflect adverse macroeconomic conditions. Section 2.2 describes the method and assumptions for the estimation of the Frye-Jacobs (FJ) LGD function. Finally, section 2.3 establishes how to generate the loss distribution trough Monte Carlo simulations using the ASRF model and the FJ LGD function. This section also shows how to compute different risk measures based on the loss distribution.

2.1. The Asymptotic Single Risk Factor (ASRF) model

The ASRF model (Vasicek 2002) assumes that the standardized asset return X of a firm i is a linear function of a single systematic risk factor, Y, and an idiosyncratic risk factor, ε_i:

$$X_i = \sqrt{r}Y + \sqrt{1-r}\varepsilon_i. \tag{1}$$

In the above equation Y and ε_i are assumed to be standard normally distributed random variables independent from each other. The systematic risk factor, Y, is unobservable and can be viewed as representing aggregate macroeconomic and financial conditions. The factor weight (or factor loading), $r \in [0,1]$, measures the sensitivity of asset returns to the risk factor. The higher the value of r, the more firms are exposed to the business cycle. This parameter introduces interdependency between defaults of any pair of firms by assuming correlation in asset returns. The correlation value equals the factor weight (r).

Based on the seminal work of Merton (1974) a default is triggered when a firm's assets value is less than debt value. That is, the default happens if the value of firm's standardized asset return, X, is below the threshold implied by the unconditional probability of default (PD) for that firm:

$$X_i \leq \Phi^{-1}(\mathrm{PD}_i), \tag{2}$$

where Φ denotes the cumulative distribution function for a standard normal random variable.[1] In this model framework the unconditional probability of default, PD, reflects expected default rates under normal business conditions. The conditional probability of default, cPD, is the probability that a firm defaults conditional on an aggregate macro-financial scenario, Y:

$$\mathrm{cPD}_i = P_r(X_i \leq \Phi^{-1}(\mathrm{PD}_i)|Y). \tag{3}$$

The intuition behind this specification is that the systematic risk factor serves to "scale up" or "scale down" the unconditional PD. Assuming an infinitely granular portfolio, i.e., the number of exposures tends to infinity and each exposure is of negligible size, and substituting equation (1) on equation (3) entails:

$$\begin{aligned} P_r(X_i \leq \Phi^{-1}(\mathrm{PD}_i)|Y) &= P_r(\sqrt{r}Y + \sqrt{1-r}\varepsilon_i \leq \Phi^{-1}(\mathrm{PD}_i)|Y) \\ &= P_r(\varepsilon_i \leq \frac{\Phi^{-1}(\mathrm{PD}_i) - \sqrt{r}Y}{\sqrt{1-r}}|Y) \\ &= \Phi(\frac{\Phi^{-1}(\mathrm{PD}_i) - \sqrt{r}Y}{\sqrt{1-r}}). \end{aligned} \tag{4}$$

1. Thus, Φ^{-1} denotes the inverse cumulative distribution function.

2.2. *The Frye-Jacobs LGD function*

The Frye-Jacobs LGD function connects the conditional LGD rate (cLGD) to the conditional default rate (cDR) under four assumptions.[2] The first assumption is that a greater rate of credit loss accompanies a greater rate of default. This assumption is much less restrictive than the common assumption that greater default rates and greater LGD rates go together. The technical assumption is that the asymptotic distributions of default and loss are comonotonic.[3] Loss and DR are comonotonic if and only if they are nondecreasing functions of the same random variable, Y. This implies that loss rate and default rate take the same quantile, q, within their respective distribution:

$$\mathrm{CDF}_{\mathrm{Loss}}[\mathrm{cLoss}] = \mathrm{CDF}_{\mathrm{DR}}[\mathrm{cDR}] = q, \tag{5}$$

where $\mathrm{CDF}_{\mathrm{Loss}}$ is the cumulative density function of the loss distribution and cLoss is a specific loss, conditional on an aggregate macro-financial scenario. Similarly, $\mathrm{CDF}_{\mathrm{DR}}$ is the cumulative density function of the default distribution and cDR is a specific default, conditional on the same aggregate macro-financial scenario. Since the loss rate is the product of default rate and loss given default rate, for any value of q, the cLGD rate equals the ratio of loss to default:

$$\mathrm{cLGD} = \frac{\mathrm{CDF}^{-1}_{\mathrm{Loss}}[q]}{\mathrm{CDF}^{-1}_{\mathrm{DR}}[q]} = \frac{\mathrm{CDF}^{-1}_{\mathrm{Loss}}[\mathrm{CDF}_{\mathrm{DR}}[\mathrm{cDR}]]}{\mathrm{cDR}}. \tag{6}$$

The model also assumes that both credit loss and default have two-parameter distributions. Within this type of distributions the model assumes the Vasicek distribution.[4] The final assumption is that the value of the factor loading, r, also applies to the loss distribution. Substituting the expressions for the Vasicek distribution into equation (6) produces the LGD function:

$$\mathrm{cLGD} = \frac{\Phi[\Phi^{-1}[\mathrm{cDR}] - \frac{\Phi^{-1}[\mathrm{PD}] - \Phi^{-1}[\mathrm{EL}]}{\sqrt{1-r}}]}{\mathrm{cDR}}, \tag{7}$$

which is fully determined by the unconditional probability of default, the factor loading and the expected loss. Thus, it only uses parameters that are already part of the standard model.

2. For a portfolio with homogeneous PDs and with equal size exposures, the cDR is equal to the cPD. For a portfolio with heterogeneous PDs and with different size exposures, the cDR is the weighted average of the cPD where the weight is the exposure of each firm in the portfolio.

3. The concept of comonotonicity has been showed to be a helpful tool for solving several research and practical problems in the domain of finance and insurance (see Deelstra *et al.* 2011).

4. Frye and Jacobs Jr (2012) recognize that this assumption is a matter of convenience since other distributions such as the Beta and the Lognormal distributions produce similar relationships but their implementation is not as practical.

2.3. *The loss distribution*

The loss distribution for a given portfolio can then be estimated through Monte Carlo simulations of the systematic factor. In each simulation/scenario, the loss, L, is the sum of the product of each firm i conditional probability of default, cPD_i, the exposure to firm i, EXP_i, and the conditional loss given default, cLGD:

$$L = \mathrm{cLGD} \cdot \sum_{i=1}^{N} \mathrm{cPD}_i \cdot \mathrm{EXP}_i \,. \tag{8}$$

Each Monte Carlo simulation can be seen as a scenario or state of the world. After simulating the common factor, the article calculates the conditional PD for each exposure and the average conditional PD of the portfolio. The latter is used to obtain the cLGD for the portfolio using equation 7.[5] Each scenario generates a particular loss for the portfolio. The frequency of various outcomes/losses after a large number of simulations generates the credit loss distribution. Figure 1 illustrates the process.

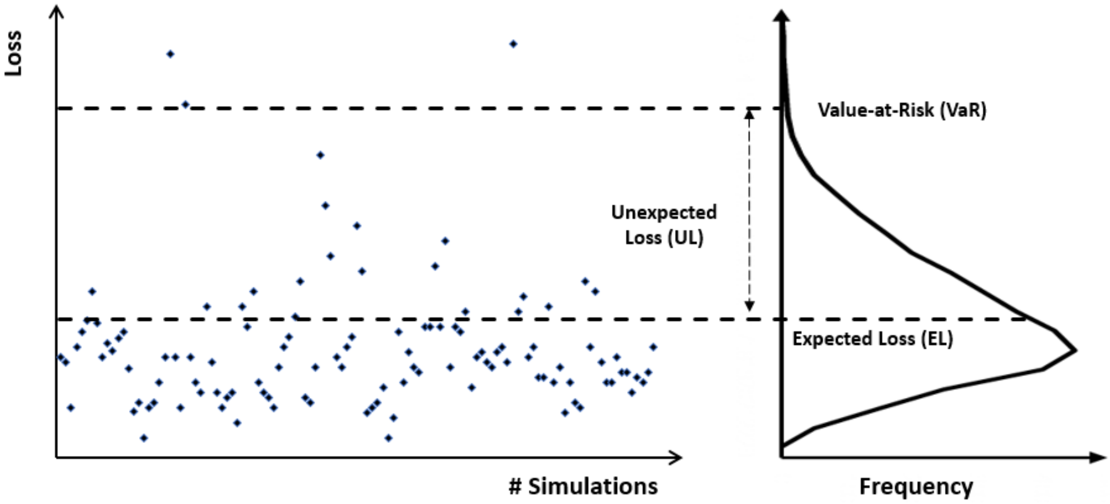


FIGURE 1: Credit Loss Distribution.

There are several risk measures that can be computed based on the portfolio loss distribution. The most commonly referred are the expected loss (EL), the value-at-risk (VaR), the unexpected loss (UL) and the expected shortfall (ES). The EL corresponds to the expected value of the portfolio loss L, which can be estimated as the mean of the simulated loss scenarios.[6] The VaR_p is the maximum possible loss if we exclude worse outcomes whose probability is less than $(1 - p)\%$. The VaR is a quantile of the

5. The reasoning for this calculation relates with empirical evidence that there is a positive relation between default rates and loss given default rates at the aggregate level. Alternatively, one could use the conditional PD for each exposure to obtain the cLGD for each exposure. However, this approach would imply a positive relation between a firm's PD and LGD at the firm level.

6. The EL can also be estimated as PD $\times$ LGD $\times$ EXP. The EL estimation does not depend on the model used.

distribution. The UL_p is the difference between the VaR_p and the EL. We can interpret the unexpected loss as the capital required to sustain losses in p% of cases. In turn, the ES measures the average loss beyond a specified quantile, the expected loss on the portfolio in the worst $(1-p)$% of cases. Basel's internal ratings-based (IRB) approach is calibrated to a probability p of 99.9%, and this is the probability level used throughout the rest of this article.

3. Data and Calibration

This article uses a unique dataset with series for non-financial corporations (NFCs) operating in Portugal between 2006 and 2019. This dataset includes individual credit exposures from the central credit register (CRC) and one-year ahead probabilities of default available from Banco de Portugal in-house credit assessment – SIAC (*Sistema Interno de Avaliação de Crédito*).[7] The initial sample covers roughly the population of NFCs that have at least one loan granted by a resident financial institution. Nevertheless, only firms whose loans are considered to be performing are included in the analysis because only those are at risk of entering default in the next year. Thus, when a firm defaults at year t it is excluded from the analysis at $t+1$ and for as long as it is considered in default.[8]

The individual exposure observed in the last month of year $t-1$ is considered as the exposure of company i at year t. In this way, all credit risk measures for year t are estimated using only information available at year $t-1$ and, consequently, can be used as early warning indicators of credit risk. The factor weight, r, is calibrated for each firm through the function determined in the Basel Accord. r is a decreasing function of the PD bounded between 0.12 (highest possible PD) and 0.24 (lowest possible PD).[9]

The last parameter needed to estimate Frye-Jacobs function is the EL. Given that the probability of default is available in the dataset, this is equivalent to saying that the model calibration requires information about the expected LGD. However, due to scarcity of data and recovery timing discontinuity there is limited information about this number. As such, this article assumes the commonly used central value of 50% as a baseline scenario and performs a sensitivity analysis considering two alternative values: 30% and 70%. Finchetto *et al.* (2019) find an average loss given default rate close to 70% for Italian firms during the same period of this analysis, while the 30% is the symmetrical value w.r.t. to 50%. Moreover, the latter might be more in line with

7. See Antunes *et al.* (2016).

8. The default is at the level of the firm and not at the level of the loan. A firm is considered to be "in default" towards the financial system if it has 2.5 per cent or more of its total outstanding loans overdue. The "default event" occurs when the firm completes its third consecutive month in default.

9. All exposures are considered as exposures to corporate - Capital Requirements Regulation article 153. A sensitivity analysis was performed on this parameter and the results are quantitatively similar.

average rates for secured exposures in accordance with banks' own estimates within the regulatory framework.[10]

4. Results

For the purpose of this exercise, either a constant value or the one given by the Frye-Jacobs function is assumed for LGD. Both approaches use the ASRF model to incorporate default risk dependency between borrowers through a unique risk factor, but only the latter captures the link between loss given default and default rates. This section starts by analyzing the loss distribution for the Portuguese banks' aggregate loan portfolio of non-financial firms using the FJ function. The results are then compared with the ones assuming a constant LGD. Finally, a sensitivity analysis is performed on the baseline scenario for an expected loss given default of 50%.

Figure 2 shows the expected loss and the three tail credit risk measures – value-at-risk, unexpected loss and expected shortfall – at 99.9% between 2006 and 2019 using the FJ LGD function.[11] In order to allow for comparisons between different years, all credit risk measures are presented as a percentage of the total exposure. Santos and Silva (2019) perform a similar exercise and find that all measures display a common pattern: a continuous increase between 2006 and 2013, followed by a decline until 2017.[12] The results presented in Figure 2 corroborate their findings and reveal that the decline after 2013 continues until 2019, following the pattern of the business cycle. In 2019, the EL was approximately at the same level as in 2008, while the UL was close, but still higher, to the minimum value reported in 2006.

When assuming a constant LGD all three tail credit risk measures decrease. This result is implicit in the construction of FJ model and should be interpreted with caution. Still, there are two important metrics that emerge from this comparison: (i) the UL difference, which evaluates by how much credit risk can be underestimated when adopting the (constant) expected LGD; and (ii) the necessary add-on to expected LGD that would provide the same required capital under both assumptions. This second measure relates with the concept of downturn LGD discussed at the beginning of this article. Figure 3 (A) illustrates the two unexpected losses and Figure 3 (B)

10. Under the Foundation IRB Approach, institutions should use LGD with values of 45% for senior exposures without eligible collateral and 75% for subordinated exposures without eligible collateral. LGD for collateralized exposures depend on the type and level of collateralization, but are bound to the 'ceiling' value of 45%. Under the Advanced IRB Approach, LGD are provided by banks based on own estimates, with some flexibility in the choice of estimation methodology. Regardless of this choice, the estimates should be calibrated to the long-run average LGD and then have an add-on to reflect the impact of downturn conditions. In order to ensure a minimum level of conservatism and to address the problem of excessive variability in risk-weighted assets, BIS (2016) proposes applying floors on estimated LGDs: 25% for unsecured exposures and between 0% and 20% for secured exposures, depending on collateral type: 0% for financial collateral, 15% for receivables, 15% for commercial or residential real estate and 20% for other physical collateral.

11. Figure A.1 in Appendix A reports the loss distributions between 2006 and 2019.

12. The results for the tail credit risk measures are quantitatively different since the authors use a multi-factor model and calibrate the exercise using their own estimates for the factor loading.

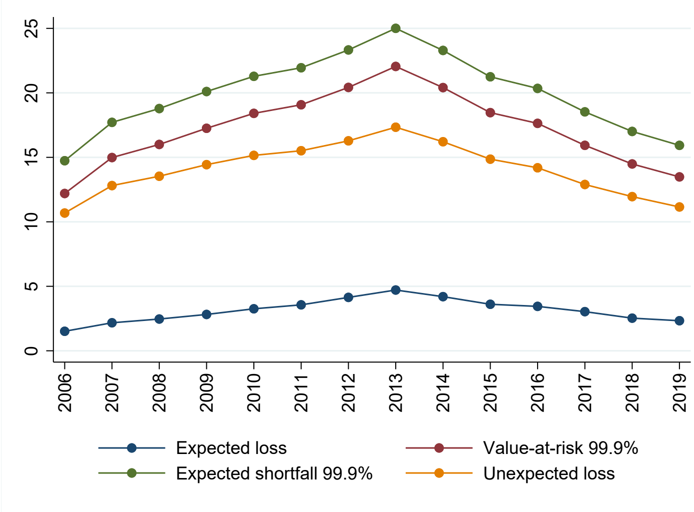


FIGURE 2: Credit risk measures as a percentage of the total exposure assuming the FJ LGD function.

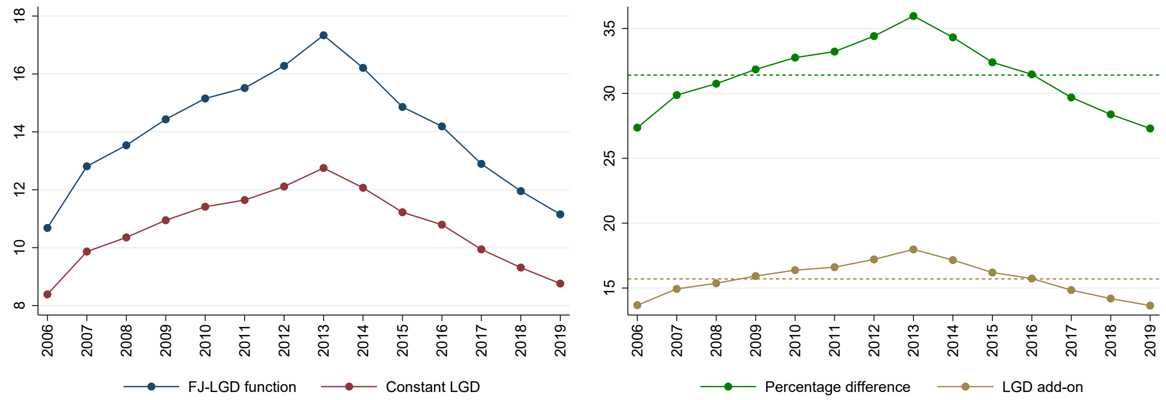
presents the percentage difference between the two measures, as well as the LGD add-on.[13] The results show that capital requirements would be underestimated by between approximately 27% to 36% under a constant LGD assumption. These numbers correspond to an add-on between 13.5 and 18 percentage points to the expected LGD. Both panels of Figure 3 also reveal a similar trend between the two metrics and the unexpected loss which indicate that the cyclicality of the tail credit risk measures is accentuated when accounting for the PD/LGD relation.

The period between 2006 and 2019 includes a full economic cycle with both an expansion and a recession. In this period the UL with the FJ LGD function is, on average, 31.4% higher than the UL under a constant LGD. This value corresponds to an average LGD add-on of 15.7 percentage points. The average value is chosen to produce a through-the-cycle measure in the spirit of the Basel framework. A through-the-cycle credit measure has a high degree of stability and smoothness which may potentially help stabilize the financial system since it creates capital during times of economic expansion that can then be utilized during economic downturns.[14]

The sensitivity analysis is performed using also the average value for each metric, as a trough-the-cycle measure. Figure 4 reports the sensitivity analysis for the base case of 50%, highlighting two alternative values for the expected LGD: 30% and 70%. Figure 4 (A) shows that the unexpected loss percentage difference is a decreasing and convex function of the average LGD. Intuitively, when the expected LGD is very high, there is not much more a lender could lose. The smaller the expected value, the greater the skew of the loss distribution. The corresponding necessary add-on to expected LGD is

13. The two metrics have a direct correspondence since the UL is proportional to the LGD values used in its calculation. This explains the identical behaviour of both series in Figure 3 (B).

14. A robustness check using through-the-cycle PDs produces the same average values for both metrics but shows less volatility throughout the years. These results corroborate the choice of averages as the benchmark values.



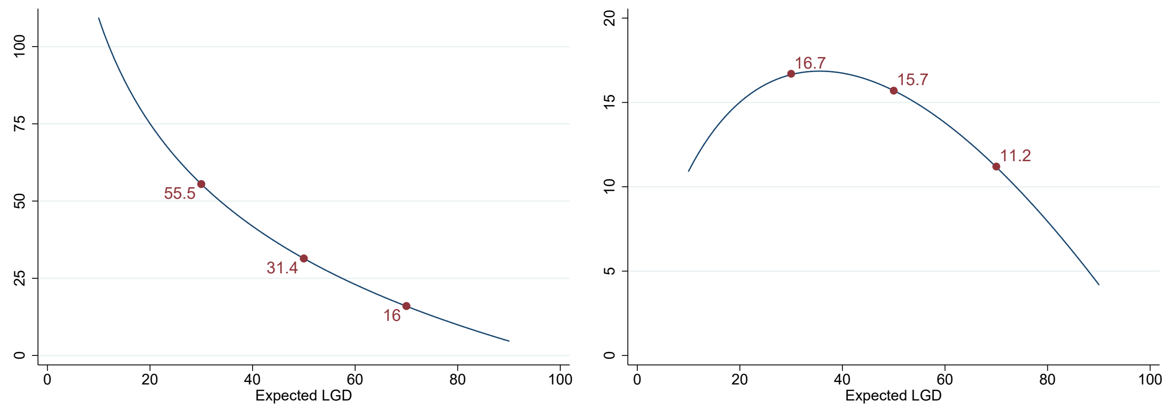
(A) UL as percentage of total exposure.

(B) UL percentage difference and LGD add-on.

FIGURE 3: Unexpected Loss under FJ-LGD function *vis-à-vis* constant LGD.

reported in Figure 4 (B). Departing from 15.7 percentage points (pp) for the baseline scenario, the 30% expected LGD implies a slightly higher add-on of 16.7 pp while the 70% only translates into 11.2 pp. Even when expanding the sensitivity analysis to a wider range of values (10%–90%), one can observe that only a very high expected LGD would lead to an add-on below 10 pp. Thus, unless the average LGD computed by banks is very high, the downturn LGD used should be substantially above its expected value.

The results suggest that in order to account for downturn conditions the expected LGD should have an add-on of approximately 15 percentage points. This value is in line with some of the applications found in the literature. Frye (2000b) argues that LGDs might increase between 20 and 25 percentage points from their normal-year average. In addition, when integrating the PD/LGD relation, Miu and Ozdemir (2006) find that economic capital increases by 35%–45% in corporate loan portfolios and 16% for a middle-market portfolio, while Altman *et al.* (2001) claim that about 30% needs to be added.



(A) UL percentage difference.

(B) LGD add-on in percentage points.

FIGURE 4: Sensitivity analysis for expected LGD. Both panels report average values for the period under analysis. Three expected LGD values are highlighted: 30%, 50% and 70%.

5. Downturn LGD in the IRB framework

The Asymptotic Single Risk Factor (ASRF) model used in the Advanced IRB-approach requires banks to estimate a loss given default. To compensate for not explicitly modeling the PD/LGD relation, Basel regulation requires the use of a downturn LGD.[15] But while banks estimate average PDs and use a supervisory mapping function to reflect economic-downturn conditions and transform them into conditional PDs, there is no explicit supervisory function to transform average LGDs into conditional/downturn LGDs. Recognizing "significant differences in practices" and "unwarranted variability in risk-weighted exposure amounts when own estimates of LGDs" are used, EBA (2019) published new technical standards that provide guidance on the types of approaches to be implemented, while still leaving flexibility with respect to the actual estimation methodology. Modeling downturn LGD is of utmost importance, as capital requirements are directly proportional to the LGD values used in the calculation.

This article obtains an estimate of 15 pp add-on to the expected LGD in order to reflect downturn conditions. This value could be used as a reference in the context of capital requirement regulation, regarding the Advanced IRB-approach. However, there are differences between some of the concepts used in this article and the ones defined by EBA: all exposures are considered as corporate[16]; the default is define at the level of the firm to the banking system and not at the level of the firm to the institution or, for firms classified as retail, at the level of the loan; probabilities of default are point-in-time and not though-the-cycle PDs; and the downturn LGD is computed using portfolio extreme losses while the regulatory downturn LGD should be computed using portfolio losses around significantly negative values for the systematic factor – possibly, not always, extreme. This conceptual exercise is applied to Portugal taking advantage of the extremely detailed statistical information covering roughly the population of NFCs. It is important to have in mind that the proportion of the loan portfolios using the Advanced IRB in Portugal is low, in European average terms, and was particularly low in the lowest phase of the economic cycle. But although the interpretation and applicability of this article's results should be interpreted with caution, they are in line with some of the applications found in the literature. Furthermore, according to EBA (2019) banks have to apply a minimum margin of conservatism (MoC) requirement of 15 percentage points on LGD estimates when using the type-3 approach, one of the three approaches proposed by EBA to estimate the downturn LGD.[17]

15. Since the implementation of Basel II, under Pillar 1 of bank capital regulation, banks can opt to either use a regulatory standardized approach to calculate credit risk capital requirements, or follow an Internal Ratings-Based (IRB) approach using their own estimated risk parameters. The IRB formula is based on the ASRF model. Portugal complies with the Credit Requirement Directive (CRD-V) and Capital Requirements Regulation (CRR II), a supervisory framework in the European Union that reflects the Basel rules.

16. It is not distinguished between SMEC, SMER, CORP e LCORP. Nevertheless, if considering all exposures as retail, the results would be similar, although slightly lower - on average, the results are approximately 2pp lower.

17. Type-1 approach calibrates downturn LGD based on the observed impact on losses of a particular downturn period; type-2 approach calibrates downturn LGD based on the estimated impact on losses

6. Conclusion

Empirical evidence has shown that default rates and loss given defaults rates are correlated. Therefore, the concept of downturn LGD in the Capital Requirements Regulation is of extreme importance to compensate for not explicitly modeling this relation. This is even more so given that capital requirements are proportional to the assumed value for LGD.

This article uses detail statistical information at the firm level to perform a conceptual exercise that, by integrating the PD/LGD relationship, provides reference values for the add-on to be applied to the long-run expected LGD. The exercise uses the Frye-Jacobs LGD function within the Basel IRB framework. This model may not be flexible enough to produce different shapes of PD/LGD correlation but, under certain assumptions, derives a relationship without any additional parameters. Due to scarcity of data, it is a parsimonious solution that attributes moderate LGD risk and works well under different scenarios. The results suggest that, except for very high figures, assuming a constant LGD leads to a significant underestimation of credit risk. In the base case it is concluded that in order to account for downturn conditions expected LGD should have an add-on of approximately 15 pp. A sensitivity analysis for a wide range of expected LGD values shows that only for high values of expected LGD – values where there is not much more a lender could lose - the add-on should be below 10 pp. These results are limited to the definition of some concepts and the parameters used but survive some robustness tests and are in line with some of the applications found in the literature.

using a limited set of methodologies; type-3 approach can be applied in rare cases, where neither type-1 nor type-2 approaches can be used. Only in exceptional cases can type-3 be approved by the Supervisory Authority since institutions should, under normal conditions, demonstrate the merits of approaches 1 and 2 when applying them.

References

Altman, Edward I, Brooks Brady, Andrea Resti, and Andrea Sironi (2005). "The link between default and recovery rates: Theory, empirical evidence, and implications." *The Journal of Business*, 78(6), 2203–2228.

Altman, Edward I, Andrea Resti, and Andrea Sironi (2001). "Analyzing and explaining default recovery rates." *A report submitted to the International Swaps & Derivatives Association*.

Antunes, António, Homero Gonçalves, and Pedro Prego (2016). "Firm default probabilities revisited." *Banco de Portugal Economic Studies*, II(2), 21–24.

BIS (2016). "Reducing variation in credit risk-weighted assets – constraints on the use of internal model approaches." *Bank for International Settlements*.

Boston, Credit Suisse First (1997). "CreditRisk+: A credit risk management framework." Tech. rep., Technical report, Credit Suisse First Boston.

Crosbie, Peter and Jeff Bohn (2003). "Modeling default risk."

Deelstra, Griselda, Jan Dhaene, and Michele Vanmaele (2011). "An overview of comonotonicity and its applications in finance and insurance." In *Advanced mathematical methods for finance*, pp. 155–179. Springer.

Düllmann, Klaus and Monika Trapp (2004). "Systematic risk in recovery rates: an empirical analysis of US corporate credit exposures." (2004,02).

EBA (2019). "Guidelines for the estimation of LGD appropriate for an economic downturn ('Downturn LGD estimation')." Tech. rep., European Banking Authority.

Finchetto, A. L., I. Guida, A. Rendina, G. Santini, and M. Scotto di Carlo (2019). "Bad loan recovery in 2018." *Notes on Financial Stability and Supervision*, (18), 1–9.

Frye, Jon (2000a). "Collateral damage." *Risk*, 13(4), 91–94.

Frye, Jon (2000b). "Depressing recoveries." *Risk*, 13(11), 108–111.

Frye, Jon (2013). "Loss given default as a function of the default rate."

Frye, Jon and Michael Jacobs Jr (2012). "Credit loss and systematic loss given default." *Journal of Credit Risk*, 8(1), 1–32.

Giese, Guido (2005). "The impact of PD/LGD correlations on credit risk capital." *Risk*, 18(4), 79–84.

Gupton, Gred M, Christopher Clemens Finger, and Mickey Bhatia (1997). *Creditmetrics: technical document*. JP Morgan & Co.

Merton, Robert C (1974). "On the Pricing of Corporate Debt: The Risk Structure of Interest Rates." *Journal of Finance*, 29(2), 449–470.

Miu, Peter and Bogie Ozdemir (2006). "Basel requirement of downturn LGD: Modeling and estimating PD & LGD correlations." *Journal of Credit Risk*, 2(2), 43–68.

Pykhtin, Michael (2003). "Recovery rates: Unexpected recovery risk." *Risk*, 16(8), 74–79.

Santos, António R and Nuno Silva (2019). "Sectoral concentration risk in Portuguese banks' loan exposures to non-financial firms." *Banco de Portugal Economic Studies*, V(1), 1–18.

Tasche, Dirk (2004). "The single risk factor approach to capital charges in case of correlated loss given default rates." Working paper.

Vasicek, Oldrich (2002). "The distribution of loan portfolio value." *Risk*, 15(12), 160–162.

Wilson, Thomas C (1997). “Portfolio credit risk.”

Appendix

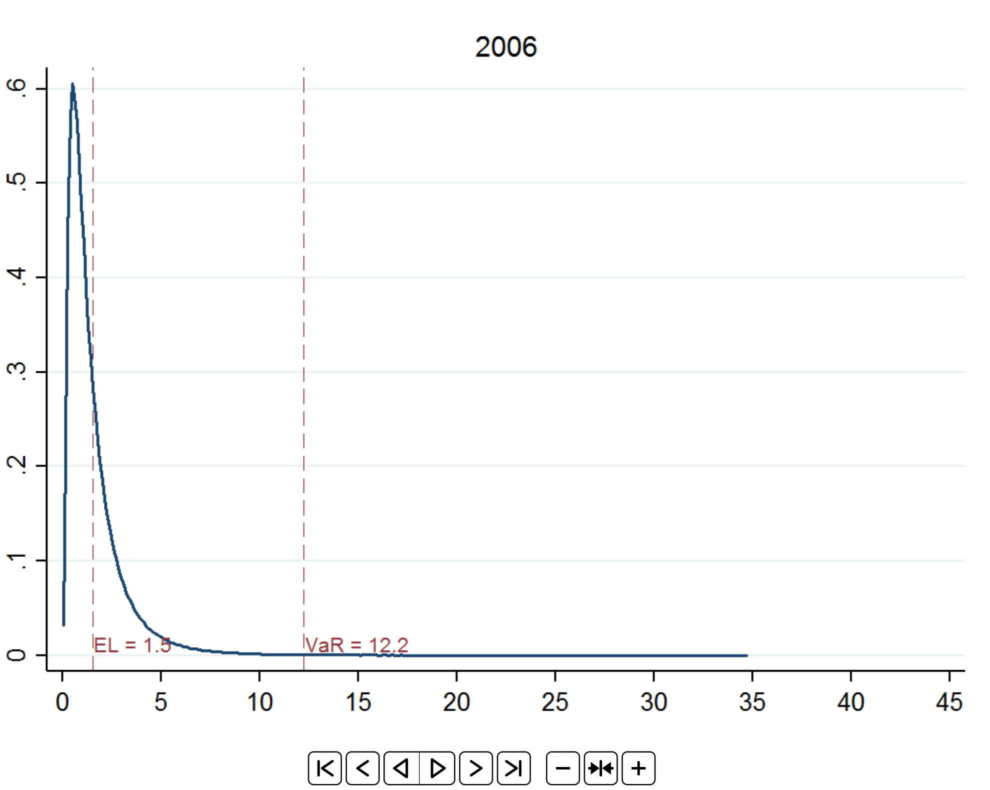


FIGURE A.1: Portfolio loss distribution as a percentage of the total exposure assuming the FJ LGD function for each year between 2006 and 2019. Results for 1,000,000 Monte Carlo simulations. **Dynamic Figure**: open the document as a PDF file.