INTER-SECTOR RELATIONS IN THE PORTUGUESE ECONOMY: AN APPLICATION OF CONTINGENT CLAIM ANALYSIS

*Nuno Silva**

1. INTRODUCTION**

Financial crises are relatively rare events. However, when they occur, they imply very high costs both in terms of economic activity in the short run and long term economic growth. In particular, the financial crisis that began in 2007 has had strong consequences on global economic activity, justifying the definition of new policies targeting the implementation of a more transparent international financial architecture, where the dominant micro-prudential vision is complemented by a broader approach. In this context, it is most important to understand the mechanisms underlying the outbreak of systemic risk. In particular, financial stability analysis needs to address the interconnections between all players in the economy. As long as these inter-linkages constitute the main channels through which shocks are propagated, understanding them can help to detect the mechanics behind shock transmission and systemic risk.

Traditionally, the literature in financial stability focuses either on financial institutions *per se* or on the relations among them. More recently, this analysis has been extended to the private non-financial and public sectors. Examples are the works of Gray (1999), Setser, Allen, Keller, Rosenberg and Roubini (2002), Gapen, Gray, Lim and Xiao (2004, 2008) and Gray (2008). These studies rely on the identification of unstable positions in sector balance-sheets. However, by focusing their analysis on emerging markets crises, especially those in Southeast Asia (1997) and Brazil (2002), these studies lack some generality as they concentrate on economies subject to currency risk. Thus, these crises should not be taken as example for countries that have most of their activity concentrated in their own currency, as is the case with countries in the euro area and the US. The last three articles stand out from the first two by incorporating contingent claim analysis as developed by Merton (1974) following the work of Black and Scholes (1973). Unlike other approaches that rely either on accounting or macroeconomic analysis, Merton's model (as it is also known) takes into account markets uncertainty and the nonlinearities intrinsic to debt valuation. The model leads to a set of objective metrics that are easy to calculate and interpret. However, none of these articles have a global view on the economy and the transmission mechanism across sectors.

Broadening the scope of the analysis, Gray, Merton and Bodie (2007) proposed to apply Merton's

* The author is thankful to Ana Cristina Leal, Nuno Alves, Mário Centeno, Nuno Ribeiro and António Antunes for comments and suggestions. The author is also in debt to the Statistics Department of the Banco de Portugal for providing information on financial accounts. The analysis, opinions and findings of this article represent the views of the author, which are not necessarily those of Banco de Portugal or the Eurosystem. All errors and omissions are the sole responsibility of the author.

** Banco de Portugal, Economics and Research Department.

model to an economy composed of five sectors, which are seen as a set of balance sheets interrelated by equity and implicit guarantees on debt payments. However, there are still few empirical applications. Recently, Castrén and Kavonius (2009) developed a network of bilateral relations for the main sectors of the euro area. Once built, this network introduces the nonlinearities common in risk transmission mechanisms through contingent claim analysis. This study broadly follows the latter. However, two major changes are introduced, namely, losses (or gains) related with credit risk are taken into account and households' real estate assets are included in their balance sheets.

This study is composed of 7 sections. Section 2 presents the data. Section 3 builds a network of bilateral relations within the Portuguese economy and explains the shock transmission system. Section 4 presents the Merton model and applies it to the Portuguese economy. Section 5 proceeds with the simulation of two shocks based on the transmission model presented in section 3, namely, a sudden loss on non-financial corporations equity and an increase in credit impairment. Section 6 discusses the limitations of the analysis. Section 7 concludes.

2. THE DATA

The data used in this study corresponds to the Portuguese non-consolidated national financial accounts compiled and published quarterly by Banco de Portugal. These accounts are a synthetic representation of the financial structure of the economy. This data is broadly organized in matrix form with eight sectors (non-financial corporations, central bank, other monetary financial institutions, other financial intermediaries, insurance companies and pension funds, general government, households and the rest of world)[1] and seven types of financial instruments (monetary gold and special drawing rights, currency and deposits, securities other than shares, loans, shares, insurance and other accounts receivable). Monetary gold and special drawing rights were excluded from the analysis since they have no counterparty sector.[2] In order to simplify the exposition, shares and insurance were joined.[3] All transactions are recorded in accordance with the double entry principle. In practice, all assets have a counterparty liability. This generates a closed system useful for studying shock propagation channels. Given its matrix form, this allows not only to assess the role of the financial sector as an intermediary in the economy, but also estimate each sector leverage ratio, which is an important resilience indicator. In addition, it is possible to determine the net financial position of resident sectors in relation with the rest of the world, revealing their degree of immunity to external shocks. Unfortunately, this data does not have any information on the real side of the economy. For instance, household real estate assets and non-financial corporations capital stock are not taken into account.

Chart 1 breaks down each sector balance-sheet instrument-by-instrument for the fourth quarter of

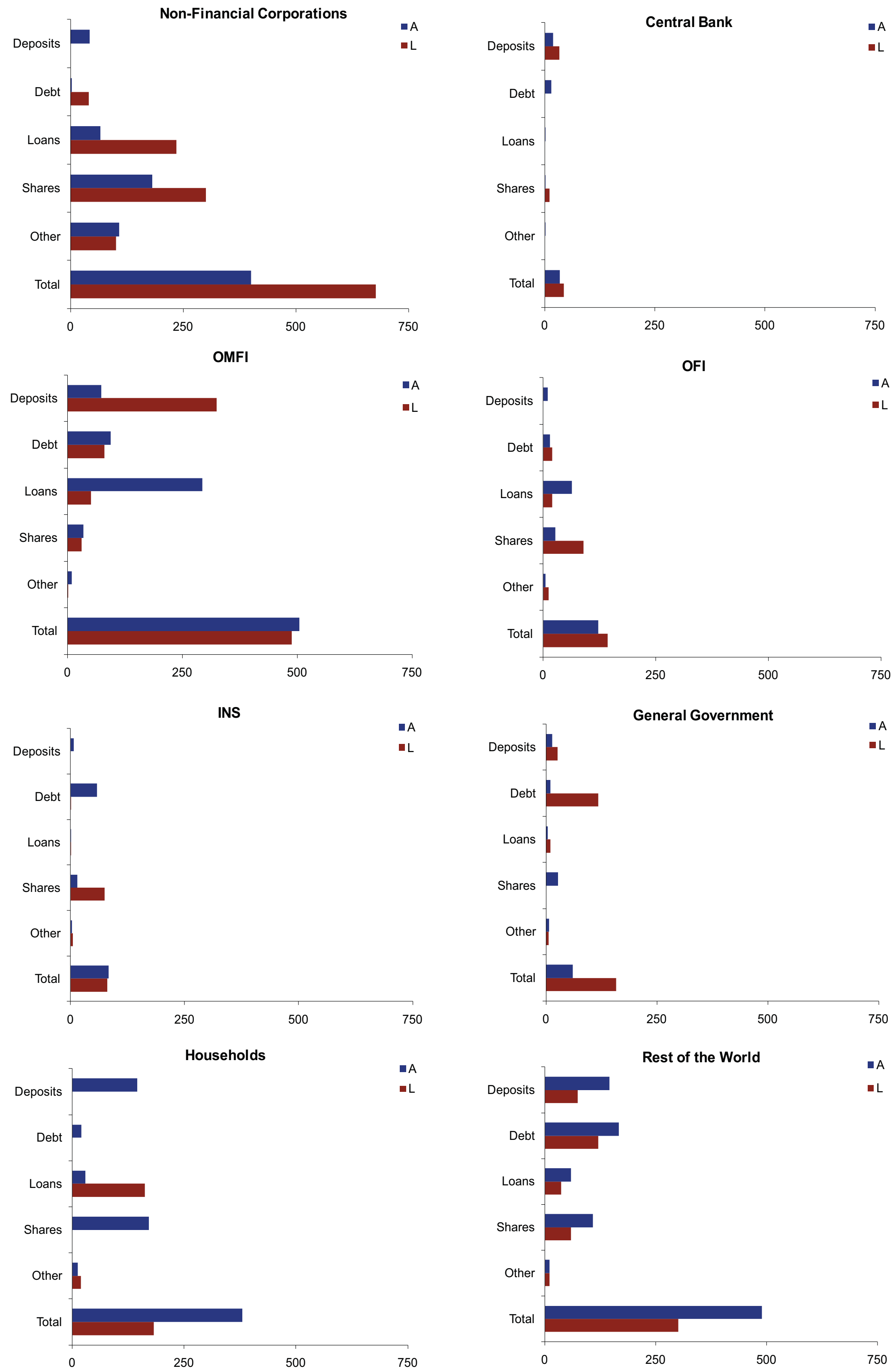
(1) The acronyms OMFI, OFI and INS will be henceforth used to refer to other monetary financial institutions, other financial intermediaries and insurance companies and pension funds, respectively. Non-financial corporations, general government and the rest of the world appear in charts as NFC, GOV and RoW, respectively.

(2) This instrument is however considered to calculate central bank's equity under the contingent claim analysis model.

(3) In order to facilitate exposition, these instruments shall be henceforth referred only as "deposits", "debt", "loans", "shares" and "other".

Chart 1

DECOMPOSITION OF THE NATIONAL FINANCIAL ACCOUNTS BY SECTOR AND INSTRUMENT FOR 2009 Q4
Values in billion Euros



Source: Banco de Portugal (National Financial Accounts).

2009.[4] Based on each sector net financial position, one can distinguish three types of sectors. Non-financial corporations and the general government have a negative net financial position. Regarding non-financial firms, this is mostly due to their relatively large capital stock, while for the general government it should roughly reflect the consecutive budget deficits incurred. On the other side, households and the rest of the world show a largely positive net financial position, which in the latter case corresponds to the accumulation of successive balance of payments deficits. Finally, all financial institutions (central bank, OMFI, OFI and INS) have a relatively balanced financial position. Among financial institutions the high value of assets and liabilities of OMFI reflects their role as financial intermediaries in the economy.

On an instrument basis, for all sectors but financial institutions, the asset side of the balance sheet consists broadly of “deposits” and “shares”. In addition, non-financial corporations have some of their assets invested in “other” and “loans”, which should correspond mainly to trade credit. The rest of the world has also an important part of its assets invested in “debt”.[5] In contrast, financial institutions assets correspond mostly to “loans” (OMFI and OFI) and “debt” (INS)[6]. The central bank has its assets spread between “debt” and “deposits”. Liability positions vary widely among sectors. For non-financial corporations, they correspond mostly to “shares” issued and “loans” from financial institutions. Among financial institutions, one can find very different situations. While the central bank and OMFI liabilities correspond mostly to “deposits” and to a lesser extent “debt”, OFI liabilities are largely composed by “shares”. Regarding the central bank, notice that the value assigned to “deposits” refers largely to liabilities under the TARGET payment system. General government liabilities correspond predominantly to “debt”. Households have most of their liabilities under mortgage “loans”. Finally, the rest of the world has its liabilities spread between “debt”, “deposits” and “shares”.

Table 1 shows each sector net financial position, i.e. the difference between financial assets and financial liabilities. The data is shown as a percentage of total financial assets of the economy for

Table 1

SECTOR NET FINANCIAL POSITION IN PERCENTAGE OF THE ECONOMY’S TOTAL FINANCIAL ASSETS IN 2009 Q4

	Portugal	Euro area
NFC	-13.8%	-8.4%
Central Bank	-0.5%	-0.3%
OMFI	0.8	0.7%
OFI	-1%	0%
INS	0.1%	0.1%
GOV	-4.5%	-4.6%
Households	9.9%	11.1%
RoW	9%	1.6%

Sources: ECB and Banco de Portugal (National Financial Accounts).

(4) In order to facilitate the analysis developed in Section 4, “shares” of non-listed companies have been adjusted to reflect price movements in financial markets.

(5) Regarding households, notice that 40% of their investments in “shares” correspond to their positions in insurance companies and pension funds.

(6) Notice that most of OFI “loans” correspond to long term loans to OMFI as counterpart of credit securitizations.

the fourth quarter of 2009.[7] The results obtained are very similar to those presented by Castrén and Kavonius (2009) for the euro area. The exceptions are non-financial corporations, which have a more negative position as compared with the euro area, and the rest of the world, which shows a more positive position. However, in this case the numbers are not comparable, since euro area values do not correspond to country averages, but the rest of the world position regarding the whole euro area.

3. THE SHOCK TRANSMISSION MECHANISM

Inter-sector exposure plays an essential role in the way shocks are transmitted in the economy. Unfortunately, for instruments other than "deposits" and "loans", national financial accounts do not contain information on bilateral balance sheet positions (also known as who-to-whom accounts). Nevertheless, these can be estimated through maximum entropy as done in several studies on the interbank loans market (e.g. Sheldon and Maurer (1998), Upper and Worms (2004) and Wells (2004)).[8] Castrén and Kavonius (2009) also use this methodology.

Consider that bilateral balance sheet positions between two sectors in a given instrument k can be represented by a $N \times N$ matrix where N represents the number of sectors and x_{ij}^k the exposure of sector i to sector j in instrument k :

$$\begin{bmatrix} x_{11} & \cdots & x_{1j} & \cdots & x_{1N} \\ \vdots & \ddots & \vdots & \iddots & \vdots \\ x_{i1} & \cdots & x_{ij} & \cdots & x_{iN} \\ \vdots & \iddots & \vdots & \ddots & \vdots \\ x_{N1} & \cdots & x_{Nj} & \cdots & x_{NN} \end{bmatrix}^k \quad with \quad \sum_{j=1}^{N} x_{ij}^k = a_i^k \quad and \quad \sum_{i=1}^{N} x_{ij}^k = l_j^k \tag{1}$$

In this case, a_i^k and l_j^k correspond to total assets and total liabilities of sector i and j in instrument k , respectively.

In addition, consider that a_i^k and l_j^k may be seen as the components of $f^k(a)$ and $f^k(l)$, the marginal distributions of assets and liabilities, respectively, and that x_{ij}^k is the realization of the joint distribution $f^k(a,l)$. Assuming independence, or maximum entropy, it comes that x_{ij}^k can be estimated as the product of the two marginal distributions. In order to improve results, two restrictions were imposed a posteriori: intra-rest of the world positions were eliminated and the central bank was considered to be entirely owned by the general government.[9] In order to preserve equality between assets and liabilities for each instrument, the RAS algorithm was applied as described by Schneider and Zenios (1990).

Define gross exposure between two sectors as the sum of bilateral assets and liabilities across instruments. Despite simple, this measure uncovers the major inter-sector and intra-sector relations

(7) In opposition to all other sections in this study, to facilitate comparison with the euro area, figures on unlisted "shares" were not adjusted to reflect stock price movements. Total financial assets in the economy include financial assets from the rest of the world.

(8) This estimation procedure is also frequently used in input-output analysis (see Lahr e De Mesnard (2004)).

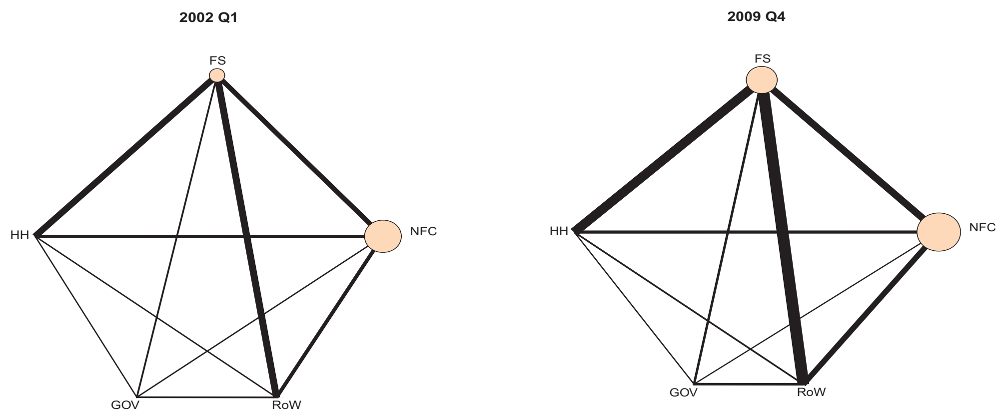
(9) Additionally, it was considered that all long-term debt issued by the central bank until the fourth quarter of 2004 was wholly owned by MFIs.

taking place in the economy. Chart 2 shows gross bilateral exposure between all sectors for the Portuguese economy in the first quarter of 2002 and the fourth quarter of 2009.[10] This representation allows us to draw three conclusions. First, the financial sector plays an essential role in the economy, not only as an intermediary of financial resources but also as a large holder of "shares" and "debt" of non-financial corporations. Moreover, this role has grown steadily over the last decade. Secondly, notice the progressive interconnection of the rest of the world with all other sectors as result of the growing process of economic and financial integration at the European and global levels. Finally, one should be aware of the high intra-sector exposure both of non-financial corporations and financial institutions. For non-financial corporations, this is mostly due to cross holdings of "shares", "other" and "loans". For financial institutions, despite the strong growth shown in the graph, this may be spurious, since it is largely associated with the introduction of IAS (International Accounting Standards) in 2005, which made the securitized assets derecognition process substantially more demanding. Since securitization vehicles are included in OFI, this change in accounting standards had implications on this sector assets growth rate.

Chart 2 is a simplification of reality in the sense that it does not distinguish neither assets and liabilities nor equity and debt, which limits its use in analysing shock transmission. Nevertheless, taking these differences into account, it is possible to have a richer characterization of the financial intermediation process. In particular, we may have more insights on the shock transmission mechanism across sectors. Regarding the latter, it should be emphasized the contributions of Kiyotaki and Moore (1997, 2002), Boissay (2006), Battiston, Delli Gatti, Gallegati, Greenwald and Stiglitz (2007) and Gray *et al*. (2007) to the rationalization of these channels. Shortly, the chain transmission mechanism works as follows. Consider that financial assets held by each sector can be classified in two types: equity ("shares") and debt ("deposits", "debt", "loans" and "other"). Additionally, assume that all these

Chart 2

BILATERAL GROSS EXPOSURE IN 2002 Q1 AND 2009 Q4



Source: Banco de Portugal (National Financial Accounts).
Note: The diameter of the circle is proportional to the intra-sector exposure; the dashes width is proportional to the inter-sector relations.

(10) In order to simplify the analysis, the central bank, OMFI, OFI and INS were all joined in a single node.

instruments are constantly valued at market prices. In this model, any shock is likely to be transmitted in either of two ways. On the one hand, equity holders of the sector that suffers the shock bear a loss (or gain) in proportion to their share. Second, given debt contingency on total assets, any shock that produce changes in debt's quality also generate immediate losses (gains) to those sectors that hold this type of assets. These losses must be then distributed across all sectors that hold equity from those sectors that have previously registered losses and so on until the shock is totally dissipated. Note however that, in theory, nothing ensures that the shock is dissipated before any sector has disappeared due to exhaustion of its equity base. When this happens it is said that the shock does not converge.

In order to better understand the transmission mechanism, consider an iterative system where losses related to any of these routes are calculated and distributed at the end of each iteration. Thus, each sector assumes not only losses corresponding to the sum of products between its exposure to the equity of each loss-registering sector and their losses but also the sum of the products between its debt exposure to each loss-registering sector and the losses caused by the deterioration of its debt quality.[11] Denominate each of these outcomes as effect 1 and effect 2, respectively. These effects have two very different economic interpretations. While effect 1 corresponds to losses actually incurred by each sector, effect 2 considers creditors' expected losses as the result of changes in borrowers' likelihood of default and losses given default. In the absence of credit risk, i.e. assuming that all economic agents are going to fulfil their contractual responsibilities, effect 2 is not present. As regards the shock transmission mechanism presented, and unlike losses related to equity holdings (effect 1), which are easy to infer based on previous iterations, losses related with changes in debt quality (effect 2) require the adoption of a debt pricing model. Section 4 presents contingent claim analysis as a way of quantifying these losses.

4. CONTINGENT CLAIM ANALYSIS

4.1. The model

Contingent claim analysis appeals to Merton's model (1974) to assess the creditworthiness of a debt issuer, which we will call the "firm", but which could be a whole economic sector. Consider a firm that issues debt at a given time with a certain maturity. The question that arises is whether the firm has enough assets to honour its obligations at maturity. The firm will honour its commitments if the value of its assets exceed, at maturity, its debt. If not, the firm declares bankruptcy and all assets are liquidated to creditors. The negative difference between assets and liabilities will then be debt holders' losses. Deciding on whether or not to pay back debt at maturity is very similar to exercising a call option. In this context, the option holder will buy the underlying asset if its market price at maturity exceeds the strike price. Otherwise, the call option is not exercised. In our case, the underlying asset corresponds to all assets of the firm while the exercise price is the nominal value of debt. It follows that the market value of debt should be equal to its face value discounted by a risk-free interest rate

(11) Note that since our analysis use non-consolidated data, a particular sector may keep building up further losses inside it. Hypothetically, in an extreme case, if the whole equity of a given sector was held by the sector itself, it would generate a cycle that invariably would end in its own destruction.

less the value of a put option on the firm. That is, in the absence of arbitrage opportunities, investors should be indifferent between taking an amount of riskless debt, or take the same amount at risk but ensuring that, in case of non-repayment, they can recover the difference between what they have received (the asset value of the firm) and what they should have received (debt repayment). This is achieved through the put option. In practice, knowing a firm's equity market value, the volatility of its equity returns, its nominal debt and the risk-free interest rate, one can use contingent claim analysis to calculate a series of risk measures, namely the distance to distress, the probability of default and the ex-ante expected loss.

Consider that A, B and E correspond respectively to assets, debt and equity market value for a given firm or sector. If there are no market frictions and assuming all assets are liquid in maturity, we have that

$$A = E + B \tag{2}$$

i.e. the market value of equity should equal the difference between assets and the market value of the risky debt. Suppose that A follows a stochastic diffusion process with a deterministic trend governed by the risk-free return. Consider that at $t = 0$, the firm issues zero coupon bonds with nominal value B_T amounting to all its liabilities. This firm is bankrupted if the value of its assets, A, is lower than B_T at maturity.

It follows that, in accordance with option pricing theory, the equity market value of the firm, E, equals an European call option on the underlying assets, A, with maturity $t = T$ and strike price equal to its nominal debt, B_T. Applying Itô's lemma, imposing no-arbitrage and frontier conditions equivalent to a call option, and defining $\tau = T - t$, one can obtain the following equation for E,

$$E = A\Phi(d_1) - B_T e^{-r\tau}\Phi(d_2) \tag{3}$$

where

$$d_1 = \frac{\ln\frac{A}{B_T} + (r + \frac{1}{2})\tau}{\sigma_A\sqrt{\tau}} \tag{4}$$

$$d_2 = \frac{\ln\frac{A}{B_T} + (r - \frac{1}{2})\tau}{\sigma_A\sqrt{\tau}} \tag{5}$$

In the above equations σ_A stands for the volatility of asset returns, r is the risk-free interest rate, which we considered to be constant, τ is the time interval up to maturity and Φ is the standardized cumulative normal function. Equation (3) has a simple interpretation. The first term evaluates assets weighted by a coefficient related to the probability of the call option being exercised; the second term weights the discounted nominal debt by a coefficient slightly smaller given that losses are limited.

In turn, the put option value, P, can be calculated as

$$P = e^{-r\tau} B_T + E - A \tag{6}$$

In a risk-free world $P = 0$ and asset value equals equity plus nominal debt discounted at the risk-free rate.

Equation (3) has two unknowns, A and σ_A. In order to obtain their value one needs to impose a second condition. One possibility is to say that E also follows a geometric Brownian motion but with different parameters than A.

Applying Itô's lemma and equating the volatility terms, we obtain

$$E\sigma_E = A\sigma_A \Phi(d_1) \tag{7}$$

where σ_E is the volatility of equity returns.

Solving the system composed of equation (3) and (7) at each point in time, it is possible to obtain a time series for A and σ_A.[12] Substituting A and E into equation (2), we can then find B and calculate the distance to distress, d_2, the probability of default, $\Phi(-d_2)$, and the expected losses, P.

4.2. Application to the whole economy

The model presented so far was designed to be applied to listed firms for which information on market value and volatility of equity returns is widely available. The application of contingent claim analysis to economic sectors, though possible, requires several assumptions regarding the definition of equity and the volatility of equity returns. In sight of this, one can broadly distinguish two groups of sectors. On the one hand, we have those sectors that issue "shares", some of them are even listed in stock exchanges. This applies to non-financial corporations, OMFI, OFI and INS[13]. For these sectors, as suggested by Gray *et al*. (2007), it makes sense to think that unlisted "shares", if listed, would follow a trend similar to those that are effectively listed. Nevertheless, the way this behaviour is actually reproduced is not clear. In this study, unlisted "shares" value were estimated as the exponential of the sum of the logarithm of unlisted "shares" with the logarithmic distance of listed "shares" to their trend. We have calculated a different trend for each sector. As already mentioned, our data already incorporates this adjustment. Thus, non-financial corporations, OMFI, OFI and INS equity were estimated assuming that their "shares" are equivalent to call options on their assets with exercise price equal to their liabilities. For the volatility of equity returns, we used the volatility of the PSI-20 and the PSI-Financial Services for non-financial corporations and OMFI, respectively, and the volatility of German 10-year bond yields for OFI and INS. For the central bank, though it issues "shares", which are fully owned by the general government, there is no index to compare. Thus, we chose to define central bank's equity

(12) Note that, unlike the original Black and Scholes (1973) model, the hypothesis of stationarity of σ_A is neglected when solving this system.

(13) For INS most of its "shares" refer to household's net equity on pension funds.

as its net welth position, which includes monetary gold and special drawing rights. This had been excluded from who-to-whom accounts since it had no counterparty sector. The volatility of Portuguese 10-year bond yields was used as a proxy for the volatility of equity returns of the central bank.

For those sectors that do not issue "shares", the situation turns harder. This is the case for the general government, households and the rest of the world. Among these, the general government is probably the most difficult case since it generally has a negative net financial position. Some authors such as Sims (1999), Keller, Kunzel and Souto (2007), Gray *et al* (2007), Gray (2008) and Gapen *et al* (2008) suggested that general government assets could be estimated based on the different priority levels of its liabilities. However, none of the several options proposed is consensual. A first hypothesis is to consider that the general government also includes the central bank. In this case, its assets would be largely made up of international currency reserves, future tax revenues and all types of real and financial assets held by the general government. Similarly, liabilities would be composed by the monetary base, the sum of future expenses and all sorts of liabilities, either in national or foreign currency. However, unlike liabilities in national currency, which are easier to control either by printing money or normatively, i.e. imposing credit restructuring; liabilities towards the rest of the world may require the acquisition of foreign currency. This leads these authors to compare national currency denominated liabilities to firm's equity. Thus, in the same way firm's equity grows according to its performance, domestic currency debt changes in value according to some exchange rate which reflects how well the economy is performing. Firm's equity devalues whenever new "shares" are issued or stock splits occur and grows whenever firms buy their own "shares". Similarly, national currencies appreciate or depreciate depending on whether the central bank issues or withdraws currency. Finally, both can serve as a buffer whenever the financial situation deteriorates. Seemingly simple, deficit monetization is usually associated with inflationary periods and lack of credibility from national institutions with consequences in the long-term growth rate of the economy. Regardless of its pros and cons, if we would follow this hypothesis and using option pricing theory, general government assets could have been calculated considering that national currency liabilities were equal to a call option on assets with an exercise price equal to foreign currency liabilities. The volatility of national currency liabilities could then be inferred from exchange rate volatility. Although interesting, this hypothesis does not make sense in the Portuguese case for three reasons. Firstly, the institutional framework of the Economic and Monetary Union (EMU) embodies monetary policy independency of the central bank. Thus, unlike firms, which are free to raise equity, the treaties governing the European Union explicitly prohibit countries participating in the euro area and, in general, all European Union states, to resort to deficit monetization. Second, most public debt in euro area states is denominated in Euros. Thus, euro area countries liabilities would be almost negligible according to the application of the criteria explained above, which would distort our conclusions. Finally, though euro area countries have most of their liabilities denominated in their own currency, each of them has little power to influence monetary policy, which again contradicts the previous arguments for defining the prioritization of liabilities.

A second alternative proposed by Castrén and Kavonius (2009) is to consider general government equity as the sum of its net financial position plus "debt", which is usually quoted in the market. This method explores the fact that non mark-to-market liabilities have, at least theoretically, to be paid at face value while mark-to-market liabilities can be obtained at below par in secondary markets. Since most countries liabilities correspond to quoted debt, this method solves the equity definition problem. Nevertheless, it should be noted that in this case, equity would be greater, the greater the proportion of assets financed by “debt”. Ultimately, this situation would mean that a country able to securitize all its liabilities would never default.

Finally, one may consider that liabilities in the hands of non-resident economic agents have priority over all other liabilities. The argument is that if a default would occur, resident economic agents would have to be more flexible. This rationale seems to be more suitable to the Portuguese case. For Portugal, unlike the euro area as a whole, where most liabilities are financed by residents in the euro area, liabilities are mainly held by non-residents. Although we have followed this principle, we recognize that this is not immune to criticism. Thus, from a legal standpoint it is difficult to justify the fact that there are two securities with similar rights where one is being fulfilled and the other is not. It may also be argued that one needs only one security to enter in default in order to all others being considered automatically in default. In spite of these critics, applying option pricing theory, general government assets can be estimated using the volatility of Portuguese 10-year government bond yields as a proxy for risk.

For the remaining two sectors, although they do not issue “shares”, they have a positive net worth, which eases the analysis. For households, it was considered that their net worth amounts to their real estate holdings plus their net financial position. Risk-adjusted assets could then be estimated considering households’ net worth as equivalent to a call option on their assets with an exercise price equal to their liabilities.[14] Similar to Castrén and Kavonius (2009), we used the volatility on 10-year national bond yields as a risk indicator. The rest of the world has a residual role in this model. Their inclusion is though necessary to close the financial system and to transmit shocks to other sectors. Therefore, it was considered that its net worth amounts to a call option on its assets with an exercise price corresponding to its financial liabilities. The VStoxx was used as a risk indicator.[15]

Based on these definitions, and assuming that each sector liabilities equal the sum of its short-term liabilities plus 50% of its long-term liabilities; we have applied contingent claim analysis to the Portuguese economy.[16] Chart 3 shows assets, the volatility of asset returns, the distance to distress and the leverage ratio for non-financial corporations, OMFI, OFI, the general government and households.

Broadly, one can find two very different patterns: before and after the mid-2007 financial crisis. Thus, the period between January 2002 and June 2007 is characterized by a very substantial increase in assets for all sectors, especially OFI, whose assets grew 138%.[17] On the other hand, households had the slowest growth (27%). This increase in assets led to a decrease in leverage ratios for all

(14) For a more detailed analysis of the method used to estimate households’ real estate assets see Cardoso, Farinha e Lameira (2008).

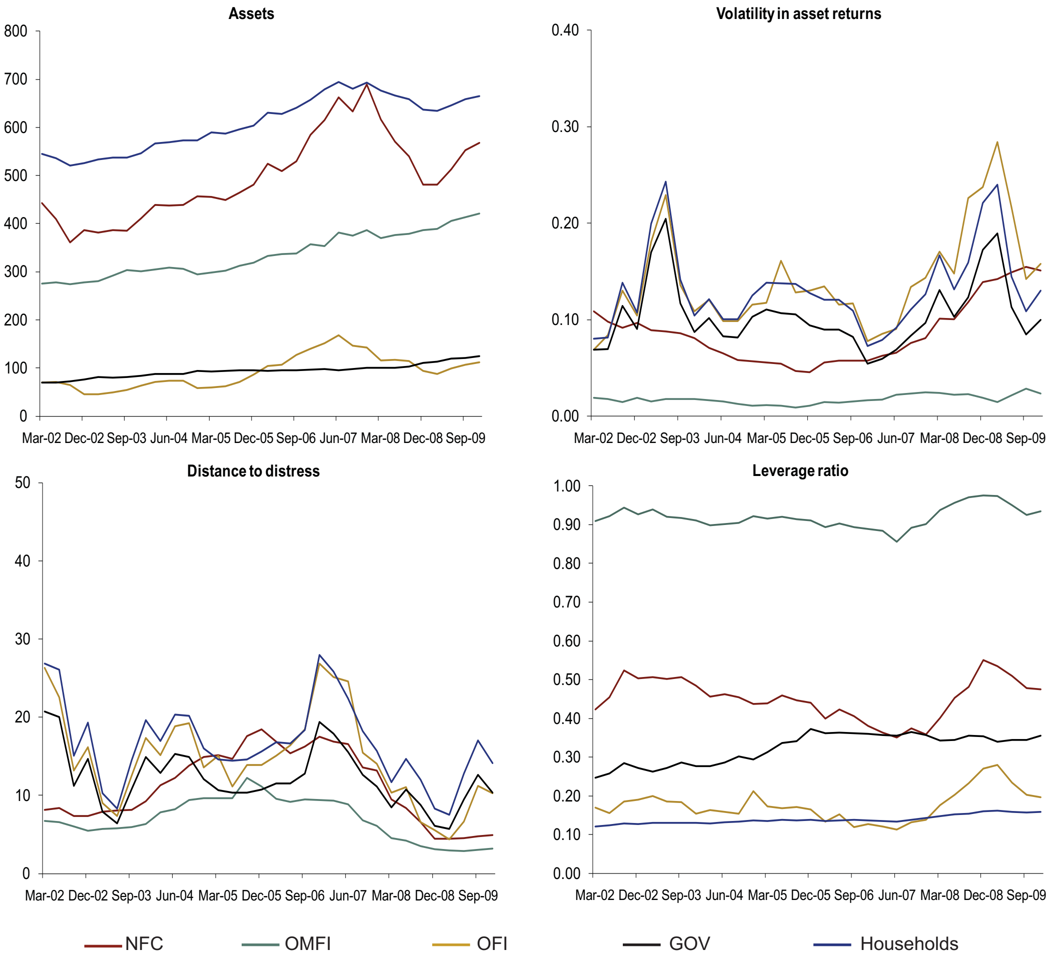
(15) VStoxx is an implicit volatility measure based on the Dow Jones Eurosotxx 50.

(16) Notice that the value used for nominal debt corresponds to the standard in the literature in contingent claim analysis, which is based on the idea that in the long run firms are able of adjusting their behavior in accordance with market developments.

(17) Note that this increase is largely motivated by the changes carried on accounting rules regarding the derecognition of securitized assets.

Chart 3

ASSETS, VOLATILITY ON ASSET RETURNS, DISTANCE TO DISTRESS, LEVERAGE RATIO
Assets in EUR billions



Source: Author's calculations.

sectors except the general government and households whose ratios grew only 11 p.p. and 1 p.p., respectively. In turn, the period after the summer of 2007 is characterized by a strong and sudden fall in most sectors assets, namely OFI (33%), non-financial corporations (14%) and households (4%). Nevertheless, the central bank and the general government increased their assets by 98% and 31%, respectively. In the first case, this reflects the non-conventional monetary policy measures implemented by the European Central Bank. The large decline in asset prices also had impact on sector leverage ratios (debt to assets). The largest increases were observed for non-financial corporations (12 p.p.), OMFI (8 p.p.) and OFI (8 p.p.). Similar to other indicators, the volatility of asset returns and the distance to distress also show a different behaviour before and after the financial crisis. Thus, the volatility of asset returns has an oscillating behaviour around relatively low values until the second quarter of 2007, when it has a strong and sudden rise. Likewise, the distance to distress evolves in accordance with assets and their volatility. It shows very high values for most of the sample, dropping dramatically after 2007. OMFI shows the lowest values for this indicator. The 2003 recession is also peculiar in terms of the volatility of asset returns and the distance to distress.

5. SIMULATION

In order to evaluate the iterative scheme presented in section 3 we defined two shocks. The first shock is a decrease in non-financial corporations growth perspectives leading to a permanent devaluation in share prices of approximately 30%. The choice of shock magnitude was based on annual return analysis (250 business days rolling windows) of the PSI-20 index between 1993 and 2010. Assuming a normal distribution, it was chosen a shock equivalent to the 10th percentile. The second shock consists of an unrecoverable loss of 1.1% in "loans" granted by OMFI to households for house purchase and a loss of 6.6% in all other "loans" granted either by OMFI, OFI and non-financial corporations to households. We will interpret this mostly as "loans" for consumption and other purposes. A 4.3% loss in "debt", "loans" and "other" granted to non-financial corporation was also assumed. This corresponds to an annual loss of 0.4%, 2.2% and 1.4% in each of these credit segments with a horizon of three years. This accounts for the persistence usually posted by credit losses. The shock magnitude was designed under the assumption of a normal distribution with expected value equal to the average annual flow of non-performing loans as a share of the total stock of credit on each segment. The values chosen correspond to the 90th percentile.

Based on the values for debt and equity estimated in Section 4 for the fourth quarter of 2009, we have simulated the impact of these shocks based on the mechanism defined in Section 3 and contingent claim analysis. Regarding the volatility of equity returns, it was considered to change according to the following function presented by Bensoussan, Crouhy, Galai, Wilkie and Dempster (1994)

$$\sigma_E = \overline{\sigma_A}\left(1 + \frac{B_T}{E} e^{-r\tau} \Phi(d_2)\right) \tag{8}$$

where $\overline{\sigma}_A$ corresponds to the expected value of σ_A.[18] The introduction of the above function is intended to incorporate the normal increase in volatility that generally occurs after a shock. Consider the following example. Assume that non-financial corporations suffer a negative shock of 1 million Euros with impact on its equity market value. In addition consider that liabilities stand at 10 million. To simplify, assume that both equity and debt of nonfinancial corporations are equally owned by other non-financial corporations, OMFI, OFI, general government and households (20% each). Finally, consider that losses caused by this shock lead nonfinancial corporation's debt to depreciate 0.1% due to an increase in its probability of default. In this case, each of the sectors involved would have an initial total loss of 210 thousand Euros, in which 200 thousand Euros correspond to effect 1 and 10 000 Euros to effect 2. In turn, this loss would then be reflected in the shareholders of those sectors affected by these losses and so on. Effect 2 is calculated using contingent claim analysis, which takes into account each sector leverage ratio and the volatility of its equity returns.

Chart 4 shows current and accumulated losses as a proportion of initial equity for our two shocks in an iterative scheme.[19] Current losses tend to zero after a few iterations signalling shock convergence.

(18) In the current application, we used B_T, E and d_2 from the previous iteration. Similarly, $\overline{\sigma}_A$ is substituted by σ_A from the previous iteration. Note that, as in many other applications of the Merton model, the hypothesis of constant volatility is violated.

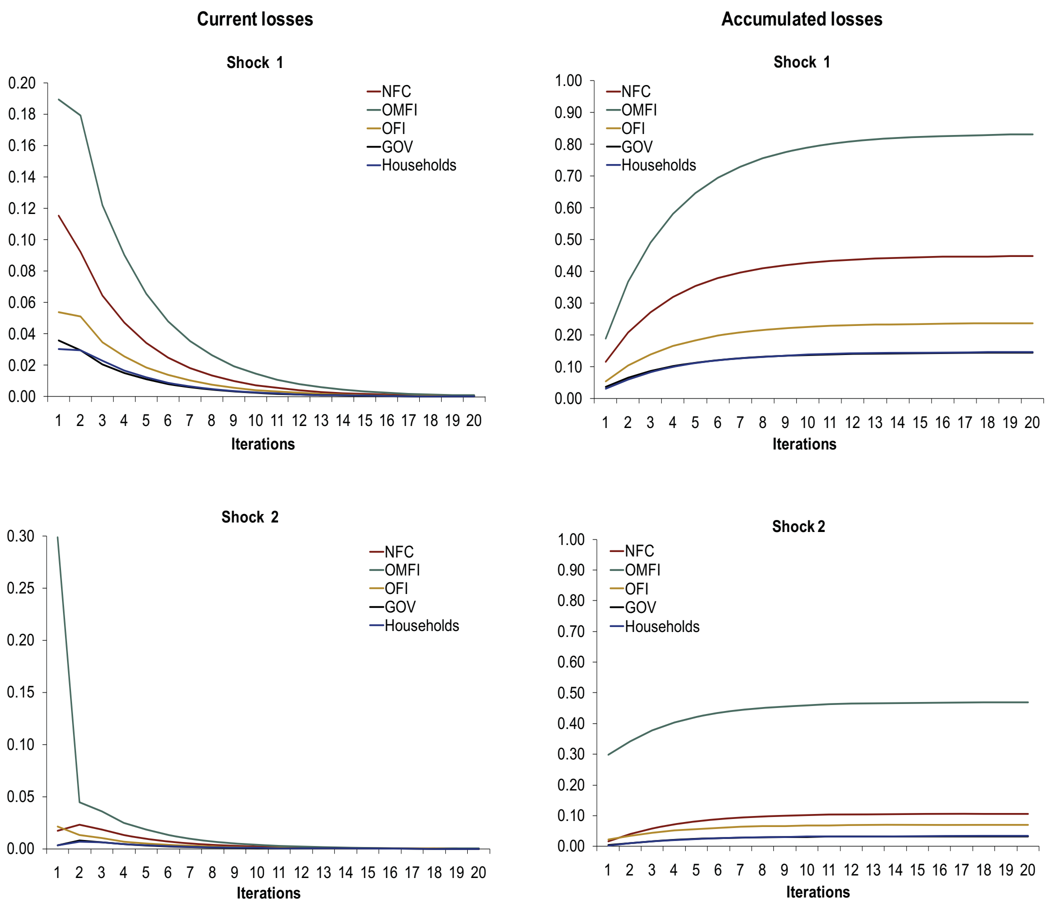
(19) The algorithm used in this study considers 20 iterations.

Likewise, accumulated losses converge to a value below initial equity indicating that all sectors are able to absorb the shock. For the first shock, the most affected sector is OMFI, whose equity suffers a loss of 83%. It follows non-financial corporations and the rest of the world with losses amounting to 45% and 40% of their initial equity, respectively. Overall, the shock led to a devaluation of around 18% of all financial assets in the economy (360 billion Euros). The second shock shows a slightly different transmission pattern in the sense that OMFI is affected before all other sectors. In the end, OMFI is once more the most affected sector with losses summing up to 47% of its equity. Losses in other sectors are considerably lower. In sum, the shock led to a total loss slightly above 4% of total assets (94 billion Euros).

In order to better understand the impact of credit risk in the economy we have decomposed total losses for both shocks in effect 1 and effect 2. Under this model, this can be done by assuming that the volatility of equity returns is zero for all sectors. In practice, this implies no losses associated with changes in debt's quality. Chart 5 compares total losses for the cases with and without credit risk. For the first shock, the difference between these two cases, i.e. losses that can be directly assigned to

Chart 4

LOSSES AS A PROPORTION OF INITIAL EQUITY BY SECTOR



Source: Author's calculations.

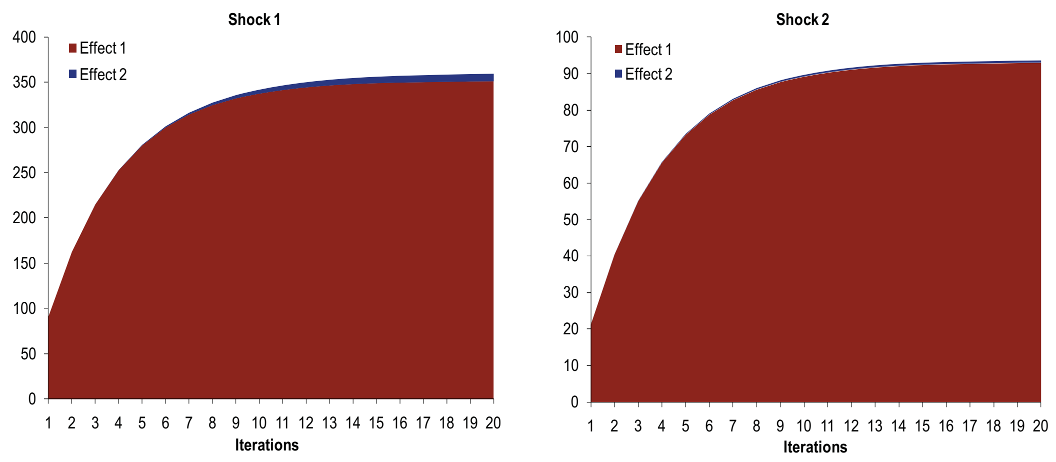
credit risk sum up to 8500 million Euros, 0.35% of initial assets and only 2.3% of total losses originated by the shock. For the second shock, this figure was less than 600 million Euros, or 0.02% of initial assets and 0.6% of the total losses. We can then conclude that effect 1 is considerably greater than effect 2 for any of these two shocks. Despite this superiority, and given the nonlinearities inherent in debt valuation models, it is appropriate to examine how much is this effect for shocks of greater magnitude. Chart 6 shows the losses caused by effect 2 after 20 iterations for different shock magnitudes.

Although effect 2 is clearly smaller as compared to shock 1 for shocks of low magnitude, it seems to have an exponential behaviour. Notice that for shocks in “shares” price in excess of 35%, effect 2 tends to infinity signalling the collapse of the system. In other words, for shocks in non-financial corporations equity above 35% there is at least one sector whose equity becomes negative before all losses are dissipated, thus preventing shock convergence. As shown in Chart 4 this sector is OMFI for our two shocks. Since contingent claim analysis cannot be estimated with negative equity, the system is said to become unsolvable.

Chart 7 shows all combinations of loss rates in each credit segment that lead the system to collapse. The system seems particularly sensitive to losses on credit to non-financial corporations (“loans”, “debt” and “other”). A loss of 11.5% in these instruments is sufficient to destabilize the system. In opposition, the economy appears to be quite resilient to shocks in “loans” for consumption and other purposes as it takes a loss of more than 50% on the whole exposure to this type of “loans” to drive down the system. Notwithstanding these findings, we must take two facts into account. Firstly, these figures represent final losses. Thus, assuming a loss-given-default of 50%, a final loss of 11.5% in non-financial corporations credit implies a flow of non-performing loans of 23% of the whole credit exposure to this sector. Secondly, given the high correlation between these credit segments, the greatest risk comes from the intermediate points rather than from the extreme ones. For instance, a combined shock of 3% in mortgage “loans”, 13% in “loans” for consumption and other purposes and

Chart 5

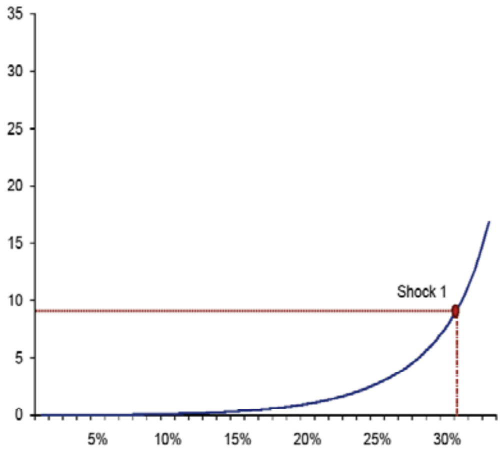
DECOMPOSITION OF TOTAL LOSSES BETWEEN EFFECT 1 AND EFFECT 2
Values in EUR billions



Source: Author’s calculations.

Chart 6

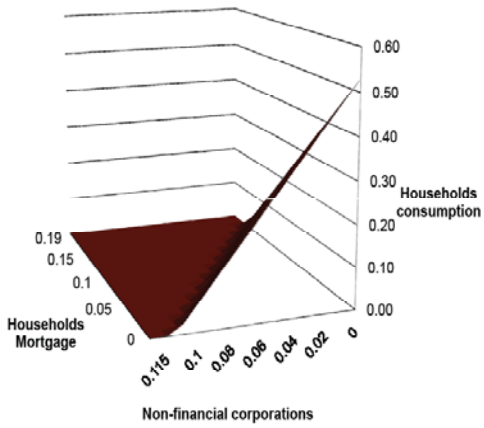
EFFECT 2 AS A FUNCTION OF INITIAL SHOCK MAGNITUDE (SHOCK 1).
Values in EUR billions



Source: Author's calculations.

Chart 7

TRIPLETS OF LOSS RATES THAT LEAD THE SYSTEM TO COLLAPSE



Source: Author's calculations.

7% in credit to non-financial corporations is sufficient to lead the system to collapse.

Chart 6 and 7 illustrate a very important phenomenon in this type of networks. After a certain point the shock transmission process becomes highly nonlinear so that it becomes difficult to stop. This is consistent with the findings of Castrén and Kavonius (2009), Allen and Gale (2000), Gallegati, Greenwald, Stiglitz and Richiardi (2008) and Haldane (2009) who argue for the existence of a tipping point. Once crossed this point, all interconnections in the economy become amplification channels instead of shock absorbers. This leads Haldane (2009) to conclude that this network, though apparently robust, is extremely fragile because the boundary between stability and depression is very weak. In this model, the existence of absorbing nodes, able of receiving negative shocks and not passing them to other sectors, is essential to stop the contagion. In advanced economies, the general government is probably the sector better prepared to play this role. This happens essentially for two reasons. Firstly, since this sector does not issue "shares", it does not produce effect 1. Secondly, general government liabilities are generally considered to be of higher quality because their resources are somehow only limited by total national wealth. Thus, even in a context of financial crisis, this sector is able of raising funds imposing taxes on those who are better off, generally, households. As a consequence, the general government usually shows low volatility in equity returns. Without a credible fiscal policy, where there are no doubts on general government's ability to appeal to households to finance its expenditure, and in sight of a non convergent shock, international intervention may be needed to prevent contagion to the whole economic system.[20]

(20) A shock is considered as non convergent if it produces sufficient losses to lead at least one sector to bankruptcy.

6. LIMITATIONS TO THE ANALYSIS

Like any other method, the model presented in this study has some limitations. The whole exercise is conditioned by contingent claim analysis own caveats, for instance the model is largely dependent, not on reality, but on markets perception of reality. The model does not sign what the market does not perceive. This is visible in cases like Enron and even on the current financial crisis, where the market had a delayed reaction. There are also other technical aspects which are often criticized. For instance, the assumption that assets follow a Brownian motion, the violation of the return on assets constant volatility assumption, and the subjectivity around the distress barrier and the horizon of the analysis.

There are also some limitations related to the degree of disaggregation of the analysis. Whenever the analysis is focused on the probability of default of a sector as a whole, we are underestimating the risks in the economy broadly for two reasons. Firstly, any analysis at the aggregate level tends to ignore the heterogeneity within each sector. For example, households have a largely positive net position. However, it is known that this wealth is unequally distributed, meaning that the risk in household's debt depends on each individual borrower. Second, when we analyze the data in aggregate we ignore what Haldane (2009) calls *small world property*. Consider, for example, that a particular economic sector makes most of its transactions with a small number of companies. The existence of this type of structure leads shocks to propagate and grow very quickly inside clusters before passing out. In other words, the existence of small worlds increases the likelihood of a local problem to swell and become global. Thus it might be useful to introduce in the model some measure correlated with the level of relational entropy within each sector.

7. CONCLUSION

This study has broadly three goals. First, it aspires to highlight the importance of inter-sector relations in the economy. In particular, it places an emphasis on the overwhelming role played by the financial system as the centre of this dense network of relations. In this context, we aimed at estimating a network of bilateral balance-sheets similar to that of Castrén and Kavonius (2009) for the euro area. The results obtained for Portugal were quite similar to those achieved for the euro area with the financial system concentrating 2/3 of all bilateral relations in the economy. Nevertheless, Portuguese non-financial corporations presented a net financial position below the euro area aggregate.

Secondly, this study sought to apply contingent claim analysis to the whole economy. The method proposed by Gray *et al.* (2007) has been adapted to the specificities of a small country belonging to the euro area. The results were in line with expectations. Assets have grown considerably until the 2007-2008 financial crises. For households and the general government, this growth has been fuelled mostly by debt, leading to an increase in leverage ratios. At the same time, the volatility of asset returns has been kept low for most of the sample, leading to high values on distance to distress. This pattern has changed suddenly in 2007 after the first rumours on the sub-prime credit crisis. Assets and distance to distress started then to decrease while the volatility of asset returns and the leverage ratio increased steeply. The worst situation was recorded for OMFI, with the distance to distress falling to below 3, followed by non-financial corporations.

Finally, we analyzed the inter-sector shock transmission mechanism and the role played by risk both on this mechanism and on each sector solvability. We have simulated two shocks: a decrease on non-financial corporations future profits and a simultaneous loss on mortgage “loans” granted by OMFI, all other “loans” granted to households and “loans”, “debt” and “other” granted to non-financial corporations. This has allowed not only to measure direct effects from loss propagation but also the nonlinear effects of the accumulation and transmission of risk in the economy. The application of both shocks to an economy with and without risk allowed us to separate both effects, leading to the conclusion that the former represent the vast majority of losses. Nevertheless, given the non-linearity associated with risk accumulation and transmission, losses related with risk should not be neglected. Depending on the shock, there is a level of losses which, once crossed, avoids the convergence of the system, leading the system to collapse. These simulations also highlight the importance of the banking system in the economy. Any shock in this sector, even if of lesser magnitude, after a certain level tends to have more impact on the overall system than a shock in any other sector. This comes both from the large exposure that all sectors have in relation with OMFI, but also from its current situation characterized by some fragility.

BIBLIOGRAFIA

Allen, F. and Gale, D. (2000), “Financial contagion”, *Journal of political economy*, 108(1), 1-33.

Battiston, S., Delli Gatti, D. Gallegati, M., Greenwald, B. and Stiglitz, J. (2007), “Credit chains and bankruptcy propagation in production networks”, J*ournal of Economic Dynamics and Control* 31(6), 2061-2084.

Bensoussan, A., Crouhy, M. Galai, D. Wilkie, A. and Dempster, M. (1994), “Stochastic equity volatility and the capital structure of the firm”, *Philosophical Transactions: Physical Sciences and Engineering*, pp. 531-541.

Black, F., and Scholes, M. (1973). “The pricing of options and corporate liabilities”. *Journal of Political Economy* , 81(2), 637-654.

Boissay, F. (2006), “Credit chains and the propagation of financial distress”, ECB *Working Paper* Series.

Cardoso, F., Farinha, L., and Lameira, R. (2008), “Household wealth in Portugal: revised series”, *Occasional Papers*. Banco de Portugal.

Castrén, O. e Kavonius, I. (2009), “Balance sheet interlinkages and macro-financial risk analysis in the euro area”, ECB *Working Paper* Series.

Gallegati, M., Greenwald, B., Richiardi, M., and Stiglitz, J. (2008), “The asymmetric effect of diffusion processes: risk sharing and contagion”, *Global Economy Journal*, 8(3), 2.

Gapen, M., Gray, D., Lim, C. and Xiao, Y. (2004), “The contingent claims approach to corporate vulnerability analysis: estimating default risk and economy-wide risk transfer”. IMF *Working Papers*.

Gapen, M., Gray, D. F., Lim, C. and Xiao, Y. (2008), "Measuring and analyzing sovereign risk with contingent claims", IMF *Staff Papers*.

Gray, D. (1999), "Assessment of corporate setor value and vulnerability: links to exchange rate and financial crises", *World Bank Publications*.

Gray, D. (2008), "A risk-based debt sustainability framework: incorporating balance sheets and uncertainty", IMF *Working Papers*.

Gray, D., Merton, R., and Bodie, Z. (2007). "New framework for measuring and managing macrofinancial risk and financial stability". NBER *Working Paper* Series.

Gray, D., and Walsh, J. (2008). "Factor model for stress testing with a contingent claims model of the chilean banking system". IMF *Working Paper*.

Haldane, A. (2009), "Rethinking the financial network", *Speech at Financial Student Association*, Amsterdam, April.

Keller, C., Kunzel, P. and Souto, M. (2007), *Measuring sovereign risk in Turkey: an application of the contingent claims approach*, Citeseer.

Kiyotaki, N. e Moore, J. (1997), "Credit Chains", *Journal of Political Economy* 105(21), 211-248.

Kiyotaki, N. and Moore, J. (2002), "Balance-sheet contagion", *The American Economic Review* 92(2), 46-50.

Lahr, M. and De Mesnard, L. (2004), "Biproportional techniques in input-output analysis: table updating and structural analysis", *Economic Systems Research* 16(2), 115-134.

Merton, R. (1974). "On the price of corporate debt". *Journal of Finance* , 29(2), 449-470.

Schneider, M. and Zenios, S. (1990), "A comparative study of algorithms for matrix balancing", *Operations Research* pp. 439-455.

Setser, B., Allen, M., Keller, C., Rosenberg, C. e Roubini, N. (2002), "A balance sheet approach to financial crisis", IMF *Working Papers*.

Sheldon, G. and Maurer, M. (1998), "Interbank lending and systemic risk: an empirical analysis for Switzerland", *Swiss Journal of Economics and Statistics* (SJES) 134(IV), 685-704.

Sims, C. (1999), "Domestic currency denominated government debt as equity in the primary surplus", *Latin American meeting of the Econometric Society*, Cancun, México. August 19.

Upper, C. and Worms, A. (2004), "Estimating bilateral exposures in the German interbank market: Is there a danger of contagion?", *European Economic Review* 48,(4), 827-849.

Wells, S. (2004), *Financial interlinkages in the United Kingdom´s interbank market and the risk of contagion*, Bank of England.