

TOWARDS A CCA-BASED SYSTEMIC RISK INDICATOR*

Nuno Silva** | Nuno Ribeiro** | António Antunes**



ABSTRACT

This paper presents the foundations of a new systemic risk indicator based on contingent claim analysis. The proposed model adapts Gray, Merton and Bodie (2007) methodology to the characteristics of euro area countries. Based on sector balance sheets and assuming a totally marked to market shock transmission mechanism, our methodology consists in estimating all sets of shocks able to deplete the equity base of at least one sector. The probability of these shocks happening is then estimated. The methodology is applied to Portugal for the period between 2002 and 2010. We considered shocks in seven dimensions, notably, shocks in some sectors equity (non-financial corporations, financial institutions, insurance companies and the general government) and liabilities (non-financial corporations, households). Shocks in households' mortgages were distinguished from the remaining. The proposed indicator points to a substantial level of systemic risk since the end of 2007.

1. INTRODUCTION

Traditionally, the literature in systemic risk focuses on financial institutions and on their relations. The financial crisis that began in the US in 2007 and, particularly, the current European sovereign debt crisis have shown that there are many more channels of contagion apart from the ones that link the banking system. As long as these inter-linkages propagate shocks, understanding them can help to detect the mechanics behind systemic risk.

The very existence of systemic risk is generally motivated by unstable balance sheet positions associated, *inter alia*, with high degrees of leverage, which are frequently accumulated during the upside of the economic cycle. When faced with large financial or real shocks, these fragilities tend to generate a cascade of losses through the financial system. Depending on each sector weaknesses and their inter-connections, these shocks end up either diluted or amplified by the network. While in the former case, losses tend to be manageable, in the latter we may be dealing with a very pronounced recession. On this concern, contingent claim analysis has been recently used to identify weak balance sheet positions at the sector level as well as possible channels for transmission. Examples are the works of Gapen *et al.* (2004, 2008), Gray, Merton and Bodie (2007) and Gray *et al.* (2008). These studies lack however some generality as they focus on economies subject to currency risk.

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** Banco de Portugal, Economics and Research Department.

Following Castrén and Kavonius (2009), Silva (2010) proposed to adapt these models to the specificities of euro area economies, namely, a common currency and no centralized public finances. Additionally, a totally marked-to-market shock transmission system is built. This study extends Silva (2010) by proposing a new systemic risk measure that explicitly estimates the one year probability of at least one sector failing its commitments at a certain point in time. This probability is then interpreted as the probability of the economy entering in financial collapse. Apart from being tailored to euro area countries, the model has the advantage of incorporating several lessons from the recent financial crisis, notably the systemic importance that comes from the link between the banking system and the sovereign. Finally, to the best knowledge of the authors, this indicator is also the first measure based on contingent claim analysis able to synthesize all information in national financial accounts data.

This study is composed of 7 sections. Section 2 presents Merton's model. Section 3 shows how national financial accounts can be used to apply contingent claim analysis at the sector level. The procedure is exemplified for the Portuguese economy. Section 4 details how exogenous shocks are transmitted in our inter-sectoral system. Section 5 introduces the concept of stability frontier in financial stability literature. The latter is then used in section 6 as part of our new systemic risk indicator. Section 7 concludes.

2. CONTINGENT CLAIM ANALYSIS

Contingent claim analysis appeals to Merton's (1974) model to assess the creditworthiness of a debt issuer, which we will call the firm, but which could be a whole economic sector. Consider a firm that issues debt at a given time with a certain maturity. The question that arises is whether the firm has enough assets to honour its obligations at maturity. The firm will honour its commitments if the value of its assets exceed, at maturity, its debt. If not, the firm declares bankruptcy and all assets are liquidated to creditors. The negative difference between assets and liabilities will then be debt holders' losses. Deciding on whether or not to pay back debt at maturity is very similar to exercising a call option. The option holder will buy the underlying asset if its market price at maturity exceeds the strike price. Otherwise, the call option is not exercised. In our case, the underlying asset corresponds to all assets of the firm while the exercise price is the nominal value of debt. It follows that the market value of debt should be equal to its face value discounted by a risk-free interest rate less the value of a put option on the firm.¹ That is, in the absence of arbitrage opportunities, investors should be indifferent between taking an amount of riskless debt, or take the same amount at risk but ensuring that, in case of non-repayment, they can recover the difference between what they have received (the asset value of the firm) and what they should have received (debt repayment). This is achieved through the put option. In practice, knowing a firm's equity market value, the volatility of its equity returns, its nominal debt and the risk-free interest rate, one can use contingent claim analysis to calculate a series of risk measures, namely the distance to distress, the probability of default and the *ex-ante* expected loss.

Consider that A , B and E correspond respectively to assets, debt and equity at market prices for a given firm or sector. If there are no market frictions and assuming all assets are liquid in maturity, we have that

$$A = E + B \quad (1)$$

i.e. the market value of equity should equal the difference between the market value of assets and the market value of the risky debt. Suppose that A follows a stochastic diffusion process with a deterministic trend governed by the risk-free return. Consider that at $t = 0$, the firm issues zero coupon bonds with nominal value B_T amounting to all its liabilities. This firm is bankrupted if the value of its assets, A , is lower than B_T at maturity.

¹ The 3-month Eurepo was used as the risk free interest rate in this study.

It follows that, in accordance with option pricing theory, the equity market value of the firm, E , equals an European call option on the underlying assets, A , with maturity $t = T$ and strike price equal to its nominal debt, B_T . Applying Itô's lemma, imposing non arbitrage and frontier conditions equivalent to a call option, and defining $\tau = T - t$, one can obtain the following equation for E ,

$$E = A\Phi(d_1) - B_T e^{-r\tau} \Phi(d_2) \tag{2}$$

where

$$d_1 = \frac{\ln \frac{A}{B_T} + (r + \frac{1}{2} \sigma_A^2) \tau}{\sigma_A \sqrt{\tau}} \tag{3}$$

$$d_2 = \frac{\ln \frac{A}{B_T} + (r - \frac{1}{2} \sigma_A^2) \tau}{\sigma_A \sqrt{\tau}} \tag{4}$$

In the above equations σ_A stands for the volatility of asset returns, r is the risk-free interest rate, which we considered to be constant, τ is the time interval up to maturity and Φ is the standardized cumulative normal function. Equation (2) has a simple interpretation. The first term evaluates assets weighted by a coefficient related to the probability of the call option being exercised; the second term weights the discounted nominal debt by a coefficient slightly smaller given that losses are limited.

In turn, the put option value, P , can be calculated as

$$P = e^{-r\tau} B_T + E - A \tag{5}$$

In a risk-free world $P = 0$ and asset value equals equity plus nominal debt discounted at the risk-free rate.

Equation (2) has two unknowns, A and σ_A . In order to obtain their value one needs to impose a second condition. One possibility is to say that E also follows a geometric Brownian motion but with parameters other than A .

Applying Itô's lemma and equating the volatility terms, we obtain

$$E\sigma_E = A\sigma_A \Phi(d_1) \tag{6}$$

where σ_E is the volatility of equity returns.

Solving the system composed of equation (2) and (6) at each point in time, it is possible to obtain a time series for A and σ_A .² Substituting A and E into equation (1), we can then find B and calculate the distance to distress, d_2 , the probability of default, $\Phi(-d_2)$, and the expected losses, P .

Firms seldom have a single issue of debt. Fortunately, Merton's model can be easily adapted to debt issues of different seniority levels. In this study we use this to divide general government's liabilities in different layers.³

² Note that, unlike the original Black and Scholes (1973) model, the hypothesis of stationarity of σ_A is neglected when solving this system.

³ For more information on this topic see Cossin and Piroette (2007).

3. APPLYING CONTINGENT CLAIM ANALYSIS AT THE SECTOR LEVEL

The model presented so far was designed to be applied to listed firms for which information on market value and volatility of equity returns is publicly available and easy to interpret. The application of contingent claim analysis to economic sectors, though possible, requires several assumptions and simplifications, which we shall present. This section has three subsections. We will start by linking the application of contingent claim analysis at the micro and sector levels. We will then show how national financial accounts can be used to estimate the so-called who-to-whom financial accounts. Finally, we will show how who-to-whom accounts, together with some market based data, can be utilized to define each sector equity, volatility of equity returns and debt barrier. The results obtained are presented in Silva, Ribeiro and Antunes (forthcoming).

3.1. From the micro to the sector level

Consider an economy composed of eight sectors: non-financial corporations, the central bank, other monetary financial institutions, other financial institutions, the insurance and pension funds sector, general government, households and the rest of the world.⁴ All these sectors present their own particularities. Nevertheless, based on their diversity, they can be broadly divided into two groups. On the one hand, we have those sectors that can be seen as single entities, such as the general government and the central bank. For these sectors, it is indifferent to analyse their risk at micro or aggregate level because they are the same. On the other hand, we have those sectors that result from the aggregation of several economic agents. This is the case of non-financial corporations, OMFI, OFI, INS, households and the rest of the word. For these sectors, their equity and debt is the sum of the equity and debt of all economic agents that integrate them. The volatility of their equity returns, however, is lower than the weighted average of the equity volatility of each of the agents included in the sector due to diversification. Additionally, by ignoring sector heterogeneity, given the non-linearities in debt valuation, the application of contingent claim analysis at the sector level underestimates the level of risk in the economy. The latter should be particularly important for those sectors with a high level of heterogeneity. This is the case of non-financial corporations and households. In this study we will not address this fact as it is fairly difficult to solve this problem without going down at the micro level. However, we will show in section 4.1 how we can minimize the impact of this problem in our systemic risk indicator.

3.2. The who-to-whom accounts

Merton's model can be applied at the sector level using Portuguese non-consolidated national financial accounts compiled and published quarterly by Banco de Portugal.⁵ This data is organized in matrix form with the eight sectors already presented and seven types of financial instruments (monetary gold and special drawing rights, currency and deposits, securities other than shares, loans, shares and other equity, insurance technical reserves and other accounts receivable). Securities other than shares and loans are divided in short-term and long-term. Securities other than shares also include financial derivatives, which we treat as a separate instrument. Shares and other equity include quoted shares, unquoted shares and mutual funds. Insurance technical reserves were divided in technical reserves related to insurance

⁴ Given that almost all pension plans in Portugal are defined benefit, we decided to allocate pension funds' assets and liabilities to those sectors that ultimately are responsible for their payment. This procedure allow us to interpret the insurance and pension funds sector as being composed only by insurance companies. For that reason we will thereafter call it INS. The acronyms OMFI and OFI will be henceforth used to refer to other monetary financial institutions and other financial intermediaries, respectively. Non-financial corporations, general government and the rest of the world appear in charts as NFC, GOV and RoW, respectively.

⁵ This data is available for all countries in the euro area, though at different detail levels.

(“insurance”) and technical reserves related to pensions (“pensions”).⁶ Except for monetary gold and special drawing rights, all other instruments are recorded in accordance with the double entry principle, meaning that all assets have a counterparty liability. This generates a closed system useful for studying shock propagation channels.

Unfortunately, in the case of Portugal, for instruments other than “deposits” and “loans”, national financial accounts do not contain information on bilateral balance sheet positions (also known as who-to-whom accounts). Nevertheless, these can be estimated through maximum entropy as done in several studies on the interbank loans market (e.g. Sheldon and Maurer (1998), Upper and Worms (2004) and Wells (2004)).

Consider that bilateral balance sheet positions between two sectors in a given instrument k can be represented by a $N \times N$ matrix where N represents the number of sectors and x_{ij}^k the exposure of sector i to sector j in instrument k :

$$\begin{bmatrix} x_{11} & \cdots & x_{1j} & \cdots & x_{1N} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ x_{i1} & \cdots & x_{ij} & \cdots & x_{iN} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ x_{N1} & \cdots & x_{Nj} & \cdots & x_{NN} \end{bmatrix}^k \quad \text{with} \quad \sum_{j=1}^N x_{ij}^k = a_i^k \quad \text{and} \quad \sum_{i=1}^N x_{ij}^k = l_j^k$$

In this case, a_i^k and l_j^k correspond to total assets and total liabilities of sector i and j in instrument k , respectively.

In addition, consider that a_i^k and l_j^k may be seen as the components of $f^k(a)$ and $f^k(l)$, the marginal distributions of assets and liabilities, respectively, and that x_{ij}^k is the realization of the joint distribution $f^k(a, l)$. Assuming independence, or maximum entropy, x_{ij}^k is the product of the two marginal distributions. In order to improve results, some restrictions were imposed a posteriori. Notably, intra-sector exposition was calculated from the difference between consolidated and non-consolidated national financial accounts and the central bank was considered to be entirely owned by the general government. Additionally, total exposure between the central bank, OMFI, OFI and INS was restricted to equal the difference between consolidated and non-consolidated accounts for the financial sector. Since the restrictions imposed are not all zeros, we defined an iterative procedure where each matrix is rebalanced immediately after imposing the restrictions. This guarantees that the equality between assets and liabilities is preserved for each instrument. This is done until convergence is obtained.⁷

⁶ This division was needed to separate insurance companies from pension funds. In order to facilitate exposition, the instruments under analysis shall be henceforth referred as “deposits”, “debt”, “loans”, “shares”, “insurance”, “pensions” and “other”.

⁷ The data concerning non-quoted shares was adjusted to reflect markets evolution. This adjustment is posterior to the estimation of who-to-whom accounts. The same is true for the separation of insurance companies from pension funds.

3.3. Sector level model assumptions

3.3.1. Equity and volatility of equity returns

At the firm level, equity is generally defined as the firm's net worth i.e. the excess value of assets over liabilities. This can be measured either at book value, which reflects only the past of the firm, or at market value, which reflects both the past of the firm and market's expectations regarding its future. For contingent claim analysis market figures are therefore preferred. For listed firms, this can be easily measured by looking at share prices. For the remaining firms, one may look at their book value and adjust it in order to reflect market trends. This procedure can also be followed at the sector level, but only for those sectors that issue "shares". This is the case of non-financial corporations, OMFI, OFI and INS. These sectors' "shares" are considered as equivalent to a call option on their assets with exercise price equal to their liabilities. Unquoted "shares" can be adjusted to reflect a trend similar to quoted "shares". For INS, given that there are no insurance companies quoted in Portugal, and given that firms from these sectors tend to invest in the same assets across Europe, we decided to multiply its book values by the price-to-book ratio implicit in the Stoxx Europe 600 insurance index. Liabilities in mutual funds, which are particularly important in the case of OFI, are also included in equity. The volatility of the returns on the PSI-20 index and the volatility of the returns on the Stoxx Europe 600 Insurance index were used as proxies for the volatility of equity returns of non-financial corporations and INS. For OMFI and OFI, we used the volatility of the returns on the PSI-Financial services index. Regarding the central bank, though it issues "shares", which are fully owned by the general government, these are not priced in the market. So, we can only use their book value, which takes into account central bank's gold holdings at market prices, but excludes future profits. Since Banco de Portugal shares are not traded in the market, it is not possible to calculate the volatility of its equity returns. As an alternative, we used the volatility of the equity returns of Banque Nationale de Belgique, which is the only central bank in the Eurosystem marked to market.

For those sectors that do not issue "shares", equity is more difficult to define. This is the case of households, the general government and the rest of the world. Fortunately, for households, it continues to make sense to consider that their equity correspond to the sum of each person net worth. In order to be consistent with the equity definitions used for other sectors, the latter should take into account households' current financial position, their real estate holdings and the present value of their future savings. Households net financial position is straightforward to calculate based on national financial accounts. As regards households' real estate holdings, there is no regularly published data for all countries in the euro area. In the case of Portugal, the most recent estimates are the ones prepared for the Banco de Portugal Annual Report 2010, which we use in this study. Lastly, we estimated the present value of future households' savings (disposable income minus consumption) as an infinite stream of cash flows with value equal to current households' savings and discount rate equal to the yield on national government 10 year bonds. We assume that this stream will grow at 2%, which is consistent with the inflation rate used by the ECB as reference when conducting monetary policy. As regards households' volatility of equity returns, we estimate the volatility of a portfolio similar to the one held by households.⁸

So far, we have treated equity and net worth as synonyms. However, for the general government and

⁸ The volatility of 6-month Euribor was used as a measure of the volatility of households' "deposits", "insurance" and "pensions" conceived. The volatility of the yields on national bonds was used for households' assets in "debt", "loans" and "other". The volatility on PSI-20 returns was used as a proxy for the volatility of households' investments in "shares". The volatility of real estate returns was estimated based on the Confidencial Imobiliário index. Finally, the volatility of households' future savings was estimated assuming that only the discount rate varies (for more information on this procedure please see section 3.3.1.1 regarding the general government). The estimates obtained were then adjusted to replicate the financing structure of the sector. Given these hypothesis, the volatility of equity returns tended to fluctuate between 20% and 45%.

the rest of the world, there is empirical evidence suggesting that this definition may not be the most suitable. In the next two subsections we appeal to a wider definition of equity to propose a new way of estimating general government and rest of the world's equity.

3.3.1.1. The general government

General government's left-hand side of the balance sheet is broadly composed of its future tax revenues plus its current financial and real assets. In opposition, the right-hand side of its balance sheet is made of future expenses and current financial liabilities. Ignoring real estate assets, for which there are no estimates, general government's net worth is thus the sum of its net financial position with the present value of its future savings. Since most governments in the euro area have a negative net financial position, and following a net wealth approach to equity, one would have either to assume that future savings more than compensate this fact or, alternatively, conclude that most governments are insolvent. Quantifying the sovereigns' future savings is nevertheless a rather complex task as it involves not only estimating future revenues but also future expenses. Notwithstanding these difficulties in estimating general government's future savings, the fact that most sovereigns permanently show fiscal deficits, turn difficult to argue that the latter will be enough to compensate the current negative financial position. Albeit all this, until recently financing did not appear as a major constraint to sovereigns increasing leverage. It happens that differently from firms, whose costs and revenues depend mainly on the evolution of markets, in the case of the general government these figures tend to be highly influenced by political decisions. At least theoretically, government's income is only conditional on the country's wealth and future GDP. Similarly, apart from some mandatory expenses, which cannot be foregone, the general government has autonomy to decide how much it wants to spend. As long as markets trust that the sovereign is able to reequilibrate its finances, they will continue to finance it even if estimates point to significant and persistent deficits. If this confidence is lost, the sovereign may be called either to raise taxes or cut spending in order to restore markets' confidence. These facts lead us to leave the traditional equivalence between net wealth and equity. Alternatively, we try to assess how large is the sovereign's leeway to adjust its financial path whenever markets lose confidence on it. Since the general idea behind contingent claim analysis is to look to the right-hand side of any firm's balance sheet and estimate its left-hand side, we will ignore general government's future revenues. Instead, we will look to general government's right-hand side of the balance sheet in search of those expenses that are not vital to the sovereign's subsistence and therefore can be potentially eliminated. Financial liabilities correspond to past expenses and therefore must be fulfilled. However, there is some flexibility regarding future expenses. We shall divide these in two categories: mandatory expenses and discretionary expenses. We interpret discretionary expenses as a set of services and goods that the sovereign wants to offer its citizens but that are not binding. Based on its revenues (how much it asks from its citizens) and political choices, the general government decides its level of discretionary expenditure. In contrast, mandatory expenses are those costs that no government can avoid, such as defence, justice, internal affairs and foreign affairs.⁹ In this study we decided to set mandatory expenses at 30% of GDP.¹⁰ Whenever markets lose their confidence in the sovereign's ability to fulfil its commitments, the general government has the option to cut on its discretionary expenses, signalling an increase in future saving. In the limit, if faced with an extremely negative shock, the sovereign may cut all its discretionary expense, but never its mandatory expenditure. Though it may happen that this option will be never exercised, and that is why it is not counted in general government's net wealth, it should be taken into account while quantifying its equity. In this study we

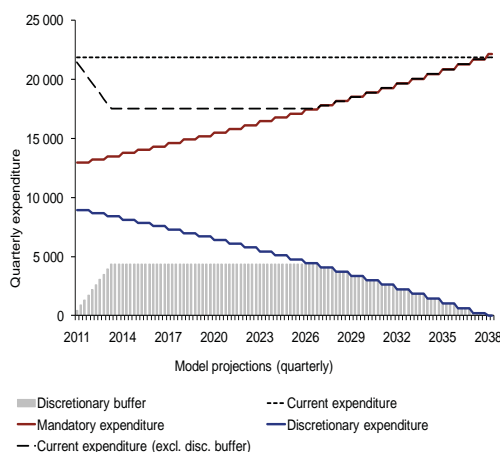
⁹ One can also argue that there is a minimum expense level associated with wealth redistribution and social cohesion, below which sovereigns' ability to fulfill its critical functions is in danger.

¹⁰ This is an arbitrary value based on personal judgment. Nevertheless, we can argue that it is approximately equal to the average government spending in upper middle income countries and high income non-OECD countries, where social welfare state is weaker than in most Euro-area countries.

will call this the discretionary buffer, which is how much the general government can save per year by reducing expenditure vis-a-vis its current level. In order to better mimic reality we imposed three restrictions on this buffer. First, in order to reproduce expenditure downward stickiness, we consider that no government is able of reducing public spending more than 2% per quarter. Second, we consider that no government is willing to reduce nominal expenditure more than 20% starting from its current level. Whenever any of these restrictions is binding, the sovereign is not able to save the whole gap between its current expenditure and its mandatory expenditure. The part that it is not able to save should be treated as mandatory expenditure. We call this the fiscal adjustment period. Lastly, we consider that mandatory expenditure grows at 2% per year, which is consistent with the definition of price stability of the ECB. In the moment total expenditure (accounting for the discretionary buffer) equals expected mandatory expenditure, the discretionary buffer starts to decrease meaning that possible savings are now smaller. The discretionary buffer becomes 0 when the mandatory expenditure equals total current expenditure. In order to evaluate the sum of all yearly discretionary buffers we need a discount rate. In this study we use the yield on 10 year national bonds. The latter introduces markets judgment on the feasibility of the current expenditure and revenue path. Chart 1 exemplifies how the discretionary buffer is estimated for the last quarter of 2010. General governments' equity is then calculated as the sum of its net financial position and its discretionary buffer.

Chart 1

DISCRETIONARY BUFFER ESTIMATION (2010 Q4)



Source: Authors' calculations.

As regards the volatility on general government's equity returns, most studies of this kind have used the volatility on 10 year national bond yields. In this study we try a different approach which we consider to be more consistent with our equity definition. Basically, we consider that general government's equity varies only depending on the discount rate of the stream of discretionary buffers. We assumed the latter follows a triangular distribution with a lower bound on the minimum yield observed during our sample period. The mode and the upper bound of the distribution were calibrated so that its standard deviation equals the volatility on national bond yields during that quarter and the expected value is as close as possible to the yield observed at the end of the quarter.

3.3.1.2. The rest of the world

As regards the rest of the world, it results from the aggregation of several economic agents with very different characteristics. This heterogeneity creates some difficulties. Additionally, the fact that we are evaluating the rest of the world only in its relation with the country under analysis tells us very little

about rest of the world's financial position. No matter the equity definition used, it will be always very difficult to interpret. This does not mean that the rest of the world is not relevant to this model. In fact, the rest of the world is very important not only as a shock absorber, but also as a potential issuer of some types of shocks. For instance, if some non-resident firm or sovereign fails its responsibilities, this may have a non-negligible impact on the economy under analysis. Given these constraints, we focused on guaranteeing that the definition used could be applied to all countries, independently of having either a negative or positive financial external position. Additionally, we defined rest of the world's equity in a manner that does not compromise the market value of its liabilities. It would not make sense to conclude that some sector assets towards the rest of the world are riskier just because the country under analysis has a strong external position. The approach followed in the case of the rest of the world is thus very similar to the one used for the general government. The left-hand side of rest of the world's balance sheet corresponds to the present value of all its future exports to the country under analysis plus all its current financial assets towards this country. On the contrary, the right-hand side comprises all future imports plus rest of the world's current financial liabilities towards the country under analysis. Differently from the general government we cannot state that the rest of the world cannot import less than some value. In the limit, if the rest of the world is not able to pay back its imports, firms from the country under analysis will stop exporting to it, restoring the equilibrium. Therefore, we will assume that rest of the world's equity correspond to the present value of all its future imports plus its current financial position towards the country under analysis. Imports are seen as a cushion, which decrease whenever they are considered unsustainable by markets. The sustainability of the aggregate level of imports is measured through a discount rate, which we set using the capital asset pricing model. The idea is that the discount rate should reflect the amount of systematic risk in exports to the rest of the world as compared with the market portfolio. Countries that have their exports concentrated in markets under some type of financial turmoil are more likely to face some type of external shock. The VStoxx, which is an implicit volatility measure based on the Dow Jones Eurostoxx 50, is used as a proxy for the volatility of the rest of the world's equity returns.

3.3.2. The default barrier

Literature in contingent claim analysis usually considers that each firm default barrier, B_T , corresponds to its short-term liabilities plus 50% of its long-term liabilities. This is based on the idea that in the long run firms are able to adjust their behaviour in accordance to market developments. Based on national financial accounts, "deposits", "loans" (short), "debt" (short) and "other" are usually considered short-term liabilities, while "loans" (long), "debt" (long), "insurance" and "pensions" are long term liabilities. For all sectors but the general government and the rest of the world, we applied the standard contingent claim analysis practice. For the remaining two sectors, the classical division between short-term and long-term liabilities was ignored since the equity definition used was already justified by these sectors' capacity to adjust. In the case of the general government, however, a slightly more complicated structure was defined in order to capture some idiosyncrasies in its behaviour. Thus, general government liabilities were divided in three layers incorporating the idea proposed by Silva (2010) that, in case of financial distress, the sovereign is able to force resident economic agents to be more flexible. This has been seen recently on public debt auctions where resident banks compensated the decrease in non-residents demand for public debt. This imposed flexibility must be taken into account while pricing general government debt towards resident economic sectors. We considered general government's senior debt as equal to the present value of its future mandatory expenses, including those unavoidable expenses during a hypothetical fiscal adjustment period. The first layer of subordinated debt is made of "debt" and "loans" in the hands of the rest of the world, "deposits", which correspond to public debt securities in the hands of households, and "others", which corresponds to liabilities towards suppliers. Finally, the second layer of subordinated debt consists of the remaining liabilities, notably, "loans" and "debt" granted by all but the rest of the world. This largely corresponds to liabilities towards resident

banks. The general government has no liabilities in “insurance” and “pensions”. We considered that “deposits” have less credit risk than “loans” and “debt” in the hands of resident banks because politically it is harder to fail on commitments towards households than force resident banks to roll-over their credit lines. All definitions are summarized in Table 1.

Table 1

SUMMARY OF MODEL ASSUMPTIONS

	Share issuing sectors	Households	Rest of the world	General government
Equity (junior claim)	NFC, OMFI, OFI, INS: Quoted shares price, adjusted unquoted shares price and mutual funds issued CB: Book value	Net financial position + Real estate + Present value future savings	Net financial position + Present value of RoW future imports	Net financial position + Present value of discretionary buffer
Volatility of equity returns	NFC: PSI-20 CB: Banque Nationale de Belgique OMFI and OFI: PSI-Financials INS: Stoxx Europe 600 Insurance	Volatility on households asset portfolio adjusted for sector leverage	VStoxx	Simulation where 10 year national bond yields are assumed to follow a triangular distribution
Liabilities (short term)	Deposits Loans (short) Debt (short) Financial derivatives Other		Deposits Loans Debt Insurance Pensions Other	Senior: Present value of future mandatory expenses 1st sub.: Deposits and Other plus Debt and Loans hold by non-residents
Liabilities (long term)	Loans (long) Debt (long) Insurance Pensions			2nd sub.: Debt and Loans hold by residents

4. APPLYING CCA IN A FORWARD LOOKING CONTEXT

Section 3 showed how contingent claim analysis can be applied at the sector level to estimate the market value of assets as well as several risk indicators. For forward looking purposes, however, since information on future equity and volatility of equity returns is no longer available, further assumptions are needed regarding how shocks are transmitted in the economy. In this section we will start by describing what type of shocks we are interested in this model. We will then build a shock transmission mechanism able to quantify the effects of these shocks.

4.1. What do we mean by a shock?

Economic agents are everyday confronted with changes that affect their decisions. These changes may be expected or unexpected. The hypothesis of rationality of economic agents assume that only unexpected changes matter because future expected changes are already taken into account in current market prices and choices. In our model, except for the central bank, all other sector’s equity is at least

partially based in market prices.¹¹ Thus, we assume in this study that only unexpected changes matter. We will call these changes shocks and consider them as exogenous events in the sense that they are determined outside the model.

For each sector, the model presented so far has two endogenous variables, A and σ_A , and several exogenous variables, notably, E , σ_E , B_T and r . For forward looking purposes, however, as will be explained in section 4.2, we will assume that σ_E is determined endogenously. Additionally, we will also assume r as fixed. Thus, we end up in a model where there may be only shocks in sectors' balance sheets, notably, in E and B_T . Given the equivalence between the left and right-hand side of the balance sheet, the latter is the same as considering shocks in A . Nevertheless, by looking only at E and B_T one avoids any duplication of shocks. As regards E , we have two distinct situations. On the one hand we have those sectors that issue "shares". For these sectors, any shock in E affects the sector that suffered the shock and all its shareholders proportionally to their share. On the other hand, we have those sectors that do not issue "shares": households, the general government and the rest of the world. For these sectors, we interpret their equity as equivalent to "shares" with no owner. In other words, these sectors equity is only a solvency measure and not an asset from any other sector. This guarantees that we are only looking to one side of the balance sheet.¹² Differently from E , B_T is composed of a number of instruments: "deposits", "debt", "loans", "insurance", "pensions" and "others". Each of these instruments can be seen as exogenous variables. Thus, it is possible to simulate shocks on some of these instruments alone.

In the last paragraph we constrained the amount of shocks one could consider to E and B_T . In the context of Merton's model, however, it makes sense only to look at shocks in E because this is a one factor model. In other words, there is only one source of uncertainty. If assets are greater than debt, all liabilities are fulfilled. Otherwise, debt holders will have to bear a loss. There is no possibility of refusing payment without going bankrupt, i.e. exhausting the sector's entire equity base. It happens that some of the economic agents included in a sector may become insolvent without that sector going bankrupt. As an example, consider that economic activity decreases unexpectedly, non-financial corporations have fewer profits, financial institutions have to accommodate more credit losses, governments have bigger budget deficits, unemployment rates increase, and imports decrease. All these events tend to decrease each sector's equity and therefore the market value of their debt. These are all shocks in E . However, in parallel, some firms will close, some banks may disappear and some households will ask for insolvency. At the aggregate level, we cannot see this unless we assume that shocks in B_T can also exist.

4.2. The shock transmission mechanism

The last section discussed which shocks are worth considering in the context of the application of contingent claim analysis at the sector level. We stated two types of shocks, notably, shocks in E and shocks in B_T . As explained in section 2, in the context of Merton's model liabilities are contingent on assets. This implies that equity is seen as a cushion to shocks on assets. In the case of shocks in E , this impact is direct. For shocks in B_T , the sector that has the corresponding asset suffers a loss that must be recognized in its equity. Both types of shocks are then transmitted in either of two ways. On the one hand, equity holders of the sector that suffers the shock bear a loss in proportion to their share. This loss must be reflected in their equity. We will call this the equity channel of transmission. On the other hand, given debt contingency on total assets, any shock that produces changes in debt quality also generates

¹¹ Households' equity is not marked-to-market. However, several of the instruments used in its estimation are either marked to market, as is the case of "shares", or marked according to some model, as is the case of real estate holdings and future savings. General government and rest of the world's equity are marked according to a model based in market prices.

¹² This would not be possible if we would think that these sectors' equity is calculated by summing and subtracting elements in the left and right-hand side of the balance sheet.

immediate losses (gains) to those sectors that hold this type of assets. We will call this the debt channel.¹³ These losses are generally not registered on balance sheets. Nevertheless, its consideration contributes to better understand how shocks are transmitted in the economy. Equity losses generated either through the equity or debt channel must then be distributed across all sectors that hold equity from those sectors, which had registered losses. Once again, debt quality deteriorates, generating more losses. This process continues until the shock is totally dissipated. This mechanism can be represented through an iterative system where losses related both to the equity and debt channels are calculated and distributed in each iteration.¹⁴ This iterative system is represented in Chart 2. Panel A illustrates how a hypothetical shock in all sectors equity propagates. Panel B shows how each sector losses are calculated and distributed.

The equity channel and the debt channel have two very different economic interpretations. While the former corresponds to losses actually incurred by each sector, the latter considers creditors' expected losses as the result of changes in borrowers' likelihood of default and losses given default. In the absence of credit risk, i.e. assuming that all economic agents are going to fulfil their contractual responsibilities, the debt channel disappears. As regards the shock transmission mechanism presented, and unlike losses transmitted through the equity channel, which can be easily estimated based on previous iterations, losses transmitted through the debt channel require the adoption of a debt pricing model, such as contingent claim analysis. Given each sector future equity estimates, which are based on our shock transmission mechanism, we only need to set a path for the evolution of the volatility of equity returns after a shock. Manipulating our system of equations¹⁵, we can find that

$$\sigma_E = \overline{\sigma_A} \left(1 + \frac{B_T}{E} e^{-rT} \Phi(d_2) \right)$$

Substituting d_2 and $\overline{\sigma_A}$, which are not available before running the model, for their values in the previous iteration, we can forecast σ_E .

In order to better represent reality, the model encloses three features that deserve some more attention. The first two features concern how OMFI and the general government interact. Section 3.3.2 stated that general government default barrier could be divided in three different categories with distinct levels of risk and therefore different prices. This differentiation must also be present in the forward looking shock transmission mechanism. For instance, shocks in general government's ability to repay its debts are stronger for those sectors that hold its junior liabilities than for those who hold general government senior liabilities. Thus, shocks on sovereigns' ability to pay back its debts should affect mainly OMFI. In parallel, we consider that the sovereign guarantees all OMFI liabilities in case of bankruptcy. Thus, variations in the market value of OMFI's debt affect only the general government and general government's debt holders indirectly. For backward purposes, we consider that this guarantee has been already taken into account in the discount rate used for calculating the present value of the discretionary buffer. However, for forward looking purposes, any variation in the market value of OMFI's debt must be subtracted explicitly on general government's equity. Together, these two features create a kind of self-fulfilling process similar to the one we have seen during the recent financial crisis (see Chart 3).

The last issue to consider is how the rest of the world interacts with all other sectors. The rest of the world, as any other sector, acts simultaneously as an asset holder and an equity and debt issuer. As an asset

¹³ The value of the shocks in E and B_T corresponds to the initial losses in Chart 2 – Panel A. Both types of shocks are counted in the equity channel. As regards shocks in B_T , notice that those sectors that failed their commitments become better off after the shock in the sense that the sector has a better debt-to-equity ratio after the shock. This gain belongs to the debt channel of transmission.

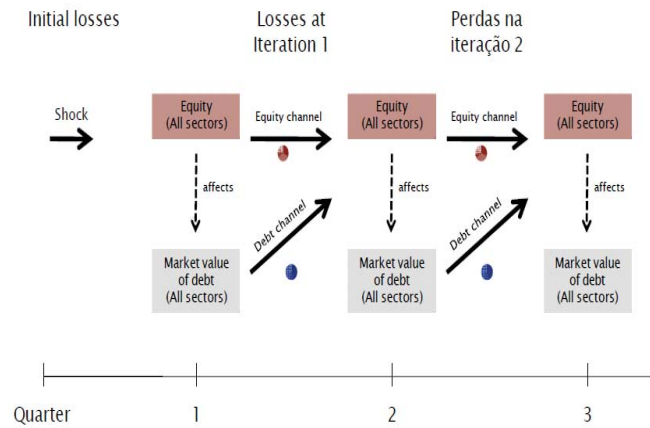
¹⁴ Though losses created by the debt channel from previous iterations are transmitted through the equity channel, they are considered as belonging to the debt channel because if the latter would not exist these losses would not exist, too.

¹⁵ See Bensoussan, Crouhy, Galai, Wilkie and Dempster (1994).

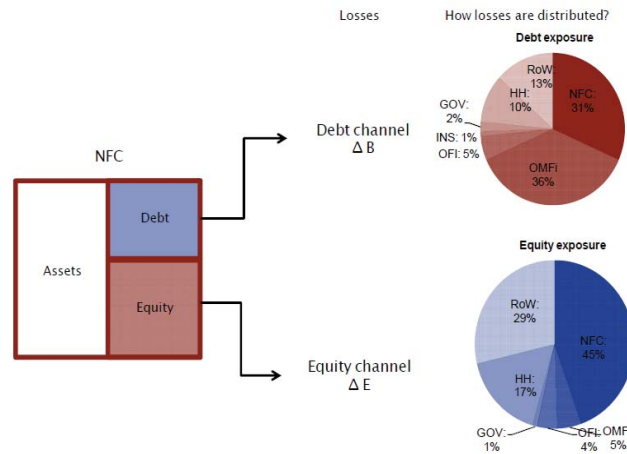
Chart 2

THE SHOCK TRANSMISSION MECHANISM

Panel A

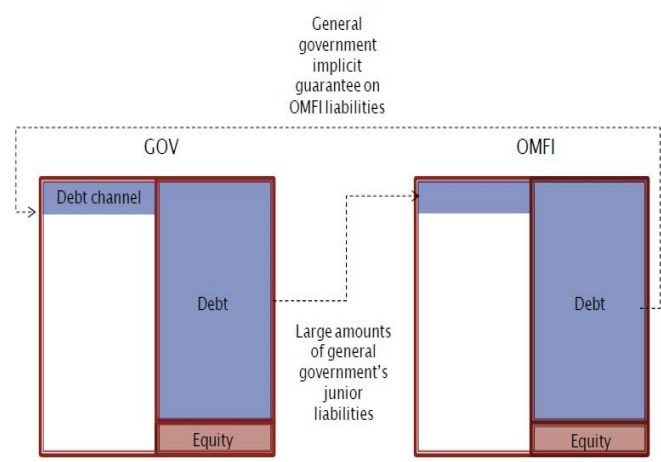


Panel B



holder it makes sense to think that it absorbs all variation in its assets' value. Similarly, as a debt issuer it is reasonable to think that if any of its constituents refuses to pay, this generates a cascade of losses in the entire financial system. However, it is not reasonable to think that losses in rest of the world's equity coming from previous losses in other sectors are sent back to resident economic sectors. The latter should dissipate and only a very small part of it will come back through resident sectors that have "shares" in the rest of the world. It happens that, as referred in section 3.3.1.2, the rest of the world encloses only non-residents economic agents in the extent that they have any economic relation with residents. Thus, any estimate of resident sectors exposure to the rest of the world based on national financial accounts would be clearly upward biased. In order to avoid this problem we opted to assume that the rest of the world does not transmit losses through the equity channel. As regards the debt channel, we do not explicitly rule out the possibility of the rest of the world transmitting this type of losses. However, this should be rather insignificant given the definition used for rest of the world's equity.

Chart 3
INTERACTIONS BETWEEN THE GENERAL GOVERNMENT AND OMFI



5. THE STABILITY FRONTIER

The previous section proposed a method to quantify future losses after a shock. One can then estimate several sector level risk indicators. Following the international financial crisis that started in the United States in 2007, regulators felt an increasing need for instruments able of monitoring systemic risk. In particular, financial regulators are increasingly interested in indicators able of summarizing how robust is their financial system in a forward looking perspective. This section presents the concept of stability frontier, which apart from being a risk indicator itself, is also a fundamental concept in our new CCA-based systemic risk indicator methodology.

According to Silva, Ribeiro and Antunes (forthcoming) losses due to the equity and debt channels evolve in a totally different way as the number of iterations increases. While the marginal variation in total losses related to the equity channel decreases as the number of iterations increases (independently of the number of iterations), the marginal variation in total losses related to the debt channel evolves in a non-monotonic manner. Silva, Ribeiro and Antunes (forthcoming) analyse the characteristics of these two channels and conclude that though the equity channel is the most important for convergent shocks, it is the evolution of losses related to the debt channel that determines if the system is able of absorbing the shock. This type of mechanism creates a wrong sensation of robustness in the system because until a certain number of iterations, the convergent equity channel dominates. However, for divergent shocks there is a point after which losses related to the debt channel dominate generating an uncontrollable stream of losses that ends up destroying the system. This is consistent with the findings of Castrén and Kavonius (2009), Allen and Gale (2000), Gallegati, Greenwald, Stiglitz and Richiardi (2008) and Haldane (2009) who argue for the existence of a tipping point. Once this point is crossed, all interconnections in the economy become amplification channels instead of shock absorbers, turning an apparently robust network in an extremely fragile system. In our model this tipping point is motivated by credit risk.

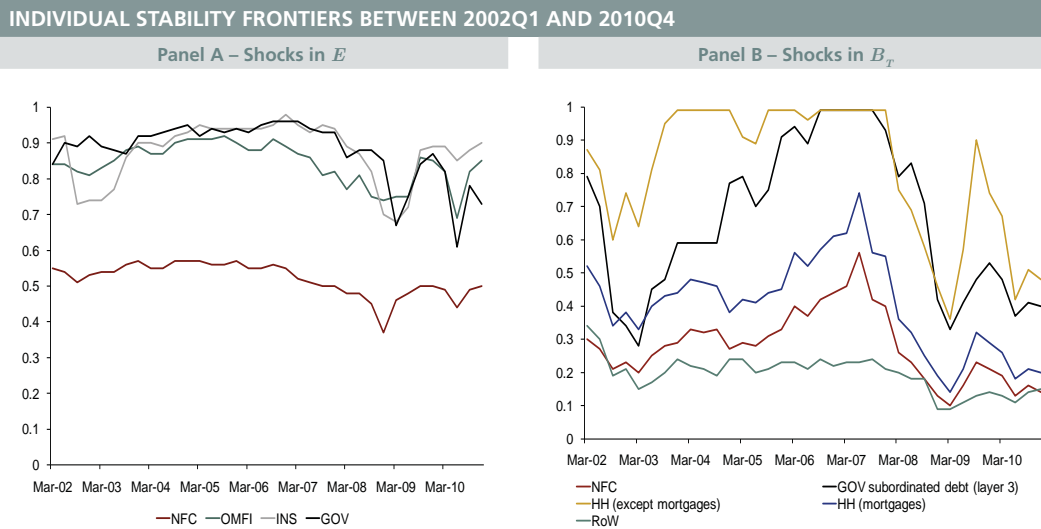
Following the idea related to the existence of a tipping point, we introduce here the concept of stability frontier. Define stability frontier as the geometric space in R_n composed of all combinations of shocks in n dimensions that lead the system to collapse in a certain moment in time. We say that the system collapses if at least one sector ends up with non-positive equity after a combination of shocks in the n dimensions has been fully absorbed. Any asset or combination of assets may form a dimension. When n is equal to one, we call this the individual stability frontier. Conditional on each sector balance sheet, inter-sector relations and markets perception of risk, individual stability frontiers state the size of the

unexpected shocks needed to collapse the economy. The result comes as percentage of all assets in that dimension.

Based on the discussion in section 4.1 we considered shocks in all sectors equity. Additionally, we considered shocks in non-financial corporations, OFI, INS, households and rest of the world's total liabilities. For households, we have also estimated the stability frontier associated with shocks in mortgages. Lastly, we have estimated the stability frontiers associated with general government's subordinated debt layers.¹⁶ Chart 4 presents the individual stability frontiers for some of these dimensions.

Chart 4 illustrates well the fact that stability frontiers are not static in time. The better the financial structure, the greater the shock needed to collapse the system suggesting that the system is more robust. The same is true in higher dimensions, which leads us to conclude that one can evaluate changes in systemic risk based on changes in the stability frontier position. The further the stability frontier is from the origin, the better the resilience of our inter-sector network to shocks on the considered dimensions. In section 6 we show how to summarize all information in stability frontiers in a single indicator.

Chart 4



Sources: Authors' calculations.

6. THE SYSTEMIC RISK INDICATOR

Section 5 presented the concept of stability frontier. As argued in section 5, the stability frontier is by itself a risk indicator. It tells us all combinations of unexpected events that are able of depleting the equity of at least one sector. The latter is however only half of the story. The other half of the story is in the probability of these events happening. The idea behind our systemic risk indicator is therefore to evaluate the probability of the economy suffering a combination of shocks beyond its stability frontier. We interpret this as the probability of a financial collapse. Mathematically, we define our systemic risk indicator as

¹⁶ We have justified stability frontiers associated with shocks in debt with intra-sector heterogeneity. In the case of the general government this heterogeneity does not exist. However, differently from firms, it is possible that sovereigns decide unilaterally not to pay part of their debt and, besides this, continue to exist. This justifies considering shocks in general government's debt.

$$SRI = \int_{(SF_1, \dots, SF_n)}^{(\infty, \dots, \infty)} \Psi(\dim 1, \dots, \dim n)$$

where Ψ is an n-dimensional density function and $\{SF_1, \dots, SF_n\}$ are the coordinates of our n-dimensional stability frontier.

For several of the dimensions stated in section 5 it is rather difficult to have an informed guess on how these distributions should be. This is the case of shocks in OMFI, OFI, INS, general government and rest of the world's liabilities. We will therefore leave these dimensions out of our systemic risk measure.¹⁷ This leaves us with seven dimensions:

1. Non-financial corporations' equity.
2. OMFI and OFI's equity (we decided to join these two dimensions).
3. INS's equity.
4. General governments' equity
5. Non-financial corporations' liabilities.
6. Households' liabilities except "loans" granted by OMFIs for house purchase.
7. Households' liabilities in "loans" granted by OMFIs for house purchase.

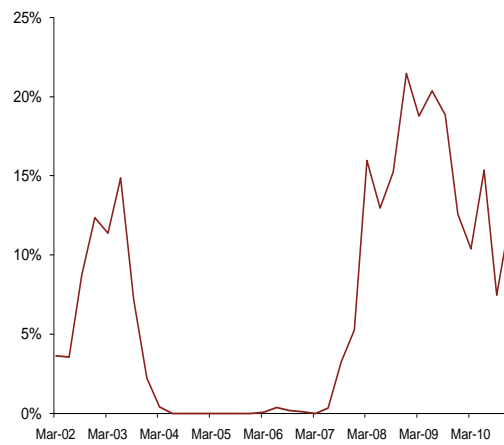
For simplicity we assume Ψ is a 7-dimensional Normal distribution with expected value 0 and variance-covariance matrix Ω . The expected value of Ψ must be 0 by definition because only unexpected changes, *i.e.* shocks, interest. Expected changes in balance sheet positions are already accounted in each sector equity value. Thus, we need only to estimate Ω , which similarly to the stability frontiers is not static. In this study we assume that Ω changes every quarter. Each variance-covariance matrix is estimated using return data on the 12 months preceding each quarter. For dimension 1, 2 and 3 Ω was estimated using the volatility of the PSI-20, PSI-Financial Services and Stoxx Europe 600 Insurance, respectively. The volatility of dimension 4 was estimated using the return on monthly estimates of general government's equity. For the remaining dimensions we used the volatility on the flow of new problem loans in % of the loans stock *vis-à-vis* its 12-month average. Based on these assumptions we obtained the following results.¹⁸ Chart 5 presents the probability of a financial collapse between March 2002 and December 2010. Our indicator points for two periods of high systemic risk, notably the period between 2002 and 2003 and the period starting in September 2007. Though the indicator pointed for probabilities of collapse well above 10% in both crises, stability frontiers in chart 4 (panel A) and Ω estimates suggest that the former crisis was not exclusively, but mainly, centred in a single sector, INS, which reached very high levels of equity volatility. In opposition, the current crisis is characterized by a strong decrease in all dimensions stability frontiers. Almost all stability frontiers presented in chart 4 reach their minimum during the current crisis. This led the indicator to reach its maximum level at approximately 20% in December 2008.

¹⁷ Those dimensions concerning central bank, households and rest of the world's equity have been also excluded from the indicator. This should have no impact given that the stability frontiers related with these dimensions is always very near 1.

¹⁸ In practice, the probability of collapse was estimated using Monte-carlo simulations. For more output on this model please see Silva, Ribeiro and Antunes (forthcoming).

Chart 5

PROBABILITY OF COLLAPSE OF THE FINANCIAL SYSTEM



Source: Authors' calculations.

7. CONCLUSION

This study proposes a new systemic risk indicator based on network analysis and contingent claim analysis. The latter intends to be a proxy for the one year probability of collapse of the financial system under analysis. Conceptually, the methodology proposed has broadly two parts. First we estimate all combinations of shocks needed to collapse the system. We call this the stability frontier. Then we evaluate the probability of shocks beyond this frontier happening in a multivariate distribution. The proposed model has the advantage of allowing the construction of a synthetic indicator of the overall level of systemic risk in the economy. Movements of the indicator can then be better understood by analyzing shifts in the stability frontiers, changes in the parameters of the multivariate distribution of risk factors and changes in the speed of convergence of the network. Finally, the model also contributes to better understand how shocks are transmitted in the economy and the role of each sector. In particular, the model explicitly considers the strong interconnection between sovereign risk and the risk in the banking sector.



REFERENCES

- Allen, F. and Gale, D. (2000), "Financial contagion", *Journal of political economy*, 108(1), 1-33.
- Bensoussan, A., Crouhy, M. Galai, D. Wilkie, A. and Dempster, M. (1994), "Stochastic equity volatility and the capital structure of the firm", *Philosophical Transactions: Physical Sciences and Engineering*, pp. 531-541.
- Black, F., and Scholes, M. (1973), "The pricing of options and corporate liabilities", *Journal of Political Economy*, 81(2), 637-654.
- Castren, O. and Kavonius, I. (2009), "Balance sheet interlinkages and macro-financial risk analysis in the euro area", *ECB Working Paper Series*.
- Cossin, D. and Pirotte, H. (2007), *Advances credit risks analysis: financial approaches and mathematical models to assess, price and manage credit risk*, John Wiley & Sons, Ltd.
- Gallegati, M., Greenwald, B., Richiardi, M., and Stiglitz, J. (2008), "The asymmetric effect of diffusion processes: risk sharing and contagion", *Global Economy Journal*, 8(3), 2.
- Gapen, M., Gray, D., Lim, C. and Xiao, Y. (2004), "The contingent claims approach to corporate vulnerability analysis: estimating default risk and economy-wide risk transfer", *IMF Working Papers*.
- Gapen, M., Gray, D. F., Lim, C. and Xiao, Y. (2008), "Measuring and analyzing sovereign risk with contingent claims", *IMF Staff Papers*.
- Gray, D., Lim, C., Loukoianova, E. and Malone, S. (2008), "A risk-based debt sustainability framework: incorporating balance sheets and uncertainty", *IMF Working Papers*.
- Gray, D., Merton, R., and Bodie, Z. (2007), "New framework for measuring and managing macrofinancial risk and financial stability", *NBER Working Paper Series*.
- Haldane, A. (2009), "Rethinking the financial network", *Speech at Financial Student Association*, Amsterdam, April.
- Merton, R. (1974), "On the price of corporate debt", *Journal of Finance*, 29(2), 449-470.
- Sheldon, G. and Maurer, M. (1998), "Interbank lending and systemic risk: an empirical analysis for Switzerland", *Swiss Journal of Economics and Statistics (SJES)* 134(IV), 685-704.
- Upper, C. and Worms, A. (2004), "Estimating bilateral exposures in the German interbank market: Is there a danger of contagion?", *European Economic Review* 48(4), 827-849.
- Wells, S. (2004), "Financial interlinkages in the United Kingdom's interbank market and the risk of contagion", *Bank of England Working Paper*.