The granularity of Portuguese firm-level exports

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Abstract
International trade flows are highly concentrated in the top units of analysis. In this paper, we study the firm-size distribution of Portuguese exports of goods over the period 1993-2017 and examine its impact on aggregate volatility. We fit power law relations using three different methods. The estimated Pareto exponents of Portuguese firm-level exports are very small, pointing to an extreme concentration of export values in the largest firms. Moreover, the Pareto estimates tend to decrease over time. We then investigate if the granular behaviour of aggregate outcomes of Gabaix (2011) is present in Portuguese firm-level exports. The results show that the idiosyncratic shocks to the largest firms may have significant effects on the dynamics of total exports. (JEL: F10, F14)

1. Introduction

Changes in total exports are usually explained by aggregate shocks to external demand and competitiveness, but often the understanding of year-to-year movements of aggregate exports requires examining individual firm behaviour. The fact that a few large firms account for a disproportionate share of activity has implications on the interpretation of several economic phenomena and international trade is no exception. Trade flows are highly concentrated in a few top firms. Such a firm-size distribution is well represented by a power law, with lower exponents associated with higher concentration and fatter tails. The seminal paper of Gabaix (2011) shows that, if the distribution of firm size is very fat-tailed, idiosyncratic shocks to a few very large firms can explain an important part of aggregate fluctuations.

Portugal is a small and open economy where large firms account for a substantial share of total exports and, hence, specific shocks to these firms may have a strong impact

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in the aggregate. There is plenty of anecdotal evidence on this type of effects in Portugal. For instance, a temporary interruption of activities in a refinery firm led to a significant decline in the volume of fuel exports in 2014, while an increase in the productive capacity of an automotive plant had a substantial contribution to the gains of Portuguese export market share in 2018.

This article analyses the granular hypothesis for Portuguese aggregate exports of goods using micro data. Can the tail of the distribution of Portuguese firm-level exports be modelled by a power law with a sufficiently low exponent? If so, do the idiosyncratic movements of the largest exporters explain a significant part of the growth rate of aggregate exports? The answer to both questions is yes, as we detail below.

We use three different methods of estimation of the Pareto exponent of the tail of the distribution of Portuguese firm-level exports in each year from 1993 to 2017. Two are the most popular methods in the literature – the conditional maximum likelihood estimator of Hill (1975) and the log-rank, log-size OLS regression with the optimal shift of Gabaix and Ibragimov (2011) – and the third is the recent estimator proposed by Nicolau and Rodrigues (2019). Irrespective of the method, the estimated Pareto exponents are very small, in some cases close to 1, pointing to an extreme concentration of export values in a few firms. Moreover, the estimates of the Pareto exponent tend to decline over time, in particular until 2009, which indicates an increase in the concentration of the values of exports in the largest firms.

Having established the significant role of top firms, we then examine how this concentration of exports in a few very large firms has an impact in the aggregate growth of Portuguese exports. Following the empirical approach of the granular residual of Gabaix (2011), we find that the idiosyncratic shocks to the largest firms have significant effects on the dynamics of total exports, explaining more than one-third of aggregate fluctuations.

The article is organised as follows. Section 2 discusses some of the related literature that frames this study. Section 3 describes the database and presents some descriptive statistics. Section 4 estimates the Pareto exponent of the tail of the distribution of Portuguese firm-level exports over time. After a short presentation of the methodology, Section 5 investigates the role of idiosyncratic shocks to the largest firms for the aggregate change in Portuguese exports. Finally, Section 6 presents some concluding remarks.

2. Related literature

The review of the vast empirical literature on power laws in economics and their implications for the behaviour of many economic and financial variables is beyond the scope of this article. In fact, there are already several extensive reviews of the literature on these issues that we mention below. Instead, this section offers a non-exhaustive list of references in different strands of the literature that are related to our study and provide a framework for our analysis.

There is a large body of literature in economics that deals with power law distributions. Gabaix (2009, 2016) provide two very intelligible surveys on empirical power laws in economics, discussing their theoretical mechanisms and their utility for the understanding of several economic phenomena, including the granular origins of aggregate fluctuations. Newman (2005) provides a more general discussion of how power law distributions occur in an extraordinarily diverse range of areas from physics, biology, computer science to economics and finance, demography and other social sciences. The great interest in these type of distributions has also led to the refinement and development of a number of tail index estimators over the years (see, for instance, Embrechts *et al.* (2012) for an extensive review of these methods).

The important role of the top units in a distribution links directly with the granular hypothesis of aggregate fluctuations of Gabaix (2011), which provides a micro-foundation for aggregate shocks. The basic idea is that idiosyncratic shocks to firms can generate aggregate fluctuations, i.e., specific events that affect the largest firms can impact the evolution of the economic aggregate of which they are part. The key factor here is the high concentration of economic activity in the top "grains" (firms, industries, or products), with size distributions that can be well fitted by power laws. The "averaging out" argument of firm-level shocks breaks down if the size distribution of firms is very fat-tailed. In this case, random shocks to the largest firms can affect total output in a noticeable way. Gabaix (2011) shows that idiosyncratic movements of the top 100 US firms explain about one-third of variations in output growth.

After the seminal paper of Gabaix (2011), there is a growing literature that looks at firms to understand aggregate outcomes. For instance, using the empirical approach of Gabaix (2011), granular firms are found to account for a significant part of business cycle fluctuations in Germany (Wagner 2012), Spain (Blanco-Arroyo and Alfarano 2017 and Blanco-Arroyo *et al.* 2018), Italy (Gnocato and Rondinelli 2018), Canada (Karasik *et al.* 2016), Australia (Miranda-Pinto and Shen 2019) and eight European countries (Ebeke and Eklou 2017). Using detailed data for sales of French firms to different markets, di Giovanni *et al.* (2014) find that firm-specific shocks contribute more to aggregate volatility than sector-destination shocks do. Friberg and Sanctuary (2016) replicate this analysis for Sweden and also find that the firm-specific component contributes substantially to the growth of total sales and exports.

Other papers have built on the granular hypothesis and developed models in which aggregate fluctuations arise from shocks to individual firms, because the firm-size distribution is extremely fat-tailed. di Giovanni and Levchenko (2012) extend the granular approach to firms in international trade and propose a new channel through which international trade affects aggregate volatility. They show that the preponderance of large firms and their role in aggregate volatility can help explain two empirical regularities: smaller countries are more volatile; and more open countries have higher volatility. Opening to trade increases the importance of large firms, thus raising macroeconomic volatility. More recently, di Giovanni *et al.* (2017, 2018) show that idiosyncratic shocks to the granular firms have an impact on international business cycle comovement; Gaubert and Itskhoki (2018) propose and quantify a granular multi-sector model of trade and show that idiosyncratic firm dynamics account for most of the evolution of a country's comparative advantage over time; Carvalho and Grassi (2019) extend the work of Gabaix (2011) and build a quantitative firm dynamics model where aggregate fluctuations are caused by firm-level disturbances alone.

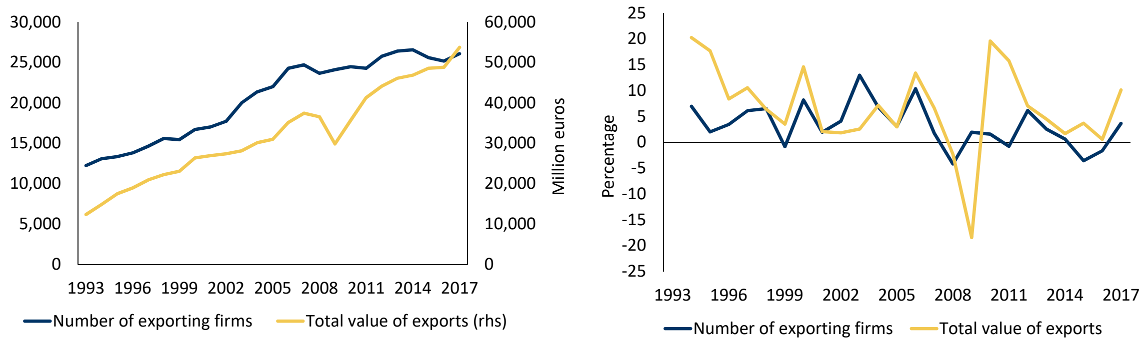
Current international trade literature has already established the dominance of large firms in aggregate exports. Empirical evidence on a highly skewed exporter-size distribution is provided, for instance, by Mayer and Ottaviano (2008) for seven European countries, Bernard *et al.* (2009) for the US, Eaton *et al.* (2011) for France, and Freund and Pierola (2015) for 32 developing countries. Our article is mostly related to Wagner (2013) that uses data on total exports at the firm-level and finds that idiosyncratic shocks to very large German manufacturing firms played a decisive role in the export collapse of 2008-2009. Our article is also connected to del Rosal (2013, 2018). We fit a power law distribution to exports as in del Rosal (2018) and estimate the granular residual using the approach of Gabaix (2011) as in del Rosal (2013), but his analyses are at product-level for several European countries and ours is at the firm-level for Portugal.

Finally, our article contributes to the empirical literature on Portuguese international trade in goods using micro-level data. Some examples of studies using the same database of this article include Amador and Opromolla (2013, 2017) who analyse the intensive and extensive margins of Portuguese exports in the firm, destination and product dimensions; Nagengast (2019) who assesses the importance of product and destination shocks to the variation of total exports; Bastos and Silva (2012) who find a positive effect of migrant networks on export participation and intensity; Mion and Opromolla (2014) who show that the export experience gained by managers in previous firms has a positive impact on the export performance of their current firm; Bastos and Silva (2010) who find that more productive firms sell larger volumes at higher prices to a given destination; Bastos *et al.* (2018b) who show that exporting to richer countries leads firms to pay higher prices for inputs; Esteves *et al.* (2018) who estimate a negative relation between domestic demand and firms' exports; Bastos *et al.* (2018a) who document new facts on the joint evolution of firm performance and prices over the life cycle.

3. Data and exploratory analysis

Statistics Portugal (*Instituto Nacional de Estatística*, Portuguese acronym: INE) collects, on a monthly basis, data on export and import transactions of goods by firms that are located in Portugal to compute the official international trade statistics. In this article, we use the total values of exports reported by firms from 1993 to 2017 aggregated at the annual level and expressed in current euros. To reach the total official value of exports published, the database comprises some estimates done by INE for non-reporting firms, which are included in fictional identifiers. We exclude this information from the analysis and, hence, use only the export values reported by firms. The firm-level data used in this article covers around 97 percent of total exports as published in the official Portuguese statistics of international trade.

Figure 1 illustrates the strong growth of the Portuguese exporting sector over this period: the total value exported more than quadrupled, from about 12,300 million euros in 1993 to around 53,700 million euros in 2017, and the number of exporting firms more than doubled, from around 12,200 in 1993 to around 26,000 in 2017. Another feature that stands out in Figure 1 is that, as found for other countries, the great trade collapse of 2009 resulted mostly from the decline in exports of firms that continued to export (i.e., the so-called intensive margin) and not from the reduction of the number of exporting firms.[1]



(A) Levels (B) Rates of change

FIGURE 1: Portuguese exports - number of exporting firms and total value (1993-2017)

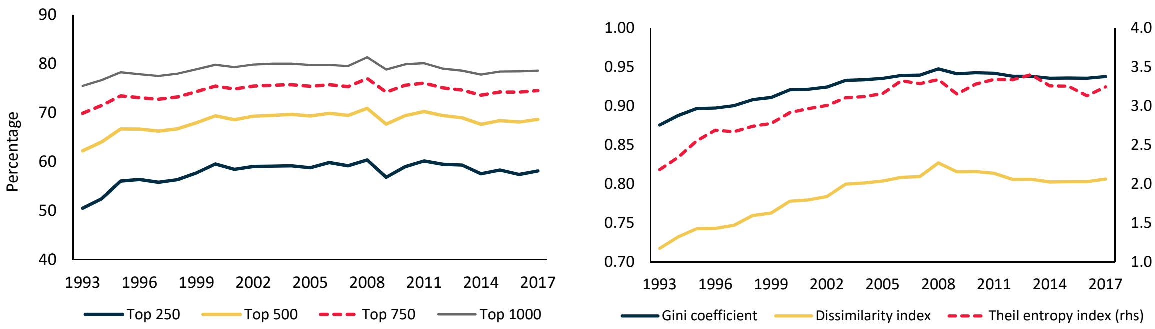
Are the values of Portuguese exports concentrated among firms? Has this feature changed over time? In the next section, we try to answer these questions by estimating the shape parameter of the tail of the distribution of firm-level exports, but some preliminary evidence is presented in Figure 2. The simplest indicators of export concentration are the shares of the major exporting firms depicted in Panel a). These shares illustrate the high concentration of Portuguese exports in a small group of firms. For instance, the top 250 exporters in 1993 (2 percent of firms) account for around half of total exports, while the main 1000 exporters (8 percent of firms) represent more than 75 percent of the total export value in 1993. In spite of the strong rise in the number of exporting firms, the shares of top firms in the total value of exports have also increased, especially until 2009. In 2017, the major 250 and 1000 exporting firms, which represent only around 1 and 4 percent of exporters, account for almost 60 and 80 percent of total exports, respectively.

Complementarily, Panel b) of Figure 2 reports some traditional indices of inequality for the whole sample, namely the Gini coefficient, the dissimilarity index, and the Theil T_T entropy index, which can also be used to assess export concentration.[2] All three indices have their minimum values at zero (if all firms exported the same value) and

1. For evidence on the importance of adjustments at the intensive margin during the 2008–2009 trade collapse, see, for instance, Bricongne *et al.* (2012) for France, Wagner (2013) for Germany, Behrens *et al.* (2013) for Belgium, Eppinger *et al.* (2018) for Spain, and Békés *et al.* (2011) for seven European countries.

2. Denote the share of firm i in total exports as p_i and N as the total number of firms, the Theil index is $T_T = lnN - \sum_{i=1}^{N} p_i * ln(1/p_i)$ and the dissimilarity index is $D = 1/2 * \sum_{i=1}^{N} |p_i - 1/N|$. After sorting firms in ascending order of their export value x_i, the Gini index is $G = \frac{2\sum_{i=1}^{N} i * x_i}{N\sum_{i=1}^{N} x_i} - \frac{N+1}{N}$.

increase as the values of exports become more concentrated in some firms. Again, the evolution of these indicators points to an increase in the concentration of export values among firms over time.



(A) Shares of top firms in total exports (B) Inequality measures

FIGURE 2: Concentration of Portuguese exports (1993-2017)

We finish this section with some additional information on the characteristics of the top Portuguese exporters. More precisely, we examine their frequency and export representativity by sector of economic activity, defined using the Portuguese industrial classification – *Classificação Portuguesa das Actividades Económicas* (CAE) Rev 3 at the 2-digits level. Table 1 illustrates the sectoral distribution of the top 250 exporters, in terms of both number of firms and value of exports over the 1993-2017 period. To facilitate the presentation, only the main fifteen sectors of activity are included in the Table 1 and sorted in descending order of their percentage shares.

Two sectors stand out in Table 1: motor vehicles and wholesale trade.[3] Each of these sectors accounts for around 11 percent of the number of largest firms in this period. The relevance of the sector of motor vehicles in terms of the value of exports of the top firms is higher (20.7 percent), while the shares of wholesalers are similar in terms of value of exports and number of firms. The importance of wholesalers in Portuguese top exporters is in line with evidence reported for other countries and also on previous studies on Portuguese international trade: many firms classified in service sectors operate and play a significant role in exports of goods.[4]

Fourteen of the fifteen major sectors of Table 1 are common in terms of their shares in the number of largest exporters and in the values of exports of these top firms. The two exceptions are footwear and leather, and coke, refined petroleum and fuels. The latter sector, in particular, represents almost 9 percent of the export value of the largest firms but less than 0.5 percent in terms of the number of top firms, pointing to the existence of a single major exporter in this sector.

3. According to CAE Rev 3, wholesale trade comprises the activity of resale (without transformation) of goods to traders (retailers or other wholesalers), to industrial, institutional and professional users, to intermediaries, but not to final consumers. The goods can be resold as they were acquired, or after carrying out some operations associated with wholesale trade, such as sorting in lots, packaging and bottling.

4. See, for instance, Crozet *et al.* (2013) for the specific role of wholesalers in exports of goods and Banco de Portugal (2016) for evidence on Portuguese service firms participating in international trade of goods.

Shares in the number of largest exporters			Shares in the value of exports of the largest firms		
Code		Share	Code		Share
29	Motor vehicles	11.1	29	Motor vehicles	20.7
46	Wholesale trade	11.0	46	Wholesale trade	10.5
13	Textiles	8.2	19	Coke, refined petroleum and fuels	8.9
14	Apparel	6.3	26	Computer and electronics	6.7
10	Food products	5.1	17	Paper and paper products	5.6
23	Other non-metallic mineral products	5.0	20	Chemicals, except pharmaceuticals	5.1
16	Wood and cork	4.9	27	Electrical equipment	4.2
27	Electrical equipment	4.6	22	Rubber and plastic products	4.1
20	Chemicals, except pharmaceuticals	4.5	13	Textiles	3.7
22	Rubber and plastic products	4.1	24	Basic metals	3.6
15	Footwear and leather	3.8	16	Wood and cork	3.0
25	Metal products, except machinery	3.7	10	Food products	2.9
17	Paper and paper products	3.1	23	Other non-metallic mineral products	2.6
24	Basic metals	3.1	14	Apparel	2.1
26	Computer and electronics	3.0	25	Metal products, except machinery	1.9
	Other sectors	18.5		Other sectors	14.5
	Total	100		Total	100

TABLE 1. Distribution of the largest 250 Portuguese exporters by sector of activity - percentage shares in the number and exports of the largest firms, 1993-2017

Notes: The sectors are defined at the 2-digits level of CAE Rev3. Only the fifteen sectors with higher shares are included and sorted in descending order of their shares. The percentage shares are computed using all 25 years from 1993 to 2017 and the largest 250 Portuguese exporting firms in each year.

4. The power-law of Portuguese firm-level exports

"When the probability of measuring a particular value of some quantity varies inversely as a power of that value, the quantity is said to follow a power law, also known as Pareto distribution." (Newman 2005, page 1).

This section begins by describing how the tail of the distribution of firm-level exports can be approximated by a power law relation. Figure 3 presents the exports of the Portuguese top 1000 firms in 1993 and 2017. Both axis are in logarithmic scales. The x-axis displays the firm rank, i.e., the first one is the firm that exported the most in the reference year and so on, whereas in the y-axis, its exports in million euros are depicted. The fact that, in both years, the relationship can be represented closely by a straight-line (R-squared in 1993 = 0.991 and R-squared in 2017 = 0.997) suggests that firm exports in Portugal follow a power law.

Analytically, let X_{it} be the export value of firm i in a given year t and $P(X_{it} > x)$ the probability that X_{it} takes a value equal to or greater than x. This variable follows a power law or Pareto distribution if the counter cumulative distribution function or empirical survival function is:

$$\overline{F}(x) = 1 - F(x) = P(X_{it} > x) = \left(\frac{x_0}{x}\right)^{\alpha}, \quad \text{with } x > x_0 > 0, \alpha > 0, \tag{1}$$

with x_0 denoting the lower bound to the power law behaviour of the distribution. The exponent α, also known as the Pareto or tail exponent, is the key parameter here. It provides a measure of how heavy the tails are, i.e., how concentrated the variable is in

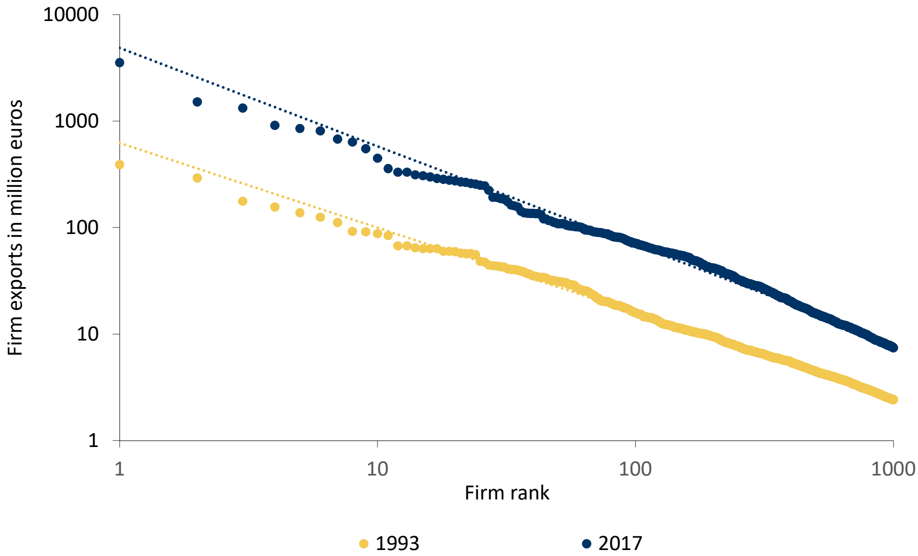


FIGURE 3: Portuguese exports of the top 1000 firms in 1993 and 2017

Note: The dots are the empirical data plotted on logarithmic scales. The lines illustrate a power law fit regressing firm exports on firm rank.

the top units. The lower the exponent α, the fatter the tails of the distribution, the greater the degree of inequality in the distribution and the higher the probability of finding very high values. For instance, a Zipf's law states that $\alpha \approx 1$.[5]

Given the number N of units in the upper tail implied by the value of x_0, we use three different methods of estimation of α in each year t from 1993 to 2017. Two are the most popular methods in the literature and the other is the recent estimator proposed by Nicolau and Rodrigues (2019).

The first method is the conditional maximum likelihood estimator (MLE) proposed by Hill (1975),

$$\hat{\alpha}_{MLE} = N \left(\sum_{i=1}^{N} lnX_{it} - lnx_0 \right)^{-1}. \tag{2}$$

The second method is the log-rank, log-size OLS regression with the shift proposed by Gabaix and Ibragimov (2011) ($\gamma = 1/2$), where α is the slope of the following regression of the log rank of firm i in year t (r_{it}) on its log size:

$$ln(r_{it} - 1/2) = \beta_0 - \alpha lnX_{it} + \varepsilon_i. \tag{3}$$

5. Zipf's law was originally formulated in terms of quantitative linguistics and is named after the American linguist George Kingsley Zipf, who popularised it. The law states that the frequency of any word in any text is inversely proportional to its rank in the frequency table, i.e., the most frequent word will occur twice as often as the second most frequent word, three times as often as the third most frequent word, etc.. This regularity exists in all languages and, for instance, the distribution of the words used in this article follows a Zipf's law.

Even if a popular way of estimating the Pareto exponent is to run a standard OLS log-log rank-size regression with $\gamma = 0$, this procedure is strongly biased in small samples. Gabaix and Ibragimov (2011) show that applying a shift $\gamma = 1/2$ is optimal in reducing the small sample bias. For future reference, we name the resulting OLS estimate of α of eq. (3) as $\hat{\alpha}_{OLS1/2}$.

A caveat of these two estimation procedures is that they assume that the observations are independent, which is not necessarily the case in most microeconomic data sets. However, there is no consensus in the literature on how to solve this issue. In practice, most applied studies present one of the former estimators, together with a warning that the observations are not necessarily independent and, hence, the standard errors probably underestimate the true standard errors (see Gabaix 2009).

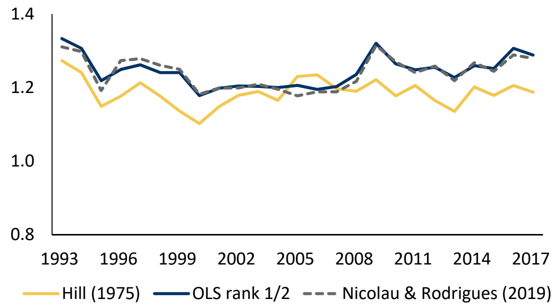
As defined in Equation (1), the Pareto distribution only applies for values equal to or greater than some minimum of value x_0, implying a cut-off still in the upper tail. However, how to choose this cut-off value is also not consensual in the literature. Given that the Hill estimator is especially sensitive to the sample size, Clauset *et al.* (2009) advocate a method for estimating x_0 that uses the Kolmogorov-Smirnov test. However, in practice, most applied studies still rely on a visual goodness of fit or use a simple rule, like the major 1000 units or the top 5 percent of the distribution.

The third method that we apply in this article is the $\hat{\alpha}_{Pareto}$, a new regression-based estimator recently proposed by Nicolau and Rodrigues (2019), which minimises the two caveats mentioned above. This estimator is more resilient to the choice of the sub-sample of large observations used to estimate the Pareto exponent and it performs well under dependence of unknown form. Moreover, the $\hat{\alpha}_{Pareto}$ estimator also provides a bias reduction when compared to the other approaches.

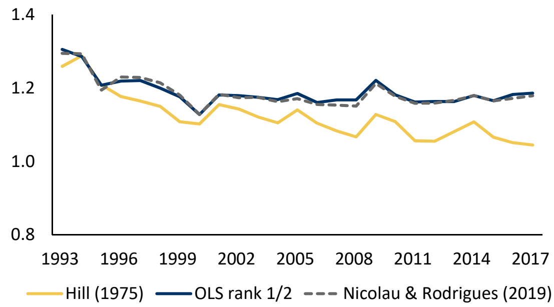
Figure 4 presents our estimation results of the Pareto exponent of the tail of the distribution of Portuguese firm-level exports from 1993 to 2017. Besides using three distinct methods, we also estimate the Pareto exponent for different tail truncation levels. Given that the sample size used in estimation should represent a significant share of the total value of exports, we use the top firms depicted in Panel a) of Figure 2 of the previous section: the main 250, 500, 750 and 1000 exporting firms in each year. An alternative strategy, which we also tested, is to choose a specific upper percentile of the annual distribution of firm-level exports (e.g., the top 5 percent exporters). In our case, the main results are similar, but, to maintain the consistency throughout the article, we chose to present the results based on a fixed number of major exporters. Moreover, we also applied the methodology of Clauset *et al.* (2009) for estimating the lower bound of power-law behaviour in each year. The implied number of firms varies strongly (from a minimum of 276 to a maximum of 1244) but the estimates of the Pareto exponents are consistent with those depicted in Figure 4.[6]

The estimated Pareto exponents of Portuguese firm-level exports are very small, always below 1.4 and in some cases very close to 1, pointing to an extreme concentration of export values in a few firms. These results are in line with evidence on firm-size

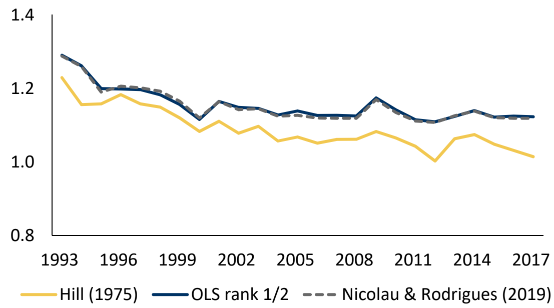
6. All estimates are available from the authors upon request.



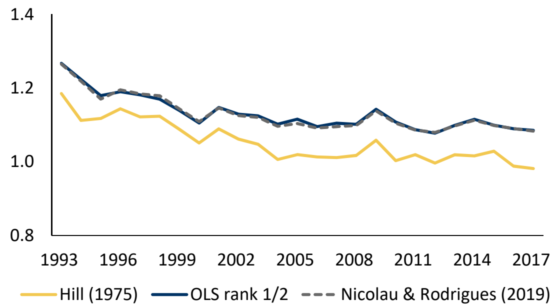
(A) Top 250 exporting firms



(B) Top 500 exporting firms



(C) Top 750 exporting firms



(D) Top 1000 exporting firms

FIGURE 4: Estimates of the Pareto exponent of the tail of the distribution of Portuguese firm-level exports (1993-2017)

Notes: Hill (1975) is the maximum likelihood estimator defined in Equation (2), OLS rank 1/2 is the OLS estimate of the log-rank, log-size regression defined in Equation (3). Four distinct tail thresholds are used in each year: top 250, 500, 750 and 1000 exporting firms.

distributions of other countries and using other proxies for firm-size, such as total sales, number of employees, assets, or revenues. Estimates of the Pareto exponent consistent with a Zipf's law are found, for instance, by Okuyama *et al.* (1999) for Japan, Axtell (2001) for the US, Fujiwara *et al.* (2004) for several European countries, Zhang *et al.* (2009) and Gao *et al.* (2015) for China, di Giovanni *et al.* (2011) for France, and da Silva *et al.* (2018) for Brazil. Our results are also consistent with those of del Rosal (2018). He studies the size distribution of exports at the product-level for the 28 European Union countries and concludes that, given the high concentration of values, power law relations with very low exponents are a good approximation to the data. For instance, considering the top 5 percent products in 2014, the Pareto exponent estimate for Portugal is 1.2, which is in the middle of the range of the estimates obtained by del Rosal (2018) for the several countries.

Another feature evident in Figure 4, in particular until 2009, is that most estimates of the Pareto exponent tend to decline over time. As discussed in Gao *et al.* (2015) that found similar results for Chinese firms using sales and equities from 2001 to 2013, an evolving Pareto coefficient implies that the relative firm sizes are changing. For large firms above the lower bound x_0, the smaller the coefficient, the greater the degree of firm-size inequality and the less homogeneous the relative firm sizes.

The downward path of Figure 4, especially until 2009, is also consistent with the rise of the measures of inequality reported in the previous section, which indicate an increase in the concentration of the values of exports, i.e., an increase in inequality in the whole sample of Portuguese exporting firms. Note that the Pareto coefficient is an inequality indicator only for the largest exporting firms included in the estimation of the power-law distribution. However, given the high concentration of Portuguese exports in these firms and the tail truncation levels used in the estimation, they account always for more than half of the total values exported.

The fact that a few large firms account for a disproportionate share of activity has implications on the interpretation of several economic phenomena, including aggregate fluctuations as discussed in Section 2. The granular hypothesis of Gabaix (2011) states that, if the distribution of firm size is very fat-tailed, then idiosyncratic shocks to large firms do not cancel out and have an impact on aggregate volatility. This is the subject of the next section of this article. In a different vein, di Giovanni and Levchenko (2013) show that the distribution of firm size is important for evaluating the gains from reductions in entry barriers and trade costs and the relative importance of intensive and extensive margins. Using firm-level data on exports from 32 developing countries, Freund and Pierola (2015) provide evidence on the very high concentration of exports in a few very large firms and on their importance in shaping sectoral trade structures and comparative advantage. This implies that small policy interventions can have big aggregate effects if they change the behaviour of the largest firms.

5. The granularity of Portuguese aggregate exports

In this section, we investigate if the extreme concentration of Portuguese exports in a few very large firms has an impact in the aggregate growth of exports. Are the idiosyncratic shocks to these large firms relevant for total export growth over time? We use the empirical strategy of the granular residual of Gabaix (2011) to try to answer this question.

Let X_{it} denote the export value of firm i in a given year t, $X_t = \sum_i X_{it}$ the value of total Portuguese exports in year t, g_{it} and g_t the respective growth rates, and K the number of granular firms. The growth rate of a firm's exports g_{it} comprises two components: one common to all firms (a macro shock) and one specific to the firm. There are several possible ways to quantify the macro shock. Gabaix (2011) uses a very simple way: the equal-weighted average growth rate of a small subset Q of very large firms. The firm-specific shock is then the portion of the growth rate g_{it} that is unaccounted for by the common shock. Finally, the granular residual Γ_t is defined as the sum of firm-specific shocks in year t, weighted by their size in the previous year. Analytically,

$$\Gamma_t = \sum_{i=1}^{K} \frac{X_{i,t-1}}{X_{t-1}} \hat{\varepsilon}_{it} = \sum_{i=1}^{K} \frac{X_{i,t-1}}{X_{t-1}} (g_{it} - \overline{g_t}) \quad \text{with} \quad \overline{g_t} = Q^{-1} \sum_{i=1}^{Q} g_{it}. \tag{4}$$

Note that, when computing the granular residual Γ_t, we are focusing only on the intensive margin of export growth, i.e., on those firms that export in t and $t-1$ so that a growth rate of their exports can be computed. In addition, the K and Q top firms considered are the largest in terms of their exported value in $t-1$. Moreover, as standard in this literature, we winsorised the top and bottom 5 percentiles of export growth rates at the firm-level, as an outlier treatment. However, the main results are very similar if we do not winsorise at all or if we use other thresholds of winsorisation, like the top and bottom 1 or 10 percentiles.

After computing the granular residual, the question of interest is whether these idiosyncratic shocks to large exporting firms can impact the growth rate of total exports. The explanatory power of the granular residual Γ_t on aggregate export growth is captured by running the following simple regression:

$$g_t = \beta_0 + \beta_1 \Gamma_t + u_t, \tag{5}$$

where β_0 and β_1 are parameters to be estimated and u_t is an error term. An extended version of the previous equation considers also the lagged values of the granular residual. The coefficient of determination, R-squared, of Equation (5) represents the proportion of the variance for the aggregate annual growth of Portuguese exports that is explained by the granular shocks.

Table 2 presents the regression results with and without a lag of the granular residual for four distinct values of K largest firms with $Q = K$. We tested several numbers for top exporting firms K and $Q \geq K$ and our results are robust to these checks. Moreover, the results are basically unchanged if the dependent variable of Equation (5), the aggregate rate of growth of Portuguese exports g_t, is computed using only the export values of continuing firms.[7]

The idiosyncratic movements of the top firms account for a large fraction of aggregate export fluctuations. If only common shocks were important for the growth rate of total exports, then the R-squared of the regressions in Table 2 would be zero, but it is not. Considering the top 250 firms, the adjusted R-squared shows that the granular residual without any lag can explain, in a statistical sense, around 29 percent of the variability of export growth and up to 40 percent with one lag.[8] The adjusted R-squared increases as the number of top firms rises, reaching around 60 percent with the top 750 and 1000 firms. This is a relatively high value in this literature, even using slightly different methodologies and distinct variables, but it is in line with the cross-country evidence of del Rosal (2013). Using exports at the product-level for several European countries, del Rosal (2013) finds that Portugal is the country with the highest explanatory power of the granular residual, with an adjusted R-squared of more than 30 percent in the regressions with one lag.

Figure 5 illustrates the goodness of fit of Equation (5) by plotting the observed rate of change of total Portuguese exports and the respective fitted values. The granular residuals of the top firms capture very well the variation in the growth of total exports in this period, suggesting that monitoring the qualitative and quantitative data on a panel of major exporters can help to predict aggregate growth.

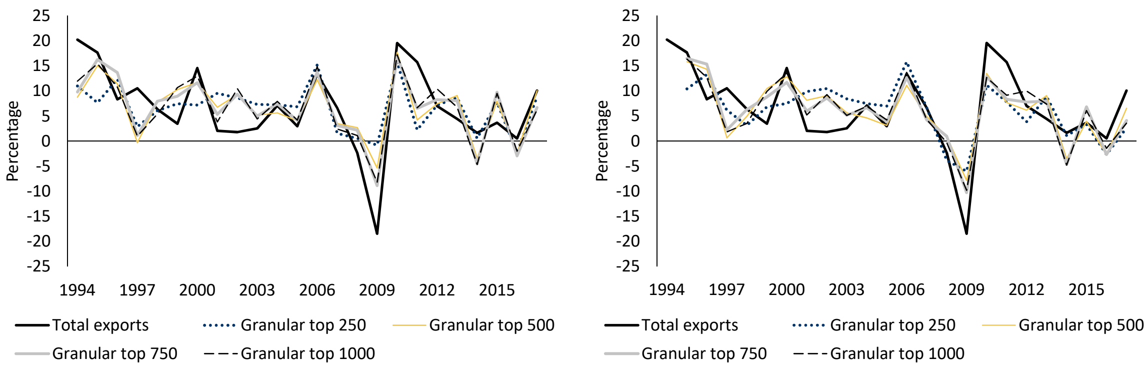
7. All results are available from the authors upon request.

8. The inclusion of additional lags does not improve the results of Table 2.

	$K=Q=250$		$K=Q=500$		$K=Q=750$		$K=Q=1000$	
Γ_t	2.433**	2.652**	2.641***	2.776***	2.757***	2.751***	2.401***	2.361***
	(0.752)	(0.698)	(0.598)	(0.551)	(0.511)	(0.479)	(0.429)	(0.409)
Γ_{t-1}		1.675*		0.960		0.613		0.643
		(0.687)		(0.554)		(0.476)		(0.402)
Intercept	6.876***	6.746***	6.574***	6.108***	5.971***	5.372***	6.203***	5.727***
	(1.419)	(1.286)	(1.253)	(1.143)	(1.137)	(1.072)	(1.108)	(1.054)
Observations	24	23	24	23	24	23	24	23
R^2	0.322	0.456	0.470	0.562	0.569	0.623	0.588	0.631
Adjusted R^2	0.292	0.402	0.446	0.518	0.549	0.586	0.569	0.594

TABLE 2. Granular residual and aggregate growth of Portuguese exports (1994-2017)

Notes: The table reports the estimation results of Equation (5) where the aggregate growth of Portuguese exports from 1994 to 2017 was regressed on the granular residual Γ_t of four different groups of top firms $K = Q = \{250, 500, 750, 1000\}$. The firms are the largest by their exports in the previous year. Standard errors are in parenthesis. Stars indicate significance levels of 5% (*), 1% (**), and 0.1%(***).



(A) Current values of the granular residual

(B) Current and lagged values of the granular residual

FIGURE 5: Actual and fitted values of the aggregate growth of Portuguese exports

Notes: The figure illustrates the estimation results of Equation (5). Panel a) plots the observerd rate of change of total Portuguese exports and the predicted values using the current value of the granular residuals of the top 250, 500, 750 and 1000 exporting firms. In Panel b), the fitted values are obtained using both the current and lagged values of the granular residual for the four groups of firms.

6. Concluding remarks

In recent years, there has been a significant improvement on the understanding of the micro-origins of aggregate fluctuations. When the firm-size distribution is very fat-tailed, idiosyncratic shocks to the largest firms directly contribute to aggregate dynamics. Hence, the role of the top units in a distribution links with the concept of granularity of Gabaix (2011).

This article shows that the tail of the distribution of Portuguese firm-level exports of goods is very heavy and adequately described by a power law with an exponent close to 1. Empirically, the existence of a power law distribution of firm size with such a low exponent indicates that Portuguese exports are very concentrated in a few large firms.

Moreover, the concentration of Portuguese export flows in the top firms tends to increase over time, in particular until 2009.

Using the empirical strategy of the granular residual of Gabaix (2011), we find that idiosyncratic shocks to the largest firms are relevant for total export growth over time, accounting for more than one-third of aggregate fluctuations. This means that volatility at the firm-level can affect aggregate export dynamics. Our findings for exports are in line with the results first discovered by Gabaix (2011): if firm-sizes in an economy are described by a Pareto distribution, then independent firm-level shocks can generate macroeconomic fluctuations, in accordance with the granular hypothesis.

This granular hypothesis has implications for monitoring and forecasting Portuguese exports. If a significant component of the dynamics of total exports originates from a small number of firms, it is essential to learn more about these top firms and the idiosyncratic shocks they are subject to. Besides studying macro shocks, monitoring the quantitative and qualitative information regarding a panel of large players might help in explaining and predicting aggregate export behaviour. These results have also policy implications, as small policy changes can have significant aggregate effects if they change the behaviour of top firms.

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